A study of EEC Evolution with Jet Internal Multiplicity

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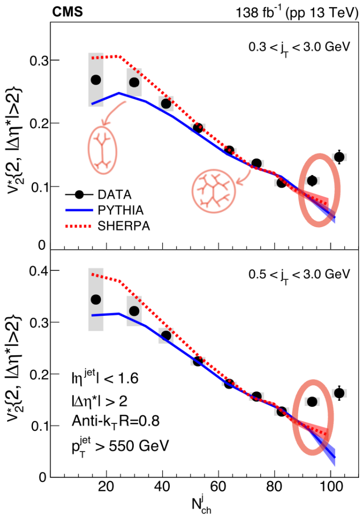
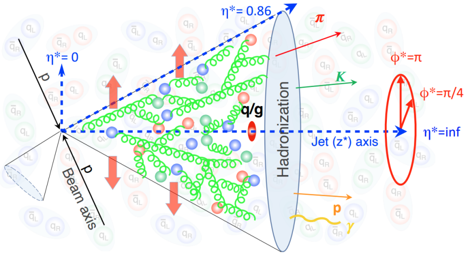
Based on:arXiv:2510.04895 and arXiv:2604.01102.

Non-trivial correlations inside high multiplicity jet

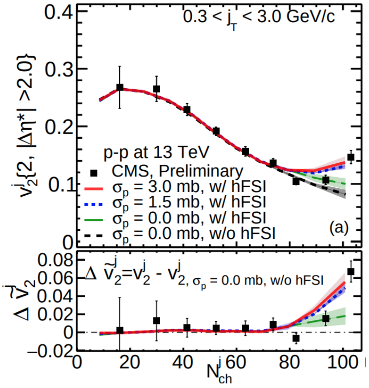


[CMS, PRL 133 (2024) 142301]

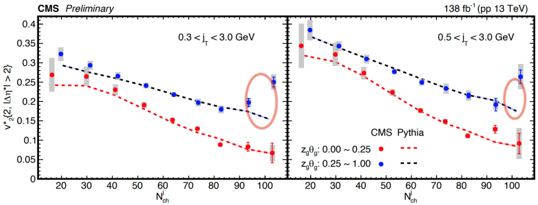


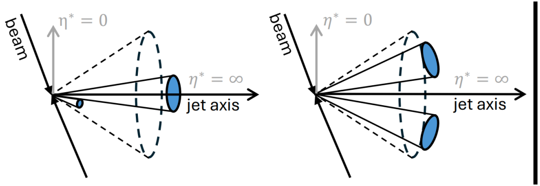
- Two-particle correlation in jet-frame.
- Event generators fail to capture the v_2 enhancement at $\nu = N_{ch}/\langle N_{ch}\rangle \gtrsim 3.5$.

Several interpretations for v_2 inside the jet



Zhao, Lin, Wang, arXiv:2401.13137 (2024)



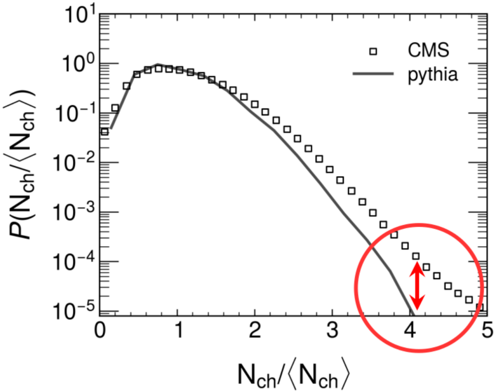


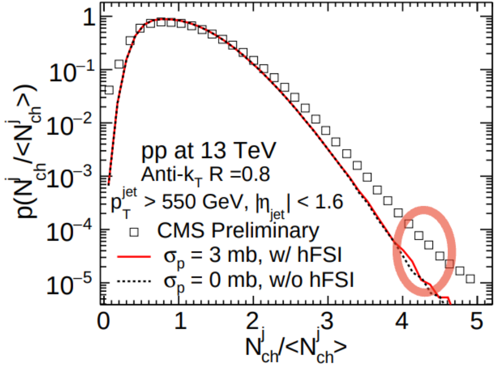
[CMS, CMS-PAS-HIN-24-024 (2025)]

[Dokshitzer, Webber, JHEP 08 (2025) 168]

- Add final-state interactions at both partonic and hadronic level.
- High-multiplicity constraint selects the jets initiated by two hard partons.

How about multiplicity distribution of jets?

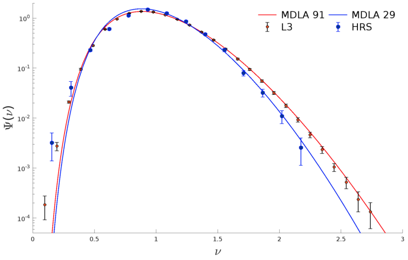




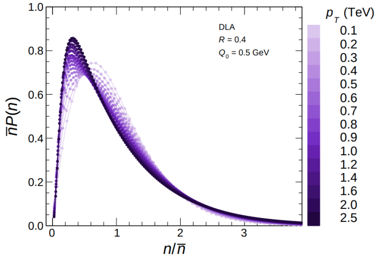
[Zhao, Lin, Wang, arXiv:2401.13137 (2024)]

- PYTHIA fails again, especially in high-multiplicity region.
- Low production probability of high-multiplicity jet.

Theoretical studies of jet multiplicity distribution



Dokshitzer, Webber, JHEP 10, 114 (2025)



Duan et al., Phys. Rev. D 112, 094022 (2025)

- Koba-Nielsen-Olesen (KNO) scaling: normalized multiplicity collapses to a universal curve $\Rightarrow \langle N\rangle P(N;\omega_J,R)=\Psi(\nu), \quad \nu = N/\langle N\rangle$.
- Approaches most rely on approximations, like DLA and MDLA.
- Hard to generalize to the study of jet substructures.

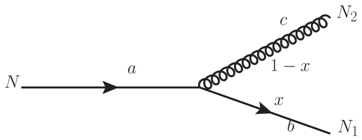
▶ **Our goal:** analytic tool for multiplicity distributions & substructure evolution.

Basic idea: multiplicity distribution $P(N)$

- Multiplicity probes full particles production history:
  perturbative evolution + physically inspired non-perturbative model.
- $P_a(N, \omega, \mu)$: parton "a" with energy ω at resolution scale μ.
- If "a" contains two finer structures at scale μ, assume **two daughters fragment independently**:

$$P_a(N,\omega,\mu) \propto \int \mathrm{d}x \hat{P}_a^{bc}(x) \sum_{N_1=0}^{N} \sum_{N_2=0}^{N} P_{bc}(N_1, x\omega; N_2, (1-x)\omega; \mu)\delta_{N_1+N_2,N}$$

$$\approx \int \mathrm{d}x \hat{P}_a^{bc}(x) \sum_{N_1=0}^{N} \sum_{N_2=0}^{N} P_b(N_1, x\omega, \mu) P_c(N_2, (1-x)\omega, \mu)\delta_{N_1+N_2,N}$$



Basic tool: generating function $Z(s)$

- Generating function of $P(N)$:

$$Z(s, \omega, \mu) = \sum_{N=0}^{\infty} P(N, \omega, \mu) e^{-Ns}$$

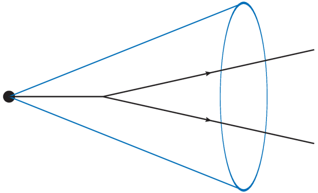
- Convolution in N becomes product in s:

$$Z_a(s, \omega, \mu) \propto \int \mathrm{d}x \hat{P}_a^{bc}(x)\, Z_b(s, x\omega, \mu) Z_c(s, (1-x)\omega, \mu)$$

- Final two partons contribute multiplicity—introduces the **non-linearity**.

Evolution equation of multiplicity function for exclusive jet

- Jet algorithm: radiations smaller than R.



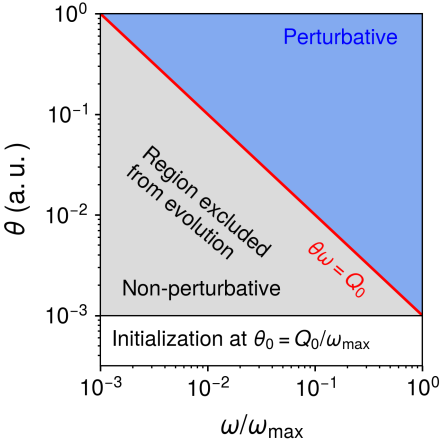
$Z_i^{(1)} \propto \alpha_s P_{ji}(x)$
$\otimes Z_j^{(0)}(s, x\omega_J) Z_k^{(0)}(s, (1-x)\omega_J)$

- Multiplicity is soft sensitive $\Rightarrow$ **angular ordering** instead of virtuality ordering.
- Perform the **non-linear** evolution in $\zeta = 1 - \cos\theta$ (θ is splitting angle):

$$\frac{\partial Z_i(s, \omega_J, \zeta)}{\partial \ln \zeta} = \int_0^1 \mathrm{d}x \frac{\alpha_s(k_\perp^2)\Theta(k_\perp^2 - Q_0^2)}{2\pi} \hat{P}_{ji}(x) Z_j(s, x\omega_J, \zeta) Z_k(s, (1-x)\omega_J, \zeta)$$

Q_0 severs as an infrared cutoff to regulate the calculation. $k_\perp^2 = 2\zeta[x(1-x)\omega_J]^2$

Non-perturbative model for initial condition of Z



- Assume Binomial or Poisson distribution

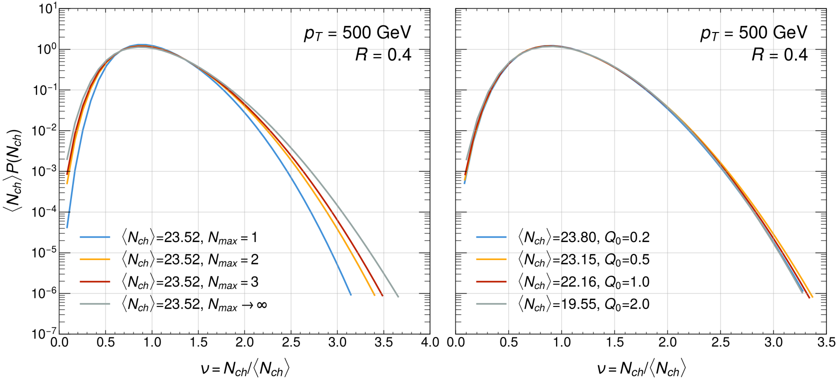
$$Z^{\mathrm{IC}}(s,\omega,Q_0) = \begin{cases} \exp\left[\langle N\rangle(\omega,Q_0)\left(e^{-s}-1\right)\right] & \text{Poisson}\,, \\ \left[1+\left(e^{-s}-1\right)\frac{\langle N\rangle(\omega,Q_0)}{N_{\max}}\right]^{N_{\max}} & \text{Binomial}\,. \end{cases}$$

- Parameterize the average particle multiplicity:

$$\langle N\rangle(\omega,Q_0) = \frac{N_0}{1+cQ_0^2/\omega^2}$$

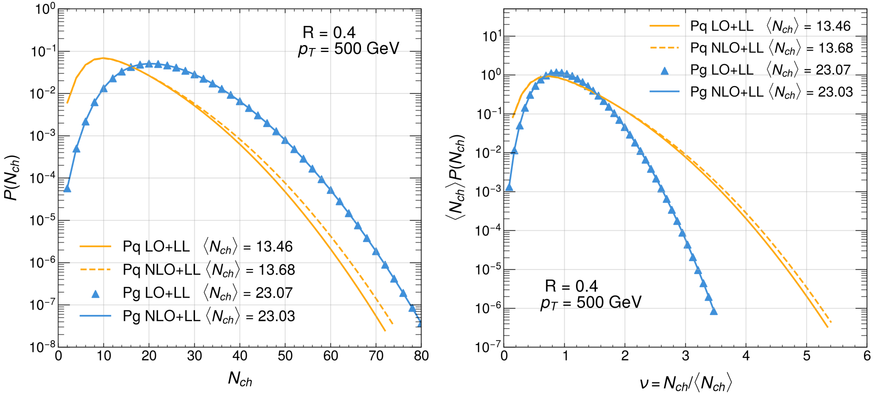
- Approaches N_0 at high energy ($\omega \gg Q_0$); Smooth suppression near threshold ($\omega \sim Q_0$).
- $c = 1$, $Q_0 = 0.224\mathrm{GeV}$, $N_0 = 0.8$, $N_{\max} = 2$ (fit CMS data).

Sensitivity on the initial non-perturbative model



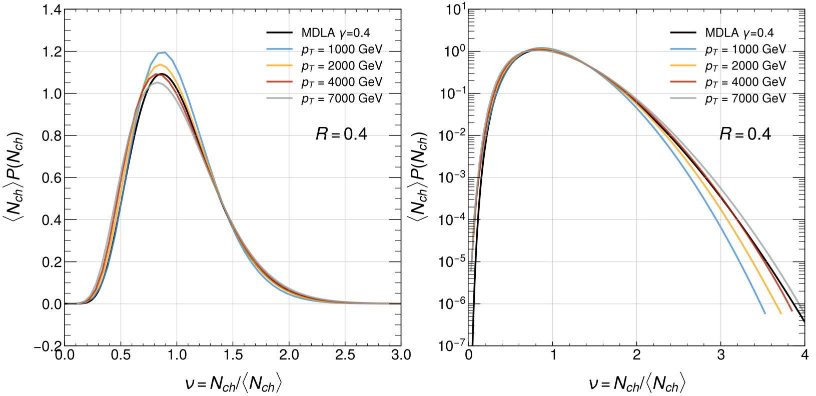
- Initial broadening propagates through the perturbative evolution.
- Shape is insensitive to the infrared scale Q_0, but dominate by perturbative evolution. $\langle N\rangle(Q_0,\omega) = \frac{N_0}{1+cQ_0^2/\omega^2}$.

Multiplicity distribution of exclusive jets



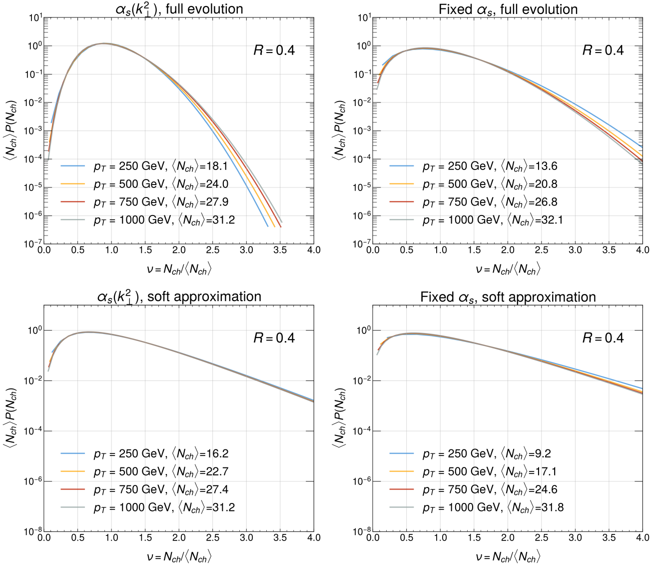
- Gluon jets exhibit higher mean multiplicity than quark jet.
- NLO effects mainly enhance the quark-jet high-N_{ch} tail.

KNO scaling and violation for exclusive gluon jet



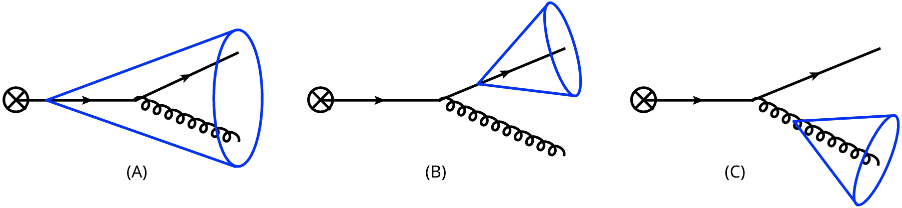
- Increasing p_T breaks KNO scaling: peak decreases and shifts to smaller ν.
- High-ν tail is enhanced at large p_T and exceeds MDLA.

The possible reasons for KNO violation



- Full evolution:
  $$\partial_{\ln\zeta} Z \propto \int \mathrm{d}x\, \hat{P}(x)\, Z(x)Z(1-x)$$
  Soft approximation:
  $$\partial_{\ln\zeta} Z \propto \int \mathrm{d}x\, \frac{1}{x}\, Z(x)Z(1)$$
- Large-ν KNO violation mainly from energy conservation.
- Running coupling regulates the size of fluctuations across different jet energy.

From exclusive jet to inclusive jet

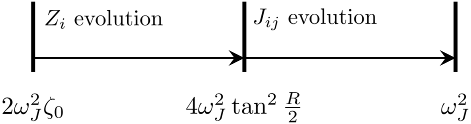


- Out-of-cone radiations: **change of energy** $\omega_J = z\omega$ or **flavor** of the parton.

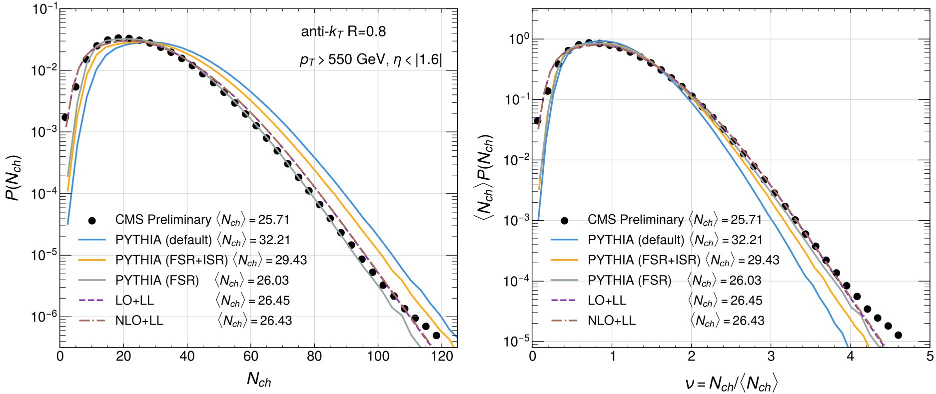
★ Ignore emissions from outside the jet re-entering the cone.

- Factorized formula to get final **cross section**:

$$\frac{\mathrm{d}\sigma_{pp\to J(N)+X}}{\mathrm{d}p_{T,J}} = \sum_{ij} \mathrm{d}\sigma_{pp\to i}(p_{T,J}/z, \mu_H) \otimes_z J_{ji}(z, R, \mu_H, \mu_J) \times P_j(N, R, \zeta_R, \zeta_0)$$

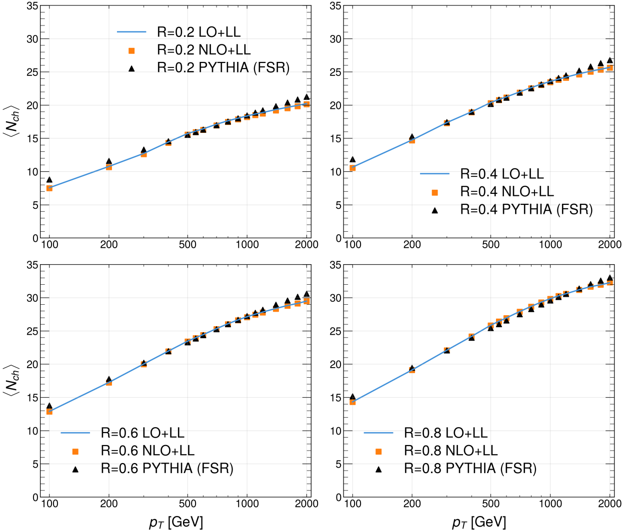


Comparison to CMS data $\nu = N/\langle N \rangle$



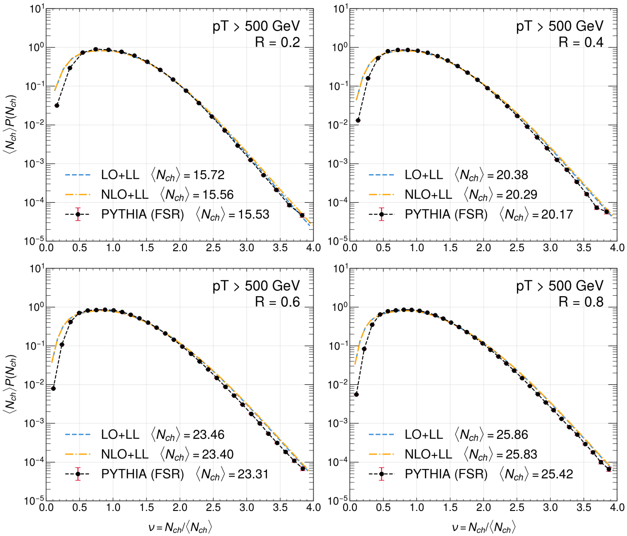
- Both our calculation and Pythia8 (FSR) reproduce the overall distribution well.
- Bot underestimate the high-multiplicity tail ($\nu \gtrsim 4$), possibly due to missing higher-order effects.

Mean multiplicity on jet p_T and R



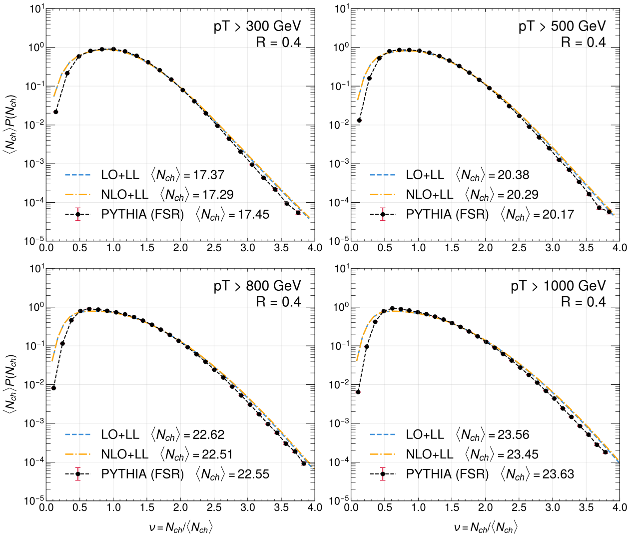
- Good agreements with Pythia across different p_T and R.
- Same non-perturbative model is used, showing good predictive power.

Dependence on jet R



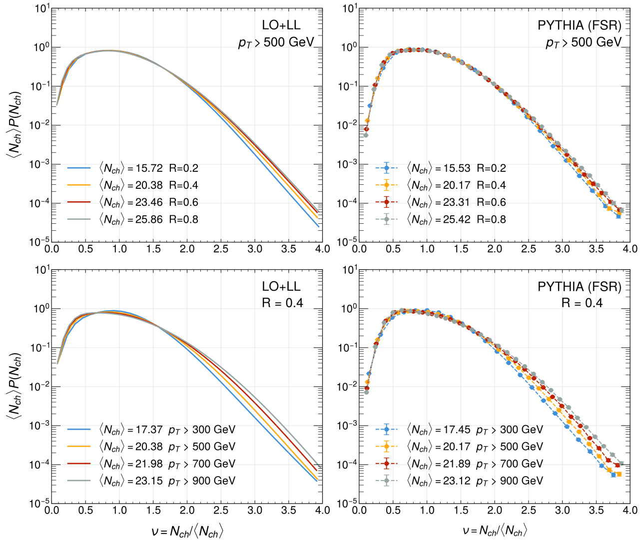
- Good agreements with Pythia across different R.
- Same non-perturbative model is used, showing good predictive power.

Dependence on jet p_T



- Good agreements with Pythia across different p_T.
- Same non-perturbative model is used, showing good predictive power.

KNO scaling and violation for inclusive jet



- KNO scaling holds for $\nu \lesssim 2$; mild violations at larger ν, consistent with PYTHIA.
- Deviations arise from energy conservation, running coupling and quark/gluon jet mixture.

Energy-energy correlator conditioned on multiplicity

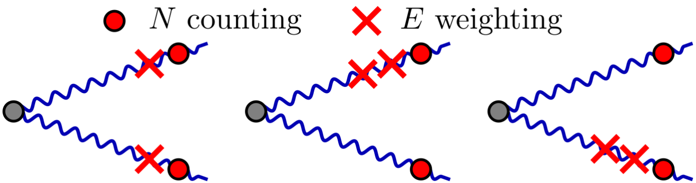
- EEC is IRC safe ⇒ test perturbative calculations.
- Energy weighted cross section of jet with multiplicity N and EEC at angle χ:

$$\frac{\mathrm{d}\Sigma(\chi, N, \omega_J)}{\mathrm{d}\chi \mathrm{d}p_{T,J}} = \sum_{ij} \mathrm{d}\sigma_{pp\to i}(p_{T,J}/z, \mu_H) \otimes_z J_{ji}(z, R, \mu_H, \mu_J) \times \mathcal{G}_j(\chi, N, R, \zeta_R, \zeta_\chi)$$

- EEC conditioned on the fixed multiplicity N:

$$\frac{\mathrm{d}\Sigma(\chi, \omega_J|N)}{\mathrm{d}\chi} = \frac{1}{P(N, \omega_J)} \frac{\mathrm{d}\Sigma(\chi, N, \omega_J)}{\mathrm{d}\chi}$$

- Way to both counting multiplicity and EEC:



The evolution equations for joint multiplicity-EEC function

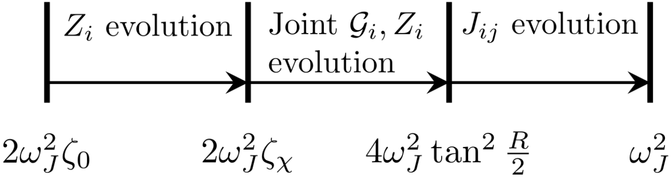
- $Z(s)$ enters the evolution of $\mathcal{G}(\chi, s)$ as an input

$$\frac{\partial \mathcal{G}_i(\chi, s, \omega_J, \zeta)}{\partial \ln \zeta} = \int \mathrm{d}x \frac{\alpha_s(k_\perp^2)\Theta(k_\perp^2 - Q_0^2)}{2\pi} \hat{P}_{ji}(x) \mathcal{G}_j(\chi, s, x\omega_J, \zeta) Z_k(s, (1-x)\omega_J, \zeta)$$

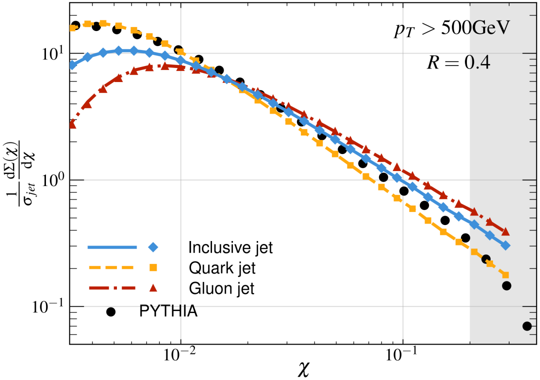
- Initial condition is perturbatively determined at EEC scale ζ_χ

$$\mathcal{G}_i^{\text{ini}}(\chi, s, \omega_J, \zeta) = \frac{1}{\chi}\frac{\alpha_s}{\pi} \int \mathrm{d}x [x(1-x)] \hat{P}_{ji}(x) Z_j(x) Z_k(1-x)$$

- Evolution picture:

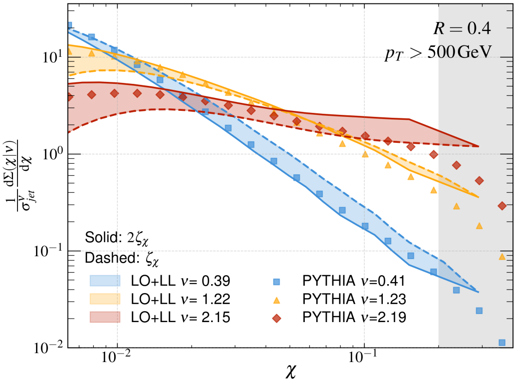


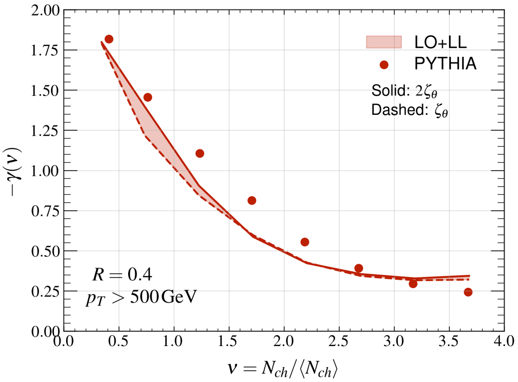
Cross check with standard EEC



- Summing the multiplicity-conditioned EEC reproduces the standard EEC.

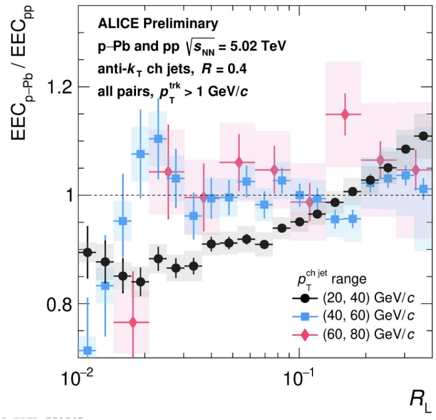
EEC conditioned on normalized multiplicity $\nu = N/\langle N \rangle$

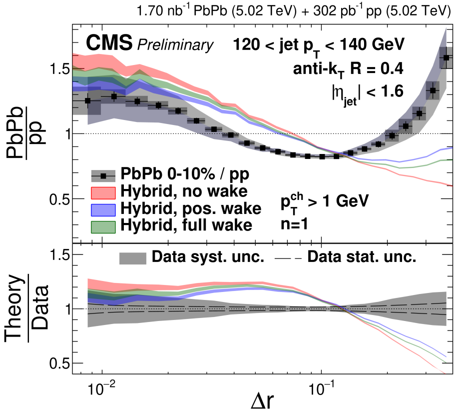




- Higher multiplicity jets show a flatter EEC.
- The multiplicity-conditioned EEC retains a power-law form, $\Sigma(\chi|\nu) \sim \chi^{\gamma(\nu)}$.
- Exponent $\gamma(\nu)$ links particle multiplicity to angular energy flow: a new probe of perturbative particle production dynamics.

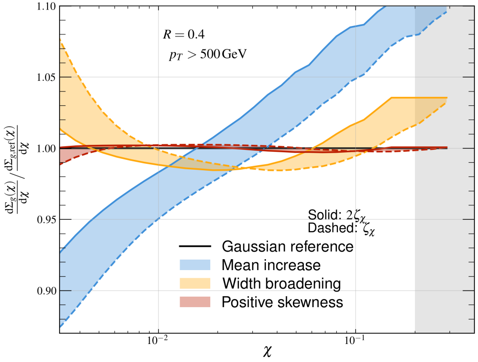
EEC ratios for pA/pp and AA/pp

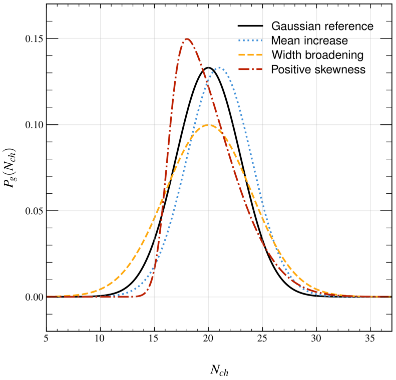




- Can multiplicity bias cause "apparent" modifications on EEC ratios?

Impact of multiplicity bias on EEC





- $\frac{d\Sigma_g(\chi)}{d\chi} = \sum_{N_{ch}} P_g(N_{ch}) \frac{d\Sigma_g(\chi|N_{ch})}{d\chi}$
- EEC is sensitive to underlying multiplicity distribution.
- Changes in multiplicity alone can induce nontrivial modifications in ratio, even if the conditional EEC is unchanged

Summary

- Developed an analytic framework for jet multiplicity, enabling substructure studies conditioned on multiplicity (though the extreme high-multiplicity region is not yet well described).
- ν-conditioned EEC reveals a multiplicity-dependent anomalous dimension, probing perturbative particle production dynamics.
- Proper control of multiplicity is crucial for reliable interpretation of EEC ratios.
- Framework can be extended to other substructure observables and to medium environments to isolate medium effects.

Thanks for your listening!