# PRECISION FRONTIER OF PQCD WITH NNLOJET

CLHCP 2025


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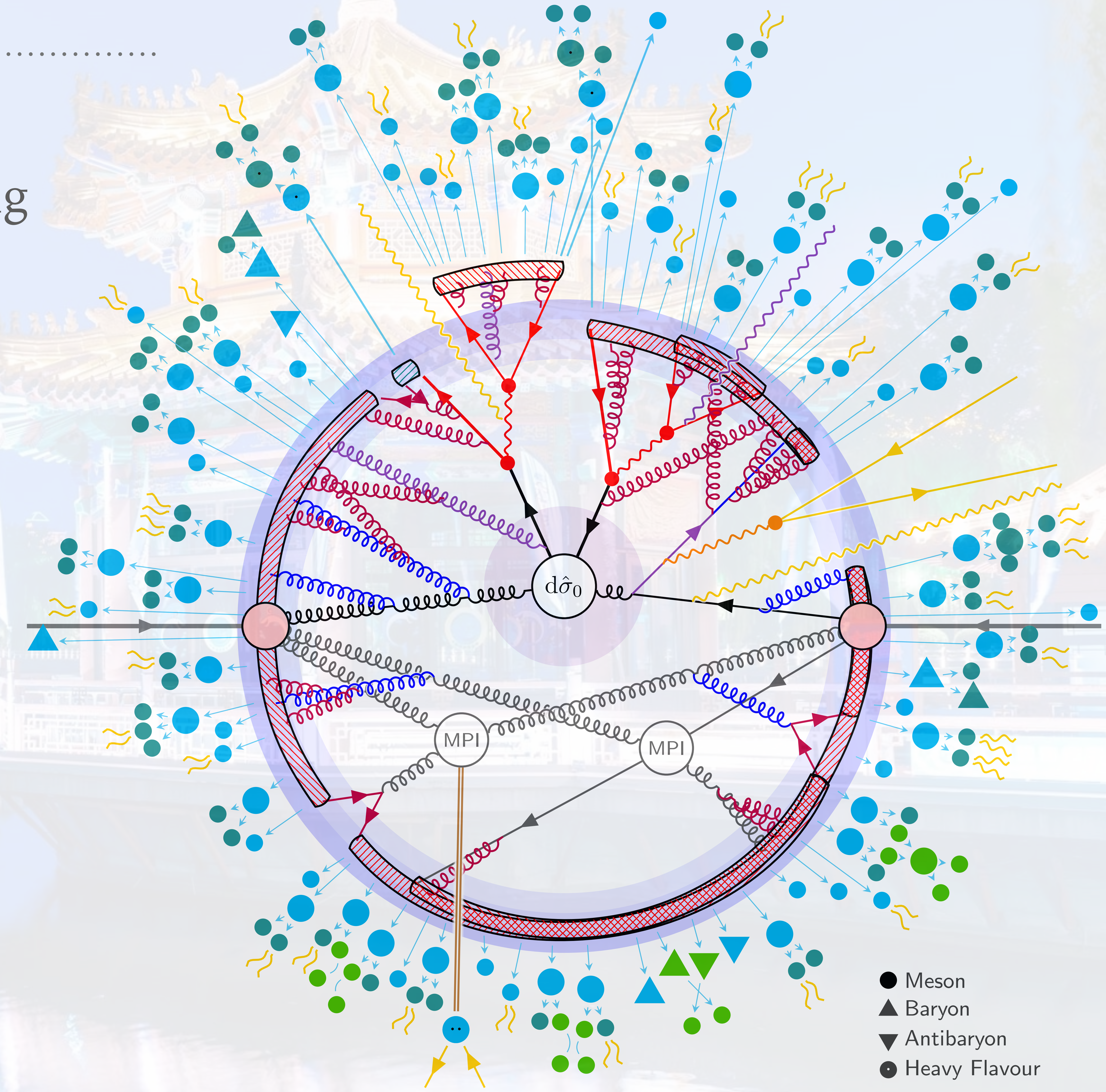
Xinxiang, 31 October, 2025

# Collider Event in Theorist's Eye

➤ The idea of factorisation in Quantum Field Theory plays important role to help theorists understanding complex high energy processes:

*PYTHIA 8.3*

# Collider Event in Theorist's Eye

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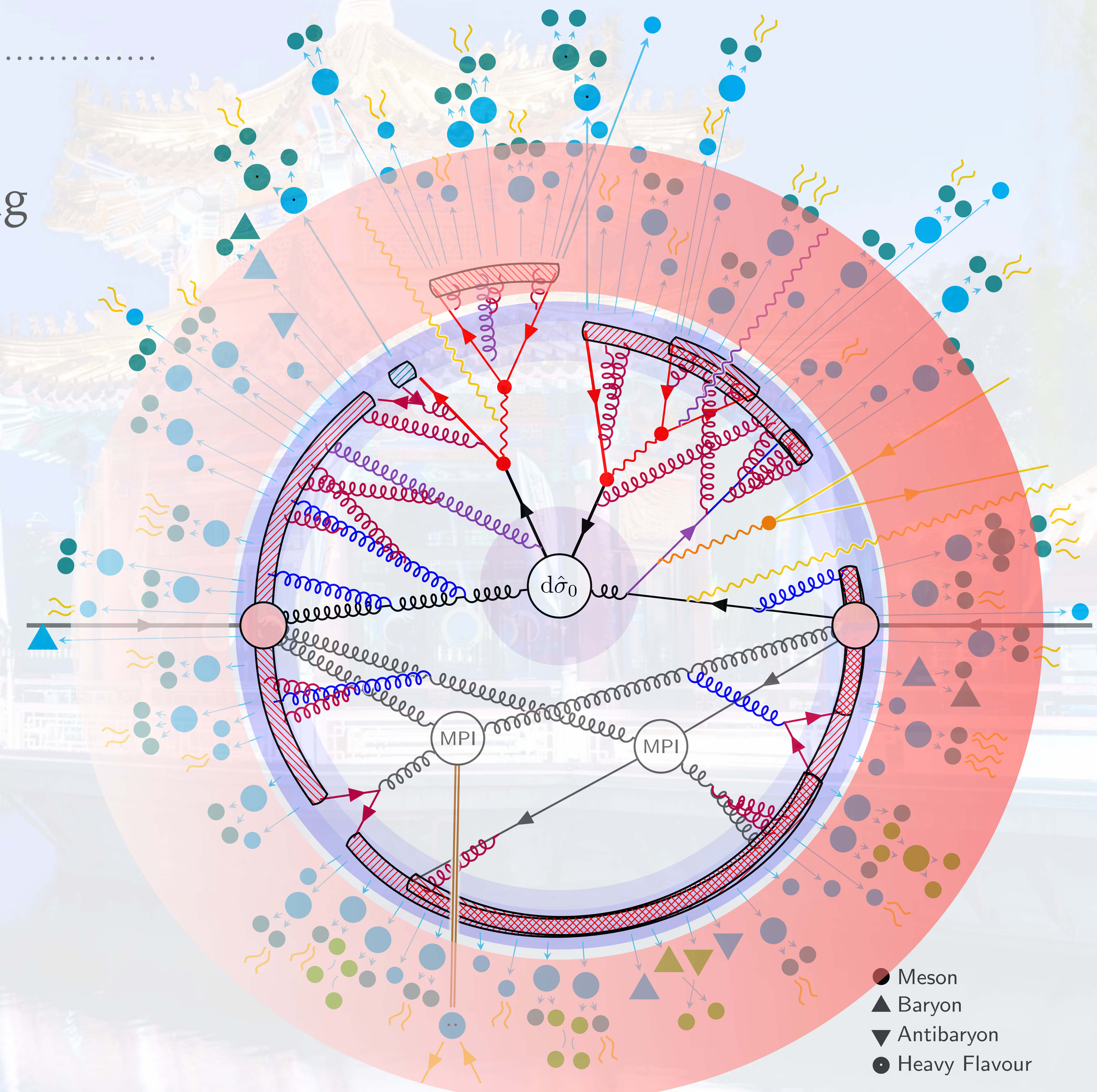
Hadronisation




*PYTHIA 8.3*

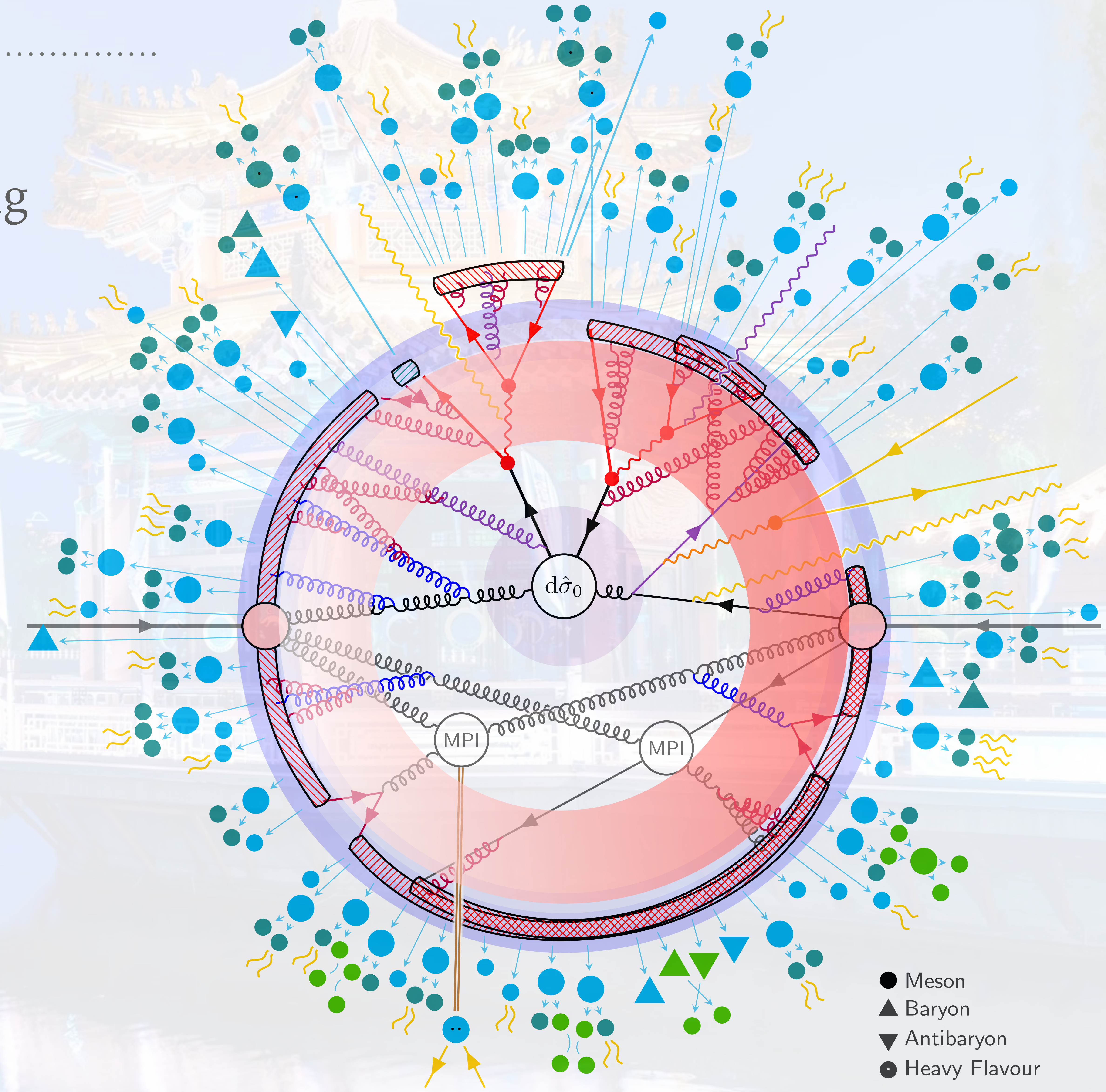
# Collider Event in Theorist's Eye

➤The idea of factorisation in Quantum Field Theory plays important role to help theorists understanding complex high energy processes:

Time ordering

Hadronisation

Parton Shower




*PYTHIA 8.3*

# Collider Event in Theorist's Eye

- ➤The idea of factorisation in Quantum Field Theory plays important role to help theorists understanding complex high energy processes:

Time ordering

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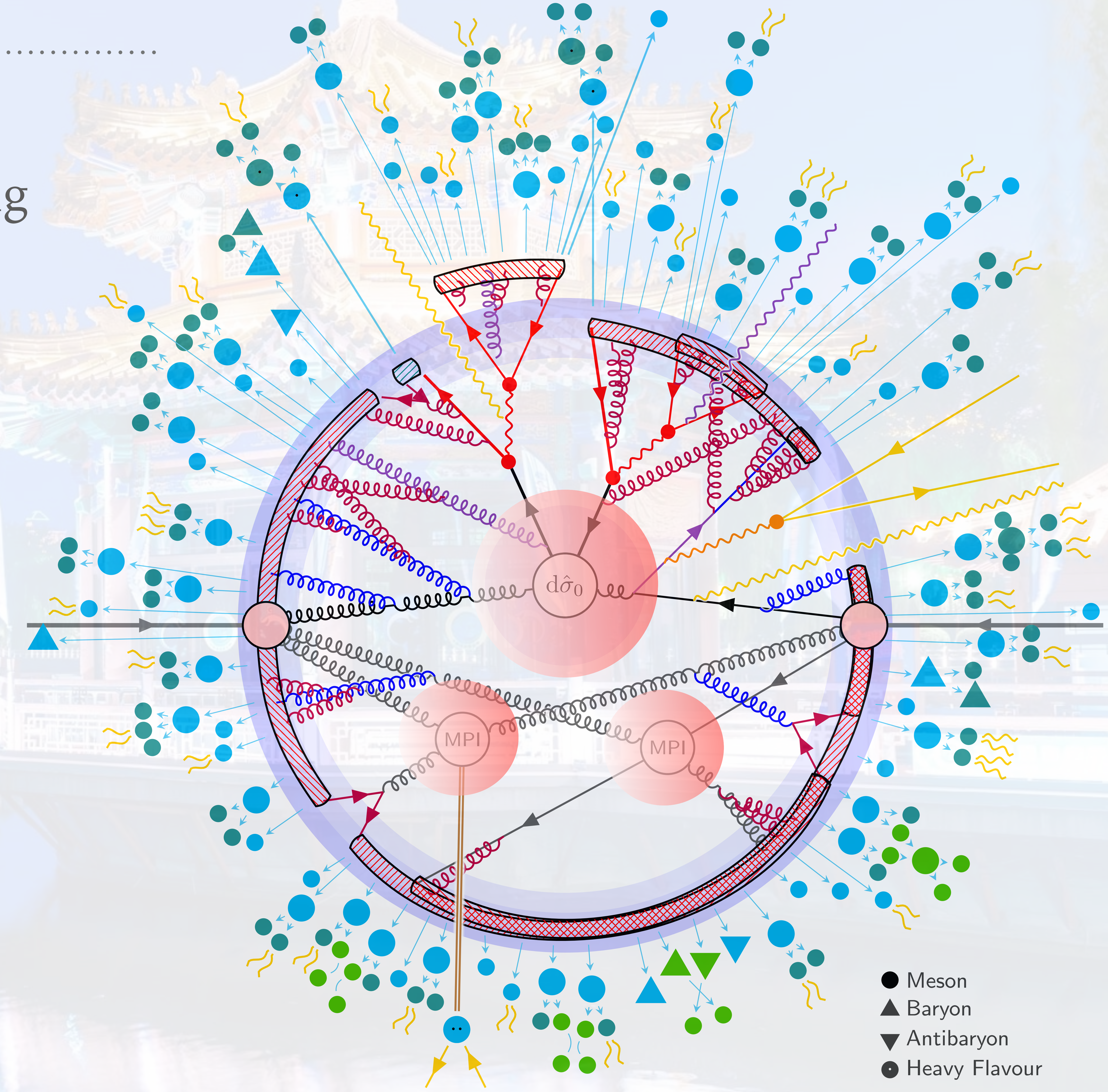
Hard Scattering



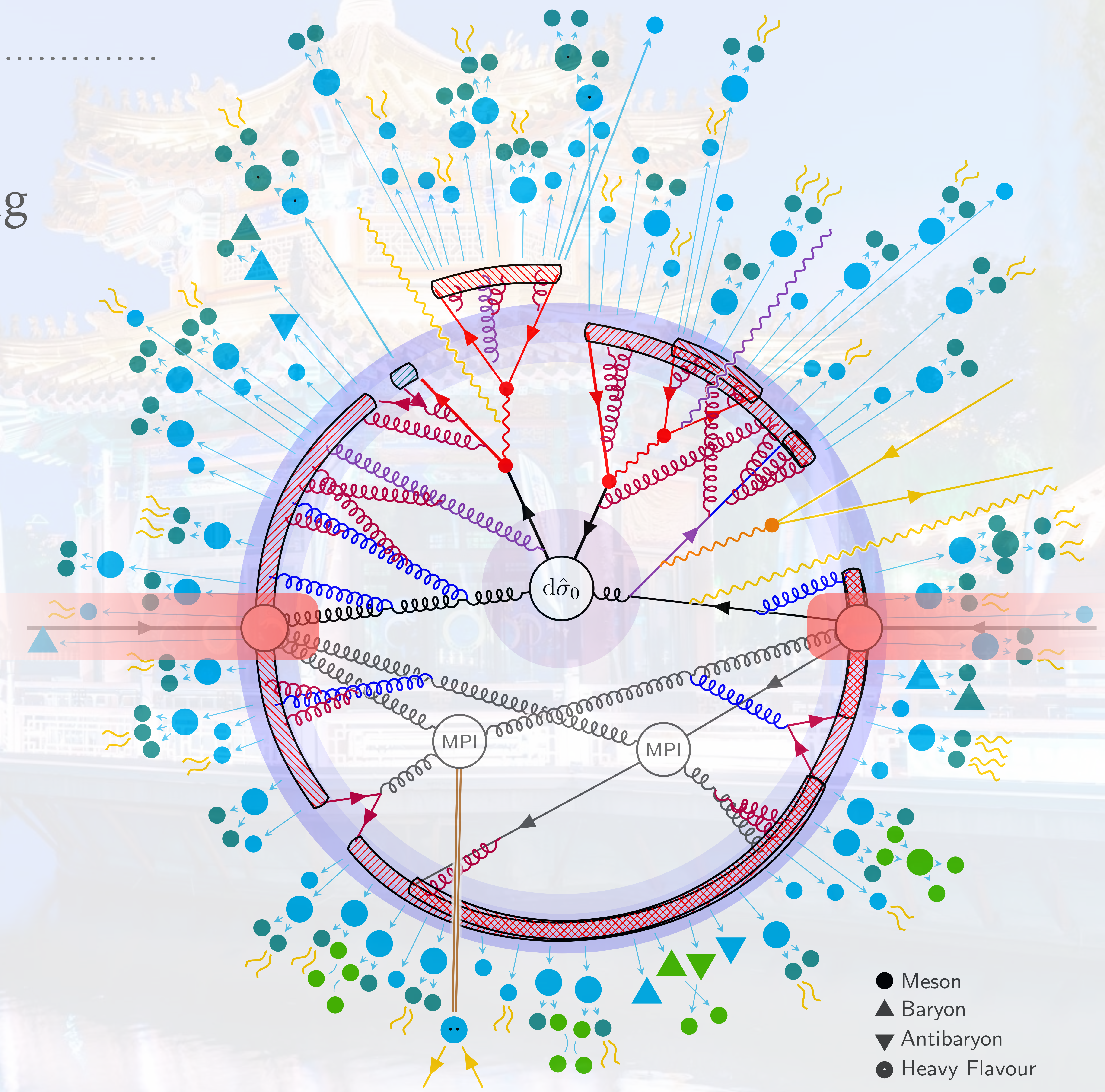

*PYTHIA 8.3*

# Collider Event in Theorist's Eye

➤ The idea of factorisation in Quantum Field Theory plays important role to help theorists understanding complex high energy processes:

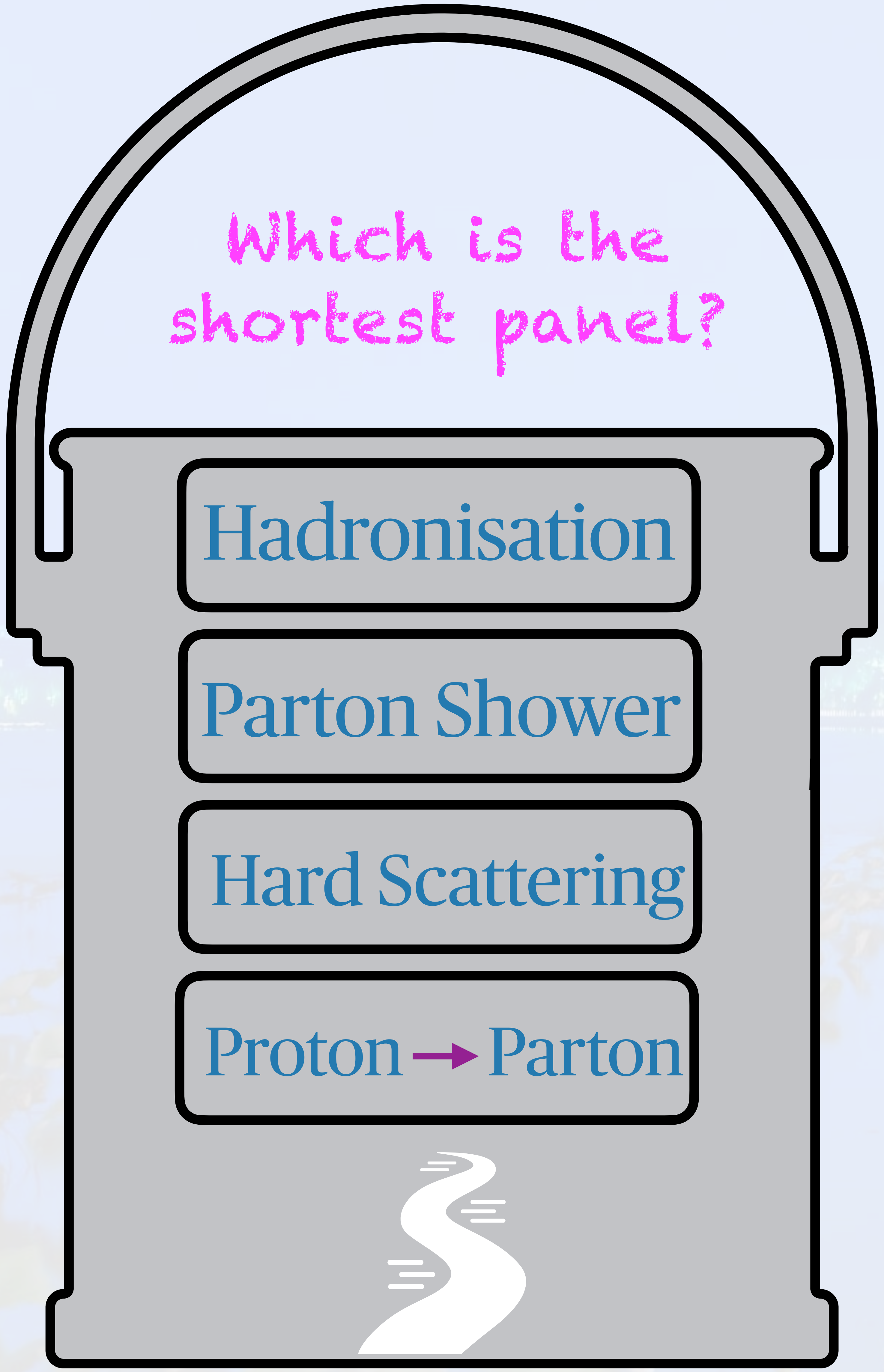
Time ordering ↑

- Hadronisation
- Parton Shower
- Hard Scattering
- Proton → Parton

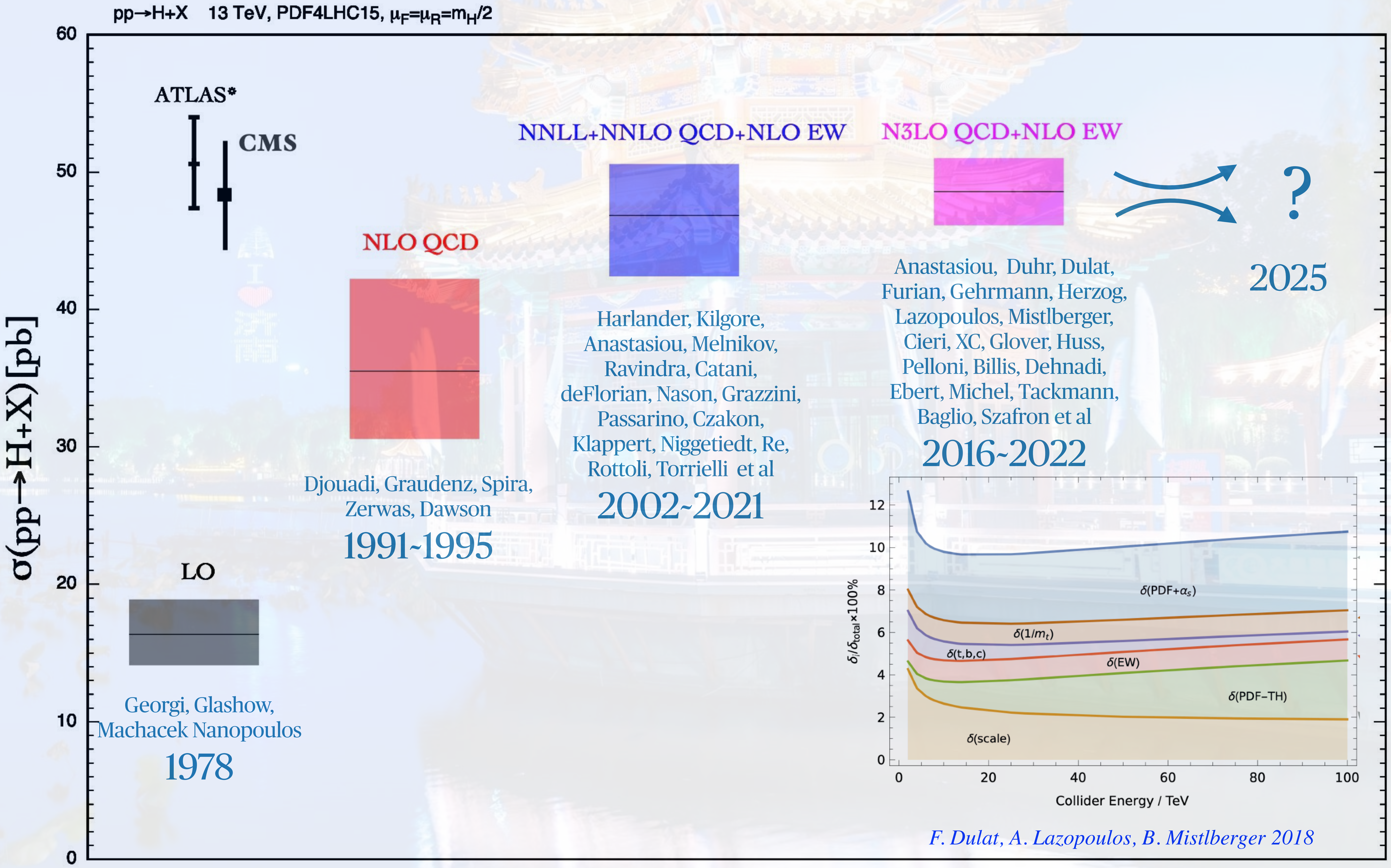



*PYTHIA 8.3*

# The Bucket Effect in $pp \to H + X$

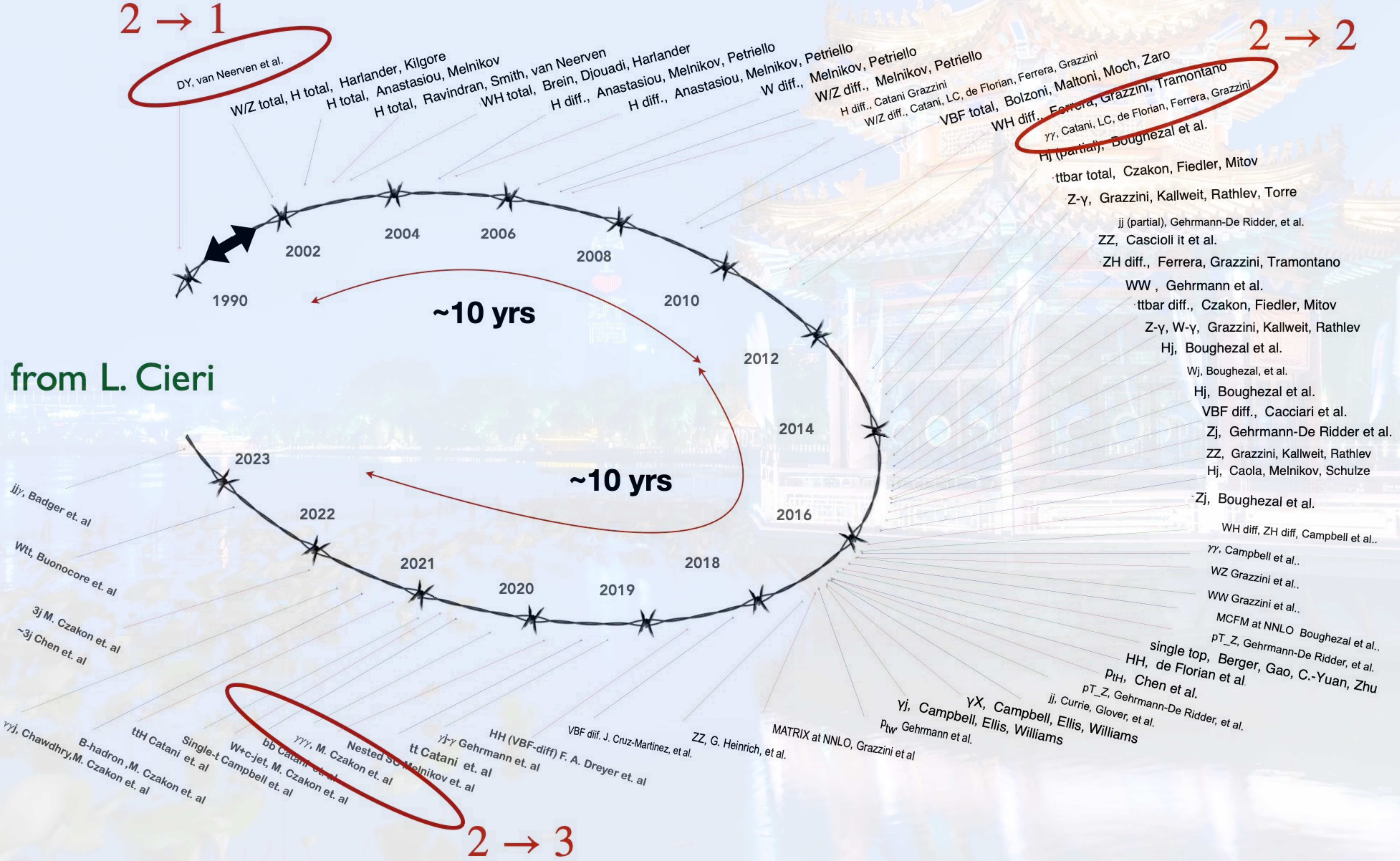


# The Bucket Effect in $pp \to H + X$


Slide based on M. Grazzini's talk @ Higgs10

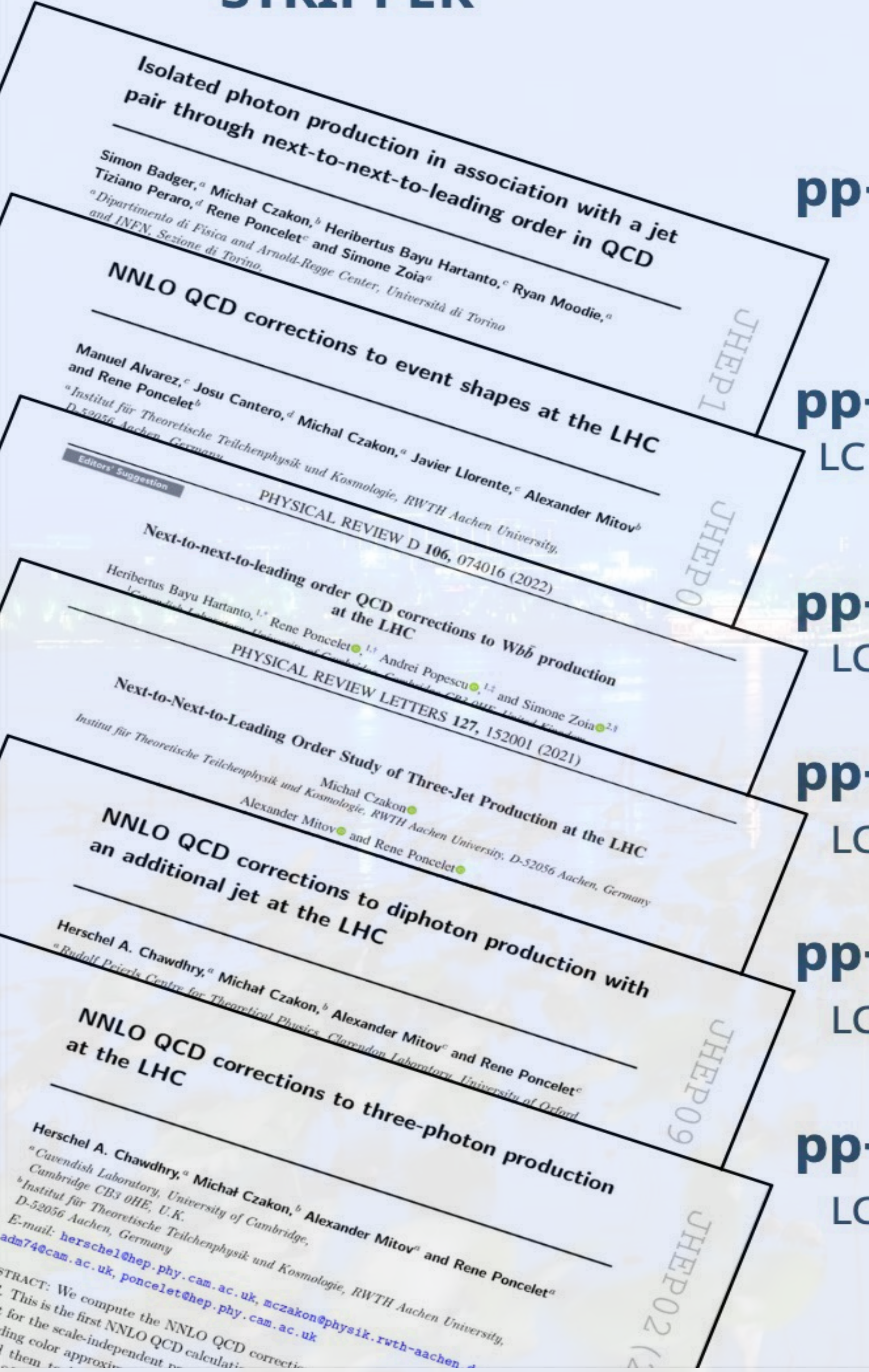
# Perturbative QCD @ NNLO



# State-of-the-Art QCD Calculations @ NNLO

Marcoli's slide @ Loop Summit 2

**Sector improved residue subtraction, STRIPPER**




pp→γjj

pp→jjj
LC

pp→Wbb
LC, mb=0

pp→jjj
LC

pp→γγj
LC

pp→γγγ
LC

**qT slicing, MATRIX**


pp→Htt
soft approx.
massification (LC)

pp→Wtt
soft approx.
massification (LC)

pp→Wbb
massification (LC)

pp→Htt
soft approx.
massification (LC)

pp→γγγ
LC

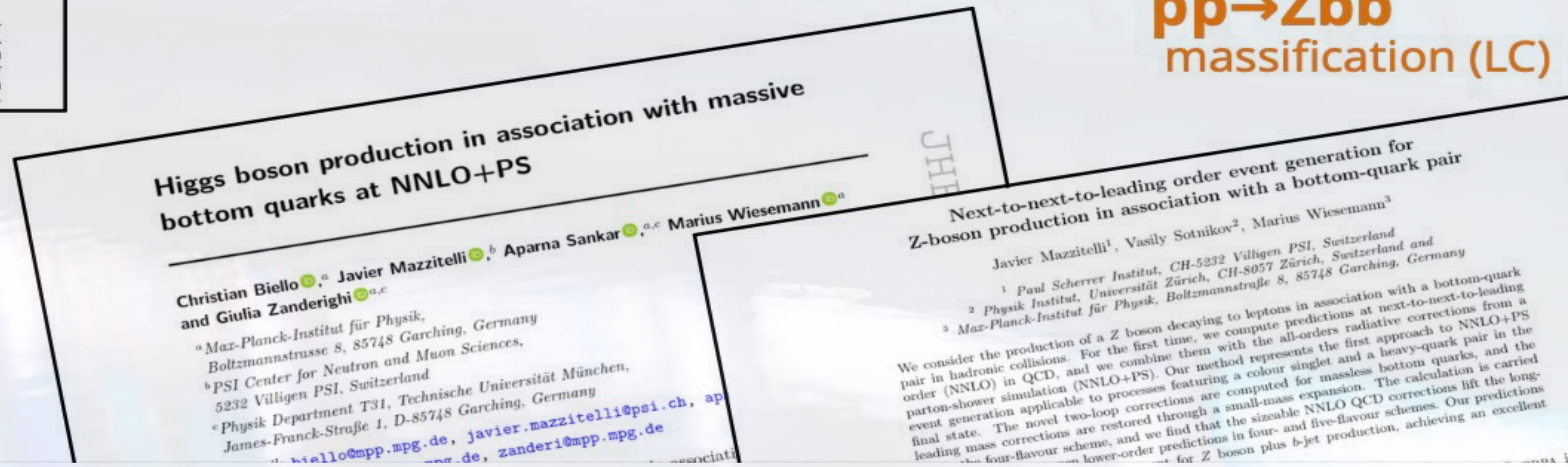
**Antenna subtraction, NNLOJET**

pp→γγj


pp→jjj
LC, gluons

**qT slicing + PS, MiNNLOPS**

pp→Zbb
massification (LC)

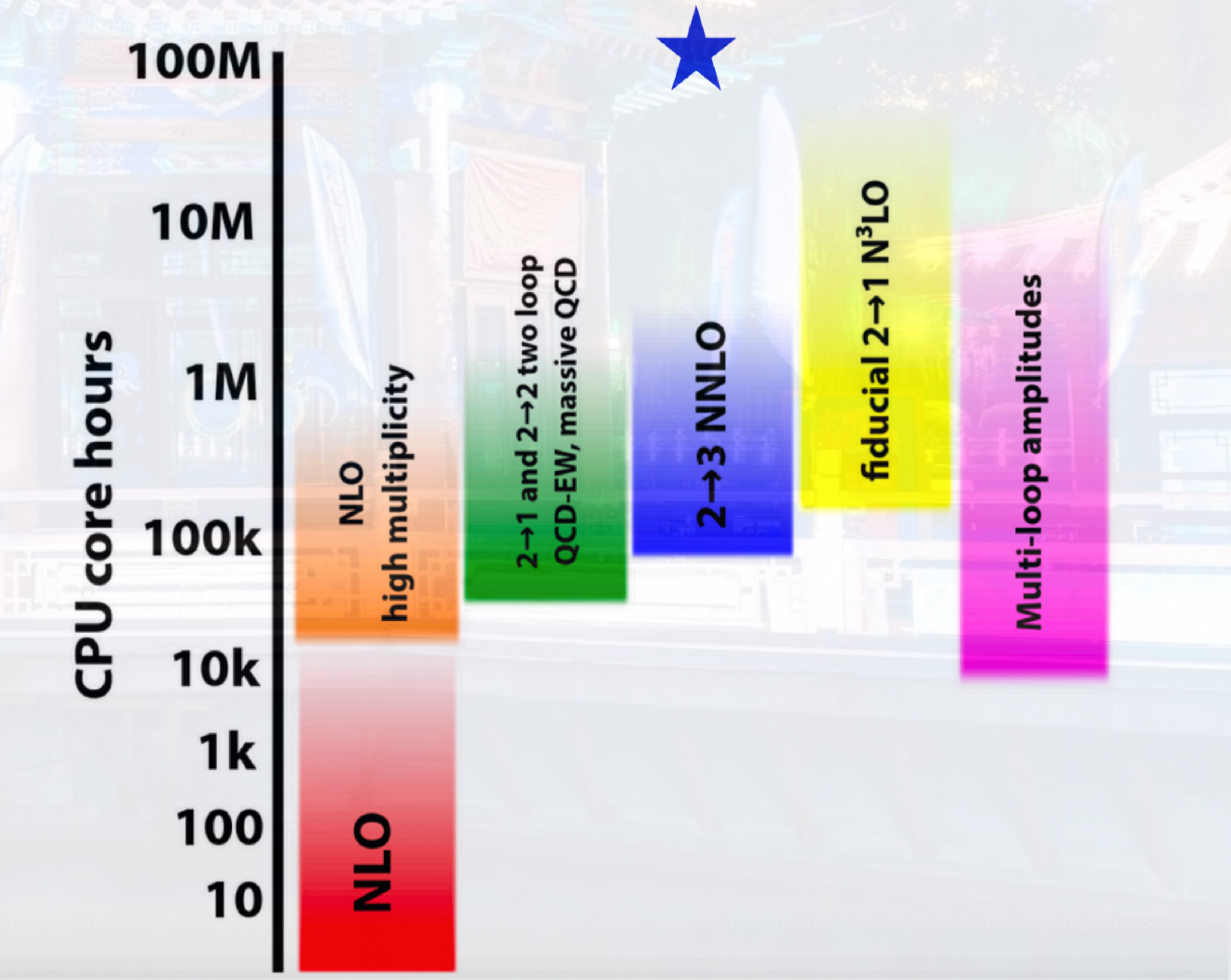



pp→Hbb
massification

# State-of-the-Art QCD Calculations @ NNLO

- NNLO QCD predictions for 2 → 2 processes (NNLO revolution since 2015 )
  - Accomplished during past 10 years on case-by-case basis
  - As parton-level event generators (fully differential final state information)
  - Current frontier at NNLO 2 → 3

pp→jjj event shapes with STRIPPER




*Snowmass White Paper, Comput. Softw. Big Sci. 6 (2022)*

- Typical size of corrections and uncertainty
  - NLO corrections: 10~100%, uncertainty: 10~30%
  - NNLO corrections: 2~15%, uncertainty: 3~8%
  - expect N3LO to yield uncertainty at level of 1%

- So, is NNLO **solved**?
  - In principle yes: STRIPPER, given the relevant amplitudes and enough computational resources, the NNLO calculation is streamlined.
  - But:
    - Prohibitive computational cost (loop AMP, IR subtraction)
    - Missing cross-validation (many years between 1st and 2nd)
    - Still a long way to automated NNLO event generation

# NNLOJET: Parton Level Event Generator

**A parton-level event generator for jet cross sections at NNLO QCD accuracy**

**About** NNLOJET is a parton-level event generator for jet cross sections using the antenna subtraction method. It can be used to compute a large number of jet cross sections and related observables in $e^+e^-$, $ep$ and $pp$ collisions at next-to-next-to-leading order in QCD. NNLOJET contains routines for Monte Carlo phase-space integration, event handling and analysis.

**Citation** If you are using NNLOJET for a scientific paper, please cite:

A. Huss et al. (NNLOJET Collaboration)
*NNLOJET: a parton-level event generator for jet cross sections at NNLO QCD accuracy*
arXiv:2503.22804 [INSPIRE]

Please also cite the revelant references for each processes (as included in the .bib file which is automatically written when running NNLOJET through the automatic workflow)

**License** GNU General Public License (GPL) v3.0

**Contact** Please send comments, questions and suggestions to nnlojet-support@cern.ch

https://nnlojet.hepforge.org/index.html

A.Huss, L.Bonino, O.Braun-White, S.Caletti, XC, J.Cruz-Martinez, J.Currie, W.Feng, G.Fontana, E.Fox, R.Gauld, A.Gehrmann-De Ridder, T. Gehrmann, E.W.N.Glover, M.Höfer, P.Jakubcik, M.Jaquier, M.Löchner, F.Lorkowski, I.Majer, M.Marcoli, P.Meinzinger, J.Mo, T. Morgan, J.Niehues, J.Pires, C.Preuss, A.Rodriguez Gracia, K.Schönwald, R.Schürmann, V.Sotnikov, G.Stagnitto, D.Walker, J.Whitehead, T.Z.Yang, H.Zhang,

- NNLO parton level event generator
  - Based on antenna subtraction
- Provides infrastructure
  - Process management
  - Phase space, histogram routines
  - Validation and testing
  - Parallel computing (MPI) support
  - Typical runtimes: 60 k ~ 250 k core-hours

# NNLOJET: Parton Level Event Generator

➤ Processes implemented:

***$e^+e^-$* scattering** Jet production

- $e^+e^- \to$ 2jets
- $e^+e^- \to$ 3jets

***ep* scattering** Jet production

- ep $\to$ lepton + 1jet
- ep $\to$ lepton + 2jets

***pp* scattering** Jet production

- pp $\to$ 1jet + X
- pp $\to$ 2jets

Vector boson (+ jet) production

- pp $\to$ ($\gamma^*$Z) + 0jet
- pp $\to$ ($\gamma^*$Z) + 1jet
- pp $\to$ $W^\pm$ + 0jet
- pp $\to$ $W^\pm$ + 1jet

Photon (+ jet) production

- pp $\to \gamma$ + X
- pp $\to \gamma$ + 1jet
- pp $\to \gamma\gamma$

Higgs (+ jet) production

- pp $\to$ H + X
- pp $\to$ H + 1jet

➤ Open-source code release: NNLOJET v1.0.2
  - ➤ Analytic matrix elements and subtraction
  - ➤ Download from nnlojet.hepforge.org

➤ Runcard options:
  - ➤ Process/sub-process selection
  - ➤ Generic histogramming
  - ➤ Multi-run feature: e.g. jet radius
  - ➤ Example runcards for published studies

➤ Cluster workflow management: Dokan
  - ➤ Automated resource allocation
  - ➤ Works with slurm and htcondor (lxplus)
  - ➤ Combination of results, quality control


https://github.com/aykhuss/dokan

**dokan (土管)**

pypi v1.0.0 python missing

A pipeline for automating the NNLOJET workflow

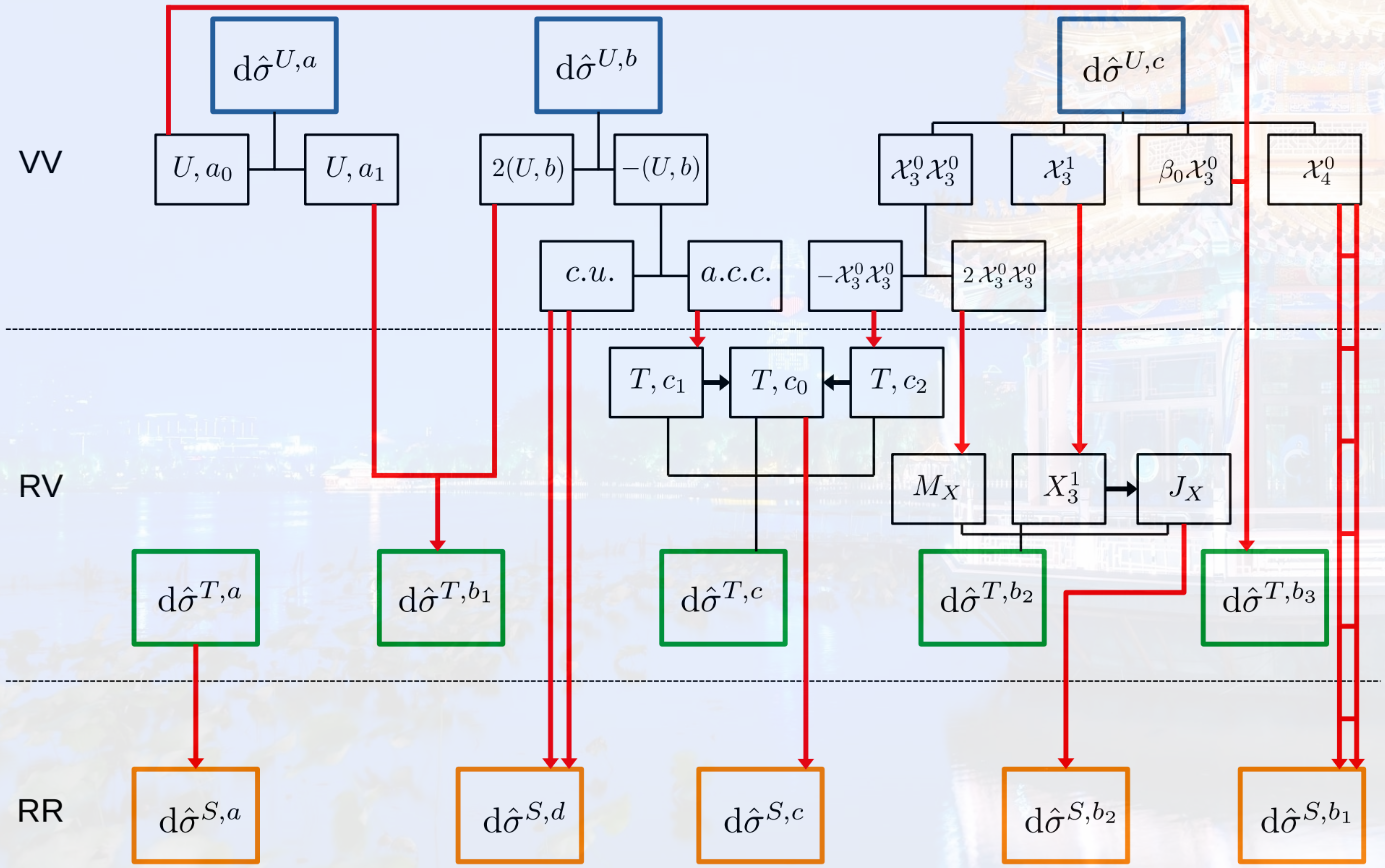
| # | LO | R | V | RR | RV | VV |
|---|---|---|---|---|---|---|
| 1 | PRD A[0] D[1] | PRD A[0] D[1] | PRD A[0] D[1] | WRM A[0] D[4] | PRD A[1] D[0] | PRD A[0] D[1] |
| 2 | PRD A[0] D[1] | PRD A[1] D[0] | PRD A[0] D[1] | PRD A[0] D[1] | WRM A[1] D[3] | PRD A[0] D[1] |
| 3 | - | PRD A[0] D[1] | PRD A[0] D[1] | PRD A[0] D[0] | WRM A[1] D[3] | PRD A[0] D[1] |
| 4 | - | PRD A[0] D[1] | PRD A[0] D[1] | PRD A[0] D[1] | PRD A[0] D[1] | PRD A[0] D[1] |
| 5 | - | PRD A[0] D[1] | PRD A[0] D[1] | PRD A[0] D[0] | PRD A[0] D[1] | PRD A[0] D[1] |
| 6 | - | PRD A[0] D[0] | PRD A[0] D[1] | PRD A[0] D[1] | PRD A[0] D[1] | PRD A[0] D[1] |
| 7 | - | - | - | PRD A[0] D[0] | PRD A[0] D[1] | PRD A[0] D[1] |
| 8 | - | - | - | PRD A[1] D[0] | PRD A[0] D[1] | PRD A[0] D[1] |
| 9 | - | - | - | PRD A[0] D[0] | PRD A[1] D[0] | PRD A[0] D[1] |
| 10 | - | - | - | PRD A[1] D[0] | PRD A[0] D[1] | PRD A[0] D[1] |
| 11 | - | - | - | PRD A[0] D[0] | PRD A[0] D[1] | PRD A[0] D[1] |
| 12 | - | - | - | PRD A[0] D[1] | - | - |
| 13 | - | - | - | PRD A[0] D[0] | PRD A[0] D[1] | PRD A[0] D[1] |
| 14 | - | - | - | PRD A[0] D[0] | - | - |
| 15 | - | - | - | PRD A[0] D[1] | WRM A[1] D[3] | PRD A[0] D[1] |
| 16 | - | - | - | PRD A[1] D[0] | WRM A[1] D[3] | PRD A[0] D[1] |
| 17 | - | - | - | PRD A[0] D[0] | WRM A[1] D[3] | PRD A[0] D[1] |
| 18 | - | - | - | PRD A[1] D[0] | - | - |
| 19 | - | - | - | PRD A[0] D[1] | - | - |
| 20 | - | - | - | PRD A[0] D[1] | - | - |
| 21 | - | - | - | WRM A[1] D[3] | - | PRD A[0] D[1] |
| 22 | - | - | - | PRD A[0] D[1] | - | PRD A[0] D[1] |
| 23 | - | - | - | PRD A[1] D[0] | - | PRD A[0] D[1] |
| 24 | - | - | - | WRM A[1] D[3] | - | PRD A[0] D[1] |
| 25 | - | - | - | WRM A[1] D[3] | - | - |
| 26 | - | - | - | PRD A[1] D[0] | - | - |
| 27 | - | - | - | PRD A[0] D[1] | - | - |

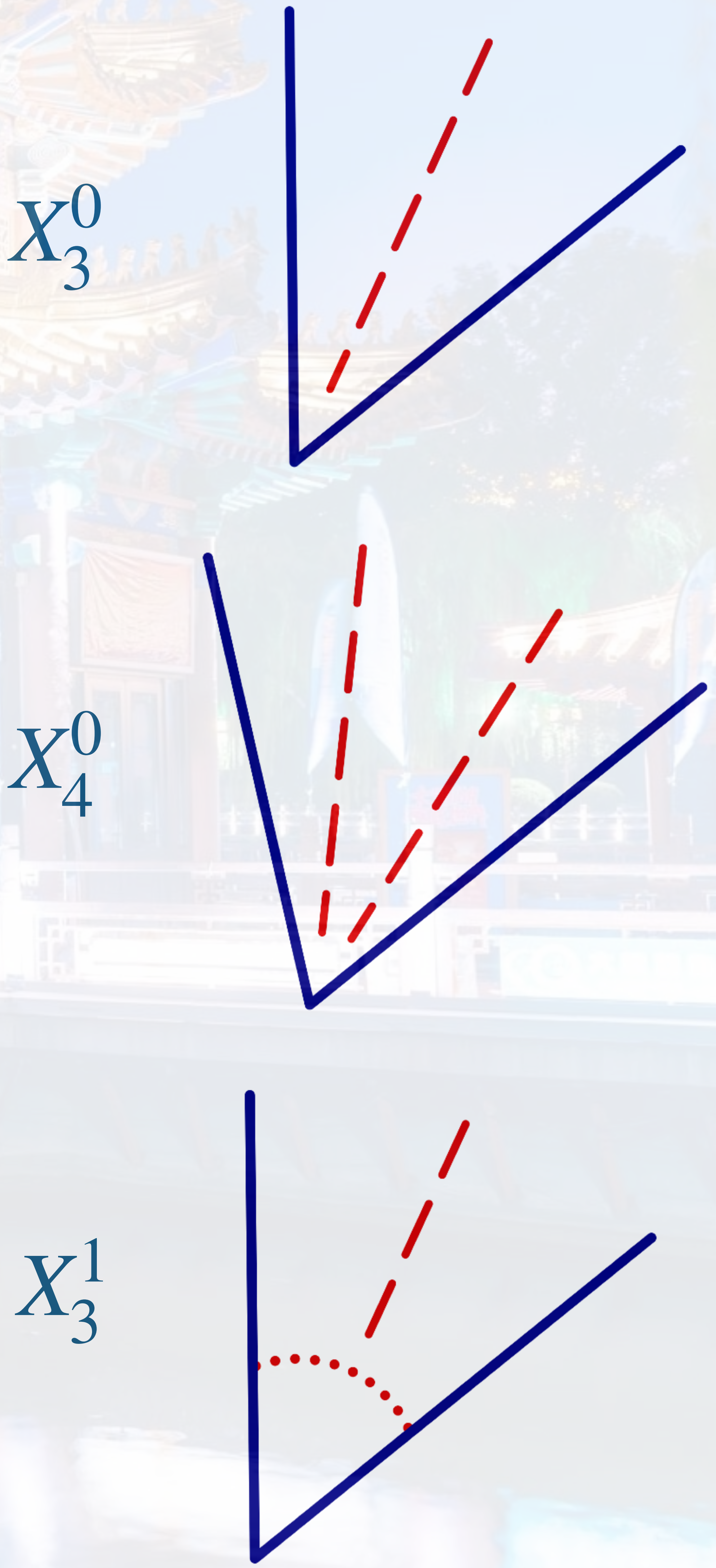
# Development of Antenna Subtraction Method

Based on Marcoli's slide @ Loop Summit 2

Successfully applied at NNLO to a variety of processes within the ***NNLOJET*** Monte Carlo framework

# Antenna Subtraction @ NNLO



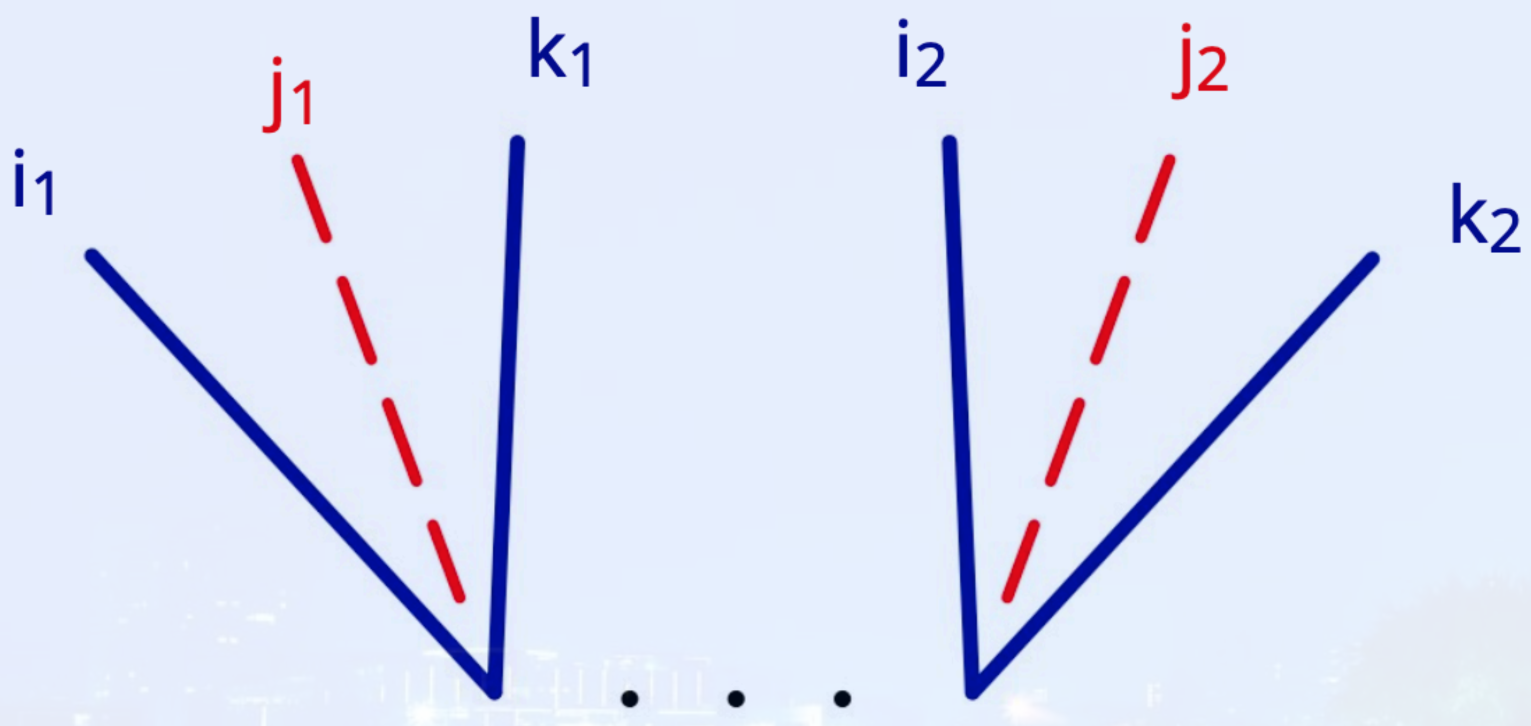

XC, Gehrmann, Glover, Huss, Marcoli JHEP 10 (2022) 099

# Antenna Subtraction @ NNLO

Marcoli's slide @ Loop Summit 2

**NNLO:** two unresolved emissions → multiple topologies

**colour-unconnected** emissions: no shared hard radiatior



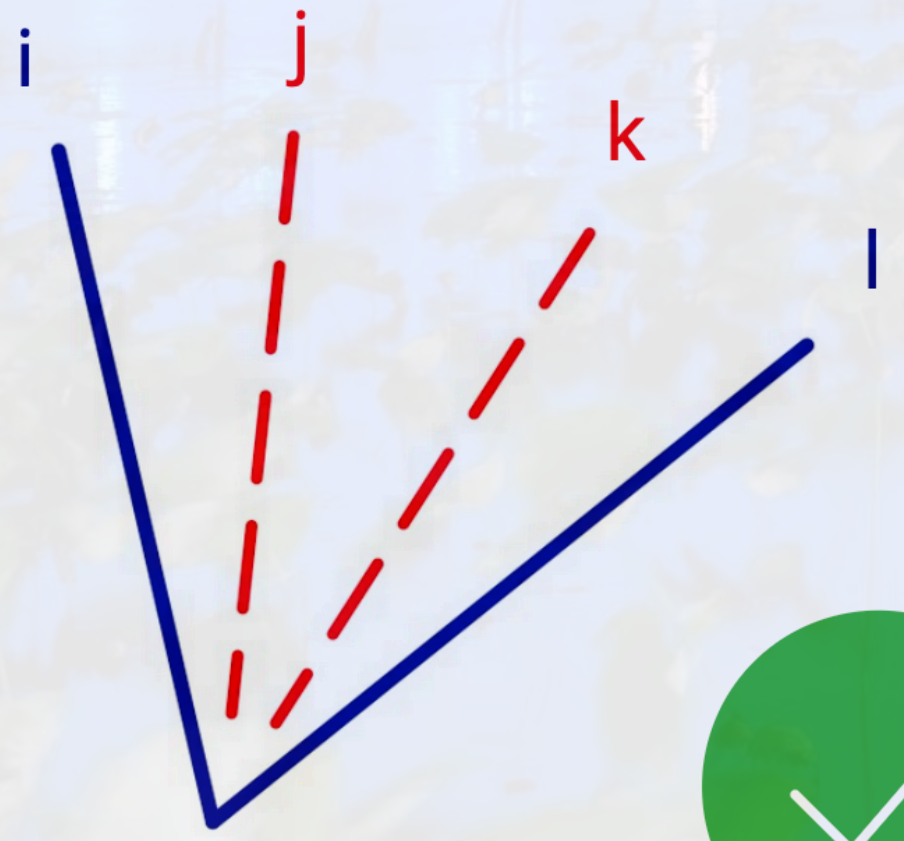

$$M^0_{n+2}(\ldots, i_1, j_1, k_1, \ldots, i_2, j_2, k_2, \ldots)$$

$$\downarrow$$

$$X^0_3(i^h_1, j_1, k^h_1) X^0_3(i^h_2, j_2, k^h_2) M^0_n(\ldots, (\widetilde{i_1 j_1}), (\widetilde{j_1 k_1}), \ldots, (\widetilde{i_2 j_2}), (\widetilde{j_2 k_2}), \ldots)$$

fully **iterated** structure

**colour-connected** emissions: both hard radiators shared




$$M^0_{n+2}(\ldots, i, j, k, l, \ldots)$$

$$\downarrow$$

$$X^0_4(i^h, j, k, l^h) M^0_n(\ldots, (\widetilde{ijk}), (\widetilde{jkl}), \ldots)$$

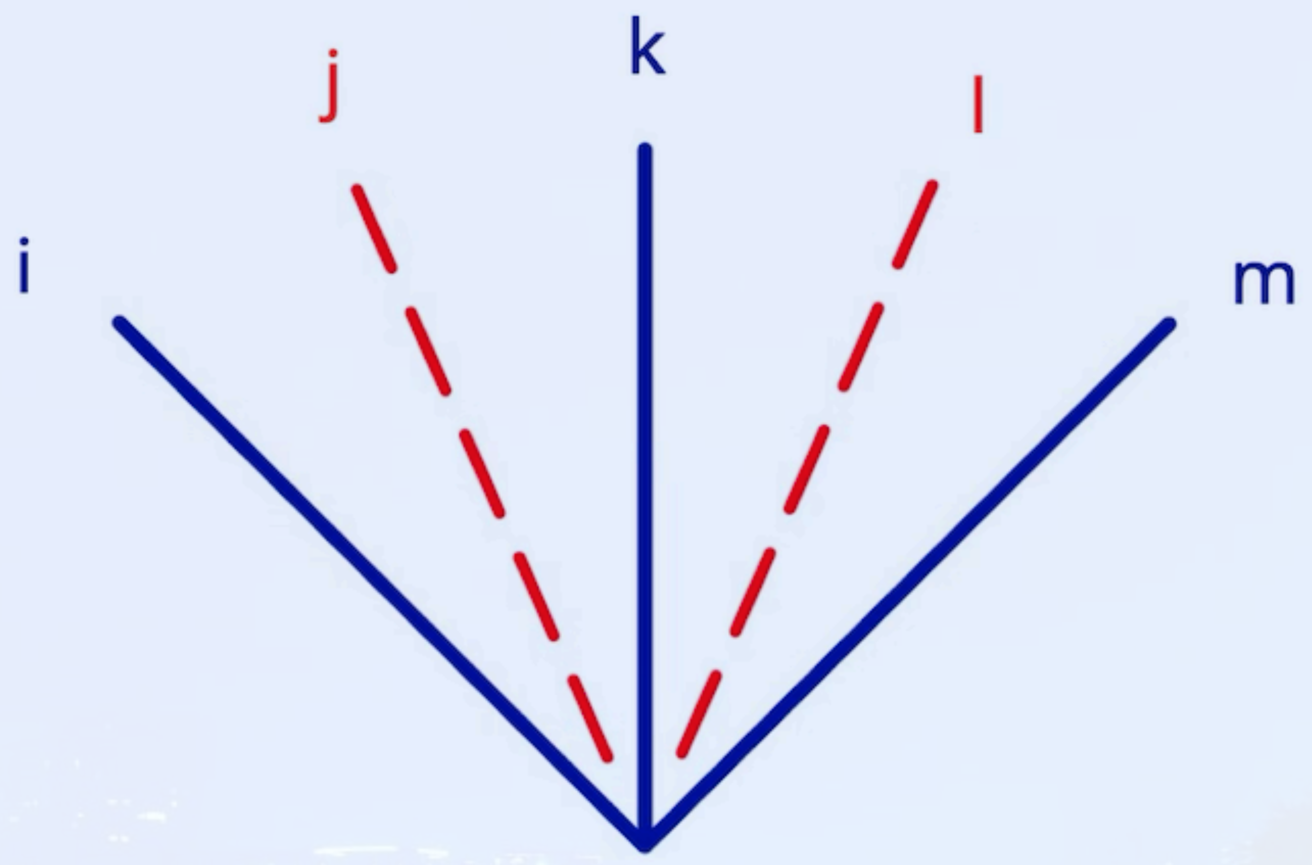
4→2 momentum mapping

$$X^0_3(i,j,k) = \frac{|\mathcal{M}^0_3(i,j,k)|^2}{|\mathcal{M}^0_2(\tilde{I},\tilde{K})|^2} \quad X^0_4(i,j,k,l) = \frac{|\mathcal{M}^0_4(i,j,k,l)|^2}{|\mathcal{M}^0_2(\widetilde{ijk},\widetilde{jkl})|^2}$$

# Generalized Antenna @ NNLO



There is more … **almost colour-connected** emissions: only one shared hard radiator



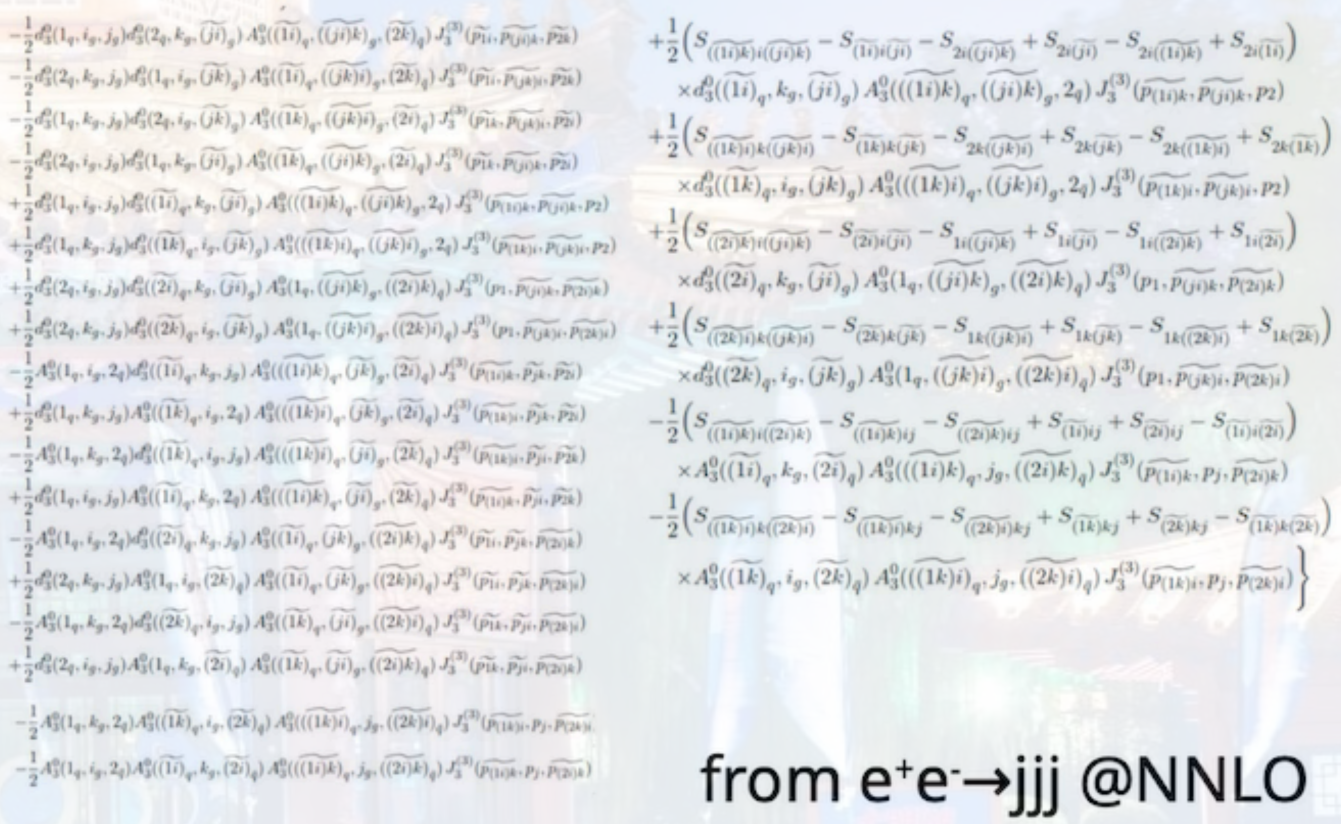

**NOT** fully iterated: the two emissions "feel" each other through the recoil on the shared radiator

traditional antenna functions can be used, but a **very complicated** sequence of iterated structures is needed, plus **Large-Angle-Soft-Terms**

[Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07]

[Weinzierl '08] [Currie, Glover, Wells '13]

most complicated and inefficient sector of antenna subtraction



from e⁺e⁻→jjj @NNLO

Ideally we want:

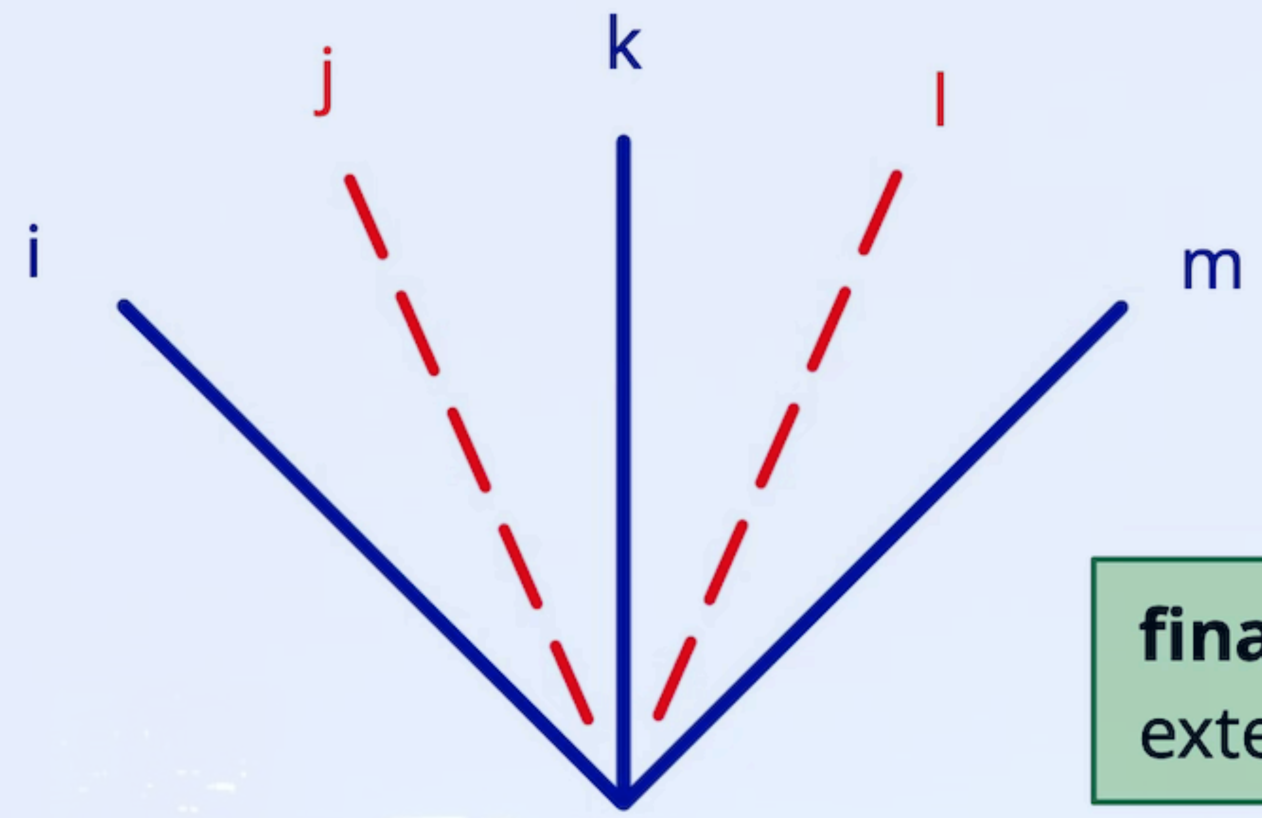
$$M^0_{n+2}(\ldots, i, j, k, l, m \ldots)$$

5→3 momentum mapping

$$X^0_{5,3}(i^h, j, k^h, l, m^h) M^0_n(\ldots, (\widetilde{ijk}), (\widetilde{ijklm}), (\widetilde{klm}), \ldots)$$

**generalized three-hard-radiator antenna function**

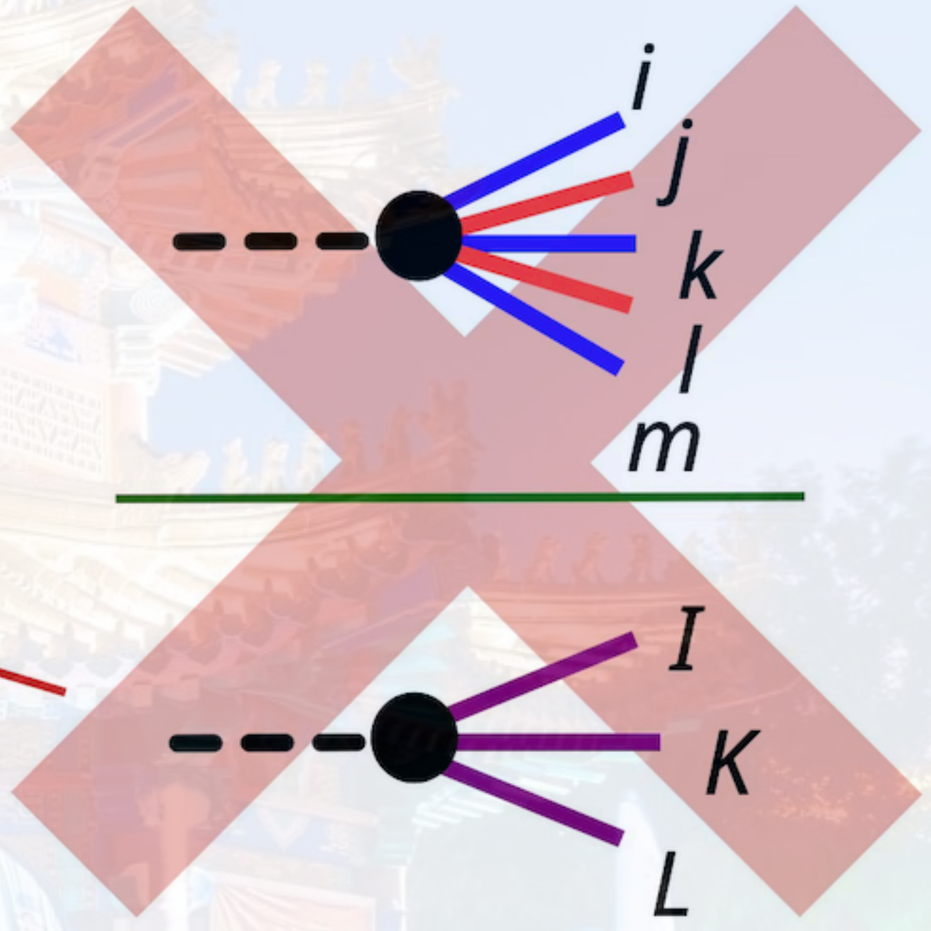
# Generalized Antenna @ NNLO




**final-state** radiation only, extension to ISR in progress

**Not possible** with matrix element-based antenna functions

non-trivial function of the three-particle phase space




With the **designer antenna algorithm**, it is possible to construct antenna functions with **more than two hard radiators: generalized antenna functions.**

[Fox, Glover, Marcoli '24]

Analytical integration made particularly simple thanks to a convenient choice of **5→3 momentum mapping**.

$$\mathrm{map}_{5\to3}: \quad \begin{aligned} p_I &= p_i + p_j - \frac{s_{ij}}{s_{ik}+s_{jk}} p_k \\ p_K &= \left(1 + \frac{s_{ij}}{s_{ik}+s_{jk}} + \frac{s_{lm}}{s_{lk}+s_{mk}}\right) p_k \\ p_M &= p_l + p_m - \frac{s_{lm}}{s_{lk}+s_{mk}} p_k \end{aligned}$$

iterated dipole mapping

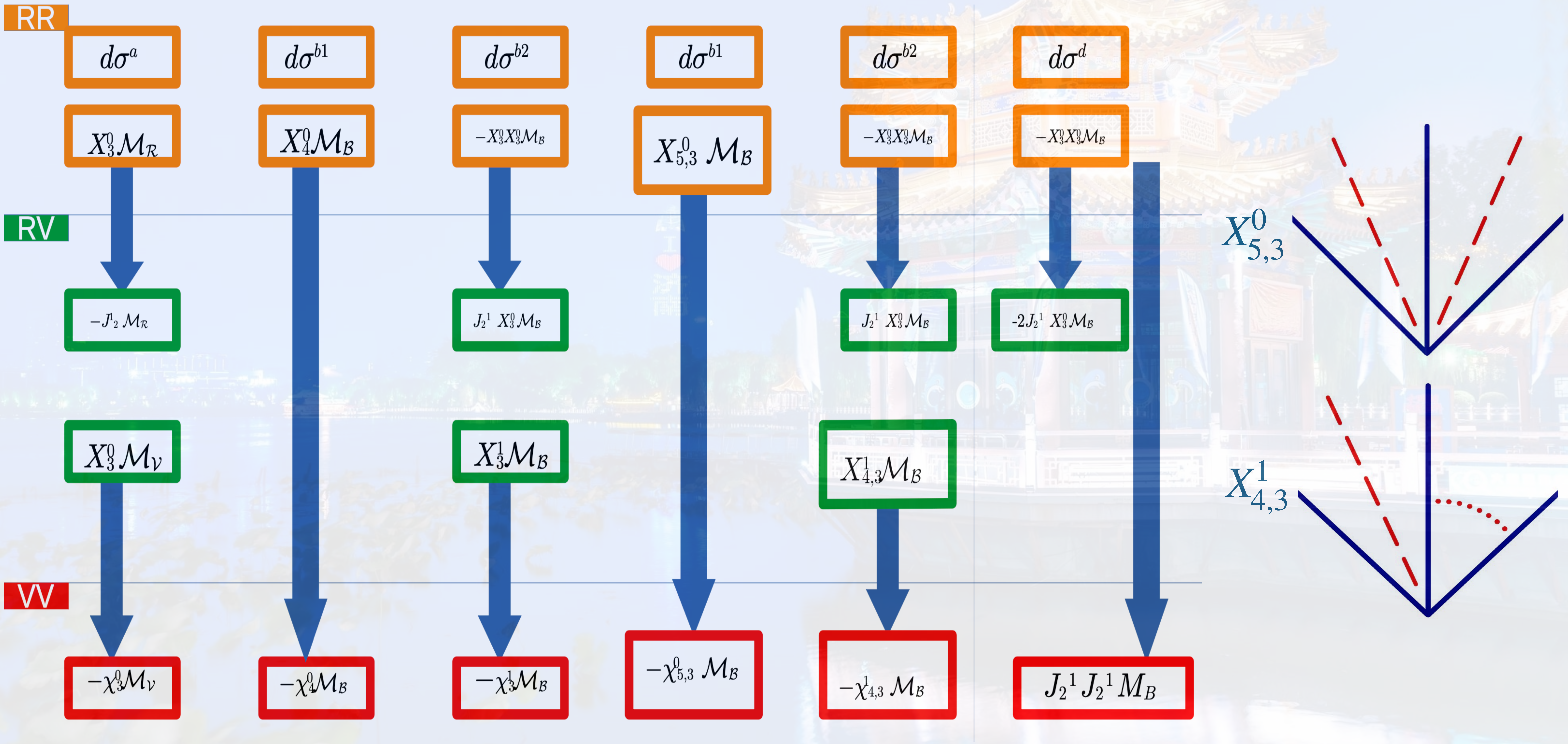
- Constructed with an iterative algorithm
  - From the desired IR limits (not from physical matrix-elements)
  - Using projector (up-down) to connect full phase space (antennae) and subspace (IR limits)
  - Can be integrated analytically (as in conventional method)

[Braun-White, Glover, Preuss '23]

[Braun-White, Glover, Preuss '23]

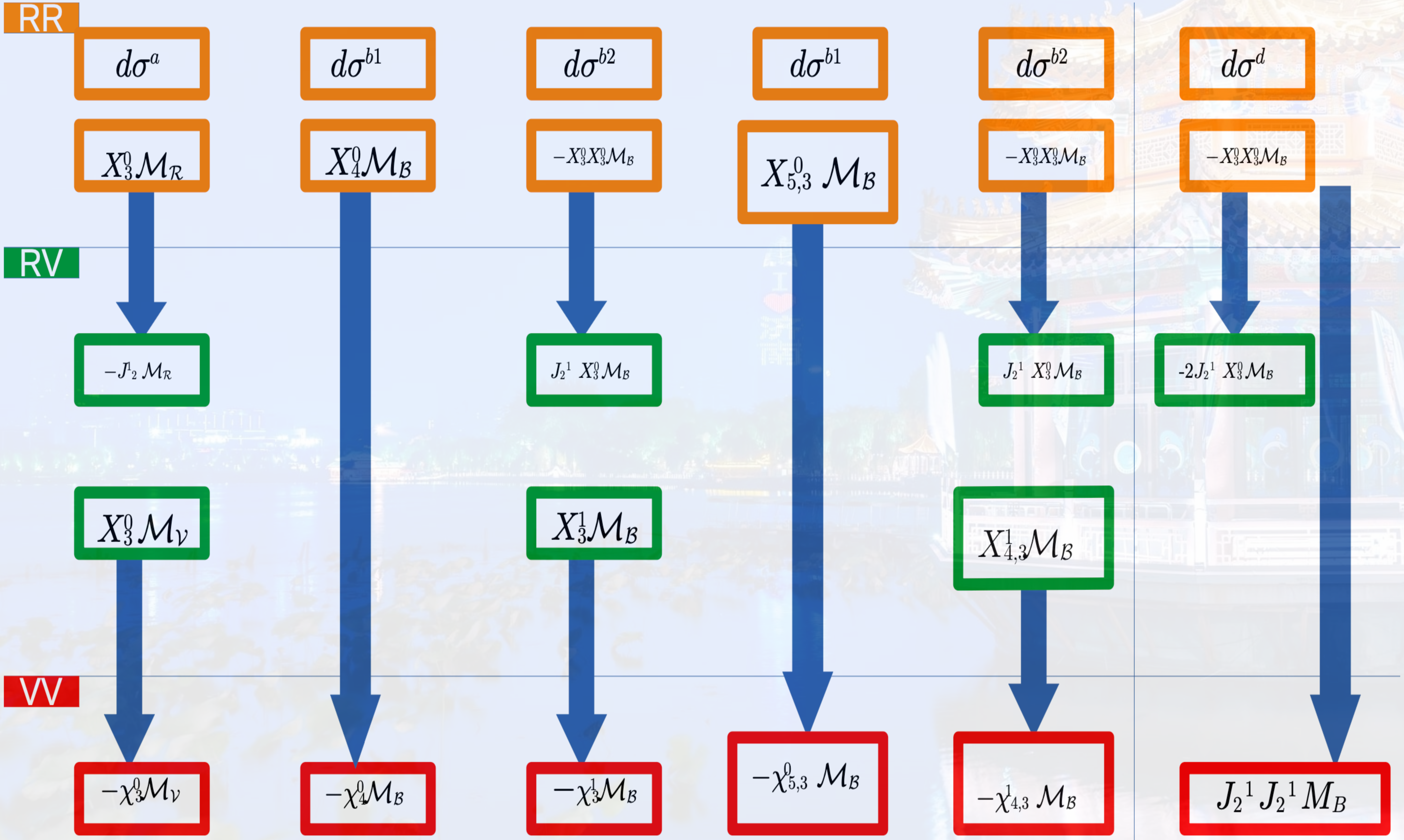
[Fox, Glover '23]

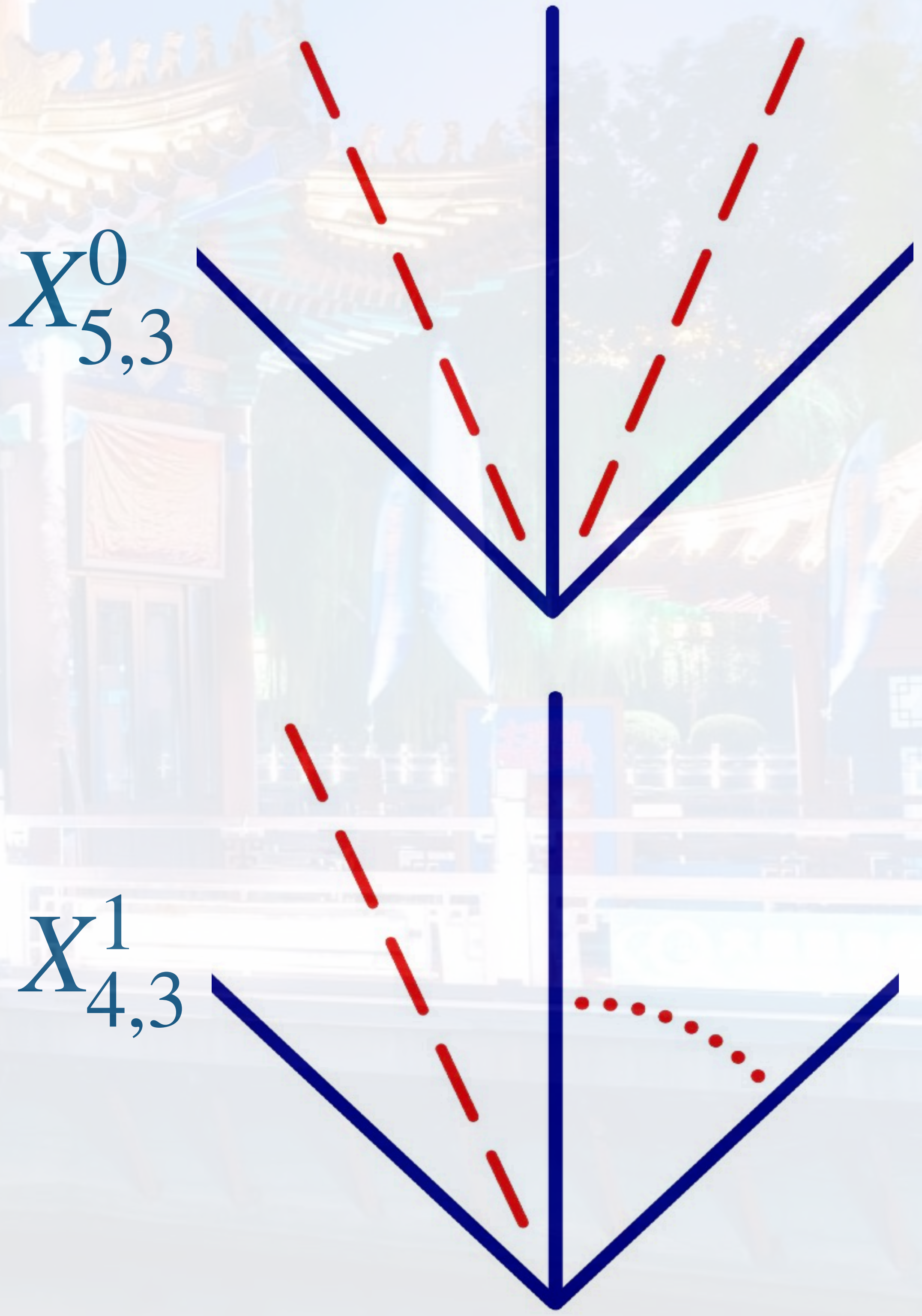
# Generalized Antenna @ NNLO




*Fox, Glover, Marcoli JHEP 12 (2024) 225*

# Generalized Antenna @ NNLO




$e^+e^- \to$ 3 jets @ NNLO

Efficiency boost by a factor of 10

*Fox, Glover, Marcoli JHEP 12 (2024) 225*

# Application of NNLOJET at the LHC

# NNLOJET: New Feature To Be Released

## PDF grids: APPLfast and PineAPPL

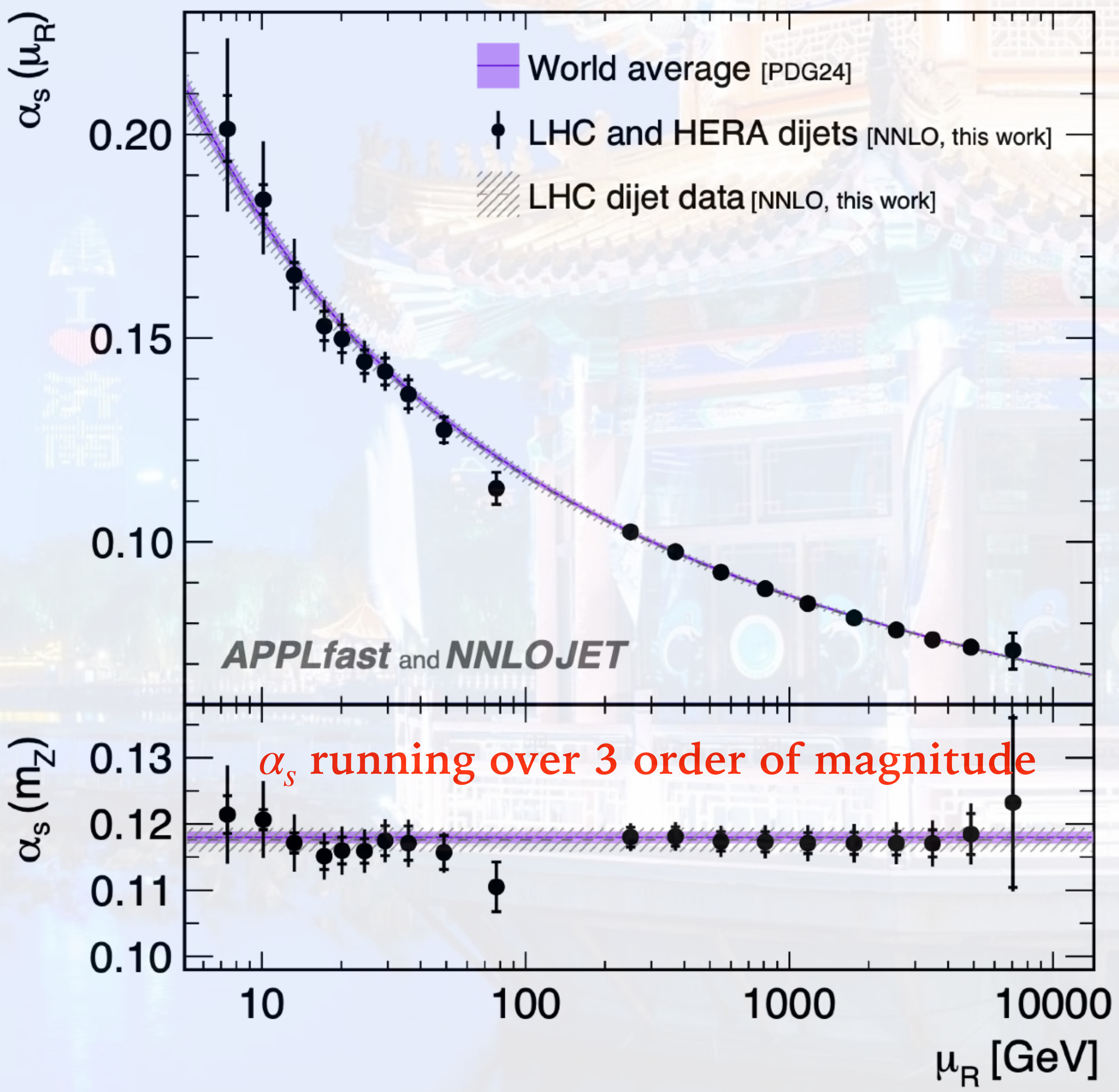
| Data | $\sqrt{s}$ [TeV] | d$\sigma$ | $R$ | $\mathcal{L}$ |
|---|---|---|---|---|
| ATLAS [10] | 7 | $\frac{\mathrm{d}^2\sigma}{\mathrm{d}m_{\mathrm{jj}}\mathrm{d}y^*}$ | 0.6 | $4.5\,\mathrm{fb}^{-1} \pm 1.8\,\%$ |
| CMS [12] | 7 | $\frac{\mathrm{d}^2\sigma}{\mathrm{d}m_{\mathrm{jj}}\mathrm{d}y_{\mathrm{max}}}$ | 0.7 | $5.0\,\mathrm{fb}^{-1} \pm 2.2\,\%$ |
| CMS [13] | 8 | $\frac{\mathrm{d}^3\sigma}{\mathrm{d}\langle p_{\mathrm{T}}\rangle_{1,2}\mathrm{d}y^*\mathrm{d}y_{\mathrm{b}}}$ | 0.7 | $19.7\,\mathrm{fb}^{-1} \pm 2.6\,\%$ |
| ATLAS [11] | 13 | $\frac{\mathrm{d}^2\sigma}{\mathrm{d}m_{\mathrm{jj}}\mathrm{d}y^*}$ | 0.4 | $3.2\,\mathrm{fb}^{-1} \pm 2.1\,\%$ |
| CMS [14] | 13 | $\frac{\mathrm{d}^2\sigma}{\mathrm{d}m_{\mathrm{jj}}\mathrm{d}y_{\mathrm{max}}}$ | 0.8 | $33.5\,\mathrm{fb}^{-1} \pm 1.2\,\%$ |
| CMS [14] | 13 | $\frac{\mathrm{d}^3\sigma}{\mathrm{d}m_{\mathrm{jj}}\mathrm{d}y^*\mathrm{d}y_{\mathrm{b}}}$ | 0.8 | $29.6\,\mathrm{fb}^{-1} \pm 1.2\,\%$ |

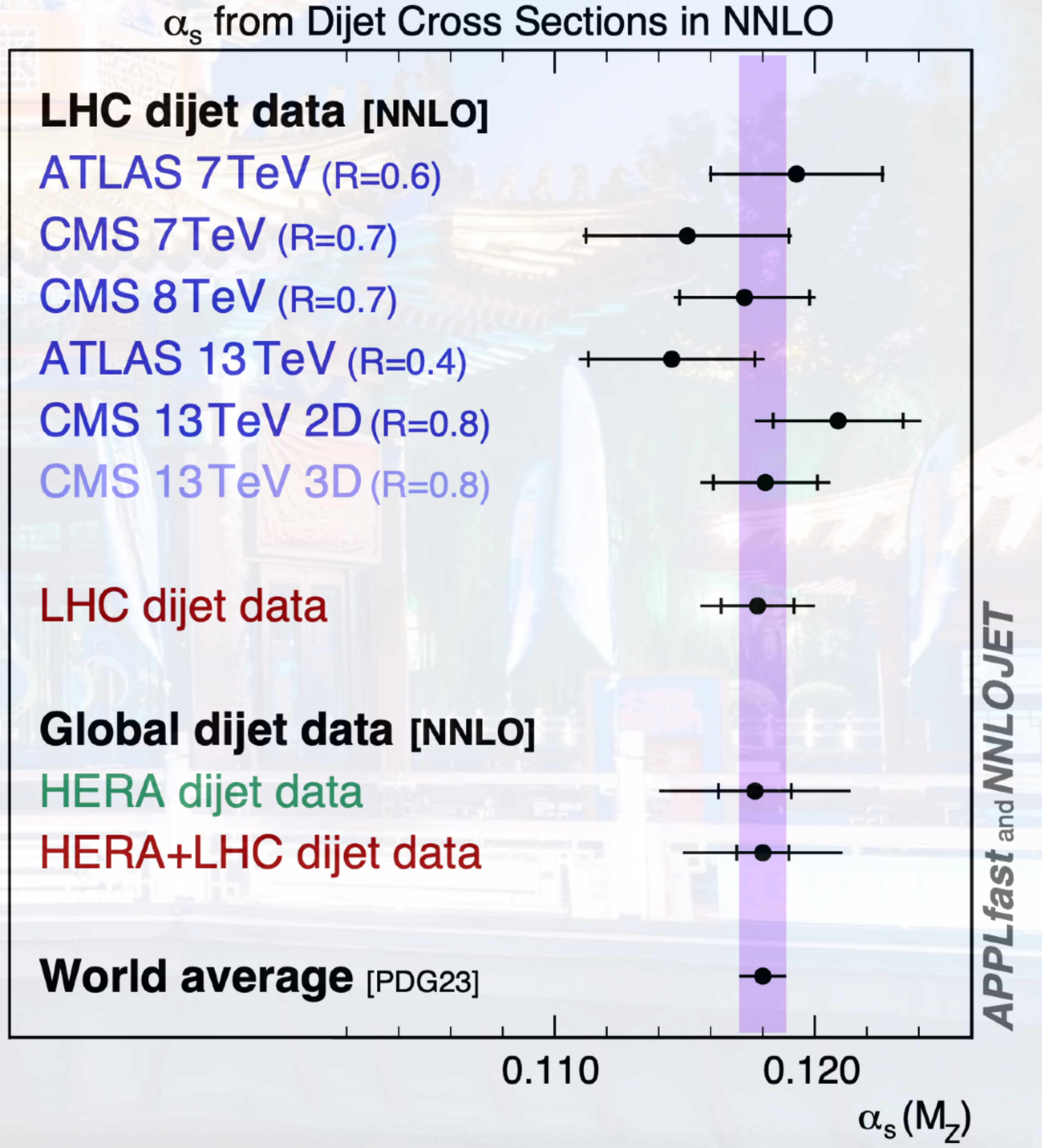
367 LHC data 119 HERA data

| Data set | $\chi^2/n_{\mathrm{dof}}$ | $\alpha_s(m_Z)$ |
|---|---|---|
| ATLAS 7 TeV | 74.7/ 77 | 0.1193 (33) (4) ( 6) |
| ATLAS 13 TeV | 87.7/106 | 0.1145 (32) (4) (16) |
| CMS 7 TeV | 50.7/ 45 | 0.1151 (39) (1) ( 9) |
| CMS 8 TeV | 37.0/ 56 | 0.1173 (25) (0) (11) |
| CMS 13 TeV (2D) | 71.6/ 78 | 0.1209 (25) (2) (20) |
| CMS 13 TeV (3D) | 137.7/112 | 0.1181 (20) (1) (15) |
| LHC dijets (CMS13-2D) | 335.3/366 | 0.1178 (14) (0) (17) |
| LHC dijets (CMS13-3D) | 397.9/400 | 0.1172 (14) (0) (14) |
| HERA | 92.8/118 | 0.1177 (14) (1) (34) |
| LHC+HERA (CMS13-2D) | 428.4/485 | 0.1180 (10) (0) (29) |
| LHC+HERA (CMS13-3D) | 491.0/519 | 0.1177 (10) (0) (27) |

$$\chi^2 = \sum_{i,j} \log\frac{\varsigma_i}{\sigma_i}\left(V_{\mathrm{exp}} + V_{\mathrm{NP}} + V_{\mathrm{NNLOstat}} + V_{\mathrm{PDF}}\right)^{-1}_{ij} \log\frac{\varsigma_j}{\sigma_j}$$

$\varsigma_i$ LHC or HERA jet data
$\sigma_i$ NNLO theory
V covariance matrices




- All LHC and HERA di-jet double- and triple-differential measurements.
- APPLfast PDF grids from NNLOJET NNLO di-jet predictions.
- Fitting of the strong coupling constant: $\alpha_s(m_Z) = 0.1178 \pm 0.0022$.

Ahmadova, Britzger, XC, Gaßler et. al., Phys. Rev. Lett. *135 (2025) 3*

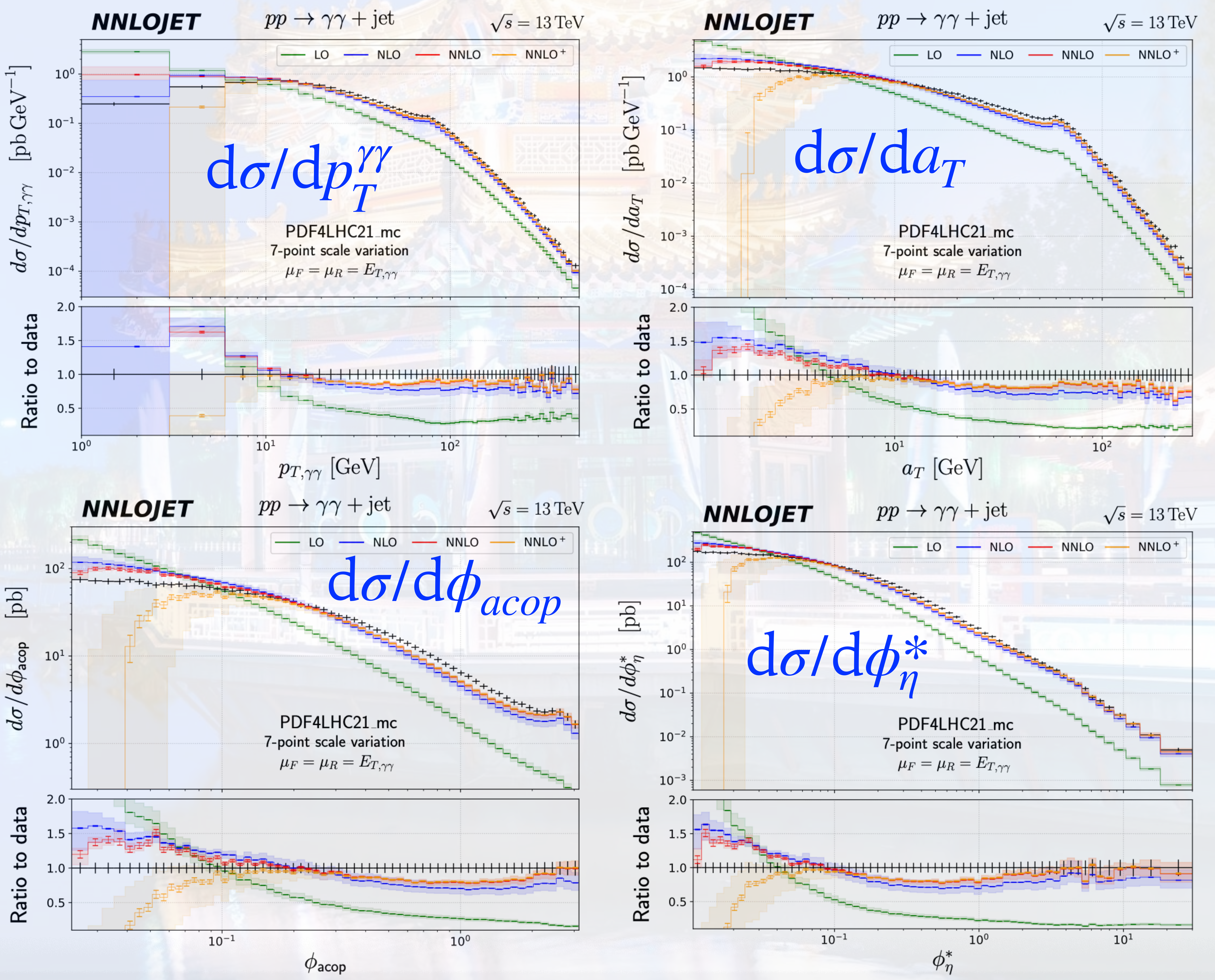
# NNLOJET: New Feature To Be Released

## $pp \to \gamma\gamma$ + recoil @ NNLO QCD

- Di-photon production at the LHC with $p_T^{\gamma\gamma} > 1$ GeV
  - The main SM background of $H \to \gamma\gamma$
  - Large pQCD and photon fragmentation corrections
  - NNLO = $\mathcal{O}(\alpha_s^3)$ terms:
    - NNLO in $q\bar{q}$ and $qg$ channels:

      Tree and 1-loop ME: OpenLoops

      Two-loop, five-point full colour contributions
    - LO loop-induced process for $gg$ channel
  - NNLO$^+$ = NNLO + NLO (loop-induced new result)
- Comparison with ATLAS 13 TeV data:
  - Better agreement but still with systematic deviation
  - Loop-induced corrections are crucial for the reduction of systematic uncertainties.
- Future plan （to be included in public release）:
  - Impact of various photon-isolation algorithm
  - Extend to $p_T^{\gamma\gamma} \in [0,1]$ GeV to complete N3LO




F. Buccioni, XC, W.J. Feng, T. Gehrmann et. al., Phys. Rev. Lett. 134 (2025) 17

# NNLOJET: New Feature To Be Released

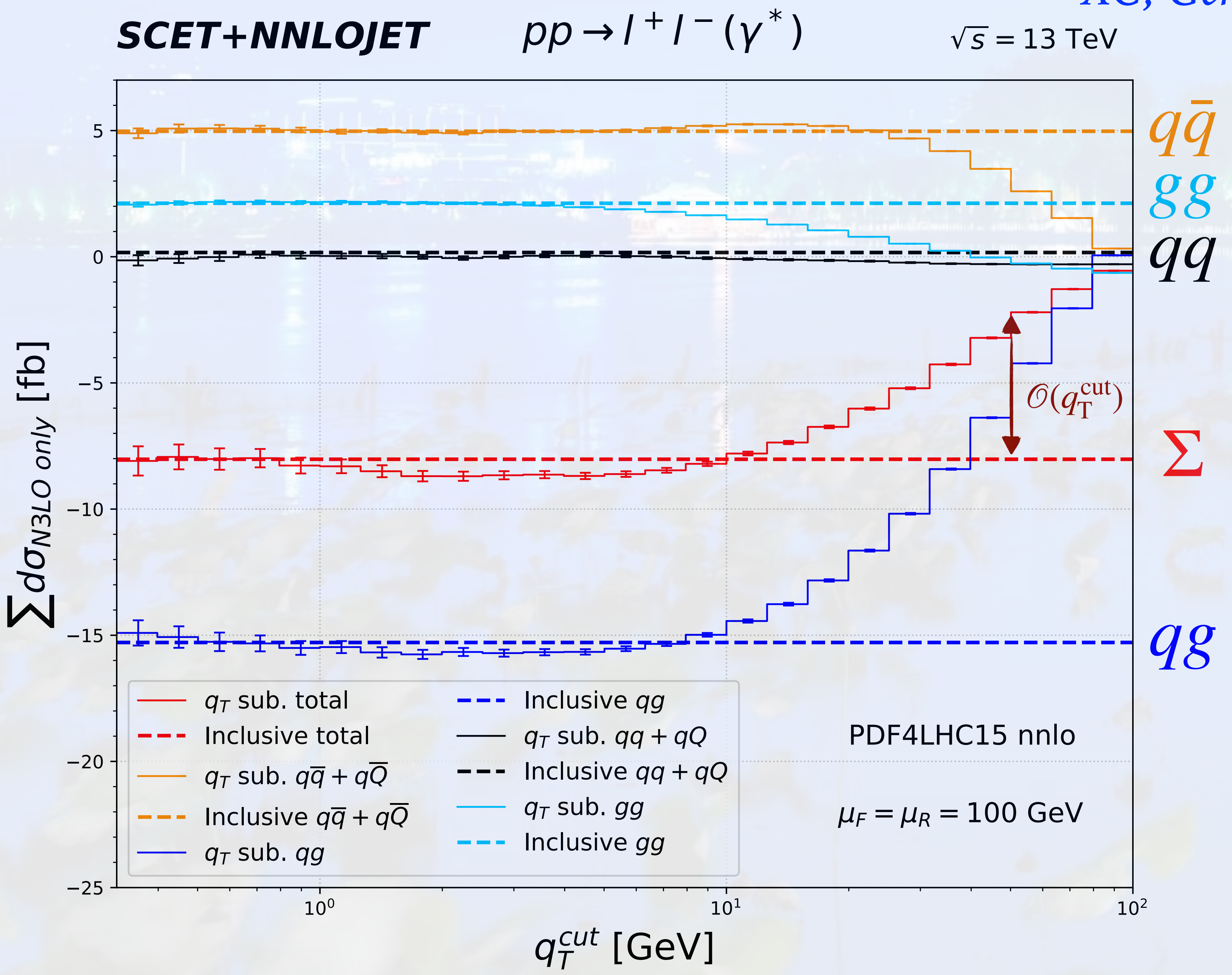
## Drell-Yan @ N3LO QCD ($q_T$ slicing)

- Fully differential N3LO correction in event generator
  - Recycle $pp \to V+J$ @ NNLO with $q_T$ slicing

$$\mathrm{d}\sigma^{F}_{N^kLO} = \mathscr{H}^{F}_{N^kLO} \otimes \mathrm{d}\sigma^{F}_{LO}\Big|_{\delta(q_T)} + \left[\mathrm{d}\sigma^{F+jet}_{N^{k-1}LO} - \mathrm{d}\sigma^{F\ CT}_{N^kLO}\right]_{q_T>q_T^{cut}} + \mathcal{O}\left((q_T^{cut}/Q)^2\right)$$

  - Fiducial power correction removed via MC recoil technique.
  - Below-cut contribution from expansion of N3LL resummation to $\mathcal{O}(\alpha_s^3)$

*XC, Gehrmann, Glover, Huss, Li, Neill et. al.. PLB 788, 425 (2019)*
*Billis, Ebert, Michel, Tackmann. EPJP 136, 214 (2021)*
*Camarda, Cieri, Ferrera. PRD 104, L111503 (2021)*




*XC, Gehrmann, et. al. PRL. 128, 052001 (2022)*

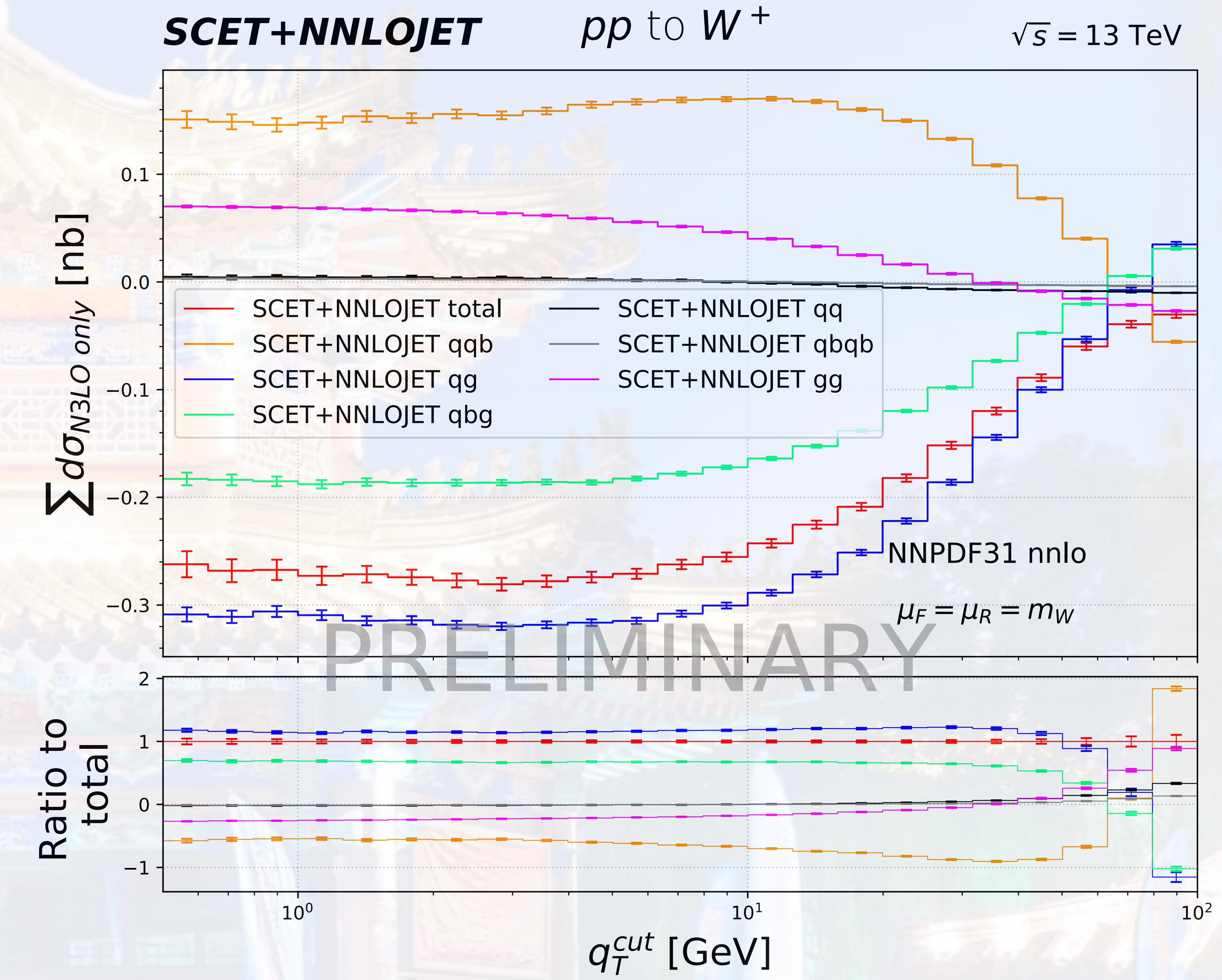
- Ingredients: 3-loop soft and beam functions:

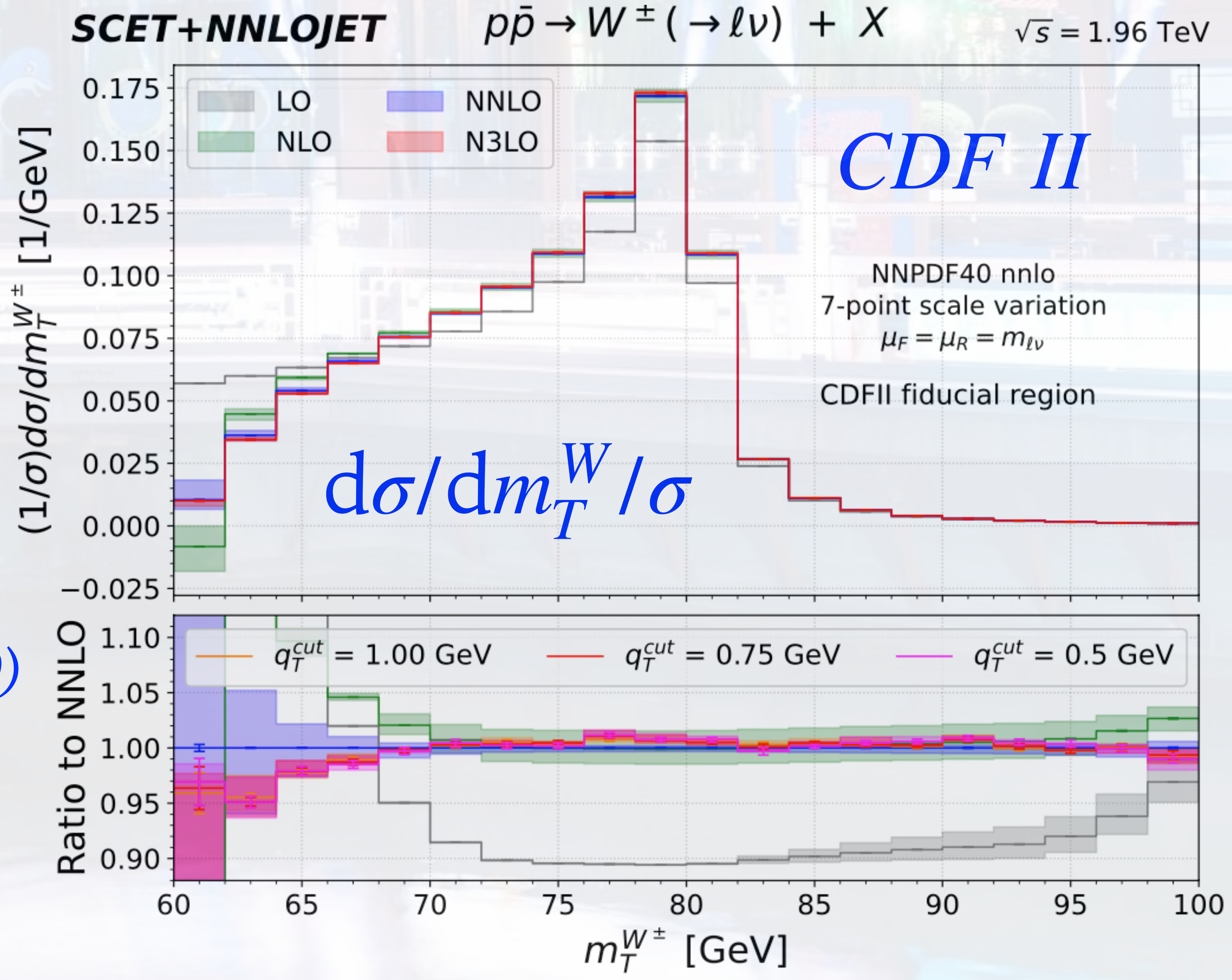
*Li, Zhu. PRL. **118**, 022004 (2017)*
*Luo, Yang, Zhu, Zhu. PRL. **124**, 092001 (2020)*
*Ebert, Mistlberger, Vita. JHEP. **09**, 146 (2020)*

- Independence on $q_T^{cut}$ slicing parameter
- Validation against inclusive total XS:

*Duhr, Dulat, Mistlberger. PRL. **125**, 172001 (2020)*

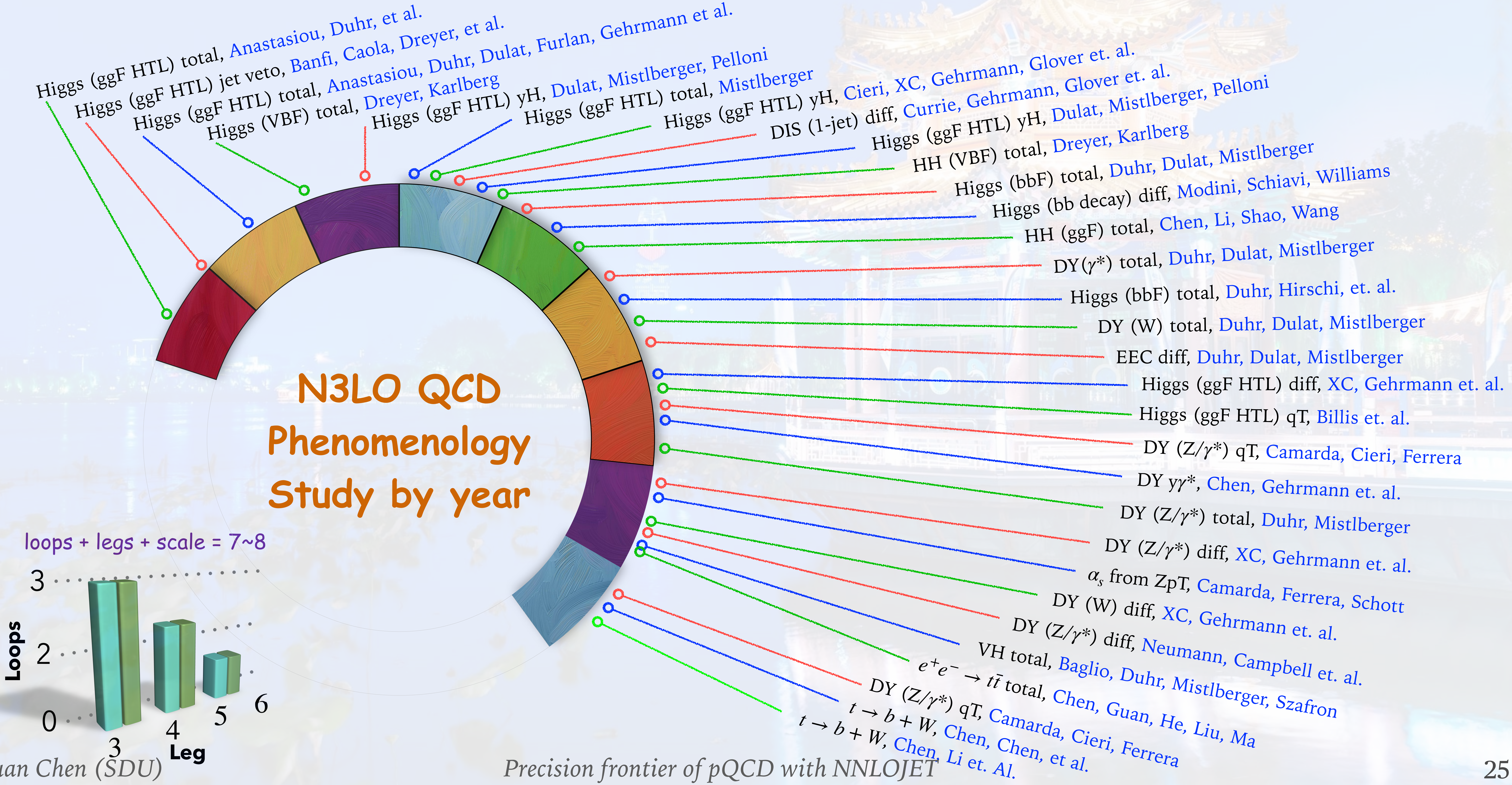
- Separated in parton channels
- Foundation of numerical Monte Carlo



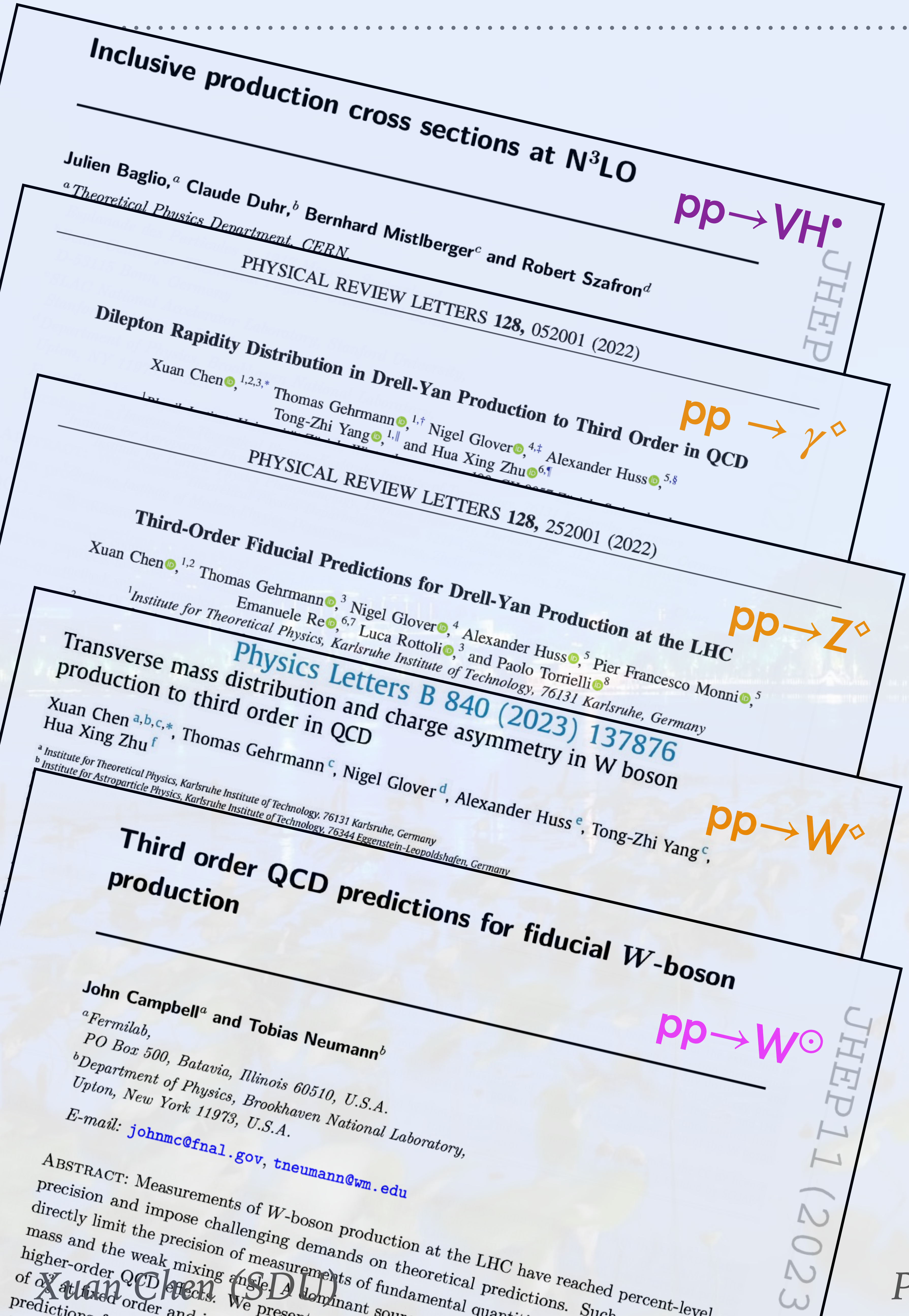




*XC, Gehrmann et. al. PLB 840 (2023)*

# Perturbative QCD @ N3LO



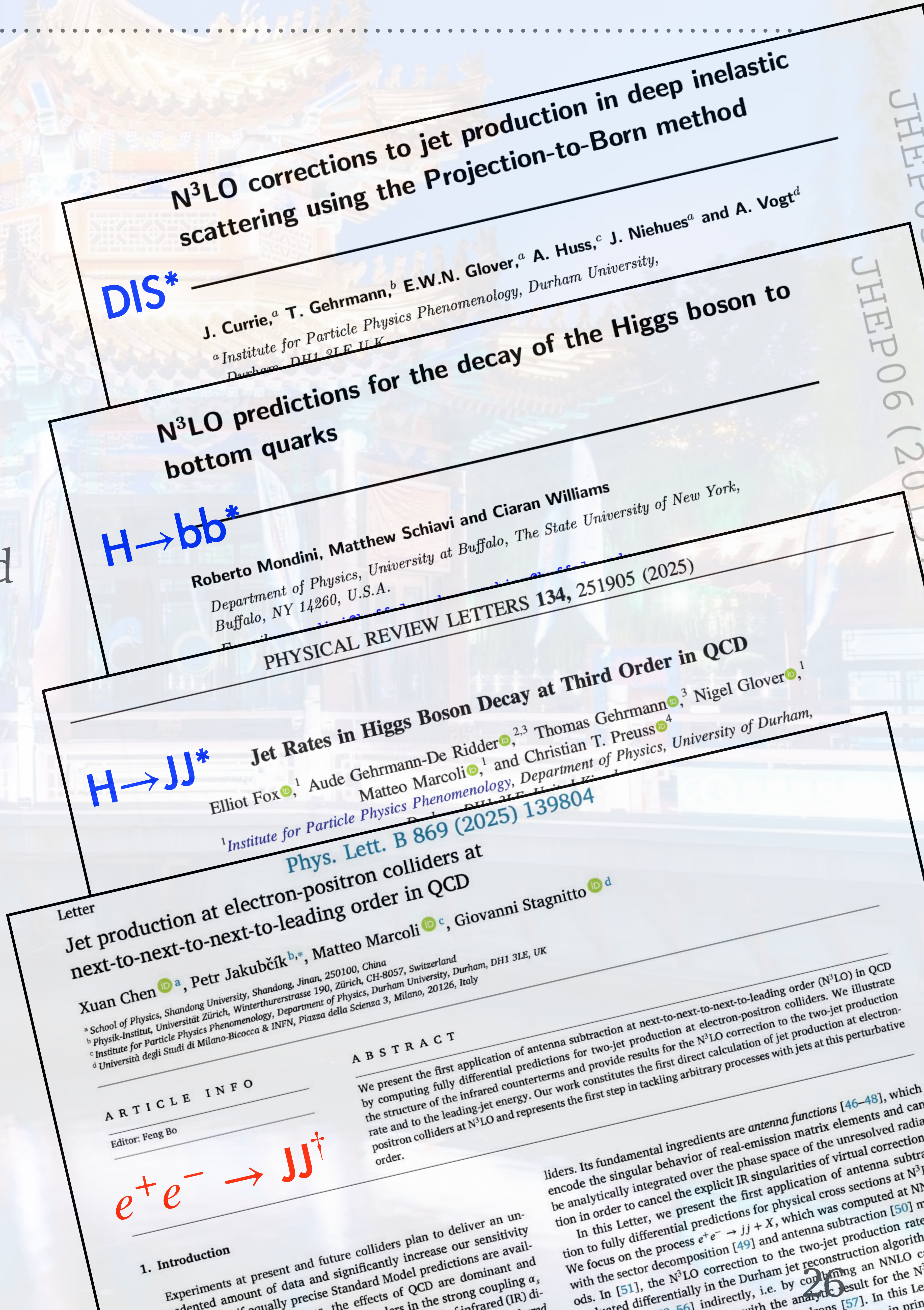
# State-of-the-Art QCD Calculations @ N3LO




- Several phenomenologically relevant results despite the extreme complexity.
- Available techniques are applicable to limited cases with high quality EXP data.
- New approaches must be developed for more complicated scattering.

(10 M → X00 k CPU hours)

- • Inclusive
- ◇ qT slicing
- ⊙ $\tau$ slicing
- * Projection-2-Born
- † Antenna subtraction



# Application of NNLOJET in $e^+e^-$ Colliders
## (Close Relation to Antenna Subtraction)

| | |
|---|---|
| $e^+e^- \to$ di-jet @ N3LO | (Antenna Subtraction) 2505.10618 |
| H decay $\to$ di-jet @ N3LO | (Generalized Antenna) 2502.17333, 2508.14282 |

# Antenna Subtraction @ N3LO

Slide from Jakubcik @ LL24

| | NLO | NNLO |
|---|---|---|
| RRR | $X_3^0\,\tilde{M}^0_{n+2}$ | $(X_4^0 - X_3^0 X_3^0)\,\tilde{M}^0_{n+1}$ |
| VRR | $X_3^0\,\tilde{M}^0_{n+2}$ | $(X_3^1 + X_3^0 X_3^0)\,\tilde{M}^0_{n+1}$ <br> $+\, X_3^0\,\tilde{M}^1_{n+1}$ |
| VVR | | $(X_4^0 + X_3^1 + X_3^0 X_3^0)\,\tilde{M}^0_{n+1}$ <br> $+\, X_3^0\,\tilde{M}^1_{n+1}$ |
| VVV | reduced matrix elem. $\tilde{M}^{\text{loops}}_{\text{hard partons}}$ <br> unintegrated antenna $X^{\text{loops}}_{\text{emissions}+2}$ <br> integrated antenna $X$ | |

# Antenna Subtraction @ N3LO

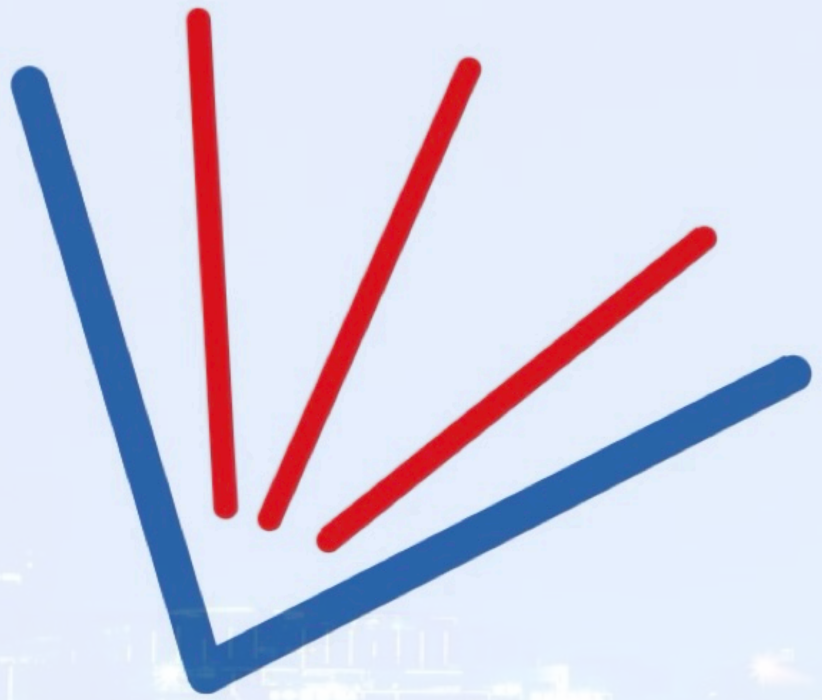
Slide from Jakubcik @ LL24

| | NLO | NNLO | N$^3$LO (a sketch) |
|---|---|---|---|
| RRR | $X_3^0\,\tilde{M}^0_{n+2}$ | $(X_4^0 - X_3^0 X_3^0)\,\tilde{M}^0_{n+1}$ | $(X_5^0 - X_4^0 X_3^0 - X_3^0 X_4^0 + X_3^0 X_3^0 X_3^0)\,\tilde{M}^0_n$ |
| VRR | $X_3^0\,\tilde{M}^0_{n+2}$ | $(X_3^1 + X_3^0 X_3^0)\,\tilde{M}^0_{n+1}$ <br> $+ X_3^0\,\tilde{M}^1_{n+1}$ | $(X_4^1 - X_3^1 X_3^0 + X_3^0 X_3^0 X_3^0 + X_3^0 X_4^0)\,\tilde{M}^0_n$ <br> $+ (X_4^0 + X_3^0 X_3^0)\,\tilde{M}^1_n$ |
| VVR | | $(X_4^0 + X_3^1 + X_3^0 X_3^0)\,\tilde{M}^0_{n+1}$ <br> $+ X_3^0\,\tilde{M}^1_{n+1}$ | $(X_3^2 + X_3^0 X_3^1 + X_3^1 X_3^0 + X_4^0 X_3^0 + X_3^0 X_3^0 X_3^0)\,\tilde{M}^0_n$ <br> $+ (-X_3^1 + X_3^0 X_3^0)\,\tilde{M}^1_n$ <br> $+ X_3^0\,\tilde{M}^2_n$ |
| VVV | reduced matrix elem. $\tilde{M}^{\text{loops}}_{\text{hard partons}}$ <br> unintegrated antenna $X^{\text{loops}}_{\text{emissions}+2}$ <br> integrated antenna $\mathcal{X}$ | | $(X_5^0 + X_4^1 + X_3^2 + X_4^0 X_3^0 + X_3^1 X_3^0 + X_3^0 X_3^0 X_3^0)\,\tilde{M}^0_n$ <br> $+ (X_4^0 + X_3^1 + X_3^0 X_3^0)\,\tilde{M}^1_n$ <br> $+ X_3^0\,\tilde{M}^2_n$ |

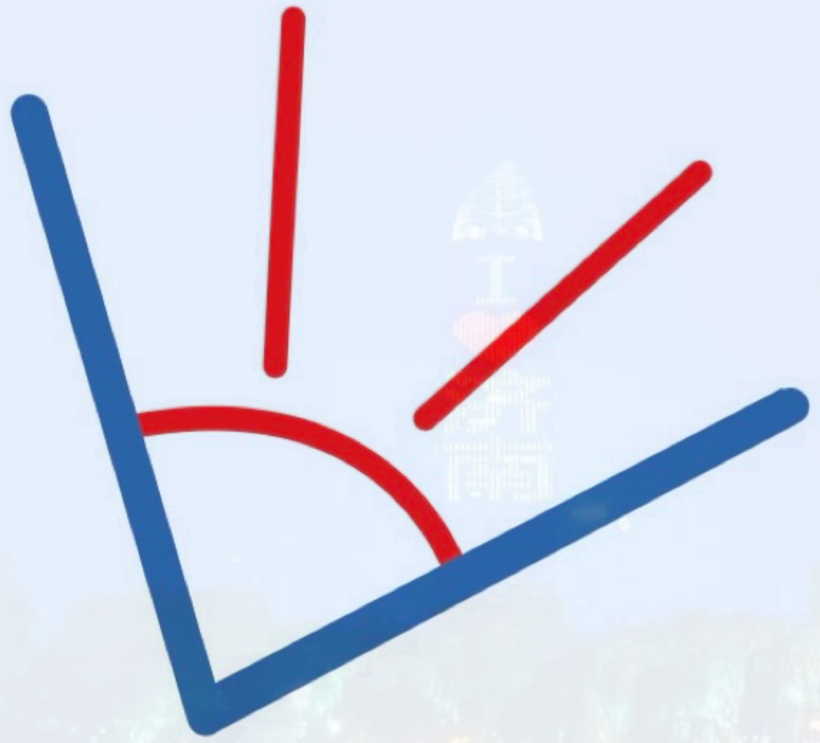
# Antenna Subtraction @ N3LO

Topology of $X_5^0$, $X_4^1$, $X_3^2$ antenna functions:

$X_5^0$

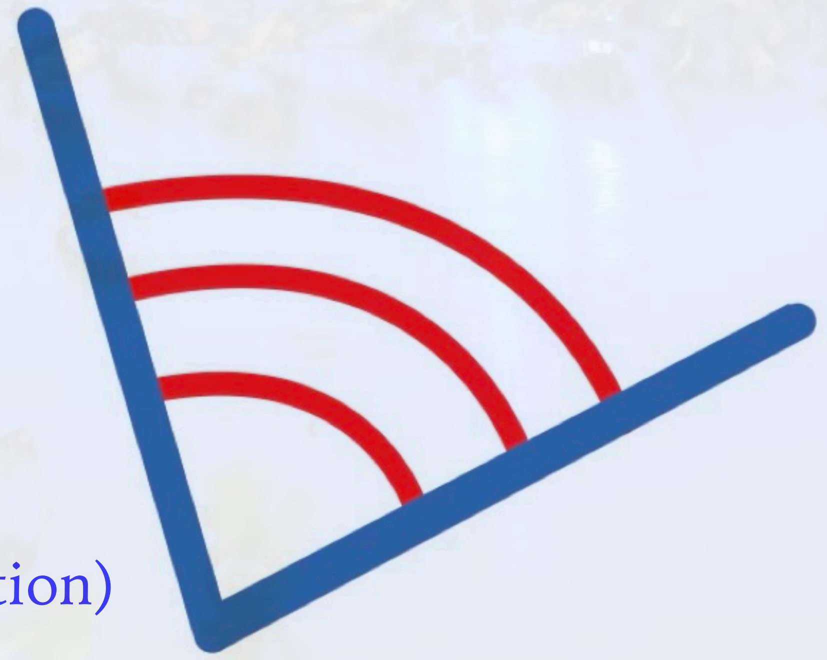


$X_4^1$



$X_3^2$

$X_2^3$



(Only for validation)

Integration of $X_5^0$, $X_4^1$, $X_3^2$ finished for all final states:


$\gamma^* \to q\bar{q}$

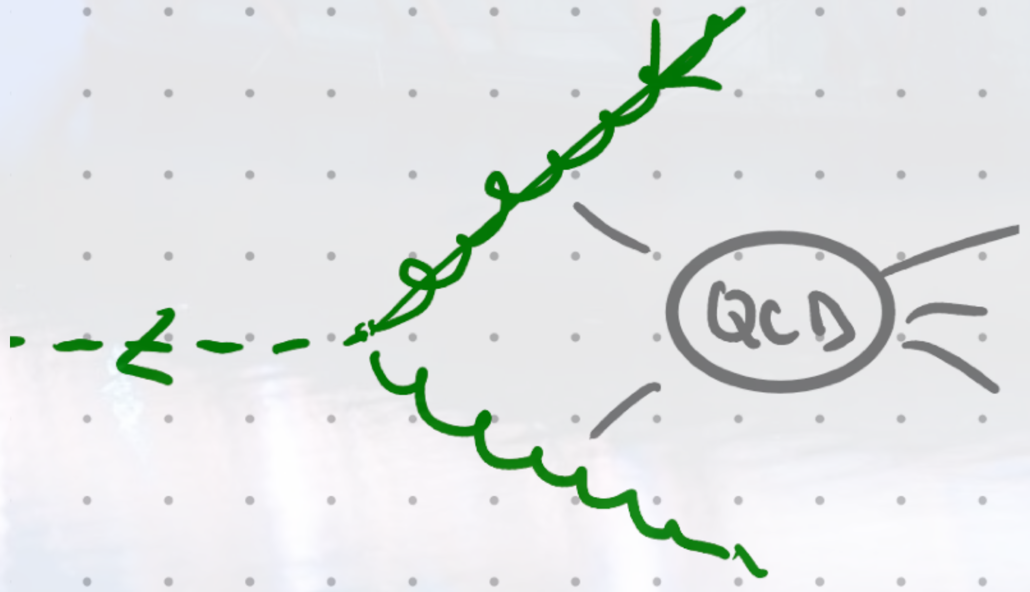
*Jakubcik, Marcoli, Stagnitto*

*JHEP 01 (2023) 168*


$H \to gg$

*XC, Jakubcik, Marcoli, Stagnitto*

*JHEP 06 (2023) 192*

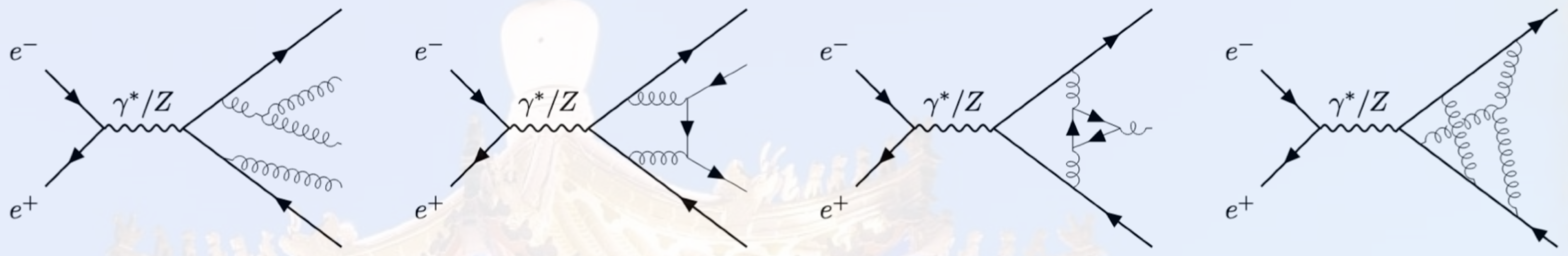



$\chi \to \tilde{g}g$

*XC, Jakubcik, Marcoli, Stagnitto*

*JHEP 12 (2023) 198*

# $e^+e^- \to$ JJ @ N3LO




(a) triple-real (b) double-real-virtual (c) real-double-virtual (d) triple-virtual

- Simple process:
  - Only $\gamma^* \to q\bar{q}$ N3LO antenna functions
  - Only dipole-like correlations at N3LO
  - Recycle ingredients from $e^+e^- \to JJJ$ @ NNLO
- Goals:
  - Establish N3LO antenna subtraction framework
    - Extension of NNLO framework
    - Introduce sector antenna mapping to remove the requirement of sub-antenna functions
  - Exploration of numerical challenges:
    - One-loop double-unresolved regions
    - Two-loop single-unresolved regions
    - Rescue-system to trigger:
      1, Quadruple precision
      2, Taylor expansion of special functions
  - Preparation of computational framework:
    - Spike tests of multiple unresolved IR limits
    - Phase space generators
    - Code generation for N3LO MC.

$$\mathrm{d}\sigma_{N^3LO} = \int_n [\mathrm{d}\sigma^{VVV} - \mathrm{d}\sigma^W] + \int_n [\mathrm{d}\sigma^{RVV} - \mathrm{d}\sigma^U]$$

triple-virtual subtraction term

double-virtual real subtraction term

$$+ \int_{n+1} [\mathrm{d}\sigma^{RRV} - \mathrm{d}\sigma^T] + \int_{n+2} [\mathrm{d}\sigma^{RRR} - \mathrm{d}\sigma^S]$$

double-real-virtual subtraction term

triple-real subtraction term

$$\mathrm{d}\sigma^S = \mathrm{d}\sigma^{S_1} + \mathrm{d}\sigma^{S_2} + \mathrm{d}\sigma^{S_3}$$

$$\mathrm{d}\sigma^T = \mathrm{d}\sigma^{V_1S_1} + \mathrm{d}\sigma^{V_1S_2} - \int_1 \mathrm{d}\sigma^{S_1}$$

$$\mathrm{d}\sigma^U = \mathrm{d}\sigma^{V_2S_1} - \int_1 \mathrm{d}\sigma^{V_1S_1} - \int_2 \mathrm{d}\sigma^{S_2}$$

$$\mathrm{d}\sigma^W = -\int_1 \mathrm{d}\sigma^{V_2S_1} - \int_2 \mathrm{d}\sigma^{V_1S_2} - \int_3 \mathrm{d}\sigma^{S_3}$$

XC, Jakubcik, Marcoli, Stagnitto, Phys. Lett. B. *869 (2025) 139804*

XC, Marcoli, 2507.12537

# $e^+e^- \to$ JJ @ N3LO

- Fully working subtraction terms for all partonic channels:
  - Spike test in IR limits of RRR, RRV and RVV

$$t_{RRR} = \log_{10}\left(\left|1 - \frac{M_{RRR}}{S_{RRR}}\right|\right)$$

  - Green → Blue → Red: deeper in IR divergence, better cancellation between ME and Subtraction terms.
  - Use sector antenna mapping in RRR and RRV:

$\tilde{X}_4$ sector: $e^+e^- \to q\bar{q}\tilde{g}\tilde{g}$ @RRV

(a) $s_{12}s_{34} \le s_{13}s_{24}$: $\{4 \to 2\}$ mapping with ordering $\{p_1^h, p_2, p_3, p_4^h\}$.

(b) $s_{12}s_{34} > s_{13}s_{24}$: $\{4 \to 2\}$ mapping with ordering $\{p_1^h, p_3, p_2, p_4^h\}$.

$\tilde{\tilde{X}}_5$ sector: $e^+e^- \to q\bar{q}\tilde{g}\tilde{g}\tilde{g}$ @RRR

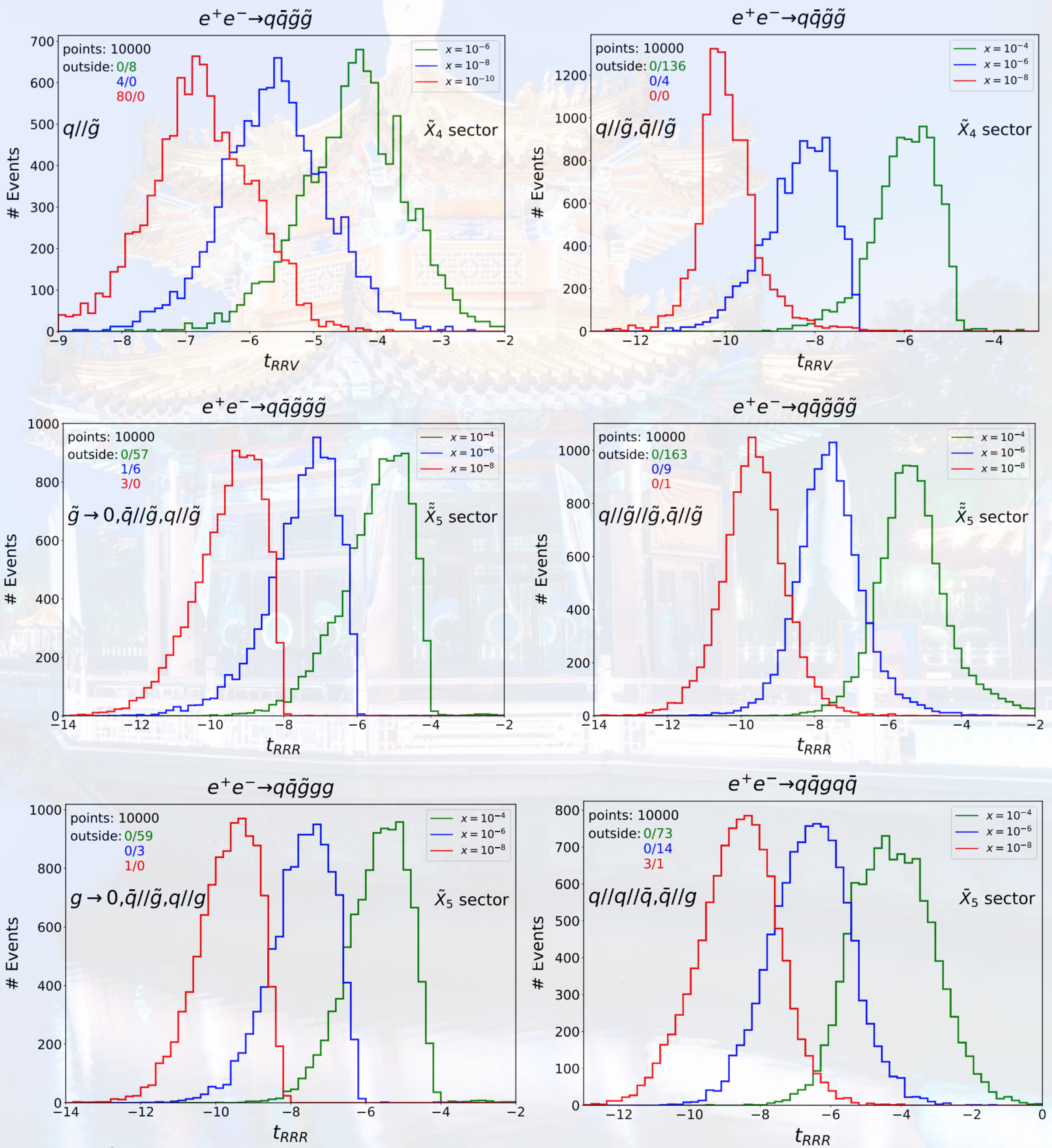
$\tilde{X}_5$ sector: $e^+e^- \to q\bar{q}\tilde{g}gg$ @RRR

$\bar{X}_5$ sector: $e^+e^- \to q\bar{q}\tilde{g}q\bar{q}$ @RRR

- Explicit pole cancellation checked analytically for RRV, RVV and VVV

XC, Jakubcik, Marcoli, Stagnitto, Phys. Lett. B. *869 (2025) 139804*

XC, Marcoli, 2507.12537



# $e^+e^- \to$ JJ @ N3LO

➤ Basic checks: inclusive cross section

N3LO coefficient:

$$\sigma^{(3)} = \sigma^{(0)}\left(\frac{\alpha_s}{2\pi}\right)^3(-105 \pm 11)$$

Monte Carlo error:

Not so small for inclusive quantities due to large cancellations.

Not the most clever way to compute inclusive cross sections.

XC, Jakubcik, Marcoli, Stagnitto, Phys. Lett. B. *869 (2025) 139804*

$$\sigma^{(3)} = \sigma^{(0)}\left(\frac{\alpha_s}{2\pi}\right)^3(-102.14\cdots)$$

Chetyrkin, Künn, Kwiatkowski, Phys. Rept. *277 (1996) 189*

N3LO 2-jet rate:

Exclusive n-jet rate @ N3LO: $$R_n^{(3)}(y_{cut}) = \frac{\Gamma^{(3)}_{\gamma^*\to n\ jets}(y_{cut})}{\Gamma^{(3)}_{\gamma^*\to hadrons}}$$

For back-to-back QCD emissions, we have at least two jets $\to n \geq 2$

$$R_2^{(3)}(y_{cut})\Gamma^{(3)}_{\gamma^*\to hadrons} = \int_0^{y_{cut}} \frac{d\sigma}{dy_{23}}dy_{23}$$

with $y_{ij} = \frac{2\min(E_i^2, E_j^2)}{Q^2}(1-\cos\theta_{ij})$

$$R_3^{(3)}(y_{cut})\Gamma^{(3)}_{\gamma^*\to hadrons} = \int_{y_{cut}}^1 \frac{d\sigma}{dy_{23}}dy_{23} - \int_{y_{cut}}^1 \frac{d\sigma}{dy_{34}}dy_{34}$$

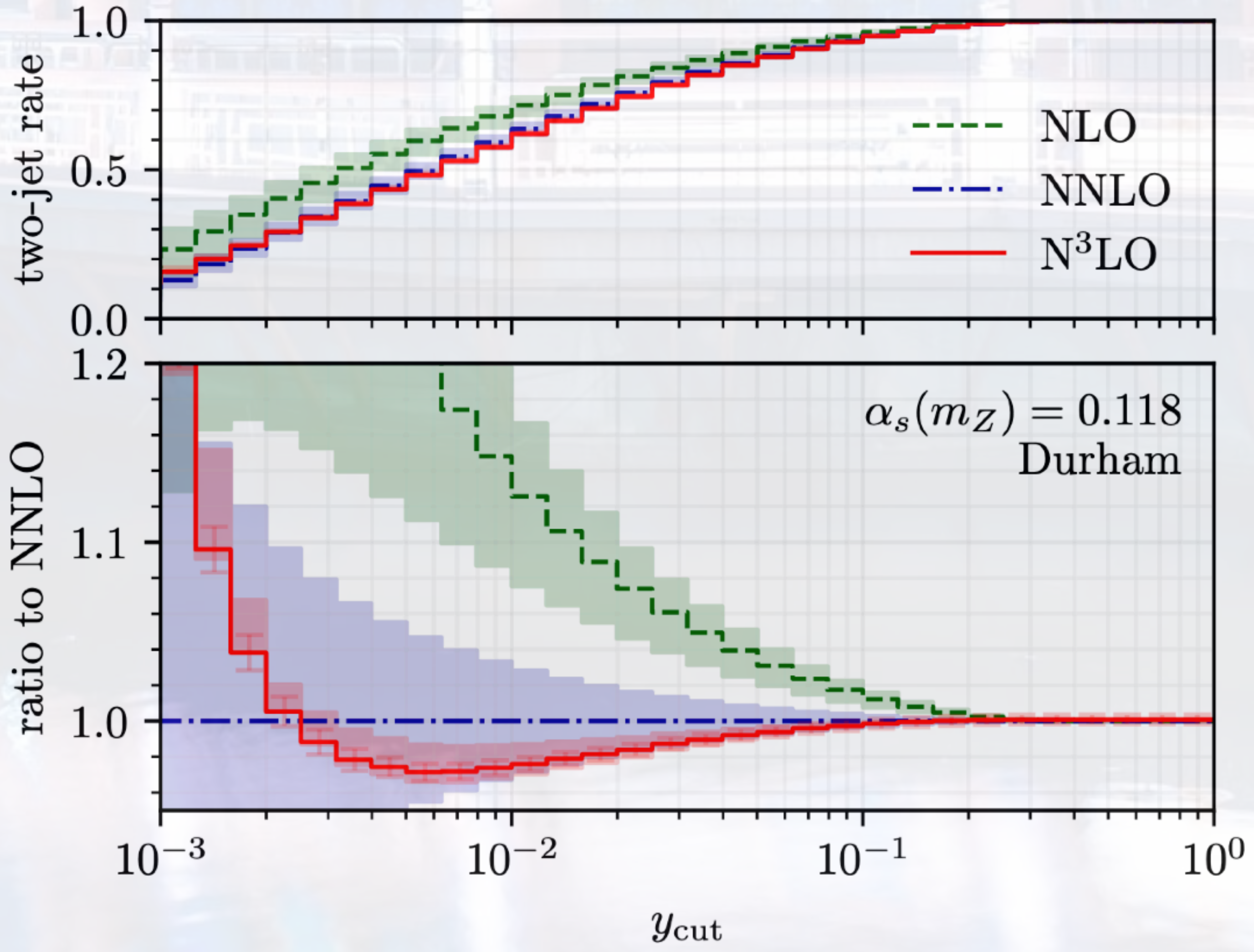
$$R_4^{(3)}(y_{cut})\Gamma^{(3)}_{\gamma^*\to hadrons} = \int_{y_{cut}}^1 \frac{d\sigma}{dy_{34}}dy_{34} - \int_{y_{cut}}^1 \frac{d\sigma}{dy_{45}}dy_{45}$$

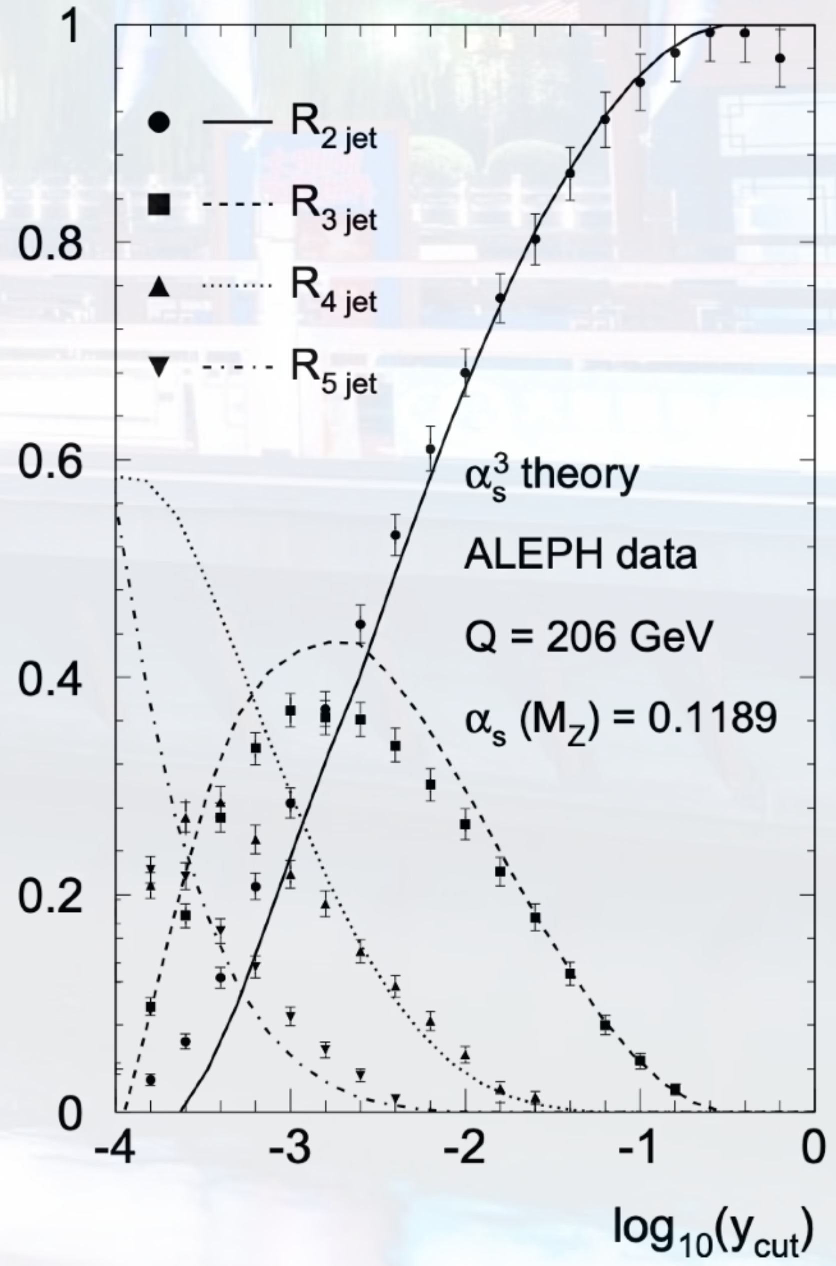
$$R_5^{(3)}(y_{cut})\Gamma^{(3)}_{\gamma^*\to hadrons} = \int_{y_{cut}}^1 \frac{d\sigma}{dy_{45}}dy_{45}$$

$$\Rightarrow \sum_{n=2}^{m+2} R_n^{(m)}(y_{cut}) = 1$$

Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, Phys. Rev. Lett. *100 (2008) 172001*

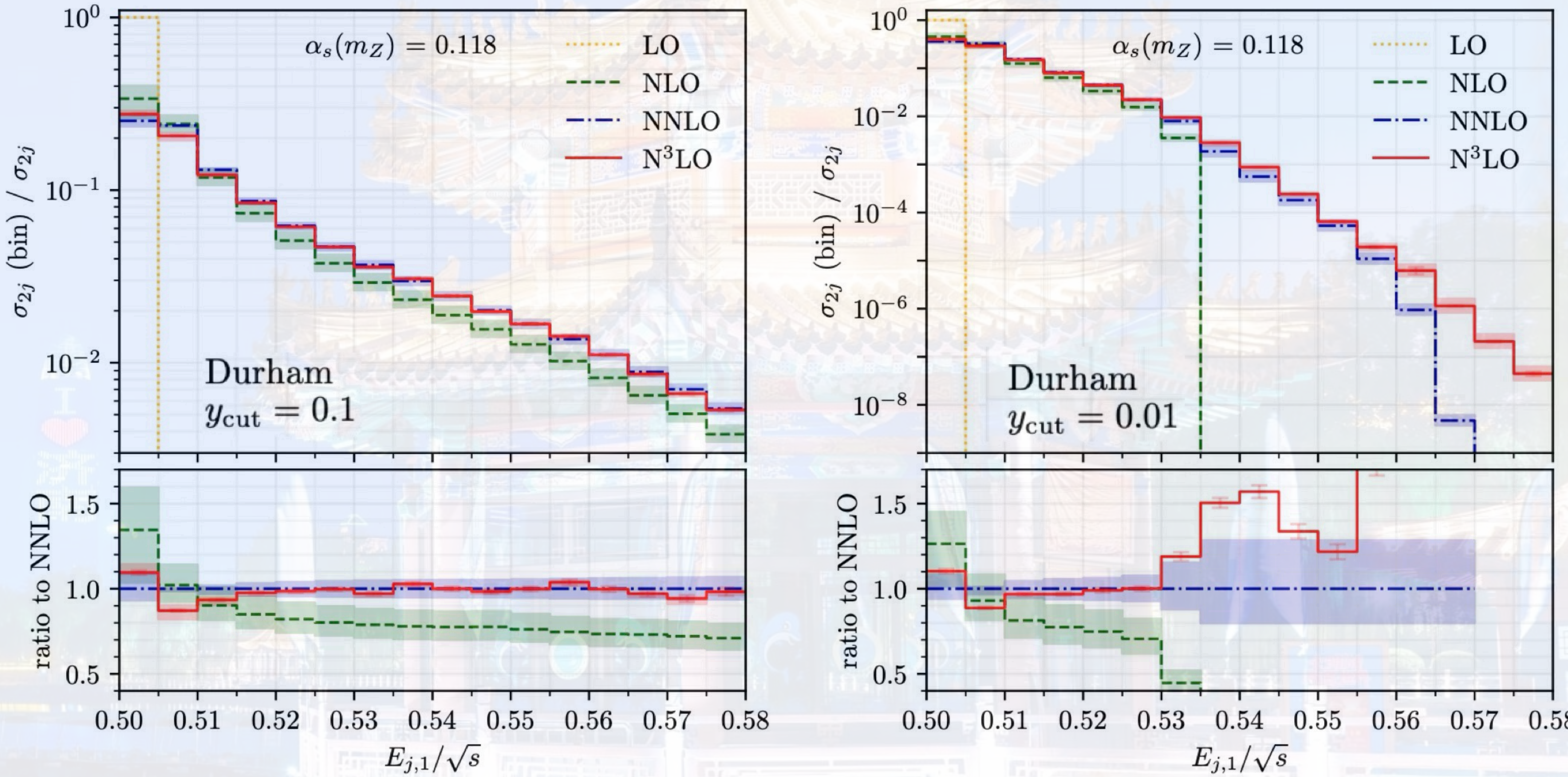
Full agreement between direct and indirect calculation of $R_2^{(3)}(y_{cut})$



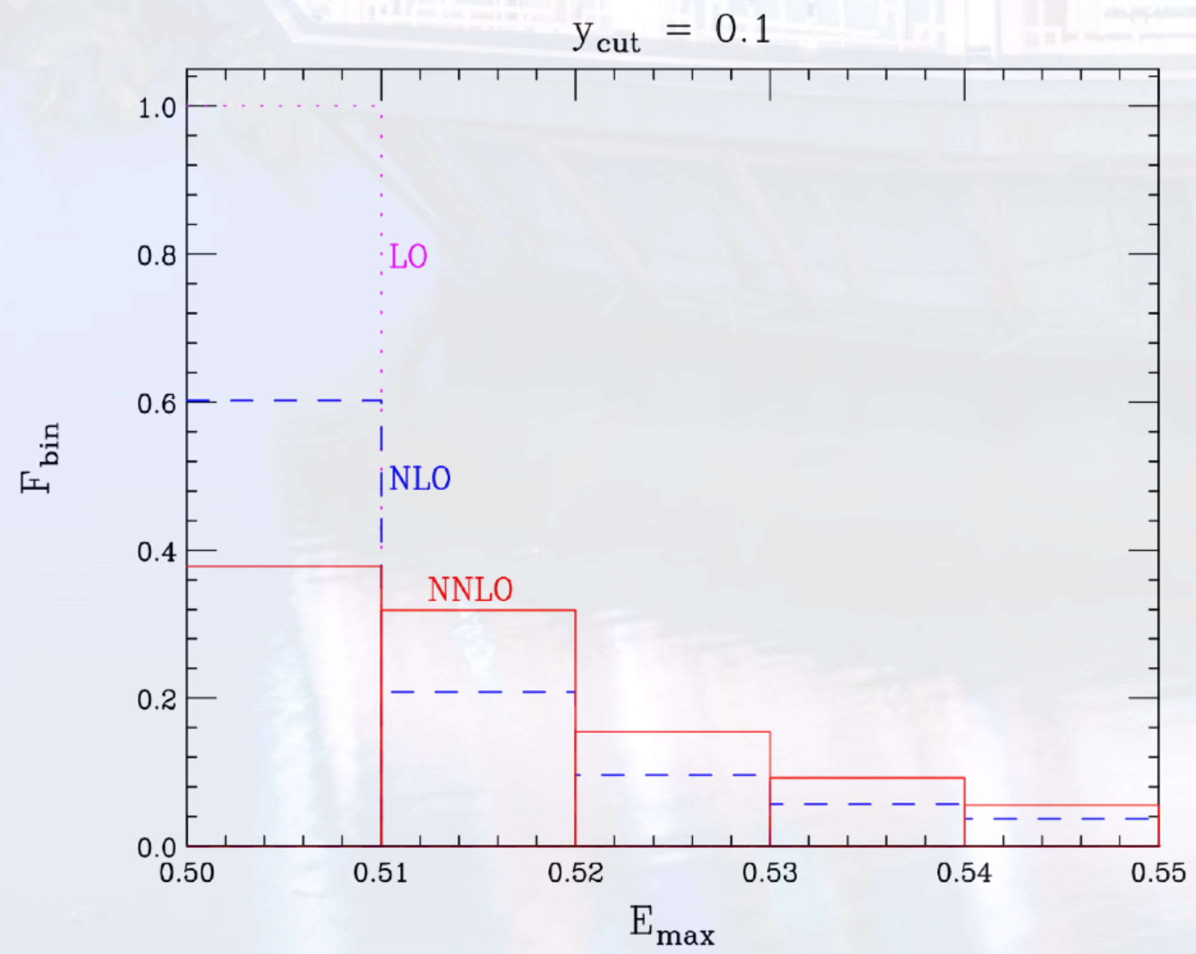



# $e^+e^- \to$ JJ @ N3LO

- Differential observable:
  - Leading-jet energy $E_{j1}$
    - Defined on 2-jet events, bin-integrated and normalized to exclusive 2-jet XS
    - Lower orders vanish faster at high $E_{j1}$ for smaller $y_{cut}$ due to energetic leading jet recoil against multiple emissions
    - Very good convergence for large $y_{cut}$
    - The distribution can be obtained by combining $e^+e^- \to$ JJ @ NNLO and N3LO inclusive XS
- Future plan:
  - Include in public release
  - Jet forward-backward asymmetry
  - $e^+e^- \to$ JJJ @ N3LO




XC, Jakubcik, Marcoli, Stagnitto, Phys. Lett. B. *869 (2025) 139804*




Full agreement up to NNLO with

Anastasiou, Melnikov, Petriello, Phys. Rev. Lett. *93 (2004) 032002*

# Application of NNLOJET in $e^+e^-$ Colliders
## (Close Relation to Antenna Subtraction)

$e^+e^- \to$ di-jet @ N3LO (Antenna Subtraction) 2505.10618

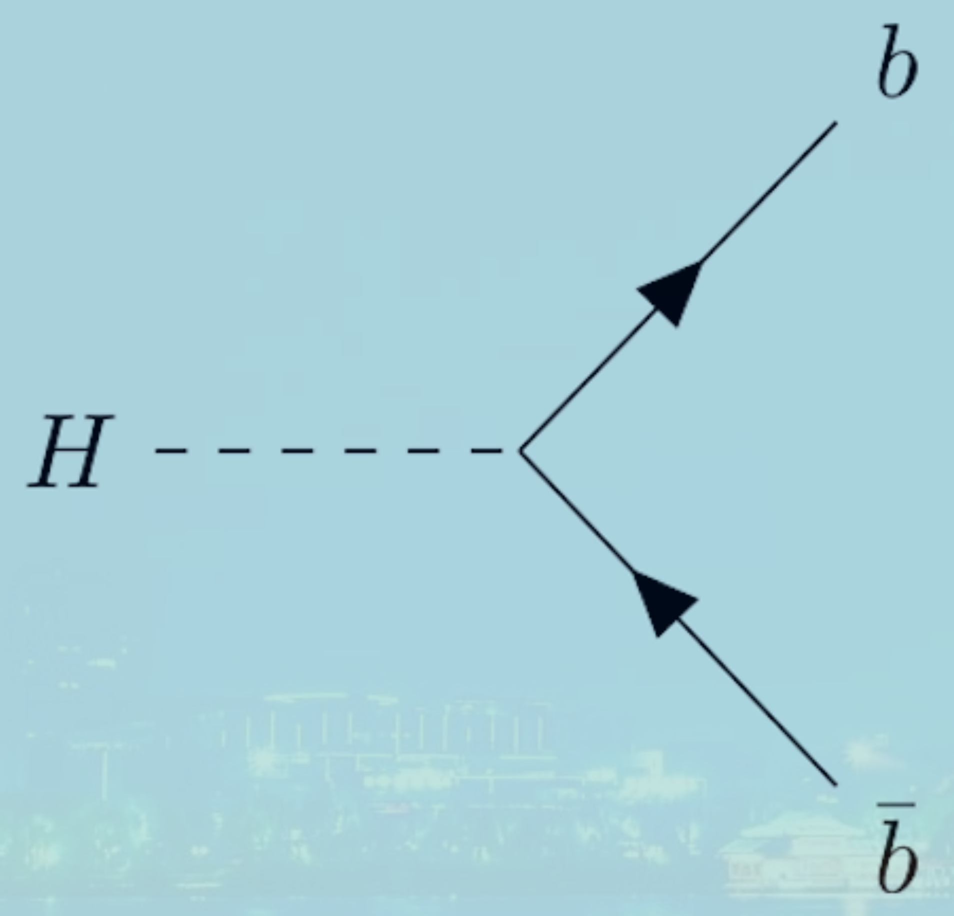
H decay $\to$ di-jet @ N3LO (Generalized Antenna) 2502.17333, 2508.14282

# Higgs → JJ @ N3LO

Marcoli's slide @ Loop Summit 2

We consider a Higgs boson at rest (neglect production mode) decaying hadronically.

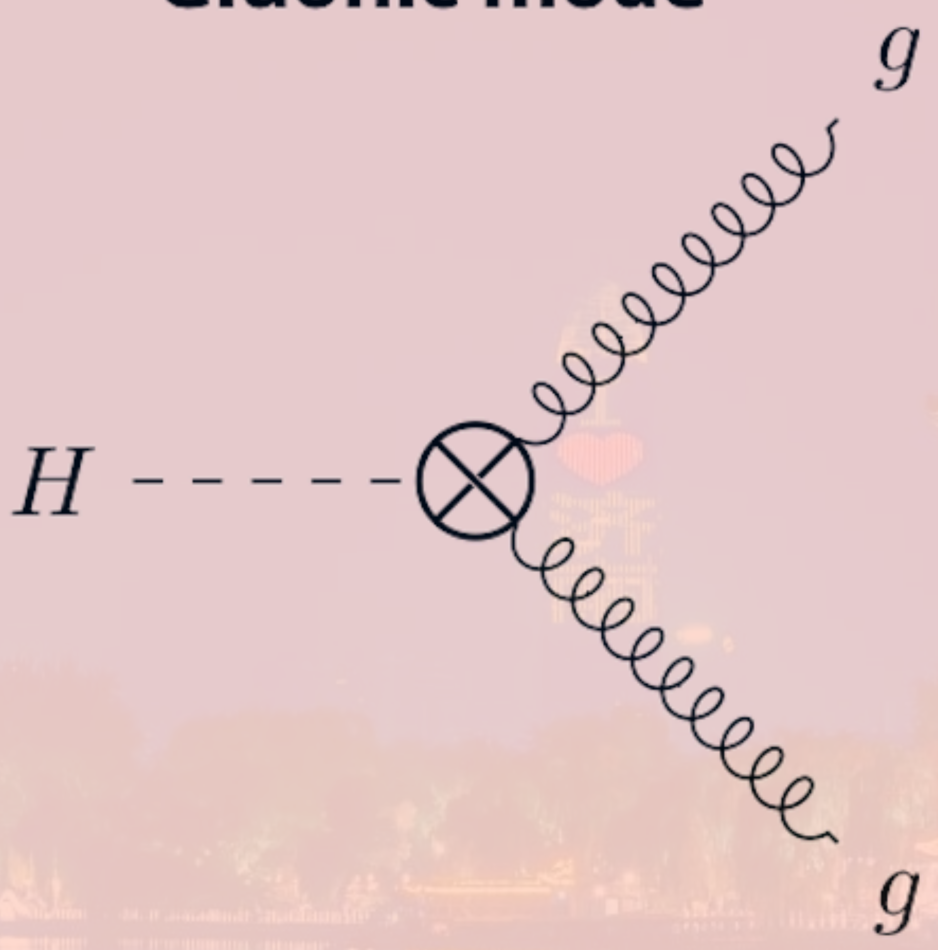
**Yuakawa mode**




$$\Gamma^{(0)}_{H\to b\bar{b}} = \frac{m_b^2(\mu_R) m_H N_c}{8\pi v^2}$$

- massless b (apart from Yuakwa interaction) ***
- analogous contribution from charm

**Gluonic mode**




$$\Gamma^{(0)}_{H\to gg} = \frac{\alpha_s^2(\mu_R) m_H^3 (N_c^2-1)}{576\pi^3 v^2}$$

- effective vertex: infinite top mass limit
- finite t, b and c mass and EW vertex corrections included by rescaling

Inclusive decay widths at order k in QCD:

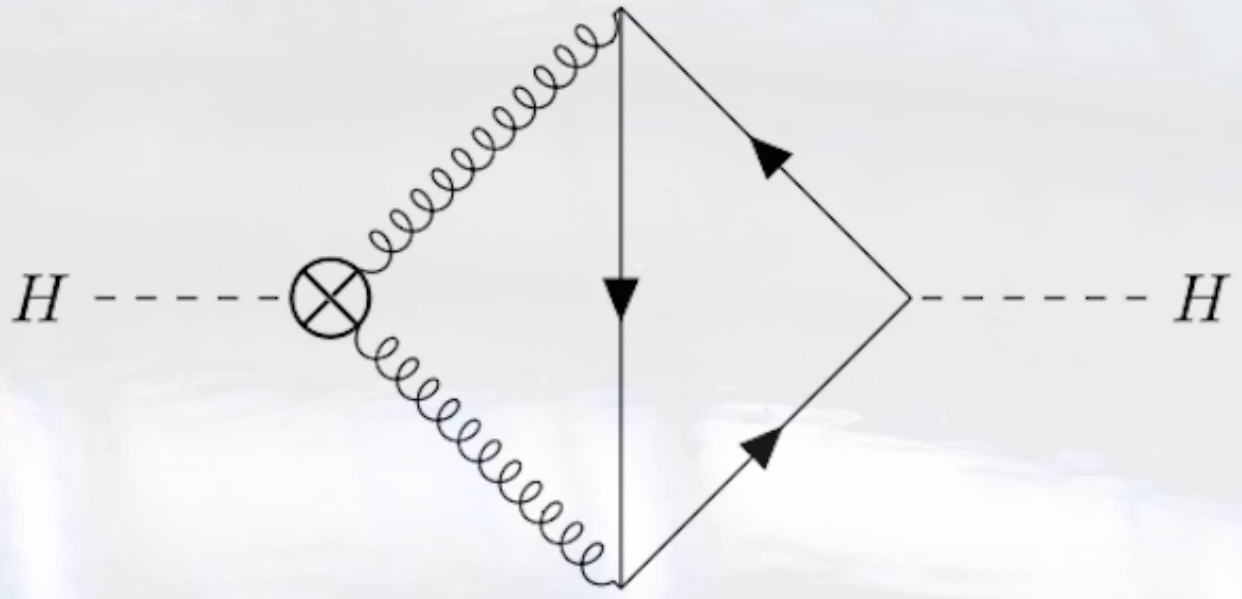
$$\Gamma^{(k)}_{H\to b\bar{b}} = \Gamma^{(0)}_{H\to b\bar{b}} \left(1 + \sum_{n=1}^{k} \alpha_s^n(\mu_R) C^{(n)}_{b\bar{b}}\right)$$

$$\Gamma^{(k)}_{H\to gg} = \Gamma^{(0)}_{H\to gg} \left(1 + \sum_{n=1}^{k} \alpha_s^n(\mu_R) C^{(n)}_{gg}\right)$$

Expansion coefficients know up to k=4

*Herzog, Ruijl, Ueda, Vermaseren, Vogt JHEP 08 (2017) 113*

*** The interference between the two modes vanishes. We verified that it is anyway negligible for the observables we consider.



= 0

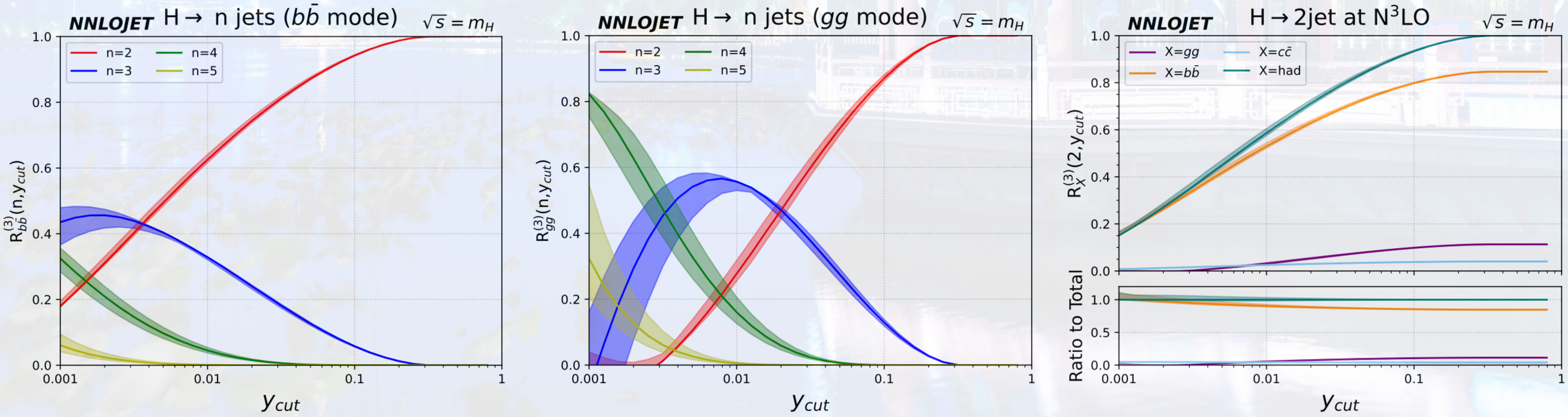
# Higgs → JJ @ N3LO

- ➤ We compute the decay of a Higgs boson into three jets up to NNLO in QCD
- ➤ From this we can extract the 3-jet rate at NNLO and 2-jet rate at N3LO
- ➤ Previous calculation in the Yukawa mode. Novel results in the gluonic mode.

*Mondini, Williams, JHEP 06 (2019) 120*
*Mondini, Schiavi, Williams, JHEP 06 (2019) 079*

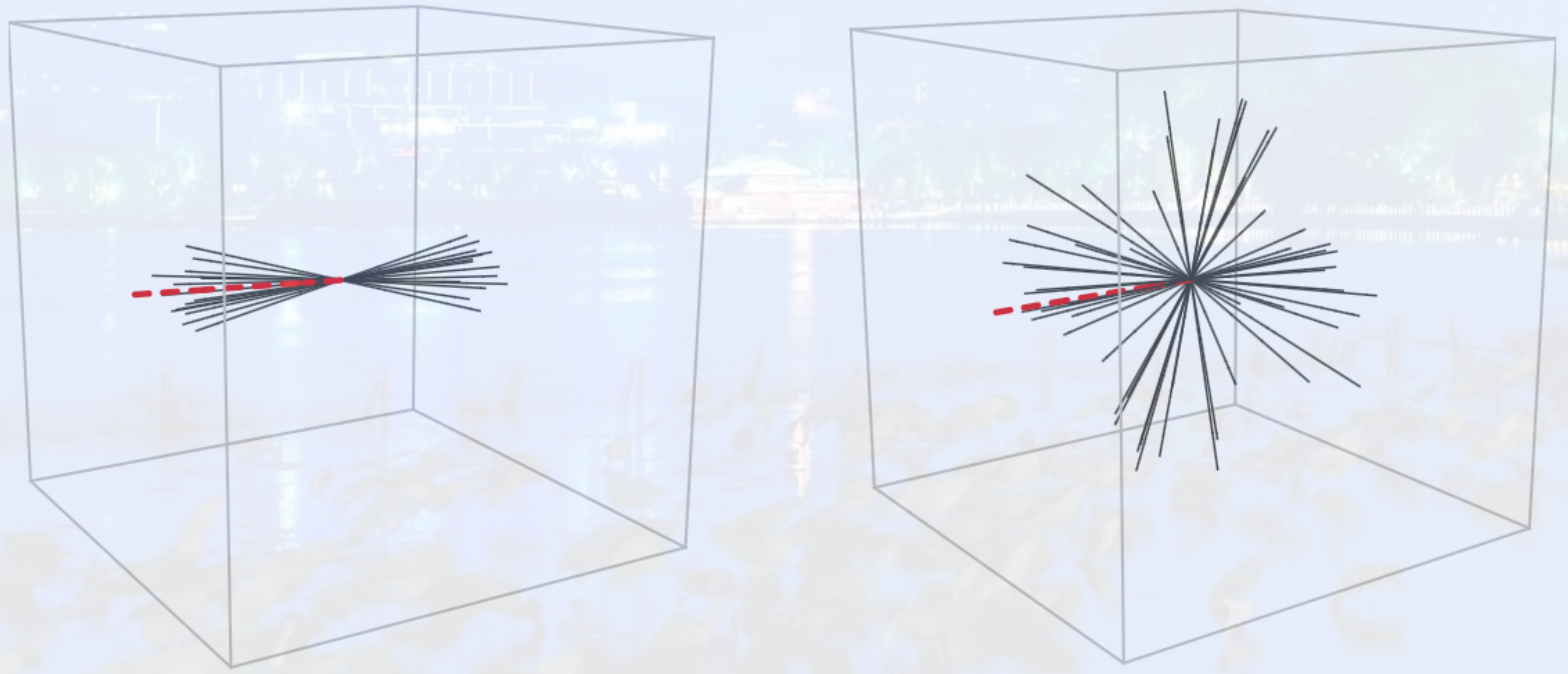
*Fox, Gehrmann-De Ridder, Gehrmann, Glover, Marcoli, Preuss*
*Phys.Rev.Lett. 134 (2025)251905*

Physical parameters: $y_b(m_H) = m_b(m_H)/vev = 0.011309$ $y_c(m_H) = m_c(m_H)/vev = 0.0024629$
$m_H = 125.09$ GeV $m_Z = 91.2$ GeV $vev = 246.22$ GeV $\alpha_s(m_z) = 0.118$ $m_t(m_H) = 166.48$ GeV



# Higgs → JJ @ N3LO

- Thrust distribution from Higgs decay
  - Fraction of gg mode enhanced in the high multiplicity hard region
  - Perturbative predictions for the gluonic mode breaks down earlier than for the Yukawa one in pencil-like regions
  - All-order resummation effects are important in the back-to-back region ($\tau \to 0$)



$$T = \max_{\vec{n}} \left( \frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i \vec{p}_i} \right)$$

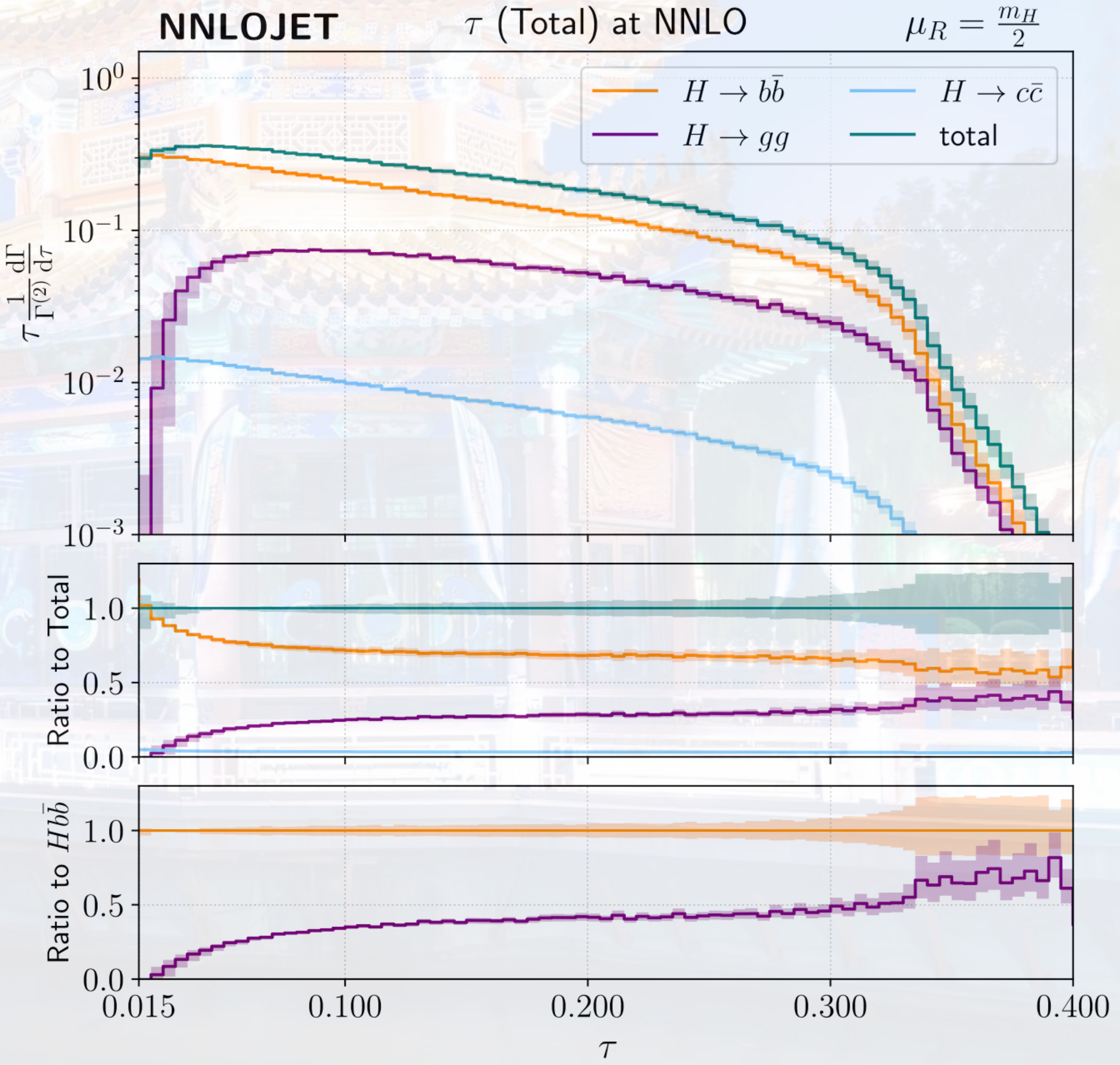
$$\tau \equiv 1 - T = \min_{\vec{n}} \left( 1 - \frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i \vec{p}_i} \right)$$

$T = 0.998\,, \quad \tau = 0.002$

pencil-like
back to back

$T = 0.65\,, \quad \tau = 0.35$

spherical
isotropic




pencil-like
back to back

spherical
isotropic

*Fox, Gehrmann-De Ridder, Gehrmann, Glover, Marcoli, Preuss [2508.14282]*

# FUTURE PROSPECTS

- Precision is not the ultimate goal → identify anomaly then understand
- The most famous **failed experiment**: Michelson–Morley in 1887, foundation of special relativity. → 1907 Nobel Prize to Albert A. Michelson . 
- "… it seems probable that **most of the grand underlying principles have been firmly established** and that further advances are to be sought chiefly in the rigorous application of these principles to all the phenomena which come under our notice. … An eminent physicist remarked that **the future truths of physical science are to be looked for in the sixth place of decimals**." —— Albert A. Michelson, 1894 University of Chicago
- NNLO QCD precision maybe **solved**, but still not easily accessible. Several ongoing efforts towards aotumation and generalization. N3LO is very challenging, but first steps have been made in this direction.
- **Generalized antenna functions** yield a simpler and more efficient formulation of final-state IR subtraction.
- First application of antenna subtraction to a **fully-differential N3LO calculation**. Gradual extension to more complicated processes is desired in the future.


Thank You for Your Attention

# BACK UP SLIDES

# STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^{3(4)}LL}$

| FO | $\alpha_s^n$ | $H(m_V,\mu)$ | $I^{(n)}_{i/j}(x,b)$ | $\ln W(x_a, x_b, m_V, \vec{b}, \mu = b_0/b) \sim \int_{\mu_h}^{\mu} d\bar{\mu}/\bar{\mu}\left(A(\alpha_s(\bar{\mu}))\ln\frac{m_V^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu}))\right)$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{d\hat{\sigma}^V_{NLO}}{dq_T}$ | NLO | ✓ | ✓ | $\ln^2(b^2m_V^2)$ | $\ln(b^2m_V^2)$ | 1 | | | | |
| $\frac{d\hat{\sigma}^V_{NNLO}}{dq_T}$ | N2LO | ✓ | ✓ | $\ln^3(b^2m_V^2)$ | $\ln^2(b^2m_V^2)$ | $\ln(b^2m_V^2)$ | 1 | | | |
| $\frac{d\hat{\sigma}^V_{N^3LO}}{dq_T}$ | N3LO | ✓ | ✓ | $\ln^4(b^2m_V^2)$ | $\ln^3(b^2m_V^2)$ | $\ln^2(b^2m_V^2)$ | $\ln(b^2m_V^2)$ | 1 | | |
| $\frac{d\hat{\sigma}^V_{N^4LO}}{dq_T}$ | N4LO | ✓ | ✗ | $\ln^5(b^2m_V^2)$ | $\ln^4(b^2m_V^2)$ | $\ln^3(b^2m_V^2)$ | $\ln^2(b^2m_V^2)$ | $\ln(b^2m_V^2)$ | 1 | |
| … | … | | | … | … | … | … | | … | … |
| $\frac{d\hat{\sigma}^V_{N^kLO}}{dq_T}$ | NKLO | | | $\ln^{k+1}(b^2m_V^2)$ | $\ln^{k}(b^2m_V^2)$ | $\ln^{k-1}(b^2m_V^2)$ | $\ln^{k-2}(b^2m_V^2)$ | $\ln^{k-3}(b^2m_V^2)$ | … | … |
| … | … | | | … | … | … | … | | … | … |
| **Resum** | | | | LL | NLL | NNLL | N3LL | N4LL | … | $N^{k+1}LL$ |
| A | | | | A1 ✓ | A2 ✓ | A3 ✓ | A4 ✓ | A5 ✗ | … | $A_{k+2}$ |
| B | | | | | B1 ✓ | B2 ✓ | B3 ✓ | B4 ✓ | … | $B_{k+1}$ |

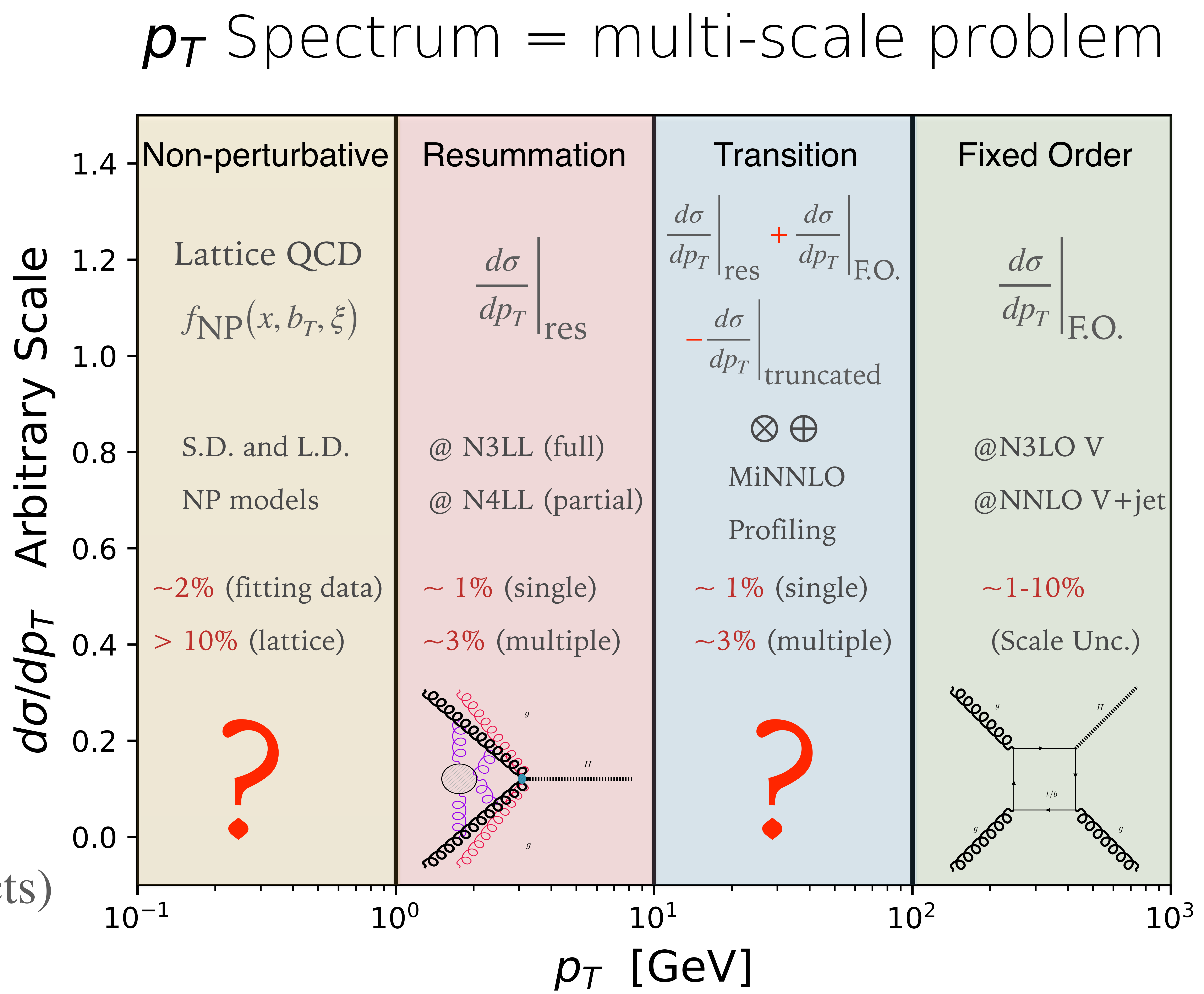
# PREDICTIONS OF COLOURLESS PT AT HADRON COLLIDER

- ➤Beyond QCD improved parton model
  - ➤pQCD describes the tail of spectrum
  - ➤Large logarithmic divergence

$$\ln\frac{p_T}{Q} \text{ as } p_T \to 1 \text{ GeV}$$

  - ➤Various LP resummation schemes
  - ➤Multiple solutions in transition region
  - ➤Non-perturbative effects ~ 1 GeV

    (Short distance and long distance effects)



# PERTURBATIVE QFT FOR PRECISION PREDICTIONS

- State-of-the-art differential N3LO predictions ($2 \to 1$)
  - Fully differential N3LO Drell-Yan production (via $\gamma^*$) (XC, T. Gehrmann, N. Glover, A. Huss, T.-Z. Yang, H. X. Zhu 2021)
  - Apply qt-slicing at N3LO with SCET factorisation and expand to N3LO:

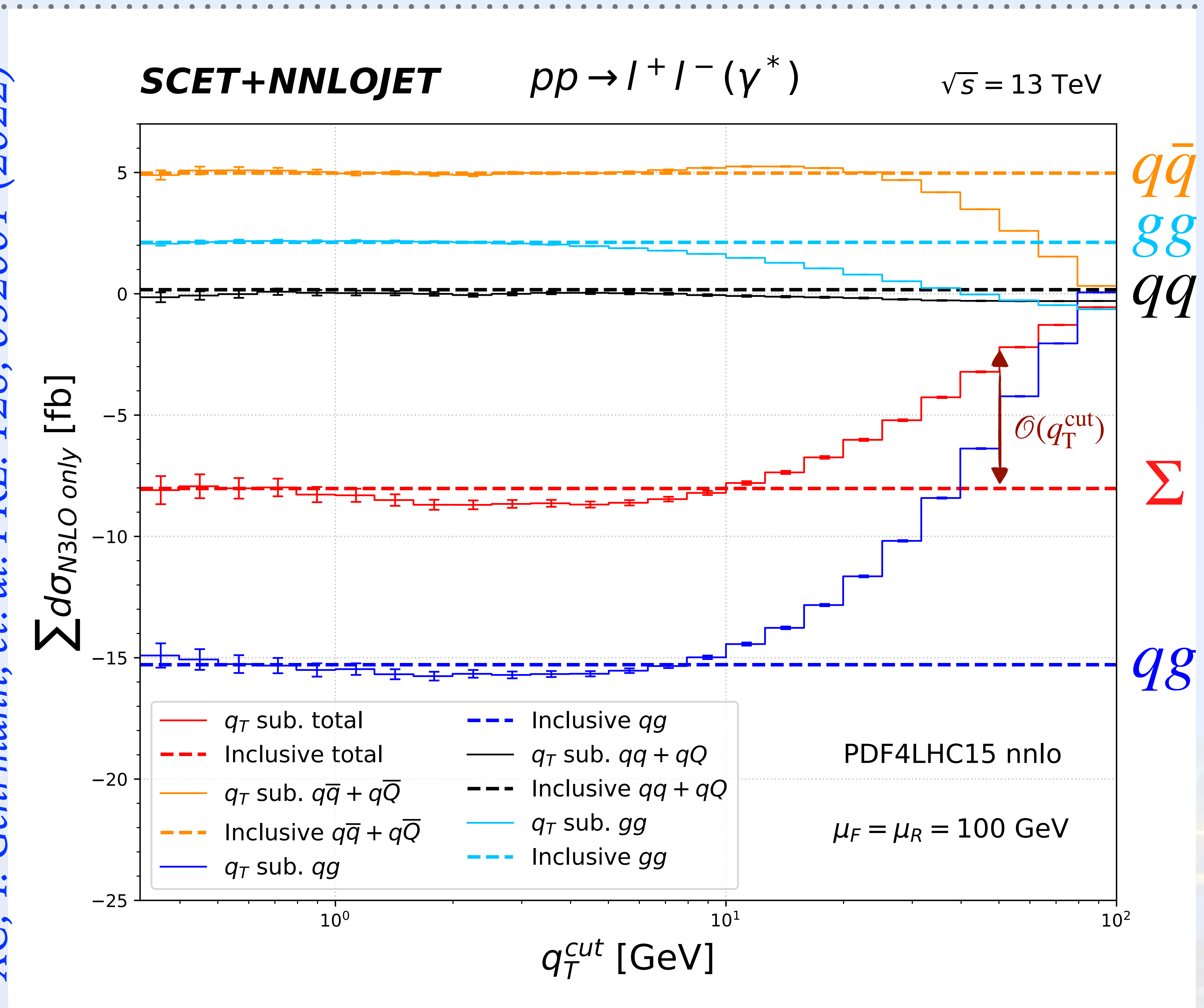
$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} \sum_q \sigma_{\mathrm{LO}}^{\gamma^*} H_{q\bar{q}} \Big[ \sum_k \int_{x_1}^1 \frac{dz_1}{z_1} \mathcal{I}_{qk}\left(z_1, b_T^2, \mu\right) f_{k/h_1}(x_1/z_1, \mu) \times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} \mathcal{I}_{\bar{q}j}\left(z_2, b_T^2, \mu\right) f_{j/h_2}(x_2/z_2, \mu) \mathcal{S}\left(b_\perp, \mu\right) + (q \leftrightarrow \bar{q}) \Big] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

  - All factorised functions are recently known up to N3LO:
    1) 3-loop hard function $H_{q\bar{q}}^{(3)}$ (T. Gehrmann, E.W.N. Glover, T. Huber, N. Ikizlerli, C. Studerus 2010)
    2) Transverse-momentum-dependent (TMD) soft function $S(b_\perp, \mu)$ at $\alpha_s^3$ (Y. Li, H.X. Zhu 2016)
    3) Matching kernel of TMD beam function $I_{qk}$ at $\alpha_s^3$ (M.-X. Luo, T.-Z. Yang, H. X. Zhu, Y. J. Zhu 2019, M. A. Ebert, B. Mistlberger, G. Vita 2020)
  - Apply qt cut to factorise N3LO contribution into two parts:

$$d\sigma_{N^3LO}^{\gamma^*} = [\mathscr{H}^{\gamma^*} \otimes d\sigma^{\gamma^*}]_{N^3LO}\Big|_{\delta(p_{T,\gamma^*})} + \left[d\sigma_{NNLO}^{\gamma^*+jet} - d\sigma_{N^3LO}^{\gamma^*\ CT}\right]_{p_{T,\gamma^*} > qt_{cut}} + \mathcal{O}(qt_{cut}^2/Q^2)$$

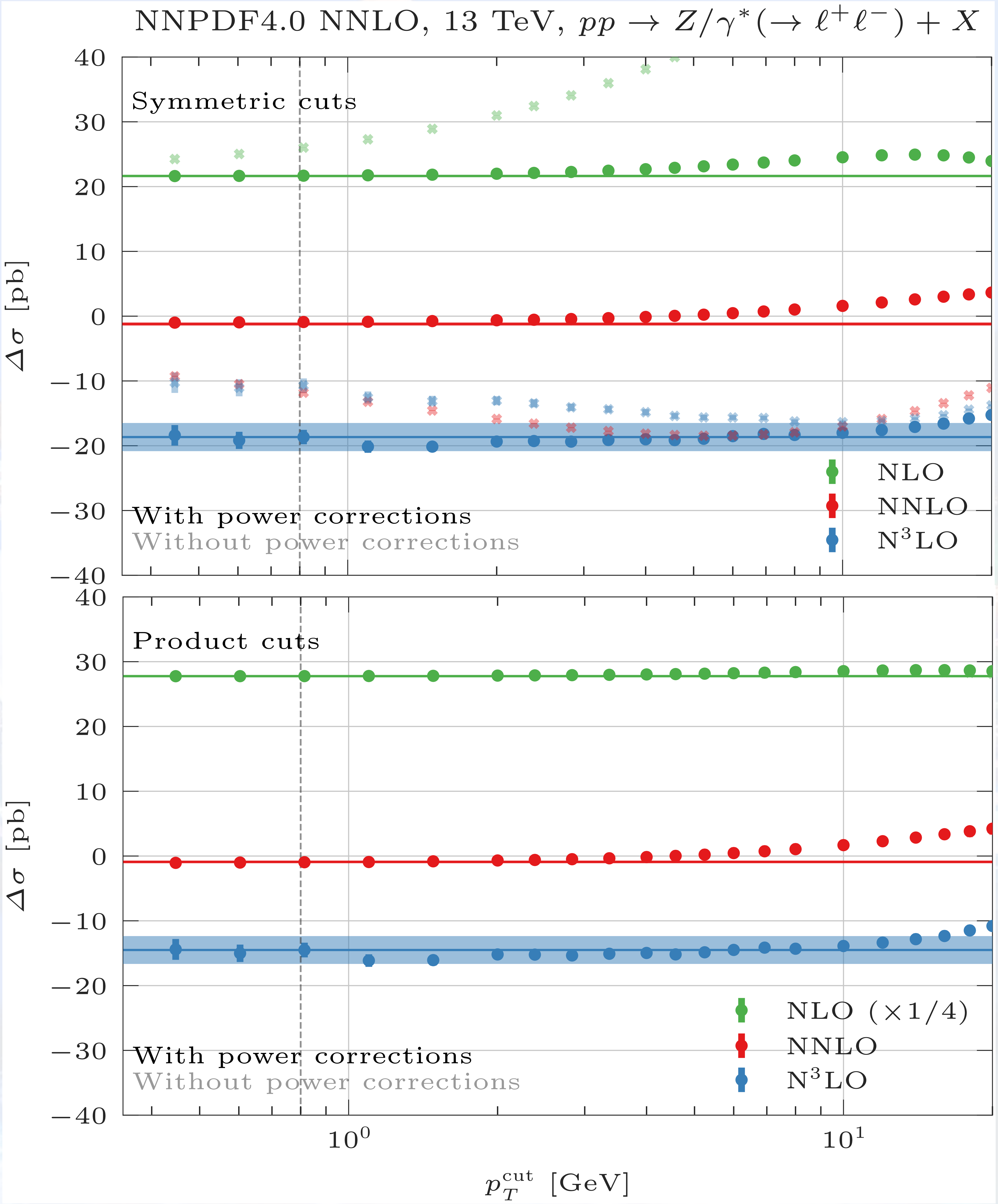
# $pp \to \gamma^*/Z$ @ N³LO

XC, T. Gehrmann, et. al. PRL. 128, 052001 (2022)




| Fixed order | $\sigma_{pp\to\gamma^*}$ (fb) | | |
|---|---|---|---|
| LO | $339.62^{+34.06}_{-37.48}$ | | |
| NLO | $391.25^{+10.84}_{-16.62}$ | | |
| NNLO | $390.09^{+3.06}_{-4.11}$ | | |
| N³LO | $382.08^{+2.64}_{-3.09}$ [14] | | |
| N³LO only | $q_T^{cut} = 0.63$ GeV | $q_T^{cut} \to 0$ fit | [14] |
| $qg$ | $-15.32(32)$ | $-15.34(54)$ | $-15.29$ |
| $q\bar{q}+q\bar{Q}$ | $+5.06(12)$ | $+5.05(12)$ | $+4.97$ |
| $gg$ | $+2.17(6)$ | $+2.19(6)$ | $+2.12$ |
| $qq+qQ$ | $+0.09(13)$ | $+0.09(17)$ | $+0.17$ |
| Total | $-7.98(36)$ | $-8.01(58)$ | $-8.03$ |

C. Duhr, F. Dulat, B. Mistlberger. PRL. **125**, 172001 (2020)




XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)

# STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

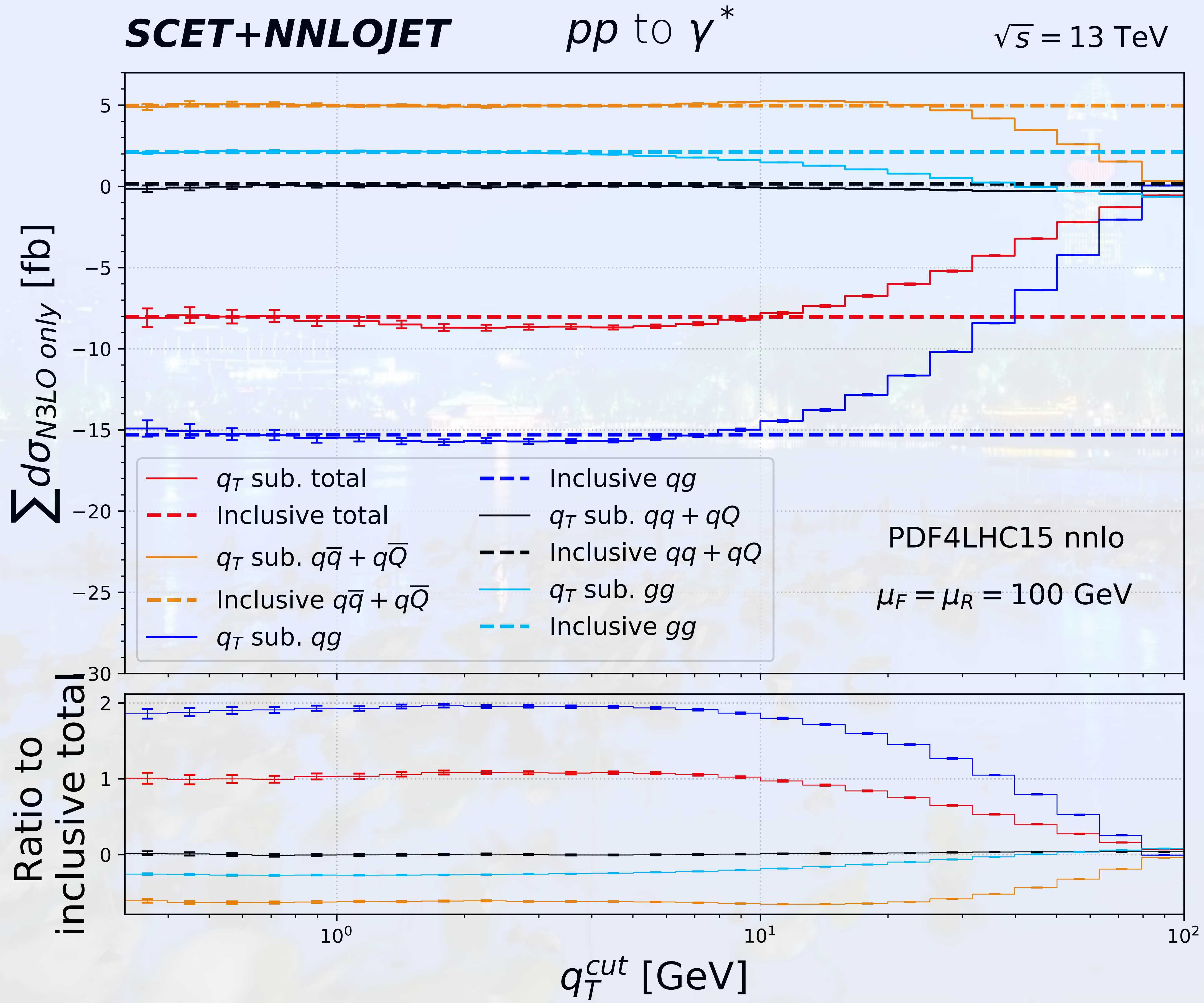
➤qT slicing at N3LO for neutral and charged current production (NNLOJET)

$$\sum d\sigma^{V}_{N3LO} \equiv \sum_{dp_{T,V}} d\sigma^{V+jet}_{NNLO}/dp_{T,V}|_{p_{T,V}>q_T^{cut}} + \sum_{dp_{T,V}} d\sigma^{V\,SCET}_{N^3LO}/dp_{T,V}|_{p_{T,V}\in[0,q_T^{cut}]}$$

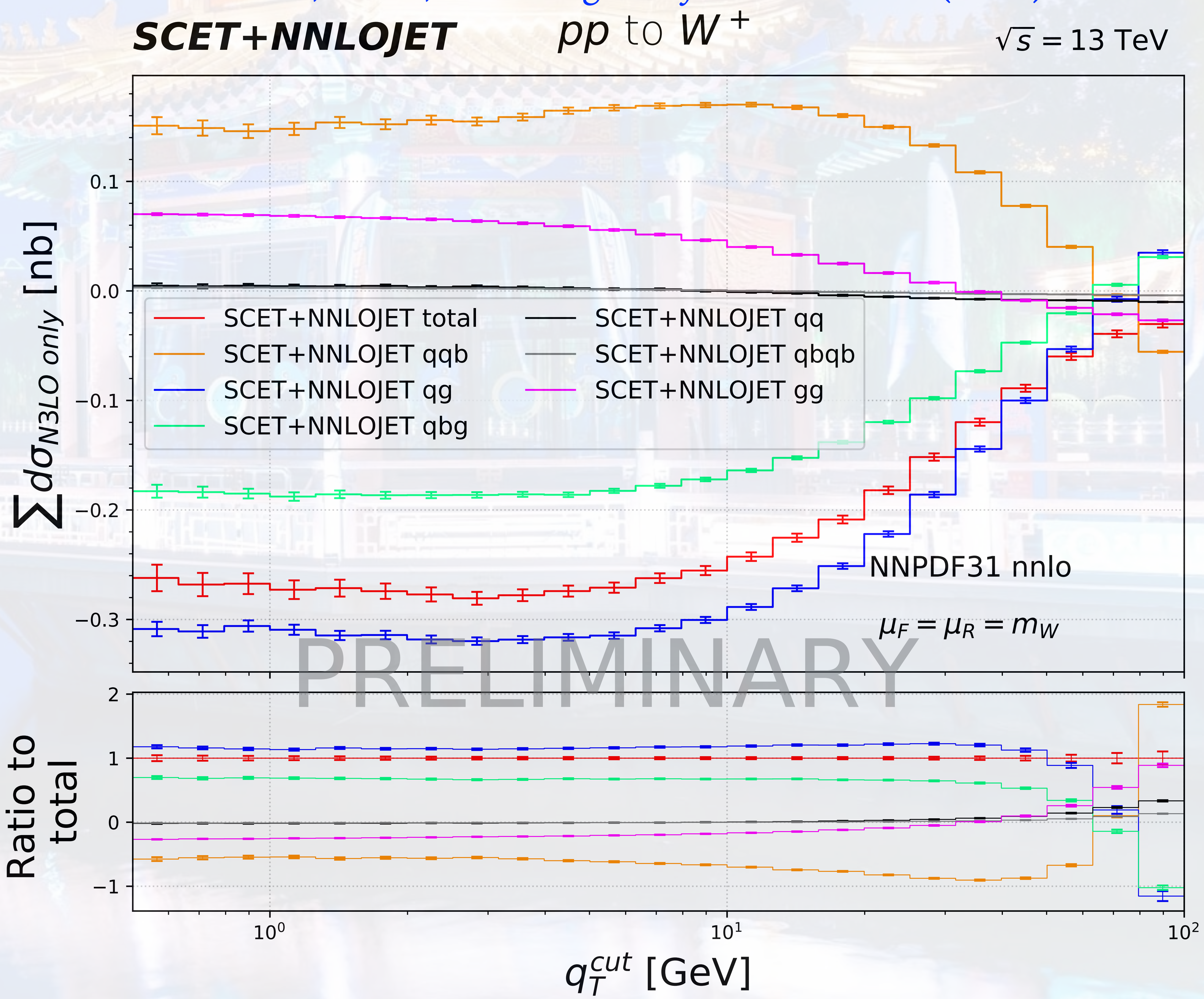
NC and CC Validated against inclusive XS within ± 5% uncertainty

$$\Delta\sigma^{\gamma^*}_{N^3LO} = -7.98 \pm 0.36\, fb \quad vs. \quad -8.03\, fb$$

Duhr, Dulat, Mistlberger *Phys.Rev.Lett.* 125 (2020)




XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Rev.Lett.* 128 (2022) 5




XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Lett.B 840 (2023)*

# STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

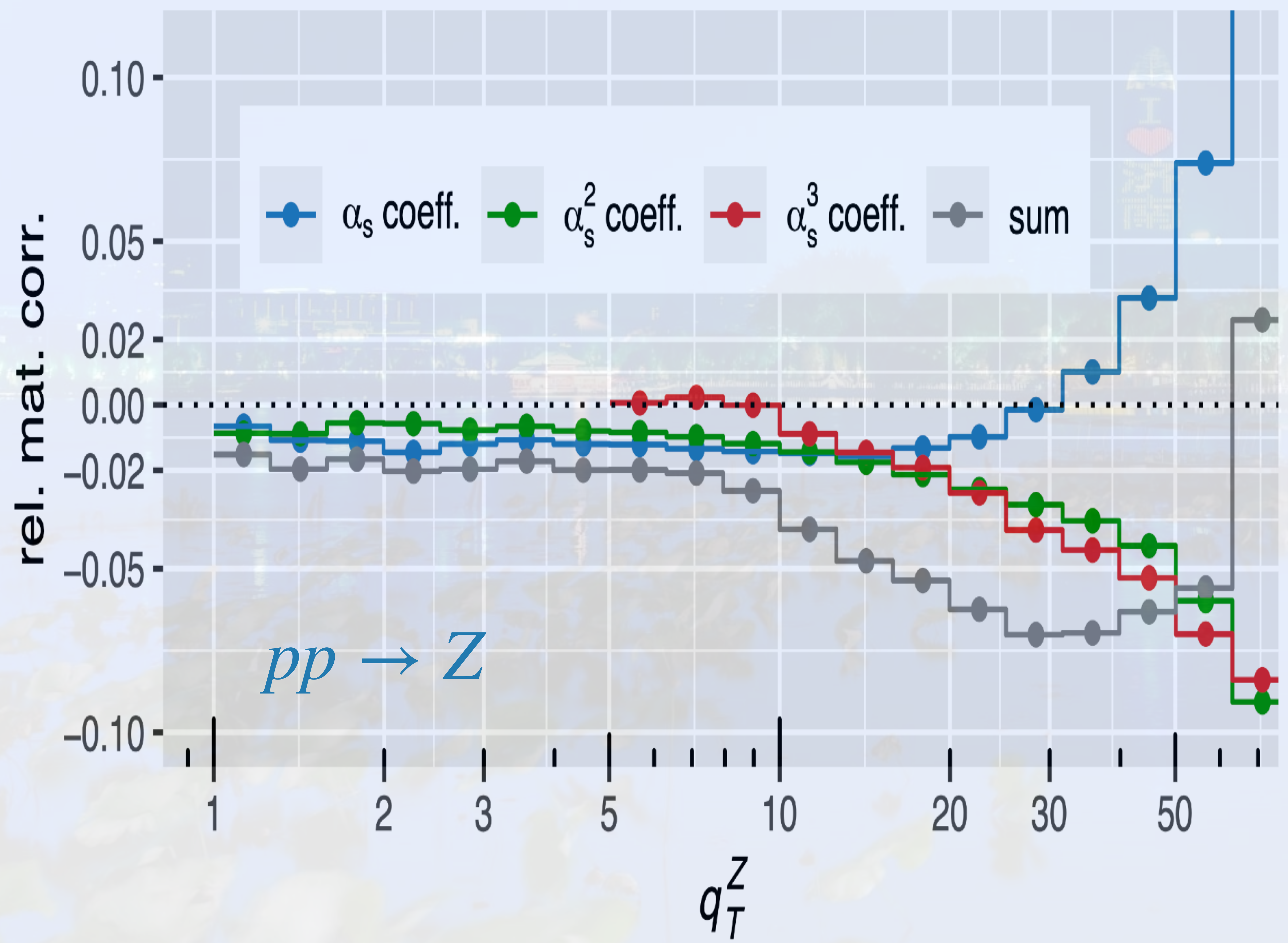
➤qT slicing at N3LO for neutral and charged current production (MCFM)

$$\sum_{\mathrm{d}p_{T,V}} \mathrm{d}\sigma^{V}_{N3LO} \equiv \sum_{\mathrm{d}p_{T,V}} \mathrm{d}\sigma^{V+jet}_{NNLO}/\mathrm{d}p_{T,V}|_{p_{T,V}>q_T^{cut}} + \sum_{\mathrm{d}p_{T,V}} \mathrm{d}\sigma^{V\,SCET}_{N^3LO}/\mathrm{d}p_{T,V}|_{p_{T,V}\in[0,q_T^{cut}]}$$

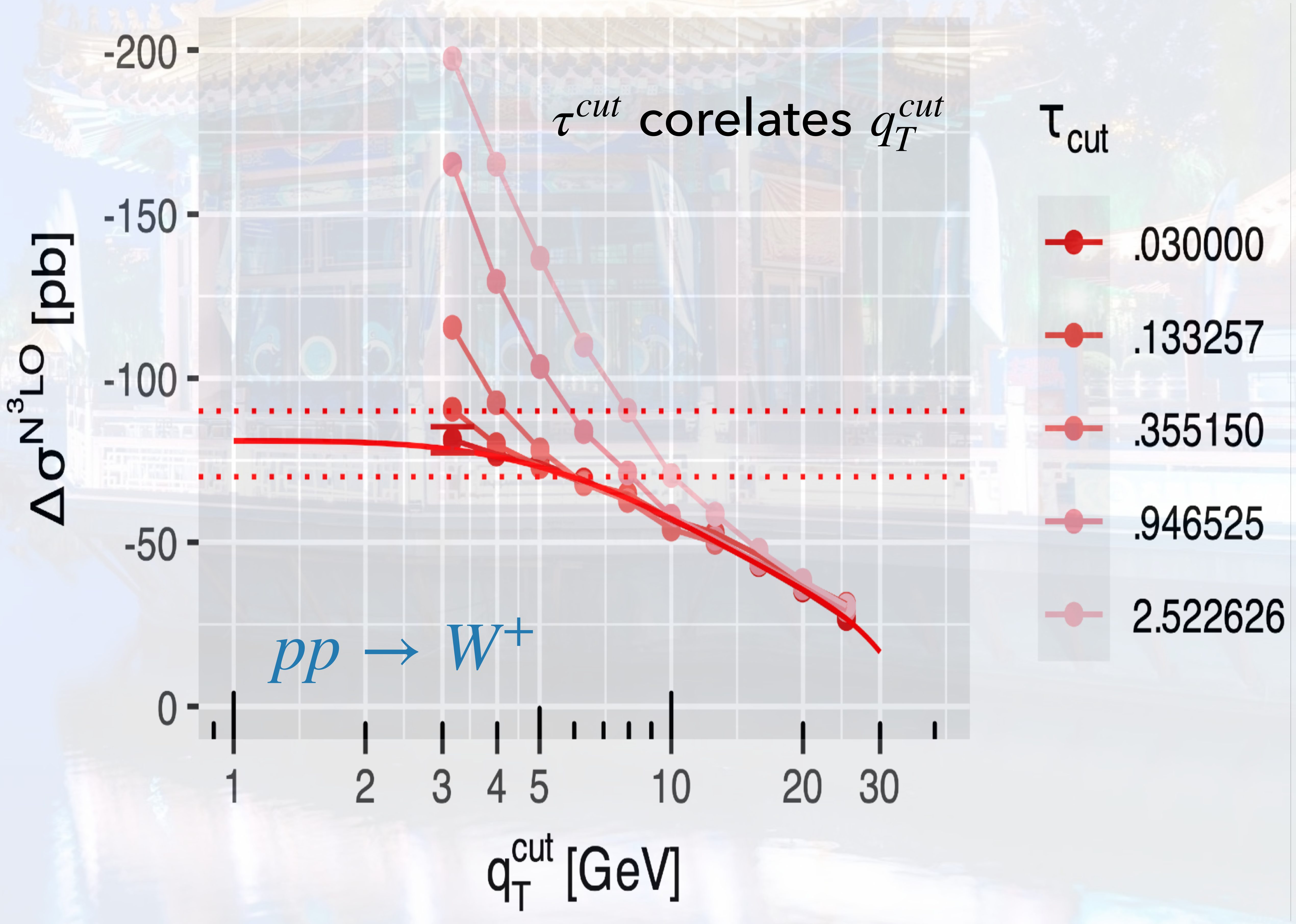
NC MCFM: $-22.6\,\mathrm{pb} \pm 1.4\,\mathrm{pb}\,(\mathrm{num.}) \pm 1\,\mathrm{pb}\,(\mathrm{slicing})$

NC NNLOJET: $-18.7\,\mathrm{pb} \pm 1.1\,\mathrm{pb}\,(\mathrm{num.}) \pm 0.9\,\mathrm{pb}\,(\mathrm{slicing})$

CC agree to inclusive XS within ± 60% uncertainty of $\Delta(\alpha_s^3)$



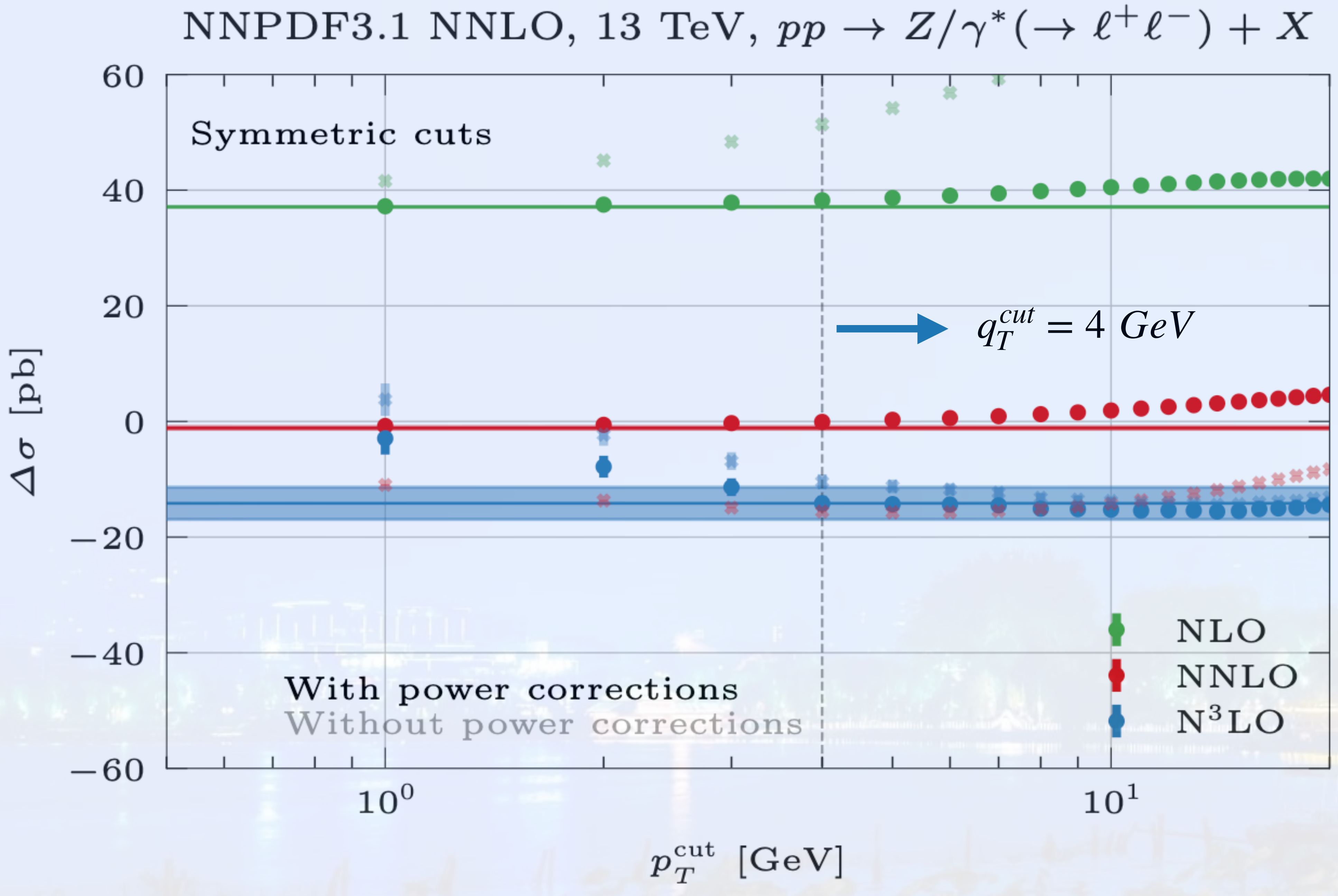

Neumann and Campbell *Phys.Rev.D* 107 (2023) 1




Neumann and Campbell *JHEP 11 (2023) 127*

# Precision Predictions at Hadron Collider

## $2 \to 1$ @ N3LO (+ N3LL) QCD




*XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)*

- Solid horizontal lines: NLO, NNLO at 1 GeV, N3LO at 4 GeV with MC error.
  - N3LO shows no plateau in 1905.05171
- Pale dots are values used by DYTurbo in 2103.04974 and 2303.12781 (taken from 1905.05171).
  - Fiducial power corrections are not included.
  - Leads to 30% difference of N3LO coefficients at $q_T^{cut} = 4\ GeV$.
- Solid dots are corrected values with fiducial power correction.
  - Central value shifts 2 pb starting from NLO (the dominant error).
  - $\pm 2.1$ pb uncertainty from MC and $q_T^{cut}$ (estimated from [3,5] GeV region).
  - Not consistent with DYTurbo update result of $\pm 0.7$ pb uncertainty.

DYTurbo result with fiducial power correction

| Order | N$^3$LO |
|---|---|
| $q_T$ subtr. ($q_T^{\rm cut} = 4\,{\rm GeV}$) | $747.1 \pm 0.7\,{\rm pb}$ |
| recoil $q_T$ subtr. | $745.7 \pm 0.7\,{\rm pb}$ |

*S. Camarda, L. Cieri, G. Ferrera Eur.Phys.J.C 82 (2022) 6*

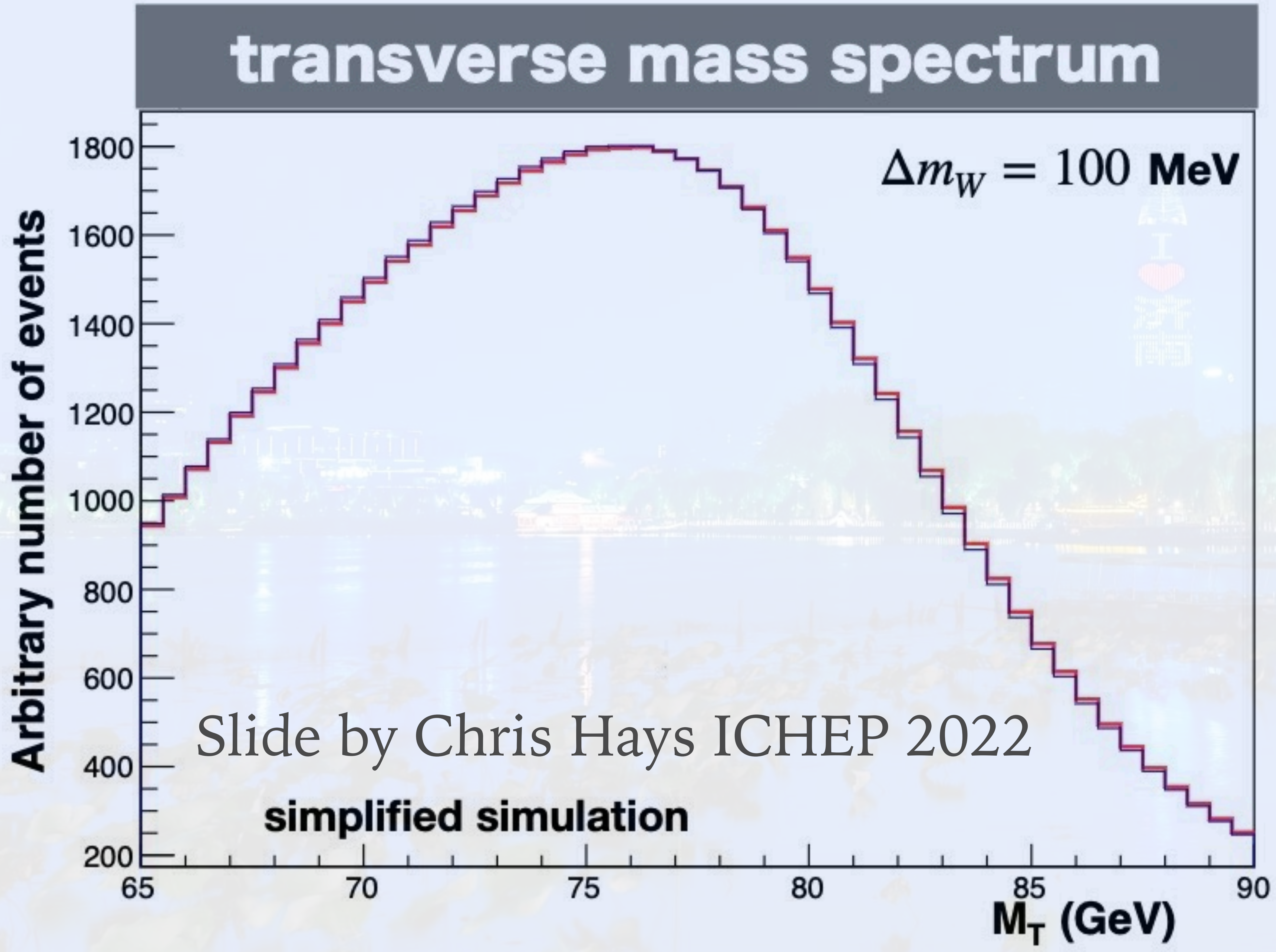
DYTurbo result without fiducial power correction cited in ATLAS $\alpha_s$ fitting

| Order | NLO | NNLO | N$^3$LO |
|---|---|---|---|
| $\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb] | $766.3 \pm 1$ | $757.4 \pm 2$ | $746.1 \pm 2.5$ |
| Order | NLL+NLO | NNLL+NNLO | N$^3$LL+N$^3$LO |
| $\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb] | $773.7 \pm 1$ | $759.8 \pm 2$ | $749.6 \pm 2.5$ |

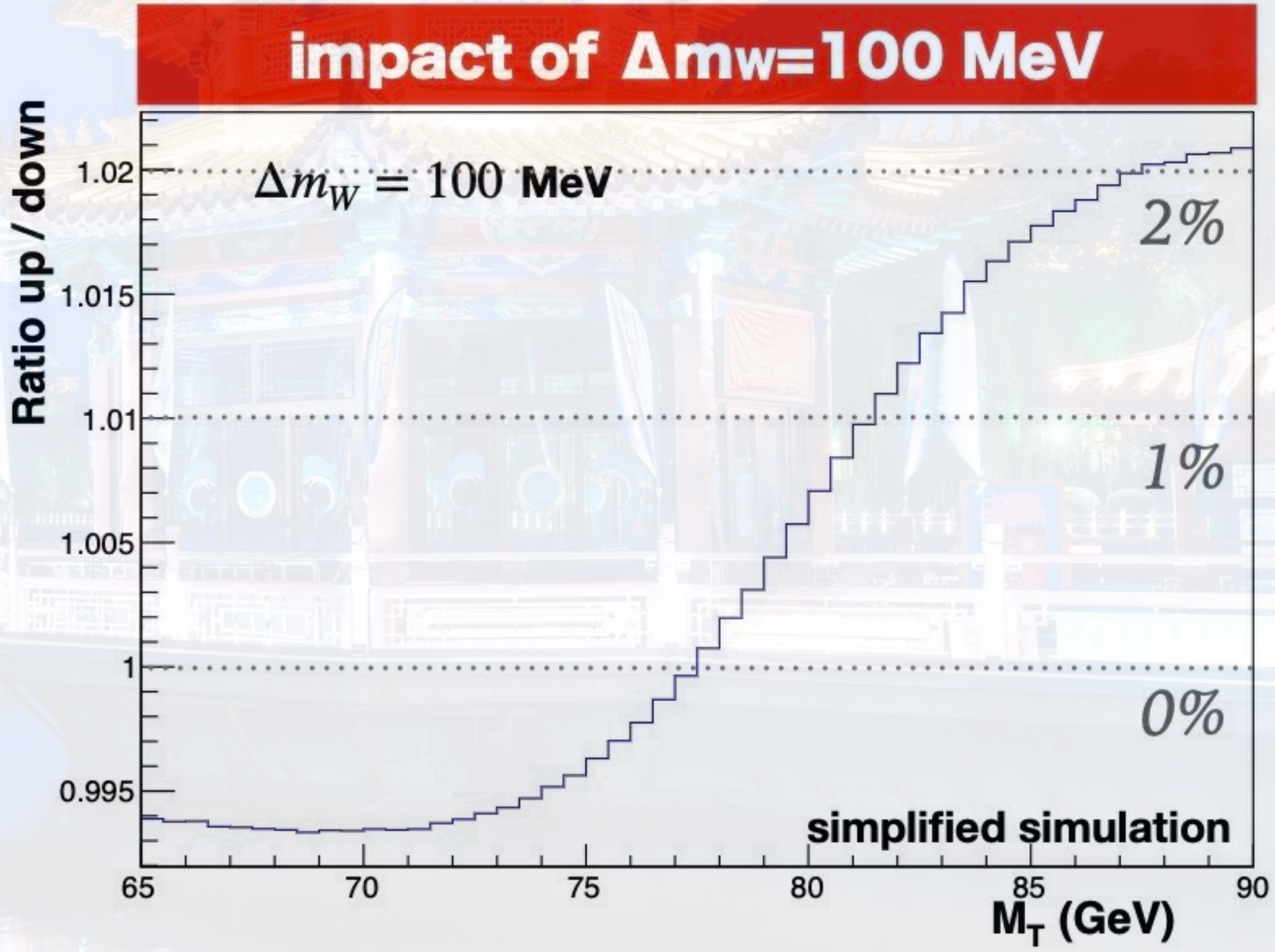
*S. Camarda, L. Cieri, G. Ferrera Phys. Rev. D **104**, L111503 (2021)*

# W MASS IN CDFII MEASUREMENT

➤$d\sigma/dm_T^W$ two templates with $\Delta m_W = 100$ MeV




Slide by Chris Hays ICHEP 2022




$\Delta m_W =$ 100 MeV ~ 0.5-2% change in $d\sigma/dm_T^W$ → $\Delta m_W =$ 10 MeV ~ 0.1% precision in $d\sigma/dm_T^W$

# PRECISION PREDICTIONS IN CDF II

➤CDF II use ResBos to generate theory templates

➤NLO+NNLL accuracy for W/Z production

Balazs, Brock, Landry, Nadolsky and Yuan `97 to `03

➤CSS factorisation and resummation of $p_T$ in $b$ space:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}^2\vec{p}_T\,\mathrm{d}y\,\mathrm{d}\cos\theta\,\mathrm{d}\phi} = \sigma_0 \int \frac{\mathrm{d}^2 b}{(2\pi)^2} e^{i\vec{p}_T\cdot\vec{b}} e^{-S(b)}$$

$$\times\, C\otimes f(x_1,\mu)\, C\otimes f(x_2,\mu) + Y(Q,\vec{p}_T,x_1,x_2,\mu_R,\mu_F)$$

Collins, Soper and Sterman `85

➤Non-perturbative effects at $\alpha_s(\Lambda)$ and large $b$:

$$S(b) = S_{\mathrm{NP}} S_{\mathrm{Pert}}\,,$$

Collins and Soper `77

$$S_{\mathrm{Pert}}(b) = \int_{C_1^2/(b^*)^2}^{C_2^2Q^2} \frac{\mathrm{d}\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2Q^2}{\bar{\mu}^2}\right) A(\bar{\mu},C_1) + B(\bar{\mu},C_1,C_2)\right]$$

$$S_{\mathrm{NP}} = \left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \ln(100 x_1 x_2)\right] b^2$$

$S_{NP}$ assumes the BLNY functional form

Brock, Landry, Nadolsky and Yuan `02

➤Use data driven method:

| Fix | g1 | g2 | g3 | $\alpha_s$ |
|---|---|---|---|---|
| $p_T^Z$ | Global fit `03 | CDFII fit | Global fit `03 | CDFII fit |
| $p_T^Z/p_T^W$ | | | Global fit `03 | |

Global fit by Brock, Landry, Nadolsky and Yuan `03

$m_T^W \sim 0.7$ MeV, $p_T^l \sim 2.3$ MeV, $p_T^\nu \sim 0.9$ MeV

CDF supplementary materials `22

➤Scale uncertainty of $p_T^Z/p_T^W$ by DYQT

Bozzi, Catani, Ferrera, de Florian, Grazzini `09 `11

$m_T^W \sim 3.5$ MeV, $p_T^l \sim 10.1$ MeV, $p_T^\nu \sim 3.9$ MeV

Not included in final result CDF sm`22