Constructing Quantum Entanglement Criteria for Collider Processes

Junle Pei
(**Institute of Physics, Henan Academy of Sciences**)

arXiv: 2505.09280, 2505.09931, 2510.08031

Co-authors: Yaqaun Fang, Xiqing Hao, Tianjun Li, Biyabi Mustapha, Xiaochuan Wang, Lina Wu, and Da Xu

CLHCP2025

October 31, 2025

Polarization state

arXiv: 2505.09280

N-particle system:

$$|P_1 P_2 \ldots P_N\rangle = \sum_{k_1=-s_1}^{s_1} \sum_{k_2=-s_2}^{s_2} \cdots \sum_{k_N=-s_N}^{s_N} \alpha_{k_1,k_2,\ldots,k_N} |k_1\rangle_{P_1} |k_2\rangle_{P_2} \ldots |k_N\rangle_{P_N} \tag{1}$$

- s_i (for $i = 1, 2, \ldots, N$) represent the spins of particles P_i (e.g., $1/2$ for fermions, 1 for vector bosons).
- k_i denote the spin projection quantum numbers for particles P_i, which are defined along the respective momentum directions, labeled as $\hat{e}_i$.
- The normalization condition:

$$\sum_{k_1=-s_1}^{s_1} \sum_{k_2=-s_2}^{s_2} \cdots \sum_{k_N=-s_N}^{s_N} |\alpha_{k_1,k_2,\ldots,k_N}|^2 = 1 \tag{2}$$

Quantum entanglement in polarization state

N-particle system:

$$|P_1 P_2 \ldots P_N\rangle = \sum_{k_1=-s_1}^{s_1} \sum_{k_2=-s_2}^{s_2} \cdots \sum_{k_N=-s_N}^{s_N} \alpha_{k_1,k_2,\ldots,k_N} |k_1\rangle_{P_1} |k_2\rangle_{P_2} \ldots |k_N\rangle_{P_N}$$

- When the N-particle system exhibits no quantum entanglement (QE), all coefficients $\alpha_{k_1,k_2,\ldots,k_N}$ satisfy the following factorization relation:

$$\alpha_{k_1,k_2,\ldots,k_N} = \tilde{\alpha}^{(1)}_{k_1} \tilde{\alpha}^{(2)}_{k_2} \ldots \tilde{\alpha}^{(N)}_{k_N} \,. \tag{3}$$

  Here, each $\tilde{\alpha}^{(i)}_{k_i}$ ($i = 1, 2, \ldots, N$) completely characterizes the polarization state of particle P_i and satisfies the normalization condition of $\sum_{k_i=-s_i}^{s_i} \left|\tilde{\alpha}^{(i)}_{k_i}\right|^2 = 1$.

- Conversely, the existence of a factorized product structure in Eq. (3) constitutes both a necessary and sufficient condition for the system to be unentangled.

Fundamental questions

To build an effective communication bridge between QE theory and high-energy phenomenology, it is essential to address the following fundamental questions:

- The conditions $\alpha_{k_1,k_2,\ldots,k_N} = \tilde{\alpha}^{(1)}_{k_1}\tilde{\alpha}^{(2)}_{k_2}\ldots\tilde{\alpha}^{(N)}_{k_N}$ serve as the necessary and sufficient criterion for no QE in multi-particle systems. The central challenge reduces to establishing experimental protocols to detect the existence of this factorization.
- For an arbitrary physical observable, under what conditions can its measured values constitute a sufficient criterion for entanglement existence in multi-particle systems? What would be the corresponding entanglement criterion based on such observable measurements?

$\Lambda\bar{\Lambda}$

arXiv: 2505.09931, 2510.08031

$$\left|\Lambda\bar{\Lambda}\right\rangle = \sum_{k,j=\pm\frac{1}{2}} \alpha_{k,j} \left|k\right\rangle_{\Lambda} \left|j\right\rangle_{\bar{\Lambda}} \; , \quad \sum_{k,j=\pm\frac{1}{2}} \left|\alpha_{k,j}\right|^2 = 1 \tag{4}$$

- The weak decay processes:

$$\Lambda \to p + \pi^- \; , \quad \bar{\Lambda} \to \bar{p} + \pi^+ \tag{5}$$

- In the rest frames of Λ and $\bar{\Lambda}$, the polar angle distributions of final-state p and $\bar{p}$ satisfy

$$\frac{1}{\Gamma_{\Lambda\to p+\pi^-}} \frac{d\Gamma_{\Lambda\to p+\pi^-}}{d\cos\theta_p} = \frac{1}{2}\left(1 + \alpha_{\Lambda}\cos\theta_p\right) \tag{6}$$

$$\frac{1}{\Gamma_{\bar{\Lambda}\to \bar{p}+\pi^+}} \frac{d\Gamma_{\bar{\Lambda}\to \bar{p}+\pi^+}}{d\cos\theta_{\bar{p}}} = \frac{1}{2}\left(1 + \alpha_{\bar{\Lambda}}\cos\theta_{\bar{p}}\right) \tag{7}$$

Decay processes

- We parameterize the momentum directions of p in the Λ rest frame and $\bar{p}$ in the $\bar{\Lambda}$ rest frame using spherical coordinates:

$$p : (\theta_1, \phi_1) \rightarrow \hat{e}_p = (\sin\theta_1 \cos\phi_1, \sin\theta_1 \sin\phi_1, \cos\theta_1) \tag{8}$$

$$\bar{p} : (\theta_2, \phi_2) \rightarrow \hat{e}_{\bar{p}} = (\sin\theta_2 \cos\phi_2, \sin\theta_2 \sin\phi_2, \cos\theta_2) \tag{9}$$

- Decay amplitudes:

$$\langle p, \pi^- | k \rangle_\Lambda = \frac{1}{\sqrt{2\pi}} e^{i(k-\lambda_p)\phi_1} d^{1/2}_{k,\lambda_p}(\theta_1) H_\Lambda(\lambda_p) \tag{10}$$

$$\langle \bar{p}, \pi^+ | j \rangle_{\bar{\Lambda}} = \frac{1}{\sqrt{2\pi}} e^{-i(j+\lambda_{\bar{p}})\phi_2} d^{1/2}_{\lambda_{\bar{p}},j}(\pi - \theta_2) H_{\bar{\Lambda}}(\lambda_{\bar{p}}) \tag{11}$$

- $\alpha_{\Lambda/\bar{\Lambda}}$ are related to $H_\Lambda(\lambda_p)/H_{\bar{\Lambda}}(\lambda_{\bar{p}})$ by

$$\alpha_{\Lambda/\bar{\Lambda}} = \frac{\left|H_{\Lambda/\bar{\Lambda}}(-\frac{1}{2})\right|^2 - \left|H_{\Lambda/\bar{\Lambda}}(\frac{1}{2})\right|^2}{\left|H_{\Lambda/\bar{\Lambda}}(\frac{1}{2})\right|^2 + \left|H_{\Lambda/\bar{\Lambda}}(-\frac{1}{2})\right|^2} \,. \tag{12}$$

Observables

Any observable constructed from the angular variables can be expressed as

$$\langle \mathcal{O}(\theta_1, \theta_2, \phi_1, \phi_2) \rangle = \sum_{k,j,m,n=\pm\frac{1}{2}} \mathcal{O}_{k,j;m,n} \alpha_{k,j} \alpha^*_{m,n} \,. \tag{13}$$

$$\begin{aligned} \mathcal{O}_{k,j;m,n} = & \frac{1}{16\pi^2} \sum_{\lambda_p, \lambda_{\bar{p}}=\pm\frac{1}{2}} (1 - 2\lambda_p \alpha_\Lambda) \left(1 - 2\lambda_{\bar{p}} \alpha_{\bar{\Lambda}}\right) \int_{-1}^{1} d\cos\theta_1 \int_{-1}^{1} d\cos\theta_2 \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \\ & \mathcal{O}(\theta_1, \theta_2, \phi_1, \phi_2) e^{i(k-m)\phi_1} e^{i(n-j)\phi_2} d^{\frac{1}{2}}_{k,\lambda_p}(\theta_1) d^{\frac{1}{2}}_{m,\lambda_p}(\theta_1) d^{\frac{1}{2}}_{\lambda_{\bar{p}},j}(\pi - \theta_2) d^{\frac{1}{2}}_{\lambda_{\bar{p}},n}(\pi - \theta_2) \,. \end{aligned} \tag{14}$$

- $\langle \mathcal{O}(\theta_1, \theta_2, \phi_1, \phi_2) \rangle$ can also be written as

$$\langle \mathcal{O}(\theta_1, \theta_2, \phi_1, \phi_2) \rangle = \mathcal{O}_0 + \mathcal{O}_1 \alpha_\Lambda + \mathcal{O}_2 \alpha_{\bar{\Lambda}} + \mathcal{O}_3 \alpha_\Lambda \alpha_{\bar{\Lambda}} \,. \tag{15}$$

Observables

$$\langle \mathcal{O}(\theta_1, \theta_2, \phi_1, \phi_2) \rangle = \mathcal{O}_0 + \mathcal{O}_1 \alpha_\Lambda + \mathcal{O}_2 \alpha_{\bar{\Lambda}} + \mathcal{O}_3 \alpha_\Lambda \alpha_{\bar{\Lambda}} \tag{16}$$

Observables	Ranges	Ranges under $\alpha_{k_1,k_2,\ldots,k_N} = \tilde{\alpha}^{(1)}_{k_1} \tilde{\alpha}^{(2)}_{k_2} \ldots \tilde{\alpha}^{(N)}_{k_N}$	Criteria for QE
$\langle \mathcal{O} \rangle$	R_1	R_2 ($\in R_1$)	$R_1 \backslash R_2$

- The critiques 2507.15947 and 2507.15949 argue that if the purpose of an experiment is to test quantum mechanics, then no conclusion derived from quantum mechanics or quantum field theory may be used at any stage of the analysis.
- In their view, present measurements rely on angular correlations among the momentum directions of different final-state particles to tomographically reconstruct correlations in the initial-state polarizations. The relation connecting the two (for example, in a fermion–antifermion system) involves the initial-state fermion's parameter governing parity (P) violation.
- Any independent determination of this parameter inevitably relies on theoretical calculations based on quantum mechanics or quantum field theory.

Observables independent of α_Λ and $\alpha_{\bar{\Lambda}}$

For $\langle \mathcal{O}(\theta_1, \theta_2, \phi_1, \phi_2) \rangle$ does not depend on α_Λ and $\alpha_{\bar{\Lambda}}$, i.e.

$$\langle \mathcal{O}(\theta_1, \theta_2, \phi_1, \phi_2) \rangle = \mathcal{O}_0 = \frac{1}{16\pi^2} \int_{-1}^{1} d\cos\theta_1 \int_{-1}^{1} d\cos\theta_2 \int_{0}^{2\pi} d\phi_1 \int_{0}^{2\pi} d\phi_2 \, \mathcal{O}(\theta_1, \theta_2, \phi_1, \phi_2) \,. \tag{17}$$

- Since $\mathcal{O}_0$ does not depend on $\alpha_{k,j}$ $(k, j = \pm\frac{1}{2})$, $\langle \mathcal{O}(\theta_1, \theta_2, \phi_1, \phi_2) \rangle$ cannot provide a quantum-entanglement criterion.
- It is impossible to extract any information about the initial system's spin polarization from angular observables when there is no violation of parity ($\alpha_\Lambda = \alpha_{\bar{\Lambda}} = 0$).

If $\mathcal{O}_p$ and $\mathcal{O}_{\bar{p}}$ depend solely on the angular variables of p (θ_1 and ϕ_1) and $\bar{p}$ (θ_2 and ϕ_2), respectively, then for pure states there exists a sufficient condition for quantum entanglement of the form

$$\langle \mathcal{O}_p \mathcal{O}_{\bar{p}} \rangle \neq \langle \mathcal{O}_p \rangle \langle \mathcal{O}_{\bar{p}} \rangle \, . \tag{18}$$

For example,

$$\langle \cos\theta_1 \rangle = \frac{\alpha_\Lambda}{3} \left(\left|\alpha_{-\frac{1}{2},-\frac{1}{2}}\right|^2 + \left|\alpha_{-\frac{1}{2},\frac{1}{2}}\right|^2 - \left|\alpha_{\frac{1}{2},-\frac{1}{2}}\right|^2 - \left|\alpha_{\frac{1}{2},\frac{1}{2}}\right|^2 \right) \, , \tag{19}$$

$$\langle \cos\theta_2 \rangle = -\frac{\alpha_{\bar{\Lambda}}}{3} \left(\left|\alpha_{-\frac{1}{2},-\frac{1}{2}}\right|^2 - \left|\alpha_{-\frac{1}{2},\frac{1}{2}}\right|^2 + \left|\alpha_{\frac{1}{2},-\frac{1}{2}}\right|^2 - \left|\alpha_{\frac{1}{2},\frac{1}{2}}\right|^2 \right) \, , \tag{20}$$

$$\langle \cos\theta_1 \cos\theta_2 \rangle = -\frac{\alpha_\Lambda \alpha_{\bar{\Lambda}}}{9} \left(\left|\alpha_{-\frac{1}{2},-\frac{1}{2}}\right|^2 - \left|\alpha_{-\frac{1}{2},\frac{1}{2}}\right|^2 - \left|\alpha_{\frac{1}{2},-\frac{1}{2}}\right|^2 + \left|\alpha_{\frac{1}{2},\frac{1}{2}}\right|^2 \right) \, . \tag{21}$$

Inequality-type criteria

For a two-component mixed state in which each component is unentangled, one finds

$$\langle \mathcal{O}_p \rangle \langle \mathcal{O}_{\bar{p}} \rangle = (p_1 \langle \mathcal{O}_p \rangle_1 + p_2 \langle \mathcal{O}_p \rangle_2) \left(p_1 \langle \mathcal{O}_{\bar{p}} \rangle_1 + p_2 \langle \mathcal{O}_{\bar{p}} \rangle_2 \right) , \tag{22}$$

$$\langle \mathcal{O}_p \mathcal{O}_{\bar{p}} \rangle = p_1 \langle \mathcal{O}_p \mathcal{O}_{\bar{p}} \rangle_1 + p_2 \langle \mathcal{O}_p \mathcal{O}_{\bar{p}} \rangle_2 = p_1 \langle \mathcal{O}_p \rangle_1 \langle \mathcal{O}_{\bar{p}} \rangle_1 + p_2 \langle \mathcal{O}_p \rangle_2 \langle \mathcal{O}_{\bar{p}} \rangle_2 . \tag{23}$$

In this case, $\langle \mathcal{O}_p \rangle \langle \mathcal{O}_{\bar{p}} \rangle$ and $\langle \mathcal{O}_p \mathcal{O}_{\bar{p}} \rangle$ need not be equal. Thus, a mixed state satisfying $\langle \mathcal{O}_p \mathcal{O}_{\bar{p}} \rangle \neq \langle \mathcal{O}_p \rangle \langle \mathcal{O}_{\bar{p}} \rangle$ is likewise unentangled (separable).

Ratio-type criteria

For two distinct physical observables A and B,

$$\frac{\langle A\rangle}{\langle B\rangle}=\frac{A_0+A_1\alpha_\Lambda+A_2\alpha_{\bar{\Lambda}}+A_3\alpha_\Lambda\alpha_{\bar{\Lambda}}}{B_0+B_1\alpha_\Lambda+B_2\alpha_{\bar{\Lambda}}+B_3\alpha_\Lambda\alpha_{\bar{\Lambda}}}\,. \tag{24}$$

If $\langle A\rangle/\langle B\rangle$ is independent of α_Λ and $\alpha_{\bar{\Lambda}}$, there are two possibilities:

- A_0 and B_0 are both nonzero. Since A_0 and B_0 do not depend on $\alpha_{k,j}$ ($k,j=\pm\frac{1}{2}$), in this case $\langle A\rangle/\langle B\rangle=A_0/B_0$ likewise does not depend on $\alpha_{k,j}$ and therefore cannot provide an entanglement criterion.
- A_0 and B_0 both vanish.

Ratio-type criteria: A_0 and B_0 both vanish

- Four points in the $\alpha_{k,j}$ space as follows:

$$P_1 : \alpha_{-\frac{1}{2},-\frac{1}{2}} = 1\,, \quad P_2 : \alpha_{-\frac{1}{2},\frac{1}{2}} = 1\,, \quad P_3 : \alpha_{\frac{1}{2},-\frac{1}{2}} = 1\,, \quad P_4 : \alpha_{\frac{1}{2},\frac{1}{2}} = 1\,. \tag{25}$$

- At P_1, we write $\langle B \rangle$ as

$$\langle B \rangle_{P_1} = \tilde{B}_1 \alpha_\Lambda + \tilde{B}_2 \alpha_{\bar{\Lambda}} + \tilde{B}_3 \alpha_\Lambda \alpha_{\bar{\Lambda}}\,, \tag{26}$$

Then, at the other points we have

$$\langle B \rangle_{P_2} = \tilde{B}_1 \alpha_\Lambda - \tilde{B}_2 \alpha_{\bar{\Lambda}} - \tilde{B}_3 \alpha_\Lambda \alpha_{\bar{\Lambda}}\,, \tag{27}$$

$$\langle B \rangle_{P_3} = -\tilde{B}_1 \alpha_\Lambda + \tilde{B}_2 \alpha_{\bar{\Lambda}} - \tilde{B}_3 \alpha_\Lambda \alpha_{\bar{\Lambda}}\,, \tag{28}$$

$$\langle B \rangle_{P_4} = -\tilde{B}_1 \alpha_\Lambda - \tilde{B}_2 \alpha_{\bar{\Lambda}} + \tilde{B}_3 \alpha_\Lambda \alpha_{\bar{\Lambda}}\,. \tag{29}$$

- It follows that

$$\sum_{i=1}^{4} \langle B \rangle_{P_i} = 0\,. \tag{30}$$

Ratio-type criteria: obstruction

$$A = \cos(\phi_1 + \phi_2) \Longrightarrow \langle A \rangle = \kappa \left(\alpha_{-\frac{1}{2},\frac{1}{2}} \alpha^*_{\frac{1}{2},-\frac{1}{2}} + \alpha^*_{-\frac{1}{2},\frac{1}{2}} \alpha_{\frac{1}{2},-\frac{1}{2}} \right) \tag{31}$$

$$B = \cos(\phi_1 - \phi_2) \Longrightarrow \langle B \rangle = \kappa \left(\alpha_{-\frac{1}{2},-\frac{1}{2}} \alpha^*_{\frac{1}{2},\frac{1}{2}} + \alpha^*_{-\frac{1}{2},-\frac{1}{2}} \alpha_{\frac{1}{2},\frac{1}{2}} \right) \tag{32}$$

$$\kappa = -\frac{\pi^2}{32} \alpha_\Lambda \alpha_{\bar{\Lambda}} \tag{33}$$

Observables	R_1	R_2	$R_1 \backslash R_2$
$\langle A \rangle / \kappa$	$[-1, 1]$	$[-1/2, 1/2]$	$[-1, -1/2) \cup (1/2, 1]$
$\langle B \rangle / \kappa$	$[-1, 1]$	$[-1/2, 1/2]$	$[-1, -1/2) \cup (1/2, 1]$
$\langle A \rangle / \langle B \rangle$	$(-\infty, +\infty)$	$(-\infty, +\infty)$	$\emptyset$

Because $\langle B \rangle$ has zeros, the value ranges of $\langle A \rangle / \langle B \rangle$ over the general and separable spaces of $\alpha_{k,j}$ (denoted R_1 and R_2) both span the entire real line. $\langle A \rangle / \langle B \rangle$ cannot be used to furnish a quantum-entanglement criterion.

Ratio-type criteria with additional spin information

- For on-shell J/Ψ mesons produced at e^+e^- colliders, experiments indicate that their spin projection quantum number along the beam axis is ± 1.
- For the process $e^+ + e^- \to J/\Psi \to \Lambda + \bar{\Lambda}$, if we restrict to $\Lambda\bar{\Lambda}$ pairs emitted back-to-back along the beam direction, then by angular-momentum conservation each event places the pair entirely in either $\left|-\frac{1}{2}\right\rangle_\Lambda \left|\frac{1}{2}\right\rangle_{\bar{\Lambda}}$ or $\left|\frac{1}{2}\right\rangle_\Lambda \left|-\frac{1}{2}\right\rangle_{\bar{\Lambda}}$.
- Consequently, for these events we obtain

$$
\begin{aligned}
&\langle \cos\theta_1 \cos\theta_2 \rangle_{\text{beam}} \\
&= -\frac{\alpha_\Lambda \alpha_{\bar{\Lambda}}}{9} \left(\left|\alpha_{-\frac{1}{2},-\frac{1}{2}}\right|^2 - \left|\alpha_{-\frac{1}{2},\frac{1}{2}}\right|^2 - \left|\alpha_{\frac{1}{2},-\frac{1}{2}}\right|^2 + \left|\alpha_{\frac{1}{2},\frac{1}{2}}\right|^2 \right) \\
&= \frac{\alpha_\Lambda \alpha_{\bar{\Lambda}}}{9} \,.
\end{aligned} \tag{34}
$$

Ratio-type criteria with additional spin information

Using the information provided by $\langle \cos\theta_1 \cos\theta_2 \rangle_{\text{beam}}$, we define

$$f_1 = -\frac{32}{9\pi^2} \frac{\langle \cos(\phi_1 + \phi_2) \rangle}{\langle \cos\theta_1 \cos\theta_2 \rangle_{\text{beam}}} = \left(\alpha_{-\frac{1}{2},\frac{1}{2}} \alpha^*_{\frac{1}{2},-\frac{1}{2}} + \alpha^*_{-\frac{1}{2},\frac{1}{2}} \alpha_{\frac{1}{2},-\frac{1}{2}} \right) , \quad (35)$$

$$f_2 = -\frac{32}{9\pi^2} \frac{\langle \cos(\phi_1 - \phi_2) \rangle}{\langle \cos\theta_1 \cos\theta_2 \rangle_{\text{beam}}} = \left(\alpha_{-\frac{1}{2},-\frac{1}{2}} \alpha^*_{\frac{1}{2},\frac{1}{2}} + \alpha^*_{-\frac{1}{2},-\frac{1}{2}} \alpha_{\frac{1}{2},\frac{1}{2}} \right) . \quad (36)$$

Observables	Ranges	Ranges under $\alpha_{k,j} = \beta_k \gamma_j$	criteria for entanglement
f_1	$[-1, 1]$	$[-1/2, 1/2]$	$[-1, -1/2) \cup (1/2, 1]$
f_2	$[-1, 1]$	$[-1/2, 1/2]$	$[-1, -1/2) \cup (1/2, 1]$

Measurements

OPEN ACCESS

Precise Measurements of Decay Parameters and CP Asymmetry with Entangled Λ–$\bar{\Lambda}$ Pairs

PDF

Share

M. Ablikim[1], M. N. Achasov[11,b], P. Adlarson[70], M. Albrecht[4], R. Aliberti[31], A. Amoroso[69a,69c], M. R. An[35], Q. An[66,53], X. H. Bai[61] *et al.* (BESIII Collaboration)

Show more

Phys. Rev. Lett. **129**, 131801 – **Published 22 September, 2022**

Export Citation

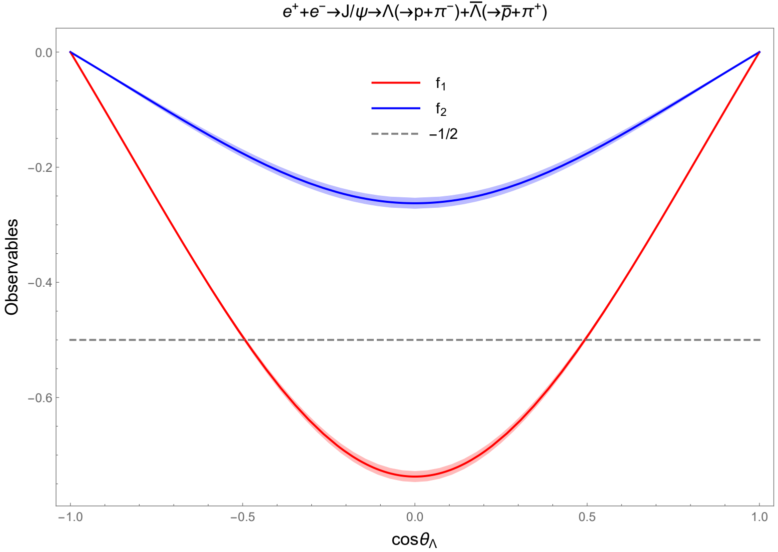
DOI: https://doi.org/10.1103/PhysRevLett.129.131801

PHYSICAL REVIEW LETTERS **129,** 131801 (2022)

TABLE I. The angular distribution parameters, $\alpha_{J/\psi}$, $\Delta\Phi$ and the asymmetry parameters α_- for $\Lambda \to p\pi^-$, α_+ for $\bar{\Lambda} \to \bar{p}\pi^+$ obtained in this Letter and in previous BESIII measurements [12] for comparison. The first uncertainty is statistical, the second one is systematic.

Parameter	This Letter	Previous results [12]
$\alpha_{J/\psi}$	$0.4748 \pm 0.0022 \pm 0.0031$	$0.461 \pm 0.006 \pm 0.007$
$\Delta\Phi$	$0.7521 \pm 0.0042 \pm 0.0066$	$0.740 \pm 0.010 \pm 0.009$
α_-	$0.7519 \pm 0.0036 \pm 0.0024$	$0.750 \pm 0.009 \pm 0.004$
α_+	$-0.7559 \pm 0.0036 \pm 0.0030$	$-0.758 \pm 0.010 \pm 0.007$
A_{CP}	$-0.0025 \pm 0.0046 \pm 0.0012$	$0.006 \pm 0.012 \pm 0.007$
α_{avg}	$0.7542 \pm 0.0010 \pm 0.0024$	$\cdots$

Measurements



Conclusion

- For the decay approach, we present a comprehensive framework to detect QE in high-energy multi-particle systems.
- By leveraging phase-space integration and the orthogonality properties of Wigner *d*-functions, we derive diverse observables through angular correlations in decay products and establish key criteria for QE by calculating the exact quantum ranges and classical ranges.
- These results establish model-independent methodologies for probing QE across collider experiments, bridging QE theory with high-energy phenomenology, while offering novel pathways to explore exotic particles and quantum properties in multi-particle systems.