Correspondence between SMEFT and HEFT in Higgs physics

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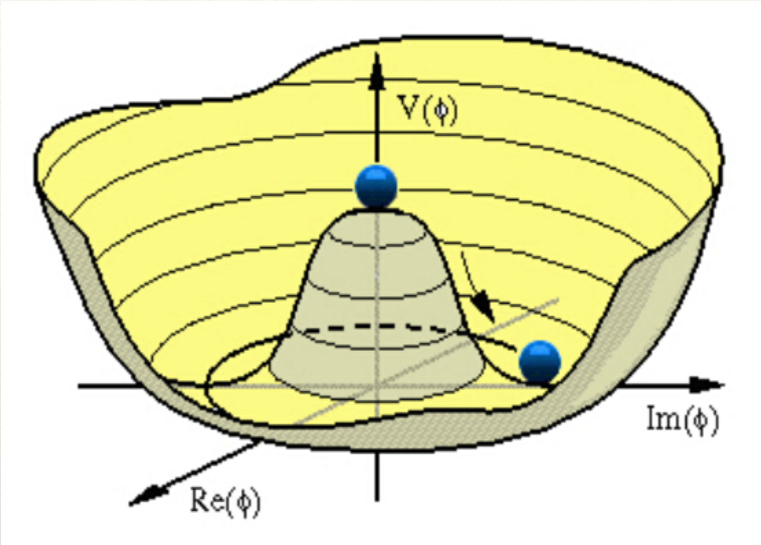
Higgs self-coupling in the SM

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$\phi = \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v+h \end{array}\right)$$

$$V(h) = \tfrac{1}{2}m_h^2h^2 + \sqrt{\tfrac{\lambda}{2}}m_H h^3 + \tfrac{1}{4}\lambda h^4$$

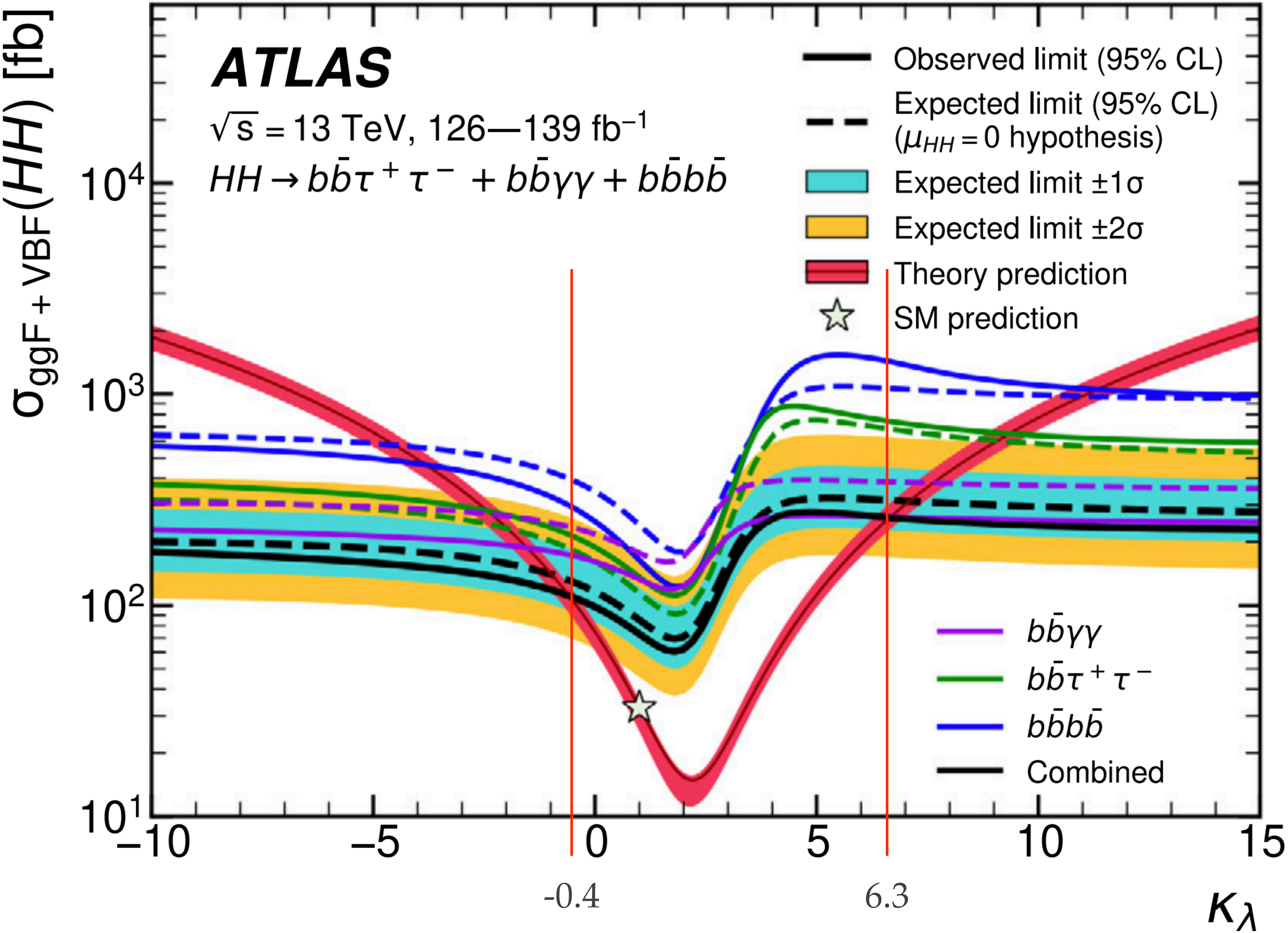
$$\lambda = m_H^2/2v^2$$

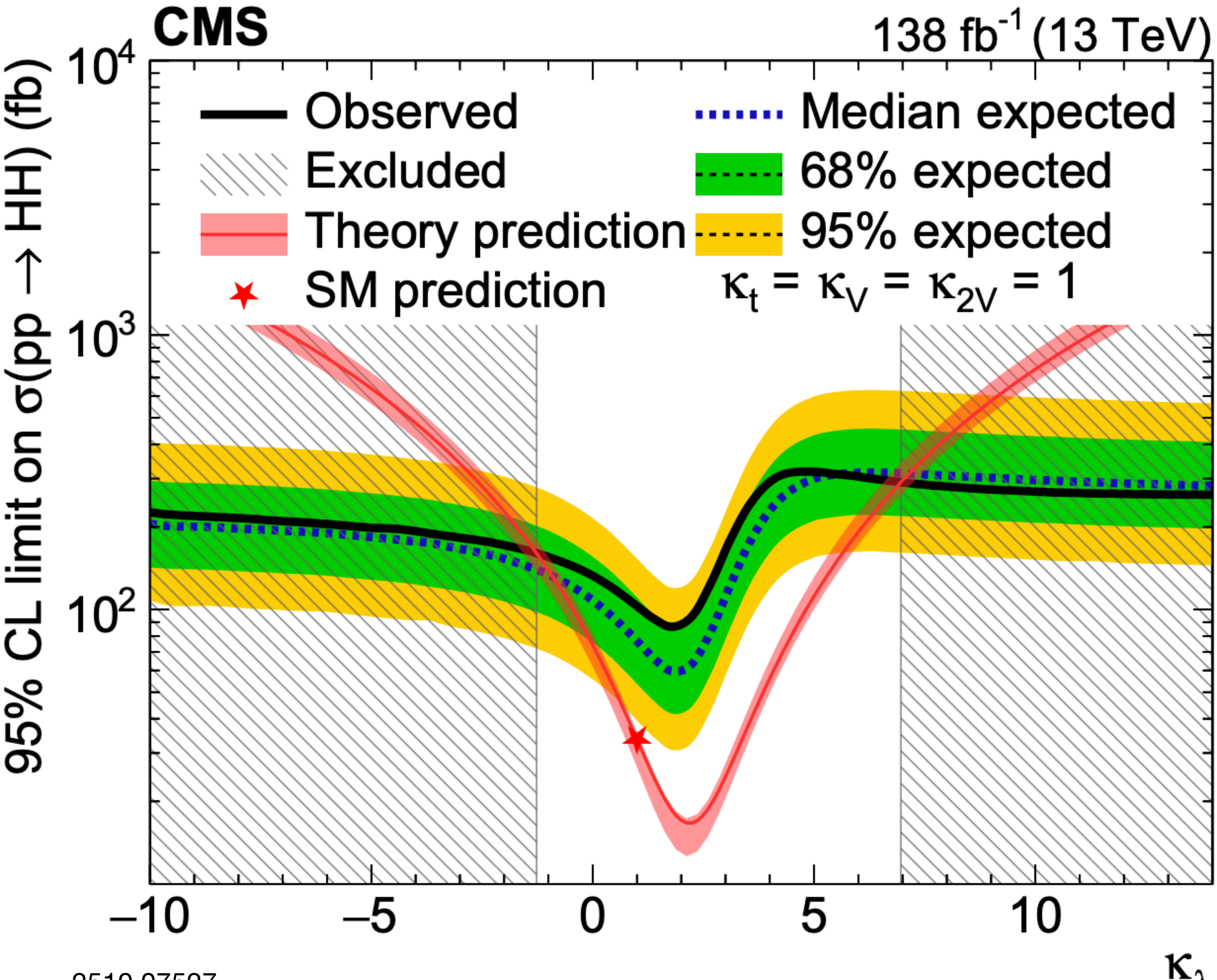


In some new physics models, the trilinear Higgs self-coupling may change by O(100)%, while the couplings with gauge bosons and fermions are still in agreement with SM.

S.Kanemura, et al, PLB558,157

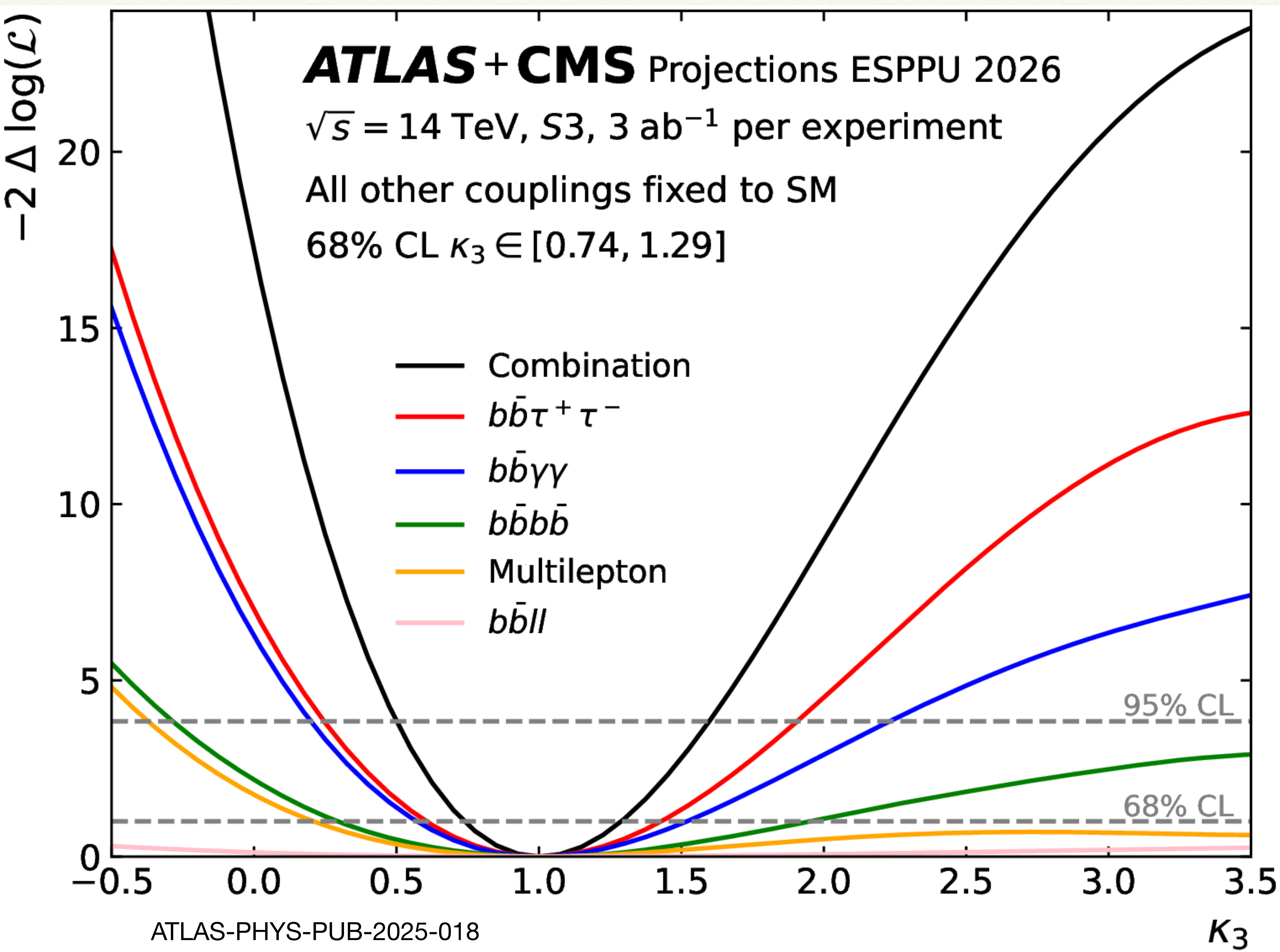
We need to measure the trilinear self coupling directly.


ATLAS
$\sqrt{s}$ = 13 TeV, 126—139 fb^{-1}
$HH \rightarrow b\bar{b}\tau^+\tau^- + b\bar{b}\gamma\gamma + b\bar{b}b\bar{b}$
Observed limit (95% CL)
Expected limit (95% CL)
(μ_{HH} = 0 hypothesis)
Expected limit ±1σ
Expected limit ±2σ
Theory prediction
SM prediction
$b\bar{b}\gamma\gamma$
$b\bar{b}\tau^+\tau^-$
$b\bar{b}b\bar{b}$
Combined
$\sigma_{ggF+VBF}(HH)$ [fb]
10^4
10^3
10^2
10^1
−10
−5
0
5
10
15
-0.4
6.3
κ_λ



CMS
138 fb^{-1} (13 TeV)
Observed
Median expected
Excluded
68% expected
Theory prediction
95% expected
SM prediction
$\kappa_t = \kappa_V = \kappa_{2V} = 1$
95% CL limit on $\sigma(pp \rightarrow HH)$ (fb)
10^4
10^3
10^2
–10
–5
0
5
10
κ_λ

2510.07527

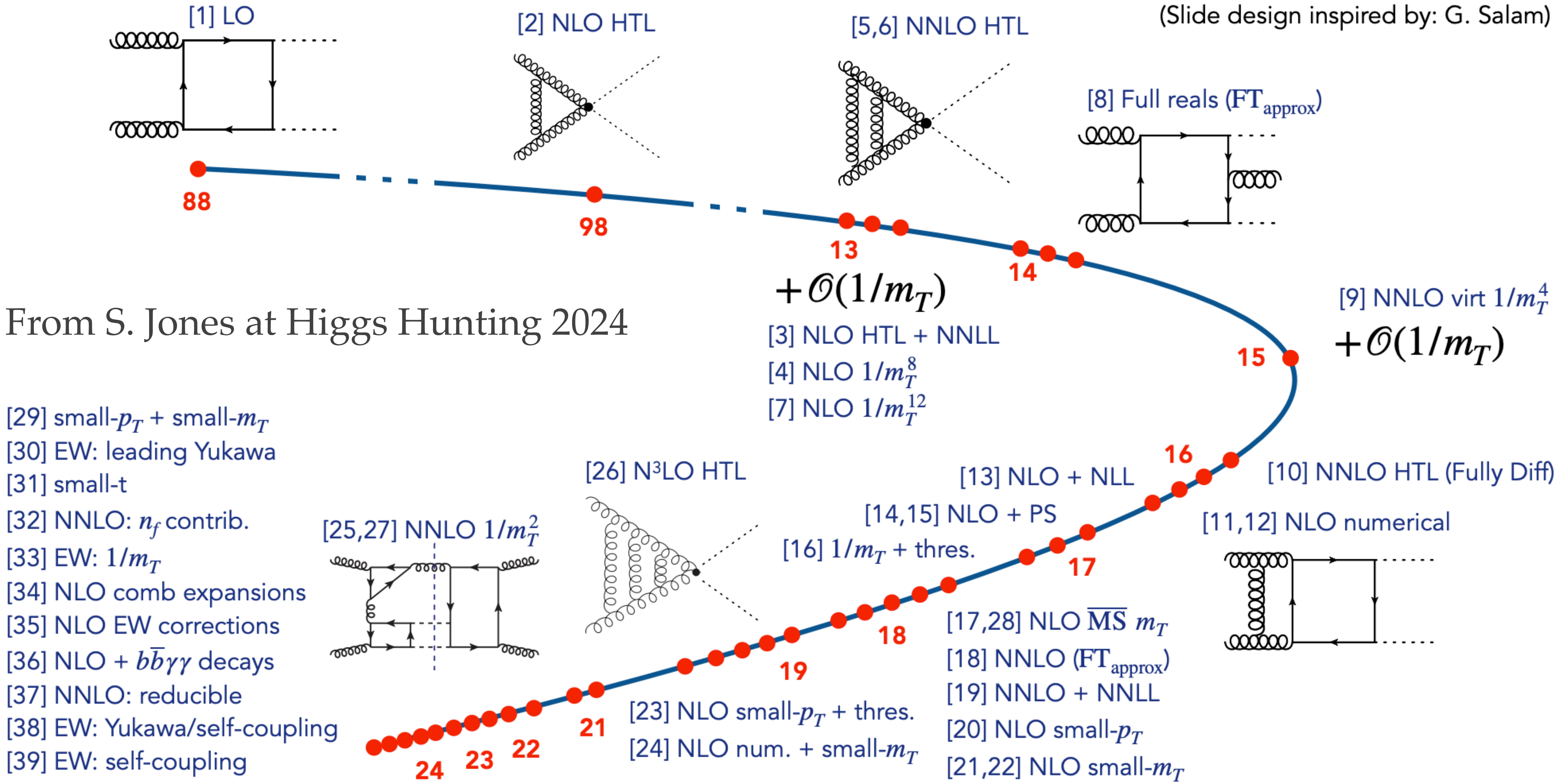


ATLAS-PHYS-PUB-2025-018

Long history in high precision calculations

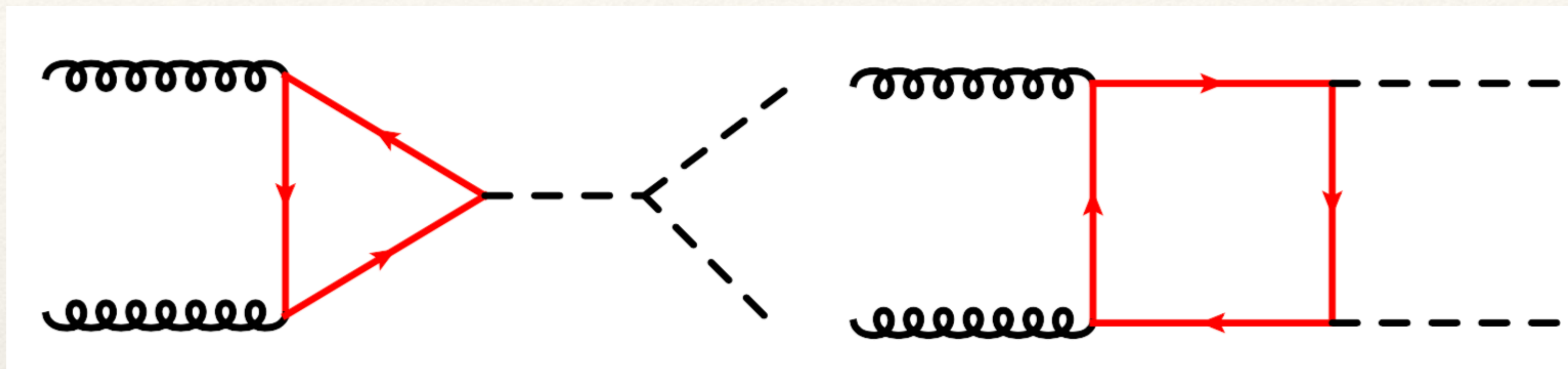
(Slide design inspired by: G. Salam)

From S. Jones at Higgs Hunting 2024



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagnaschi, Degrassi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23 [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, SPJ, Kerner, Stone, Vestner [39] Li, Si, Wang, Zhang, Zhao 24

gg →HH as a function of κ



$$\sigma_{HH} = A + B\kappa + C\kappa^2$$

computation	A [fb]	A/A(LO)	B [fb]	B/B(LO)	C [fb]	C/C(LO)
LO m_t fin	35.0		-23.0		4.73	
NLO m_t fin	62.6	1.79	-44.4	1.93	9.64	2.04
NLO m_t fin × NNLO SM FTApprox	70.0	2.00	-49.6	2.16	10.8	2.28
NNLO + NNLL $m_t \to \infty$ × NNLO+NLL SM (partial m_t fin)	71.3	2.04	-47.7	2.08	9.93	2.10

1910.00012

The meaning of κ

$$(\sigma \cdot \mathbf{BR})\,(gg \to H \to \gamma\gamma) \;=\; \sigma_{SM}(gg \to H) \cdot \mathbf{BR}_{SM}(H \to \gamma\gamma) \cdot \frac{\kappa_g^2 \cdot \kappa_\gamma^2}{\kappa_H^2}$$

Handbook of LHC Higgs Cross Sections,1307.1347

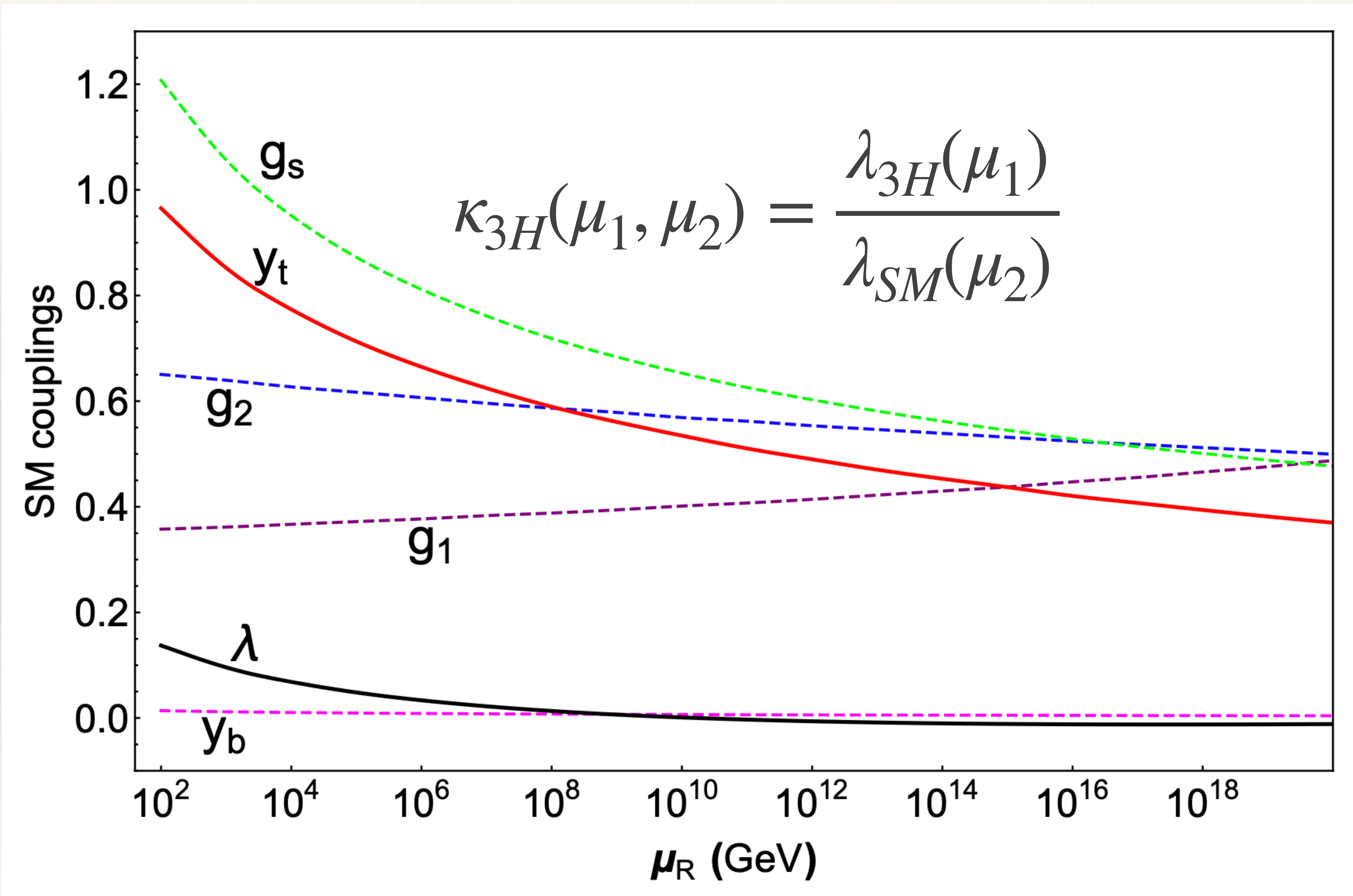
It can be considered as a rescaling factor of the corresponding parameter in the Lagrangian.

It is a ratio of two parameters in two Lagrangians ($\mathscr{L}_{NP}$ and $\mathscr{L}_{SM}$).

$$\kappa_{3H} = \frac{\lambda_{3H}}{\lambda_{SM}} \qquad \kappa_{4H} = \frac{\lambda_{4H}}{\lambda_{SM}}$$

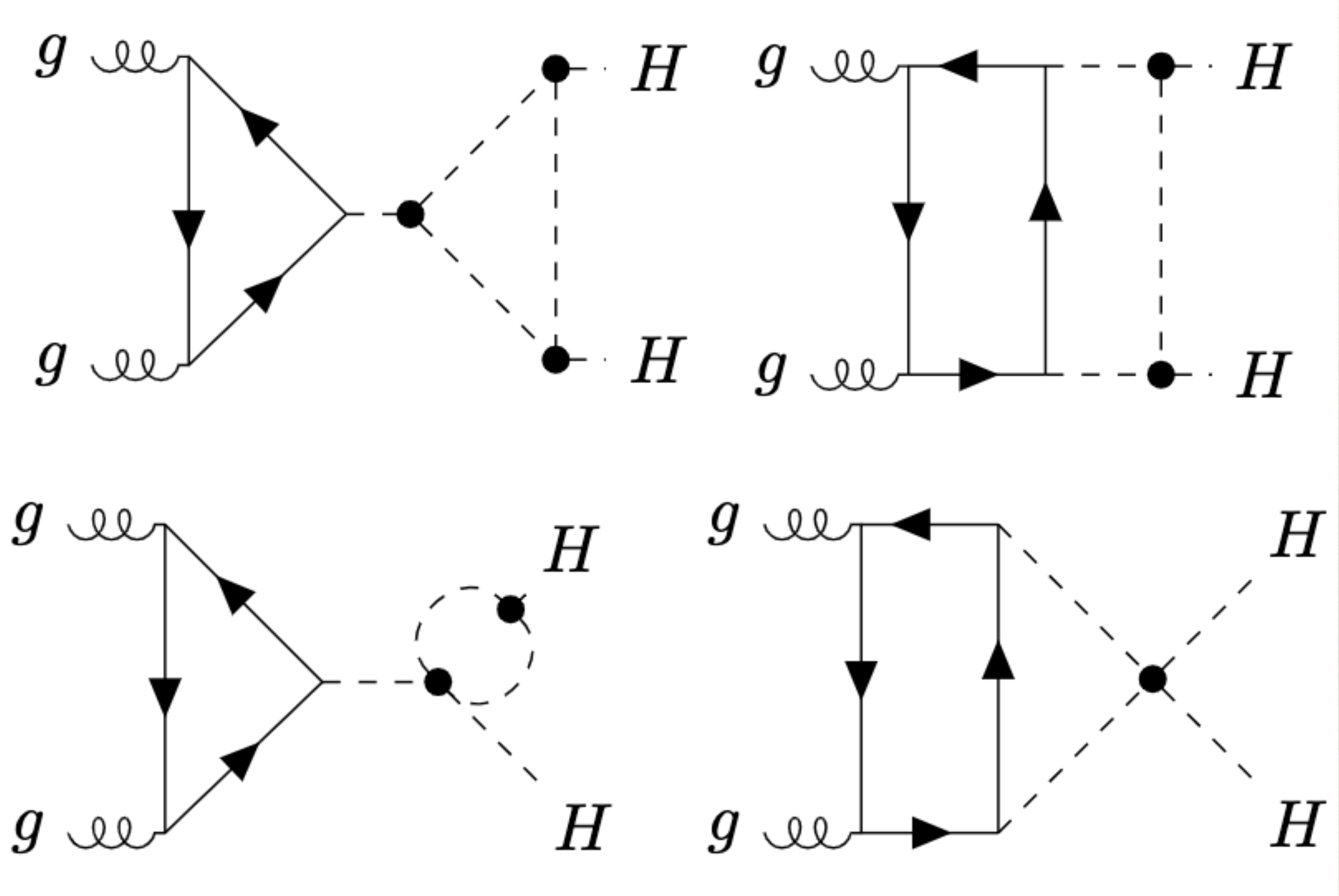
How to rescale the single parameter in $\mathscr{L}_{SM}$?

The meaning of κ



$$\kappa_{3H}(\mu_1, \mu_2) = \frac{\lambda_{3H}(\mu_1)}{\lambda_{SM}(\mu_2)}$$

A more realistic function form



$$\sigma_{HH} = A + B\kappa_3 + C\kappa_3^2 + D\kappa_3^3 + E\kappa_3^4 + F\kappa_3^2\kappa_4 + G\kappa_3\kappa_4 + H\kappa_4 + \cdots$$

HEFT

- To describe the new physics beyond the SM, we often take a consistent effective field theory framework.
- HEFT: Higgs field as a singlet, non-linear, no explicit constraints on couplings

$$\mathcal{L}_{\mathrm{HEFT}} = \mathcal{L}_2 + \mathcal{L}_4$$

$$\mathcal{L}_2 = \frac{v^2}{4}\left(1 + 2a\frac{H}{v} + b\left(\frac{H}{v}\right)^2 + \cdots\right)\mathrm{Tr}[D_\mu U^\dagger D^\mu U]$$
$$+\frac{1}{2}\partial_\mu H \partial^\mu H - V(H) - \frac{1}{2g^2}\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]$$
$$-\frac{1}{2g'^2}\mathrm{Tr}[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}] + \mathcal{L}_{GF} + \mathcal{L}_{FP}.$$

$$U(\pi^a) = e^{i\pi^a\tau^a/v}$$

$$V(H) = (-\mu^2 + \lambda v^2)vH + \frac{1}{2}(-\mu^2 + 3\lambda v^2)H^2$$
$$+\kappa_3 \lambda v H^3 + \kappa_4 \frac{\lambda}{4}H^4.$$

Renormalization

The renormalized Lagrangian in the κ framework after EW gauge symmetry breaking:

$$\mathscr{L}_{\mathrm{H}}^{\kappa} = \frac{1}{2} Z_\phi (\partial_\mu H)^2 - \left(-\frac{1}{2} Z_{\mu^2} Z_\phi Z_v^2 \mu^2 v^2 + \frac{1}{4} Z_\lambda Z_\phi^2 Z_v^4 \lambda v^4 \right) - (Z_\lambda Z_\phi^2 Z_v^3 \lambda v^3 - Z_{\mu^2} Z_\phi Z_v \mu^2 v) H$$
$$- \left(\frac{3}{2} Z_\lambda Z_\phi^2 Z_v^2 \lambda v^2 - \frac{1}{2} Z_{\mu^2} Z_\phi \mu^2 \right) H^2 - Z_{\kappa_{3\mathrm{H}}} Z_\lambda Z_\phi^2 Z_v \lambda_{3\mathrm{H}} v H^3 - \frac{1}{4} Z_{\kappa_{4\mathrm{H}}} Z_\lambda Z_\phi^2 \lambda_{4\mathrm{H}} H^4 + \cdots$$

The linear term is

$$(\mu^2 v - \lambda v^3) H + [(\delta Z_{\mu^2} + \delta Z_\phi + \delta Z_v) \mu^2 v - (\delta Z_\lambda + 2\delta Z_\phi + 3\delta Z_v) \lambda v^3] H$$

We choose the renormalization scheme in which there is no tadpole contributions.

$\mu^2 = \lambda v^2$ and $(\delta Z_{\mu^2} - \delta Z_\lambda - \delta Z_\phi - 2\delta Z_v) \mu^2 v + T = 0$ with T the one-loop diagrams.

$$T = \frac{3\lambda_{3\mathrm{H}} v}{16\pi^2} m_H^2 \left(\frac{1}{\epsilon} + \ln \frac{\mu_R^2}{m_H^2} + 1 \right)$$

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Renormalization

The quadratic term is

$$\frac{1}{2}(\partial_\mu H)^2 - \mu^2 H^2 + \frac{1}{2}\delta Z_\phi(\partial_\mu H)^2 - \left(\frac{3}{2}\delta Z_\lambda + \frac{5}{2}\delta Z_\phi - \frac{1}{2}\delta Z_{\mu^2} + 3\delta Z_v\right)\mu^2 H^2$$
$$\equiv \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}m_H^2 H^2 + \frac{1}{2}\delta Z_\phi(\partial_\mu H)^2 - \frac{1}{2}(\delta Z_{m_H^2} + \delta Z_\phi)m_H^2 H^2$$

We choose the on-shell renormalization scheme.

$$\delta Z_{m_H^2} = \frac{3\lambda_{4\mathrm{H}}}{16\pi^2}\left(\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{m_H^2} + 1\right) + \frac{9\lambda_{3\mathrm{H}}^2 v^2}{m_H^2}\frac{1}{8\pi^2}\left(\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{m_H^2} + 2 - \frac{\pi}{\sqrt{3}}\right)$$
$$\delta Z_\phi = \frac{9\lambda_{3\mathrm{H}}^2 v^2}{8\pi^2}\frac{\sqrt{3} - 2\pi/3}{\sqrt{3}m_H^2}$$

Since we focus on the corrections induced by the Higgs self-couplings, we can simply take $\delta Z_v + \delta Z_\phi/2 = 0$

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Renormalization

The result of one-particle reducible diagrams and counter-terms:

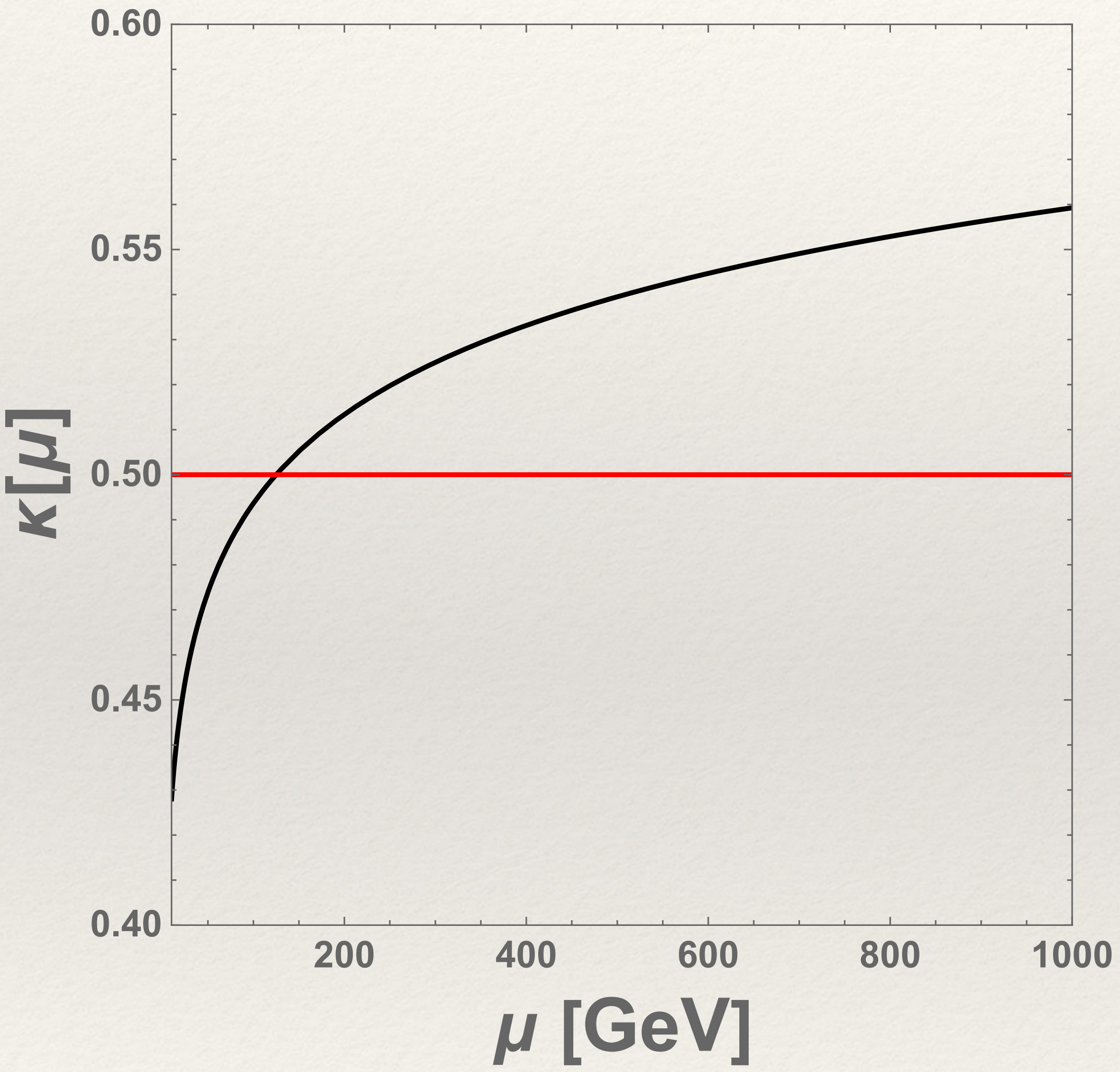
$\overline{\text{MS}}$ scheme

$$
\begin{aligned}
&\mathcal{M}^{\rm LO}_{gg\to H^*\to HH}\times\left\{\underline{\frac{3}{16\pi^2}\frac{1}{\epsilon}\left(-2\lambda_{\rm 4H}-\lambda_{\rm 3H}+6\lambda_{\rm 3H}^2\frac{v^2}{m_H^2}\right)+\delta Z_{\kappa_{\rm 3H}}}\right.\\
&+\frac{3}{16\pi^2}\ln\frac{\mu_R^2}{m_H^2}\left[-2\lambda_{\rm 4H}-\lambda_{\rm 3H}+6\lambda_{\rm 3H}^2\frac{v^2}{m_H^2}\right]\\
&-\frac{9\lambda_{\rm 3H}^2}{8\pi^2}\frac{v^2}{s-m_H^2}\left[\beta\left(\ln\left(\frac{1-\beta}{1+\beta}\right)+i\pi\right)+\frac{s}{m_H^2}\left(1-\frac{2\pi}{3\sqrt{3}}\right)+\frac{5\pi}{3\sqrt{3}}-1\right]\\
&+\frac{3\lambda_{\rm 3H}^2}{8\pi^2}\frac{v^2}{m_H^2}\left(12-\frac{7\pi}{\sqrt{3}}\right)-\frac{9\lambda_{\rm 3H}^2v^2}{4\pi^2}C_0[m_H^2,m_H^2,s,m_H^2,m_H^2,m_H^2]\\
&\left.-\frac{3\lambda_{\rm 4H}}{16\pi^2}\left[\beta\left(\ln\left(\frac{1-\beta}{1+\beta}\right)+i\pi\right)+5-\frac{2\pi}{\sqrt{3}}\right]-\frac{3\lambda_{\rm 3H}}{16\pi^2}\right\},
\end{aligned}
$$

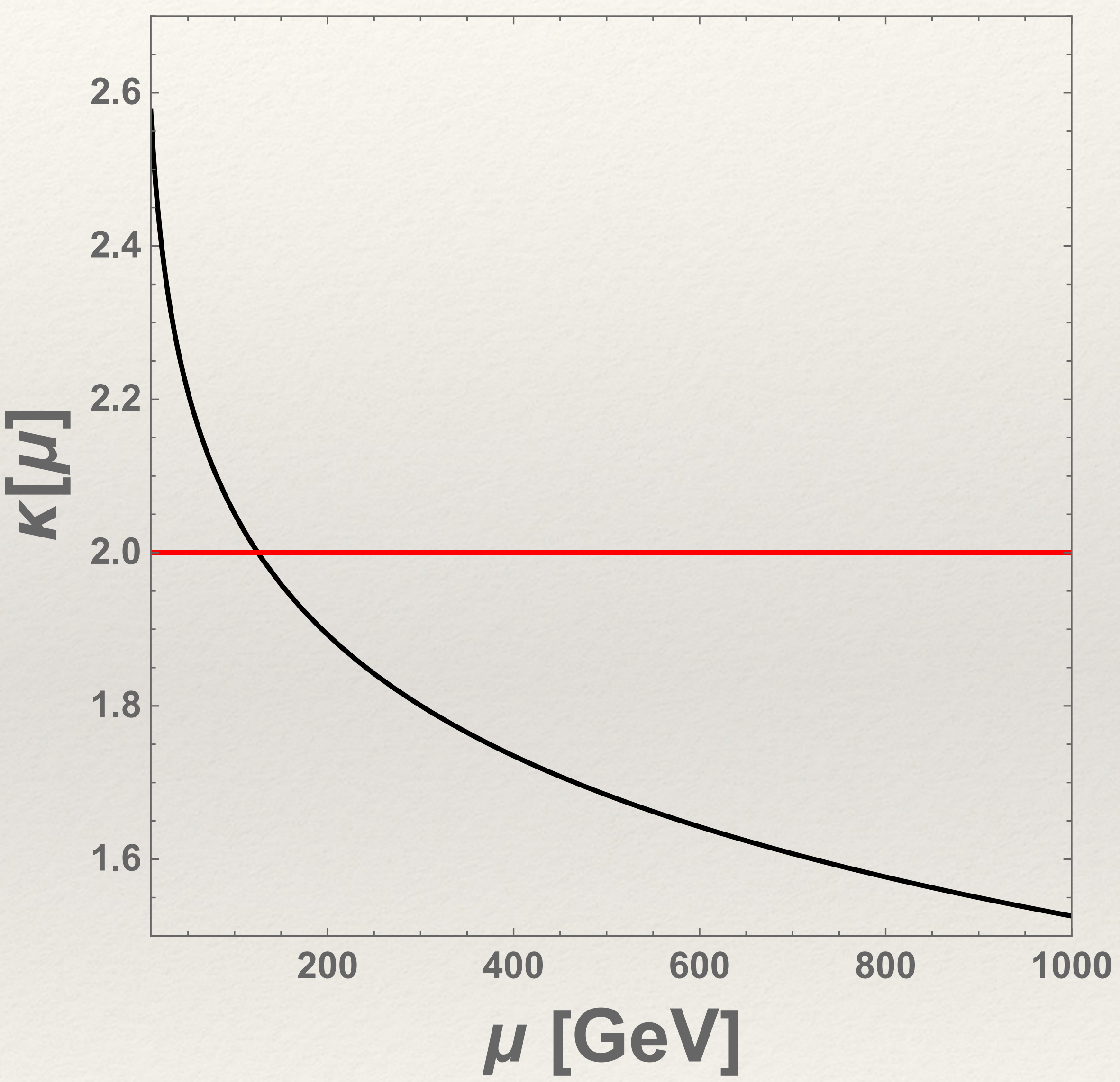
H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Running of $\kappa(\mu)$

$\kappa(\mu = 125 \text{ GeV}) = 0.5$



$\kappa(\mu = 125 \text{ GeV}) = 2$



Updated function forms

The λ dependent correction is

$$\delta\sigma^{\kappa_\lambda}_{\mathrm{ggF,EW}} = (0.075\kappa^4_{\lambda_{3H}} - 0.158\kappa^3_{\lambda_{3H}} - 0.006\kappa^2_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.058\kappa^2_{\lambda_{3H}} + 0.070\kappa_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.149\kappa_{\lambda_{4H}})\ \ \mathrm{fb}$$

$$\delta\sigma^{\kappa_\lambda}_{\mathrm{VBF,EW}} = (0.0215\kappa^4_{\lambda_{3H}} - 0.0324\kappa^3_{\lambda_{3H}} - 0.0019\kappa^2_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.0043\kappa^2_{\lambda_{3H}} + 0.0151\kappa_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.0211\kappa_{\lambda_{4H}})\ \ \mathrm{fb}$$

$\kappa_{\lambda_{3H}}$	$\kappa_{\lambda_{4H}}$	ggF			VBF		
		$\sigma^{\kappa_\lambda}_{\mathrm{LO}}$	$\sigma^{\kappa_\lambda}_{\mathrm{NNLO-FT}}$	$\delta\sigma^{\kappa_\lambda}_{\mathrm{EW}}$	$\sigma^{\kappa_\lambda}_{\mathrm{LO}}$	$\sigma^{\kappa_\lambda}_{\mathrm{NNNLO}}$	$\delta\sigma^{\kappa_\lambda}_{\mathrm{EW}}$
1	1	16.7	31.2	-0.225	1.71	1.69	-2.30×10^{-2}
3	1	8.59	18.4	1.28	3.59	3.53	8.35×10^{-1}
6	1	67.3	161	60.6	25.1	24.6	20.7
1	3	16.7	31.2	-0.393	1.71	1.69	-3.89×10^{-2}
1	6	16.7	31.2	-0.646	1.71	1.69	-6.27×10^{-2}
3	3	8.59	18.4	1.30	3.59	3.53	8.50×10^{-1}
6	6	67.3	161	61.0	25.1	24.6	20.7

The QCD corrections are significant in ggF, but not sensitive to κ_{3H}.

The EW corrections are 91% (82%) in ggF (VBF) for $\kappa_{3H} = 6$.

The dependence on λ_{4H} is weak.

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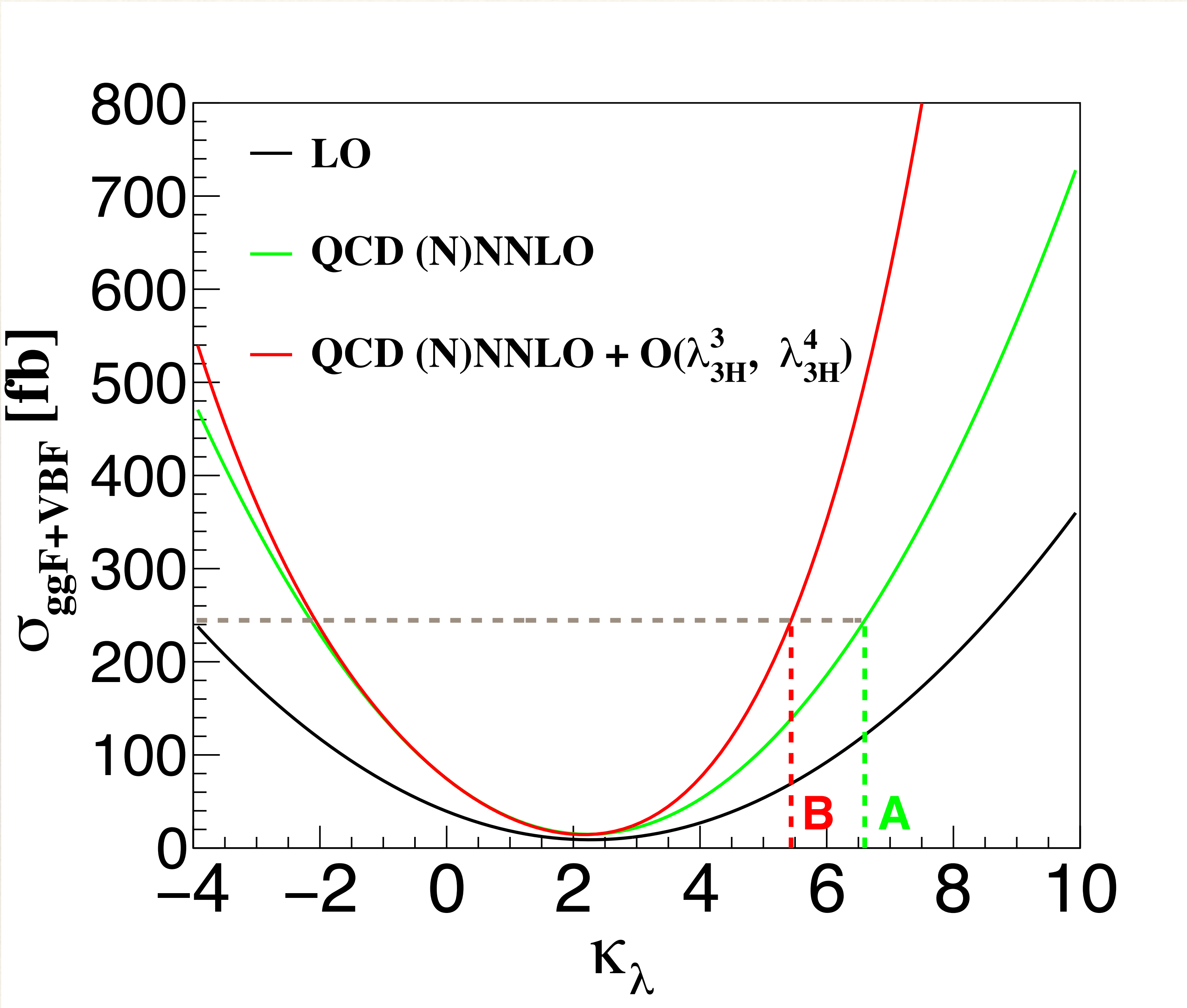
More stringent constraint



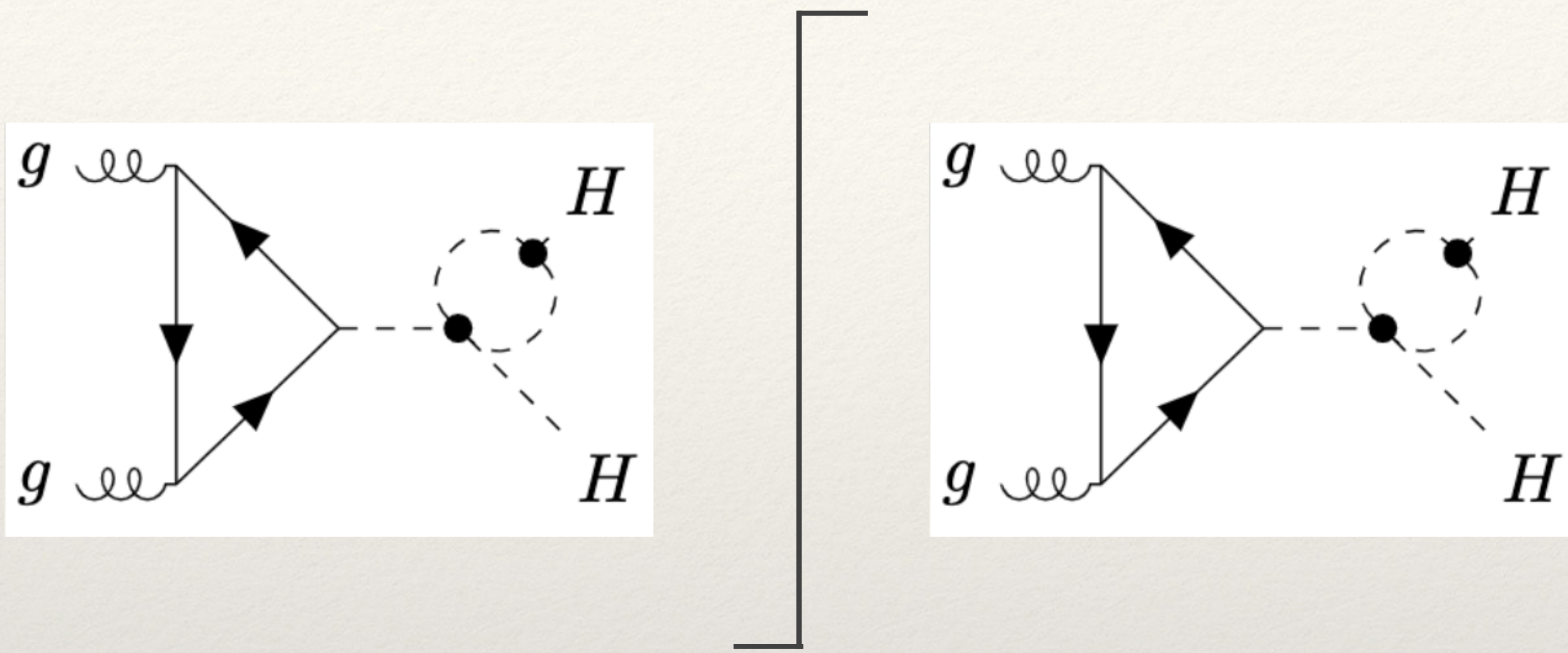
ATLAS (CMS) limit

6.6 (6.49)

5.4 (5.37)

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Sensitivity to κ_4



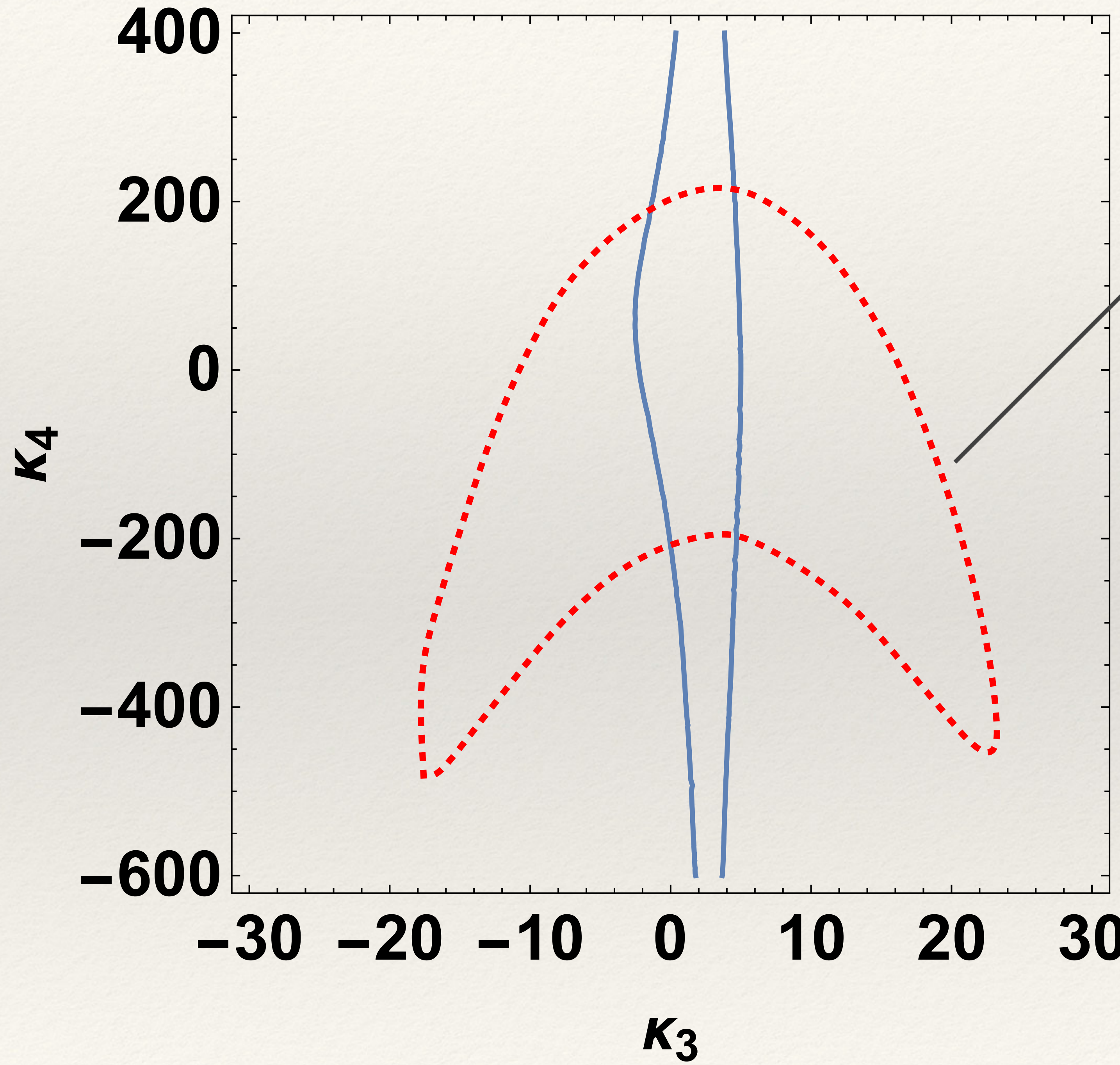
$$\delta\sigma^{\kappa_\lambda}_{\text{ggF,EW}} = (0.075\kappa^4_{\lambda_{3H}} - 0.158\kappa^3_{\lambda_{3H}} - 0.006\kappa^2_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.058\kappa^2_{\lambda_{3H}} + 0.070\kappa_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.149\kappa_{\lambda_{4H}}) \quad \text{fb}$$

$$+[0.887\kappa^6_{\lambda_{3H}} - 2.439\kappa^5_{\lambda_{3H}} + 3.200\kappa^3_{\lambda_{3H}} + \kappa_{\lambda_{4H}}(-0.579\kappa^4_{\lambda_{3H}} + 2.821\kappa^3_{\lambda_{3H}} - 2.870\kappa^2_{\lambda_{3H}})$$

$$+\kappa^2_{\lambda_{4H}}(0.135\kappa^2_{\lambda_{3H}} - 0.742\kappa_{\lambda_{3H}} - 1.179)] \times 10^{-3} \quad \text{fb}$$

This is only partial NNLO correction!

Sensitivity to κ_4



HHH constraint
2411.02040

A query from experimentalists

From: Zhengliang Guo <zhengliang.guo@cern.ch>
Sent: Monday, June 2, 2025 8:16 PM
To: j.wang@sdu.edu.cn <j.wang@sdu.edu.cn>

``While applying the correction for the ggF to HH cross section, I noticed that the formula presented in your report at the HH meeting differs from another reference. Specifically, when k3 =1, the effect of k4 appears negative in your report, whereas it is positive in the reference. I was wondering if you could clarify the reason for this discrepancy.''

Comparison with SMEFT

- SMEFT: Higgs field as a doublet, Wilson coefficients suppressed by Λ

$$V(\phi_b) = -\mu_b^2(\phi^\dagger\phi)_b + \lambda_b(\phi^\dagger\phi)_b^2 + \frac{d_6^b}{\Lambda^2}\left[(\phi^\dagger\phi)_b - \frac{v^2}{2}\right]^3 + \frac{d_8^b}{\Lambda^4}\left[(\phi^\dagger\phi)_b - \frac{v^2}{2}\right]^4 + \frac{d_{10}^b}{\Lambda^6}\left[(\phi^\dagger\phi)_b - \frac{v^2}{2}\right]^5 .$$

Borowka et al, 1811.12366

$$\mathcal{L}_{3H} = -\kappa_3\lambda v H^3 - (\delta Z_\lambda + \frac{3}{2}\kappa_3\delta Z_\phi + \bar{d}_6\delta Z_{d_6})\lambda v H^3$$

$$\kappa_3 \equiv 1 + \bar{d}_6 \qquad d_6^b = Z_{d_6}\frac{\lambda\Lambda^2}{v^2}\bar{d}_6$$

J.L. Ding, H.T. Li, JW, 2511.xxxxx

Comparison with SMEFT

$$\mathcal{L}_{4H} + \mathcal{L}_{5H} = -\frac{1}{4}\kappa_4 \lambda H^4 - \kappa_5 \frac{\lambda}{v} H^5 + \cdots$$

$$\kappa_4 \equiv 1 + 6\bar{d}_6 + \bar{d}_8$$

$$\kappa_5 \equiv \tfrac{1}{4}(3\bar{d}_6 + 2\bar{d}_8 + \bar{d}_{10})$$

If the new operators are renormalized in the $\overline{\text{MS}}$ scheme. We find that the amplitude of $gg \to H^* \to HH$ is different from that in HEFT.

$$\frac{i\mathcal{M}_{\text{SMEFT2}} - i\mathcal{M}_{\kappa_5}}{i\mathcal{M}_{\text{LO}}} = \frac{3m_H^2(\kappa_3 - 1)\Big[-(3-\sqrt{3}\pi)\kappa_3^2 + (\kappa_3 - 3\kappa_3^2 - \kappa_4)(1 + \ln\frac{\mu_R^2}{m_H^2})\Big]}{32\pi^2 v^2 \kappa_3}$$

J.L. Ding, H.T. Li, JW, 2511.xxxxx

Comparison with SMEFT

If we require that the amplitude remains the same, we have to change the renormalization condition. The renormalization of the dimension-six operator is not in $\overline{\text{MS}}$ any more.

$$(\delta Z_{d_6})_{\text{finite}} = (\delta Z_\lambda)_{\text{finite}}$$

$$\delta Z_\lambda = -\frac{3\lambda_3}{16\pi^2}\left[\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{m^2} + 1\right] + \frac{3\lambda_4}{16\pi^2}\left[\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{m^2} + 1\right] + \frac{3\lambda_3^2 v^2}{8\pi^2 m^2}\left[\frac{3}{\epsilon} + 3\ln\frac{\mu_R^2}{m^2} + 6 - \sqrt{3}\pi\right],$$

$\overline{\text{MS}}$ in HEFT is not $\overline{\text{MS}}$ in SMEFT!

J.L. Ding, H.T. Li, JW, 2511.xxxxx

Summary

The SM is a master piece in human history. It has been tested by a lot of experiments at very high precision level.

However, the Higgs sector still needs more precise comparison between theories and experiments.

Higher-order corrections provide more precise estimate of the dependence on Higgs self-couplings.

Thanks a lot for your attention!

Back-up slides

HEFT

$$\begin{aligned}\mathcal{L}_4 = & -a_{dd\mathcal{V}\mathcal{V}1}\frac{\partial^\mu H \partial^\nu H}{v^2}\mathrm{Tr}[\mathcal{V}_\mu \mathcal{V}_\nu] - a_{dd\mathcal{V}\mathcal{V}2}\frac{\partial^\mu H \partial_\mu H}{v^2}\mathrm{Tr}[\mathcal{V}^\nu \mathcal{V}_\nu] \\
& + \left(a_{11} + a_{H11}\frac{H}{v} + a_{HH11}\frac{H^2}{v^2}\right)\mathrm{Tr}[\mathcal{D}_\mu \mathcal{V}^\mu \mathcal{D}_\nu \mathcal{V}^\nu] - \frac{m_{\mathrm{H}}^2}{4}\left(2a_{H\mathcal{V}\mathcal{V}}\frac{H}{v} + a_{HH\mathcal{V}\mathcal{V}}\frac{H^2}{v^2}\right)\mathrm{Tr}[\mathcal{V}^\mu \mathcal{V}_\mu] \\
& - \left(a_{HWW}\frac{H}{v} + a_{HHWW}\frac{H^2}{v^2}\right)\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] + i\left(a_{d2} + a_{Hd2}\frac{H}{v}\right)\frac{\partial^\nu H}{v}\mathrm{Tr}[\hat{W}_{\mu\nu}\mathcal{V}^\mu] \\
& + \left(a_{\Box\mathcal{V}\mathcal{V}} + a_{H\Box\mathcal{V}\mathcal{V}}\frac{H}{v}\right)\frac{\Box H}{v}\mathrm{Tr}[\mathcal{V}_\mu \mathcal{V}^\mu] + \left(a_{d3} + a_{Hd3}\frac{H}{v}\right)\frac{\partial^\nu H}{v}\mathrm{Tr}[\mathcal{V}_\nu \mathcal{D}_\mu \mathcal{V}^\mu] \\
& + \left(a_{\Box\Box} + a_{H\Box\Box}\frac{H}{v}\right)\frac{\Box H \Box H}{v^2} + a_{dd\Box}\frac{\partial^\mu H \partial_\mu H \Box H}{v^3} \\
& + \left(a_{Hdd}\frac{m_{\mathrm{H}}^2}{v^2} + a_{ddW}\frac{m_{\mathrm{W}}^2}{v^2} + a_{ddZ}\frac{m_{\mathrm{Z}}^2}{v^2}\right)\frac{H}{v}\partial^\mu H \partial_\mu H.\end{aligned}$$

Brivio et al, JHEP, 1403, 024
Gavela, Kanshin, Machado, Saa, JHEP, 1503, 043

Renormalization:

- Background field method: Guo, Ruiz-Femenia, Sanz-Cillero, PRD92,074005(2015)
  Buchalla, et al, NPB, 928,93 (2018), PRD104,076005(2021)
- Scattering amplitude: Herrero, Morales, PRD106,073008 (2022)

Renormalization

In the SM, the Lagrangian for the Higgs sector can be written as

$$\mathscr{L}_{\mathrm{H}} = (D_\mu \phi_0)^\dagger (D^\mu \phi_0) + \mu_0^2 (\phi_0^\dagger \phi_0) - \lambda_0 (\phi_0^\dagger \phi_0)^2$$

where ϕ_0 denotes the bare Higgs doublet and D_μ is the covariant derivative. The relations between the bare fields and couplings, and their renormalized counterparts, are given by $\phi_0 = Z_\phi^{1/2} \phi$, $\mu_0^2 = Z_{\mu^2} \mu^2$, and $\lambda_0 = Z_\lambda \lambda$.

The EW gauge symmetry is spontaneously broken once the Higgs field develops a non-vanishing vacuum expectation value v. Taking the unitary gauge, we write the Higgs field as

$$\phi = \frac{1}{\sqrt{2}} \left(0,\, Z_v v + H\right)^T$$

Our strategy is equivalent to the application of HEFT in Higgs boson pair production.