The nonleptonic B weak decays to radially excited D mesons

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2025 年 10 月 30 日

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Introduction

- The ground states $D_{(s)}$ and $D^*_{(s)}$ have been well determined in experiments, their produtions and decays have been extensively studied.
- While the first radially excited states $D_{(s)}(2S)$ and $D^*_{(s)}(2S)$ have not been well confirmed.
- For example both $D(2550)$ and $D_J(2580)$ are considered as the candidates of $D(2^1S_0)$ state.
- $D_{s0}(2590)$ observed by LHCb is suggested to a candidate of $D_s(2^1S_0)$ state. Some theoretical works suggest that it may not be a pure state but rather has D^*K component.

Introduction

- $D_1^*(2680)^0, D^*(2650)^0$ and $D_1^*(2600)^0$ observed by LHCb are considered as the candidated of netrual $D(2^3S_1)$. The $D^*(2640)^\pm$ discovered by Delphi have masses consistent with the predictions of charged $D(2^3S_1)$, while it has not been confirmed in any other experiments.
- Except for the assignment of $D_s(2^3S_1)$ to $D_{s1}^*(2700)^\pm$, $D_s(1^3D_1)$ and a mixture of $D_s(2^3S_1)$ and $D_s(1^3D_1)$ are also been proposed.
- We will assume these several particles discovered in experiments as the corresponding first excited D mesons in the calcualtions:
  $D_0(2550) \to D(2^1S_0),\ D_{s0}(2590)^\pm \to D_s(2^1S_0),$
  $D_1^*(2600)^0, D^*(2640)^\pm \to D(2^3S_1),\ D_{s1}^*(2700)^\pm \to D_s(2^3S_1).$

Introduction

At the end of the twentieth century, Jaus put forward the covariant light-front quark model (CLFQM). The CLFQM has some unique advantages:

- The light-front wave functions descrbing the hadron through quark and gluon degrees of freedom can preserve a Lorentz invariant formalism.
- The final state meson at $q^2 = 0$ is usually relativistic. The CLFQM with relativistic effects involved is suitable to study hadronic transition form factors.

The CLFQM general calculation procedure

The CLFQM general calculation procedure

The Bauer-Stech-Wirble (BSW) form factors for $B \to D$ and $B \to D^*$ transitions are defined as follows,

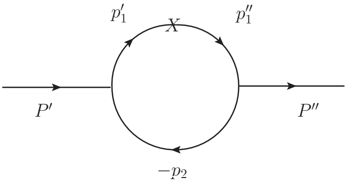
$$
\begin{aligned}
\left\langle D\left(P''\right)|V_\mu|B\left(P'\right)\right\rangle &= \left(P_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu\right) F_1^{BD}\left(q^2\right) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0^{BD}\left(q^2\right), \\
\left\langle D^*\left(P'', \varepsilon''\right)|V_\mu|B\left(P'\right)\right\rangle &= -\frac{1}{m_{D^*} + m_B} \epsilon_{\mu\nu\alpha\beta} \varepsilon''^{*\nu} P^\alpha q^\beta V^{BD^*}\left(q^2\right), \\
\left\langle D^*\left(P'', \varepsilon''\right)|A_\mu|B\left(P'\right)\right\rangle &= i\Bigg\{\left(m_{D^*} + m_B\right) \varepsilon''^{*}_\mu A_1^{BD^*}\left(q^2\right) - \frac{\varepsilon''^{*} \cdot P}{m_{D^*} + m_B} P_\mu A_2^{BD^*}\left(q^2\right) \\
&\quad -2m_{D^*} \frac{\varepsilon''^{*} \cdot P}{q^2} q_\mu \left[A_3^{BD^*}\left(q^2\right) - A_0^{BD^*}\left(q^2\right)\right]\Bigg\},
\end{aligned}
$$

where $P = P' + P''$, $q = P' - P''$, ϵ is the polarization vector. The four-momentum of the initial (final) meson is $P' = p_1' + p_2$ $(P'' = p_1'' + p_2)$.

The CLFQM general calculation procedure

The decay amplitude in the lowest order for the transitions $B \to D$ and $B \to D^*$

$$\mathcal{B}_\mu^{BD} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{h_B' h_D''}{N_1' N_1'' N_2} S_\mu^{BD},$$
$$\mathcal{B}_\mu^{BD^*} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{h_B' \left(i h_{D^*}''\right)}{N_1' N_1'' N_2} S_{\mu\nu}^{BD^*} \varepsilon''^{*\nu}, \tag{1}$$



- $N_1^{\prime(\prime\prime)} = p_1^{\prime(\prime\prime)2} - m_1^{\prime(\prime\prime)2}$, $N_2 = p_2^2 - m_2^2$ arise from the quark propagators.

The traces S_{μ}^{BD} and $S_{\mu\nu}^{BD^*}$ can be obtained directly using Lorentz contraction,

$$
\begin{aligned}
S_{\mu}^{BD} &= \mathrm{Tr}\left[\gamma_5\left(\not p_1'' + m_1''\right)\gamma_\mu\left(\not p_1' + m_1'\right)\gamma_5\left(-\not p_2 + m_2\right)\right],\\
S_{\mu\nu}^{BD^*} &= \left(S_V^{BD^*} - S_A^{BD^*}\right)_{\mu\nu}\\
&= \mathrm{Tr}\left[\left(\gamma_\nu - \frac{1}{W_V''}\left(p_1'' - p_2\right)_\nu\right)\left(\not p_1'' + m_1''\right)\left(\gamma_\mu - \gamma_\mu\gamma_5\right)\left(\not p_1' + m_1'\right)\gamma_5\left(-\not p_2 + m_2\right)\right],
\end{aligned}
\tag{2}
$$

The covariant vertex function h_M'' with $M = D(1S, 2S), D^*(1S, 2S)$ is defined as

$$
\begin{aligned}
h_M'' &= \left(M''^2 - M_0''^2\right)\sqrt{\frac{x_1 x_2}{N_c}}\frac{1}{\sqrt{2}\widetilde{M}_0''}\varphi,\\
M_0''^2 &= \left(e_1'' + e_2\right)^2 = \frac{p_\perp'^2 + m_1''^2}{x_1} + \frac{p_\perp^2 + m_2^2}{x_2}, \quad \widetilde{M}_0'' = \sqrt{M_0''^2 - \left(m_1'' - m_2\right)^2}.
\end{aligned}
\tag{3}
$$

The CLFQM general calculation procedure

The phenomenological Gaussian-type wave function φ depicts the light-front momentum distribution amplitude for the S-wave mesons,

$$\varphi(1S) = 4\left(\frac{\pi}{\beta^2}\right)^{\frac{3}{4}}\sqrt{\frac{dp_z}{dx_2}}\exp\left(-\frac{p_z^2+p_\perp^2}{2\beta^2}\right),$$

$$\varphi(2S) = 4\left(\frac{\pi}{\beta^2}\right)^{\frac{3}{4}}\sqrt{\frac{dp_z}{dx_2}}\exp\left(-\frac{p_z^2+p_\perp^2}{2\beta^2}\right)(3-2\frac{p_z^2+p_\perp^2}{\beta^2}),$$

- β is a phenomenological parameter and can be fixed by fitting the corresponding decay constant.

Transition Form Factors

Input parameters:

- The constituent quark masses(GeV): $m_c = 1.4,\ m_s = 0.37,\ m_{u,d} = 0.25;$
- The masses of the initial and the final mesons(GeV):

  $m_\pi = 0.140, m_K = 0.494,\ m_\rho = 0.775,\ m_{K^*} = 0.892,\ m_D = 1.86966,$

  $m_{D(2S)} = 2.549,\ m_{D_s} = 1.96835,\ m_{D_s(2S)} = 2.591,$

  $m_{D_s^{\star\pm}} = 2.1122, m_{D^{\star 0}} = 2.0068, m_{D^{\star\pm}} = 2.0102,\ m_B = 5.279,$

  $m_{D_s^{\star\pm}(2S)} = 2.732,\ m_{D^{\star 0}(2S)} = 2.627, m_{D^{\star\pm}(2S)} = 2.637, m_{\bar{B}_s^0} = 5.4154.$
- The CKM matrix elements: $V_{cd} = 0.221 \pm 0.004,\ V_{cs} = 0.975 \pm 0.006,$

  $V_{ud} = 0.97373 \pm 0.00031;$

Transition Form Factors

Input parameters:

- The shape parameters fitted by the decay constants:

$$\beta_{D_s^{\star\pm}(1S)} = 0.534^{+0.014}_{-0.014}, \beta_{D^{\star 0}(1S)} = 0.500^{+0.140}_{-0.187}, \beta_D = 0.466^{+0.022}_{-0.021},$$

$$\beta_{D_s^{\star\pm}(2S)} = 0.473^{+0.041}_{-0.041}, \beta_{D^{\star 0}(2S)} = 0.456^{+0.004}_{-0.003}, \beta_{D_s} = 0.600^{+0.026}_{-0.025},$$

$$\beta_{D^{\star\pm}(1S)} = 0.502^{+0.041}_{-0.041}, \beta_{D^{\star\pm}(2S)} = 0.453^{+0.004}_{-0.003},\ \beta_{\bar{B}_s^0} = 0.626^{+0.045}_{-0.045},$$

$$\beta_{D(2S)} = 0.297^{+0.041}_{-0.041}, \beta_{D_s(2S)} = 0.422^{+0.026}_{-0.025}, \beta_B = 0.555^{+0.060}_{-0.060}.$$

- mean life$(10^{-12}s)$: $\tau_{B^0} = (1.519 \pm 0.004)$, $\tau_{B_s^0} = (1.520 \pm 0.005)$,

$\tau_{B^\pm} = (1.638 \pm 0.004)$.

Transition Form Factors

- All the calculations are carried out within the $q^+ = 0$ reference frame, where the form factors can only be obtained at spacelike momentum transfers $q^2 = -q_\perp^2 \leq 0$,
- The parameterized form factors are extrapolated from the space-like region to the time-like region by using

$$F\left(q^2\right) = \frac{F(0)}{\left(1 - q^2/m^2\right)\left[1 - a\left(q^2/m^2\right) + b\left(q^2/m^2\right)^2\right]}. \tag{4}$$

- $F(q^2)$ denotes different form factors, such as $F_1(q^2)$, $F_0(q^2)$, $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$.

Transition Form Factors

- The form factors of the transtions $B_{(s)} \to D_{(s)}(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of $B_{(s)}$ and final state mesons.

	$F(0)$	$F(q^2_{max})$	a	b
F_1^{BD}	$0.66^{+0.00+0.01}_{-0.01-0.01}$	$0.81^{+0.01+0.01}_{-0.00-0.01}$	$0.80^{+0.01+0.04}_{-0.02-0.04}$	$0.86^{+0.01+0.02}_{-0.01-0.02}$
F_0^{BD}	$0.66^{+0.00+0.01}_{-0.01-0.01}$	$0.70^{+0.02+0.01}_{-0.03-0.01}$	$0.46^{+0.14+0.00}_{-0.12-0.01}$	$0.77^{+0.01+0.05}_{-0.01-0.05}$
$F_1^{BD(2S)}$	$0.26^{+0.01+0.01}_{-0.02-0.02}$	$0.34^{+0.02+0.01}_{-0.03-0.03}$	$0.99^{+0.04+0.16}_{-0.10-0.18}$	$0.66^{+0.07+0.17}_{-0.12-0.15}$
$F_0^{BD(2S)}$	$0.26^{+0.01+0.02}_{-0.01-0.02}$	$0.32^{+0.03+0.02}_{-0.00-0.04}$	$0.65^{+0.03+0.05}_{-0.01-0.04}$	$-0.24^{+0.02+0.01}_{-0.03-0.03}$
$F_1^{B_sD_s}$	$0.67^{+0.00+0.01}_{-0.00-0.01}$	$0.81^{+0.00+0.00}_{-0.01-0.01}$	$0.82^{+0.00+0.02}_{-0.01-0.02}$	$0.96^{+0.01+0.03}_{-0.02-0.03}$
$F_0^{B_sD_s}$	$0.67^{+0.00+0.01}_{-0.00-0.01}$	$0.71^{+0.01+0.01}_{-0.02-0.01}$	$0.48^{+0.00+0.08}_{-0.01-0.08}$	$0.85^{+0.02+0.06}_{-0.02-0.06}$
$F_1^{B_sD_s(2S)}$	$0.26^{+0.02+0.02}_{-0.02-0.02}$	$0.29^{+0.02+0.01}_{-0.02-0.02}$	$0.59^{+0.12+0.05}_{-0.16-0.01}$	$0.35^{+0.11+0.04}_{-0.13-0.02}$
$F_0^{B_sD_s(2S)}$	$0.26^{+0.01+0.01}_{-0.02-0.02}$	$0.25^{+0.02+0.01}_{-0.02-0.02}$	$-0.09^{+0.22+0.22}_{-0.14-0.26}$	$-0.07^{+0.13+0.18}_{-0.21-0.08}$

Transition Form Factors

- The form factors of the transtions $B \to D^*(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of B and final state mesons.

F	F(0)	$F(q^2_{max})$	a	b
V^{BD^*}	$0.77^{+0.00+0.01}_{-0.00-0.01}$	$0.94^{+0.00+0.01}_{-0.00-0.01}$	$0.78^{+0.00+0.03}_{-0.01-0.03}$	$0.82^{+0.04+0.14}_{-0.08-0.16}$
$A_0^{BD^*}$	$0.75^{+0.00+0.06}_{-0.01-0.02}$	$0.79^{+0.00+0.05}_{-0.02-0.02}$	$0.17^{+0.01+0.01}_{-0.01-0.00}$	$0.12^{+0.07+0.02}_{-0.06-0.03}$
$A_1^{BD^*}$	$0.67^{+0.00+0.01}_{-0.01-0.11}$	$0.76^{+0.00+0.01}_{-0.01-0.01}$	$0.38^{+0.01+0.01}_{-0.01-0.01}$	$0.19^{+0.05+0.09}_{-0.05-0.09}$
$A_2^{BD^*}$	$0.58^{+0.00+0.00}_{-0.01-0.02}$	$0.69^{+0.01+0.01}_{-0.01-0.02}$	$0.68^{+0.02+0.00}_{-0.02-0.00}$	$0.66^{+0.25+0.10}_{-0.12-0.16}$
$V^{BD^*(2S)}$	$0.19^{+0.05+0.01}_{-0.06-0.01}$	$0.19^{+0.01+0.03}_{-0.04-0.01}$	$0.11^{+0.05+0.03}_{-0.06-0.05}$	$0.34^{+0.00+0.03}_{-0.04-0.06}$
$A_0^{BD^*(2S)}$	$0.27^{+0.04+0.01}_{-0.05-0.00}$	$0.28^{+0.00+0.01}_{-0.02-0.00}$	$0.24^{+0.05+0.00}_{-0.06-0.00}$	$-0.07^{+0.15+0.03}_{-0.29-0.06}$
$A_1^{BD^*(2S)}$	$0.16^{+0.04+0.01}_{-0.05-0.00}$	$0.15^{+0.05+0.02}_{-0.04-0.02}$	$-0.23^{+0.04+0.00}_{-0.04-0.00}$	$0.17^{+0.03+0.01}_{-0.01-0.01}$
$A_2^{BD^*(2S)}$	$-0.05^{+0.04+0.01}_{-0.04-0.00}$	$0.01^{+0.00+0.00}_{-0.00-0.00}$	$-1.10^{+0.02+0.01}_{-0.01-0.01}$	$-0.63^{+0.02+0.01}_{-0.01-0.00}$

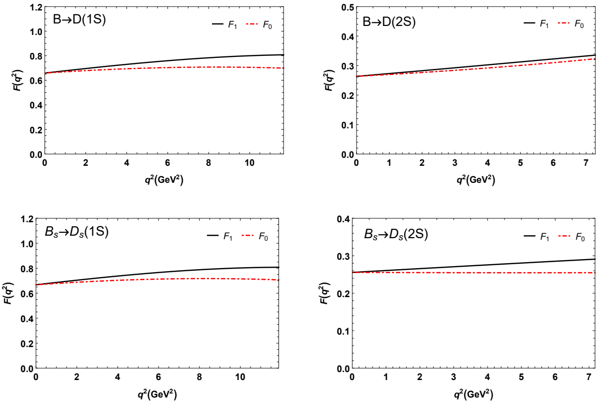
Transition Form Factors

- The form factors of the transtions $B_s \to D_s^*(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of B_s and final state mesons..

F	F(0)	$F(q^2_{max})$	a	b
$V^{B_sD_s^*}$	$0.78^{+0.01+0.01}_{-0.01-0.01}$	$0.87^{+0.00+0.00}_{-0.00-0.01}$	$0.86^{+0.01+0.04}_{-0.01-0.04}$	$1.11^{+0.01+0.02}_{-0.01-0.02}$
$A_0^{B_sD_s^*}$	$0.74^{+0.01+0.01}_{-0.01-0.01}$	$0.69^{+0.01+0.01}_{-0.01-0.00}$	$0.23^{+0.01+0.01}_{-0.01-0.01}$	$0.21^{+0.00+0.01}_{-0.00-0.01}$
$A_1^{B_sD_s^*}$	$0.66^{+0.01+0.01}_{-0.02-0.02}$	$0.95^{+0.00+0.00}_{-0.00-0.01}$	$0.81^{+0.01+0.02}_{-0.01-0.02}$	$0.93^{+0.01+0.01}_{-0.01-0.01}$
$A_2^{B_sD_s^*}$	$0.57^{+0.00+0.00}_{-0.01-0.00}$	$0.68^{+0.01+0.00}_{-0.01-0.00}$	$0.80^{+0.00+0.03}_{-0.01-0.03}$	$0.96^{+0.01+0.03}_{-0.01-0.02}$
$V^{B_sD_s^*(2S)}$	$0.26^{+0.03+0.04}_{-0.03-0.04}$	$0.28^{+0.00+0.02}_{-0.09-0.10}$	$0.25^{+0.02+0.04}_{-0.01-0.04}$	$0.30^{+0.03+0.04}_{-0.01-0.00}$
$A_0^{B_sD_s^*(2S)}$	$0.31^{+0.02+0.01}_{-0.03-0.02}$	$0.33^{+0.00+0.08}_{-0.01-0.07}$	$0.21^{+0.01+0.03}_{-0.05-0.00}$	$-0.09^{+0.04+0.10}_{-0.02-0.14}$
$A_1^{B_sD_s^*(2S)}$	$0.21^{+0.02+0.03}_{-0.03-0.03}$	$0.20^{+0.00+0.01}_{-0.07-0.06}$	$-0.16^{+0.20+0.04}_{-0.17-0.02}$	$0.12^{+0.00+0.12}_{-0.01-0.07}$
$A_2^{B_sD_s^*(2S)}$	$-0.01^{+0.02+0.06}_{-0.02-0.06}$	$0.01^{+0.00+0.03}_{-0.01-0.01}$	$-4.03^{+0.51+0.75}_{-0.47-1.38}$	$4.31^{+0.00+2.13}_{-0.00-2.44}$

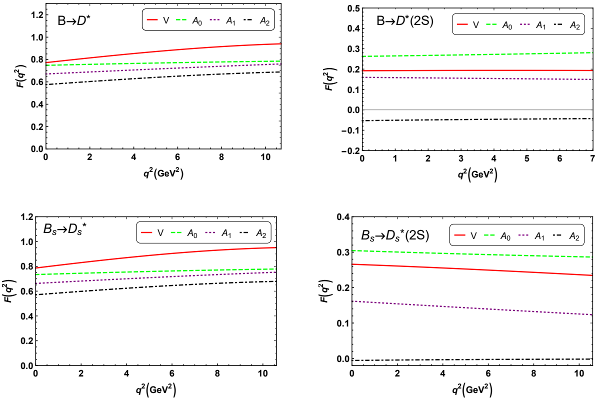
Transition Form Factors

图: The q^2-dependence of the $B_{(s)} \to D_{(s)}(1S, 2S)$ transtion form factors.



Transition Form Factors

图: The q^2-dependence of the $B_{(s)} \to D^*_{(s)}(1S,2S)$ transtion form factors.



Non-leptonic Decays

$$B \to D^{(*)}(1S, 2S)M$$

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

The effective Hamiltonian for the decays $B_{(s)} \to D^{(*)}_{(s)}(1S, 2S)M$ with $M = \pi, K, \rho, K^*$ can be written as

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V^*_{cb} V_{uq} \{ C_1 Q_1 + C_2 Q_2 \} , \tag{5}$$

The local tree four-quark operators $Q_{1,2}$ are defined by :

$$Q_1 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha] , \tag{6}$$

$$Q_2 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\beta] , \tag{7}$$

where α and β are color indices.

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

Combining with the form factors, we can obtain the partial widths for our considered non-leptonic decays, which are written as

$$\Gamma\left(B \to DP\right) = \frac{G_F^2\left(m_B^2 - m_D^2\right)^2 |\vec{p}_c|}{16\pi m_B^2} \left|V_{uq}^* V_{cb}\right|^2 |a_1|^2 f_P^2 F_0^2\left(m_P^2\right), \tag{8}$$

$$\Gamma\left(B \to D^* P\right) = \frac{G_F^2 |\vec{p}_c|^3}{4\pi} \left|V_{uq}^* V_{cb}\right|^2 |a_1|^2 f_P^2 A_0^2\left(m_P^2\right), \tag{9}$$

$$\Gamma\left(B \to DV\right) = \frac{G_F^2 |\vec{p}_c|^3}{4\pi} \left|V_{uq}^* V_{cb}\right|^2 |a_1|^2 f_V^2 F_+^2\left(m_V^2\right), \tag{10}$$

$$\Gamma\left(B \to D^* V\right) = \frac{G_F^2 |\vec{p}_c|}{16\pi m_B^2} \left|V_{uq}^* V_{cb}\right|^2 \left(|H_0|^2 + |H_+|^2 + |H_-|^2\right), \tag{11}$$

Here $\vec{p}_c$ is the momentum of either of the two final state meson in the B rest frame and $H_{0,\pm}$ are the helicity amplitudes,

$$H_0 = \frac{\mathrm{i} f_V a_1}{2m_{D^*}} \Big[\left(m_B^2 - m_{D^*}^2 - m_V^2\right)\left(m_B + m_{D^*}\right) A_1^{BD^*}\left(m_V^2\right) - \frac{4m_B^2 p_c^2}{m_B + m_{D^*}} A_2^{BD^*}\left(m_V^2\right) \Big], \tag{12}$$

$$H_\pm = \mathrm{i} f_V m_V a_1 \left[-\left(m_B + m_{D^*}\right) A_1^{BD^*}\left(m_V^2\right) \mp \frac{2m_B p_c}{m_B + m_{D^*}} V^{BD^\star}\left(m_V^2\right) \right]. \tag{13}$$

The polarization fractions are defined as $f_{L,\parallel,\perp} = \frac{H_{0,\parallel,\perp}}{H_0 + H_\parallel + H_\perp}$, where $H_\parallel$ and $H_\perp$ are parallel and perpendicular amplitudes, respectively, and can be obtained through $H_{\parallel,\perp} = \frac{(H_- \pm H_+)}{\sqrt{2}}$.

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

- The branching ratios of the decays $B \to D(1S, 2S)M$ with $M = \pi, K, \rho, K^*$.

Modes	This work	Bethe-Salpeter	QCDF	PQCD	Exp.	Unit
$\bar{B}^0 \to D^+(1S)\pi^-$	$4.37^{+0.01+0.01+0.01}_{-0.01-0.09-0.01}$	3.24	3.93	2.69	2.52	(10^{-3})
$\bar{B}^0 \to D^+(1S)K^-$	$0.34^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$	0.245	0.30	0.243	0.186	
$\bar{B}^0 \to D^+(1S)\rho^-$	$10.09^{+0.03+0.10+0.25}_{-0.03-0.20-0.28}$	7.91	10.42	6.96	7.6	
$\bar{B}^0 \to D^+(1S)K^{*-}$	$0.56^{+0.00+0.01+0.01}_{-0.00-0.00-0.01}$	0.431	0.53	0.407	0.45	
$\bar{B}^0 \to D^+(2S)\pi^-$	$4.76^{+0.01+0.48+0.33}_{-0.01-0.62-0.57}$	0.458	–	–	–	(10^{-4})
$\bar{B}^0 \to D^+(2S)K^-$	$0.37^{+0.00+0.04+0.04}_{-0.00-0.05-0.03}$	0.034	–	–	–	
$\bar{B}^0 \to D^+(2S)\rho^-$	$10.54^{+0.03+1.06+0.73}_{-0.03-1.37-1.26}$	1.03	–	–	–	
$\bar{B}^0 \to D^+(2S)K^{*-}$	$0.58^{+0.00+0.06+0.04}_{-0.00-0.08-0.07}$	0.0557	–	–	–	
$B^- \to D^0(1S)\pi^-$	$4.71^{+0.01+0.02+0.12}_{-0.01-0.09-0.13}$	3.49	–	5.11	4.68	(10^{-3})
$B^- \to D^0(1S)K^-$	$0.37^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$	0.264	–	0.400	0.363	
$B^- \to D^0(1S)\rho^-$	$10.88^{+0.03+0.00+0.27}_{-0.03-0.20-0.31}$	8.40	–	11.3	13.4	
$B^- \to D^0(1S)K^{*-}$	$0.60^{+0.00+0.00+0.02}_{-0.00-0.01-0.02}$	0.466	–	0.696	0.53	
$B^- \to D^0(2S)\pi^-$	$5.14^{+0.01+0.52+0.36}_{-0.01-0.67-0.61}$	0.488	–	–	–	(10^{-4})
$B^- \to D^0(2S)K^-$	$0.40^{+0.00+0.04+0.03}_{-0.00-0.05-0.05}$	0.0362	–	–	–	
$B^- \to D^0(2S)\rho^-$	$11.36^{+0.03+1.14+0.79}_{-0.03-1.48-1.35}$	1.09	–	–	–	
$B^- \to D^0(2S)K^{*-}$	$0.62^{+0.00+0.06+0.04}_{-0.00-0.08-0.07}$	0.0595	–	–	–	

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

- The branching ratios(10^{-3}) of the decays $B \to D^{*}(1S, 2S)(\pi, \rho, K^{(*)})$.

Modes	$\bar{B}^0 \to D^{*+}(1S)\pi^-$	$\bar{B}^0 \to D^{*+}(1S)K^-$	$\bar{B}^0 \to D^{*+}(1S)\rho^-$	$\bar{B}^0 \to D^{*+}(1S)K^{*-}$
This work	$5.25^{+0.01+0.05+0.02}_{-0.01-0.19-0.40}$	$0.40^{+0.00+0.00+0.00}_{-0.00-0.01-0.03}$	$14.32^{+0.04+0.14+0.07}_{-0.04-0.50-1.05}$	$0.83^{+0.00+0.01+0.00}_{-0.00-0.03-0.06}$
BS	3.80	0.281	8.73	0.758
RIQ	–	–	14.24	0.83
PQCD	2.60	0.237	7.94	0.488
Exp.	2.66	0.216	6.8	0.33
Modes	$\bar{B}^0 \to D^{*+}(2S)\pi^-$	$\bar{B}^0 \to D^{*+}(2S)K^-$	$\bar{B}^0 \to D^{*+}(2S)\rho^-$	$\bar{B}^0 \to D^{*+}(2S)K^{*-}$
This work	$0.45^{+0.00+0.16+0.01}_{-0.00-0.16-0.01}$	$0.03^{+0.00+0.01+0.00}_{-0.00-0.01-0.01}$	$1.09^{+0.00+0.40+0.02}_{-0.00-0.41-0.02}$	$0.06^{+0.00+0.02+0.00}_{-0.00-0.00?2-0.00}$
BS	0.038	0.00274	0.0267	0.00162
RIQ	–	–	1.12	0.07

Modes	$B^- \to D^{*0}(1S)\pi^-$	$B^- \to D^{*0}(1S)K^-$	$B^- \to D^{*0}(1S)\rho^-$	$B^- \to D^{*0}(1S)K^{*-}$
This work	$5.67^{+0.01+0.05+0.98}_{-0.01-0.19-2.53}$	$0.43^{+0.00+0.00+0.07}_{-0.00-0.01-0.19}$	$15.45^{+0.04+0.13+2.30}_{-0.04-0.53-6.52}$	$0.89^{+0.00+0.01+0.13}_{-0.00-0.03-0.37}$
BS	4.11	0.304	8.73	0.846
RIQ	–	–	15.39	0.90
PQCD	5.04	0.398	11.7	0.682
Exp.	5.17	0.419	9.8	0.81
Modes	$B^- \to D^{*0}(2S)\pi^-$	$B^- \to D^{*0}(2S)K^-$	$B^- \to D^{*0}(2S)\rho^-$	$B^- \to D^{*0}(2S)K^{*-}$
This work	$0.46^{+0.00+0.16+0.01}_{-0.00-0.17-0.01}$	$0.03^{+0.00+0.01+0.00}_{-0.00-0.01-0.00}$	$1.10^{+0.00+0.04+0.02}_{-0.00-0.04-0.02}$	$0.06^{+0.00+0.02+0.00}_{-0.00-0.02-0.00}$
BS	0.041	0.00295	0.0287	0.00173
RIQ	–	–	1.181	0.070

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

- One can find that the branching ratios of the decays with the ground $D^{(*)}$ involved are about one order larger than those of the corresponding decays with the first radially excited $D^{(*)}$ involved.
- The small branching ratios for the latter are related to the node structure of the wave function of $D(2S)$ state.
- Our predictions for the neutral channels $\bar{B}^0 \to D^+(\pi, K, \rho, K^*)^-$ are consistent with the QCDF results but exceed the data. By comparison, the results from the PQCD approach are closer to the experimental data, where except for the factorizable emission diagram amplitde, the nonfactorizable one should also give nonnegligible contribution. Furthermore, these two kinds of amplitdues partially cancel each other.
- It is strange that the difference between our results and the data for the charged B decays is not significant.

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

- The branching ratios of the decays $\bar{B}_s^0 \to D_s^+(1S, 2S)(\pi, \rho, K^{(*)})^-$.

Modes	This work	BS	RQM	NCQM	QCDSRs	Instant-BS	PQCD	QCDF	Exp.	Unit
$\bar{B}_s^0 \to D_s^+(1S)\pi^-$	$4.57^{+0.02+0.04+0.07}_{-0.02-0.09-0.09}$	2.92	3.5	5.3	5	2.7	1.7	4.39	2.98 ± 0.14	
$\bar{B}_s^0 \to D_s^+(1S)K^-$	$0.36^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$	0.221	0.28	0.4	0.4	0.21	0.13	0.33	0.225 ± 0.012	(10^{-3})
$\bar{B}_s^0 \to D_s^+(1S)\rho^-$	$10.55^{+0.04+0.09+0.16}_{-0.04-0.22-0.21}$	7.04	9.4	12.6	13	6.4	4.7	–	6.8 ± 1.4	
$\bar{B}_s^0 \to D_s^+(1S)K^{*-}$	$0.59^{+0.00+0.01+0.01}_{-0.00-0.01-0.01}$	0.392	0.47	0.8	0.6	0.38	0.281	–	–	
$\bar{B}_s^0 \to D_s^+(2S)\pi^-$	$4.70^{+0.02+0.61+0.50}_{-0.02-0.68-0.59}$	1.13	7	–	–	–	–	–	–	
$\bar{B}_s^0 \to D_s^+(2S)K^-$	$0.37^{+0.00+0.05+0.04}_{-0.00-0.05-0.05}$	0.084	0.5	–	–	–	–	–	–	(10^{-4})
$\bar{B}_s^0 \to D_s^+(2S)\rho^-$	$10.43^{+0.04+1.36+1.11}_{-0.04-1.52-1.31}$	2.49	17	–	–	–	–	–	–	
$\bar{B}_s^0 \to D_s^+(2S)K^{*-}$	$0.57^{+0.00+0.07+0.06}_{-0.00-0.08-0.07}$	0.134	0.8	–	–	–	–	–	–	

- The branching ratios (10^{-3}) of the decays $\bar{B}_s^0 \to D_s^{*+}(1S, 2S)(\pi, \rho, K^{(*)})^-$.

Modes	This work	RQM	QCDSR	3P QCDSR	PQCD	BS	RIQ	Exp.
$\bar{B}_s^0 \to D_s^{*+}(1S)\pi^-$	$4.04^{+0.02+0.03+0.14}_{-0.02-0.09-0.15}$	2.7	2	2.11	1.89	3.37	–	1.9
$\bar{B}_s^0 \to D_s^{*+}(1S)K^-$	$0.31^{+0.00+0.00+0.01}_{-0.00-0.00-0.01}$	0.21	0.2	0.159	0.164	0.249	–	0.132
$\bar{B}_s^0 \to D_s^{*+}(1S)\rho^-$	$11.22^{+0.05+0.08+0.36}_{-0.05-0.25-0.38}$	8.7	13	–	5.23	7.26	11.73	9.5
$\bar{B}_s^0 \to D_s^{*+}(1S)K^{*-}$	$0.66^{+0.00+0.00+0.02}_{-0.00-0.01-0.02}$	0.48	0.56	0.163	0.322	0.688	0.69	–
$\bar{B}_s^0 \to D_s^{*+}(2S)\pi^-$	$0.61^{+0.00+0.10+0.06}_{-0.00-0.11-0.09}$	0.8	–	–	–	0.108	–	–
$\bar{B}_s^0 \to D_s^{*+}(2S)K^-$	$0.05^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	0.05	–	–	–	0.00777	–	–
$\bar{B}_s^0 \to D_s^{*+}(2S)\rho^-$	$1.51^{+0.00+0.27+0.19}_{-0.00-0.29-0.25}$	2.2	–	–	–	0.0475	1.05	–
$\bar{B}_s^0 \to D_s^{*+}(2S)K^{*-}$	$0.09^{+0.00+0.02+0.01}_{-0.00-0.02-0.01}$	0.08	–	–	–	0.00332	0.06	–

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

- Polarization fractions (10^{-2}) of the decays $B_{(s)} \to D^*_{(s)}(1S, 2S)(\rho, K^*)$.

Channels	$\bar{B}^0 \to D^{*+}(1S)\rho^-$	$\bar{B}^0 \to D^{*+}(1S)K^{*-}$	$\bar{B}^0 \to D^{*+}(2S)\rho^-$	$\bar{B}^0 \to D^{*+}(2S)K^{*-}$
$f_L[\%]$	90.59	88.11	93.52	91.56
PDG	88.5 ± 2.0	92^{+38}_{-32}	-	-
$f_{\|\|}[\%]$	8.27	10.47	6.42	8.36
Channel	$B^- \to D^{*0}(1S)\rho^-$	$B^- \to D^{*0}(1S)K^{*-}$	$B^- \to D^{*0}(2S)\rho^-$	$B^- \to D^{*0}(2S)K^{*-}$
$f_L[\%]$	90.60	88.12	93.25	91.21
PDG	89.2 ± 2.4	86.0 ± 6.7	-	-
$f_{\|\|}[\%]$	8.26	10.46	6.68	8.70
Channels	$\bar{B}^0_s \to D^{*+}_s(1S)\rho^-$	$\bar{B}^0_s \to D^{*+}_s(1S)K^{*-}$	$\bar{B}^0_s \to D^{*+}_s(2S)\rho^-$	$\bar{B}^0_s \to D^{*+}_s(2S)K^{*-}$
$f_L[\%]$	90.68	88.22	92.68	90.52
BS+FA	87.40	84.10	–	–
BS+PQCD	85.40	85.70	–	–
PQCD	87	83	–	–
Bell	105^{+6}_{-11}	–	–	–
$f_{\|\|}[\%]$	8.22	10.41	7.32	9.48
BS+FA	10.40	13.30	–	–
BS+PQCD	11.30	10.40	–	–

Non-leptonic Decays $B \to D^{(*)}(1S, 2S)M$

- Although there exist significant difference between the form factors of the transitions $B_{(s)} \to D^*_{(s)}(1S)$ and $B_{(s)} \to D^*_{(s)}(2S)$, the similar polarization behaviors can be observed in these decays $B_{(s)} \to D^*_{(s)}(1S, 2S)(\rho, K^*)$.
- The longitudinal polarization is dominant, reaching approximately 90%, while the transverse parallel and perpendicular polarization factions are only a few percent or roughly 10%.

Summary

- The form factors of the transitions $B_{(s)} \to D_{(s)}(2S)$, $D^*_{(s)}(2S)$ are much smaller than those of the corrresponding transtions $B_{(s)} \to D_{(s)}(1S)$, $D^*_{(s)}(1S)$ because of the different node structures of the wave functions between the ground and radilly excited states of $D_{(s)}$ mesons.
- Except for the branching ratios of the neutral decays $B^0_{(s)} \to D^{(*)\pm}_{(s)}(1S)(\pi, \rho, K^{(*)})^{\mp}$, which have some excess compared to the experimental data, overall our predicitions for the decays $B_{(s)} \to D^{(*)}_{(s)}(1S)(\pi, \rho, K^{(*)})$ are consistent with the experimental measurements.
- Most the branching ratios of the decays $B_{(s)} \to D^{(*)}_{(s)}(2S)(\pi, \rho, K^{(*)})$ lie in the range of 10^{-5} to 10^{-4}, which are likely to be detected by the present LHCb and Belle II experiments. Our preditions for these decays are larger than the results given by the BS equation, but agree well with the RQM and RIQ calculations.
- Although there exist obvious difference in the branching ratios between the decays $B_{(s)} \to D^*_{(s)}(1S)(\rho, K^*)$ and $B_{(s)} \to D^*_{(s)}(2S)(\rho, K^*)$, the similar polarization behavious can be observed in these decays $B_{(s)} \to D^*_{(s)}(1S, 2S)(\rho, K^*)$.
- Study of the decays $B_{(s)} \to D^{(*)}_{(s)}(2S)(\pi, \rho, K^{(*)})$ are helpful to recognize the radially excited charmed mesons.

Thank you for your attention!