Quantum Information at High Energy Colliders

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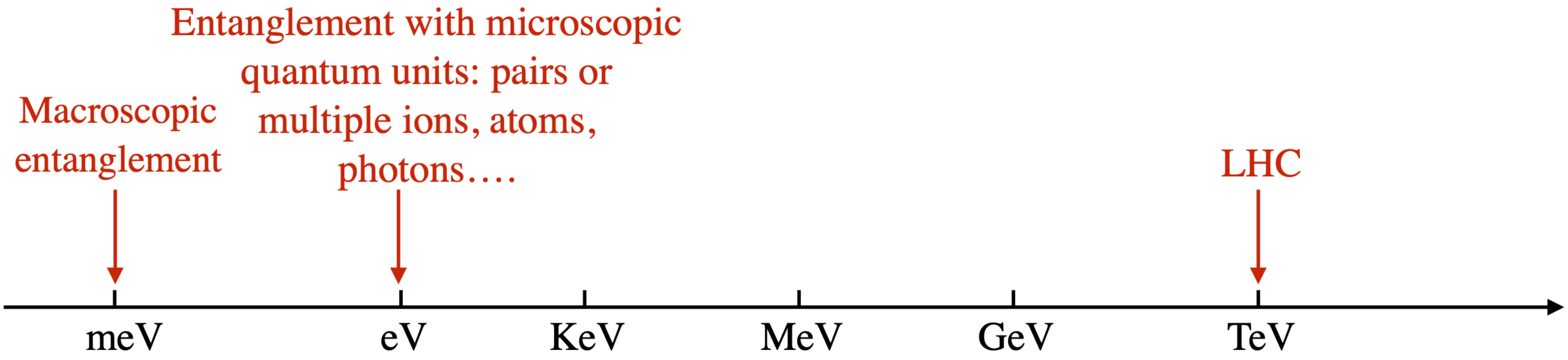
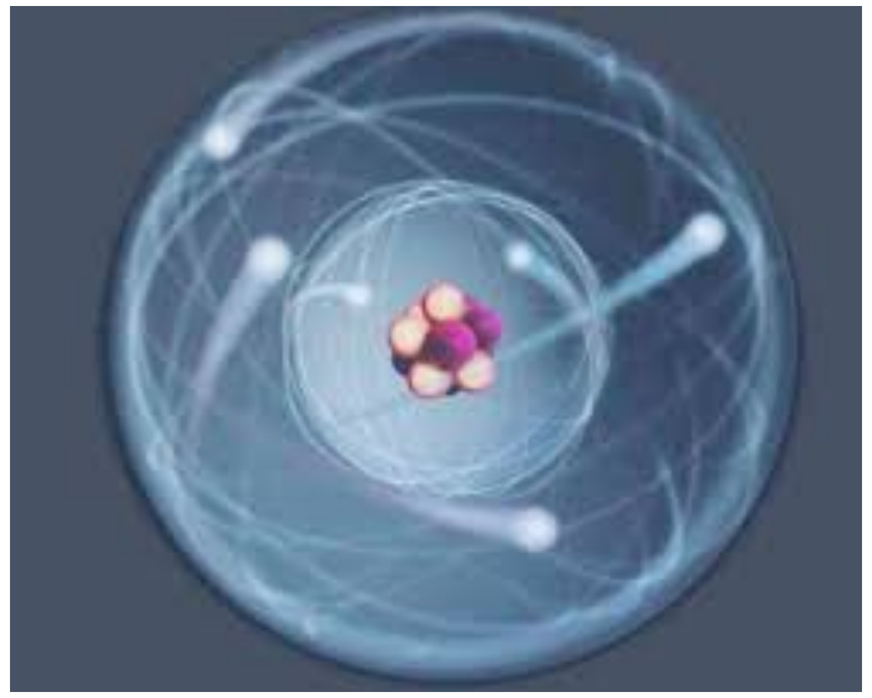
Recent development at Quantum ideas

- Quantum information ideas/concept applied to several systems: photons, atoms, molecules, tardigrade

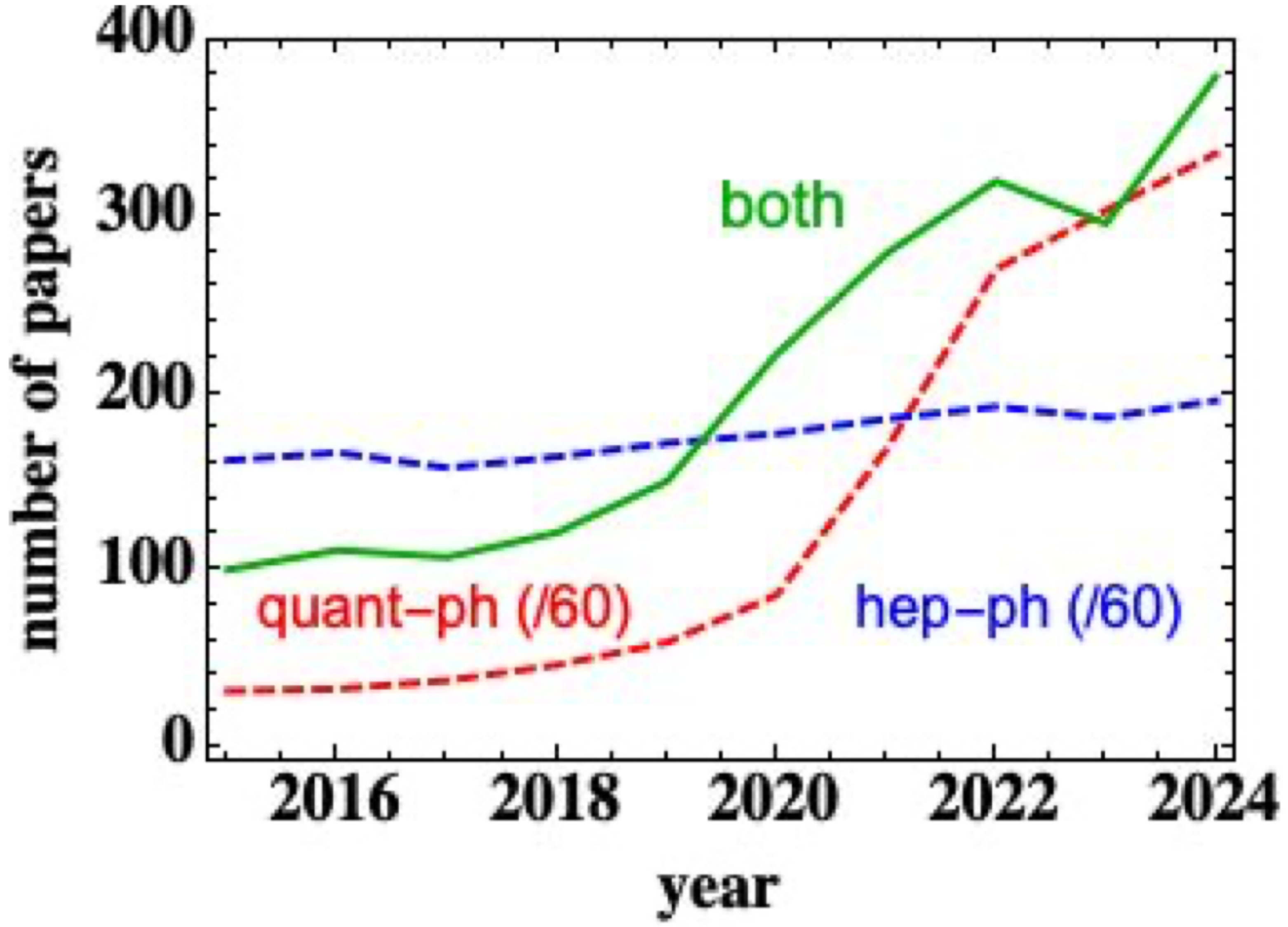




High-Energy Physics and QI Science

- New quantum systems: decaying particles
- Enormous amounts of data

High-Energy Physics



Quantum- Information Science

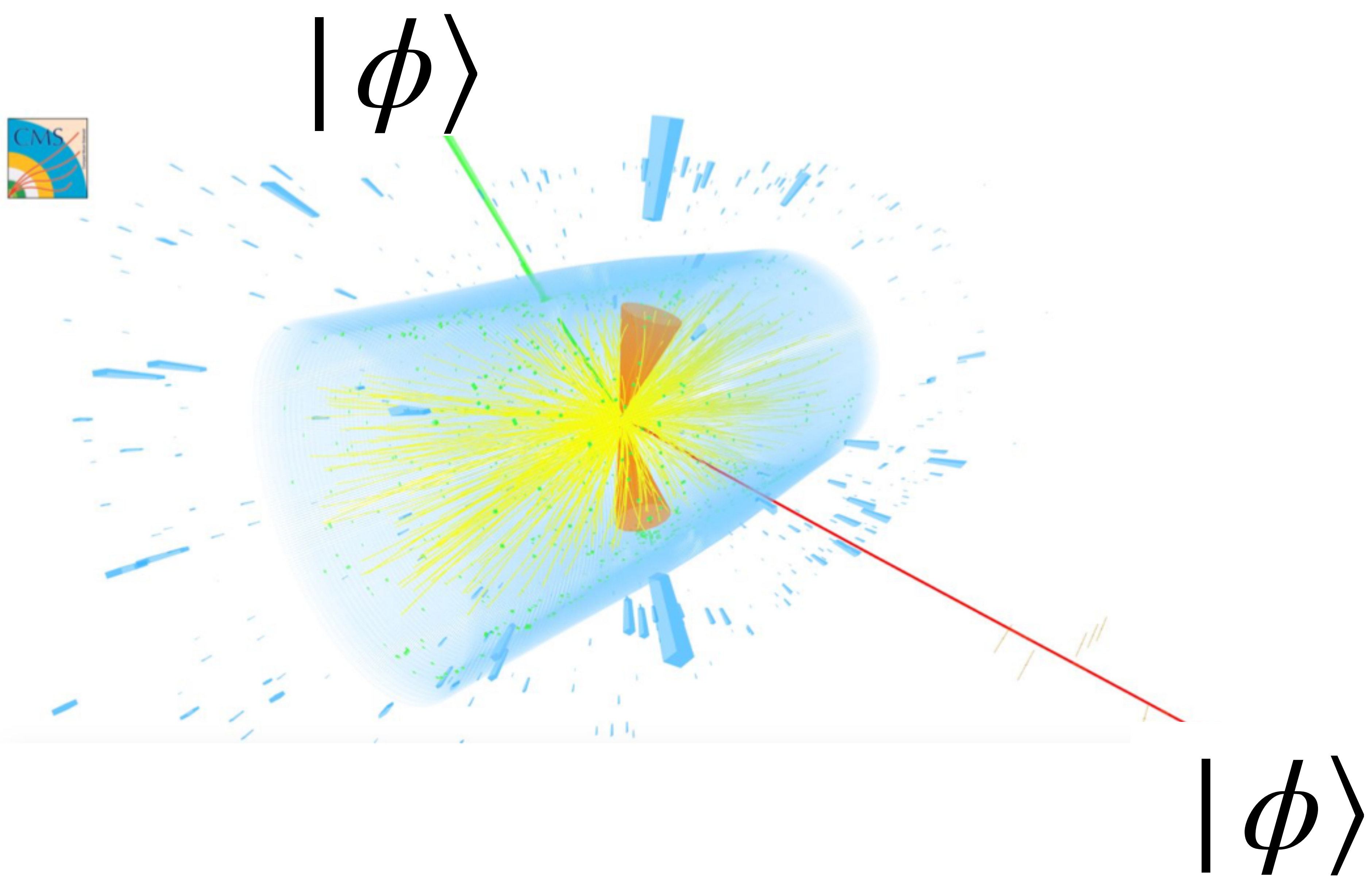
- Quantum mechanics explicitly at work
- New approaches to spin correlations

Outlines

- Entanglement is there
- Realization of QFI at collider?
- General Framework
- Case Studies
- Summary

Quantum Tomography @ Colliders

- If a collider is our source of quantum particles, what are the **qubits**?
  1. Color charge?
  2. Electroweak charge?
  3. Flavor?
  4. **Spin?**



- Should be conserved and observable?
  1. **Spin**
  2. Flavor

Quantum Tomography @ Colliders

- For a state vector $\left|\phi_i\right\rangle$
- Density matrix $\rho = \sum_i n_i \left|\phi_i\right\rangle \left\langle\phi_i\right|$ (pure state: $n_i = 1$; mixed state:$\Sigma_i n_i = 1$)
- For a **single qubit** (i.e., a doublet of spin, iso-spin etc.):

$$\rho = \frac{1}{2}\left(I_2 + \sum_i B_i\sigma_i\right)$$

- For a **biparticle system**

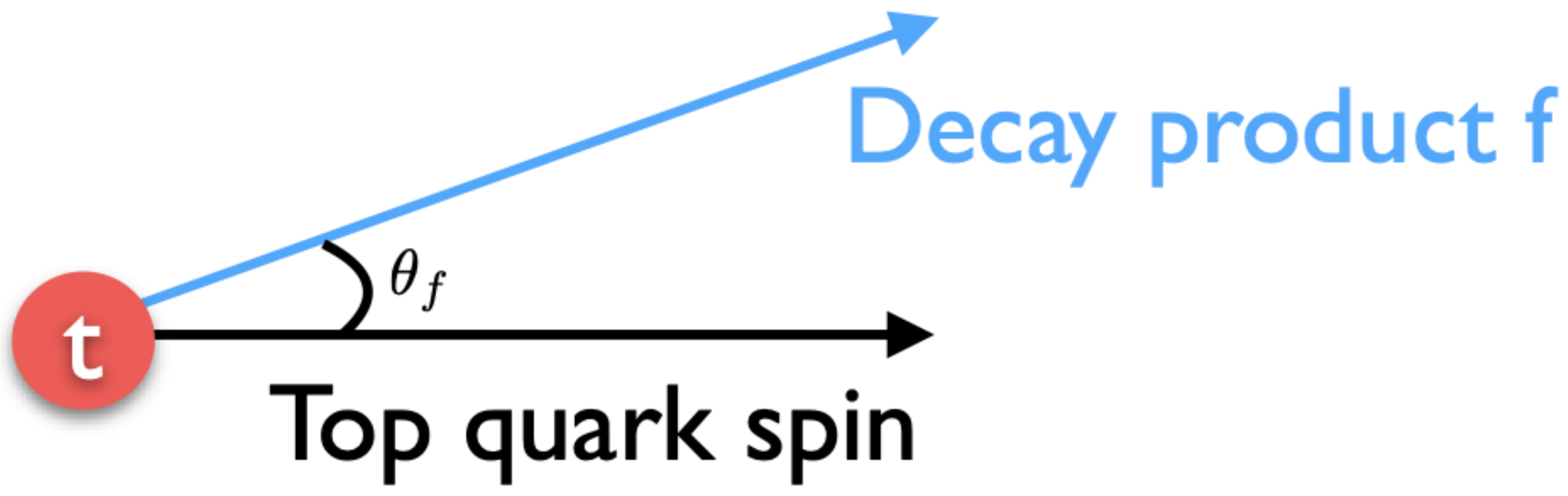
$$\rho = \frac{1}{4}\left(I_4 + \sum_i \left(B_i^A\left(\sigma_i \otimes I_2\right) + B_i^B\left(I_2 \otimes \sigma_i\right)\right) + \sum_{i,j} C_{ij}\left(\sigma_i \otimes \sigma_j\right)\right)$$

$B_i^{A,B}$the polarizations, $C_{ij}$the spin-correlation matrix

The 15 coefficients→ **Quantum Tomography** for the bipartite.

Spin correlation in Top pair

- Top quark:
  1. The most massive particle in the Standard Model.
  2. **Lifetime**: $\sim 10^{-25}$ s.
  3. **Spin-decorrelation**: $\sim 10^{-21}$ s



- Spin information →decay products.

- Spin-correlations between top-quark pairs can be measured.

- Spin-Correlations can be a classical property. For example, Spin-Correlations ≠Quantum Entanglement. However, Quantum Entanglement ⊂ Spin-Correlations.

Entanglement in ttbar system

- Single observable

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi), \text{ with } D = \frac{\mathrm{tr}[\mathbf{C}]}{3}$$

- Entanglement condition： $D < -\frac{1}{3}$
- Top pair displays entanglement in two disconnected regimes: threshold and boosted regions
  - Dilepton:

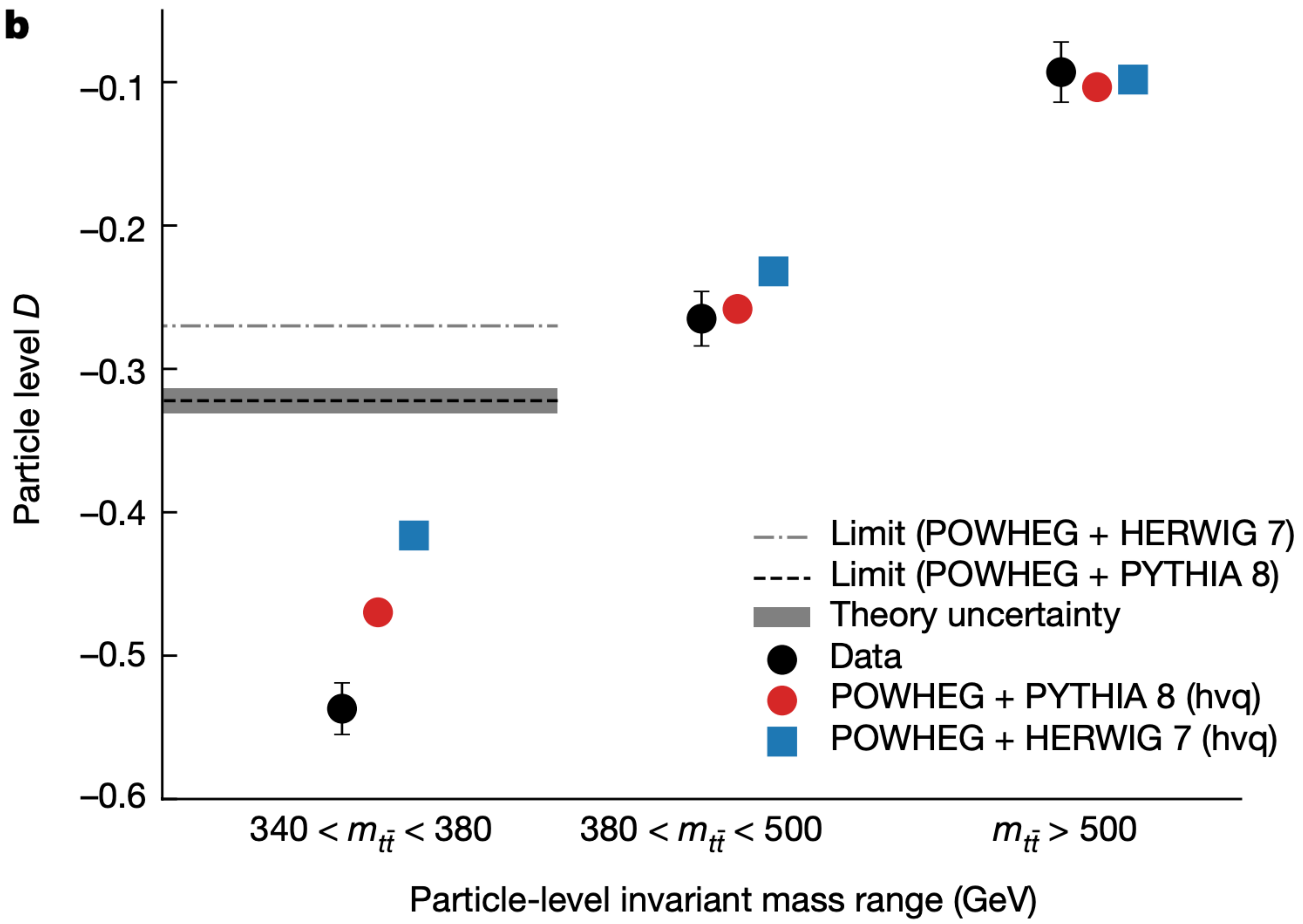
    Afik, Nova ’20
    Fabbrichesi, Floreanini, Panizzo ’21 Severi, Boschi, Maltoni, Sioli ’21
    Saavedra, Casas ’22; Severi, Vryonidou ’22

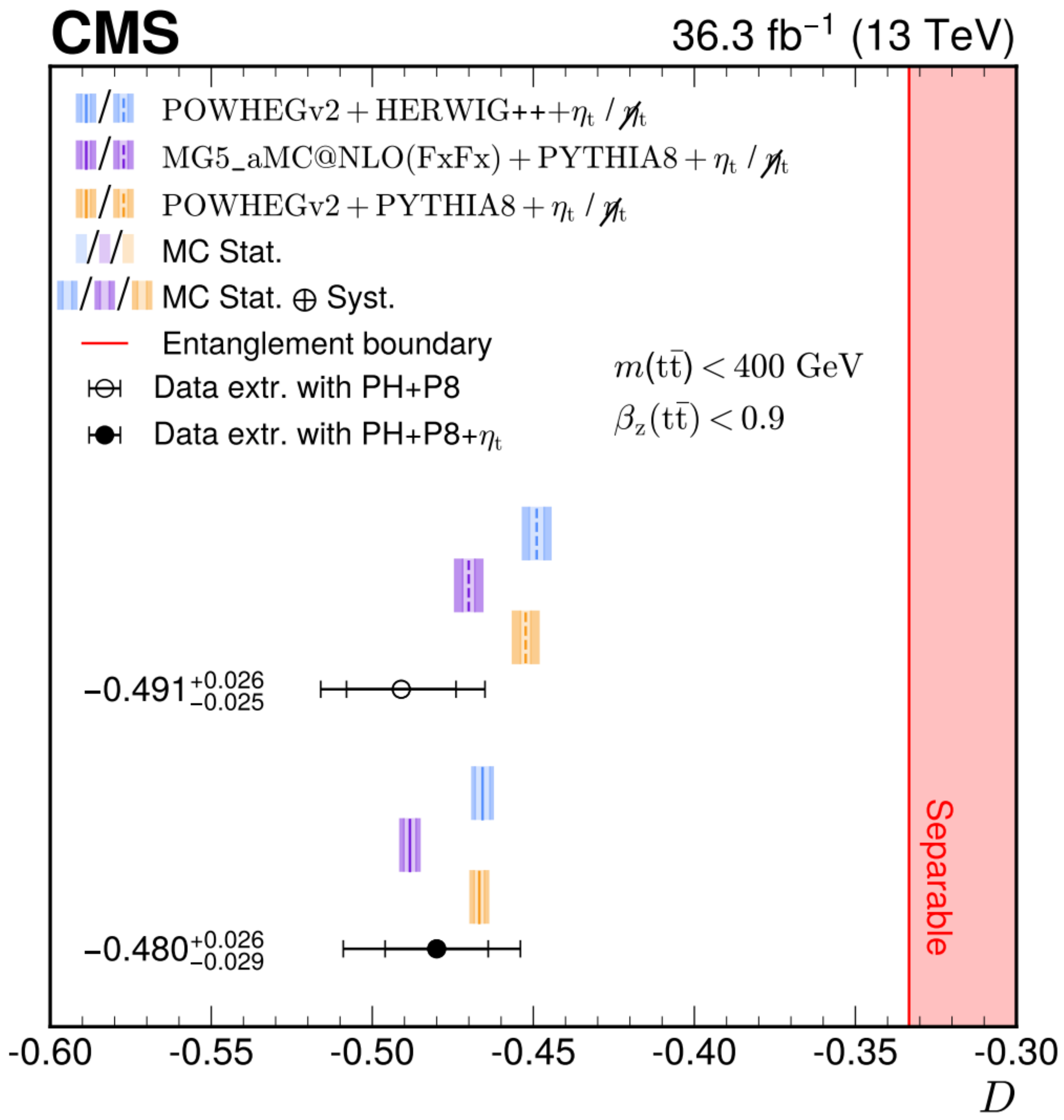
  - lepton+Jets:

    Dong, DG, Kong, Navarro ’23

    Han, Low, Wu ’23

LHC results





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CMS Rep. Prog. Phys. 87 (2024) 117801

Difference from concurrence to QFI?

- LHC confirms the entanglement is there.
- Could we use quantum information to give sensitivity to some SM parameters?
- Can collider experiments reach the Ultimate Quantum Limit?
- Challenge: colliders only access to classical observables (momenta)

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Start with Classical Fisher Information (CFI)

- CFI: quantifies how much information an observable random variable carries about an unknown parameter d on which its probability distribution $f(q, d)$ depends $$F_c(d) = \int dq\ f(q\,|\,d)\left[\frac{\partial}{\partial d}\log f(q\,|\,d)\right]^2$$
- Cramér–Rao Bound: for any classical observable O_d for parameter d $$\mathrm{Var}(O_d) \geq \left(\partial_d \langle O_d \rangle\right)^2 / F_c(d)$$
- Only classical optimal observable O_d^{opt} reaches equality

Classical Fisher Information (CFI)

- Classical optimal observable reaching CFI
  - In perturbative sense: $f(q\,|\,d) = f_0 + d \cdot f_1 + \mathcal{O}(d^2)$
- Classical optimal observable $O_d^{\rm opt} = f_1/f_0$
  - Saturating CR bound: $F_c(d) = \int dq\; f_0 \left(O_d^{\rm opt}\right)^2$
- E.g. BELLE search of τ Electric Dipole Moments PLB2003, JHEP04(2022)110
- E.g. machine learning techniques at colliders are optimizing sensitivities towards CFI

Quantum Fisher Information (QFI)

- QFI $F_q(d)$: quantum precision limit for estimating parameters d encoded in quantum density matrix ρ_d
- Consider small parameter d (perturbative sense): $\rho_d = \rho_0 + d \cdot \rho_1 + \mathcal{O}(d^2)$
  - There exists Symmetric Logarithmic Derivative (SLD) operator $\hat{Q}^{\mathrm{opt}}$:
    $$\rho_1 = \frac{1}{2}\left\{\rho_0, \hat{Q}^{\mathrm{opt}}\right\}$$
- QFI in a perturbative form: $F_q(d) = \mathrm{Tr}\left[\rho\left(\hat{Q}^{\mathrm{opt}}\right)^2\right] \simeq \mathrm{Tr}\left[\rho_0\left(\hat{Q}^{\mathrm{opt}}\right)^2\right]$

Ultimate Quantum Limit: Quantum Cramér–Rao bound

- Quantum Cramér–Rao bound: for any quantum operator $\hat{Q}$ for measuring d
  - $\mathrm{Var}\left(\hat{Q}\right) \geq \left(\partial_d \langle \hat{Q} \rangle\right)^2 / F_q(d)$
  - Mean $\langle \hat{Q} \rangle$ and Variance $\mathrm{Var}\left(\hat{Q}\right)$
  - A higher QFI corresponds to a more precise measurement limit
- QFI saturation ⇔ projective measurements $\hat{\Pi}_i = |\psi_i\rangle\langle\psi_i|$

in SLD $\hat{Q}^{\mathrm{opt}} = \sum_i \lambda_i \hat{\Pi}_i$ eigenbasis

(S. L. Braunstein et al *Phys.Rev.Lett*. 1994)
(J. Liu, H. Yuan, X.-M. Lu, and X. Wang, *J. Phys.A* 2020)

The generalized quantum measurement at colliders

- Colliders perform classical measures on momenta of particles $|\vec{p}\rangle$
- But particle decay can serve as generalized quantum measurement
- Suppose a fermion decay $A \to BC$, with decay amplitude $M(A_\alpha \to BC)$
  - Spin density matrix: ρ_A
  - Generalized measurement : $\hat{D}_{\alpha\alpha'} \equiv N_D^{-1} M(A_\alpha \to BC) M^\dagger(A_{\alpha'} \to BC)$
  - Completeness condition: $\mathbf{I} = \int d\Omega_B \hat{D}(p_B)$
  - Normalized differential distribution and conservation:

$$1 = \mathrm{Tr}[\rho_A] = \int d\Omega_B \mathrm{Tr}[\rho_A \hat{D}(p_B)] = \int d\Omega_B f(p_B) = 1$$

Q. Wang et al, 2402.16574, CPL
C.F. Qiao et al, 2002.04284, PRD
R. Ashby-Pickering et al, 2209.13990, JHEP

The relation between QFI and CFI

- A classical observable $O(p)$ corresponds to a quantum measurement $\hat{Q}_p$

$$\hat{Q}_O = \int d\Omega_p \hat{D}(p) O(p)$$

where $\hat{D}(p)$ is the generalized quantum measurement operator

- Same expectation value but different variance
  - $\langle \hat{Q}_O \rangle = \mathrm{Tr}\left[\rho \hat{Q}_O\right] = \langle O(p) \rangle$
  - Using Cauchy inequality: $\mathrm{Var}(O) \geq \mathrm{Var}(\hat{Q}_O)$ (J. Liu, H. Yuan, X.-M. Lu, and X. Wang, *J. Phys.A* 2020)
- Quantum optimal measurement precision is always better than classical optimal precision
  - $F_q(d) \geq \dfrac{\left(\partial_d \langle \hat{Q}_{O_d^{\mathrm{opt}}} \rangle\right)^2}{\mathrm{Var}(\hat{Q}_{O_d^{\mathrm{opt}}})} \geq \dfrac{\left(\partial_d \langle O_d^{\mathrm{opt}} \rangle\right)^2}{\mathrm{Var}(O_d^{\mathrm{opt}})} = F_c(d)$

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τ Decay as Quantum Measurement

- $\tau^+\tau^-$ production and spin density matrix:

$$\rho_{\alpha\alpha';\beta\beta'}(\hat{k}) = \frac{1}{|\overline{M}|^2} \overline{\sum_{\text{initial}}} M(\text{initial} \to \tau^+_\alpha(\hat{k})\tau^-_\beta(-\hat{k}))M^*(\text{initial} \to \tau^+_{\alpha'}(\hat{k})\tau^-_{\beta'}(-\hat{k}))$$

  - Introduce spin density matrix $\rho_d = \rho_0 + d \cdot \rho_1 + \mathcal{O}(d^2)$

- Two-body τ decay: $\tau^\pm \to \pi^\pm \overset{(-)}{\nu}$

  - Single τ spin measurement operator

$$\hat{D}^\pm_{\alpha\alpha'}(\hat{q}_\pm) = \frac{f_\pi^2}{2m_\tau^3|q|} M(\tau^\pm_{\alpha'} \to \pi^\pm(\hat{q}_\pm)X)M^*(\tau^\pm_\alpha \to \pi^\pm(\hat{q}_\pm)X)$$

τ Decay with maximal spin analyzing power

- Single τ spin measurement operator for general decay

$$\hat{D}^{\pm}_{\alpha\alpha'}(\hat{q}_{\pm}) = \frac{1}{2}(\mathbf{I}_2 \mp a\hat{q}_{\pm} \cdot \overrightarrow{\sigma})_{\alpha\alpha'}$$

  - With $0 \leq |a| \leq 1$, $|a| = 1$ maximal spin analyzing power; $|a| = 0$ no spin analyzing power

- Two-body τ decay: $\tau^{\pm} \to \pi^{\pm} \overset{(-)}{\nu}$

  - Maximal spin analyzing power $\to \mathrm{Rank}[\hat{D}] = 1$

  - Projector: $\hat{D} = |\hat{q}\rangle\langle\hat{q}|$, $|\hat{q}\rangle = \{\cos(\theta/2), \sin(\theta/2)e^{i\phi}\}$

  - Properties: $\mathrm{Tr}[\hat{D}^{\pm}] = 1, \quad (\hat{D}^{\pm})^2 = \hat{D}^{\pm}, \quad \int d\Omega_{\pm}\hat{D}^{\pm}(\hat{q}_{\pm}) = 2\pi\mathbf{I}_2$

Collider Reality: Separable (local) Measurements

- $\tau^+\tau^-$, each decays independently

$$M^{\text{tot}}_{\alpha\beta} \propto M(\text{ini} \to \tau^+_\alpha(k_+)\tau^-_\beta(k_-)) \times M(\tau^+_\alpha(k_+) \to \pi^+\bar{\nu})M(\tau^-_\beta(k_-) \to \pi^-\nu)$$

- Their decay forms separable quantum measurements

- $\hat{E}_{\alpha\alpha';\beta\beta'}(\hat{q}_+, \hat{q}_-) = \hat{D}^+_{\alpha\alpha'}(\hat{q}_+) \otimes \hat{D}^-_{\beta\beta'}(\hat{q}_-)$

- Properties:

$$\text{Tr}[\hat{E}] = 1, \quad \hat{E}^2(\hat{q}_+, \hat{q}_-) = \hat{E}(\hat{q}_+, \hat{q}_-), \quad \frac{1}{4\pi^2}\int d\Omega_+ d\Omega_- \hat{E}(\hat{q}_+, \hat{q}_-) = \mathbf{I}_4 \,.$$

- General collider measurements *cannot* access entangled states

Reaching QFI at collider is very difficult

- Recall to saturate QFI, one should measure with all four optimal projectors $\hat{\Pi}_i = |\psi_i\rangle\langle\psi_i|$, with orthonormal eigenstates $|\Psi_i\rangle$ of SLD $\hat{Q}^{\mathrm{opt}}$

- Collider separable (non-entangled) measurements: $\hat{E} = |\hat{q}_+, \hat{q}_-\rangle\langle\hat{q}_+, \hat{q}_-|$

  - Define first measurement: $|E_1\rangle = |\hat{q}_+, \hat{q}_-\rangle \equiv |\hat{q}_+\rangle \otimes |\hat{q}_-\rangle$

  - The other orthonormal separable measurements are forced to be $|E_2\rangle = |-\hat{q}_+, \hat{q}_-\rangle, \;\; |E_3\rangle == |\hat{q}_+, -\hat{q}_-\rangle, \;\; |E_4\rangle == |-\hat{q}_+, -\hat{q}_-\rangle$

- Conditions for QFI saturation:

$$[\hat{Q}, \hat{E}_j] = 0 \quad \Leftrightarrow \quad \hat{Q}|E_j\rangle = \lambda_j |E_j\rangle$$

Reaching QFI at collider is very difficult

- Conditions for QFI saturation: $[\hat{Q}, \hat{E}_j] = 0 \quad \Leftrightarrow \quad \hat{Q}\,|E_j\rangle = \lambda_j\,|E_j\rangle$
- $\hat{Q}^{\text{opt}}$ from production is independent with the measurement $\hat{E}$
  - If eigenstate $|\psi_i\rangle$ is entangled → *cannot* reach quantum optimal at colliders
- Schmidt Condition to be separated:
  - $|\psi\rangle = a|\uparrow\uparrow\rangle + b|\uparrow\downarrow\rangle + c|\downarrow\uparrow\rangle + d|\downarrow\downarrow\rangle$
    $|\psi\rangle$ separable $\longleftrightarrow ad - cb = 0$
  - Zero measure, very difficult to satisfy

Rank-deficient ρ saves the Day

- If ρ_0 is not full-rank, SLD is not unique——Most general form of SLD:

$$\hat{Q} = \sum_{\substack{i,j \\ p_i + p_j \neq 0}} \frac{\langle p_i | \rho_1 | p_j \rangle}{p_i + p_j} |p_i\rangle\langle p_j| + \sum_{\substack{i,j \\ p_i = p_j = 0}} r_{ij} |p_i\rangle\langle p_j|$$

- $|p_i\rangle$: the eigenstate of ρ_0 with eigenvalue (probability) p_i

- r_{ij} : arbitrary complex numbers satisfying $r_{ij} = r_{ij}^*$

- Null space freedom provides the flexibility of $\hat{Q}$, *which provides hope to match* $\hat{E}_m(\hat{q}) = \hat{\Pi}_m(r)$

$$F_q(d) \xleftarrow{\hat{Q}^{\text{opt}}} \rho_d \xrightarrow{\hat{E}_m(\hat{q}_+,\hat{q}_-)=\hat{\Pi}_m(r)} \frac{df_m}{d\Omega_+ d\Omega_-} \xrightarrow{O_d^{\text{opt}}(\hat{q}_+,\hat{q}_-)} F_c(d)$$

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Three scenarios

- Higgs decay with CP-violating coupling: $h \to \tau^+\tau^-$
  - $\mathscr{L}_{\text{CPV}} = -\frac{m_\tau}{v} h\,\bar{\tau}\left[\cos\delta_h + i\gamma_5 \sin\delta_h\right]\tau$
- τ Magnetic/Electric Dipole Moment: $e^+e^- \to \tau^+\tau^-$
  - $\mathscr{L}_{\text{MDM/EDM}} = \frac{1}{2}F_{\mu\nu}\,\bar{\tau}\sigma^{\mu\nu}\left[-\frac{e}{2m_\tau}a_\tau - i\gamma_5 d_\tau\right]\tau$
- We use helicity basis for spin density matrix

Higgs CP-violating scenario

- Spin density matrix perturbatively: $\rho_0 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \rho_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

- Most simple and direct SLD: $Q_h = 2\rho_1$,
  - easy to check $\rho_1 = \frac{1}{2}\{\rho_0, Q_h\}$, and QFI: $F_q(\delta_h) = 4$

- Q_h is rank 2, with two entangled eigenstates:
$\hat{\Pi}_1 = \frac{1}{\sqrt{2}}(i|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \hat{\Pi}_2 = \frac{1}{\sqrt{2}}(-i|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

- $\hat{\Pi}_{1,2}$ cannot match to collider measurements

Higgs CP-violating scenario

- Most general SLD satisfying $\rho_1 = \dfrac{1}{2}\{\rho_0, \hat{Q}^{\text{gen}}\}$:

$$\hat{Q}^{\text{gen}}(r) = \begin{pmatrix} r_{44} & -\frac{r_{43}}{\sqrt{2}} & \frac{r_{43}}{\sqrt{2}} & r_{42} \\ -\frac{r_{34}}{\sqrt{2}} & \frac{r_{33}}{2} & -\frac{r_{33}}{2}-2i & -\frac{r_{32}}{\sqrt{2}} \\ \frac{r_{34}}{\sqrt{2}} & -\frac{r_{33}}{2}+2i & \frac{r_{33}}{2} & \frac{r_{32}}{\sqrt{2}} \\ r_{24} & -\frac{r_{23}}{\sqrt{2}} & \frac{r_{23}}{\sqrt{2}} & r_{22} \end{pmatrix}$$

- Scan all solutions across the auxiliary variables r's and particle momentum choices $\hat{q}_+$ and $\hat{q}_-$
    - to match $\hat{\Pi}_{1,2,3,4}(r) = \hat{E}_{1,2,3,4}(\hat{q}_+, \hat{q}_-)$

Two Construction Methods

- Matrix-based method
  - Brute force linear algebra: $[\hat{Q}, \hat{E}_j] = 0 \quad \Leftrightarrow \quad \hat{Q}\,|\,E_j\rangle = \lambda_j\,|\,E_j\rangle$
    - Direct, but computational challenge
  - Work in Pauli basis:

$$\hat{Q} = a\mathbf{I}_4 + \sum_i b_i^+(\sigma_i \otimes \mathbf{I}_2) + \sum_j b_j^-(\mathbf{I}_2 \otimes \sigma_j) + \sum_{i,j} c_{ij}\,\sigma_i \otimes \sigma_j$$

    - If $\hat{Q}$ is separable and can match to $\hat{E}(\hat{q}_+, \hat{q}_-)$, if and only if,

$$a = P_1\,, \qquad \overrightarrow{b}^{\pm} = P_{2/3}\,\hat{q}_\pm\,, \qquad c_{ij} = P_4\,\hat{q}_+^i\hat{q}_-^j$$

    - Requiring separable spin-spin correlation matrix: $\overrightarrow{c} \propto \overrightarrow{\hat{q}}_+ \overrightarrow{\hat{q}}_-$
- Amplitude-based method (focus on the transition amplitude M)
  - Closer to particle interactions and physics, see Appendix of arXiv:2506.10673

Higgs CP-violating scenario

- We found $\hat{\Pi}_{1,2,3,4}(r) = \hat{E}_{1,2,3,4}(\hat{q}_+, \hat{q}_-)$ for Higgs CPV scenario!

- Collider accessible SLD: $Q_h^{\text{opt}} = \dfrac{2}{\sin\varphi}\begin{pmatrix} -\cos\varphi & 0 & 0 & e^{-i\Phi} \\ 0 & -\cos\varphi & e^{-i\varphi} & 0 \\ 0 & e^{i\varphi} & -\cos\varphi & 0 \\ e^{i\Phi} & 0 & 0 & -\cos\varphi \end{pmatrix} \neq Q_h^{\text{old}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -2i & 0 \\ 0 & 2i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

- With $\varphi = \phi_1 - \phi_2$ and $\Phi = \phi_1 + \phi_2$; $\phi_{1,2}$ are free phase parameters

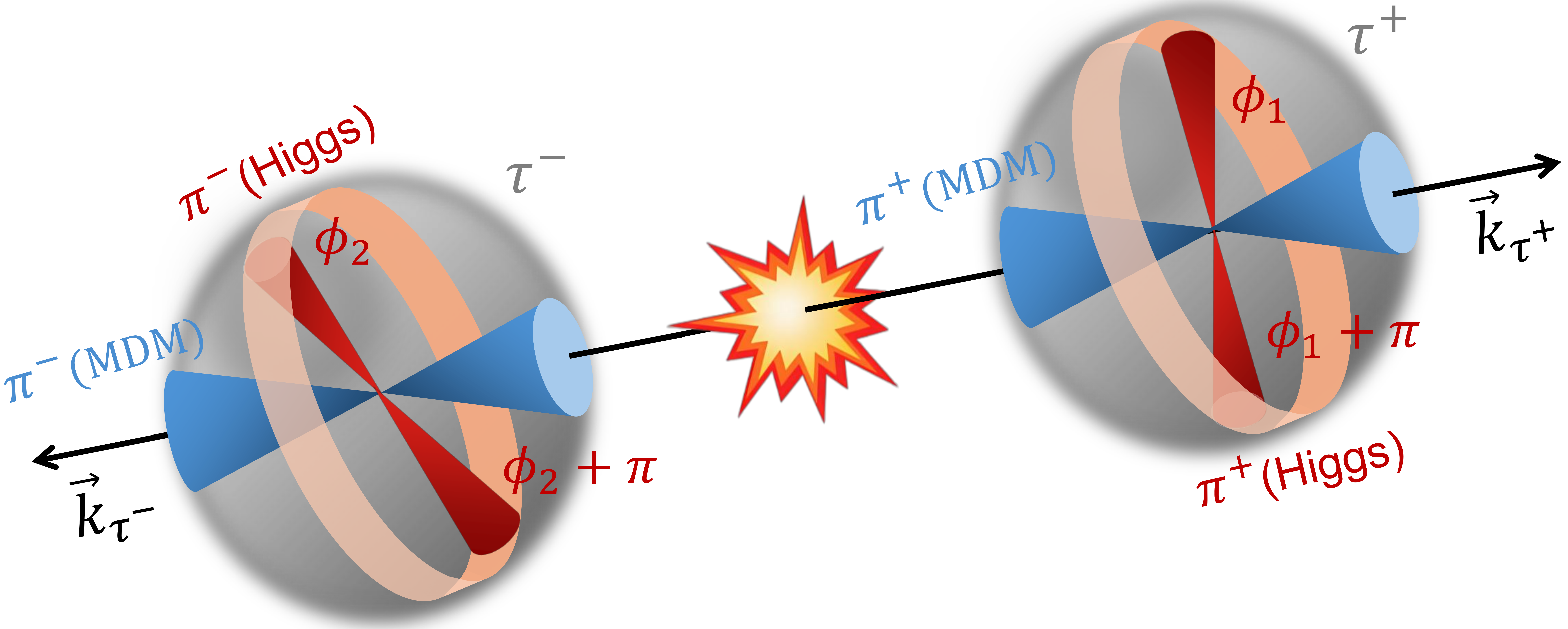
Quantum optimal collider measurements $\hat{E}_{1,2,3,4}$

CPV h	θ_+	θ_-	ϕ_+	ϕ_-
$\lvert E_1\rangle$	$\pi/2$	$\pi/2$	ϕ_1	ϕ_2
$\lvert E_2\rangle$	$\pi/2$	$\pi/2$	$\phi_1+\pi$	ϕ_2
$\lvert E_3\rangle$	$\pi/2$	$\pi/2$	ϕ_1	$\phi_2+\pi$
$\lvert E_4\rangle$	$\pi/2$	$\pi/2$	$\phi_1+\pi$	$\phi_2+\pi$

$\theta_\pm$: polar angle of $\hat{q}_\pm$

$\phi_\pm$: azimuthal angle of $\hat{q}_\pm$

Optimal collider measurements: four sets of directions



MDM scenario

- The spin density matrix for MDM:

$$\rho_0 = c_0 \begin{pmatrix} \frac{1}{2}(\cos 2\theta + 3) & -\frac{im}{\sqrt{s}}\sin 2\theta & -\frac{im}{\sqrt{s}}\sin 2\theta & -\sin^2\theta \\ \frac{im}{\sqrt{s}}\sin 2\theta & \frac{4m^2}{s}\sin^2\theta & \frac{4m^2}{s}\sin^2\theta & \frac{im}{\sqrt{s}}\sin 2\theta \\ \frac{im}{\sqrt{s}}\sin 2\theta & \frac{4m^2}{s}\sin^2\theta & \frac{4m^2}{s}\sin^2\theta & \frac{im}{\sqrt{s}}\sin 2\theta \\ -\sin^2\theta & -\frac{im}{\sqrt{s}}\sin 2\theta & -\frac{im}{\sqrt{s}}\sin 2\theta & \frac{1}{2}(\cos 2\theta + 3) \end{pmatrix}$$

$$\rho_1 = (1 - \frac{4m^2}{s})c_0^2 \begin{pmatrix} -2\sin^2\theta(\cos 2\theta + 3) & -\frac{ic_1\sqrt{s}}{4m}\sin 2\theta & -\frac{ic_1\sqrt{s}}{4m}\sin 2\theta & 4\sin^4\theta \\ \frac{ic_1\sqrt{s}}{4m}\sin 2\theta & 2\sin^2\theta(\cos 2\theta + 3) & 2\sin^2\theta(\cos 2\theta + 3) & \frac{ic_1\sqrt{s}}{4m}\sin 2\theta \\ \frac{ic_1\sqrt{s}}{4m}\sin 2\theta & 2\sin^2\theta(\cos 2\theta + 3) & 2\sin^2\theta(\cos 2\theta + 3) & \frac{ic_1\sqrt{s}}{4m}\sin 2\theta \\ 4\sin^4\theta & -\frac{ic_1\sqrt{s}}{4m}\sin 2\theta & -\frac{ic_1\sqrt{s}}{4m}\sin 2\theta & -2\sin^2\theta(\cos 2\theta + 3) \end{pmatrix}$$

MDM and EDM scenarios

- Quantum optimal collider-accessible SLD for MDM:

$$\hat{Q} = 2\left(\frac{s}{4m^2} - 1\right) c_0 \begin{pmatrix} -\frac{8m^2}{s}\sin^2\theta & 0 & 0 & 0 \\ 0 & \cos 2\theta + 3 & 0 & 0 \\ 0 & 0 & \cos 2\theta + 3 & 0 \\ 0 & 0 & 0 & -\frac{8m^2}{s}\sin^2\theta \end{pmatrix}$$

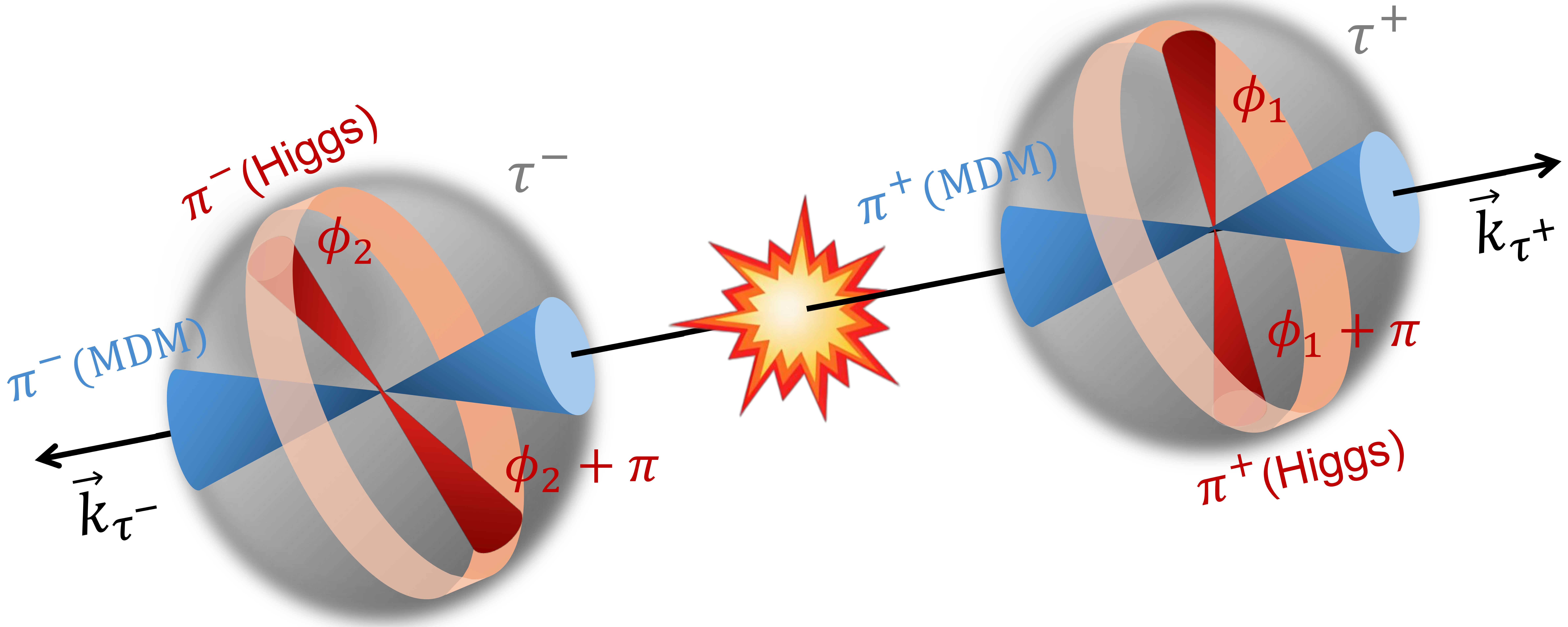
$$F_q^{\mathrm{MDM}}(\theta) = \frac{2s\sin^2\theta\,(\cos 2\theta + 3)\left(s - 4m^2\right)^2}{m^2\left(8m^2\sin^2\theta + s\cos 2\theta + 3s\right)^2}$$

- Corresponding classical optimal collider measurements:

MDM	θ_+	θ_-	ϕ_+	ϕ_-
$\lvert E_1\rangle = \lvert\downarrow\downarrow\rangle$	π	π	$-$	$-$
$\lvert E_2\rangle = \lvert\downarrow\uparrow\rangle$	π	0	$-$	$-$
$\lvert E_3\rangle = \lvert\uparrow\downarrow\rangle$	0	π	$-$	$-$
$\lvert E_4\rangle = \lvert\uparrow\uparrow\rangle$	0	0	$-$	$-$

- EDM has no solution

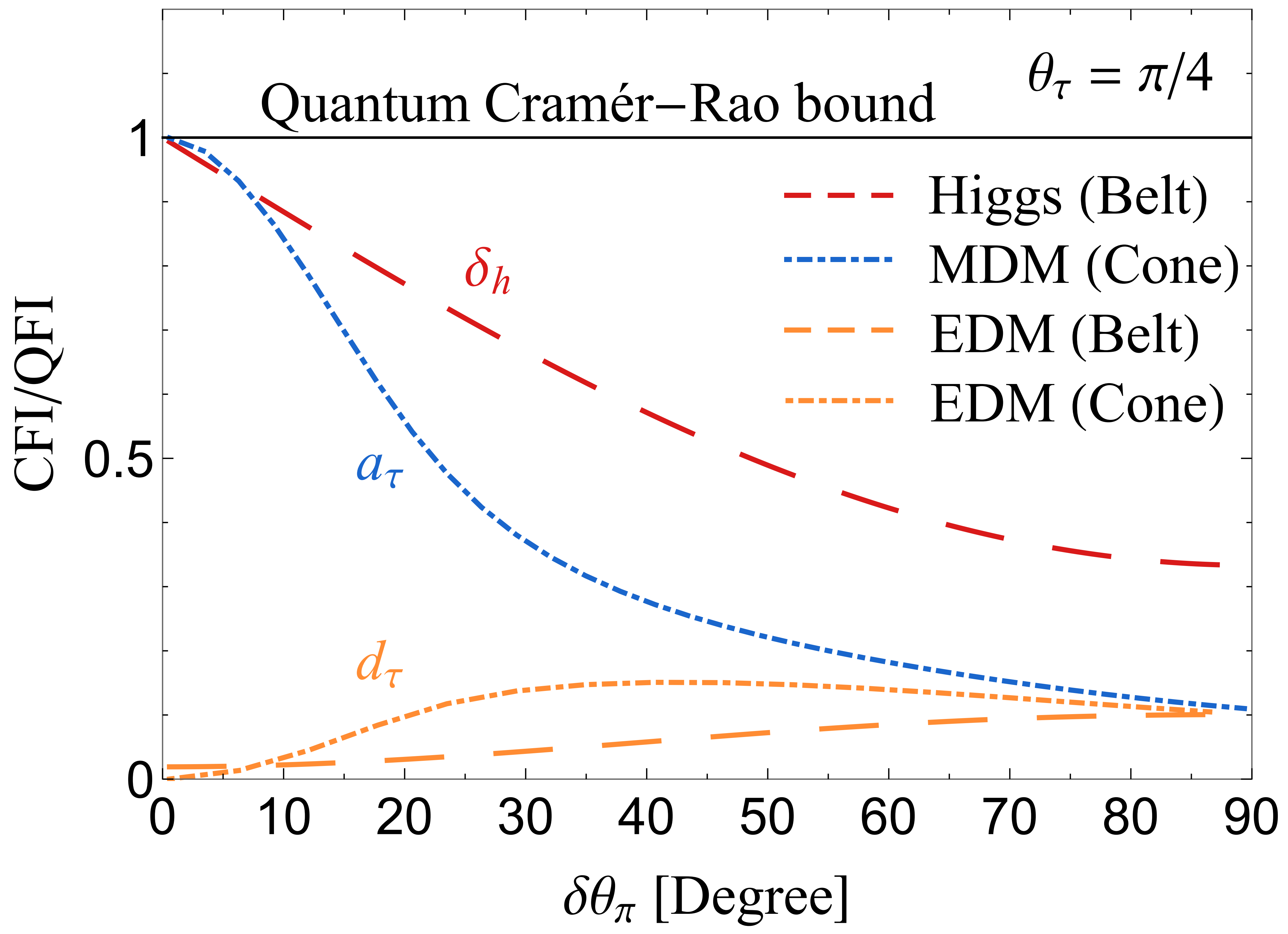
Optimal collider measurements: four sets of directions

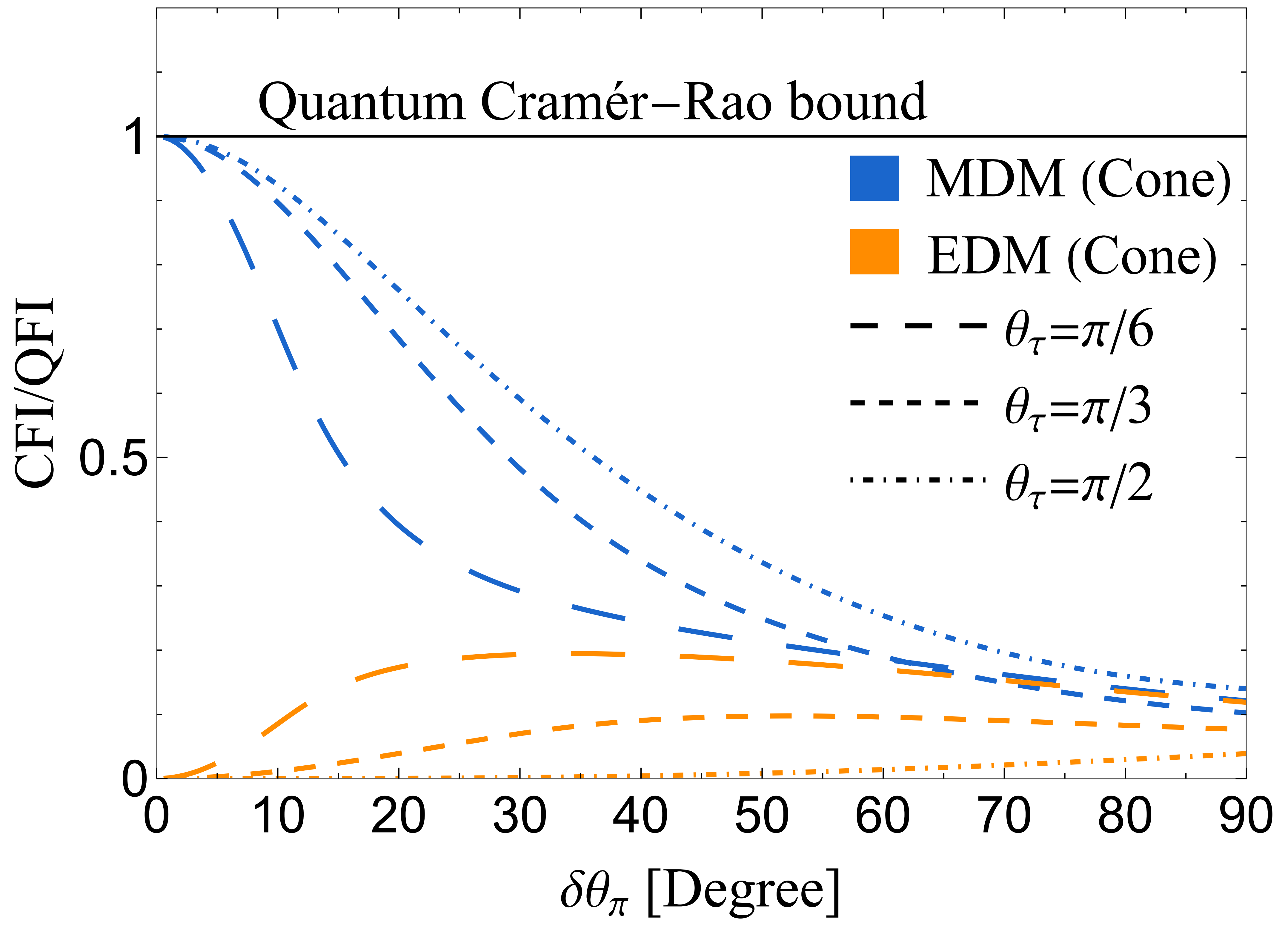


Collider Strategy: Phase-Space Restricted PS

- Expand each optimal direction into a cone of angular size $\delta\theta_\pi$
  - Allowing non-zero events
  - CFI asymptotic saturates QFI

$$F_c(d) = \int d\mathrm{PS}_{\mathrm{Belt/Cone}} \times \frac{\Sigma_1^2}{\Sigma_0}$$





Summary and Outlook

- QFI represents the ultimate precision limit in parameter estimating, QFI ≥ CFI

- Quantum measurement connect quantum states to classical distribution
  Particle decay serves as generalized quantum measurements

- Collider measurements can only access separable projectors $\hat{E} = \hat{D}(q_+)\hat{D}(q_-)$

- Condition for QFI saturation: (specific directions)

$$F_q(d) \xleftarrow{\hat{Q}^{\rm opt}} \rho_d \xrightarrow{\hat{E}_m(\hat{q}_+,\hat{q}_-)=\hat{\Pi}_m(r)} \frac{df_m}{d\Omega_+ d\Omega_-} \xrightarrow{O_d^{\rm opt}(\hat{q}_+,\hat{q}_-)} F_c(d)$$

- Rank deficiency of $\rho \longrightarrow$ Flexibility of SLD

- QFI saturation is achievable for Higgs CPV decay and MDM cases, but not for EDM

- Framework easily applicable to other entangled biparticle systems:
  $t\bar{t}$ from ee, $t\bar{t}$ pseudo-scalar resonance, baryon pairs $(\Delta, \Lambda, \Lambda_b)$, gauge boson pairs

Thank you!