第七届粒子物理天问论坛

武汉， 2025.09.18-22

Classification of Coupled-Channel Near-Threshold Structures

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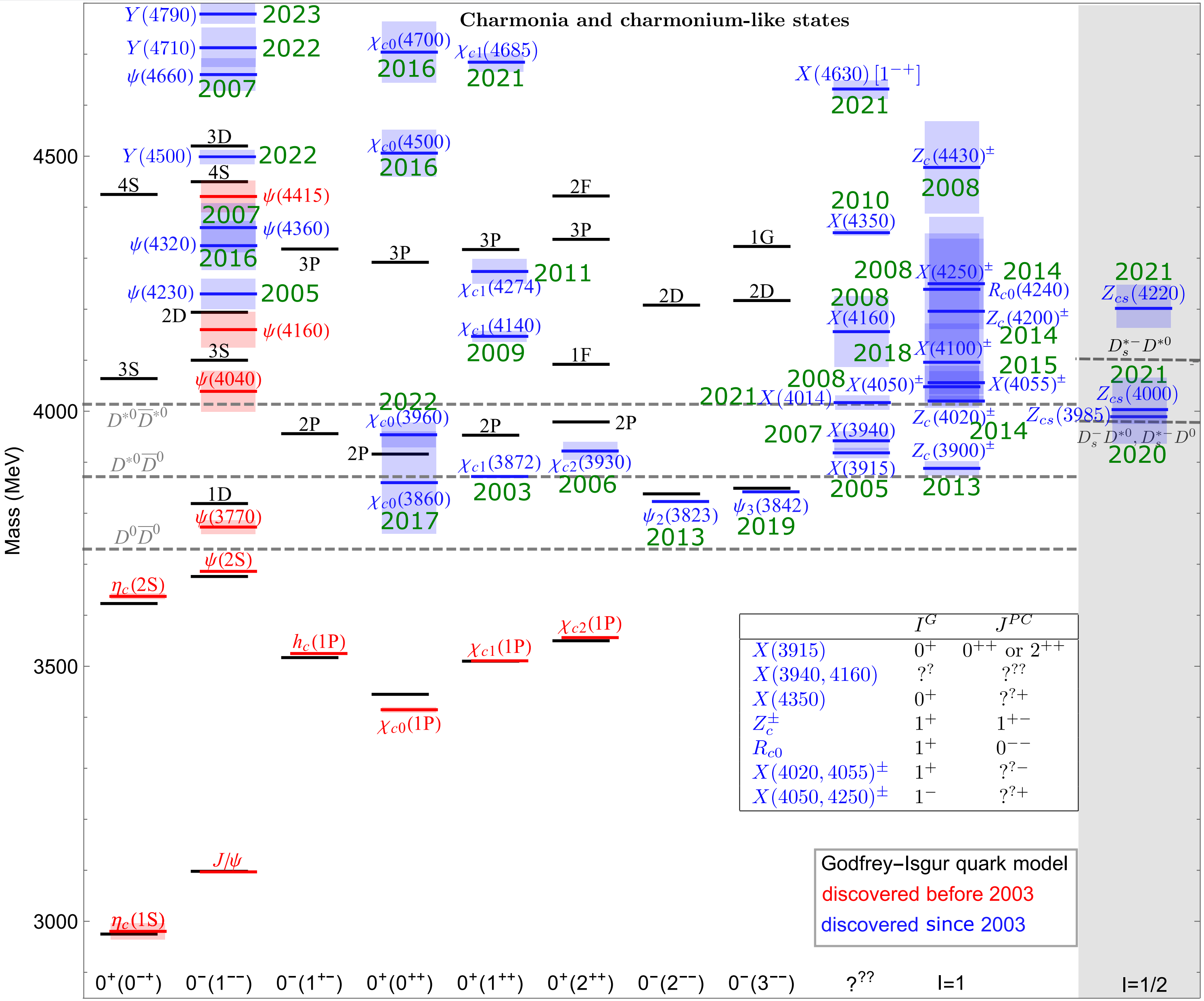
X.-K. Dong, FKG, B.-S. Zou, PRL 126, 152001 (2021)

V. Baru, FKG, C. Hanhart, A. Nefediev, PRD 109, L111501 (2024)

Zhen-Hua Zhang, FKG, PLB 863 (2025) 139387

2025.09.19

Charmonia and charomium-like states



Effective range expansion

$$f_0^{-1}(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 - ik + \mathcal{O}\left(\frac{k^4}{\beta^4}\right)$$

a_0: S-wave scattering length; negative: repulsion or attraction w/ a bound state

positive: attraction w/o bound state

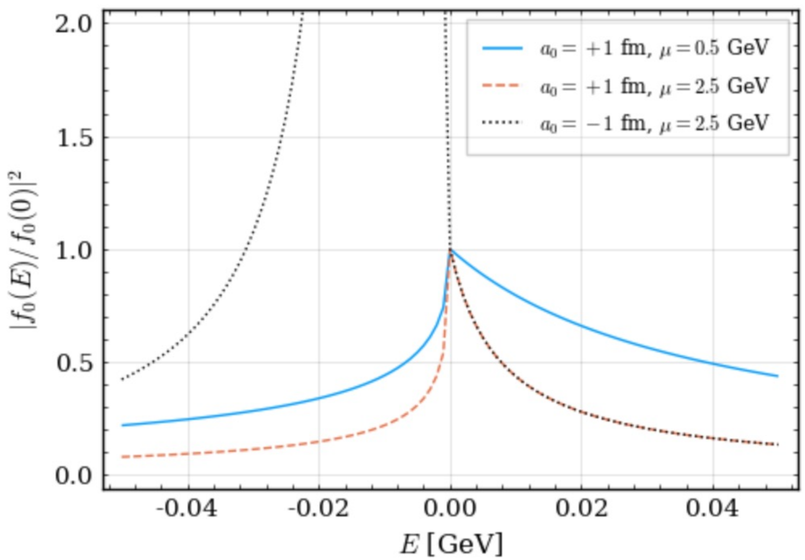
Very close to threshold, then scattering length approximation: $f_0^{-1}(E) = \frac{1}{a_0} - i\sqrt{2\mu E}$

$$|f_0(E)|^2 = \begin{cases} \dfrac{1}{1/a_0^2 + 2\mu E} & \text{for } E \geq 0 \\ \dfrac{1}{\left(1/a_0 + \sqrt{-2\mu E}\right)^2} & \text{for } E < 0 \end{cases}$$

- Cusp at threshold (E=0)
- Maximal at threshold for positive a_0 (attraction)
- Half-maximum width: $\frac{2}{\mu a_0^2}$;

  virtual state pole at $E_{\text{virtual}} = -1/(2\mu a_0^2)$

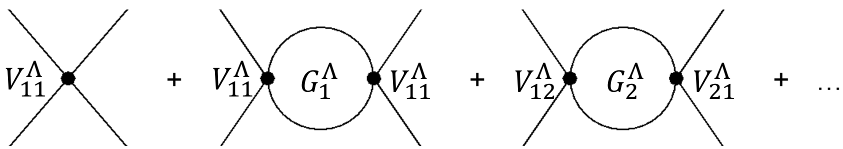
  Example of virtual state: ${}^1S_0$ NN system
- Strong interaction, a_0 becomes negative, pole below threshold, peak below threshold



Near-threshold structures

X.-K. Dong, FKG, B.-S. Zou, PRL126,152001(2021)

- Full threshold structure needs to be measured in a lower channel ⇨ coupled channels
- Consider a two-channel system, construct a "nonrelativistic" effective field theory (NREFT)
  - ➢ Energy region around the higher threshold, Σ_2
  - ➢ Expansion in powers of $E = \sqrt{s} - \Sigma_2$
  - ➢ Momentum in the lower channel can also be expanded



$$\det = \left(\frac{1}{a_{11}} - ik_1\right)\left(\frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon}\right) - \frac{1}{a_{12}^2}$$

$$T(E) = 8\pi\Sigma_2 \begin{pmatrix} -\frac{1}{a_{11}} + ik_1 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - \sqrt{-2\mu_2 E - i\epsilon} \end{pmatrix}^{-1} = -\frac{8\pi\Sigma_2}{\det} \begin{pmatrix} \frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon} & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \frac{1}{a_{11}} - ik_1 \end{pmatrix}$$

For invertible V

Effective scattering length with open-channel effects becomes complex, $\mathrm{Im}\,\frac{1}{a_{22,\mathrm{eff}}} \leq 0$

$$T_{22}(E) = -\frac{8\pi}{\Sigma_2}\left[\frac{1}{a_{22,\mathrm{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1}$$

$$\frac{1}{a_{22,\mathrm{eff}}} = \frac{1}{a_{22}} - \frac{a_{11}}{a_{12}^2(1 + a_{11}^2 k_1^2)} - i\frac{a_{11}^2 k_1}{a_{12}^2(1 + a_{11}^2 k_1^2)}.$$

See also, K. Sone, T. Hyodo, arXiv:2405.08436

Near-threshold structures

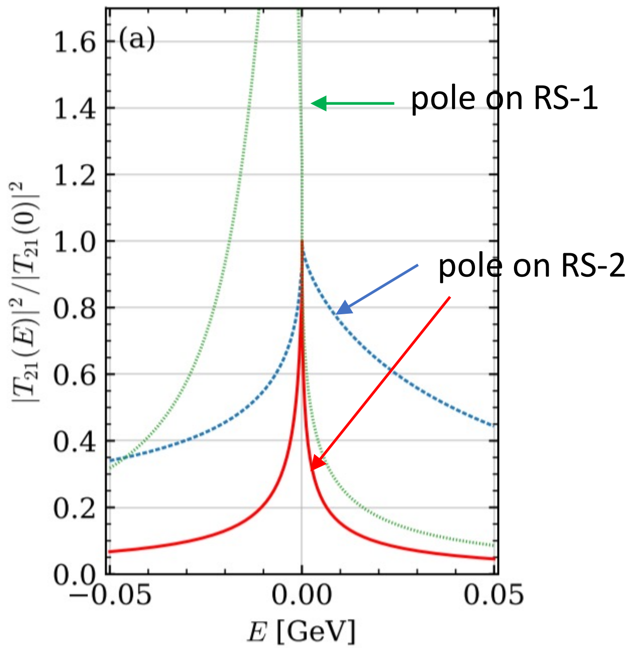
- Full threshold structure needs to be measured in a lower channel ⇨ coupled channels

$$T_{21}(E) = \frac{-8\pi\Sigma_2}{a_{12}(1/a_{11} - ik_1)}\left[\frac{1}{a_{22,\text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1}.$$

$$|T_{21}(E)|^2 \propto |T_{22}(E)|^2 \propto$$

≤ 0 due to unitarity

$$\begin{cases} \left[\left(\text{Re}\,\frac{1}{a_{22,\text{eff}}}\right)^2 + \left(\text{Im}\,\frac{1}{a_{22,\text{eff}}} - \sqrt{2\mu E}\right)^2\right]^{-1} & \text{for } E \geq 0 \\ \left[\left(\text{Im}\,\frac{1}{a_{22,\text{eff}}}\right)^2 + \left(\text{Re}\,\frac{1}{a_{22,\text{eff}}} + \sqrt{-2\mu E}\right)^2\right]^{-1} & \text{for } E < 0 \end{cases}$$



- Large $|a_{22,\text{eff}}|$ means a near-threshold pole
- Maximal at threshold for positive $\text{Re}(a_{22,\text{eff}})$ (attraction), FWHM $\propto 1/\mu$, "virtual" state pole
- Peaking at pole for negative $\text{Re}(a_{22,\text{eff}})$: "bound" state pole

$$\frac{1}{\mu}\left(\frac{4}{|a_0|^2} - \sum_x x\sqrt{\frac{3}{|a_0|^2} + x^2}\right),$$

the sum runs over $x = \text{Im}(1/a_0)$ and $\text{Re}(1/a_0)$

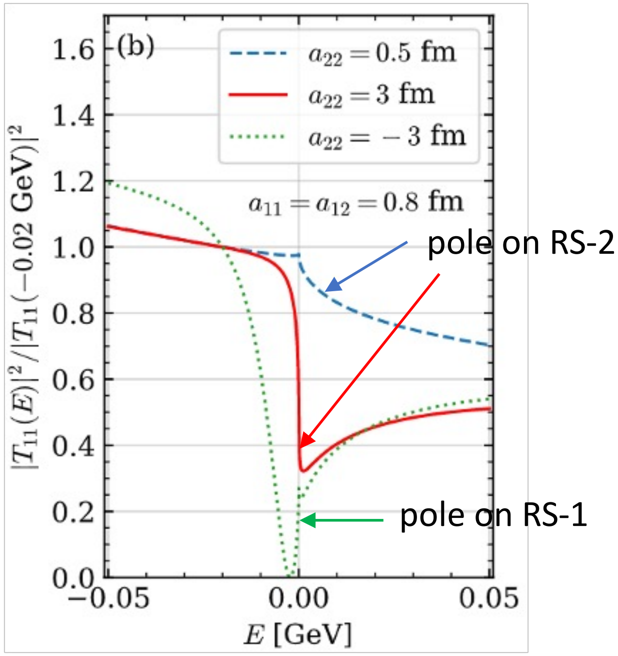
Near-threshold structures

X.-K. Dong, FKG, B.-S. Zou, PRL 126, 152001 (2021)

- An amplitude with a pole can also produce a dip ⇨ coupled channel T-matrix element

  1: lower ch.; 2: higher ch.

$$T_{11}(E) = \frac{-8\pi\Sigma_2\left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E}\right)}{\left(\frac{1}{a_{11}} - i\,k_1\right)\left[\frac{1}{a_{22,\mathrm{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]}$$



  - One pole and one zero
  - Universality for large scattering length:

    For strongly interacting channel-2 (large a_{22}), there must be a dip around threshold (zero close to threshold)
  - It can be rewritten in an interference form:

    V. Baru, FKG, C. Hanhart, A. Nefediev, PRD 109, L111501 (2024)

$$T_{11}(E) = -8\pi E_2^{\mathrm{thr}}\left(\frac{1}{a_{11}^{-1} - ik_1} + \frac{a_{12}^{-2}(a_{11}^{-1} - ik_1)^{-2}}{a_{22,\mathrm{eff}}^{-1} - ik_2}\right)$$

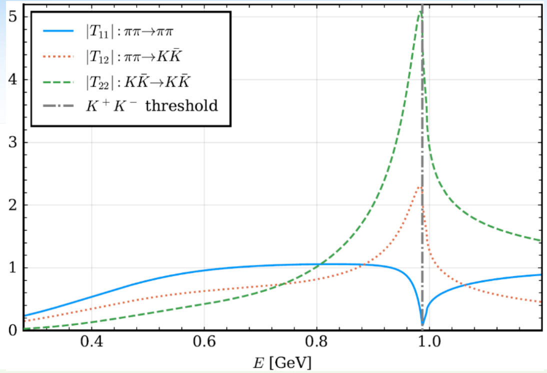
coupled-channel amp. in a 2-potential form

background   pole term

The interfering phase is fixed by unitarity!

- Strong interaction can lead to highly nontrivial near-threshold structures!

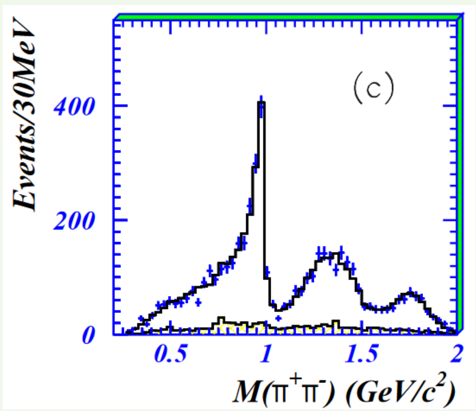
Peak versus dip



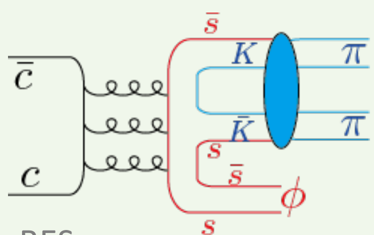
✓ T-matrix for $\pi\pi$ and $K\bar{K}$ coupled channels

with the T-matrix from
L.-Y. Dai, M. R. Pennington, PRD90(2014)036004

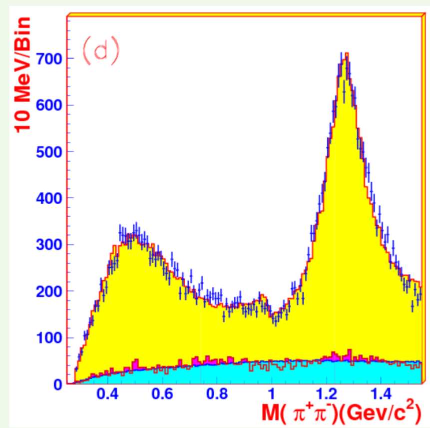
✓ $J/\psi \to \phi K\bar{K} \to \phi\pi^+\pi^-$ v.s. $J/\psi \to \omega\pi\pi \to \omega\pi^+\pi^-$



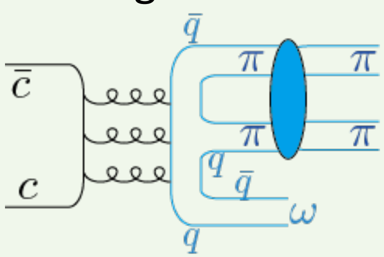
Driving ch.: $K\bar{K}$



BES, PLB607(2005)243

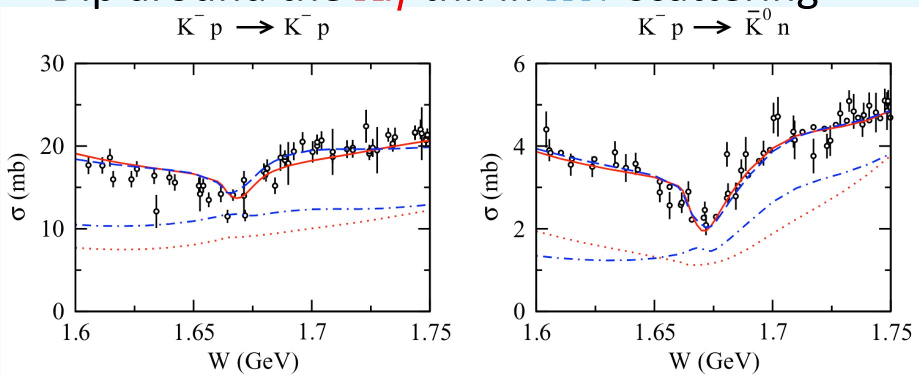


Driving ch.: $\pi\pi$



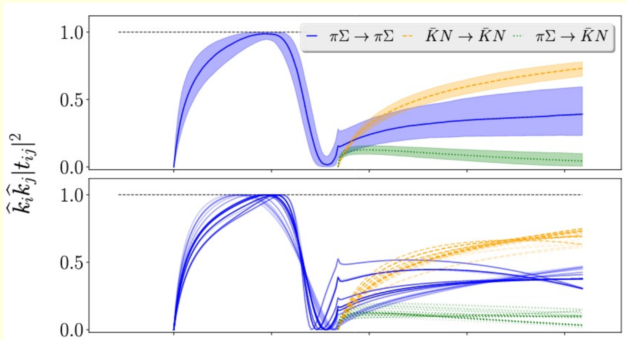
BES, PLB598(2004)149

✓ Dip around the $\Lambda\eta$ thr. in $\bar{K}N$ scattering



H. Kumano et al., PRC 92, 025205 (2015)

✓ Dip below $\bar{K}N$ thr. in $\pi\Sigma$ scattering

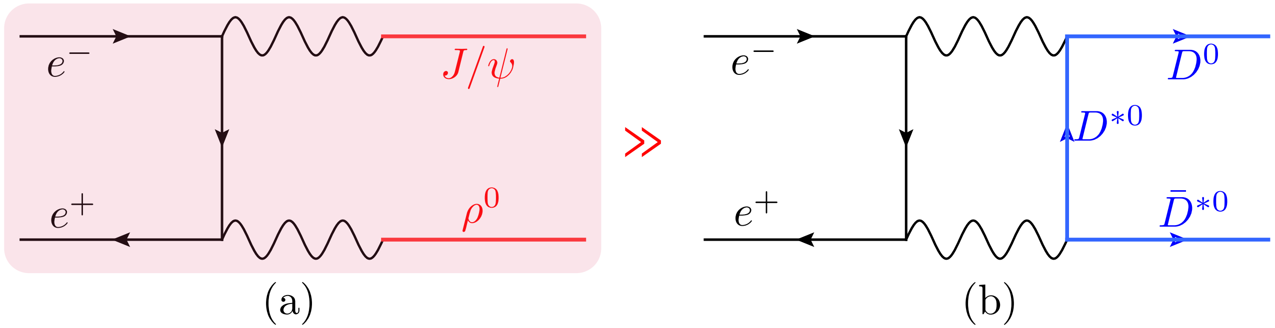


LQCD, J. Bulava et al., PRL 132, 051901 (2024)

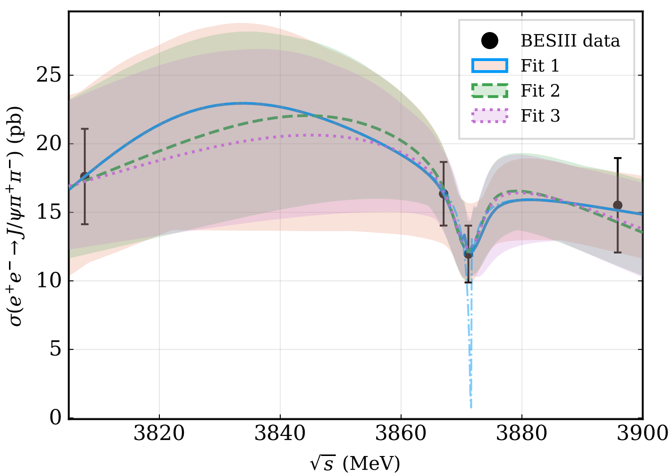
Peak versus dip

- Direct production of $X(3872)$ in $e^+e^- \to X(3872) \to J/\psi\pi^+\pi^-$

V. Baru, FKG, C. Hanhart, A. Nefediev, PRD 109, L111501 (2024)



- It is reasonable to assume that the $X(3872)$ roots in the amplitude of $J/\psi\rho \to J/\psi\rho$
  - Channel-1: $J/\psi\rho^0$; channel-2: $D\bar{D}^*$
  - Production amplitude: $\mathcal{A}(\sqrt{s}) \propto T_{11}(E)$

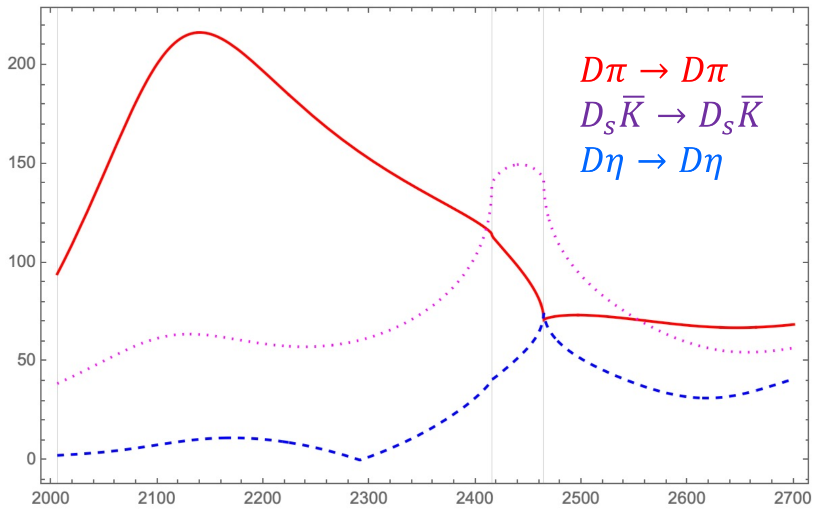


All fits lead to a large, negative (bound state) $D\bar{D}^*$ single-channel scattering length!

- Expect a dip around $D^*\bar{D}^*$ thr., too, corresponding to the X_2, spin partner of $X(3872)$

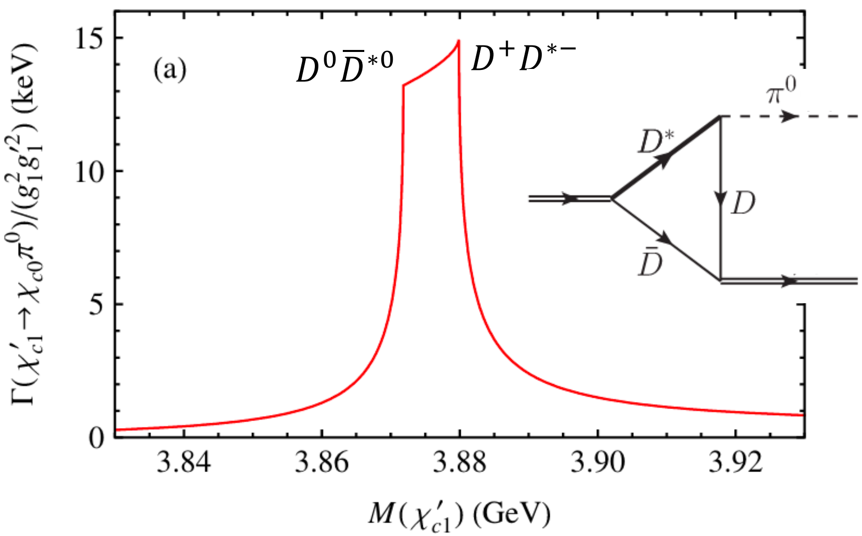
With more channels: some theory curves

w/ a pole



Calculated using UCHPT w/ parameters fixed in
L. Liu et al., PRD87 (2013) 014508;

w/o a pole



FKG, Hanhart, Meißner, G. Li, Q. Zhao,
PRD83 (2011) 034013

- Thresholds are dense, close to one another ⇒ more complicated line shapes
- Some nearby thresholds are from channels related to each other by symmetries: isospin, SU(3), heavy quark spin

➡️ Two nonrelativistic channel system related by some symmetry

Two NR channels: RGE analysis

V. Lensky, M.C. Birse, EPJA 47 (2011) 142

- Coupled-channel EFT for contact-range S-wave interactions

$$\mathsf{T}(p,\delta) = \mathsf{V}(p,\delta) + \mathsf{V}(p,\delta)\mathsf{J}(p,\delta)\mathsf{T}(p,\delta),$$

$$J_i(p,\delta) = 2M_i \int \frac{d^3\vec{q}}{2\pi^3}\frac{1}{p_i^2 - q^2 + i\epsilon} = -\frac{M_i}{2\pi}(\mu + i p_i).$$

δ: threshold difference
M_i: reduced mass in channel-i
μ: PDS scale

- Renormalization group equations: $\frac{\partial \mathsf{V}}{\partial \mu} = -\mathsf{V}\frac{\partial \mathsf{J}}{\partial \mu}\mathsf{V}. \Rightarrow \mu\frac{\partial \hat{\mathsf{V}}}{\partial \mu} = \hat{p}\frac{\partial \hat{\mathsf{V}}}{\partial \hat{p}} + \hat{\delta}\frac{\partial \hat{\mathsf{V}}}{\partial \hat{\delta}} + \hat{\mathsf{V}} + \hat{\mathsf{V}}^2.$

$$\hat{p} = p/\mu, \quad \hat{\delta} = \delta/\mu, \quad \hat{\mathsf{V}} = \frac{\mu}{2\pi}\mathsf{M}^{1/2}\mathsf{V}\,\mathsf{M}^{1/2},$$

- Fixed point (FP) solutions:
  - Trivial one: weak interacting limit: $\hat{V}_0 = 0$
  - Two bound/virtual states at threshold: $\hat{V}_2 = -\mathbb{I}_{2\times 2}$
  - One bound state at threshold coupled with both channels:

$$\hat{\mathbf{V}}_1 = \begin{pmatrix} -c & \pm\sqrt{c(1-c)} \\ \pm\sqrt{c(1-c)} & -(1-c) \end{pmatrix}$$

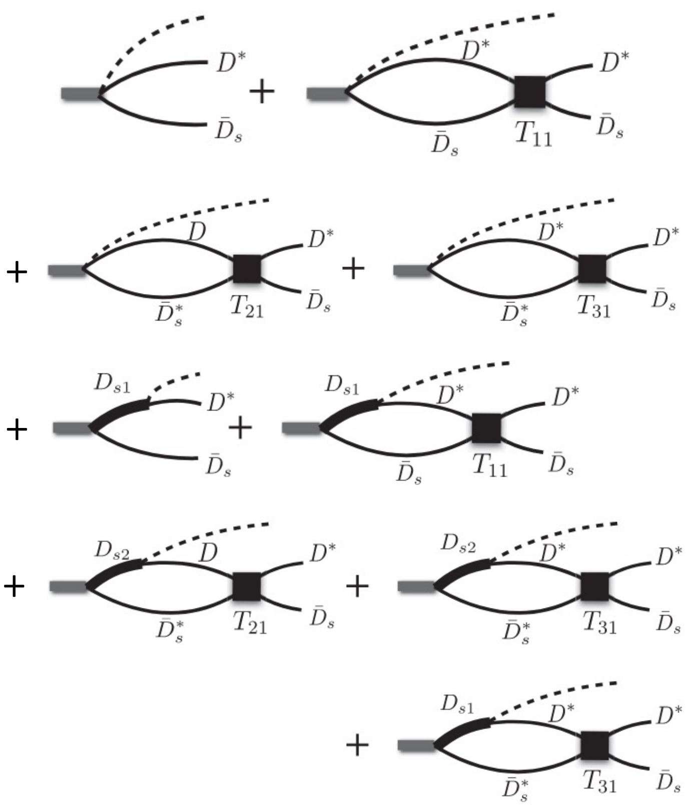
V is not invertible in this case

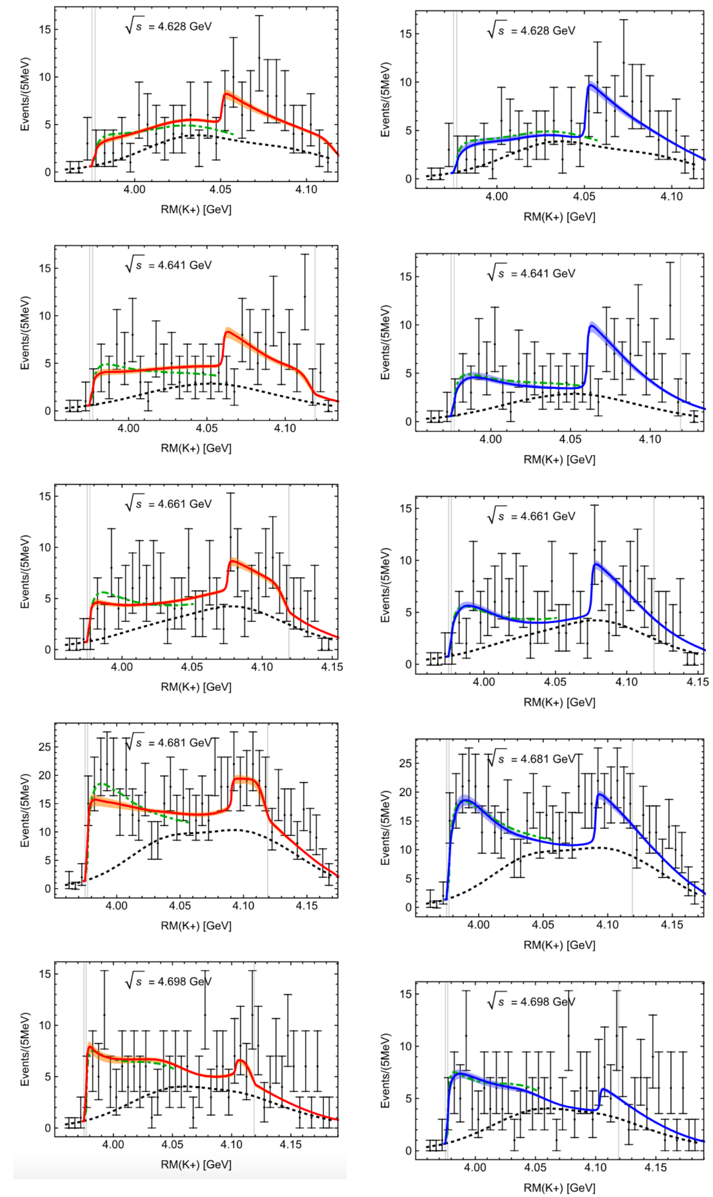
$$\mathbf{T} = 2\pi\mathbf{M}^{-1/2}\mathbf{R}\begin{pmatrix} -\frac{1}{a_{11}} + ip_{11} & \frac{1}{a_{12}} + ip_{12} \\ \frac{1}{a_{12}} + ip_{12} & -\frac{1}{a_{22}} + ip_{22} \end{pmatrix}^{-1}\mathbf{R}^T\mathbf{M}^{-1/2}$$

Example: Z_{cs}

- $Z_{cs}(3985)$ as a $(D_s^- D^{*0} + D_s^{*-} D^0)$ molecular state V. Baru et al., PRD 105, 034014 (2022)
  - Fit to BESIII data in the full range
  - Scenarios: two

    or one Z_{cs} (strong ch. coupling)
  - Both $D_{s2}\bar{D}_s^* D^{(*)}$ and $D_{s1}\bar{D}_s^{(*)} D^*$ triangles



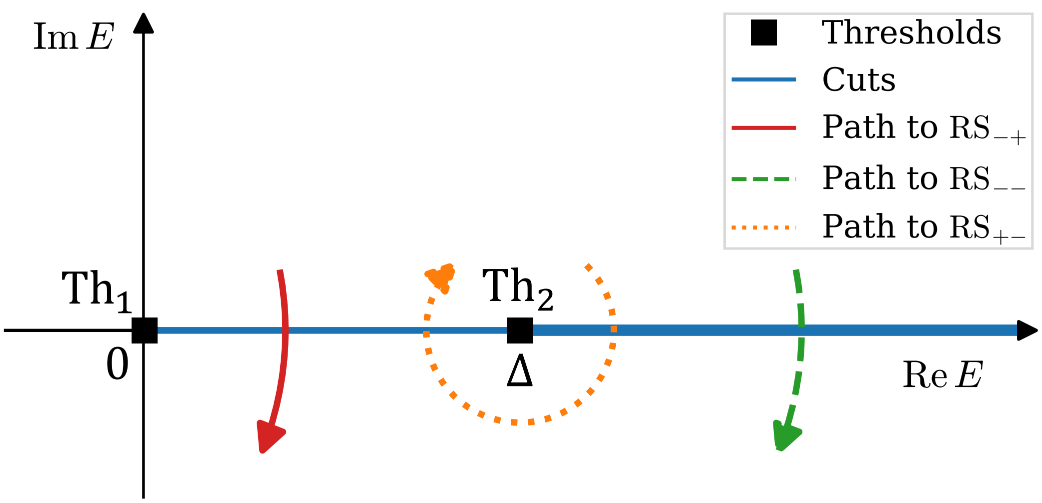


Riemann sheets

- Will understand various possible line shapes through pole locations

Z.-H. Zhang, FKG, PLB 863 (2025) 139387

- For two channels, 4 Riemann sheets (RSs)
  - RS_{++} (RS-1): $\mathrm{Im}\, p_1 > 0, \mathrm{Im}\, p_2 > 0$
  - RS_{-+} (RS-2): $\mathrm{Im}\, p_1 < 0, \mathrm{Im}\, p_2 > 0$
  - RS_{--} (RS-3): $\mathrm{Im}\, p_1 < 0, \mathrm{Im}\, p_2 < 0$
  - RS_{+-} (RS-4): $\mathrm{Im}\, p_1 > 0, \mathrm{Im}\, p_2 < 0$
- Physical region: upper edge of cut on RS_{++}
  - Paths from the physical region to various unphysical RSs:

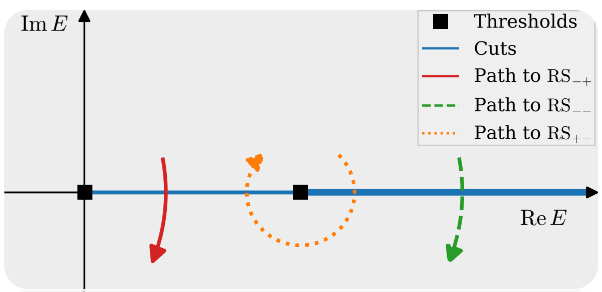


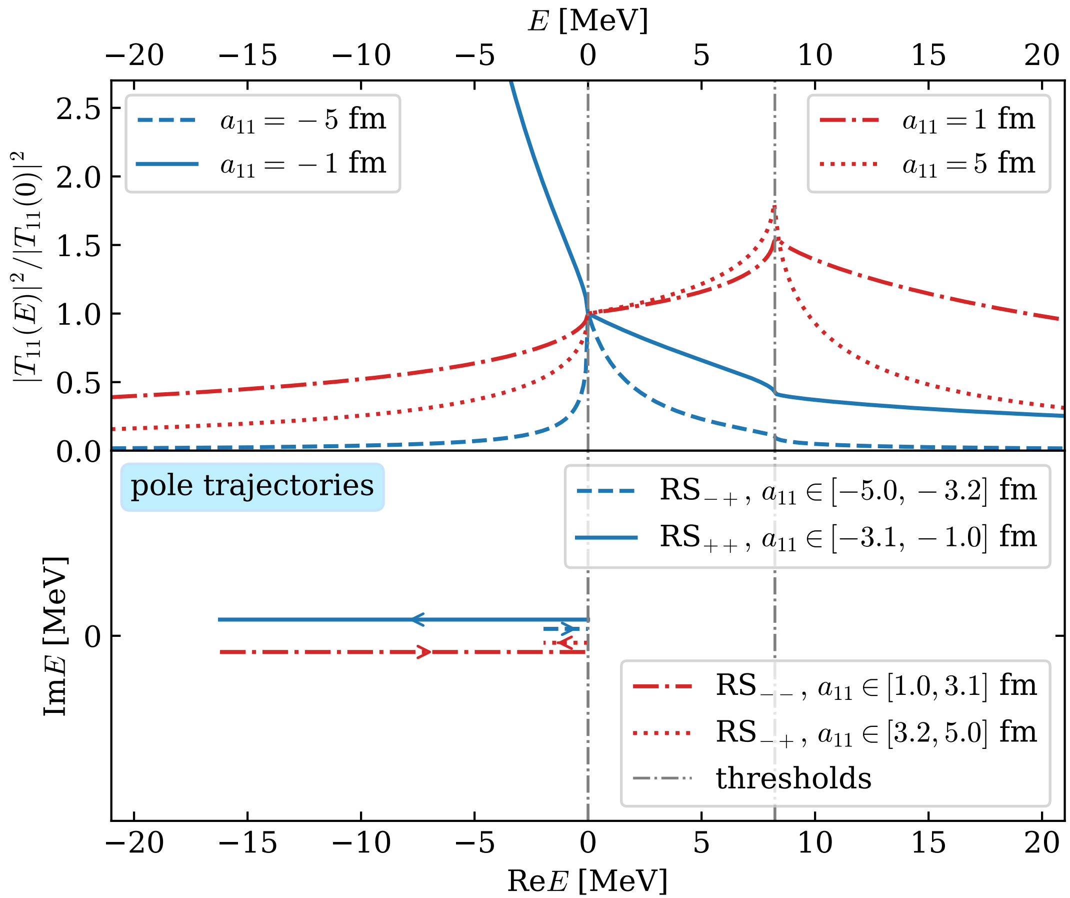
Single-pole cases

- From now on, we consider the two channels related by some symmetry, $a_{22} = a_{11}$
- Start from the 1-pole FP, $a_{11} = -\infty$, and the consider evolution of line shape and pole by varying a_{11}
  - Pole equation:

$$\frac{2}{a_{11}} - ir_1p_1 - ir_2p_2 = 0,$$

$r_i = \pm$ controls RS





Here we take masses of $D^0\bar{D}^{*0}$ and D^+D^{*-} as those for channel-1 and channel-2

Two-channel cases

- Again consider symmetry-related channels for simplicity
  - Pole equation

$$\left(\frac{1}{a_{11}} - ir_1p_1\right)\left(\frac{1}{a_{11}} - ir_2p_2\right) - \frac{1}{a_{12}^2} = 0$$

  - Can be rewritten as an order-4 polynomial equation in ω

$$p_1 = \sqrt{\frac{\mu_1\Delta}{2}}\left(\omega + \frac{1}{\omega}\right), \quad p_2 = \sqrt{\frac{\mu_2\Delta}{2}}\left(\omega - \frac{1}{\omega}\right)$$

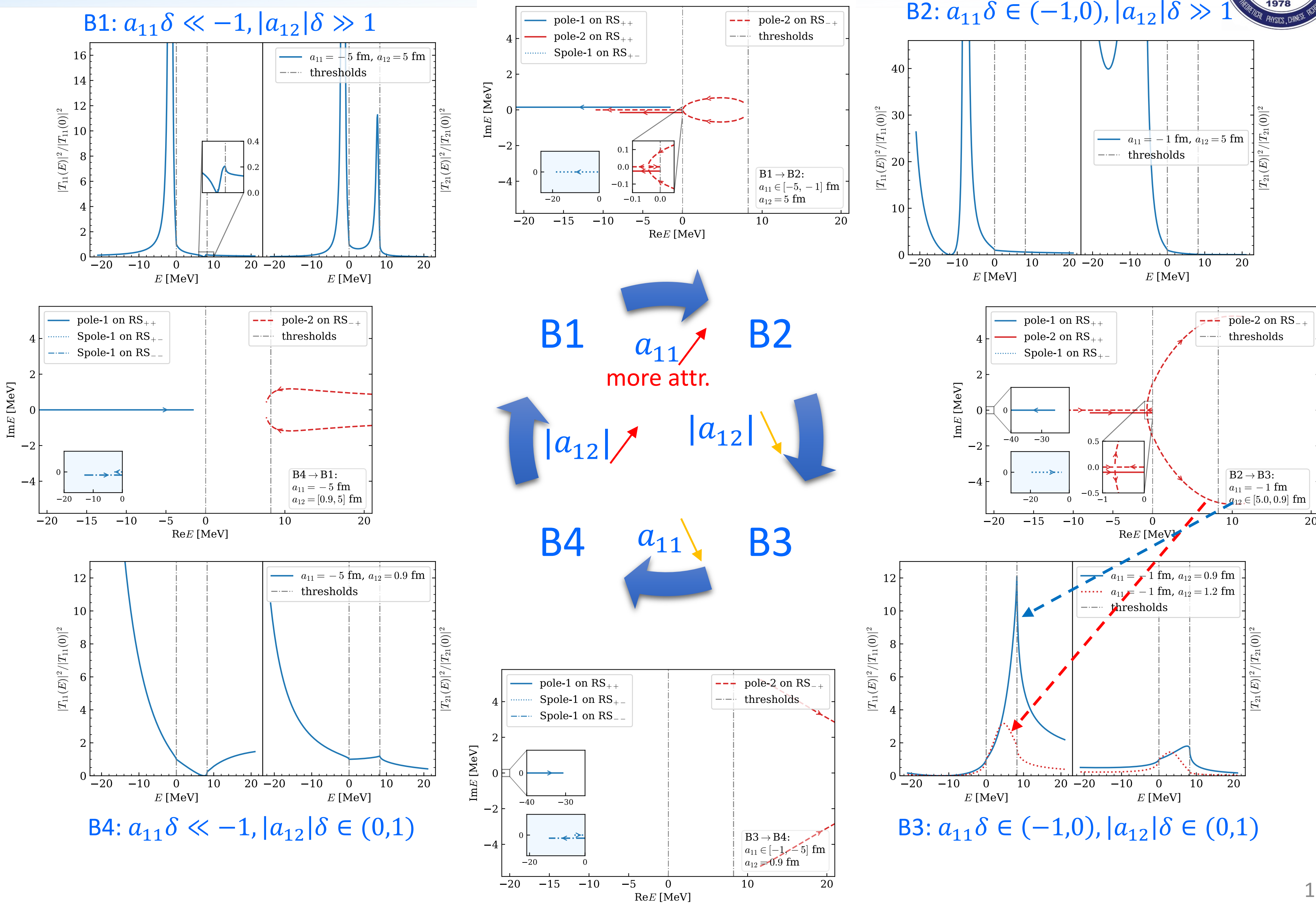
M. Kato, Annals Phys. 31, 130 (1965)

    - 4 solutions, thus 4 poles
    - But not all independent. Only 2 independent ones:
      - One bound or virtual state pole + its shadow pole
      - One complex conjugate pair

- Vary a_{11} (single-channel interaction strength) and a_{12} (channel coupling parameter), check the correspondence between the line shapes and pole locations in various RSs

Two-channel cases (B): $a_{11} < 0$

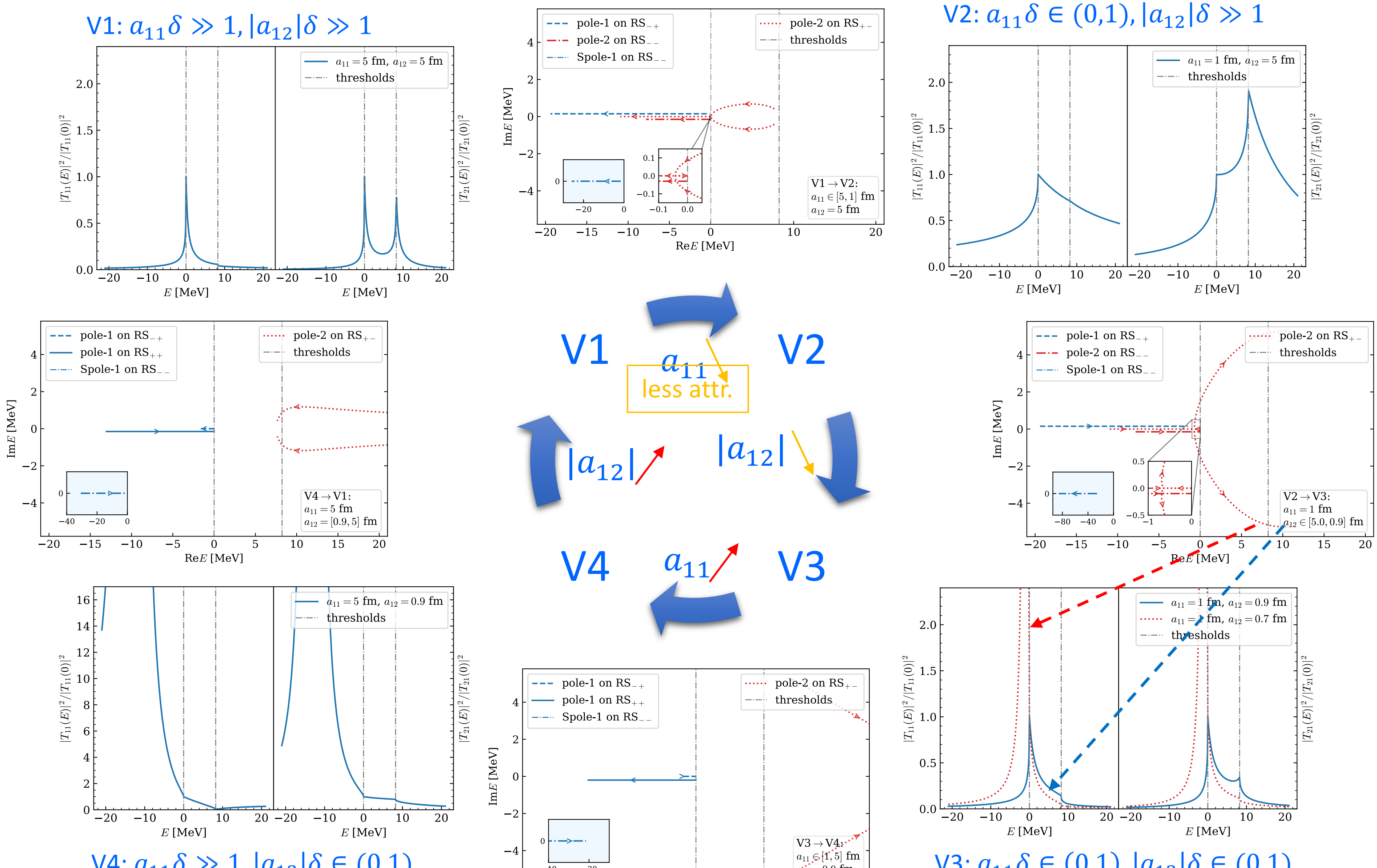
Here, $\delta \equiv \sqrt{2\mu_2\Delta}$, Spole: shadow pole



Two-channel cases (V): $a_{11} > 0$

V1: $a_{11}\delta \gg 1, |a_{12}|\delta \gg 1$

V2: $a_{11}\delta \in (0,1), |a_{12}|\delta \gg 1$



V4: $a_{11}\delta \gg 1, |a_{12}|\delta \in (0,1)$

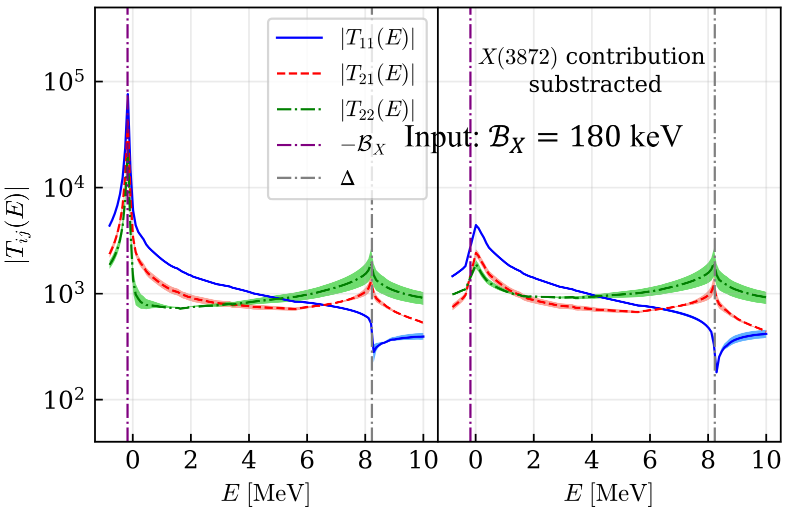
V3: $a_{11}\delta \in (0,1), |a_{12}|\delta \in (0,1)$

Poles dual to those in the B cases, on opposite RSs.

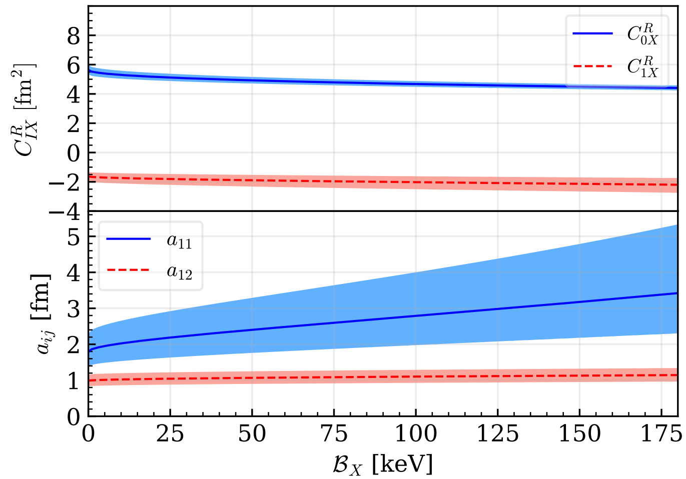
Two-channel cases: $a_{11} > 0$

- Example: $X(3872)$ and W_{c1}^0 (another pole for $J^{PC} = 1^{++}$ in $D^0\bar{D}^{*0} - D^+D^{*-}$ coupled channel systems, isovector as it has two charged partners) Z.-H. Zhang et al., JHEP 08 (2024) 130

  - W_{c1}^0 in $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering amplitudes



  - W_{c1}^0 in $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering amplitudes corresponds to case V4



V4: $a_{11}\delta \gg 1, |a_{12}|\delta \in (0,1)$

$\Rightarrow$ X(3872) is a bound state unavoidably!
Shape sensitive the poles rel. to thresholds, not that to total mass.

For a full analysis consider both $X(3872)$ and W_{c1} simultaneously, T. Ji et al., arXiv:2502.04458

Summary

- Universal threshold dip for large $|a_{22}|$ in 2-channel system
- Correspondence between coupled-channel line shapes and poles; pole trajectories can understood from evolving the interaction strength and channel coupling from RG fixed points

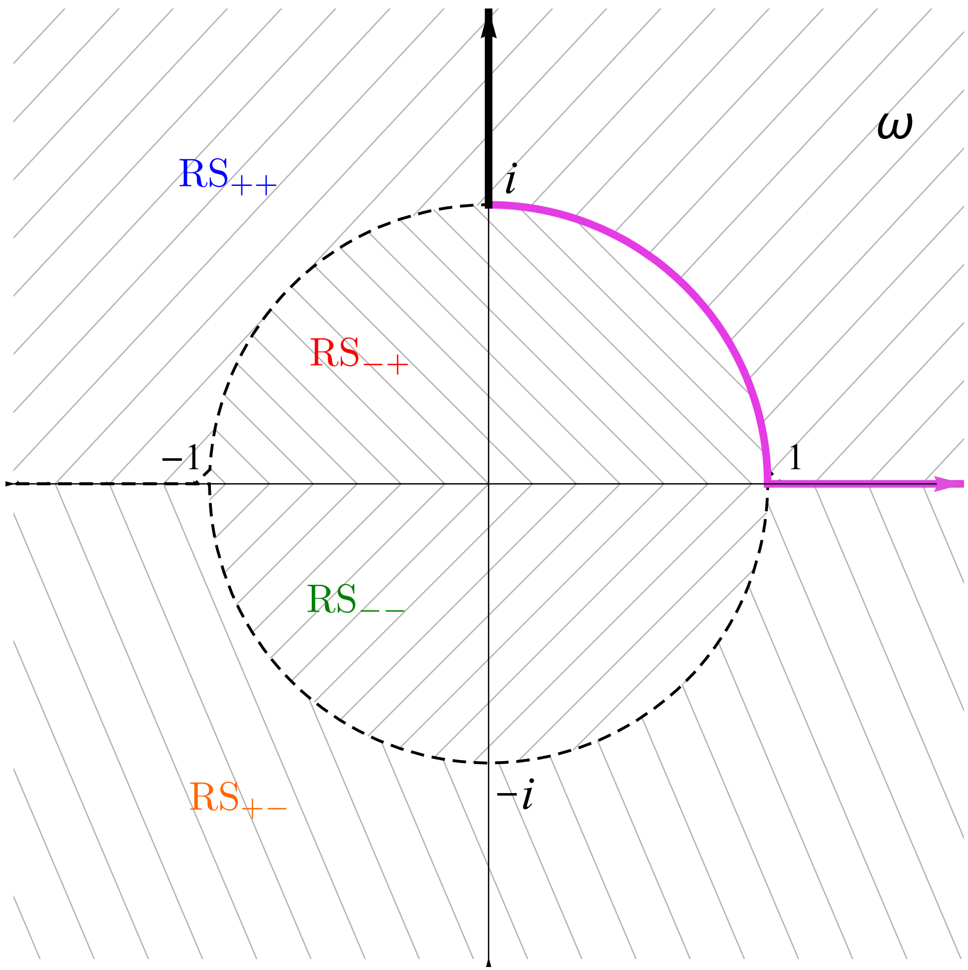
Thank you for your attention!

Riemann sheets

- Conformal mapping of all 4 FSs to a single ω plane

M. Kato, Annals Phys. 31, 130 (1965)

$$p_1 = \sqrt{\frac{\mu_1 \Delta}{2}} \left(\omega + \frac{1}{\omega}\right), \quad p_2 = \sqrt{\frac{\mu_2 \Delta}{2}} \left(\omega - \frac{1}{\omega}\right)$$



Threshold-1: $\omega = \pm i$

Threshold-2: $\omega = \pm 1$