

The **top mass** at the $t\bar{t}$ threshold with CEPC

Approval meeting of Physics Benchmarks for RefTDR

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on behalf of

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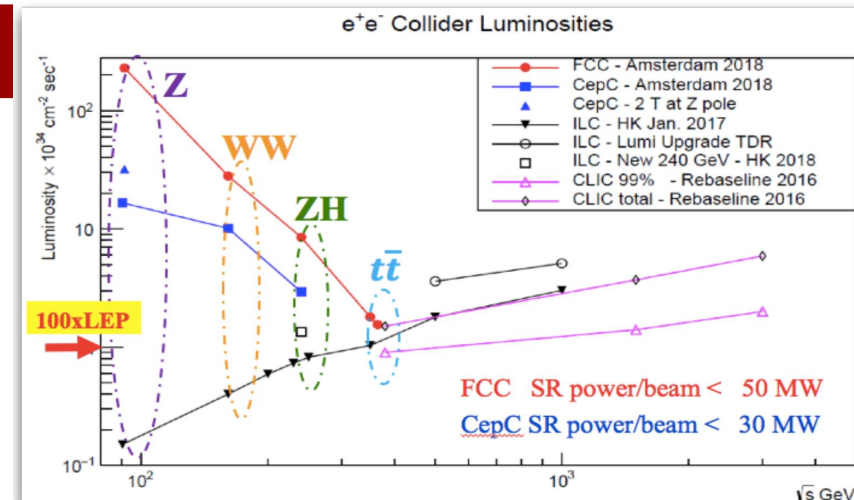
Reference: [Eur. Phys. J. C \(2023\) 83:269](#), [arXiv:2207.12177](#)

- 1. Motivation [Eur. Phys. J. C \(2023\) 83:269](#) [arXiv:2207.12177](#)
- 2. Fast-simulation
- 3. Comparison with FCC-ee result <https://arxiv.org/pdf/2503.18713>
- 4. Summary

Motivation

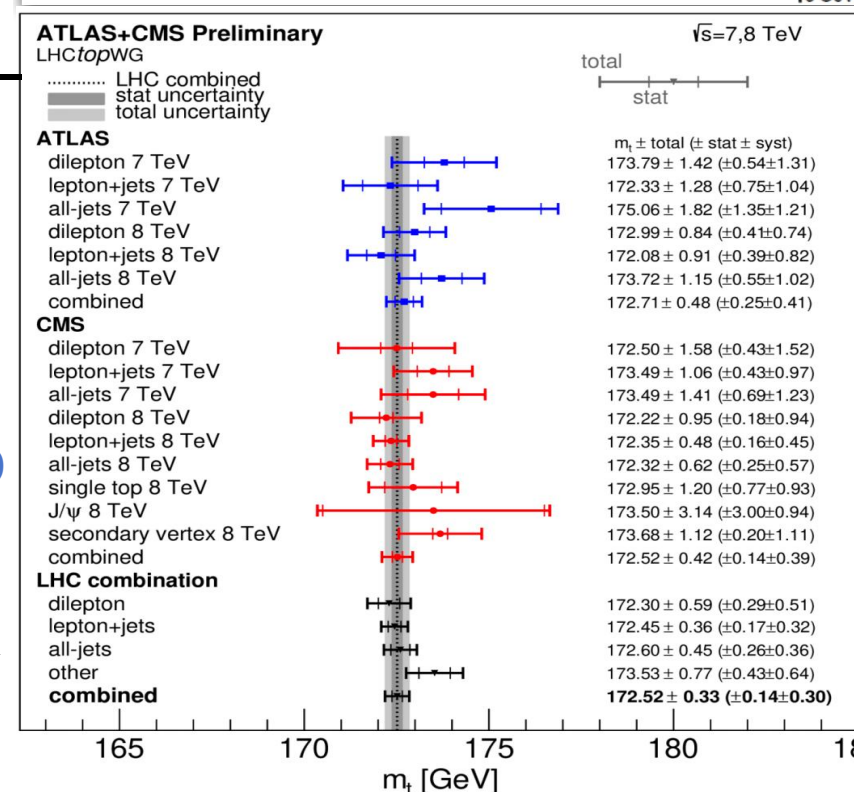
Introduction

- CEPC will be a versatile machine with many opportunities
 - Higgs factory @~240 GeV, Diboson factory @~160 GeV, Z factory @~90 GeV
- @~360 GeV it can also be a playground for
 - Top quark precision measurements
 - Higgs complementary measurements
 - BSM searches



Top quark mass measurements

- The top “pole” mass is measured using top reconstruction at hadron colliders
- Heavily relies on the performance of MET (the neutrino) and JER & JES
- ATLAS+CMS combined measurements (15) reached a level of uncertainties of 330 MeV dominated by systematic uncertainties
- Precision improvements are limited by dominant systematic uncertainties in hadron collider environments.

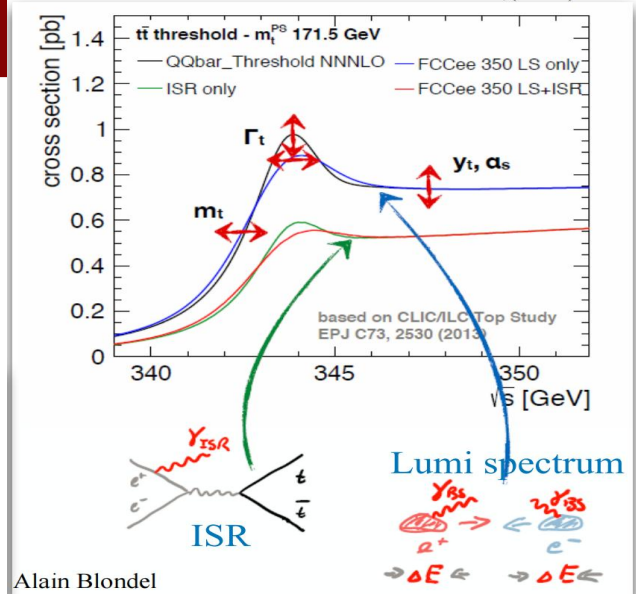
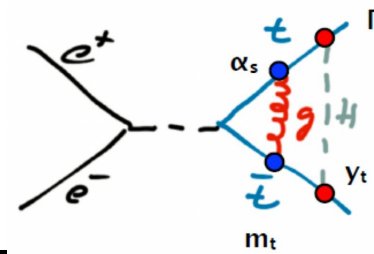


[ATLAS-CONF-2023-066, CMS-PAS-TOP-22-001 for Run1](#)

[New results such as CMS Eur. Phys. J. C 83 \(2023\) 963 with 370 MeV using Run2](#)

ttbar threshold scan

- ee-colliders enable both top reconstruction and ttbar threshold scan.
- The scan is made against $\sqrt{s}$ and cross-section is the direct observable
- This brings measurements of top mass and a couple of other parameters
 - Γ_t , y_t , α_s

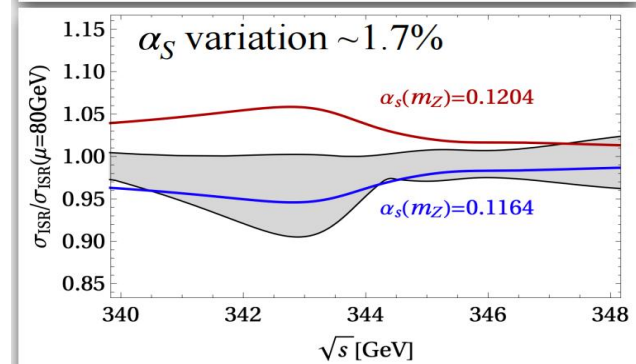
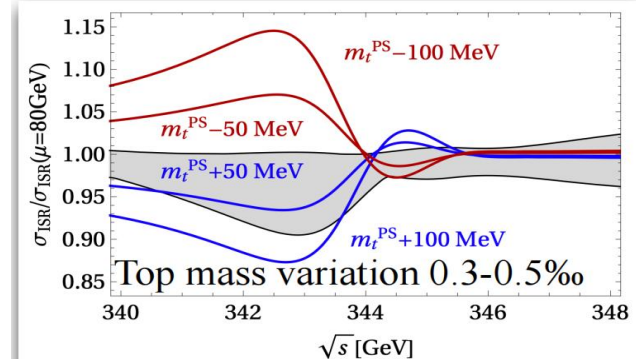


Our setup in [Eur. Phys. J. C \(2023\) 83:269](#), [arXiv:2207.12177](#)

- Use the package “QQbar_threshold” to calculate $\sigma(e^+e^- \rightarrow t\bar{t})$ near threshold in ee-colliders at N³LO precision

$$m_t^{\text{PS}} = 171.5 \text{ GeV}, \quad \alpha_s(m_Z) = 0.1184$$

- ISR effects are also included in the package
- We integrate LS by a Gaussian function with the CEPC expected beam energy spread (~ 500 MeV) as a function of $\sqrt{s}$

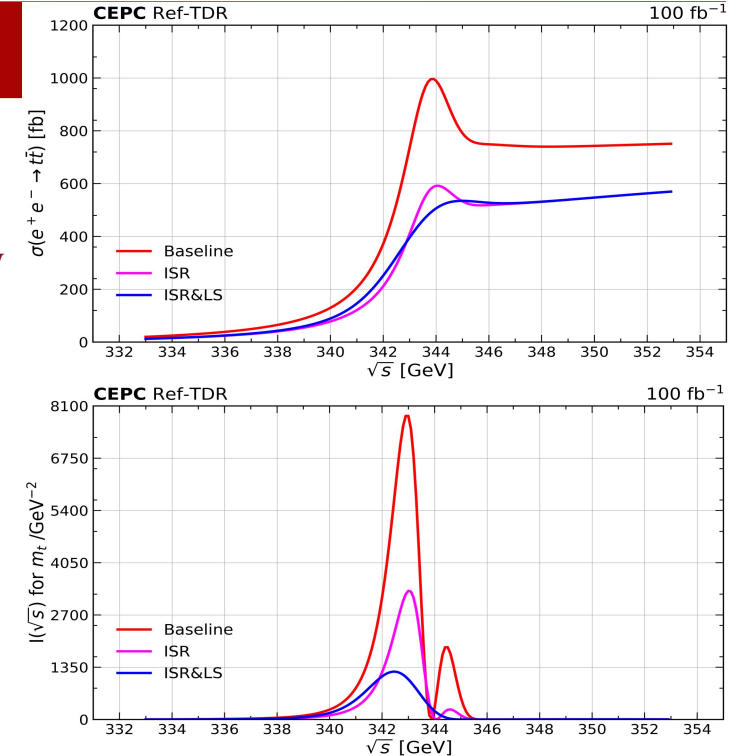


Determining the best collision energy

- CEPC plans to collect **100 fb⁻¹** of data by ramping up the center-of-mass energy to the $t\bar{t}$ threshold.
- Around the $t\bar{t}$ threshold, we need to identify the **energy point(s)** that contain(s) the **most sensitivity**
- Construct **Fisher** information to test the energy point(s)

$$I(\sqrt{s}) = \int \left(\frac{\partial \log(G(\sigma | \sigma_0(\sqrt{s}, \theta), \sqrt{\sigma_0(\sqrt{s}, \theta))})}{\partial \theta} \right)^2 \times G(\sigma | \sigma_0(\sqrt{s}, \theta), \sqrt{\sigma_0(\sqrt{s}, \theta)}) d\sigma.$$

- **Larger amplitudes implies richer information and higher sensitivities**



- Aiming at measuring one parameter at a time (1D), given limited total luminosity:
 - **Only colliding at one optimal energy point would give the best sensitivity**
 - This is tested with many different situations: one vs multiples energy points, un-even luminosity allocation etc.
 - We also have tried 2D fit at a time, but it yields a worse precision on m_{top}
- Statistical precision at CEPC shows that **342.75 GeV** is optimal for m_{top} , giving a **9 MeV** uncertainty.

statistical and systematical uncertainties of m_{top}

$$\mathcal{L} = \prod_{i=1}^N P(D|\sigma_{t\bar{t}}(m_{top}, \Gamma_{top}, \alpha_S, \sqrt{s_i}) \times L_i \times \epsilon)$$

- **Statistics:** 9MeV are calculated under the total luminosity of 100 fb⁻¹, All luminosity is allocated to a single energy point optimized via Fisher information.
- **Theory:** Assuming 1% and 3% theory uncertainties on the cross section, which leads to 8 MeV and 24 MeV uncertainty on m_{top} , respectively.
- **Quick scan & Beam energy:** A quick scan is used to determine the optimal energy point, and the CEPC beam energy precision (~1 MeV) results in a 2 MeV uncertainty on m_{top}
- **α_S & width :** are the inputs for this 1D top mass measurement, α_S uncertainty (0.0007) and top width variation (± 0.14 GeV) lead to 16 MeV and 10 MeV uncertainties on m_{top}
- **Experimental efficiency:** in this paper, Experimental efficiency of the CEPC is yet to know. Assuming experimental efficiency uncertainties of 0.5%, 1%, 3%, and 5% results in corresponding top mass uncertainties of 4, 10, 27, and 44 MeV.
- **Background:** Assuming background uncertainties of 1% and 5% leads to top mass uncertainties of 2 MeV and 14 MeV, respectively, despite clean background subtraction.(refer to [t \$\bar{t}\$ bar threshold scan at CLIC](#))
- **Luminosity spectrum:** LS is varied for 10% and 20% that result in uncertainties of 3 MeV and 6 MeV on top mass

	Top mass uncertainties (MeV)	
	Optimistic	Conservative
Statistics	9	9
Theory	8	24
Quick scan	2	2
α_S	16	16
Width	10	10
Experimental efficiency	4	44
Background	2	14
Beam energy	2	2
Luminosity spectrum	3	6
Total	24	57

[Eur. Phys. J. C \(2023\) 83:269](#), [arXiv:2207.12177](#)

New work towards RefTDR: Fast simulation -> Improve Uncertainties

- Using CEPC RefTDR detector simulation, **experimental uncertainties** can be more accurately assessed, we focusing only on the most relevant sources:
 - **JES** and its uncertainties (WIP)
 - **an excellent btagger** and its uncertainties (WIP)
 - **JER** and its impact on b-tagging (WIP)
- The **statistical and background uncertainties** are primarily driven by the final ϵ_{sig} and σ_{bkg_total} . To obtain a more realistic evaluation:
 - Employ Delphes with a CEPC-specific detector configuration
 - Generate full signal and background samples to optimize the final ϵ_{sig} and σ_{bkg_total}
- The Other systematic uncertainties are held consistent with our previous work

Fast-simulation

- We have finished the analysis semi-leptonic & full hadronic channel of $e^+e^- \rightarrow t\bar{t} \rightarrow W^+b W^-b$
- Use **MadGraph5_aMC@NLO**, **Pythia8**, **Delphes** to generate fast-simulation sample
- Background samples are generated at **LO**, with cross sections computed using MadGraph5_aMC@NLO
- Signal sample (sl,hh), **NLO** samples are used, and the cross sections are calculated by QQbar_threshold tool (NNNLO precision)
- A total of 11 samples were generated at different center-of-mass energies: {338 GeV, 339 GeV, 340 GeV, 341 GeV, 342 GeV, 342.75 GeV, 343 GeV, 344 GeV, 345 GeV, 346 GeV, and 347 GeV}. Set $m_{\text{top}}=171.5\text{GeV}$, $\Gamma_{\text{top}}=1.33\text{GeV}$, $\alpha_s=0.1184$
- Turn ISR & FSR on in Pythia8

Background	Cross Section (fb)	Events
W^+W^-	11585.09	1000000
ZZ	702.62	1000000
ZW^+W^-	11.20	100000
ZZZ	0.6063	100000
$q\bar{q}$	4729.90	1000000
$b\bar{b}$	854.91	1000000
SingleTop	8.247	1000000

Table 18: Backgrounds with corresponding cross sections and generated events at 342.75GeV

- The delphes card is based on [CEPC Ref TDR model](#) by zhangkl@ihep.ac.cn
- Jet-clustering: Using the $e^+e^- - kt$ algorithm, the number of reconstructed jets is required to be 4 for the sl channel, and 6 for the hh channel, the other parameters is based on algorithm in delphes.
- JES : set ScaleFormula {1.0075} for 0.75% scale
- Btagging efficiency: using 95% working point refer to [JOI model from zhangkl@ihep.ac.cn](#)

```
# default efficiency formula (misidentification rate)  
add EfficiencyFormula {0} {0.01}
```

```
# efficiency formula for c-jets (misidentification rate)  
add EfficiencyFormula {4} {0.01}
```

```
# efficiency formula for b-jets  
add EfficiencyFormula {5} {0.95}
```

Object selection - isolated muon/electron

- To distinguish between semi-leptonic (sl) and fully hadronic (hh) events, the number of isolated leptons in the event is a key factor. Leptons with $E > 23\text{GeV}$ and $\text{IPS} < 2.7$ are selected as isolated leptons.

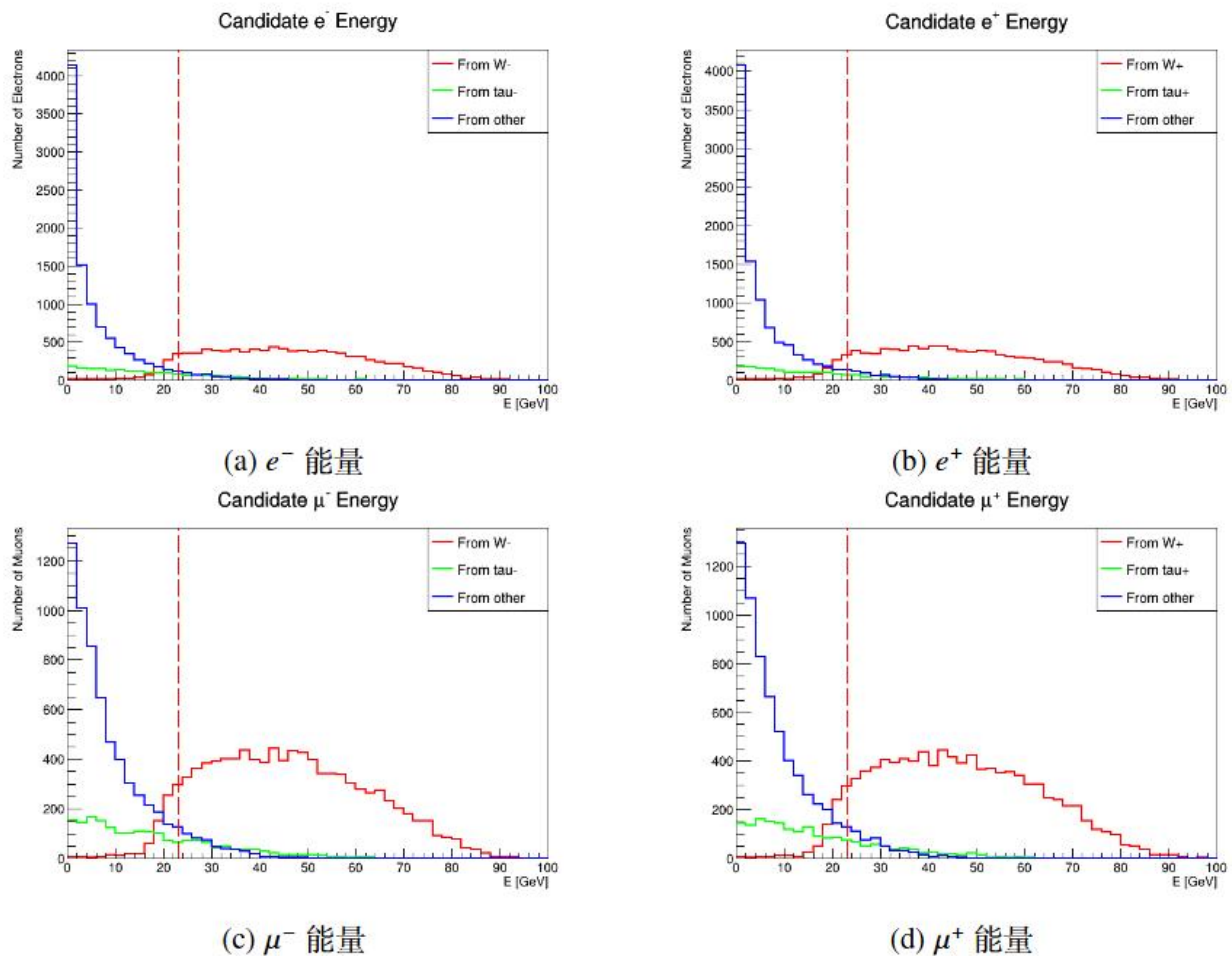


图 3-1 W 玻色子衰变得到的轻子与其他来源的轻子的能量分布

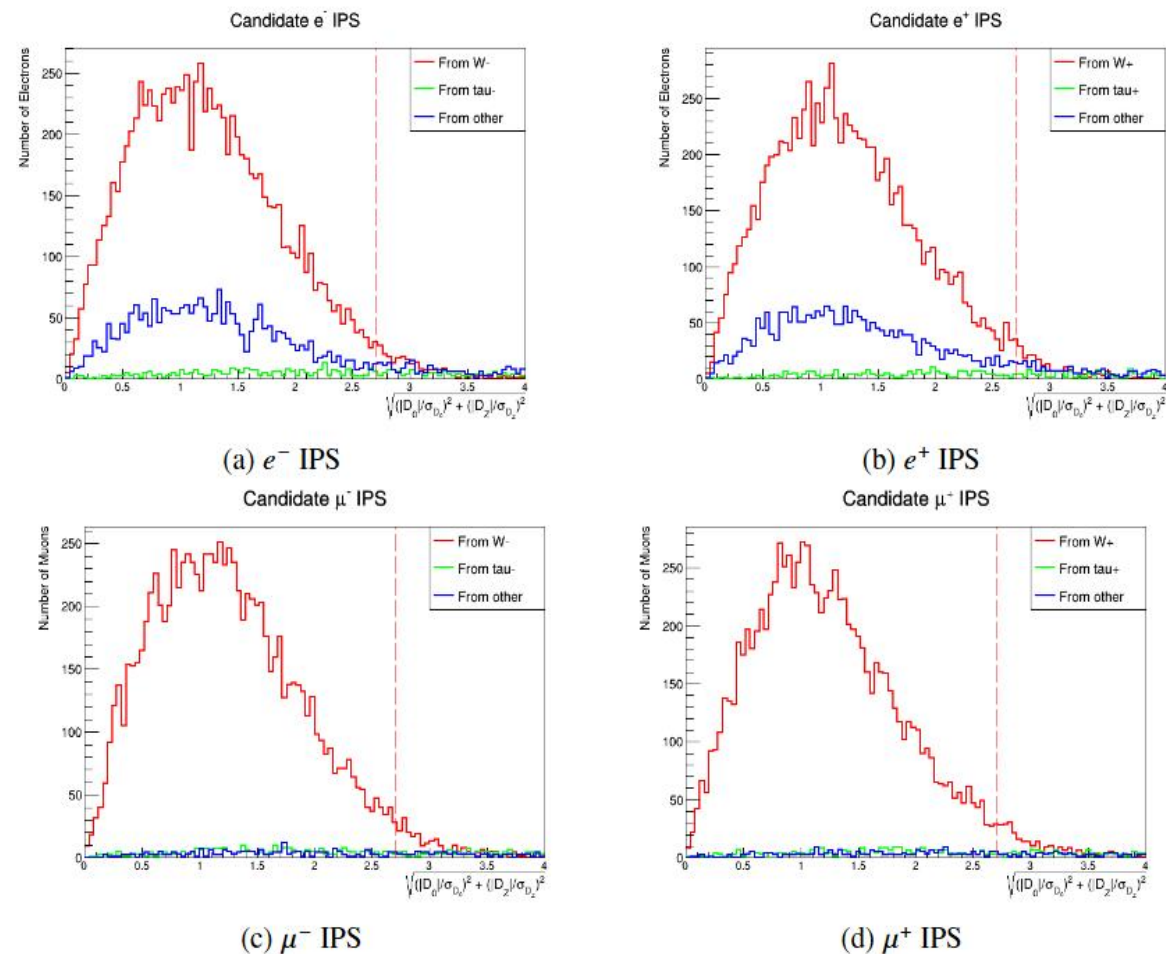


图 3-2 W 玻色子衰变得到的轻子与其他来源的轻子的 IPS 分布

Object selection - Jets

- The sl events require 4 jets, originating from a pair of b quarks and a quark-antiquark pair from a W boson decay;
- The hh events require 6 jets, from a b quark pair and two W bosons each decaying into a quark-antiquark pair.
- A strict requirement of exactly 2 b-jets is applied for sl events, while at least 2 b-jets are required for hh events.

cut	$t\bar{t}$ (semi-leptonic)		
	events	Rel. Eff	Accu. Eff
No cut	1000000	1.0000	1.0000
1 lepton	665221	0.6652	0.6652
4 jets	665221	1.0000	0.6652
2 bjets	469401	0.7056	0.4694

表 3-1 $\sqrt{s} = 342.75\text{GeV}$ 时经过基本筛选后半轻信号的事例数、相对效率以及绝对效率

cut	$t\bar{t}$ (fully -hadronic)		
	events	Rel. Eff	Accu. Eff
No cut	999999	1.0000	1.0000
0 lepton	996550	0.9966	0.9966
6 jets	996550	1.0000	0.9966
2 or more bjets	837555	0.8405	0.8376

表 3-2 $\sqrt{s} = 342.75\text{GeV}$ 时经过基本筛选后全强信号的事例数、相对效率以及绝对效率

Events selection - event shape variables : y_{ij}

- In $e^+e^- - kt$ algorithm: $d_{ij} = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$, $y_{ij} = d_{ij} / Q$, where Q is the total energy in the event
- For sl channel: use y_{34} and y_{45} , For hh channel: use y_{34} , y_{45} and y_{56}
- Optimize the best cut threshold between background & signal by scanning the signal significance : $SOB = \sqrt{2(s+b)\ln(1+s/b)} - s$

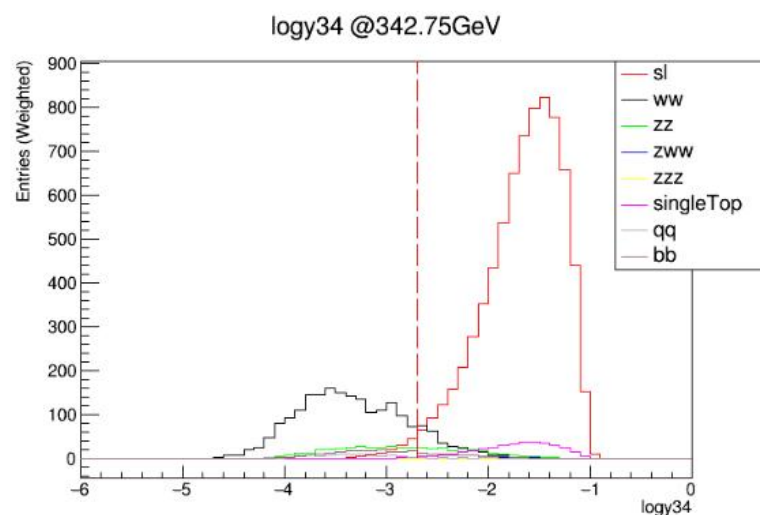


图 3-3 $\sqrt{s} = 342.75\text{GeV}$ 时半轻信号和本底的 $\lg y_{34}$ 分布, cut: $\lg y_{34} > -2.7$ (“sl” 代表半轻信号, 红色虚线代表 cut 位置, 下同)

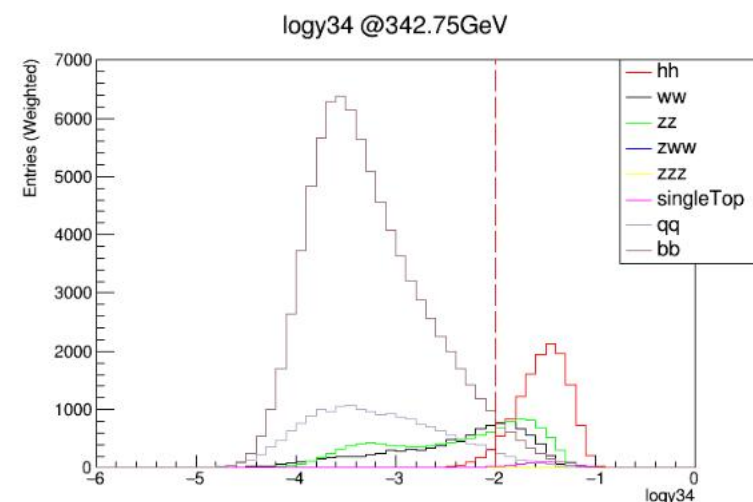


图 3-5 $\sqrt{s} = 342.75\text{GeV}$ 时全强信号和本底的 $\lg y_{34}$ 分布, cut: $\lg y_{34} > -2.0$ (“hh” 代表全强信号, 红色虚线代表 cut 位置, 下同)

Events selection - event shape variables : Sphericity & Thrust

- **Sphericity**: Measures how uniformly momentum is distributed in space (spherical shape), $S \approx 1$ for spherical events; $S \approx 0$ for jet-like events

$$S^{\alpha\beta} = \frac{\sum_i \frac{p_i^\alpha p_i^\beta}{|\mathbf{p}_i|}}{\sum_i |\mathbf{p}_i|}$$

- **Thrust**: Measures how concentrated momentum is along a certain direction, Thrust ≈ 1 indicates strong directionality (e.g., two-jet events)

$$T = \max_{|\mathbf{n}|=1} \left[\frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|} \right]$$

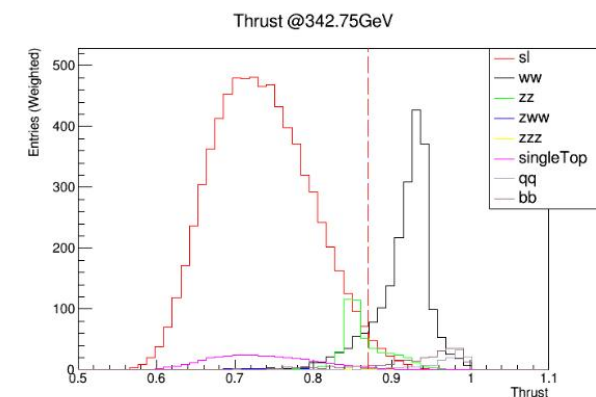
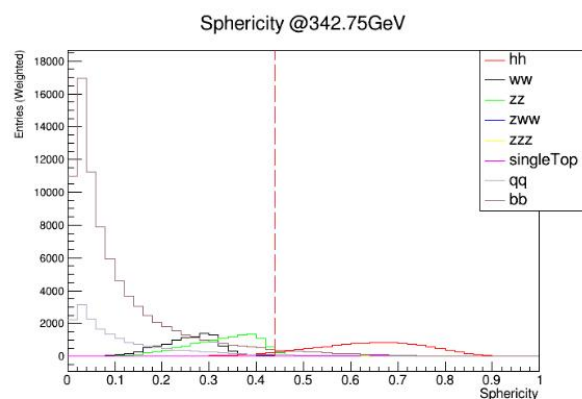
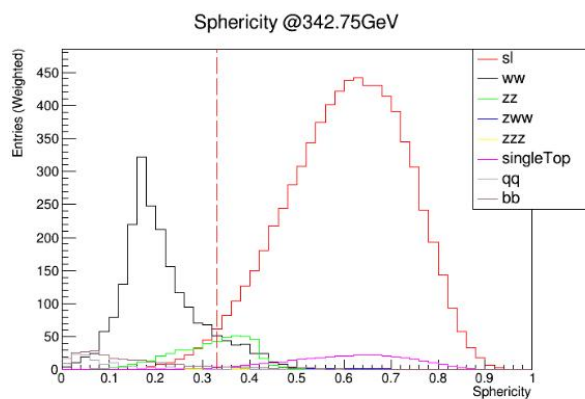


图 3-16 $\sqrt{s} = 342.75\text{GeV}$ 时半轻信号和本底的 Sphericity 分布, cut: Sphericity > 0.32

图 3-17 $\sqrt{s} = 342.75\text{GeV}$ 时全强信号和本底的 Sphericity 分布, cut: Sphericity > 0.44

图 3-18 $\sqrt{s} = 342.75\text{GeV}$ 时半轻信号和本底的 Thrust 分布, cut: Thrust < 0.87

Events selection - event shape variables : PFOs & Charged PFOs

Event selection is based on the total number of **particles (PFOs)** and **charged particles (Charged PFOs)**, which are strongly correlated with the number of jets in the event.

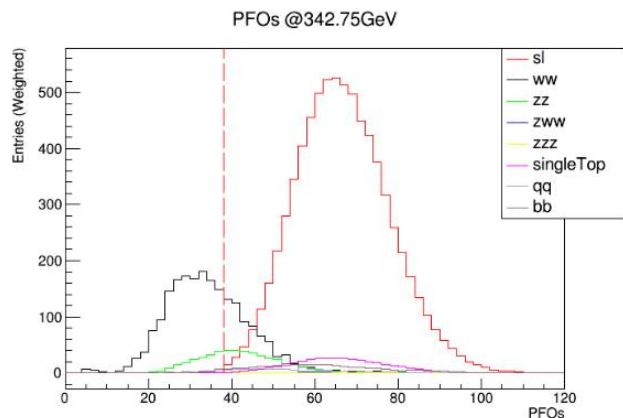


图 3-8 $\sqrt{s} = 342.75\text{GeV}$ 时半轻信号和本底的 PFOs 分布, cut: PFOs > 38

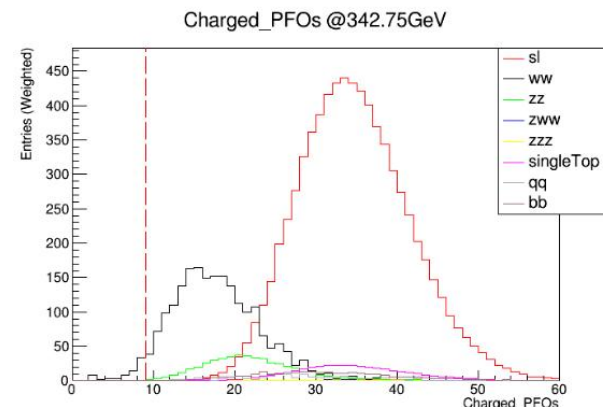


图 3-10 $\sqrt{s} = 342.75\text{GeV}$ 时半轻信号和本底的 Charged PFOs 分布, cut: Charged PFOs > 9

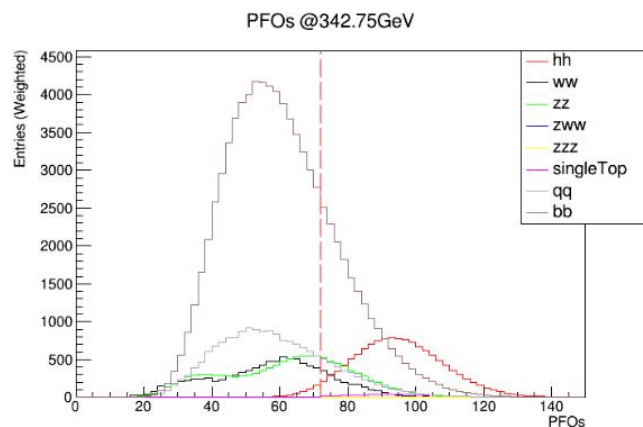


图 3-9 $\sqrt{s} = 342.75\text{GeV}$ 时全强信号和本底的 PFOs 分布, cut: PFOs > 72

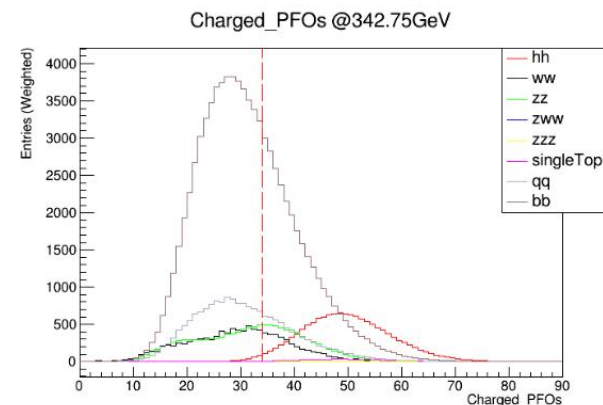


图 3-11 $\sqrt{s} = 342.75\text{GeV}$ 时全强信号和本底的 Charged PFOs 分布, cut: Charged PFOs > 34

Events selection - event shape variables : $P_{\max}$ & E_{Total}

- **Maximum particle momentum ($P_{\max}$):** In semi-leptonic events, the **isolated lepton** typically carries the highest momentum; in fully hadronic events, $P_{\max}$ is more related to the **number of particles or jets**.
- **Total event energy (E_{Total}):** Affected by initial state radiation (ISR) and jet reconstruction quality.

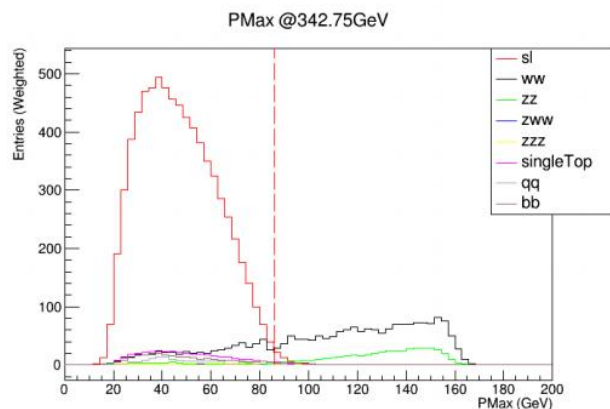


图 3-12 $\sqrt{s} = 342.75\text{GeV}$ 时半轻信号和本底的 $P_{\max}$ 分布, cut: $P_{\max} < 86\text{GeV}$

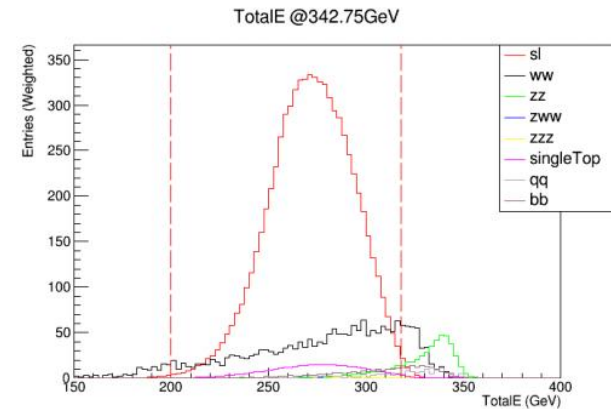


图 3-14 $\sqrt{s} = 342.75\text{GeV}$ 时半轻信号和本底的 TotalE 分布, cut: $200\text{GeV} < \text{TotalE} < 318\text{GeV}$

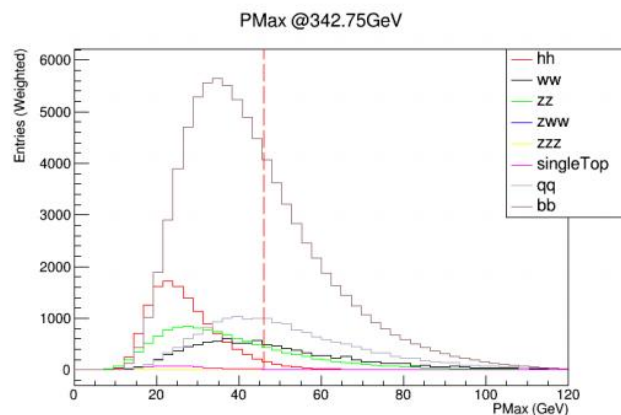


图 3-13 $\sqrt{s} = 342.75\text{GeV}$ 时全强信号和本底的 $P_{\max}$ 分布, cut: $P_{\max} < 46\text{GeV}$

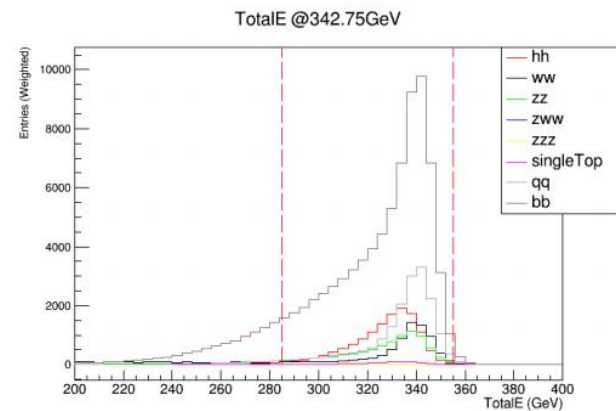


图 3-15 $\sqrt{s} = 342.75\text{GeV}$ 时全强信号和本底的 TotalE 分布, cut: $285\text{GeV} < \text{TotalE} < 355\text{GeV}$

- χ^2 serves as the criterion for pairing in top mass reconstruction, selecting the combination that minimizes it
- It effectively suppresses single top backgrounds that mimic the signal

Semi-leptonic fit parameters: sf_{bH} 、 sf_{bL} 、 sf_{jj} 、 sf_l 、 $P_{x\nu}$ 、 $P_{y\nu}$ 、 $P_{z\nu}$

$$\begin{aligned} \chi^2 = & \left(\frac{E_{bHbLjjl\nu} - E_{com}}{\sigma_E} \right)^2 + \left(\frac{\sum_{bHbLjjl\nu} P_{xi}}{\sigma_{P_x}} \right)^2 + \left(\frac{\sum_{bHbLjjl\nu} P_{yi}}{\sigma_{P_y}} \right)^2 + \left(\frac{\sum_{bHbLjjl\nu} P_{zi}}{\sigma_{P_z}} \right)^2 \\ & + \left(\frac{M_{bHjj} - M_{tH}}{\sigma_{M_{tH}}} \right)^2 + \left(\frac{M_{bLl\nu} - M_{tL}}{\sigma_{M_{tL}}} \right)^2 + \left(\frac{M_{jj} - M_{W_H}}{\sigma_{M_{W_H}}} \right)^2 + \left(\frac{M_{l\nu} - M_{W_L}}{\sigma_{M_{W_L}}} \right)^2 \\ & + (sf_{bH} - 1)^2 + (sf_{bL} - 1)^2 + (sf_{jj} - 1)^2 + (sf_l - 1)^2 \end{aligned} \quad (3-9)$$

Full-hadronic fit parameters: sf_{b_1} 和 sf_{b_2} sf_{j_1} 、 sf_{j_2} 、 sf_{j_3} 、 sf_{j_4}

$$\begin{aligned} \chi^2 = & \left(\frac{E_{b_1b_2j_1j_2j_3j_4} - E_{com}}{\sigma_E} \right)^2 + \left(\frac{\sum_{b_1b_2j_1j_2j_3j_4} P_{xi}}{\sigma_{P_x}} \right)^2 + \left(\frac{\sum_{b_1b_2j_1j_2j_3j_4} P_{yi}}{\sigma_{P_y}} \right)^2 + \left(\frac{\sum_{b_1b_2j_1j_2j_3j_4} P_{zi}}{\sigma_{P_z}} \right)^2 \\ & + \left(\frac{M_{b_1j_1j_2} - M_{t_1}}{\sigma_{M_{t_1}}} \right)^2 + \left(\frac{M_{b_2j_3j_4} - M_{t_2}}{\sigma_{M_{t_2}}} \right)^2 + \left(\frac{M_{j_1j_2} - M_{W_1}}{\sigma_{M_{W_1}}} \right)^2 + \left(\frac{M_{j_3j_4} - M_{W_2}}{\sigma_{M_{W_2}}} \right)^2 \\ & + (sf_{b_1} - 1)^2 + (sf_{b_2} - 1)^2 + (sf_{j_1} - 1)^2 + (sf_{j_2} - 1)^2 + (sf_{j_3} - 1)^2 + (sf_{j_4} - 1)^2 \end{aligned} \quad (3-10)$$

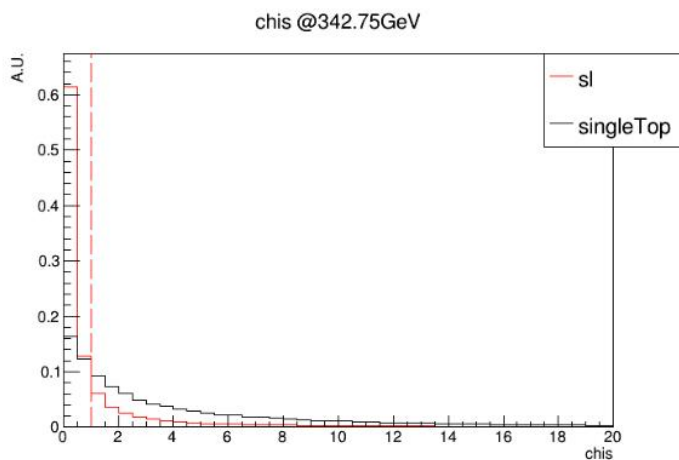


图 3-20 $\sqrt{s} = 342.75\text{GeV}$ 时半轻信号和 singleTop 本底 χ^2 的归一化分布

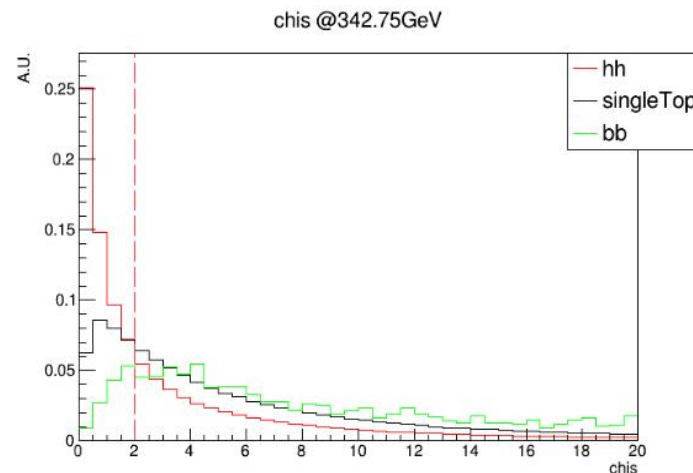


图 3-22 $\sqrt{s} = 342.75\text{GeV}$ 时全强信号和 singleTop 本底、 $b\bar{b}$ 本底的 χ^2 归一化分布

Cutflow table

cut	$t\bar{t}$ (semi-leptonic)			singleTop (semi-leptonic)			W^+W^-			ZZ			ZW^+W^-			ZZZ			$q\bar{q}$			$b\bar{b}$		
	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff
No cut	1000000	1.0000	1.0000	800000	1.0000	1.0000	2000000	1.0000	1.0000	1688980	1.0000	1.0000	100000	1.0000	1.0000	77860	1.0000	1.0000	1000000	1.0000	1.0000	1000000	1.0000	1.0000
1 lepton	665221	0.6652	0.6652	533895	0.6674	0.6674	775407	0.3877	0.3877	240461	0.1424	0.1424	27998	0.2800	0.2800	16441	0.2112	0.2112	11763	0.0118	0.0118	9750	0.0097	0.0097
4 jets	665221	1.0000	0.6652	533895	1.0000	0.6674	615470	0.7937	0.3077	183610	0.7636	0.1087	26475	0.9456	0.2647	15451	0.9398	0.1984	11763	1.0000	0.0118	9750	1.0000	0.0097
2 bjets	469401	0.7056	0.4694	369372	0.6918	0.4617	3611	0.0059	0.0018	11912	0.0649	0.0071	2140	0.0808	0.0214	1611	0.1043	0.0207	281	0.0239	0.0003	3243	0.3326	0.0032
$\lg y_{34} > -2.7$	461129	0.9824	0.4611	358496	0.9706	0.4481	492	0.1363	0.0002	4879	0.4096	0.0029	1820	0.8505	0.0182	1431	0.8883	0.0184	52	0.1851	0.0001	1038	0.3201	0.0010
$\lg y_{45} > -4.5$	461129	1.0000	0.4611	358496	1.0000	0.4481	492	1.0000	0.0002	4878	0.9998	0.0029	1820	1.0000	0.0182	1431	1.0000	0.0184	52	1.0000	0.0001	1038	1.0000	0.0010
PFOs > 38	460354	0.9983	0.4604	357743	0.9979	0.4472	353	0.7175	0.0002	3987	0.8173	0.0024	1713	0.9412	0.0171	1352	0.9448	0.0174	52	1.0000	0.0001	1034	0.9961	0.0010
Charged PFOs > 9	460354	1.0000	0.4604	357743	1.0000	0.4472	353	1.0000	0.0002	3987	1.0000	0.0024	1713	1.0000	0.0171	1352	1.0000	0.0174	52	1.0000	0.0001	1034	1.0000	0.0010
PMAX < 86	456314	0.9912	0.4563	341686	0.9551	0.4271	121	0.3428	0.0001	939	0.2355	0.0006	1371	0.8004	0.0137	1070	0.7914	0.0137	51	0.9808	0.0001	995	0.9623	0.0010
200 < TotalE < 318	453553	0.9939	0.4536	336981	0.9862	0.4212	73	0.6033	0.0000	370	0.3940	0.0002	1276	0.9307	0.0128	196	0.1832	0.0025	10	0.1961	0.0000	557	0.5598	0.0006
Sphericity > 0.33	444727	0.9805	0.4447	330249	0.9800	0.4128	28	0.3836	0.0000	215	0.5811	0.0001	1245	0.9757	0.0124	175	0.8929	0.0022	0	0.0000	0.0000	185	0.3321	0.0002
Thrust < 0.87	444150	0.9987	0.4441	329713	0.9984	0.4121	22	0.7857	0.0000	205	0.9535	0.0001	1245	1.0000	0.0124	175	1.0000	0.0022	0	0.0000	0.0000	184	0.9946	0.0002
$\chi^2 < 1$	288383	0.6493	0.2884	87499	0.2654	0.1094	0	0.0000	0.0000	3	0.0146	0.0000	78	0.0627	0.0008	5	0.0286	0.0001	0	0.0000	0.0000	1	0.0054	0.0000
σ (fb)	157.630			8.247			11585.09			702.62			11.20			0.606			4729.90			854.91		
eff. σ (fb)	45.460			0.902			0.000			0.001			0.009			0.000			0.000			0.001		

cut	$t\bar{t}$ (fully-hadronic)			singleTop (fully-hadronic)			W^+W^-			ZZ			ZW^+W^-			ZZZ			$q\bar{q}$			$b\bar{b}$		
	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff	events	Rel. Eff	Accu. Eff
No cut	999999	1.0000	1.0000	1000000	1.000	1.000	1000000	1.0000	1.0000	845092	1.0000	1.0000	100000	1.0000	1.0000	77918	1.0000	1.0000	1000000	1.0000	1.0000	1000000	1.0000	1.0000
0 lepton	996550	0.9966	0.9966	996873	0.9969	0.9969	631372	0.6314	0.6314	725284	0.8582	0.8582	59527	0.5953	0.5953	62004	0.7958	0.7958	995632	0.9956	0.9956	994433	0.9944	0.9944
6 jets	996550	1.0000	0.9966	996873	1.0000	0.9969	618640	0.9798	0.6186	679766	0.9372	0.8044	59254	0.9954	0.5925	61000	0.9838	0.7829	995632	1.0000	0.9956	994427	1.0000	0.9944
≥ 2 bjets	837555	0.8405	0.8376	817336	0.8199	0.8173	8798	0.0142	0.0088	146986	0.2162	0.1739	8526	0.1439	0.0853	16087	0.2637	0.2065	38785	0.0390	0.0388	974219	0.9797	0.9742
$\lg y_{34} > -2.0$	794223	0.9483	0.7942	770596	0.9428	0.7706	3027	0.3452	0.0030	57292	0.3898	0.0678	7118	0.8349	0.0712	12157	0.7557	0.1560	1865	0.0481	0.0019	32506	0.0334	0.0325
$\lg y_{45} > -2.4$	734484	0.9248	0.7345	692292	0.8984	0.6923	254	0.0839	0.0003	6462	0.1128	0.0076	5411	0.7602	0.0541	8098	0.6661	0.1039	446	0.2391	0.0004	7158	0.2202	0.0072
$\lg y_{56} > -3.2$	710571	0.9674	0.7106	664513	0.9599	0.6645	118	0.4646	0.0001	3034	0.4695	0.0036	4839	0.8943	0.0484	7470	0.9224	0.0959	318	0.7130	0.0003	5210	0.7279	0.0052
Sphericity > 0.44	681478	0.9591	0.6815	640742	0.9642	0.6407	12	0.1017	0.0000	274	0.0903	0.0003	4697	0.9707	0.0470	7218	0.9663	0.0926	203	0.6384	0.0002	3372	0.6472	0.0034
Charged PFOs > 34	670049	0.9832	0.6700	628950	0.9816	0.6290	8	0.6667	0.0000	204	0.7445	0.0002	4445	0.9463	0.0445	6857	0.9500	0.0880	199	0.9803	0.0002	3353	0.9944	0.0034
PFOs > 72	654478	0.9768	0.6545	612493	0.9738	0.6125	6	0.7500	0.0000	181	0.8873	0.0002	4151	0.9339	0.0415	6441	0.9393	0.0827	191	0.9598	0.0002	3326	0.9919	0.0033
285 < TotalE < 355	644716	0.9851	0.6447	602001	0.9829	0.6020	6	1.0000	0.0000	129	0.7127	0.0002	3816	0.9193	0.0382	5736	0.8905	0.0736	182	0.9529	0.0002	3072	0.9236	0.0031
PMAX < 46	623895	0.9677	0.6239	574576	0.9544	0.5746	0	0.0000	0.0000	119	0.9225	0.0001	3507	0.9190	0.0351	5344	0.9317	0.0686	160	0.8791	0.0002	2886	0.9395	0.0029
$\chi^2 < 2$	309101	0.4954	0.3091	153616	0.2674	0.1536	0	0.0000	0.0000	8	0.0672	0.0000	248	0.0707	0.0025	777	0.1454	0.0100	4	0.0250	0.0000	245	0.0849	0.0002
σ (fb)	157.870			8.250			11585.09			702.62			11.20			0.606			4729.90			854.91		
eff. σ (fb)	48.798			1.267			0.000			0.007			0.028			0.006			0.019			0.209		

- After event selection & kinematic fit:

hh ($\chi^2 < 2$): Sigeff = 30.91 % Xsec of bkg = 1.536 fb

sl ($\chi^2 < 1$): Sigeff = 28.84 % Xsec of bkg = 0.913 fb

$$\mathcal{L} = \prod_{i=1}^N P(D|\sigma_{t\bar{t}}(m_{top}, \Gamma_{top}, \alpha_S, \sqrt{s_i}) \times L_i \times \epsilon)$$

[Eur. Phys. J. C \(2023\) 83:269](#)

- In the formula, P denotes the Poisson distribution, D is the observed number of signal events, $\sigma_{t\bar{t}}$ is the expected signal cross section, L_i is the expected CEPC luminosity at the given center-of-mass energy, and ϵ is the signal selection efficiency. The mean of the Poisson distribution $E = \sigma_{t\bar{t}}(\sqrt{s_i}, m_{top}) \times L_i \times \epsilon$ corresponds to the expected number of signal events.
- **At a fixed center-of-mass energy**, each top quark mass m_{top} corresponds to an expected number of signal events E . Combined with the observed number of events D , one can compute $P(D|E)$. The larger the value of $P(D|E)$, the more likely it is that the top quark mass is m_{top} .
- Since the observed signal count is fixed to the expected value at $m_{top} = 171.5 \text{ GeV}$, the likelihood is maximized at this mass, and the measured m_{top} is therefore fixed at 171.5 GeV. The statistical uncertainty is then extracted from the **1 σ width** of the likelihood curve.

Final result of each Uncertainty of m_{top}

- final uncertainty result (combine with sl & hh)
 hh ($\chi^2 < 2$): Sigeff = 30.91 % Xsec of bkg = 1.536 fb
 sl ($\chi^2 < 1$): Sigeff = 28.84 % Xsec of bkg = 0.913 fb

Source	m_{top} precision (MeV)	
	Optimistic	Conservative
Statistics	9	9
Theory	8	24
Quick scan	2	2
α_s	16	16
Top width	5 (-5)	5 (-5)
Experimental efficiency	4	44
Background	1 (-1)	2 (-12)
Beam energy	2	2
Luminosity spectrum	3	6
Total	22 (-2)	54 (-3)

- final uncertainty result (combine with sl & hh, without χ^2 cut)
- hh : Sigeff = 62.39 % Xsec of bkg = 4.736 fb
- sl : Sigeff = 44.41 % Xsec of bkg = 3.399 fb

Source	m_{top} precision (MeV)	
	Optimistic	Conservative
Statistics	7 (-2)	7 (-2)
Theory	8	24
Quick scan	2	2
α_s	16	16
Top width	5 (-5)	5 (-5)
Experimental efficiency	4	44
Background	1 (-1)	3 (-11)
Beam energy	2	2
Luminosity spectrum	3	6
Total	21 (-3)	54 (-3)

	Top mass uncertainties (MeV)	
	Optimistic	Conservative
Statistics	9	9
Theory	8	24
Quick scan	2	2
α_s	16	16
Width	10	10
Experimental efficiency	4	44
Background	2	14
Beam energy	2	2
Luminosity spectrum	3	6
Total	24	57

- Compare 2 version(with/without χ^2 cut) with result in [Eur. Phys. J. C \(2023\) 83:269](#), [arXiv:2207.12177](#)
 - Statistics:** ϵ_{sig} is the main contributing factor, with σ_{bkg_total} having a lesser impact
 - Width:** The previous version contained a calculation error, which has been corrected in this update
 - Background:** σ_{bkg_total} are the main contributing factors
- Since single top, already the dominant background after event shape cuts, has a small cross section, applying the χ^2 cut further suppresses it but at the cost of signal efficiency. Thus, omitting the χ^2 cut may be preferable

Comparison with FCC-ee result

<https://arxiv.org/pdf/2503.18713>

Fcc-ee vs CEPC - 1. Expermental

Source	m_{top} precision (MeV)	
	Optimistic	Conservative
Statistics	7	7
Theory	8	24
Quick scan	2	2
α_s	16	16
Top width	5	5
Experimental efficiency	4	44
Background	1	3
Beam energy	2	2
Luminosity spectrum	3	6
Total	21	54

Uncertainty source	m_t^{PS} [MeV]	Γ_t [MeV]	Input values
Experimental (stat. $\times 1.2$)	4.3	10.4	$L = 410 \text{ fb}^{-1}$ (FCC-ee)
Parametric y_t	4.2	3.6	$\delta y_t = 3\%$
Parametric α_s	2.2	1.7	$\delta \alpha_s(m_Z^2) = 10^{-4}$
Luminosity calibration (uncorr.)	0.5	1.0	$\delta L/L = 0.1\%$
Luminosity calibration (corr.)	0.4	0.4	$\delta L/L = 0.05\%$
Beam energy calibration (uncorr.)	1.2	1.8	$\delta \sqrt{s} = 5 \text{ MeV}$ [36, 37]
Beam energy calibration (corr.)	1.2	0.1	$\delta \sqrt{s} = 2.5 \text{ MeV}$
Beam energy spread (uncorr.)	0.3	0.8	$\delta \Delta E = 1\%$ [36]
Beam energy spread (corr.)	0.1	1.1	$\delta \Delta E = 0.5\%$
Total profiled	6.8	11.5	
Theory, unprofiled (scale)	35	25	$\text{N}^3\text{LO NR-QCD}$ [11]

Uncertainty source	Impact on σ_{WbWb} [%]		
	340 GeV	345 GeV	365 GeV
Integrated luminosity	0.12	0.11	0.02
b tagging	0.11	0.06	0.01
ZZ had. norm.	0.46	0.19	0.04
ZZ semihad. norm.	0.23	0.07	0.03
WW had. norm.	0.17	0.09	0.02
WW semihad. norm.	0.06	0.04	0.03
q $\bar{q}$ had. norm.	0.12	0.09	0.02
q $\bar{q}$ semihad. norm.	0.18	0.06	0.01
WWZ norm.	0.03	0.01	0.01
Total (incl. stat)	2.31	0.89	0.12

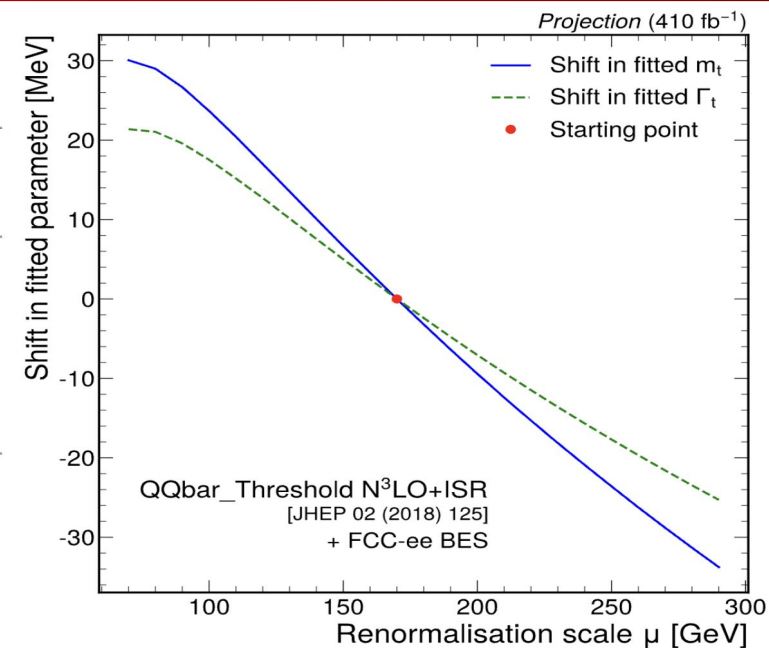
Table 2. Total uncertainty and impact of various systematic uncertainty sources on the measured WbWb production cross section (σ_{WbWb}) at different centre-of-mass energies.

- **Lumi:** 100 fb⁻¹ in 342.75 GeV vs 41 fb⁻¹ $\times$ 10 energy points (340–345 GeV)
- **Expermental:** 4.3 vs 4/44
 - **FCC-ee:** stat-only Lumi & btagging & background impact on cross-section(ee->tt), then give an conservative factor of 20%
 - **CPEC:** totally asumed systematic uncertainty (including , but from optimistic to a very very Conservative asuming(from 0.5% ~ 5%, which cover FCC-ee 1% ~ 2%) this is dominant in our Conservative estimation

Fcc-ee vs CEPC - 2.Theory

Source	m_{top} precision (MeV)	
	Optimistic	Conservative
Statistics	7	7
Theory	8	24
Quick scan	2	2
α_s	16	16
Top width	5	5
Experimental efficiency	4	44
Background	1	3
Beam energy	2	2
Luminosity spectrum	3	6
Total (without theory)	19	48
Total	21	54

Uncertainty source	m_t^{PS} [MeV]	Γ_t [MeV]	Input values
Experimental (stat. $\times 1.2$)	4.3	10.4	$L = 410 \text{ fb}^{-1}$ (FCC-ee)
Parametric y_t	4.2	3.6	$\delta y_t = 3\%$
Parametric α_s	2.2	1.7	$\delta \alpha_s(m_Z^2) = 10^{-4}$
Luminosity calibration (uncorr.)	0.5	1.0	$\delta L/L = 0.1\%$
Luminosity calibration (corr.)	0.4	0.4	$\delta L/L = 0.05\%$
Beam energy calibration (uncorr.)	1.2	1.8	$\delta \sqrt{s} = 5 \text{ MeV}$ [36, 37]
Beam energy calibration (corr.)	1.2	0.1	$\delta \sqrt{s} = 2.5 \text{ MeV}$
Beam energy spread (uncorr.)	0.3	0.8	$\delta \Delta E = 1\%$ [36]
Beam energy spread (corr.)	0.1	1.1	$\delta \Delta E = 0.5\%$
Total profiled	6.8	11.5	
Theory, unprofiled (scale)	35	25	$N^3\text{LO NR-QCD}$ [11]



<https://arxiv.org/pdf/2503.18713>

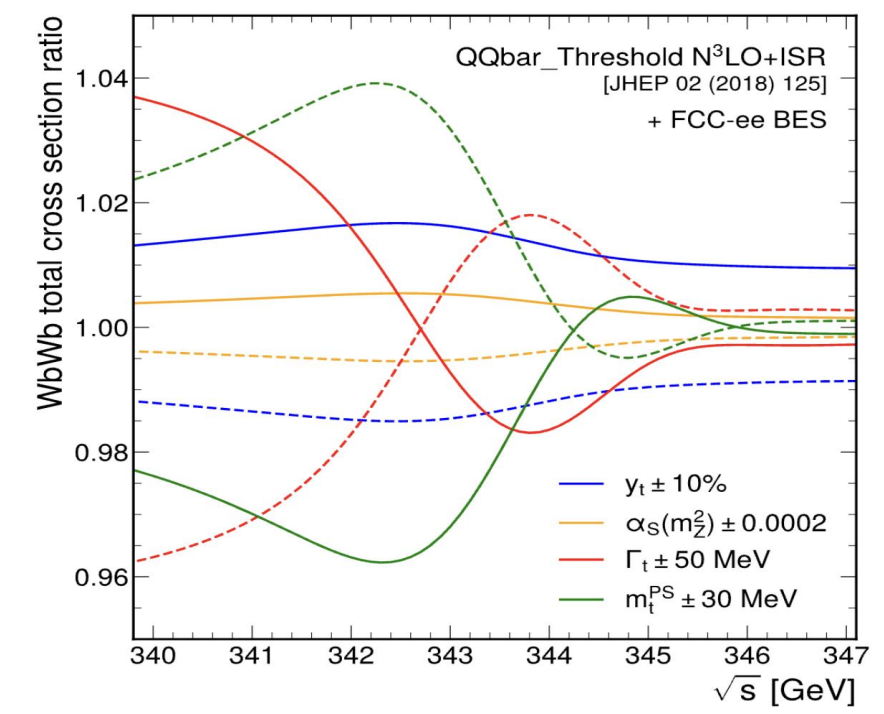
- **Theory:** **35** vs **8/24**
- **FCC-ee:** Directly varies the renormalisation scale μ in the $N^3\text{LO}$ theoretical prediction, dominant
- **CPEC:** follow CLIC's way ([arXiv:1111.4486](https://arxiv.org/abs/1111.4486)), assume a $\pm 1\%$ or $\pm 3\%$ theoretical uncertainty on crosssection normalization

Fcc-ee vs CEPC - 3. α_s & width

Source	m_{top} precision (MeV)	
	Optimistic	Conservative
Statistics	7	7
Theory	8	24
Quick scan	2	2
α_s	16	16
Top width	5	5
Experimental efficiency	4	44
Background	1	3
Beam energy	2	2
Luminosity spectrum	3	6
Total	21	54

Uncertainty source	m_t^{PS} [MeV]	Γ_t [MeV]	Input values
Experimental (stat. $\times 1.2$)	4.3	10.4	$L = 410 \text{ fb}^{-1}$ (FCC-ee)
Parametric y_t	4.2	3.6	$\delta y_t = 3\%$
Parametric α_s	2.2	1.7	$\delta \alpha_s(m_Z^2) = 10^{-4}$
Luminosity calibration (uncorr.)	0.5	1.0	$\delta L/L = 0.1\%$
Luminosity calibration (corr.)	0.4	0.4	$\delta L/L = 0.05\%$
Beam energy calibration (uncorr.)	1.2	1.8	$\delta \sqrt{s} = 5 \text{ MeV}$ [36, 37]
Beam energy calibration (corr.)	1.2	0.1	$\delta \sqrt{s} = 2.5 \text{ MeV}$
Beam energy spread (uncorr.)	0.3	0.8	$\delta \Delta E = 1\%$ [36]
Beam energy spread (corr.)	0.1	1.1	$\delta \Delta E = 0.5\%$
Total profiled	6.8	11.5	
Theory, unprofiled (scale)	35	25	$N^3\text{LO NR-QCD}$ [11]

Uncertainty source	m_t^{PS} [MeV]	Γ_t [MeV]	y_t [%]
Experimental (stat. $\times 1.2$)	4.2	10.0	1.5
Parametric m_t	—	5.3	1.2
Parametric Γ_t	3.0	—	0.8
Parametric m_t	3.8	4.8	—
Parametric α_s	2.2	1.6	0.2
Luminosity calibration (uncorr.)	0.6	1.1	0.2
Luminosity calibration (corr.)	1.0	0.7	0.9
Beam energy calibration (uncorr.)	1.3	1.9	0.1
Beam energy calibration (corr.)	1.3	< 0.1	< 0.1
Beam energy spread (uncorr.)	0.3	0.9	< 0.1
Beam energy spread (corr.)	< 0.1	1.1	< 0.1
Total profiled	6.5	11.7	2.1



• α_s : 2.2 vs 16 width : 3.0 vs 5

FCC-ee: $\alpha_s(m_Z^2)$ is varied by 1×10^{-4} Vs **CEPC:** $\alpha_s(m_Z^2)$ is varied by 7×10^{-4} ([arXiv:1111.4486](https://arxiv.org/abs/1111.4486))

FCC-ee: 3D fit no width variation Vs **CEPC:** 1D fit width is varied by 0.14 GeV

Fcc-ee vs CEPC - 4. Beam energy & LS

Source	m_{top} precision (MeV)	
	Optimistic	Conservative
Statistics	7	7
Theory	8	24
Quick scan	2	2
α_s	16	16
Top width	5	5
Experimental efficiency	4	44
Background	1	3
Beam energy	2	2
Luminosity spectrum	3	6
Total	21	54

Uncertainty source	m_t^{PS} [MeV]	Γ_t [MeV]	Input values
Experimental (stat. $\times 1.2$)	4.3	10.4	$L = 410 \text{ fb}^{-1}$ (FCC-ee)
Parametric y_t	4.2	3.6	$\delta y_t = 3\%$
Parametric α_s	2.2	1.7	$\delta \alpha_s(m_Z^2) = 10^{-4}$
Luminosity calibration (uncorr.)	0.5	1.0	$\delta L/L = 0.1\%$
Luminosity calibration (corr.)	0.4	0.4	$\delta L/L = 0.05\%$
Beam energy calibration (uncorr.)	1.2	1.8	$\delta \sqrt{s} = 5 \text{ MeV}$ [36, 37]
Beam energy calibration (corr.)	1.2	0.1	$\delta \sqrt{s} = 2.5 \text{ MeV}$
Beam energy spread (uncorr.)	0.3	0.8	$\delta \Delta E = 1\%$ [36]
Beam energy spread (corr.)	0.1	1.1	$\delta \Delta E = 0.5\%$
Total profiled	6.8	11.5	
Theory, unprofiled (scale)	35	25	N ³ LO NR-QCD [11]

- **Beam energy:** 1.2 vs 2
 - FCC-ee can the beam energy with a precision about $\delta \sqrt{s} = 5/2.5 \text{ MeV}$ ($3 \times 10^{-5}/1.5 \times 10^{-5}$)
 - CEPC can control the beam energy with a precision down to 10^{-5} , corresponding to $\sim O(1) \text{ MeV}$ at $t\bar{t}$ threshold
- **Lumiosity spectrum (LS) vs Beam energy spread (BES):** 0.1/0.3 vs 3/6
 - FCC-ee: BES is varied for 0.5% or 1.0%
 - CEPC: LS is varied for 10% and 20% that result in uncertainties 3/6 MeV of m_{top}

“The authors of Ref. [20] project machine calibration uncertainties of 2–6 MeV, in agreement with our findings.” <https://arxiv.org/pdf/2503.18713>

Summary

- The expected top mass uncertainty at **CEPC** is 21 MeV (optimistic) and 54 MeV (conservative), both significantly better than the 330 MeV at the LHC, and comparable to the **FCC-ee** projection of ~ 36 MeV.
- This highlights the advantage of the threshold scan method used in this study, which yields higher precision than the LHC's direct reconstruction approach, as it avoids systematics from energy loss and jet reconstruction. The improvement also benefits from **CEPC's superior beam energy control**.
- In this analysis, the **experimental efficiency** Uncertainty from btagging efficiency, JER and JES are still can't calculate:
 - In the current CEPC Delphes card, these effects are implemented as fixed constants.
 - These constants are partly derived from limited full simulation samples. These developments are still ongoing.
- Future improvements include training a jet tagger at 360 GeV and adding variables like momentum peak and forward-backward asymmetry to refine the top mass measurement.

Back Up

Q1: Are the experimental efficiency scenarios reasonable? There is no full simulation sample to even make a guess?

The experimental efficiency of the future detectors are yet to know. We assume several possible scenarios for the level of this uncertainty: 0.5%, 1%, 3% and 5%. This uncertainty impacts directly on the signal yields and results in a measurement uncertainty of the top mass of 4 MeV, 9 MeV, 27MeV and 44 MeV, respectively

At this stage, using fast simulation, it is not able to calculate it now, so we can only rely on a full assumption.

Uncertainty source	Impact on σ_{WbWb} [%]		
	340 GeV	345 GeV	365 GeV
Integrated luminosity	0.12	0.11	0.02
b tagging	0.11	0.06	0.01
ZZ had. norm.	0.46	0.19	0.04
ZZ semihad. norm.	0.23	0.07	0.03
WW had. norm.	0.17	0.09	0.02
WW semihad. norm.	0.06	0.04	0.03
$q\bar{q}$ had. norm.	0.12	0.09	0.02
$q\bar{q}$ semihad. norm.	0.18	0.06	0.01
WWZ norm.	0.03	0.01	0.01
Total (incl. stat)	2.31	0.89	0.12

Table 2. Total uncertainty and impact of various systematic uncertainty sources on the measured WbWb production cross section (σ_{WbWb}) at different centre-of-mass energies.

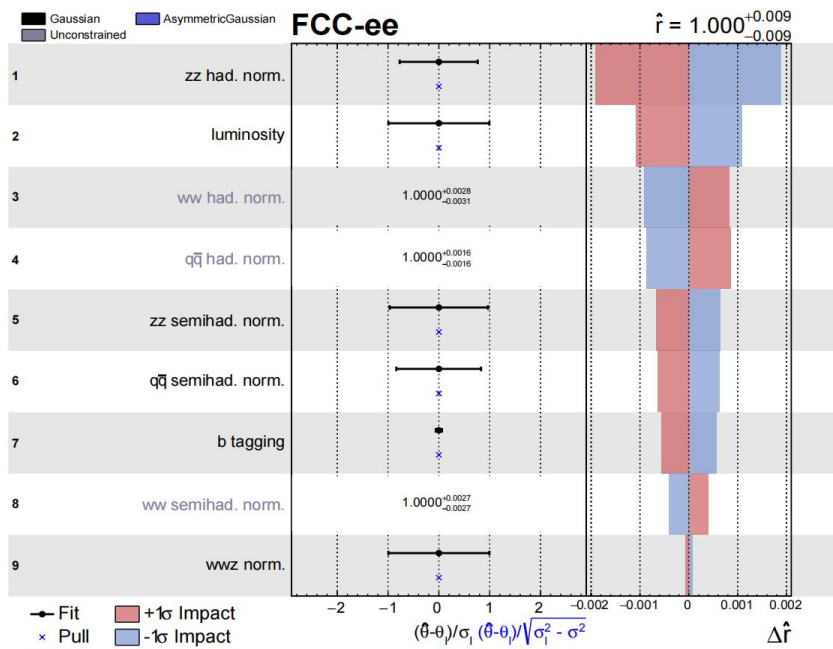


Figure 11. Impacts and constraints on the background normalisation and systematic uncertainties in the fit of the WbWb production cross section at $\sqrt{s} = 345$ GeV.

Uncertainties: quick scan and beam energy ²⁸

	Top mass uncertainties (MeV)	
	Optimistic	Conservative
Statistics	9	9
Theory	8	24
Quick scan	2	2
α_s	16	16
Width	10	10
Experimental efficiency	4	44
Background	2	14
Beam energy	2	2
Luminosity spectrum	3	6
Total	24	57

- The quick scans of CEPC beam energy are used to locate the optimal energy point before the high-luminosity runs
- CEPC can control the beam energy with a precision down to 10^{-5} corresponding to $\sim O(1)$ MeV at tt threshold
- This leads to an uncertainty of 2 MeV, as a small contribution to the total
- CLIC has a control of 10^{-4} on the beam energy, but still gives an impact on top mass less than the statistical uncertainty

- A quick scan preselects the optimal energy point for the high-luminosity run.
- The optimal collision energy is unknown in advance. A low-luminosity quick scan (e.g., 1 fb^{-1} per point) estimates an m_{top} close to the true value.
- Using this m_{top} , the predicted cross-section curve is fixed, allowing the optimal energy to be located.

Uncertainties: background

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$$\mathcal{L} = \prod_{i=1}^N P(D|\sigma_{t\bar{t}}(m_{top}, \Gamma_{top}, \alpha_S, \sqrt{s_i}) \times L_i \times \epsilon)$$

	Top mass uncertainties (MeV)	
	Optimistic	Conservative
Statistics	9	9
Theory	8	24
Quick scan	2	2
α_S	16	16
Width	10	10
Experimental efficiency	4	44
Background	2	14
Beam energy	2	2
Luminosity spectrum	3	6
Total	24	57

- Background is considered to be **subtracted cleanly** from the observed data. But their uncertainties could affect the measurement
- Assuming background uncertainties of 1% and 5% will give **2** and **14 MeV** on top mass measurement
 - This is similar to CLIC that has 18 MeV uncertainty of top mass from 5% background variations, given the low level of background

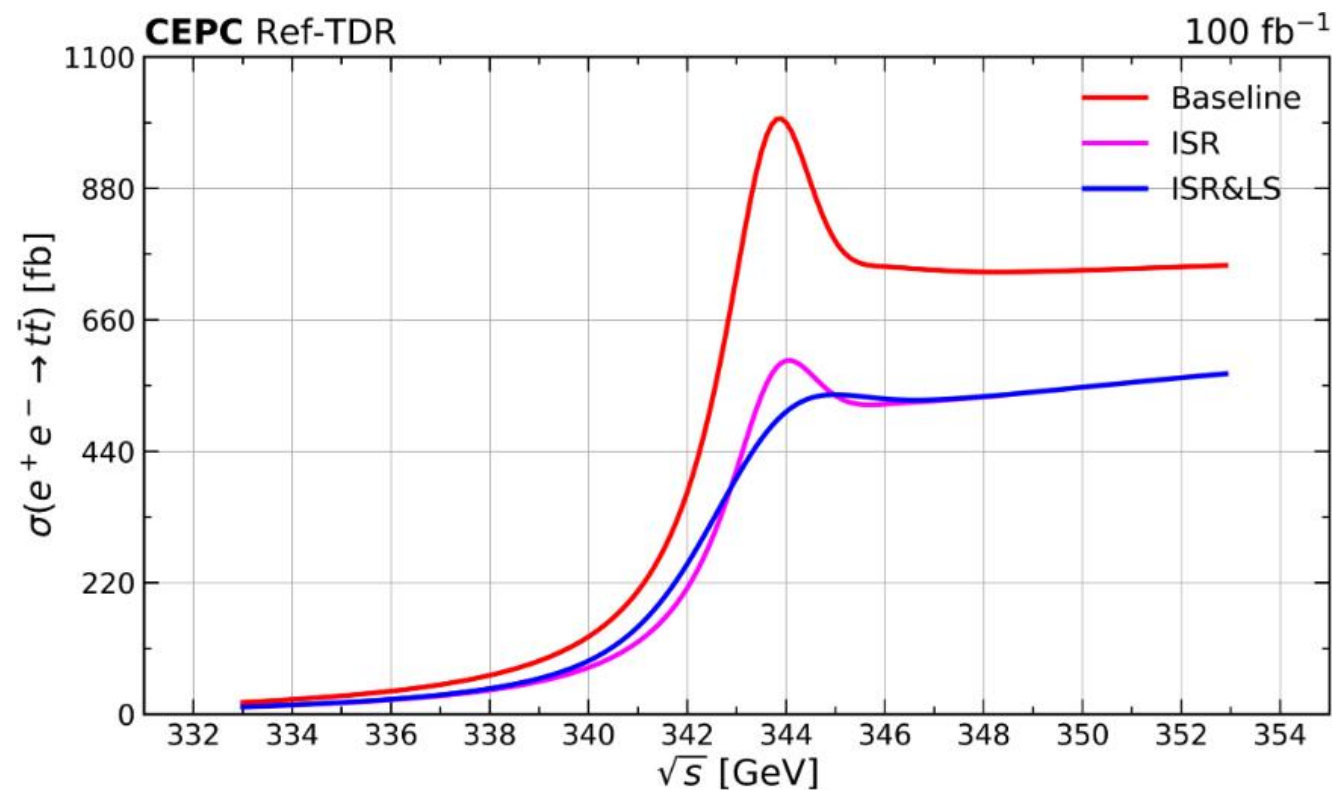
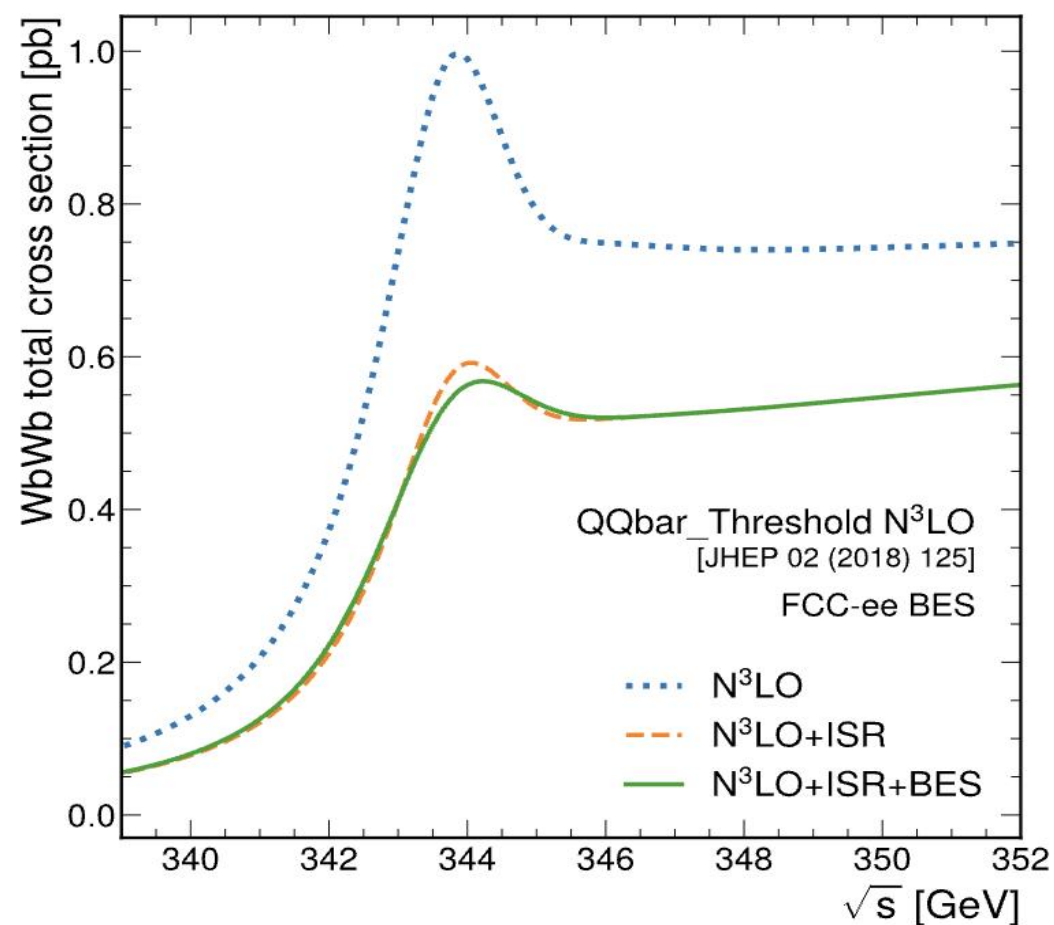
- Background uncertainties are incorporated into the likelihood function (Eq. 2) as nuisance parameters constrained by Gaussian priors.
- Background efficiencies are taken from [2013 CLIC result](#), and the cross sections are calculated at leading order (LO), including initial-state radiation
- Assuming a 1% background uncertainty (optimistic) or 5% (conservative), the top quark mass uncertainty is found to be 2 MeV and 14 MeV, respectively, demonstrating that background uncertainty plays a critical role.

Uncertainties: luminosity spectrum

	Top mass uncertainties (MeV)	
	Optimistic	Conservative
Statistics	9	9
Theory	8	24
Quick scan	2	2
α_S	16	16
Width	10	10
Experimental efficiency	4	44
Background	2	14
Beam energy	2	2
Luminosity spectrum	3	6
Total	24	57

- LS is varied for 10% and 20% that result in uncertainties of 3 and 6 MeV on top mass
- This is very different than CLIC (75 MeV from 20% LS uncertainty), given the different controls of the luminosity spectrum in circular and linear colliders

same cross section calculate result between FCC-ee & CEPC



Kinematic fit -> full hadronic channel chi square formula

$$\begin{aligned}\chi^2 = & \left(\frac{\sum_{6 \text{ jets}} E_i - 342.75}{\sigma_E} \right)^2 + \left(\frac{\sum_{6 \text{ jets}} P_{xi}}{\sigma_{P_x}} \right)^2 + \left(\frac{\sum_{6 \text{ jets}} P_{yi}}{\sigma_{P_y}} \right)^2 + \left(\frac{\sum_{6 \text{ jets}} P_{zi}}{\sigma_{P_z}} \right)^2 \\ & + \left(\frac{M_{t_1} - 171.5}{\sigma_{M_{t_1}}} \right)^2 + \left(\frac{M_{t_2} - 171.5}{\sigma_{M_{t_2}}} \right)^2 + \left(\frac{M_{W_1} - 80.3692}{\sigma_{M_{W_1}}} \right)^2 + \left(\frac{M_{W_2} - 80.3692}{\sigma_{M_{W_2}}} \right)^2 \\ & + (\text{Sf}_{b_1} - 1)^2 + (\text{Sf}_{b_2} - 1)^2 + (\text{Sf}_{j_1} - 1)^2 + (\text{Sf}_{j_2} - 1)^2 + (\text{Sf}_{j_3} - 1)^2 + (\text{Sf}_{j_4} - 1)^2\end{aligned}$$

$$p_T(j_1) > p_T(j_2) > p_T(j_3) > p_T(j_4)$$

$$p_T(b_1) > p_T(b_2)$$

6 unknown parameters: $\text{Sf}_{b_1}, \text{Sf}_{b_2}, \text{Sf}_{j_1}, \text{Sf}_{j_2}, \text{Sf}_{j_3}, \text{Sf}_{j_4}$

for reco level, there will be more than 2 bjets, so j_1, j_2, j_3, j_4 can be bjet in some events

For example, one possible pairing:

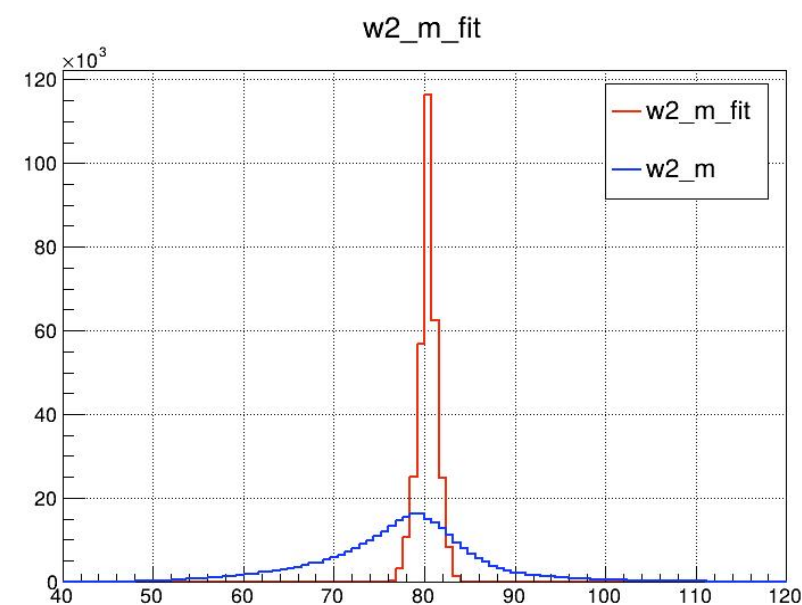
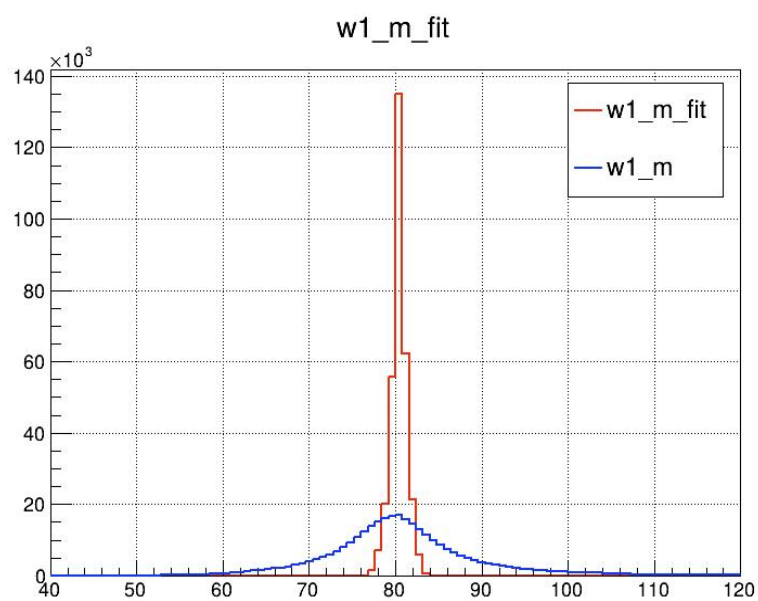
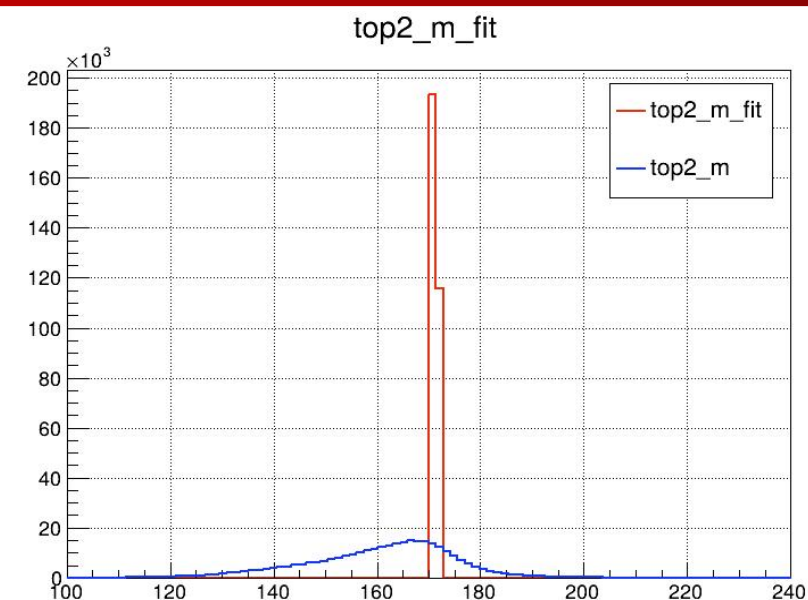
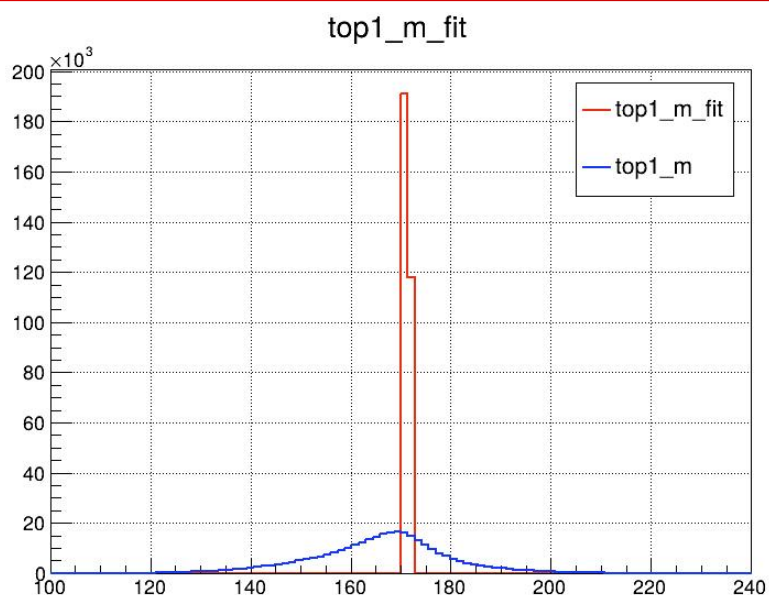
$$M_{t_1} = \text{Sf}_{j_1} \cdot M_{j_1} + \text{Sf}_{j_2} \cdot M_{j_2} + \text{Sf}_{b_1} \cdot M_{b_1}$$

$$M_{t_2} = \text{Sf}_{j_3} \cdot M_{j_3} + \text{Sf}_{j_4} \cdot M_{j_4} + \text{Sf}_{b_2} \cdot M_{b_2}$$

Kinematic fit $\rightarrow$ pairing in top mass reconstruction $\rightarrow$ improve top mass resolution

with χ^2 or not

hh channel

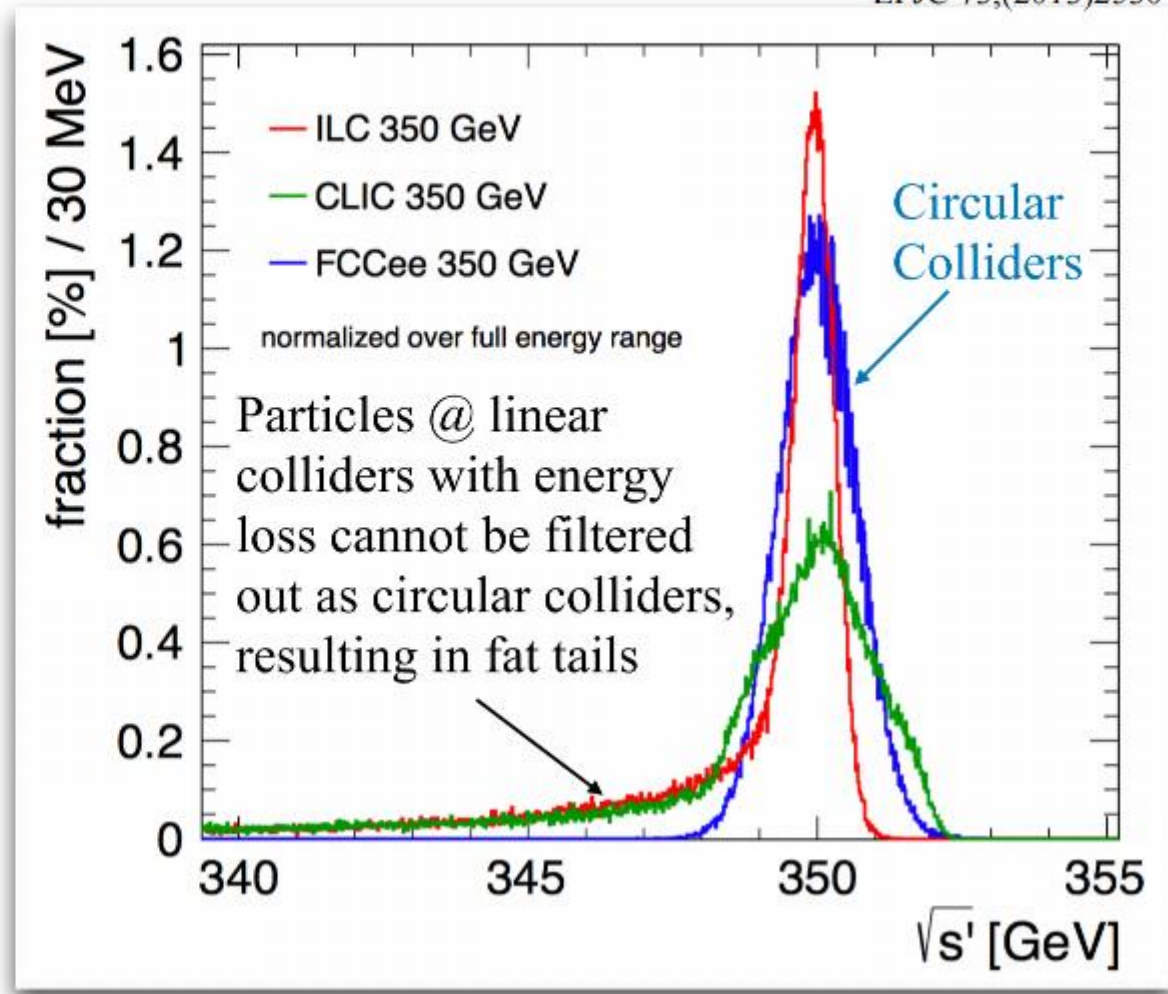


LS in linear/circular colliders

6

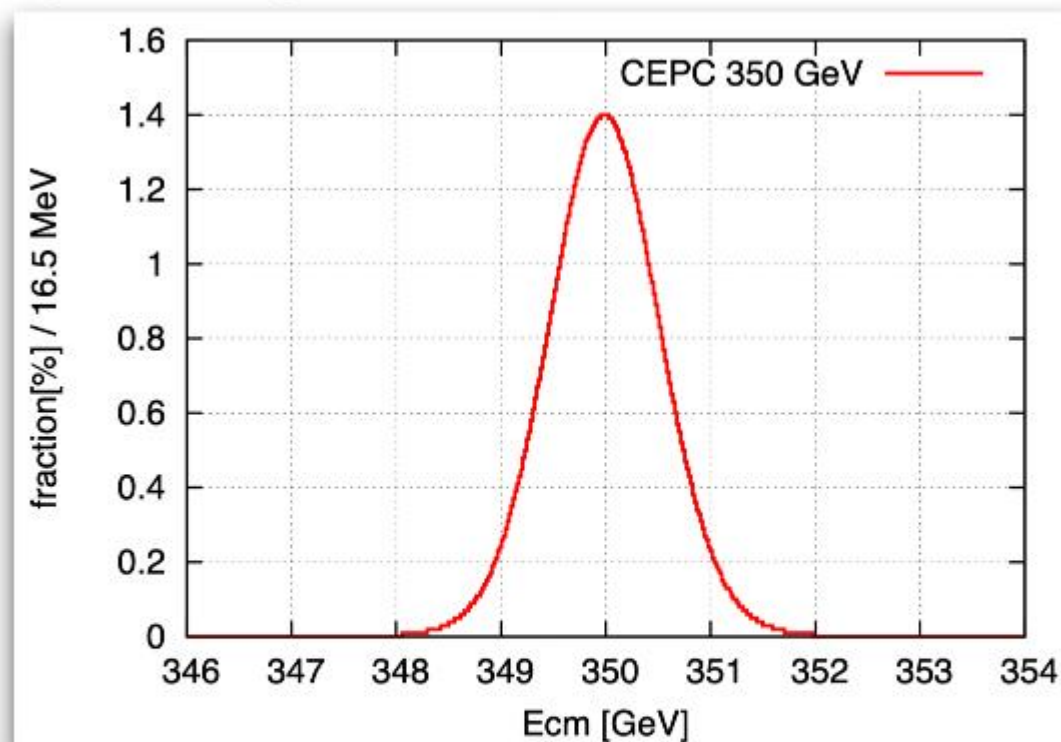
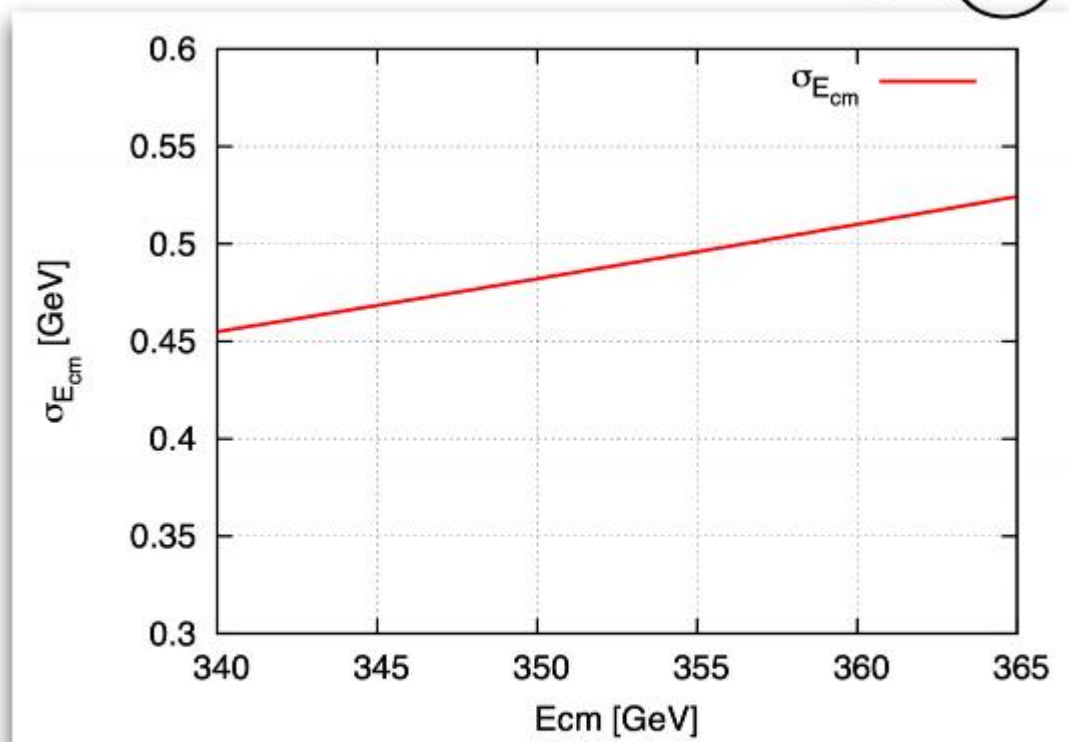
EPJC 73,(2013)2530

- The luminosity spectrum at linear colliders is obviously worse than circular colliders given that the particles with energy loss are not removed by the **bending magnets**
- This can substantially change the cross-section curve at around the $t\bar{t}$ threshold



LS @ CEPC

7



- The beam energy resolution increases as a function of $\sqrt{s}$
- The luminosity spectrum is shown for $\sqrt{s} = 350$ GeV with a width of ~ 480 MeV
- Similar to the FCC-ee scenario

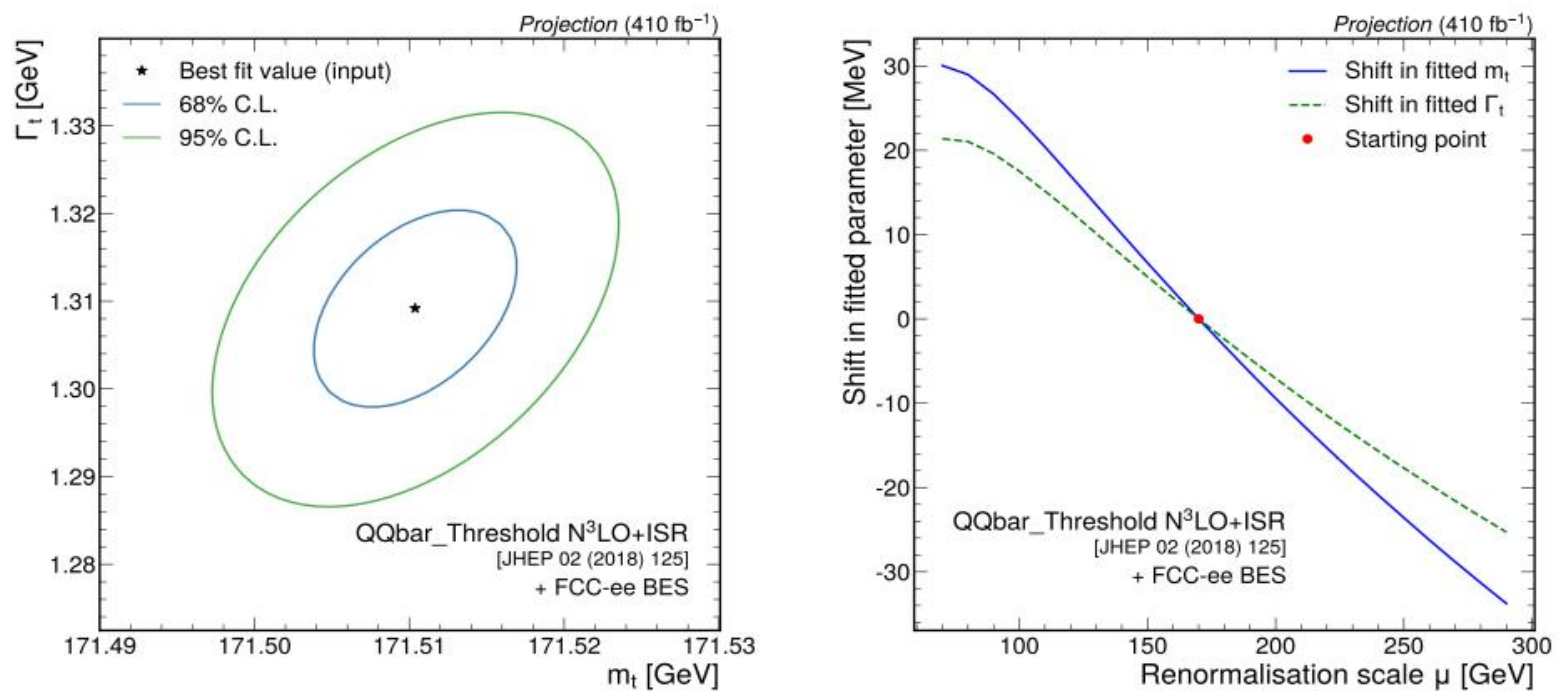


Figure 5. Left: two-dimensional confidence intervals in m_t and Γ_t , corresponding to 68% (inner ellipse) and 95% (outer ellipse) confidence level (C.L.). Right: shift in fitted m_t and Γ_t as a function of the choice of the renormalisation scale in the N³LO calculation, with respect to a reference value (starting point) of 170 GeV.

- The expected top mass uncertainty at CEPC is 21 MeV (optimistic) and 54 MeV (conservative), both significantly better than the 330 MeV at the LHC, and comparable to the FCC-ee projection of ~ 36 MeV.
- This highlights the advantage of the threshold scan method used in this study, which yields higher precision than the LHC's direct reconstruction approach, as it avoids systematics from energy loss and jet reconstruction. The improvement also benefits from **CEPC's superior beam energy control**.
- In this analysis, the **experimental efficiency** Uncertainty from btagging efficiency, JER and JES are still can't calculate:
 - In the current CEPC Delphes card, these effects are implemented as fixed constants.
 - These constants are partly derived from limited full simulation samples. These developments are still ongoing.
 - To accurately estimate such systematic uncertainties, one would need to:
 - Generate full simulation samples for both signal and background processes;
 - Train a jet tagger that can be imported into Delphes.
- Future improvements include training a jet tagger at 360 GeV and adding variables like momentum peak and forward-backward asymmetry to refine the top mass measurement.