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Spin polarization and the QCD plasma

OUTLINE

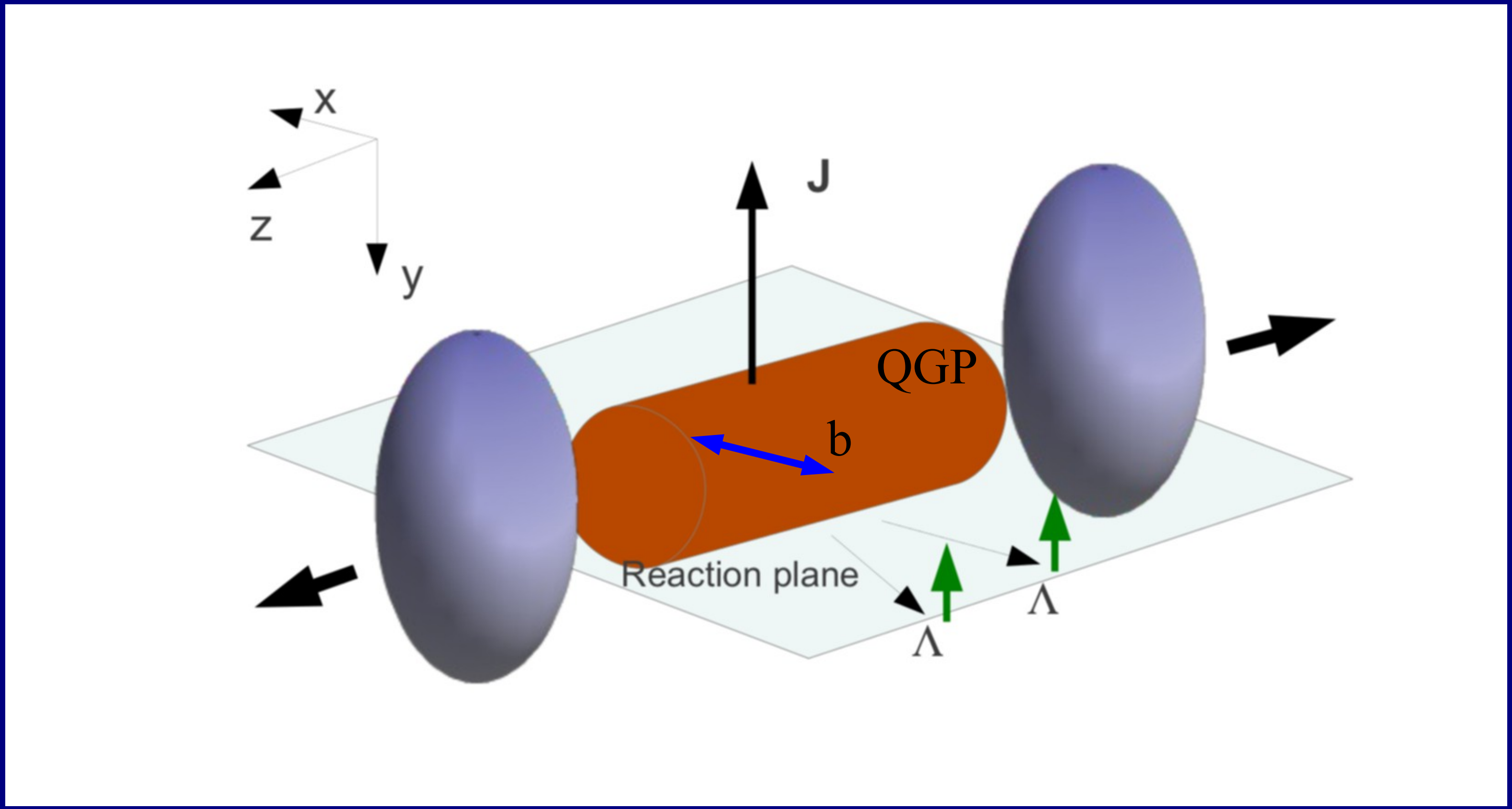
- Introduction
- Brief theory summary
- Numerical calculations and bulk viscosity
- Spin hydro and pseudo-gauge invariance
- Conclusions

UCAS Beijing, June 20 2025

Global polarization in relativistic nuclear collisions

Peripheral collisions ⇒ Angular momentum ⇒ Global polarization w.r.t reaction plane

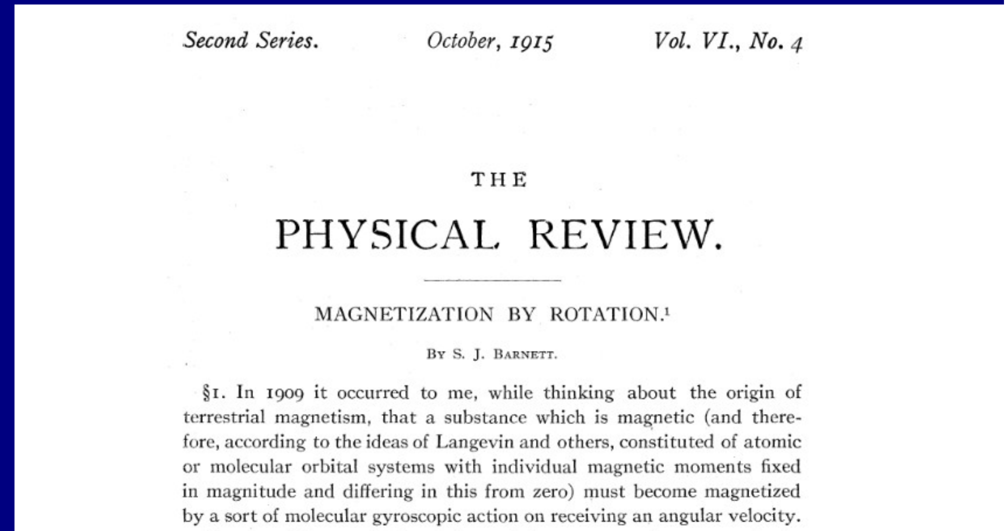
By parton spin-orbit coupling: Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301
By local equilibration: F. B., F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906



Barnett effect

S. J. Barnett, *Magnetization by Rotation,*

Phys. Rev. 6, 239–270 (1915).



Second Series. October, 1915 Vol. VI., No. 4

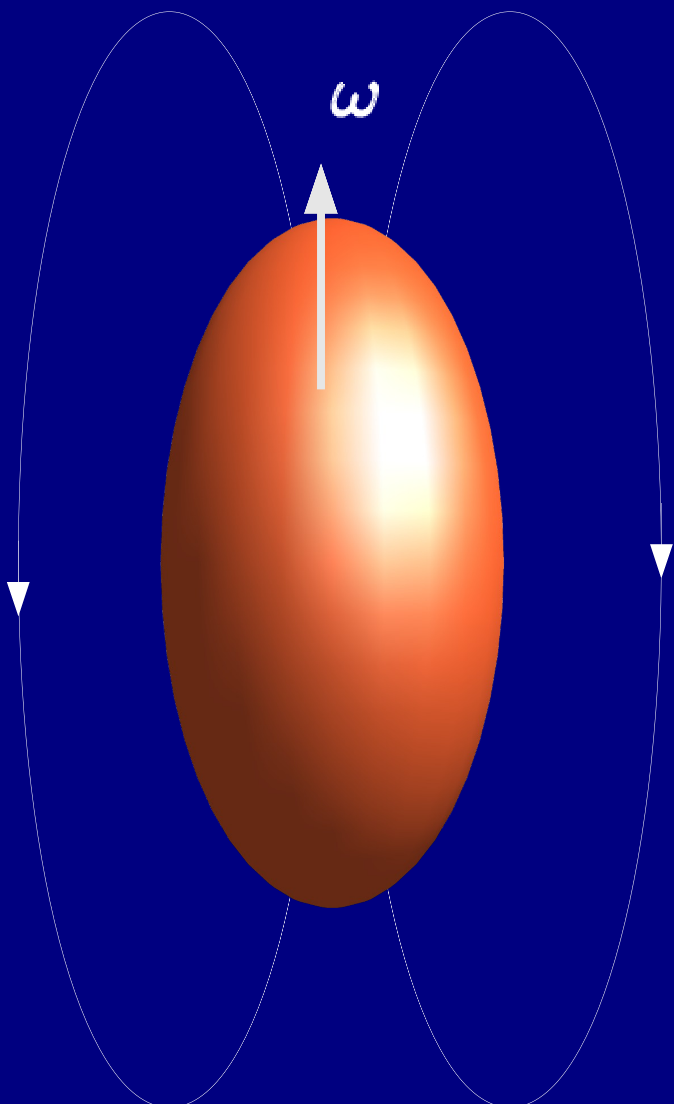
THE

PHYSICAL REVIEW.

MAGNETIZATION BY ROTATION.[1]

By S. J. Barnett.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.



Spontaneous magnetization of an uncharged body when spun around its axis

$$P \simeq \frac{S+1}{3}\frac{\hbar\omega}{KT} \Rightarrow M = \frac{\chi}{g}\omega$$

It can be seen as a dissipative transformation of the orbital angular momentum into spin of the constituents. The angular velocity decreases and a small magnetic field appears; this phenomenon is accompanied by a heating of the sample. Requires a spin-orbit coupling.

Polarization and vorticity

Local equilibrium at the freeze-out implies a connection between spin polarization and (thermal) vorticity

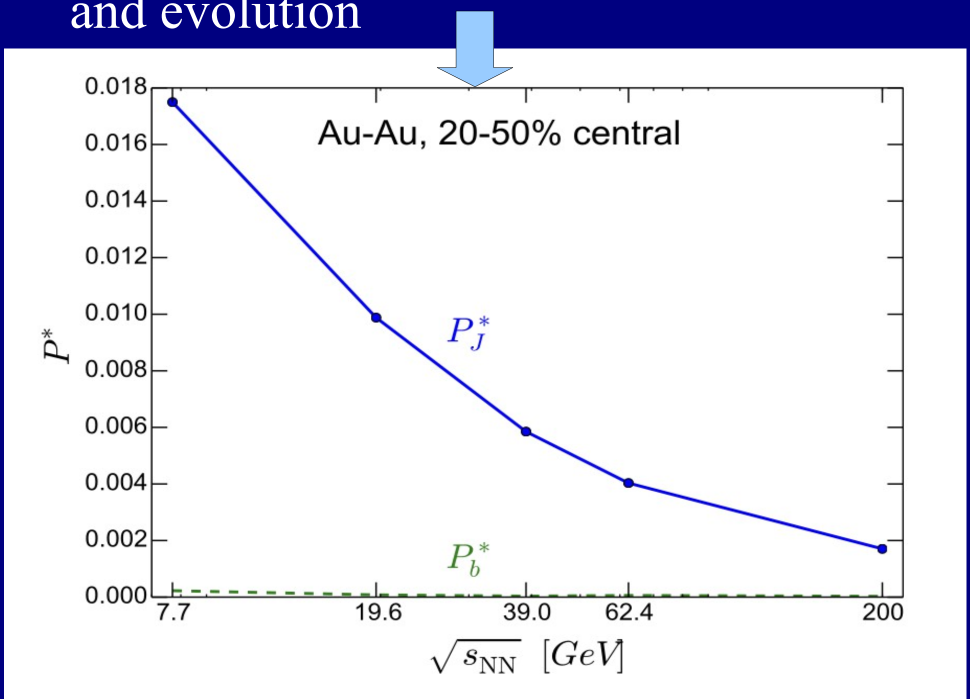
F.B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)

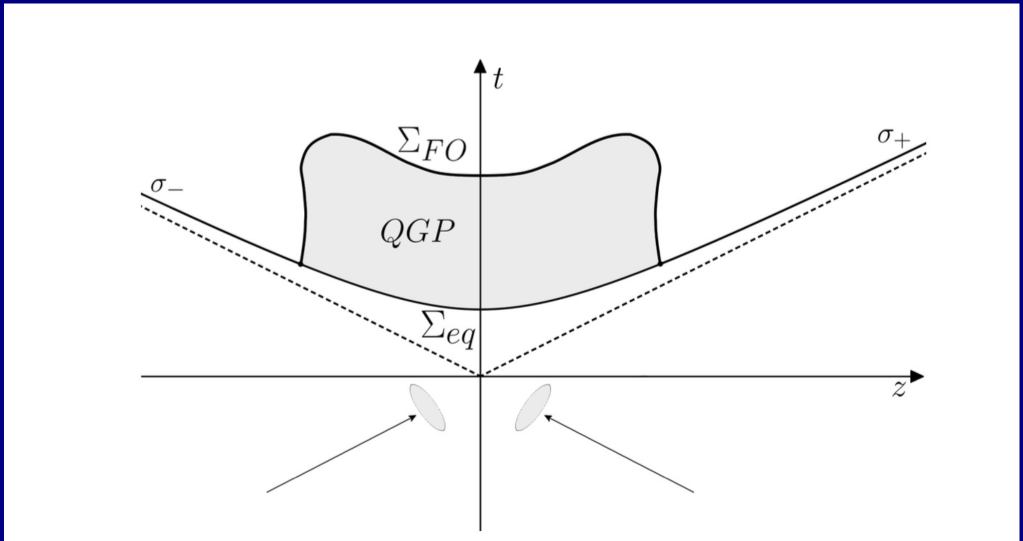
$$n_F = (\mathrm{e}^{\beta \cdot p - \xi} + 1)^{-1}$$

$$S^\mu(p) = \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F (1 - n_F) \partial_\nu \beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \; n_F}$$

$$\beta = \frac{1}{T} u$$

Quantitative prediction of 3+1D hydrodynamic model of QGP production and evolution

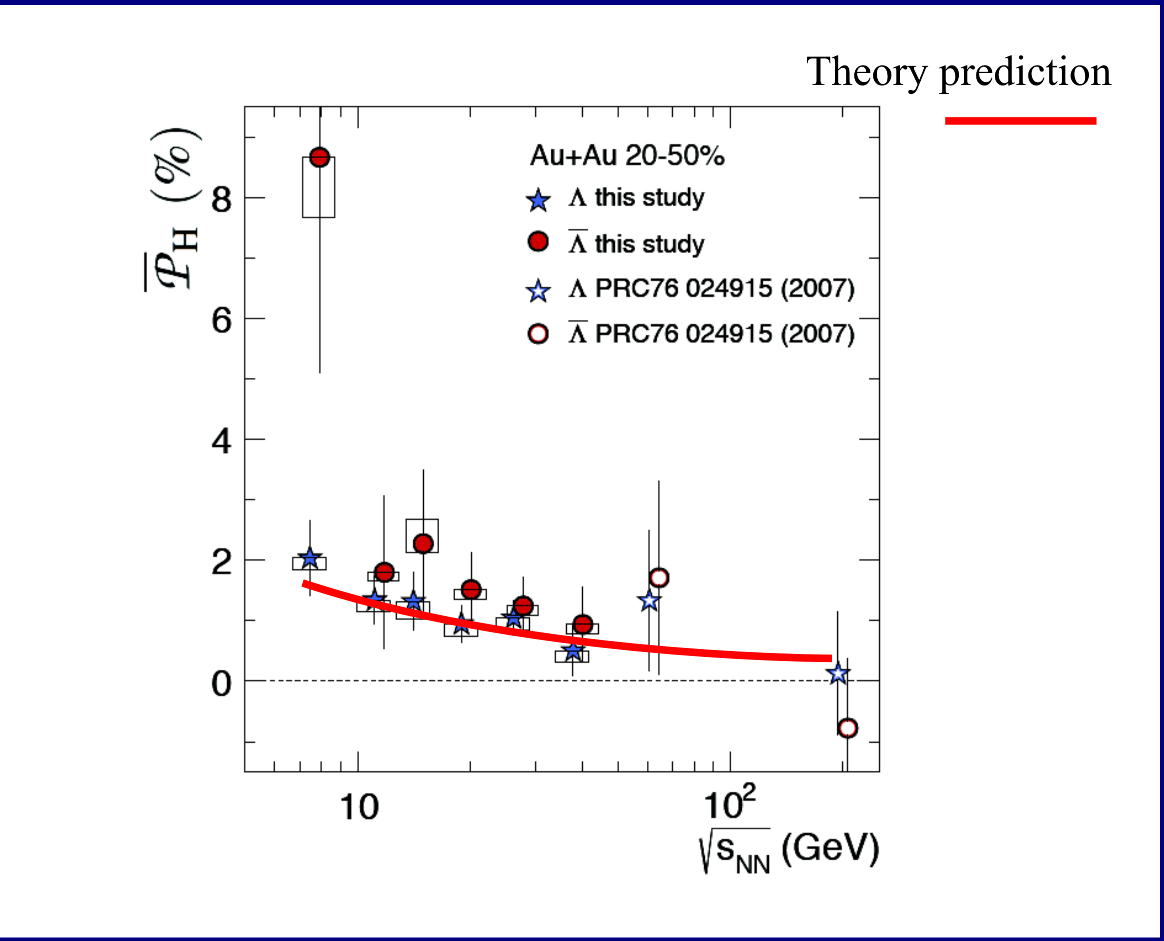


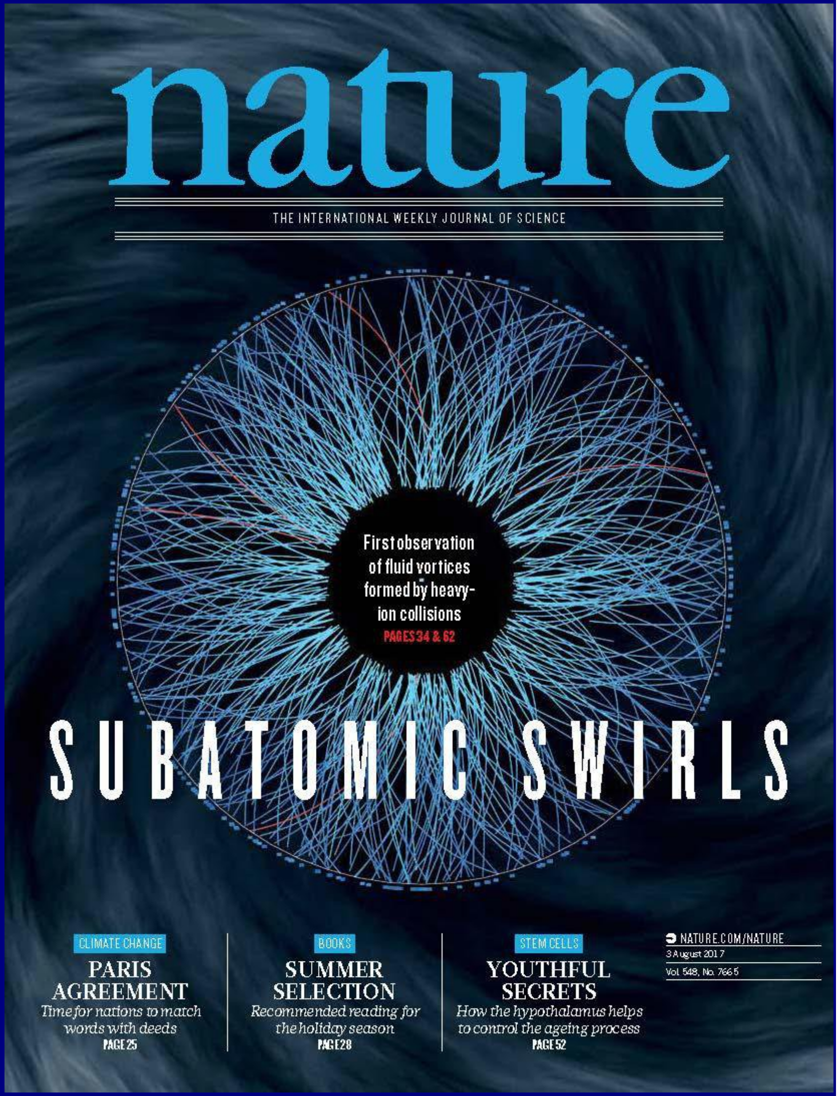


I. Karpenko, F.B., Eur. Phys. J. C 77 (2017) 213

Discovery of polarization in heavy ion collisions

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017

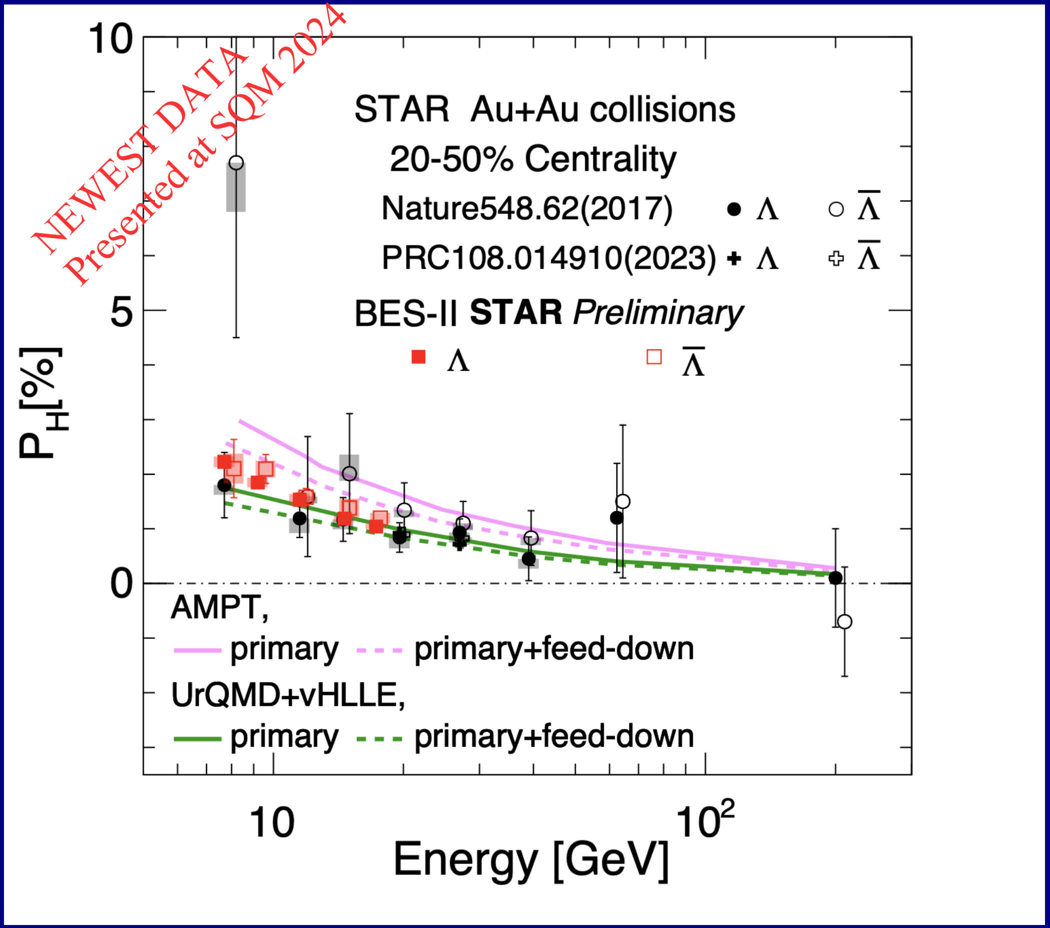


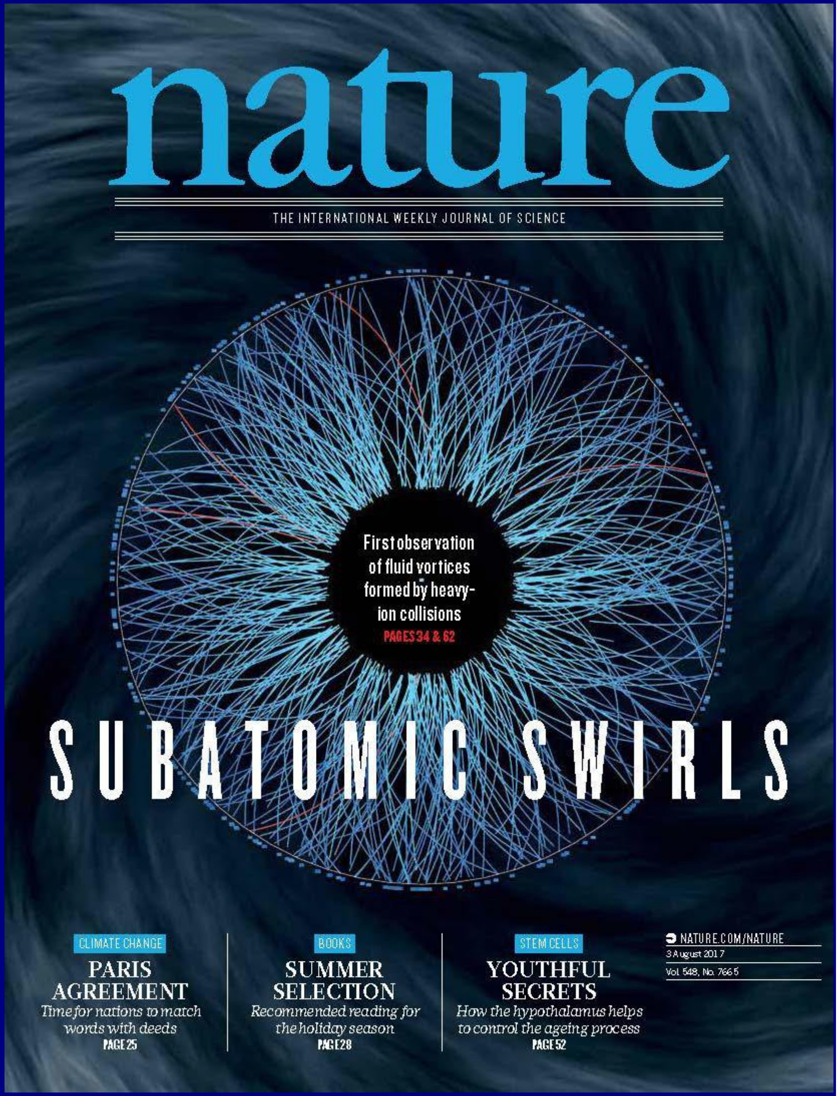


Particle and antiparticle have the same polarization sign.
This shows that the phenomenon cannot be driven
by a mean field (such as EM) whose coupling is *C-odd.*
In agreement with the predictions based on spin-vorticity formula

Discovery of polarization in heavy ion collisions

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017





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This shows that the phenomenon cannot be driven
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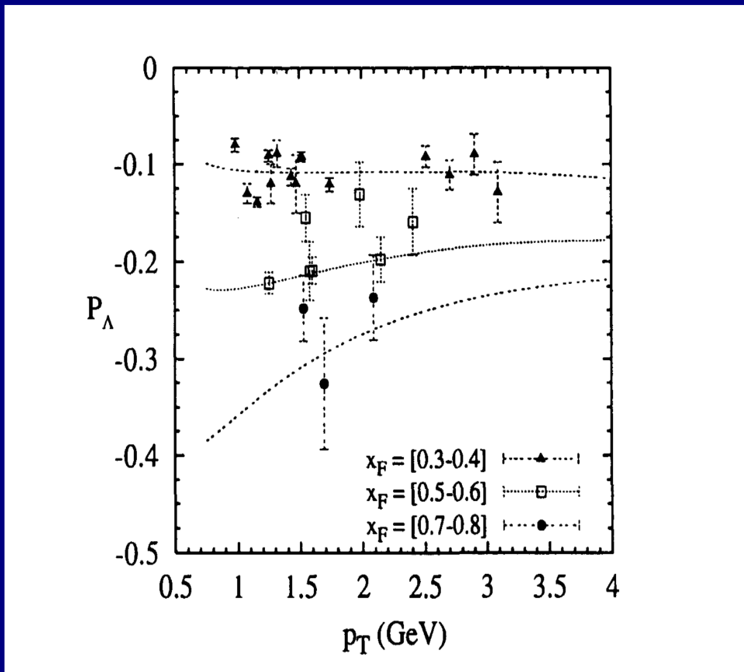
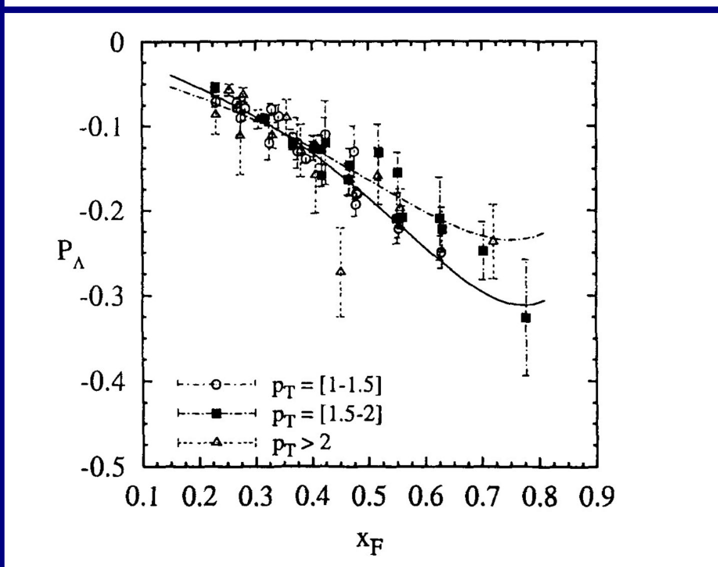
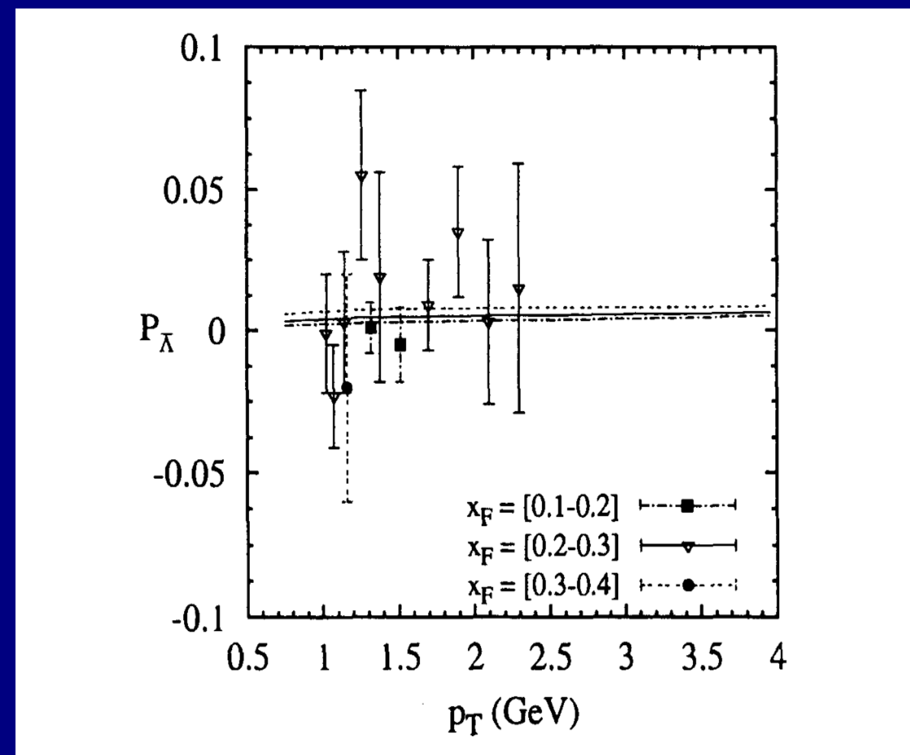
Comparison with NN collisions



Λ is polarized perpendicular to the production plane (no global polarization)

$$x_F = \frac{p_z}{|p_{zMAX}|}$$



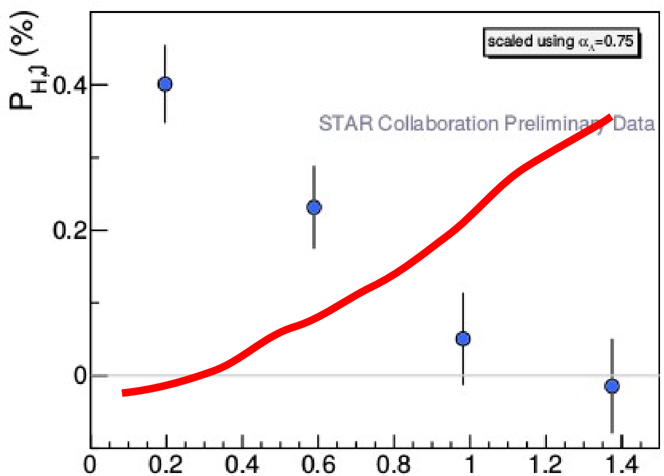


Polarization of anti-Λ almost vanishing compared to Λ

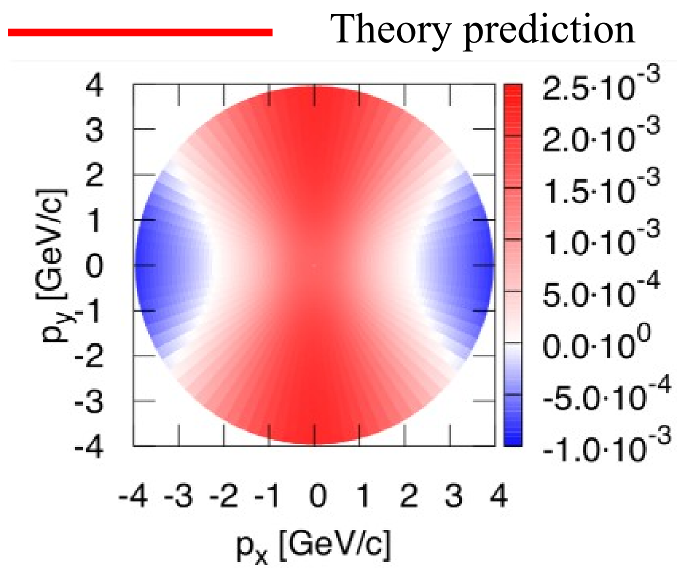
M. Anselmino et al., Czech. J. Phys. 51 (2001) Suppl. A, compilation of data from pBe collisions

(old) Puzzle: momentum dependence of polarization

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F (1-n_F)\partial_\nu \beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \ n_F}$$

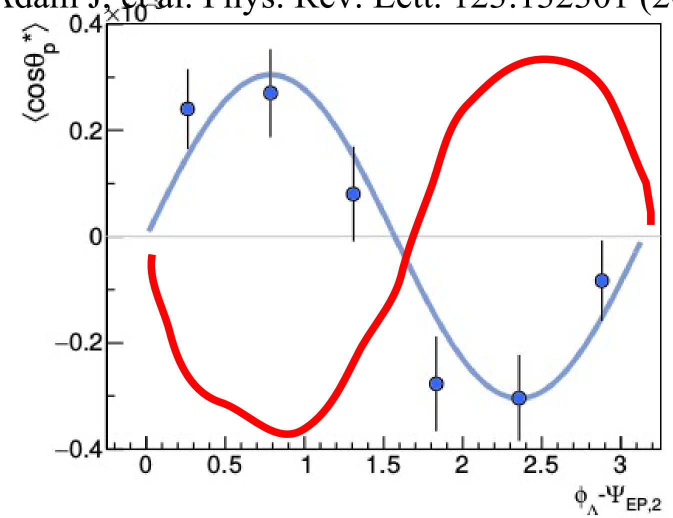


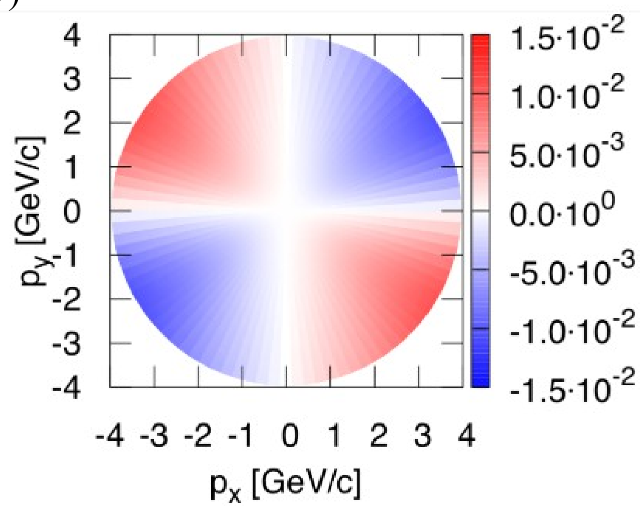
Niida T. Nucl. Phys. A982:511 (2019)



Spin component along J at p_z=0

Adam J, et al. Phys. Rev. Lett. 123:132301 (2019)





Spin component along beam line at p_z=0

A brief theory summary

F. B., Lecture Notes in Physics 987, 15 (2021) arXiv:2004.04050

There are two methods to calculate the spin polarization of final particles
1- A quantum statistical method built on local equilibrium density operator
2- An extension of relativistic kinetic theory to particles with spin

Spin polarization vector for spin ½ particles:

$$S^\mu(p) = \sum_{i=1}^{3} [p]_i^\mu \operatorname{tr}(D^{1/2}(J_i)\Theta(p)) \qquad \Theta(p)_{sr} = \frac{\operatorname{Tr}(\widehat{\rho}\widehat{a}^\dagger(p)_r \widehat{a}(p)_s)}{\sum_t \operatorname{Tr}(\widehat{\rho}\widehat{a}^\dagger(p)_t \widehat{a}(p)_t)}$$

$$\Downarrow$$

$$S^\mu(p) = \frac{1}{2} \frac{\int \mathrm{d}\Sigma \cdot p \operatorname{tr}_4(\gamma^\mu \gamma^5 W_+(x,p))}{\int \mathrm{d}\Sigma \cdot p \operatorname{tr}_4 W_+(x,p)}$$

$$\begin{aligned} \widehat{W}(x,k)_{AB} &= -\frac{1}{(2\pi)^4} \int \mathrm{d}^4 y \, \mathrm{e}^{-ik\cdot y} : \Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) : \\ &= \frac{1}{(2\pi)^4} \int \mathrm{d}^4 y \, \mathrm{e}^{-ik\cdot y} : \overline{\Psi}_B(x+y/2)\Psi_A(x-y/2) : \end{aligned}$$

$$W(x,k) = \operatorname{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

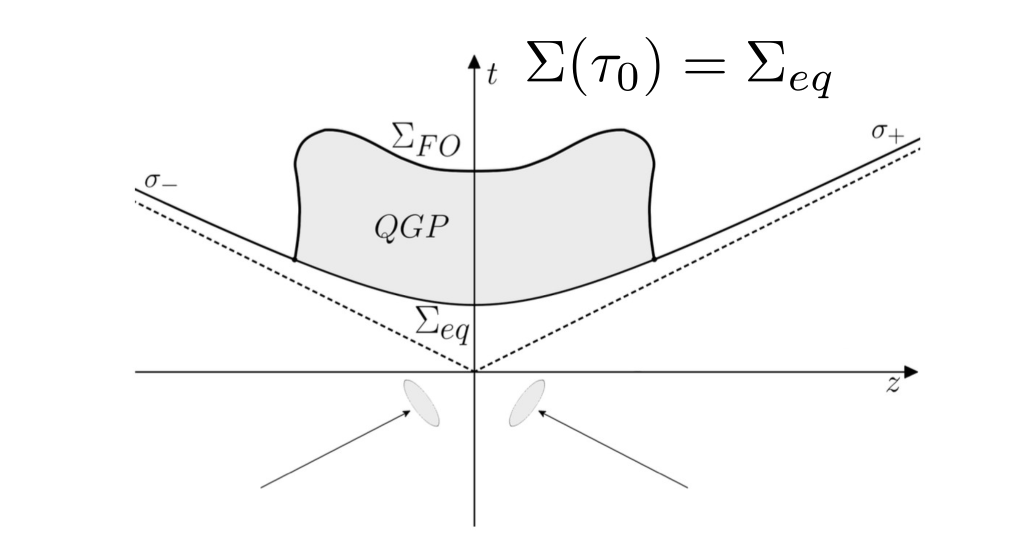
Density operator

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right].$$

Density operator: local equilibrium at the initial time

T_B is the Belinfante stress-energy tensor

$$\beta = \frac{1}{T} u \quad \zeta = \frac{\mu}{T}$$



With the Gauss theorem: calculate at Freeze-out

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) + \int_\Theta \mathrm{d}\Theta \left(\widehat{T}_B^{\mu\nu} \nabla_\mu \beta_\nu - \widehat{j}^\mu \nabla_\mu \zeta \right) \right],$$

Local equilibrium, non-dissipative terms

Dissipative terms

Local equilibrium and hydrodynamic limit

$$W(x,k)_{\rm LE} = \frac{1}{Z}{\rm Tr}\left(\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y)\widehat{T}_B^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\widehat{j}^\mu(y)\right]\widehat{W}(x,k)\right)$$

Expand the β and ζ fields from the point *x* where the Wigner operator is to be evaluated

$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \ldots$$

$$\int_\Sigma {\rm d}\Sigma_\mu T_B^{\mu\nu}(y)\beta_\nu(x) = \beta_\nu(x)\int_\Sigma {\rm d}\Sigma_\mu T_B^{\mu\nu}(y) = \beta_\nu(x)\widehat{P}^\nu$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp\left[-\beta_\mu(x)\widehat{P}^\mu - \frac{1}{4}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}_x^{\mu\nu} - \frac{1}{4}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}_x^{\mu\nu} + \ldots\right]$$

$$\widehat{J}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda(y-x)^\mu\widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu\widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda(y-x)^\mu\widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu\widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x) \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu}(x) \widehat{J}^{\mu\nu}_x - \frac{1}{2} \xi_{\mu\nu}(x) \widehat{Q}^{\mu\nu}_x + \ldots]$$

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

Thermal vorticity
Adimensional in natural units

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$$

Thermal shear
Adimensional in natural units

At global equilibrium the thermal shear vanishes because of the Killing equation

Linear response theory

$$e^{\widehat{A}+\widehat{B}} = e^{\widehat{A}} + \int_0^1 dz\; e^{z(\widehat{A}+\widehat{B})}\widehat{B}e^{-z\widehat{A}}e^{\widehat{A}} \simeq e^{\widehat{A}} + \int_0^1 dz\; e^{z\widehat{A}}\widehat{B}e^{-z\widehat{A}}e^{\widehat{A}}$$

$$\widehat{A} = -\beta_\mu(x)\widehat{P}^\mu$$

$$\widehat{B} = \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}_x^{\mu\nu} - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}_x^{\mu\nu} + \ldots]$$

$$W(x,k) \simeq \frac{1}{Z}\mathrm{Tr}(e^{\widehat{A}+\widehat{B}}\widehat{W}(x,k)) \simeq \ldots$$

CORRELATORS

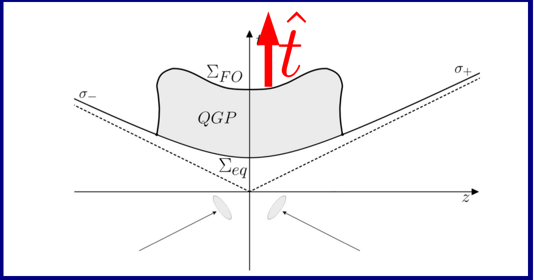
$$\langle \widehat{Q}_x^{\mu\nu}\widehat{W}(x,p)\rangle \qquad \langle \widehat{J}_x^{\mu\nu}\widehat{W}(x,p)\rangle$$

Spin mean vector at leading order

$$\widehat{\rho}_{\mathrm{LE}} \simeq \frac{1}{Z} \exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \; n_F}$$

Neglected by “prejudice” until 2021



See also

R. H. Fang, L. G. Pang, Q. Wang, X. N. Wang, Phys. Rev. C 94 (2016) 024904
W. Florkowski, A. Kumar and R. Ryblewski, Phys. Rev. C 98 (2018) 044906
Y. C. Liu, L. L. Gao, K. Mameda and X. G. Huang, Phys. Rev. D 99 (2019) 085014
N. Weickgenannt, X. L. Sheng, E. Speranza, Q. Wang and D. H. Rischke, Phys. Rev. D 100 (2019) 056018

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\; n_F(1-n_F)\hat{t}_\nu\xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\; n_F},$$

$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right).$$

F. B., M. Buzzegoli, A. Palermo, Phys. Lett. B 820 (2021) 136519
S. Liu, Y. Yin, JHEP 07 (2021) 188
Confirmed by C. Yi, S. Pu, D. L. Yang, Phys.Rev.C 104 (2021) 6, 064901
Y. C. Liu, X. G. Huang, Sci.China Phys.Mech.Astron. 65 (2022) 7, 272011

Second order terms in LE expansion

X. L. Sheng, F. B., X. G. Huang, Z. H. Zhang, Phys. Rev. C 110 (2024) 064908
Z. Yang, X. Q. Xie, S. Pu, J. H. Gao and Q. Wang, [arXiv:2412.19400 [hep-ph]].
Z. H. Zhang, X. G. Huang, F. B. and X. L. Sheng, [arXiv:2412.19416 [hep-ph]].

$$\widehat{A}_x = -\beta(x)\cdot\widehat{P}$$
$$\widehat{B}_x = \frac{1}{2}\varpi(x) : \widehat{J}_x + \frac{1}{2}\xi : \widehat{Q}_x$$

Quadratic in the gradients

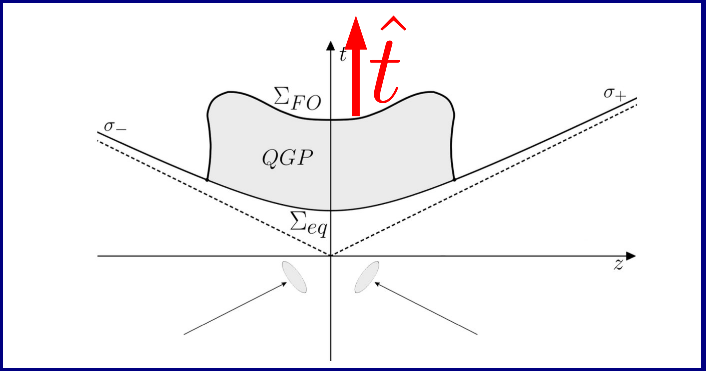
$$\begin{aligned} \mathrm{e}^{\widehat{A}_x+\widehat{B}_x} \approx\ & \mathrm{e}^{\widehat{A}_x} + \int_0^1 \mathrm{d}z\ \mathrm{e}^{z\widehat{A}_x}\widehat{B}_x \mathrm{e}^{-z\widehat{A}_x} \\ & + \int_0^1 \mathrm{d}z_1 \int_0^{z_1} \mathrm{d}z_2\ \mathrm{e}^{z_1\widehat{A}_x}\widehat{B}_x \mathrm{e}^{(z_1-z_2)\widehat{A}_x}\widehat{B}_x \mathrm{e}^{-z_1\widehat{A}_x} + \mathcal{O}(\widehat{B}^3)\,. \end{aligned}$$

Linear in the second-order derivatives

$$\widehat{\rho}_{\mathrm{LE}} \simeq \frac{1}{Z}\exp\left[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \partial_\lambda\partial_\mu\beta^\nu(x)\widehat{\Theta}^{\lambda\mu\nu}_x + \dots\right]$$

Linear terms

Calculation with canonical stress-energy tensor and spin potential



$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}_C \beta_\nu - \zeta \widehat{j}^\mu(y) + \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu\lambda\nu}\right)\right]$$

$$S^{(1)\mu}(p) \;=\; -\frac{1}{8mN}\int {\rm d}\Sigma\cdot p_+\, n_F(x,p)\,[1-n_F(x,p)] \times \left\{\epsilon^{\mu\nu\lambda\sigma}\Omega_{\nu\lambda}p_\sigma - \frac{2}{p\cdot\hat{t}}\hat{t}_\nu \epsilon^{\mu\nu\lambda\sigma}p_\lambda\left[(\xi_{\sigma\rho}+\Omega_{\sigma\rho}-\varpi_{\sigma\rho})\,p^\rho - \partial_\sigma\zeta\right]\right\},$$

Confirms previous findings, with an extension to the canonical spin potential (M. Buzzegoli PRC 105 (2022) 4, 044907)

$$S^{(2)\mu}_{\rm lin}(p) \;=\; \frac{1}{4m(p\cdot\hat{t})^2N}\int {\rm d}\Sigma\cdot p_+\, n_F(x,p)\,[1-n_F(x,p)]\,(y_\Sigma(0)-x)\cdot\hat{t} \times\hat{t}_\alpha p_\rho\left[\epsilon^{\mu\sigma\alpha\rho}p^\lambda p^\nu\partial_\sigma\xi_{\nu\lambda} + \left(\frac{1}{2}p^\alpha\epsilon^{\mu\nu\lambda\rho} - \epsilon^{\mu\alpha\lambda\rho}p^\nu\right)p^\sigma\partial_\sigma\varpi_{\nu\lambda} - \epsilon^{\mu\sigma\alpha\rho}p^\lambda\partial_\sigma\partial_\lambda\zeta + \frac{1}{2}\epsilon^{\alpha\nu\lambda\sigma}\partial^\rho(\Omega_{\nu\lambda}-\varpi_{\nu\lambda})(p^\mu p_\sigma - m^2 g^\mu_\sigma)\right],$$

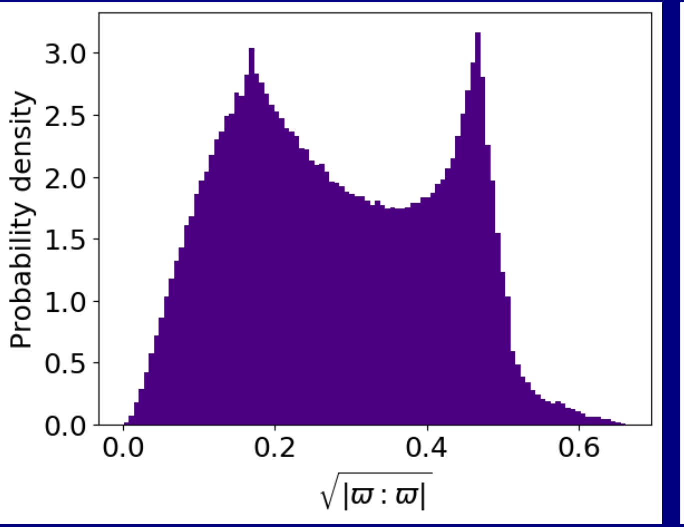
Quadratic terms

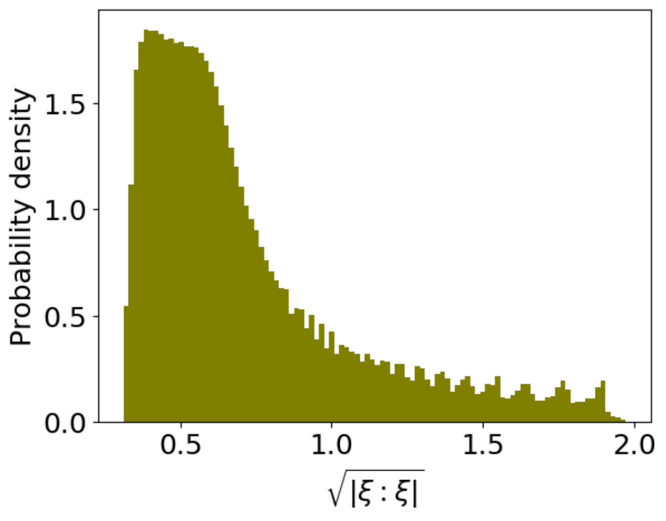
$$S^{(2)\mu}_{\rm quad}(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot p_+ \,\mathrm{tr}\left[\gamma^\mu\gamma^5 W_{\rm quad}(x,p)\right]}{\int \mathrm{d}\Sigma\cdot p_+ \,\mathrm{tr}[W^{(0)}(x,p)]} - S^{(1)\mu}(p)\frac{\int \mathrm{d}\Sigma\cdot p_+ \,\mathrm{tr}[W^{(1)}(x,p)]}{\int \mathrm{d}\Sigma\cdot p_+ \,\mathrm{tr}[W^{(0)}(x,p)]},$$

$$\mathrm{tr}\left[\gamma^\mu\gamma^5 W_{\rm quad}(x,p)\right] = \frac{[1-2n_F(x,p)]\,\mathrm{tr}\left[\gamma^\mu\gamma^5 W^{(1)}(x,p)\right]\mathrm{tr}\left[W^{(1)}(x,p)\right]}{[1-n_F(x,p)]\,\mathrm{tr}\left[W^{(0)}(x,p)\right]}.$$

$$W_0(x,p) = \frac{\delta(p^2-m^2)\mathrm{sgn}(p^0)}{(2\pi)^3}(\not p + m)n_F(x,p),$$

$$\mathrm{tr}\left[W^{(1)}(x,p)\right] = -\frac{\delta(p^2-m^2)}{(2\pi)^3|p^0|}n_F(x,p)\left[1-n_F(x,p)\right](y^0_\Sigma - x^0)4mp^\lambda\partial_\lambda[p^\sigma\beta_\sigma(x)-\zeta(x)],$$





Expectation: should be small

Amplitude distribution (one entry per cell) at the freeze-out

Numerical computation: sensitivity to initial conditions and bulk viscosity

A. Palermo, F.B., E. Grossi, I. Karpenko, Eur. Phys. J. 84 (2024) 9, 920

Recent hydro calculations of Λ polarization in relativistic heavy ion collisions

S. Alzhrani, S. Ryu, and C. Shen, Phys. Rev. C **106**, 014905 (2022), arXiv:2203.15718 [nucl-th].
F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, and A. Palermo, Phys. Rev. Lett. **127**, 272302 (2021), arXiv:2103.14621 [nucl-th].
B. Fu, S. Y. F. Liu, L. Pang, H. Song, and Y. Yin, Phys. Rev. Lett. **127**, 142301 (2021), arXiv:2103.10403 [hep-ph].
X.-Y. Wu, C. Yi, G.-Y. Qin, and S. Pu, Phys. Rev. C **105**, 064909 (2022), arXiv:2204.02218 [hep-ph].
Z.-F. Jiang, X.-Y. Wu, H.-Q. Yu, S.-S. Cao, and B.-W. Zhang, Acta Phys. Sin. **72**, 072504 (2023).
Z.-F. Jiang, X.-Y. Wu, S. Cao, and B.-W. Zhang, Phys. Rev. C **108**, 064904 (2023), arXiv:2307.04257 [nucl-th].
V. H. Ribeiro, D. Dobrigkeit Chinellato, M. A. Lisa, W. Matioli Serenone, C. Shen, J. Takahashi, and G. Torrieri, Phys. Rev. C **109**, 014905 (2024), arXiv:2305.02428 [hep-ph].

C. Yi, X. Y. Wu, J. Zhu, S. Pu and G. Y. Qin, Phys. Rev. C 111 (2025) no.4, 044901

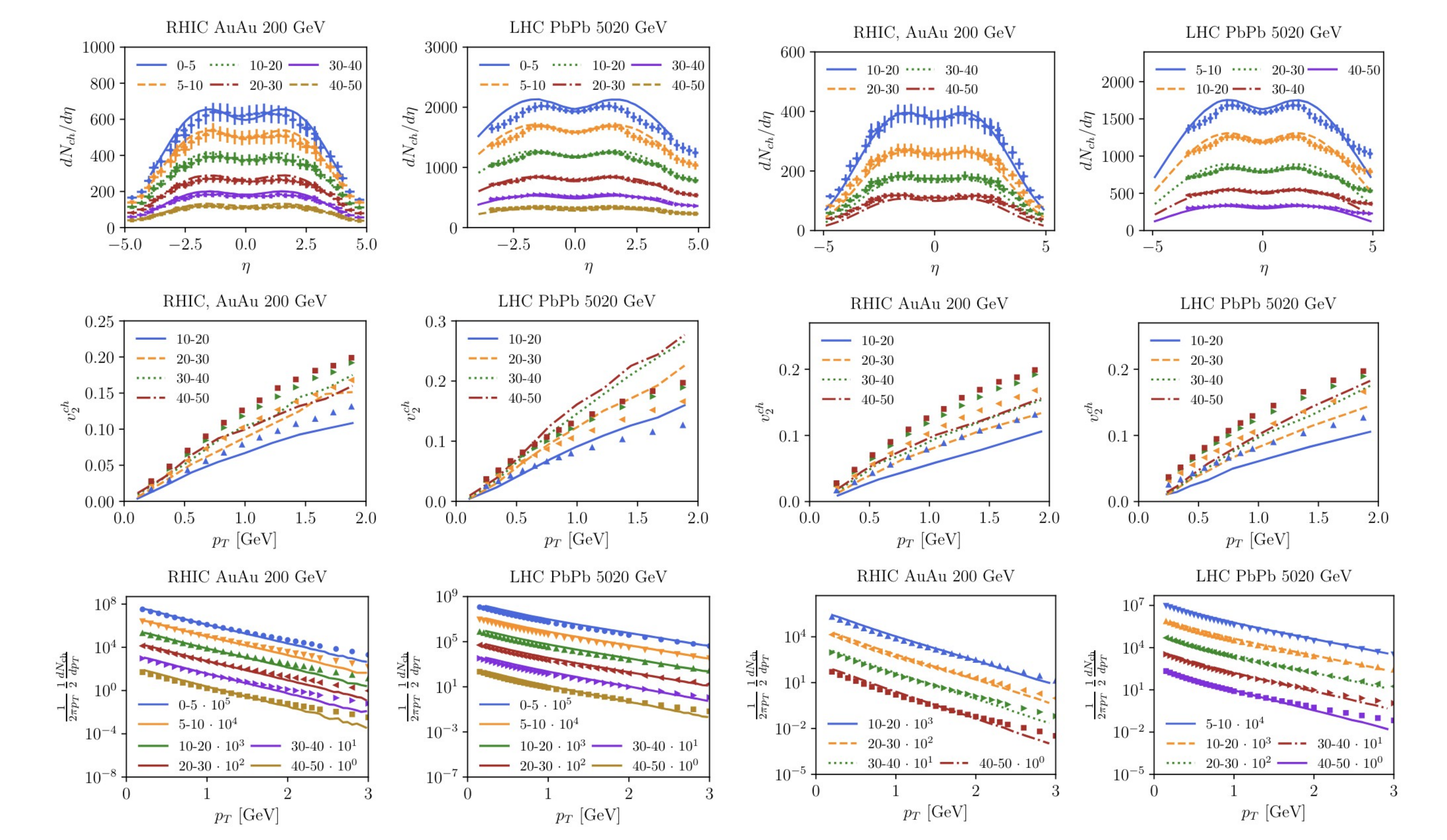
Numerical implementation of 3+1 D causal viscous hydrodynamics (VHLLE) with statistical hadronization and particle rescattering (afterburner SMASH)

Initial state model: SUPERMC (C. Shen et al.), GLISSANDO (Monte-Carlo Glauber)

Polarization transferred to Λ in secondary decays of Σ^0 and Σ^* taken into account

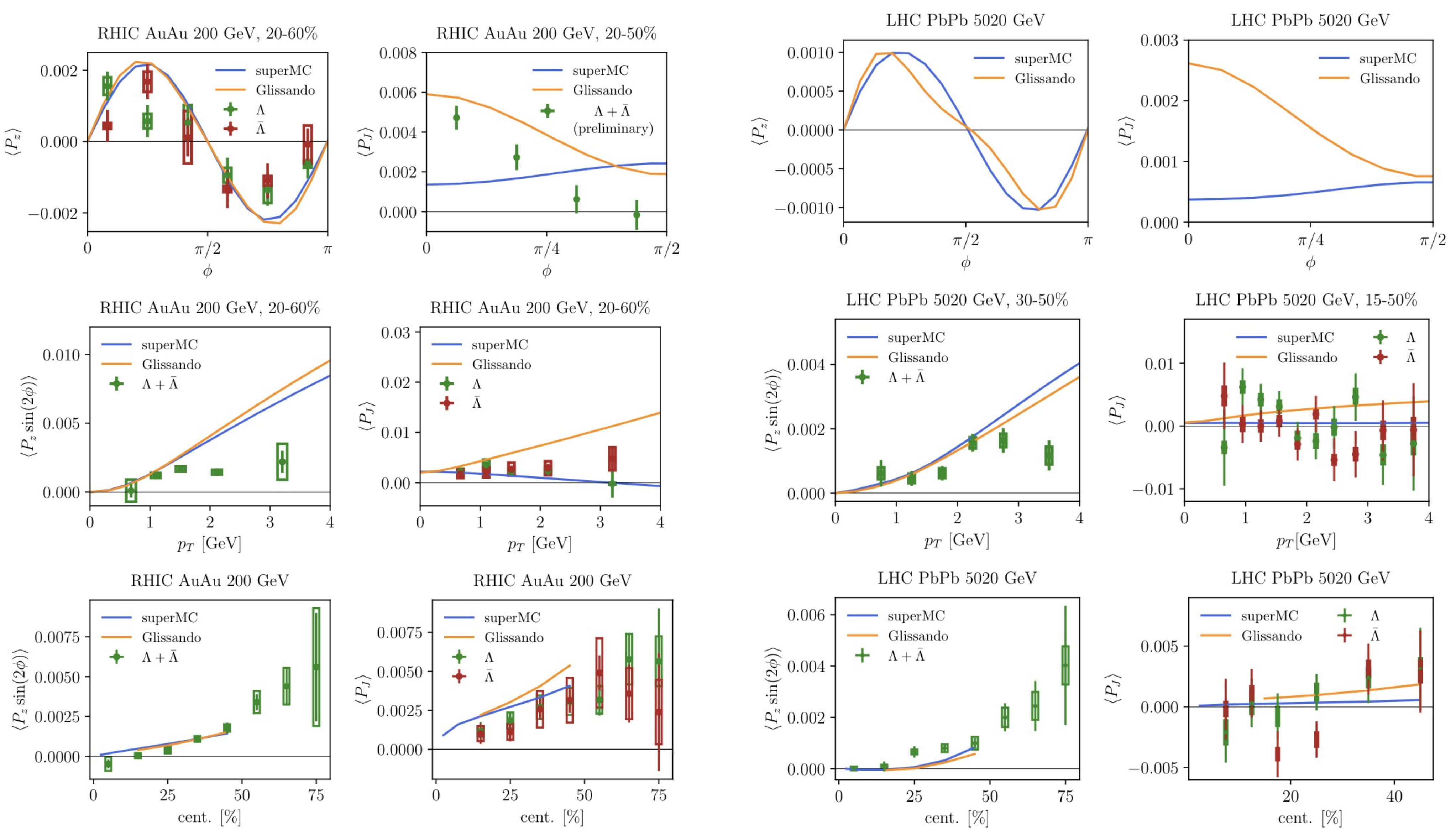
Qualification of the code

Benchmark distributions



SUPERMC GLISSANDO

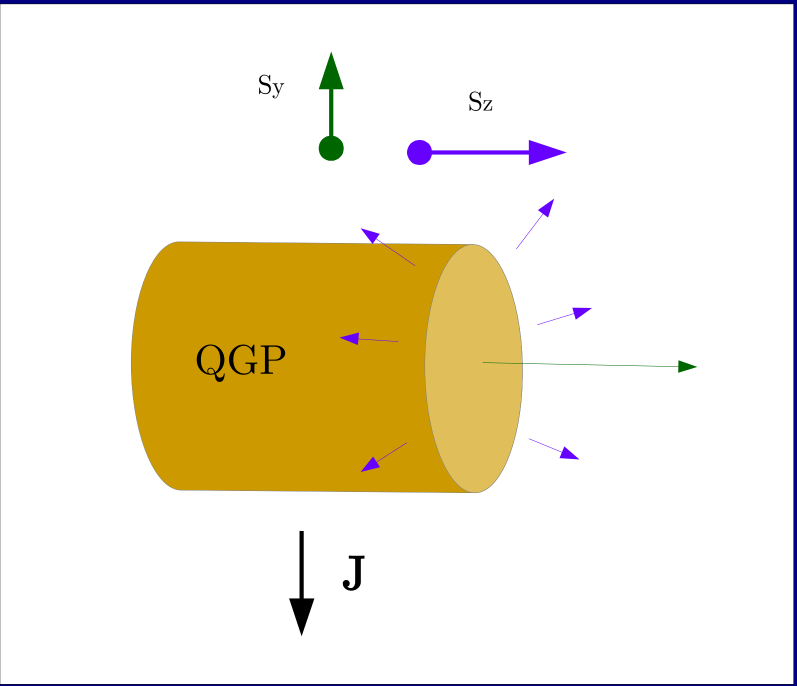
RESULTS

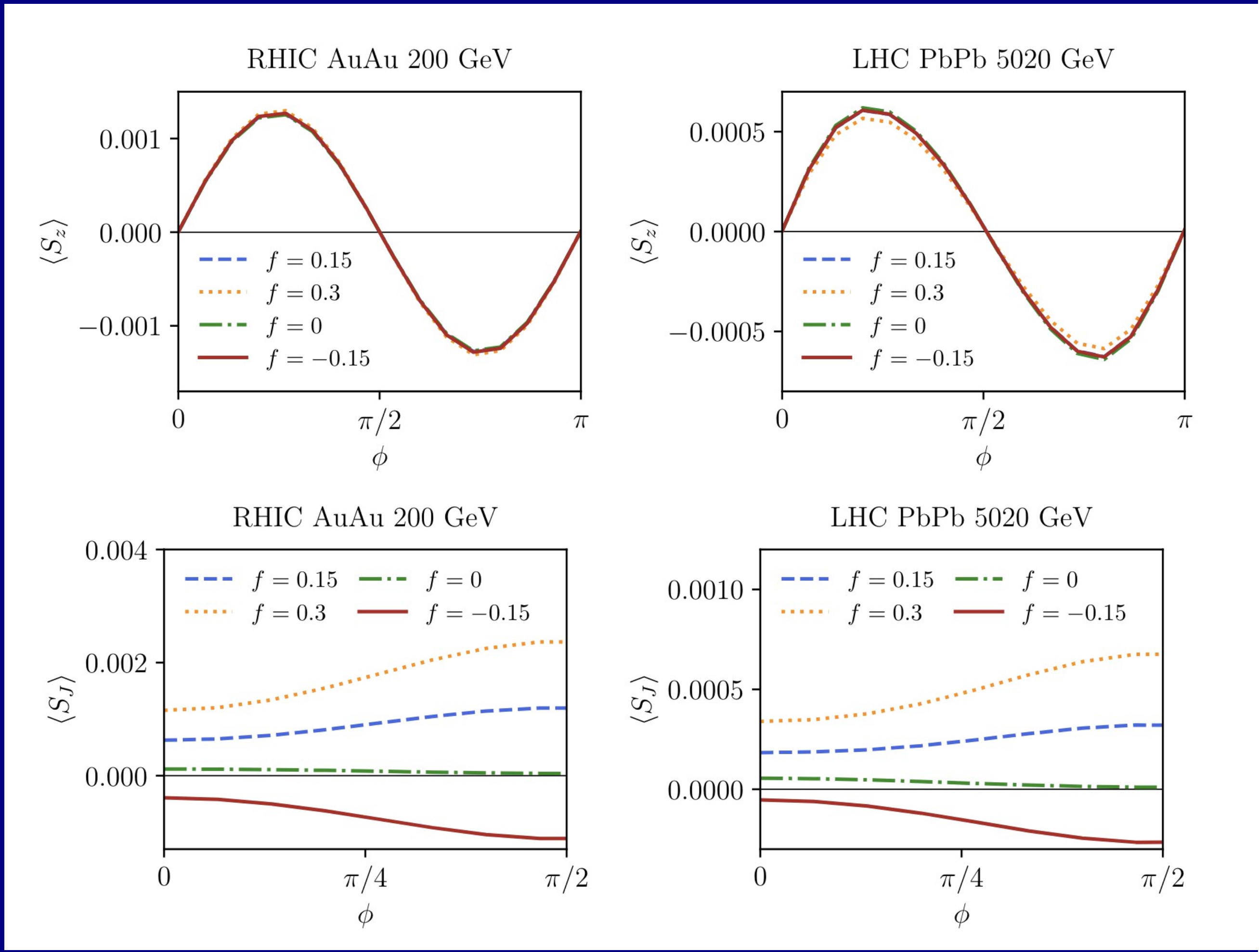

RHIC AuAu 200 GeV, 20-60%
RHIC AuAu 200 GeV, 20-50%
LHC PbPb 5020 GeV
LHC PbPb 5020 GeV
RHIC AuAu 200 GeV, 20-60%
RHIC AuAu 200 GeV, 20-60%
LHC PbPb 5020 GeV, 30-50%
LHC PbPb 5020 GeV, 15-50%
RHIC AuAu 200 GeV
RHIC AuAu 200 GeV
LHC PbPb 5020 GeV
LHC PbPb 5020 GeV
superMC
Glissando
Λ
$\bar{\Lambda}$
$\Lambda + \bar{\Lambda}$
(preliminary)
$\langle P_z \rangle$
$\langle P_J \rangle$
$\langle P_z \sin(2\phi) \rangle$
ϕ
p_T [GeV]
cent. [%]

Sensitivity to initial longitudinal flow

Variation of SUPERMC flow parameter

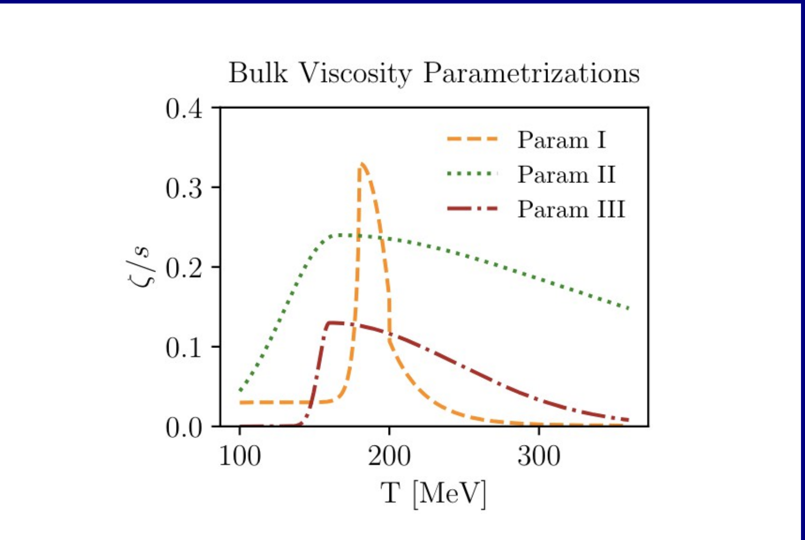
$$T^{\tau\tau} = \rho \cosh(f\, y_{CM})$$
$$T^{\tau\eta} = \frac{\rho}{\tau} \sinh(f\, y_{CM})$$

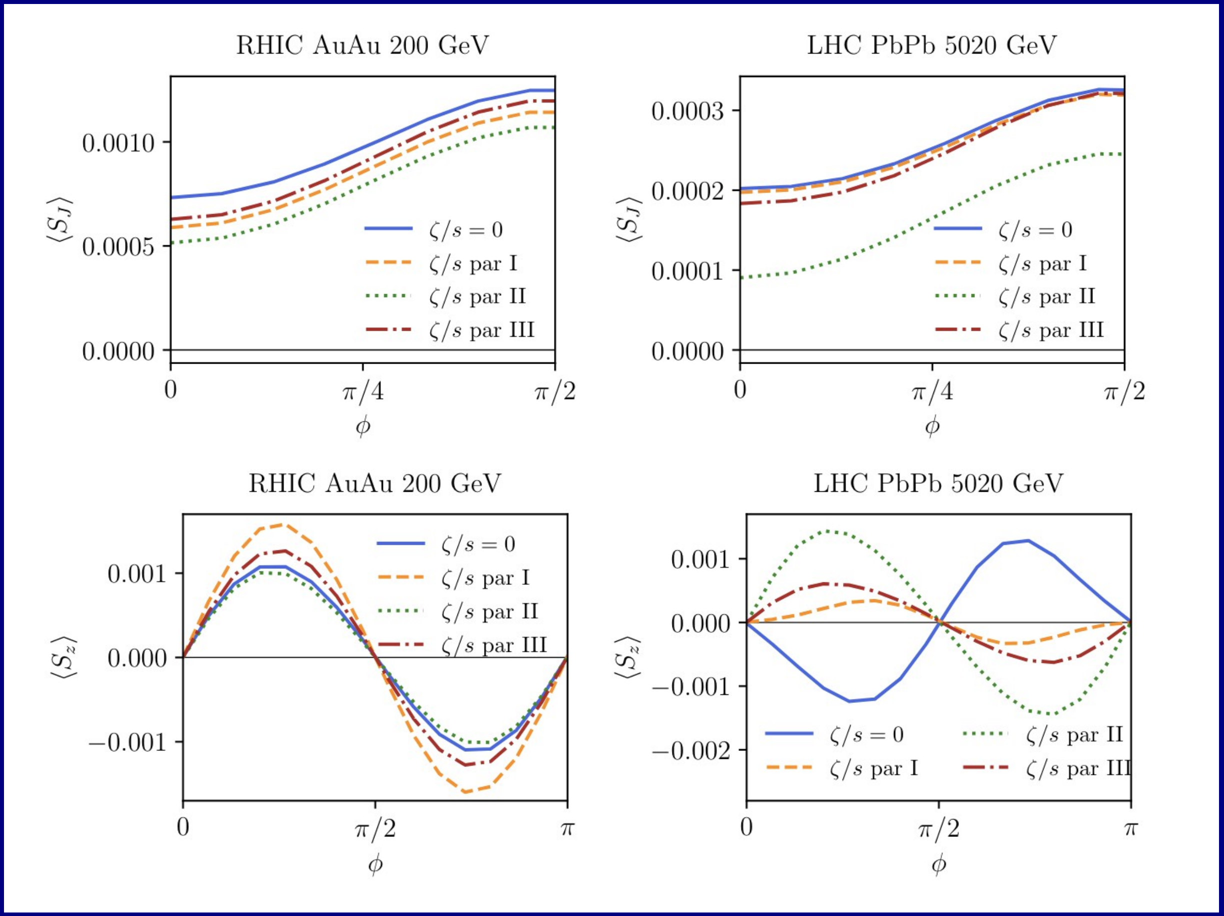




Sensitivity of polarization to bulk viscosity

While polarization seems not to depend much on shear viscosity, it turns out to be very sensitive to bulk viscosity at the highest LHC energy





Spin and pseudo-gauge invariance

F. W. Hehl, Rept. Math. Phys. 9 (1976) 55 (see also F. B., L. Tinti Phys. Rev. D 84 (2011) 025013)

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha \left(\widehat{\Phi}^{\alpha,\mu\nu} - \widehat{\Phi}^{\mu,\alpha\nu} - \widehat{\Phi}^{\nu,\alpha\mu} \right)$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu}$$

They leave the conservation equations and spacial integrals (=generators, or total energy, momentum and angular momentum operators) invariant

EXAMPLE: Belinfante symmetrization

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha \left(\widehat{\mathcal{S}}^{\alpha,\mu\nu} - \widehat{\mathcal{S}}^{\mu,\alpha\nu} - \widehat{\mathcal{S}}^{\nu,\alpha\mu} \right)$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = 0$$

Morale: cannot uniquely separate orbital from spin angular momentum

Free Dirac field:

$$\widehat{T}^{\mu\nu} = \frac{1}{2}\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \Psi$$

$$\widehat{\mathcal{S}}^{\lambda,\mu\nu} = \frac{1}{2}\overline{\Psi}\{\gamma^\lambda, \Sigma^{\mu\nu}\}\Psi = \frac{i}{8}\overline{\Psi}\{\gamma^\lambda[\gamma^\mu, \gamma^\nu]\}\Psi$$

Canonical pseudo-gauge

$$\widehat{T}'^{\mu\nu} = \frac{i}{4}\left[\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \Psi + \overline{\Psi}\gamma^\nu \overleftrightarrow{\partial}^\mu \Psi\right]$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = 0$$

Belinfante pseudo-gauge

Pseudo-gauge dependent LTE

If the spin tensor is non-zero (non-Belinfante) angular momentum constraints must be additionally implemented and we find the local thermodynamic equilibrium density operator

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \zeta \widehat{j}^\mu \right) \right] .$$

$$\Omega_{\lambda\nu} \equiv \text{reduced spin potential}$$

Can to Bel transformation

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu}\beta_\nu - \frac{1}{2}(\Omega_{\lambda\nu} - \varpi_{\lambda\nu})\widehat{\mathcal{S}}^{\mu,\lambda\nu} + \frac{1}{2}\xi_{\lambda\nu}\left(\widehat{\mathcal{S}}^{\lambda,\mu\nu} + \widehat{\mathcal{S}}^{\nu,\mu\lambda}\right) - \zeta \widehat{j}^\mu \right) \right] ,$$

It is the same as the previous (in form)

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu}\beta_\nu - \zeta \widehat{j}^\mu \right) \right] ,$$

only if

$$\varpi = \Omega$$

$$\xi_{\mu\nu} = \frac{1}{2}(\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu) = 0$$

GLOBAL THERMODYNAMIC EQUILIBRIUM

Spin polarization and spin hydrodynamics

There is no direct relation between spin tensor and spin of the particles.
Spin polarization can be non-vanishing even if there is no spin tensor contributing to the angular momentum current (that is, Belinfante PG).

The spin polarization vector *operator* does NOT depend on the pseudo-gauge

$$S^\mu(p) = \sum_{i=1}^{3} [p]_i^\mu \operatorname{tr}(D^{1/2}(J_i)\Theta(p)) \qquad \Theta(p)_{sr} = \frac{\operatorname{Tr}(\widehat{\rho}\widehat{a}^\dagger(p)_r \widehat{a}(p)_s)}{\sum_t \operatorname{Tr}(\widehat{\rho}\widehat{a}^\dagger(p)_t \widehat{a}(p)_t)}$$

Spin polarization acquires a dependence on the pseudo-gauge choice because the *quantum state* depends on the pseudo-gauge: this is the problem

Local thermodynamic equilibrium density operator:

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \zeta \widehat{j}^\mu \right) \right].$$

$$\Omega_{\lambda\nu} \equiv \text{ reduced spin potential}$$

Spin vector: leading order expressions at LTE

M. Buzzegoli, Phys. Rev. C 105 (2022) 4, 044907

1) Belinfante PG

$$S^\mu_{\rm B}(k) \simeq S^\mu_\varpi(k) + S^\mu_\xi(k);$$

$$S^\mu_\varpi(k) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} k_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\varpi_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}},$$

$$S^\mu_\xi(k) = -\frac{1}{4m}\epsilon^{\mu\lambda\sigma\tau} \frac{k_\tau k^\rho}{\varepsilon_k} \frac{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\hat{t}_\lambda \xi_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}},$$

2) Canonical PG

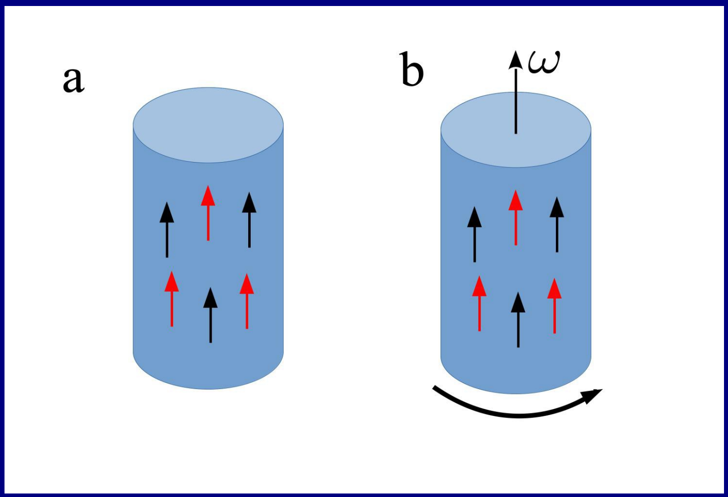
$$S^\mu_{\rm C}(k) \simeq S^\mu_\varpi(k) + S^\mu_\xi(k) + \Delta^{\rm C}_\Theta S^\mu(k).$$

$$\Delta^{\rm C}_\Theta S^\mu(k) = \frac{\epsilon^{\lambda\rho\sigma\tau}\hat{t}_\lambda(k^\mu k_\tau - \eta^\mu_\tau m^2)}{8m\varepsilon_k} \times \frac{\int_\Sigma {\rm d}\Sigma(x) \cdot k\, n_{\rm F}\,(1-n_{\rm F})\,(\varpi_{\rho\sigma} - \Omega_{\rho\sigma})}{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}}.$$

3) GLW-HW PG (spin tensor conserved)

$$S^\mu_{\rm GLW,HW}(k) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} k_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\Omega_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}}.$$

What does the previous LTE physically represent?



a) Non-rotating globally neutral meta-stable state with both particles and anti-particles polarized and zero velocity

b) Global equilibrium state

F. B., W. Florkowski and E. Speranza, Phys. Lett. B 789 (2019), 419-425

M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov and H. U. Yee, JHEP 11 (2021), 150

For a quantum state to represent a), a spin tensor is needed.

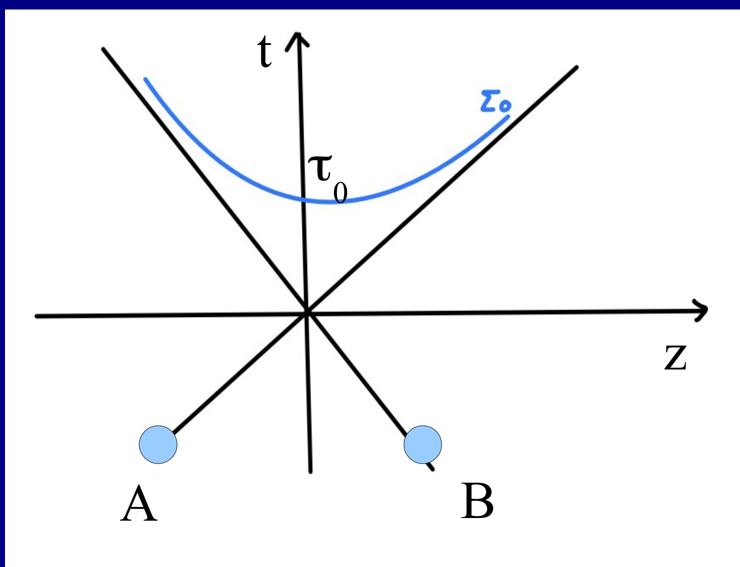
Question: can we prepare a state like a *???*

A pseudo-gauge invariant local equilibrium state

F. B., C. Hoyos, arXiv:2506.xxxx

The actual quantum state (in the Heisenberg picture) of the system are the two colliding nuclei:

$|P_A\rangle|P_B\rangle$ this is a pseudo-gauge invariant quantum state



Evolving the initial quantum state in the Schroedinger Picture

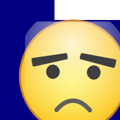
$$\lim_{t\to-\infty} \exp[-i\widehat{H}(\tau_0 - t)]|P_A\rangle|P_B\rangle$$

still yields a pseudo-gauge invariant state

Basic (tacit) assumption in heavy ion collisions:

$$\lim_{t\to-\infty} \exp[-i\widehat{H}(\tau_0 - t)]|P_A\rangle|P_B\rangle\langle P_B|\langle P_A|\exp[i\widehat{H}(\tau_0 - t)] \Rightarrow \widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma_0} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \zeta\widehat{j}^\mu\right)\right].$$

Pseudo-gauge invariant state



Non pseudo-gauge invariant state

We need to find a Local Thermodynamic Equilibrium State which is PG invariant!

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\varpi_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \xi_{\lambda\nu}\widehat{\mathcal{S}}^{\lambda,\mu\nu}\right)\right]$$

This is invariant under:

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha\left(\widehat{\Phi}^{\alpha,\mu\nu} - \widehat{\Phi}^{\mu,\alpha\nu} - \widehat{\Phi}^{\nu,\alpha\mu}\right)$$
$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu}$$

- It is the most general expression linear in the stress-energy tensor and spin tensor which is PG invariant

- It implies that LTE requires the reduced spin potential to be equal to the thermal vorticity

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_\Sigma {\rm d}\Sigma_\mu \widehat{T}^{\mu\nu}_B\beta_\nu\right]$$

Pseudo-gauge transformations of currents

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \zeta \widehat{j}^\mu \right) \right] .$$

$$\widehat{j}'^\mu = \widehat{j}^\mu + \partial_\lambda \widehat{M}^{\lambda\mu}$$

The LE quantum state depends on the PG unless ζ =const

Example for the Dirac field

$$\bar\Psi \gamma^\mu \Psi \to \bar\Psi \gamma^\mu \Psi + C \partial_\lambda \left(\bar\Psi [\gamma^\lambda, \gamma^\mu] \Psi \right)$$

QED is a gauge theory which is not invariant under a PG transformation of the currents.
For instance, the Hamiltonian, integrating the gauge-invariant stress-energy tensor,
is not PG invariant

$$\widehat{T}^{\mu\nu} = \frac{i}{4} \bar\Psi \gamma^\mu \overleftrightarrow{\partial}^\nu \Psi - \frac{q}{2} \bar\Psi \gamma^\mu \Psi \widehat{A}^\nu + (\mu \leftrightarrow \nu) \equiv \widehat{T}^{\mu\nu}_F - \frac{1}{2} \left(\widehat{j}^\mu \widehat{A}^\nu + \widehat{j}^\nu \widehat{A}^\mu \right)$$

In fact, there is a measurable field which is not invariant under a PG transformation of the current:

$$\partial_\lambda \widehat{F}^{\lambda\nu} = \widehat{j}^\nu$$

and singles out a UNIQUE current operator: $\bar\Psi \gamma^\mu \Psi$

Summary

• Calculation of spin polarization corrections at local equilibrium (non-dissipative contribution) at first and second-order in the gradient expansion

• Numerical results in hydrodynamic simulation up to 1^{st} order terms show the high sensitivity of spin polarization to bulk viscosity; something to be taken advantage of to determine bulk viscosity of the QGP

• Pseudo-gauge invariant local thermodynamic equilibrium state

Shear and bulk viscosity of the QGP

Measuring the shear and bulk viscosity of the Quark Gluon Plasma is one of the most important objectives

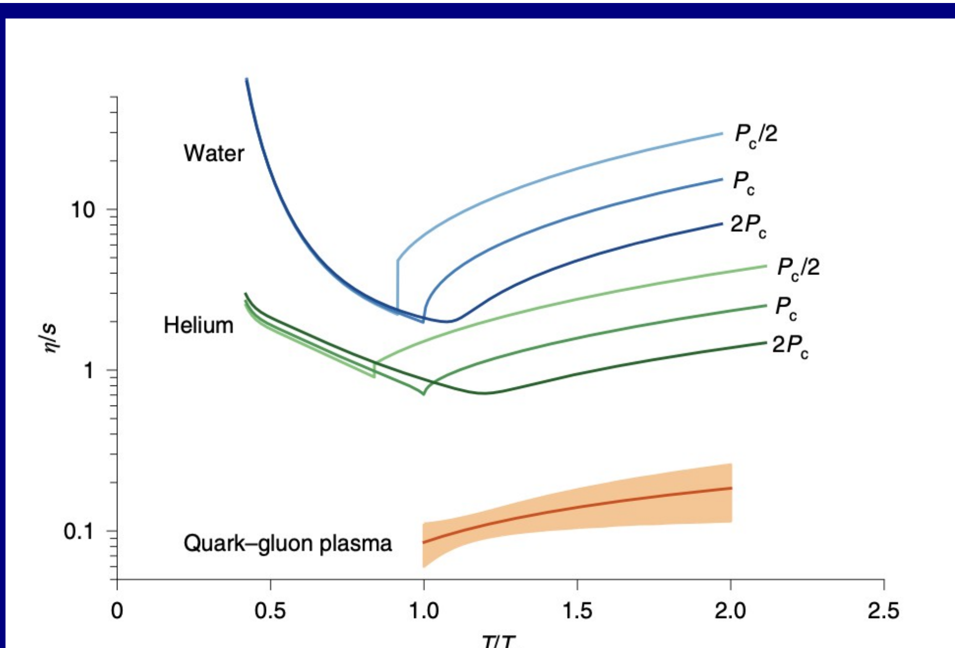
nature physics

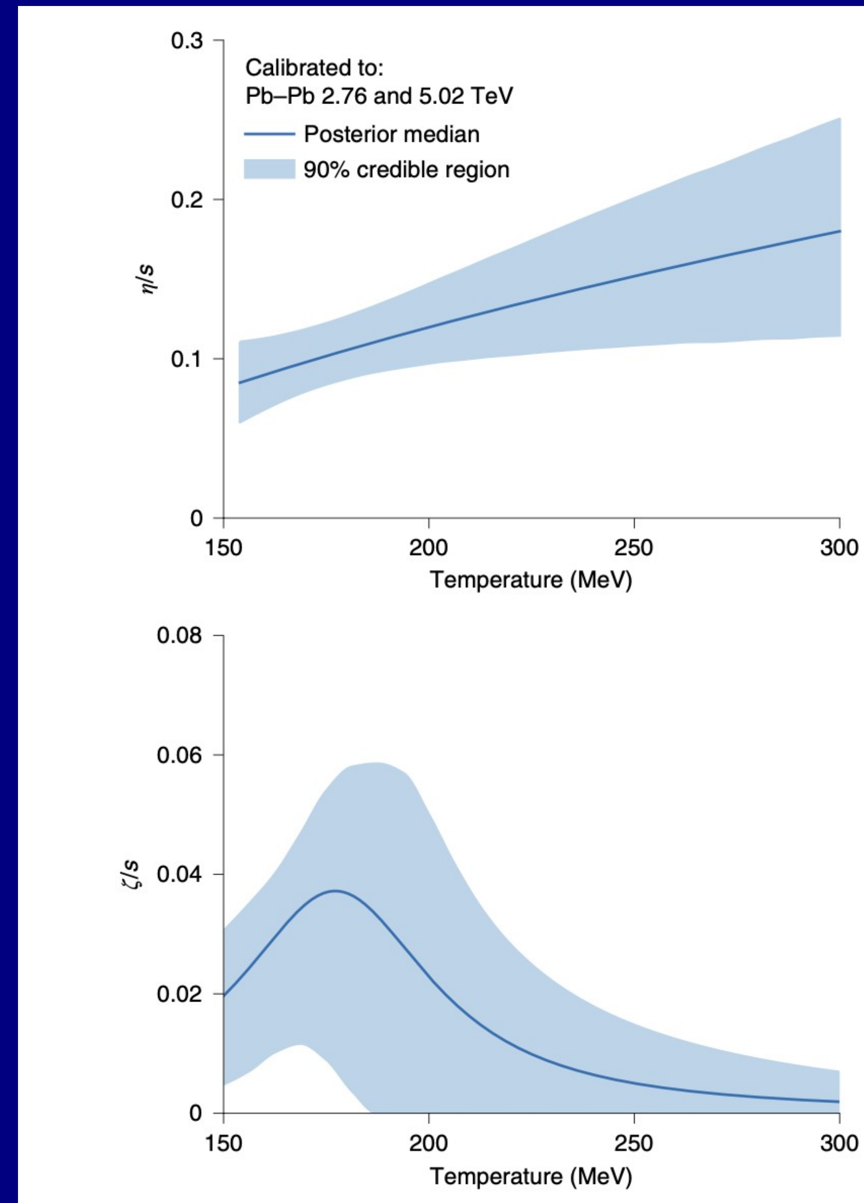
LETTERS

https://doi.org/10.1038/s41567-019-0611-8

Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma

Jonah E. Bernhard*, J. Scott Moreland and Steffen A. Bass





Fit by using momentum-related observables

Spin polarization and spin hydrodynamics

In QFT in flat space-time, from translational and Lorentz invariance one obtains two conserved Noether currents:

$$\widehat{T}^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi^a)} \partial^\nu \Psi^a - g^{\mu\nu} \mathcal{L}$$

$$\widehat{\mathcal{S}}^{\lambda,\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\lambda \Psi^a)} D^A (J^{\mu\nu})^a_b \Psi^b$$

$$\partial_\mu \widehat{T}^{\mu\nu} = 0$$

$$\partial_\lambda \widehat{\mathcal{J}}^{\lambda,\mu\nu} = \partial_\lambda \left(\widehat{\mathcal{S}}^{\lambda,\mu\nu} + x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} \right) = \partial_\lambda \widehat{\mathcal{S}}^{\lambda,\mu\nu} + \widehat{T}^{\mu\nu} - \widehat{T}^{\nu\mu} = 0$$

However, the Lagrangian density can be changed and so, are those tensors objectively defined? (well known problem already for the EM stress-energy tensor)

Why do we have a dependence on Σ ?

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int_{\Sigma_{FO}} \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

The divergence of the integrand of $J^{I\ K}$ vanishes, therefore it does not depend on the integration hypersurface (it is a constant of motion) and

$$\widehat{\Lambda} \widehat{J}_x^{\mu\nu} \widehat{\Lambda}^{-1} = \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{J}_x^{\alpha\beta}$$

The divergence of the integrand of $Q^{I\ K}$ does not vanish, therefore it does depend on the integration hypersurface and

$$\widehat{\Lambda} \widehat{Q}^{\mu\nu} \widehat{\Lambda}^{-1} \neq \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{Q}^{\alpha\beta}$$

Exact formulae at global equilibrium

Resummation of the power series expansion in (constant) thermal vorticity

A. Palermo, F.B., Eur. Phys. J. Plus 138 (2023) 6, 547

Global equilibrium density operator

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} \right]$$

$$\langle \widehat{O} \rangle = \mathrm{Tr}\left(\widehat{\rho} \widehat{O} \right)$$

$$\beta^\mu(x) = b^\mu + \varpi^{\mu\nu} x_\nu \equiv \frac{u^\mu}{T}$$

1) Analytic continuation to imaginary thermal vorticity

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu - \frac{i}{2} \phi_{\mu\nu} \widehat{J}^{\mu\nu} \right]$$

3) Factorization of the density operator

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\widetilde{b}_\mu(\phi)\widehat{P}^\mu\right] \exp\left[-i\frac{\phi_{\mu\nu}}{2}\widehat{J}^{\mu\nu}\right] \equiv \frac{1}{Z} \exp\left[-\widetilde{b}_\mu(\phi)\widehat{P}^\mu\right]\widehat{\Lambda}$$

$$\widetilde{b}^\mu(\phi) = \sum_{k=0}^{\infty} \frac{1}{(k+1)!} \underbrace{(\phi^\mu{}_{\alpha_1}\phi^{\alpha_1}{}_{\alpha_2}\ldots\phi^{\alpha_{k-1}}{}_{\alpha_k})}_{k \text{ times}} b^{\alpha_k}$$

4) TEV of creation/annihilation quadratic combination obtained by iterations

$$\langle \widehat{a}^\dagger_s(p)\widehat{a}_t(p')\rangle = 2\varepsilon' \sum_{n=1}^{\infty} (-1)^{2S(n+1)} \delta^3(\Lambda^n \boldsymbol{p} - \boldsymbol{p}') D^S(W(\Lambda^n, p))_{ts} \mathrm{e}^{-\widetilde{b}\cdot\sum_{k=1}^n \Lambda^k p}$$

$$W(\Lambda, p) = [\Lambda p]^{-1}\Lambda[p]$$

5) Calculate the Wigner function

$$W(x,k) = \frac{1}{(2\pi)^3} \int \frac{\mathrm{d}^3 p}{2\varepsilon} \sum_{n=1}^{\infty} (-1)^{n+1} \mathrm{e}^{-n\tilde{\beta}(in\phi)\cdot p} \times$$
$$\left[\mathrm{e}^{-in\frac{\phi:\Sigma}{2}}(m + \not{p})\delta^4\left(k - (\Lambda^n p + p)/2\right) + (m - \not{p})\mathrm{e}^{in\frac{\phi:\Sigma}{2}}\delta^4\left(k + (\Lambda^n p + p)/2\right)\right]$$

Spin vector

$$S^\mu(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}\left(\gamma^\mu\gamma_5 W_+(x,p)\right)}{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}\left(W_+(x,p)\right)}$$

$$S^\mu(p) = \frac{1}{2m}\frac{\sum_{n=1}^{\infty}(-1)^{n+1}\mathrm{e}^{-n\widetilde{b}(in\phi)\cdot p}\mathrm{tr}\left(\gamma^\mu\gamma_5\mathrm{e}^{-in\frac{\phi:\Sigma}{2}}\not{p}\right)\delta^3(\Lambda^n p - p)}{\sum_{n=1}^{\infty}(-1)^{n+1}\mathrm{e}^{-n\widetilde{b}(in\phi)\cdot p}\mathrm{tr}\left(\mathrm{e}^{-in\frac{\phi:\Sigma}{2}}\right)\delta^3(\Lambda^n p - p)}$$

The series can be resummed:

$$S^\mu(p) = \frac{-i\xi^\mu}{2\sqrt{-\xi^2}}\frac{\sin\left(\sqrt{-\xi^2}/2\right)}{\cos\left(\sqrt{-\xi^2}/2\right) + \mathrm{e}^{-b\cdot p+\zeta}}$$

and analytically continued

$$S^\mu(p) = \frac{1}{2}\frac{\theta^\mu}{\sqrt{-\theta^2}}\frac{\sinh\left(\frac{\sqrt{-\theta^2}}{2}\right)}{\cosh\left(\frac{\sqrt{-\theta^2}}{2}\right) + e^{-b\cdot p+\zeta}}$$

$$\xi^\mu \mapsto \theta^\mu = -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\varpi_{\nu\rho}p_\sigma$$

Extending the formula to local equilibrium with ϖ(x)

$$S^\mu(p) = -\frac{1}{4m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int \mathrm{d}\Sigma\cdot p\, n_F \frac{\varpi_{\nu\rho}}{\sqrt{-\theta^2}}\, \frac{\sinh\left(\sqrt{-\theta^2}/2\right)}{\cosh\left(\sqrt{-\theta^2}/2\right)+\mathrm{e}^{-b\cdot p+\zeta}}}{\int \mathrm{d}\Sigma\cdot p\, n_F}$$