Nuclear deformation effects of vector mesons in UPC
AND
New nonlinear response of anisotropic flow to eccentricity

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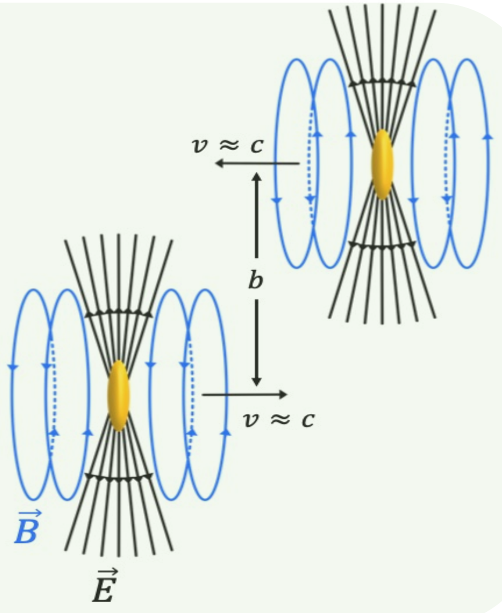
Outline

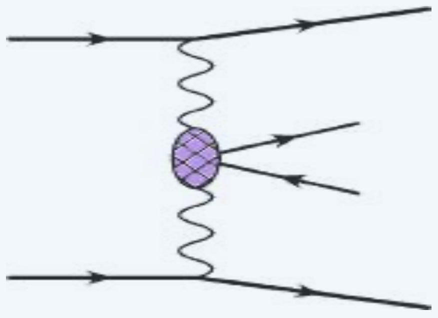
- **Nuclear deformation effects in UPC**
- **New nonlinear response of anisotropic flow to eccentricity (in hydrodynamics)**
- **Summary**

Nuclear deformation effects in UPC

Lin, Hu, Xu, SP, Wang, PRD (2024)

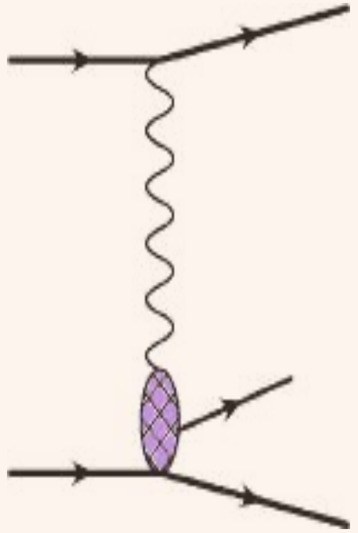
Ultra-Peripheral Collisions





Photon-photon interactions

+



Photon-nucleus interactions

Ann. Rev. Nucl. Part. Sci. 55, 271 (2005)

- **Ultra-Peripheral Collisions (UPC):**

  The impact parameter is larger than 2 times the radius of a nucleus

- **Since the QCD effects are higher orders and QED effects are enhanced by the Ze, UPC provides a nice platform to study the strong EB effects.**

  $Z\alpha \approx 1$ → High photon density

- **Because the relativity, the photon (EB fields) are almost real. Photon-photon, photon-nuclear interactions**

Measure mass distribution

- **Charge distribution:**

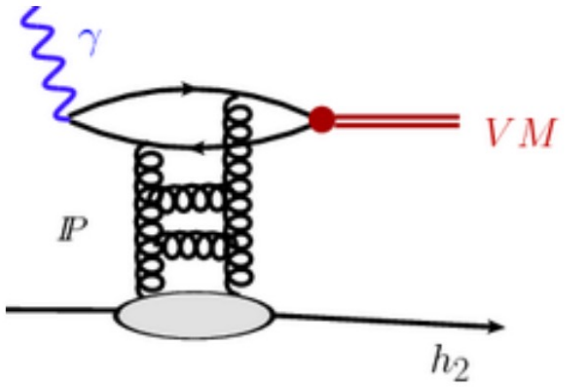
$$\gamma + A \rightarrow \gamma + A$$

to get the form factor $F(q^2)$ and the charge radius

$$\sqrt{< r^2 >} = -6 \lim_{q^2 \rightarrow 0} \frac{\partial F(q^2)}{\partial q^2}$$

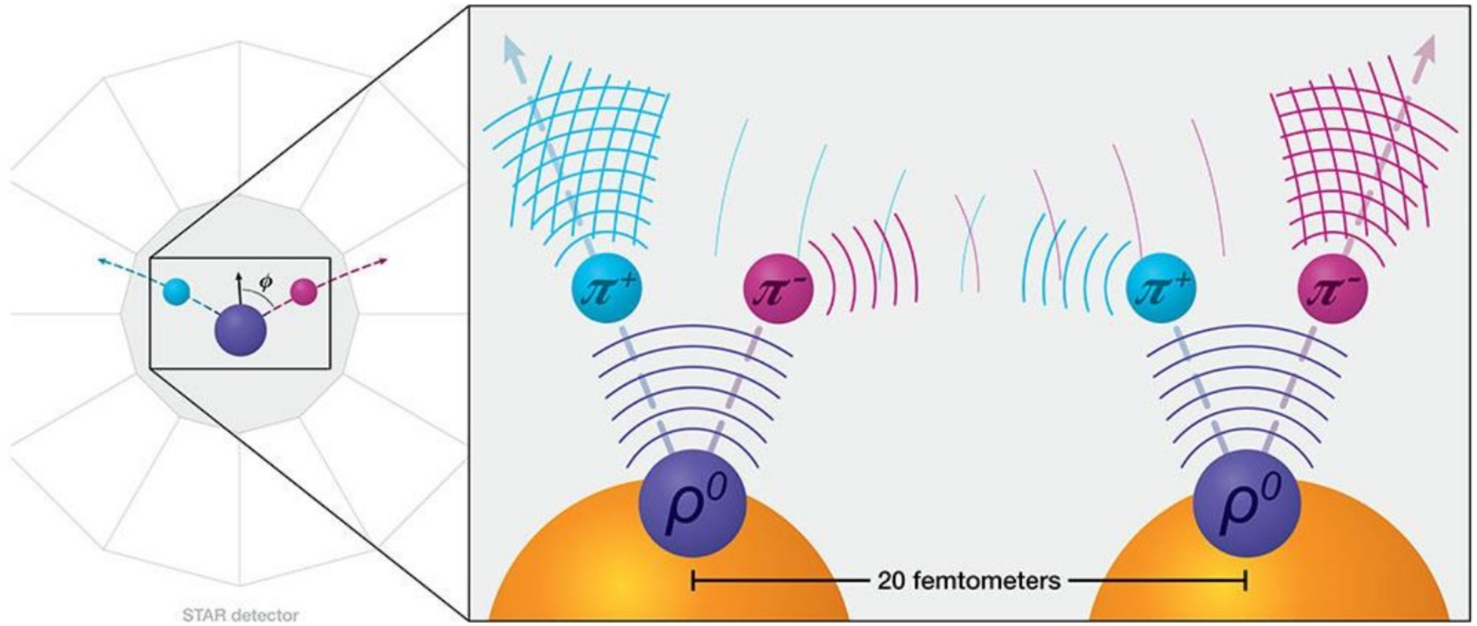
- **Mass distribution:**

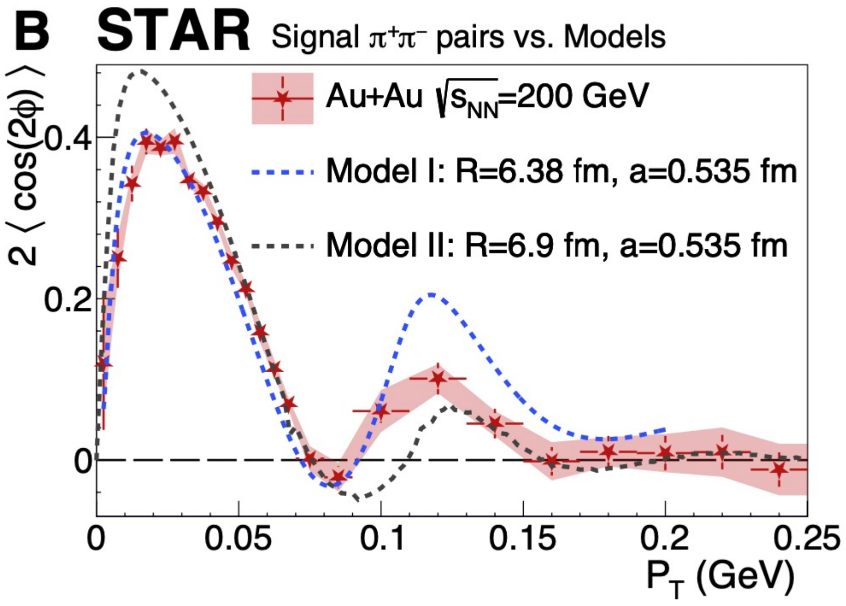
$$\gamma + A \rightarrow A + \rho^0,$$
$$\rho^0 \rightarrow \pi^+ + \pi^-$$



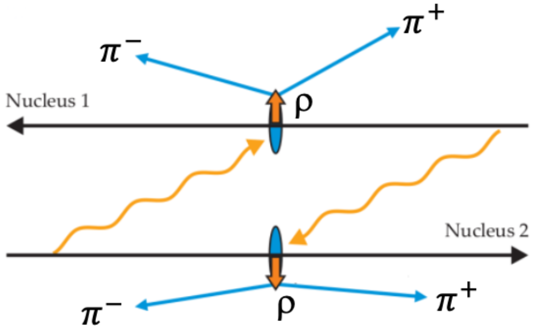
The cross section includes the information of mass distribution.

Double-slit experiment in Fermi scale



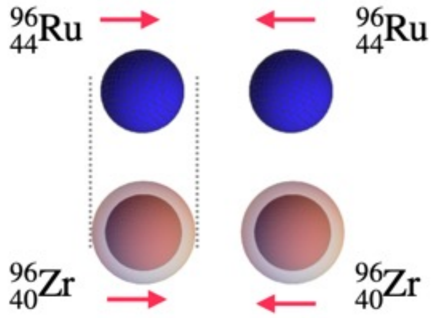


STAR: Sci. Adv. 9, abq3903 (2023)

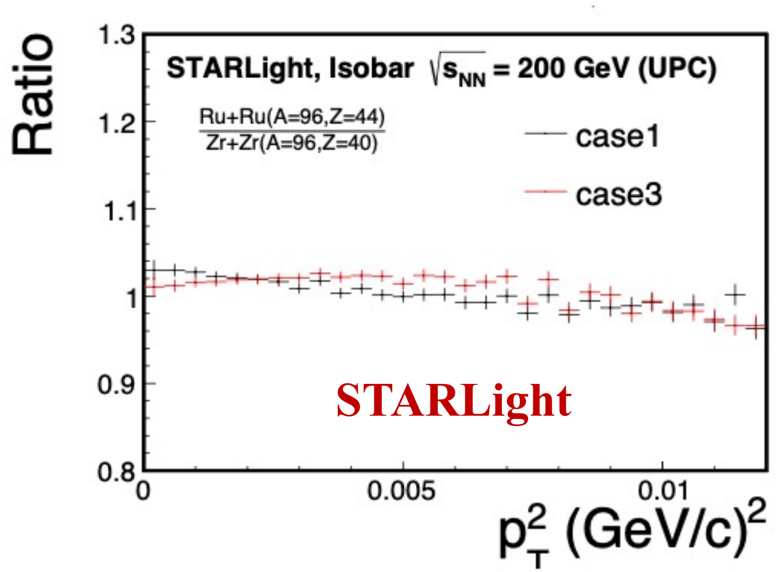


- Azimuthal asymmetries $\cos(2\phi)$ in diffractive vector meson production in UPC
  - Model I: Zha, Brandenburg, Ruan, Tang, Xu, PRD 2021
  - Model II: Xing, Zhang, Zhou, Zhou, JHEP 2020
- For $\cos(\phi)$ and $\cos(3\phi)$ related to ρ^0: Hagiwara, Zhang, Zhou, Zhou, PRD 2021
- Also see studies for J/ψ: Brandenburg, Xu, Zha, Zhang, Zhou, Zhou, PRD 2022

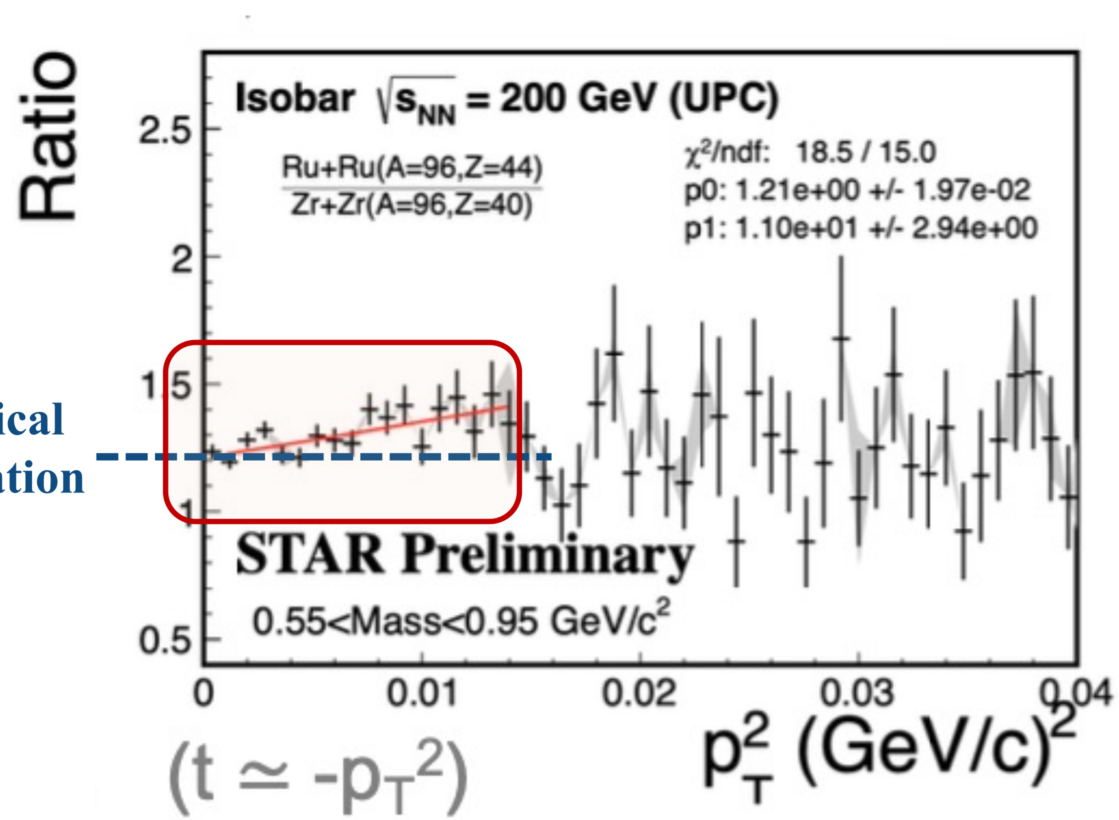
Puzzle in isobaric collisions



$$\gamma + A \rightarrow A + \rho^0,$$
$$\rho^0 \rightarrow \pi^+ + \pi^-$$



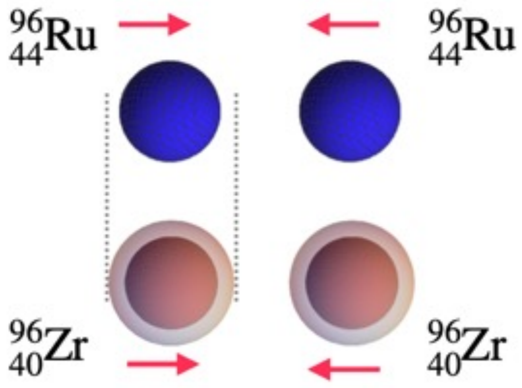
Theortical expectation 1.2



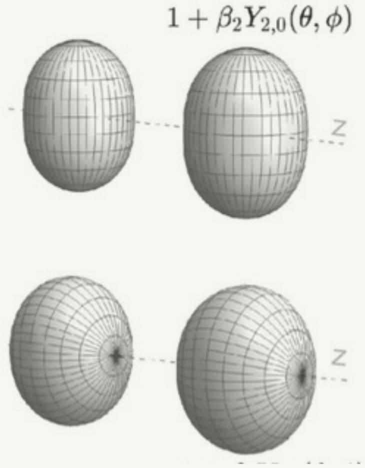
Theortical expectation:

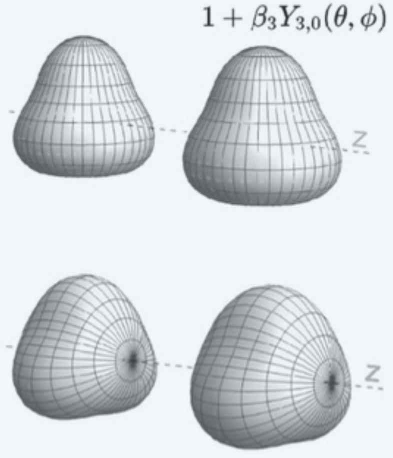
Without deformation effects, Ru and Zr are almost the same, therefore the ratio of the differential cross section for Ru+Ru an Zr+Zr should be a constant and equal to $(44/40)^2$=1.2

Nuclear deformation effects



Ru





Zr

$$\rho(r,\theta,\phi)=\frac{\rho_0}{1+\exp\{[r-R_0(\theta,\phi)]/a\}},\qquad R_0(\theta)=R\left[1+\beta_2 Y_{2,0}(\theta)+\beta_3 Y_{3,0}(\theta)+\ldots\right],$$

(a) with deformation	R	a	β_2	β_3
Ru	5.093 fm	0.471 fm	0.16	0
Zr	5.021 fm	0.517 fm	0	0.20

PRC 105, 014905 (2022)

Theoretical model

- For simplicity, we take the **equivalent photon approximation**

$$\sigma = \int dy[n_1(y)\sigma_{\gamma A_2 \to \rho A_2}(y) + n_2(-y)\sigma_{\gamma A_1 \to \rho A_1}(-y)],$$

- We implement the **dipole model** to study this process

Ryskin, Z. Phys. C 57, 89 (1993); Brodsky, Frankfurt, Gunion, Mueller, Strikman, PRD 50, 3134 (1994); Kowalski, Teaney, PRD 68, 114005 (2003); Kowalski, Motyka, Watt, PRD 74, 074016 (2006)

$$\frac{d\sigma^{\gamma A \to \rho^0 A}}{dq_T^2} = \frac{1}{16\pi}|\mathcal{A}|^2$$

In the amplitude A, we input the the nuclear mass distribution to the thickness function.

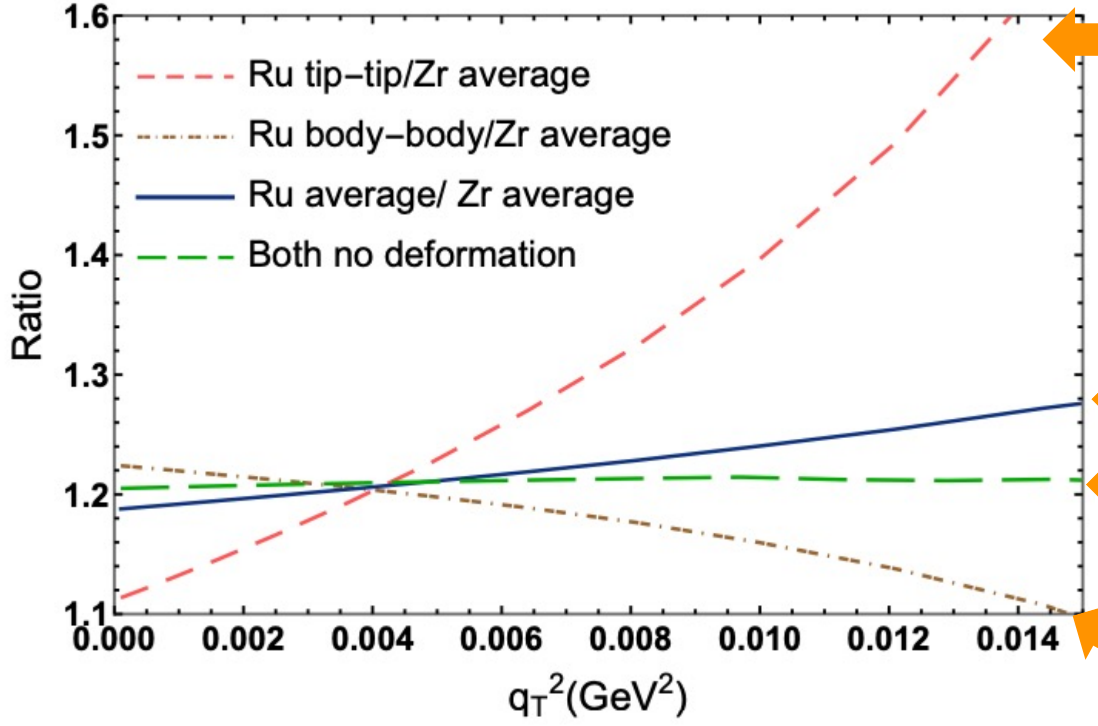
$$T_A(\mathbf{b}_T) = \int dz \boxed{\rho(z, \mathbf{b}_T)}$$

$$\rho(r,\theta,\phi) = \frac{\rho_0}{1 + \exp\{[r - R_0(\theta,\phi)]/a\}},$$

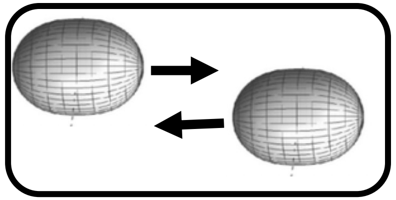
Also see more comprehensive studies beyond EPA in:
Mäntysaari, B. Schenke, C. Shen, and W. Zhao, PRL 131, 062301 (2023)

Results

Nuclear deformation effect is the key to understand the data!



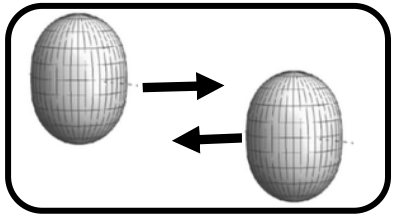
Lin, Hu, Xu, SP, Wang, PRD (2024)



Tip-tip collisions

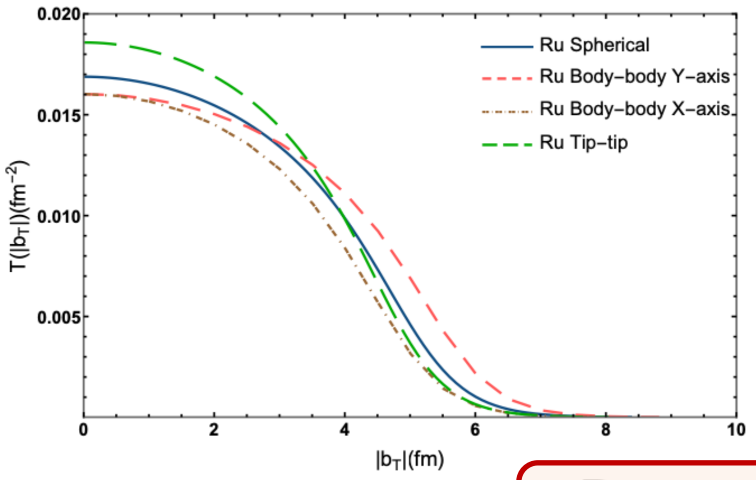
With deformation

Without deformation



Body-body collisions

How the nuclear deformation influence the ratio?



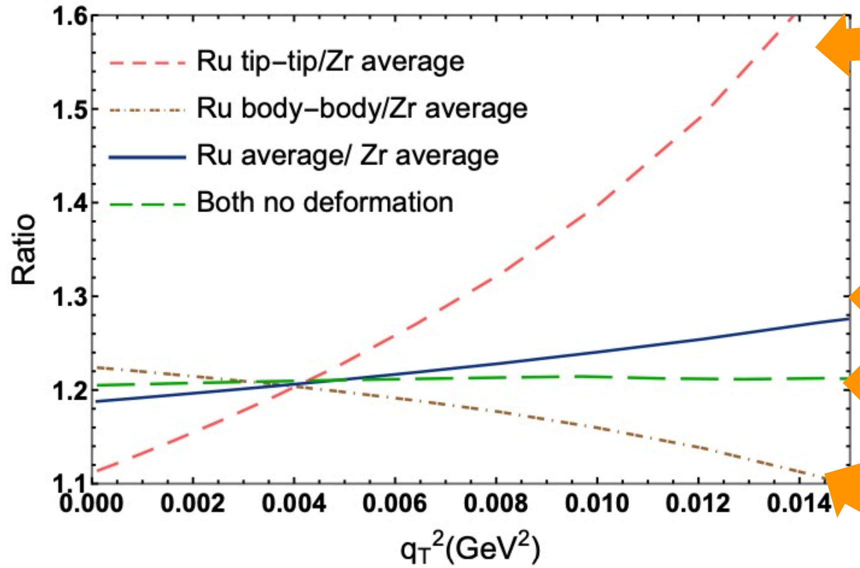
We parameterize the thickness function as

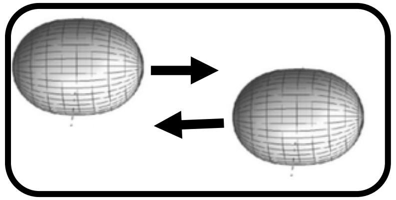
$$T_A(\mathbf{b}_T) \sim \exp\left(-\frac{\mathbf{b}_T^2}{\boxed{w_T^2}}\right)$$

where w_T is a parameter that can be interpreted as the effective width of the nucleus.

$$\frac{d\sigma_{\mathrm{Ru}}/dq_T^2}{d\sigma_{\mathrm{Zr}}/dq_T^2} \propto e^{\delta w_T q_T^2}, \qquad \delta w_T \equiv -\frac{1}{2}\left[(w_T^{\mathrm{Ru}})^2 - (w_T^{\mathrm{Zr}})^2\right]$$

$$w_{T,\mathrm{tip}}^{\mathrm{Ru}} < w_{T,\mathrm{spherical}}^{\mathrm{Ru}} \approx w_{T,\mathrm{spherical}}^{\mathrm{Zr}} < w_{T,\mathrm{body}}^{\mathrm{Ru}}.$$

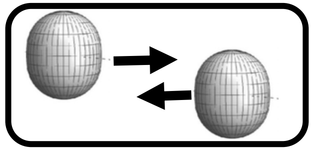




Tip-tip collisions

With deformation

Without deformation



Body-body collisions

New nonlinear response of anisotropic flow to eccentricity (in preparation)

Xiang Ren, Jin-Yu Hu and Hao-Jie Xu

Eccentricity vs anisotropic flow

- **Eccentricity**

$$\epsilon_n \equiv -\frac{\int r^n e^{in\phi}\epsilon(r,\phi) r dr d\phi}{\int r^n \epsilon(r,\phi) r dr d\phi}$$

- **Anisotropic flow**

$$(2\pi)^3 \frac{dN}{dY p_T dp_T d\phi_p} = A_0 \left[1 + \sum_{n=1}^{\infty} 2v_n(p_T)\cos n\phi\right]$$

$$v_2 \propto \epsilon_2$$

$$v_4 = \boxed{\chi} v_2^2 + v_{4L} \propto \epsilon_2^2 + \ldots$$

How to understand this non-linear response from the hydrodynamics analytically?

Conservation equations in hydrodynamics

- **Energy-momentum conservation equation**

$$\partial_\mu T^{\mu\nu} = 0$$

- **Constitution equation**

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \pi^{\mu\nu}$$
$$\pi^{\mu\nu} = -\Pi g^{\mu\nu} + 2\eta_s \sigma^{\mu\nu}$$

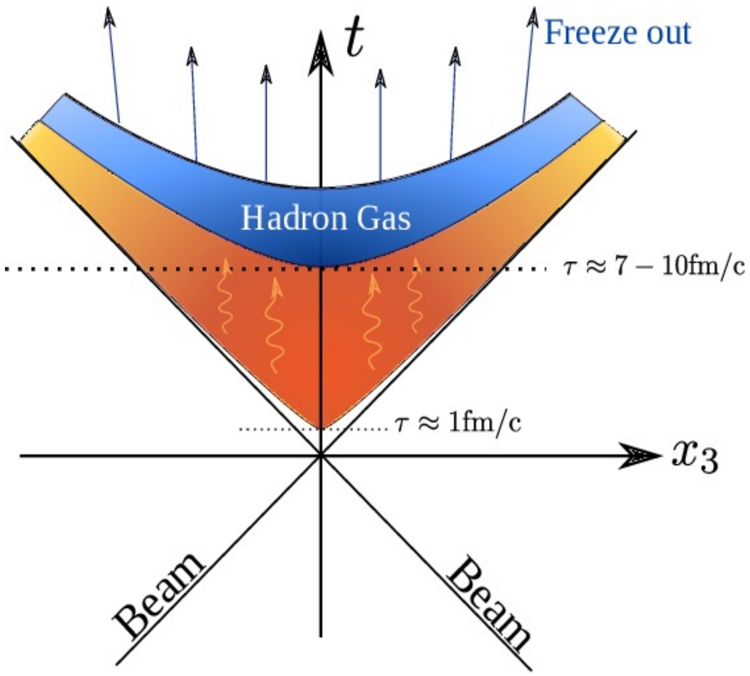
- **Equation of state**

$$P = P(\epsilon)$$

Pressure as a function of energy density

Bjorken flow

- **Bjorken flow:** **a well-known 0+1 dimensional analytic solution for the relativistic hydrodynamics with longitudinal boost invariance**



- **Physical picture:**
  - **All the particles are generated at the same time in the rest frame of the fluid cell.**
  - **The system has longitudinal boost invariance, all thermodynamic variables depend on proper time only.**

- **Solutions**
  - **in (t, x, y, z) coordinates:**

    Scaling law for Energy density $$\frac{e(t)}{e(t=0)} = \frac{1}{\tau^{4/3}}$$

    τ: proper time

    Fluid velocity $$u^{\mu} = \left(\frac{t}{\tau}, 0, 0, \frac{z}{\tau}\right)$$

  - **in (τ, x, y, η) coordinates (η space rapidity):**

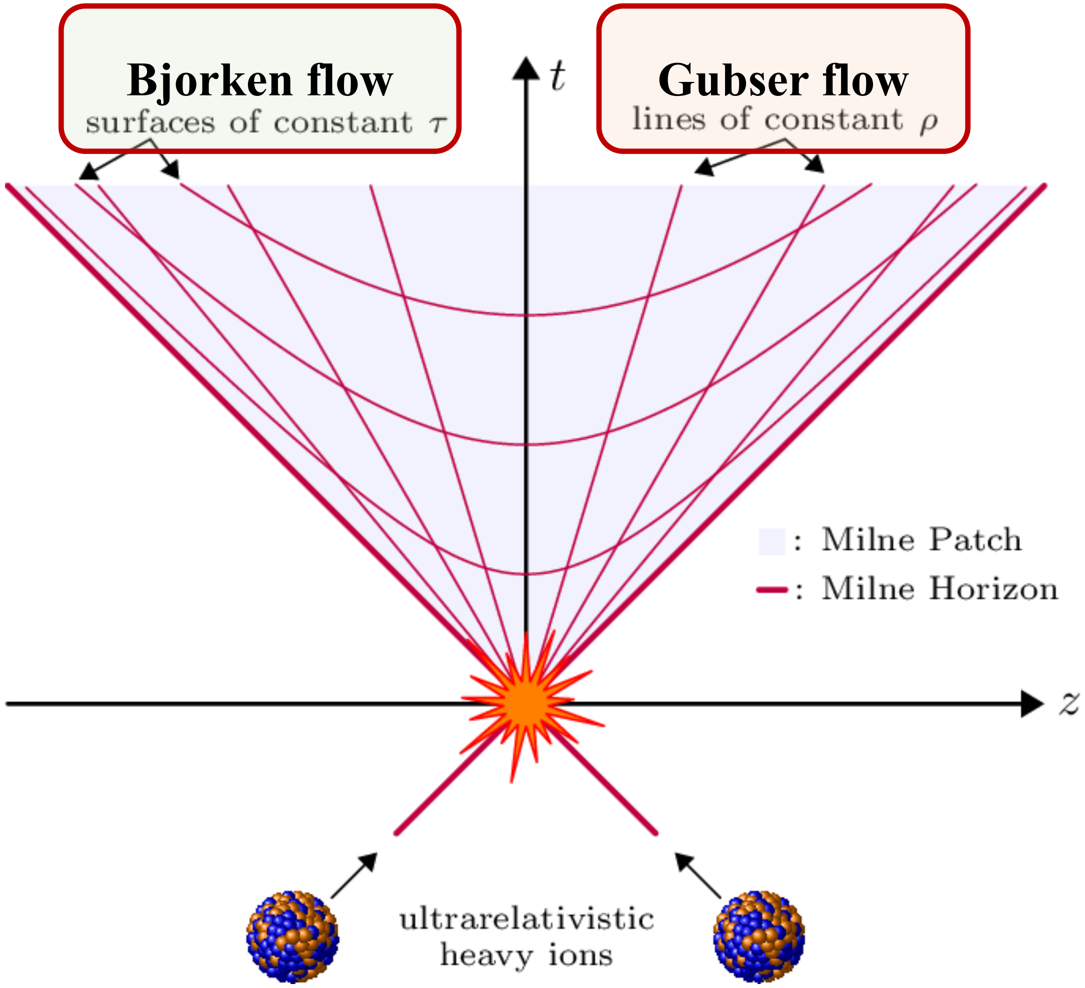
    $$u^{\mu} = (1, 0, 0, 0)$$

Bjorekn flow: the fluid cell is at rest in (τ, x, y, η) coordinates

No radial flow in a Bjorken flow!

Gubser flow (I)

- **Gubser flow:** a 1+1 dimensional analytic solution for relativistic hydrodynamics with **transverse flows**



- **Motivation:**
  Extended Bjorken flow with radial flow

- **Basic idea:**
  Search for the analytic solution as $u^{\mu} = (1,0,0,0)$ in special coordinates?

- **Answer:**
  Taking conformal (Weyl) transformation

Gubser, PRD (2010); Gubser, Yarom, NPB (2011)

Gubser flow (II)

- **Gubser flow:** **a 1+1 dimensional analytic solution for relativistic hydrodynamics with transverse flows**

$$(\tau, x_\perp, \phi, \eta)$$

τ: proper time
$x_\perp$: transverse length
ϕ: azimuthal angle in transverse plane
η: space rapidity

Scaling law for Energy density

$$\varepsilon = \frac{CL^{8/3}}{\tau^{4/3}} \frac{1}{\left[L^4 + (x_\perp^2 - \tau^2)^2 + 2L^2(x_\perp^2 + \tau^2)\right]^{4/3}}$$

C: integration constant
L: size of the system

Fluid velocity

$$u_\mu = \left(-\frac{1}{\cosh\rho}\frac{L^2 + x^2 + \tau^2}{2L\tau}, \boxed{-\frac{1}{\cosh\rho}\frac{x_\perp}{L}}, 0, 0\right)$$

$$\cosh\rho = \frac{1}{2L\tau}\left[L^4 + (x_\perp^2 - \tau^2)^2 + 2L^2(x_\perp^2 + \tau^2)\right]^{1/2}$$

The radial flow survive! But, no ϕ dependence!

Perturbation to Gubser flows

- **Perturbation in temperature:**

$$T_0 \to T = T_0[1 + \boxed{\delta(\rho) S(\Theta, \phi)}],$$

Accordingly, from the conservation equations, one can find that the fluid velocity becomes,

$$\begin{aligned} u^\mu &\to u^\mu + \delta u^\mu \\ \delta u^\mu &= \left(0, \frac{2L\tau}{L^2 + x_\perp^2} \nu_s(\rho) \boxed{\partial_\Theta S(\Theta, \phi)}, \tau \nu_s(\rho) \boxed{\partial_\phi S(\Theta, \phi)}, 0\right) \\ &\quad + \mathcal{O}(\tau^2) \end{aligned}$$

ϕ dependence is introduced through the perturbations

where

$$\begin{aligned} \delta &= \delta_0 + \delta_0 \frac{\eta_0}{2\lambda C} (2e^\rho)^{-2/3} + \mathcal{O}(e^{2\rho}), \\ \nu_s &= -\frac{3}{2}\delta_0 + k_0 \frac{\eta_0}{\lambda C} (2e^\rho)^{-2/3} + \mathcal{O}(e^{2\rho}), \end{aligned}$$

Hatta, Noronha, Torrieri, Xiao PRD (2014); Hatta, Monnai, Xiao, PRD (2015)

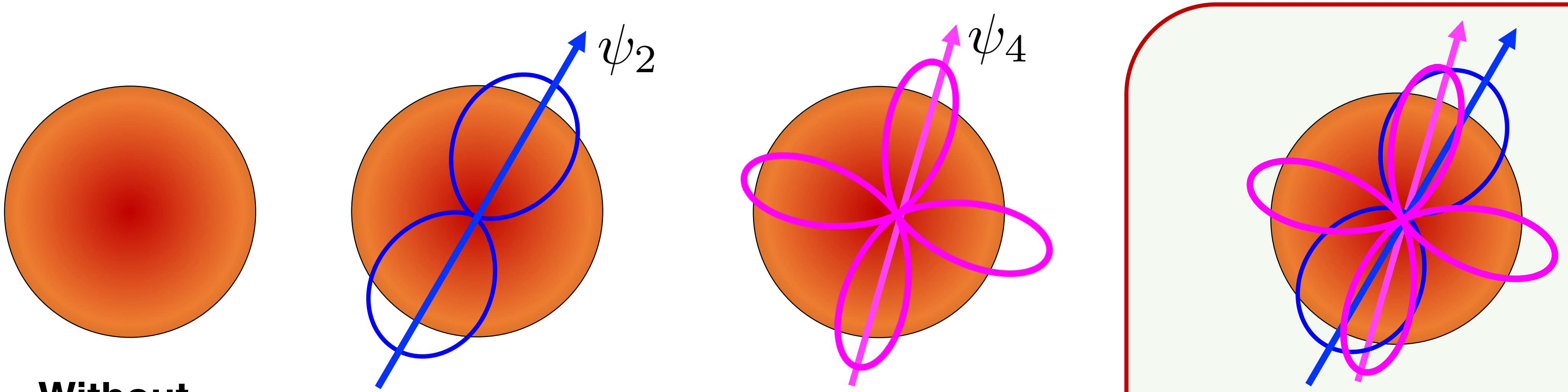
Perturbation and eccentricity (I)

- **Fluctuations:** **In this work, we introduce the fluctuations as**

$$S(\Theta,\phi)=a_2\left(\frac{2x_\perp L}{L^2+x_\perp^2}\right)^2\cos 2\phi+a_4\left(\frac{2x_\perp L}{L^2+x_\perp^2}\right)^4\cos 4\phi,$$

For simplicity, here we set $\boldsymbol{\psi_2=\psi_4=0}$

- **The parameters $\boldsymbol{a_2}$ and $\boldsymbol{a_4}$ are related to eccentricity,**



Without fluctuations

Finite a2 → ϵ_2

Finite a4 → ϵ_4

Finite a2 and a4 → **Coupling between a2 and a4 contributes to ϵ_4**

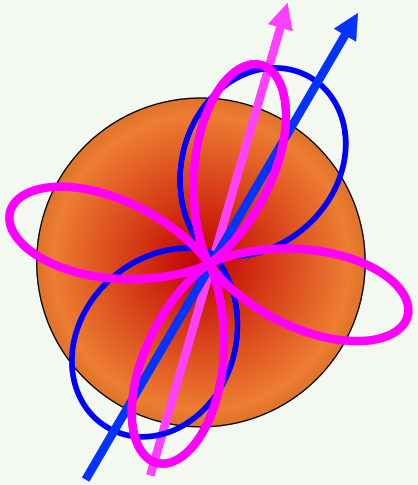
Perturbation and eccentricity (II)

- **Fluctuations: In this work, we introduce the fluctuations as**

$$S(\Theta,\phi) = a_2\left(\frac{2x_\perp L}{L^2+x_\perp^2}\right)^2 \cos 2\phi + a_4\left(\frac{2x_\perp L}{L^2+x_\perp^2}\right)^4 \cos 4\phi,$$

For simplicity, here we set $\boldsymbol{\psi_2 = \psi_4} = 0$

- **The parameters $\boldsymbol{a_2}$ and $\boldsymbol{a_4}$ are related to eccentricity,**



Finite a2 and a4

Coupling between a2 and a4 contributes to ϵ_4

$$\epsilon_n \propto -\frac{\int d^2x_\perp \varepsilon^{3/4}\frac{x_\perp^n}{(L^2+x_\perp^2)^{n-1}}\cos n\phi}{\int d^2x_\perp \varepsilon^{3/4}\frac{x_\perp^n}{(L^2+x_\perp^2)^{n-1}}}$$

$$\begin{aligned}
\epsilon_2 &= -a_2 - \frac{24}{35}a_2a_4 + \ldots \\
\epsilon_4 &= -\frac{36}{35}a_4 \boxed{-\frac{18}{35}a_2^2} + \ldots \\
a_2 &= -\epsilon_2 + \mathcal{O}(\epsilon_2^2, \epsilon_4^2, \epsilon_2\epsilon_4), \\
a_4 &= -\frac{35}{36}\epsilon_4 \boxed{-\frac{1}{2}\epsilon_2^2} + \mathcal{O}(\epsilon_2^2, \epsilon_4^2, \epsilon_2\epsilon_4).
\end{aligned}$$

Cooper-Frye formula and v_2, v_4

• Cooper-Frye formula:

For a given freezeout proper time, we insert the temperature and fluid velocity into the Cooper-Frye formula

$$(2\pi)^3 \frac{dN}{dY p_T dp_T d\phi_p} = -\int_\Sigma p^\mu d\Sigma_\mu f\left(\frac{p^\mu u_\mu}{T}\right) = A_0\left[1 + \sum_{n=1}^{\infty} 2v_n(p_T)\cos n\phi\right]$$

• Anisotropic flow:

$$\begin{aligned} v_2 &= A_2\epsilon_2, \\ v_4 &= A_4\epsilon_4 + \boxed{C_4\epsilon_2^2}. \end{aligned}$$

A2, A4, C4 are functions of the p_T, freezeout temperature T and $x_\perp$.

Small and large p_T limits

- **In small p_T limit,**

$$v_2 = \frac{2^{10}p_T^2\left(K_1 m_T + 4K_0 T\right)}{21B^6 K_1 m_T T^2}\epsilon_2,$$

$$v_4 = \frac{2^{16}p_T^4\left(7K_1 m_T + 56K_0 T\right)}{663B^{12} K_1 m_T T^4}\epsilon_4 + \frac{2^{18}p_T^4\left(67K_1 m_T + 549K_0 T\right)}{7735B^{12} K_1 m_T T^4}\epsilon_2^2.$$

$$K_n\left(\frac{m_T}{T}\right) = \int_0^\infty dy \exp\left(-\frac{m_T}{T}\cosh(y-Y)\right)\cdot\cosh(ny-nY)$$

$$\boldsymbol{v_2 / \epsilon_2 \propto p_T^2}$$

- **In large p_T limit,**

$$v_2 \approx \frac{p_T}{2T}\delta u^*_{\perp 2} = \epsilon_2 \frac{p_T}{T}\frac{500\sqrt{5}}{B^3 81}.$$

$$v_4 \approx \frac{1}{2}\left(\frac{p_T}{2T}\right)^2 \delta u^{*2}_{\perp 2} = \frac{1}{2}v_2^2.$$

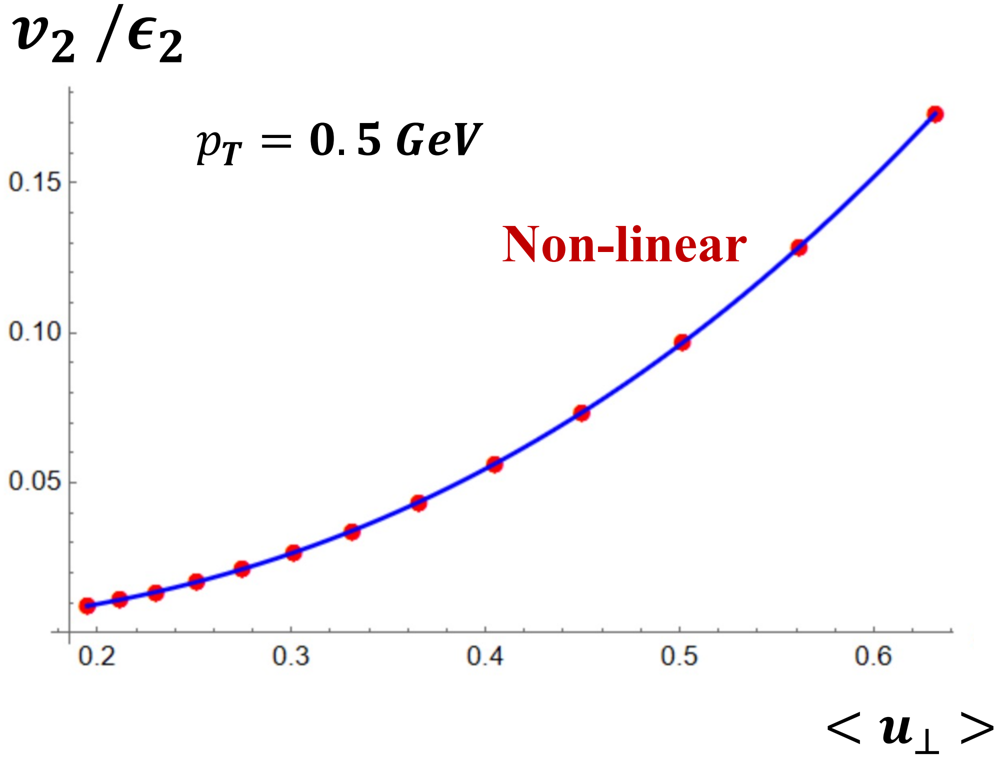
$$\boldsymbol{v_2 / \epsilon_2 \propto p_T}$$

$$\boldsymbol{v_4 / v_2^2 = 1/2}$$

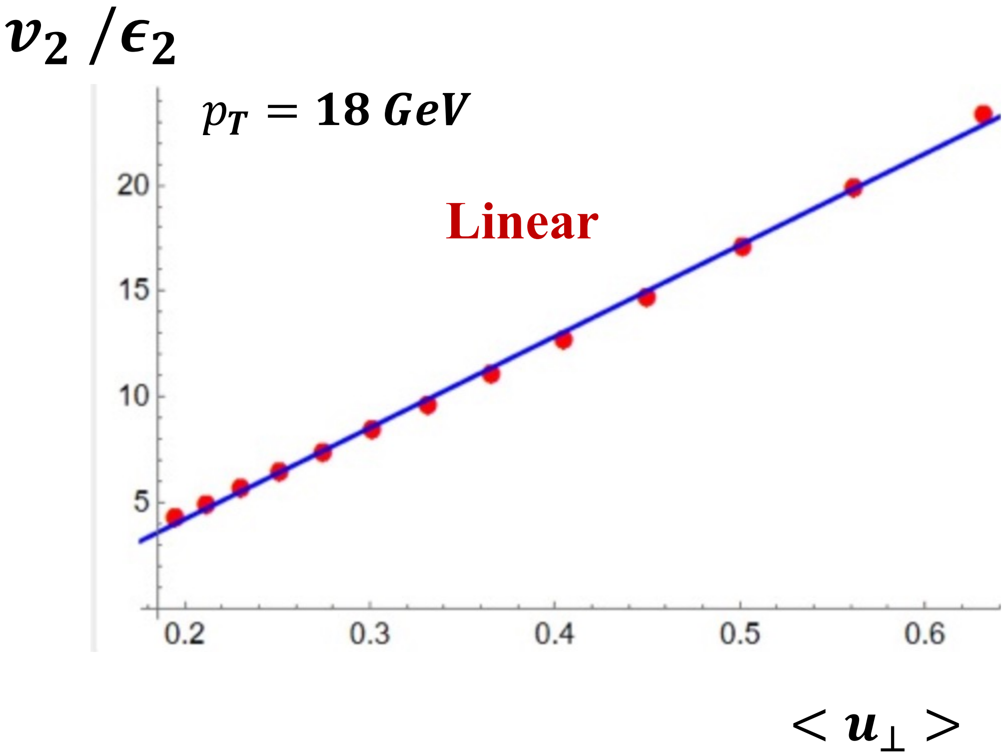
Agree with Teaney, Li, PRC (2011)

v_2/ϵ_2 as function of averaged transverse velocity $< u_\perp >$

Small p_T



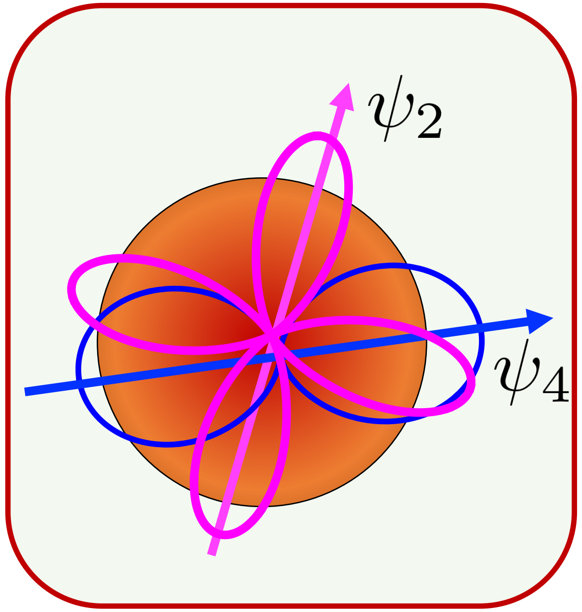
Large p_T

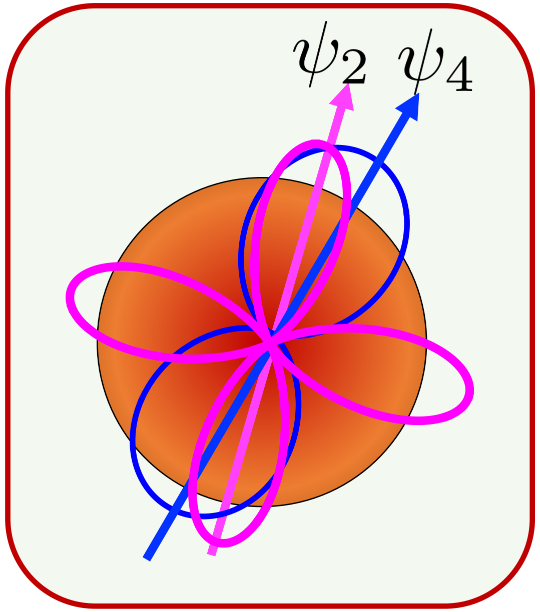


Averaged transverse velocity: $$< u_\perp >= \frac{\int x_\perp dx_\perp d\theta u_\perp}{\int x_\perp dx_\perp d\theta}$$

Effects of different participant and event planes

- **Different participant planes:** **finite and different ψ_2 and ψ_4**





$$\epsilon_2 = -a_2 - \frac{24}{35}a_2 a_4 + \ldots$$

$$\epsilon_4 = -\frac{36}{35}a_4 \boxed{- \frac{18}{35}\cos(\psi_2 - \psi_4)a_2^2} + \ldots$$

- **Introduce the event plane Ψ_2 and Ψ_4 in Cooper-Frye formula:**

$$(2\pi)^3 \frac{dN}{dY p_T dp_T d\phi_p} = -\int_\Sigma p^\mu d\Sigma_\mu f\left(\frac{p^\mu u_\mu}{T}\right) = A_0\left[1 + \sum_{n=1}^{\infty} 2v_n(p_T)\boxed{\cos[n(\phi - \Psi_n)]}\right]$$

Nonlinear response for anisotropic flows

- Ψ_2 and Ψ_4: measured from experiments
- ψ_2 and ψ_4 : correspond to ϵ_2 and ϵ_4

$\psi_2 \approx \Psi_2 \quad \psi_4 \not\approx \Psi_4$

Commonly used in experiments:

$$v_4\{\Psi_4\}^2 = v_{4L}^2 + v_4\{\Psi_2\}^2 \approx v_{4L}^2 + \boxed{\left(\chi v_2^2\{\psi_2\}\right)^2}$$

Our results in Gubser flow:

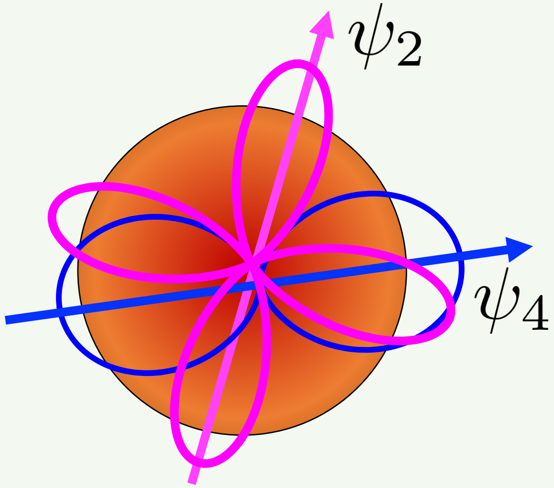
$$v_4\{\Psi_4\} \quad = \quad v_{4L} + \boxed{\chi\cos[4(\psi_2 - \psi_4)]} v_2^2\{\psi_2\}$$

Nonlinear response coefficient is related to the initial participant planes.

Nonlinear response for anisotropic flows

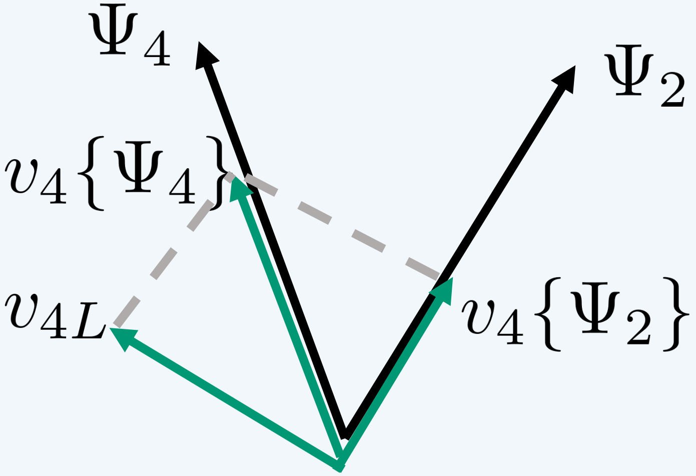
- $\boldsymbol{\Psi_2}$ **and** $\boldsymbol{\Psi_4}$**: measured from experiments**
- $\boldsymbol{\psi_2}$ **and** $\boldsymbol{\psi_4}$ **: correspond to** $\boldsymbol{\epsilon_2}$ **and** $\boldsymbol{\epsilon_4}$

$\boldsymbol{\psi_2 \approx \Psi_2 \quad \psi_4 \not\approx \Psi_4}$



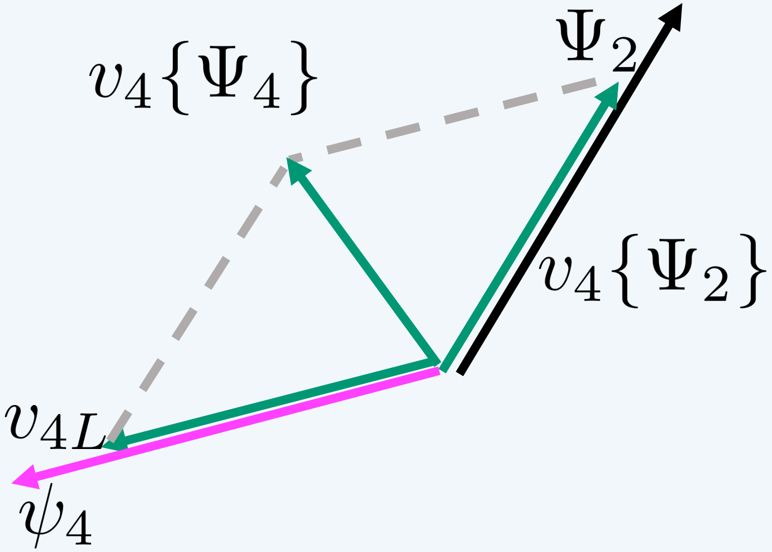
perturbations given by initial conditions, e.g. deformations, fluctuations

Commonly used in experiments:



$$
\begin{aligned}
v_4\{\Psi_4\}^2 &= v_{4L}^2 + v_4\{\Psi_2\}^2 \\
&\approx v_{4L}^2 + \boxed{\chi v_2^2\{\psi_2\}}
\end{aligned}
$$

Our results in Gubser flow:

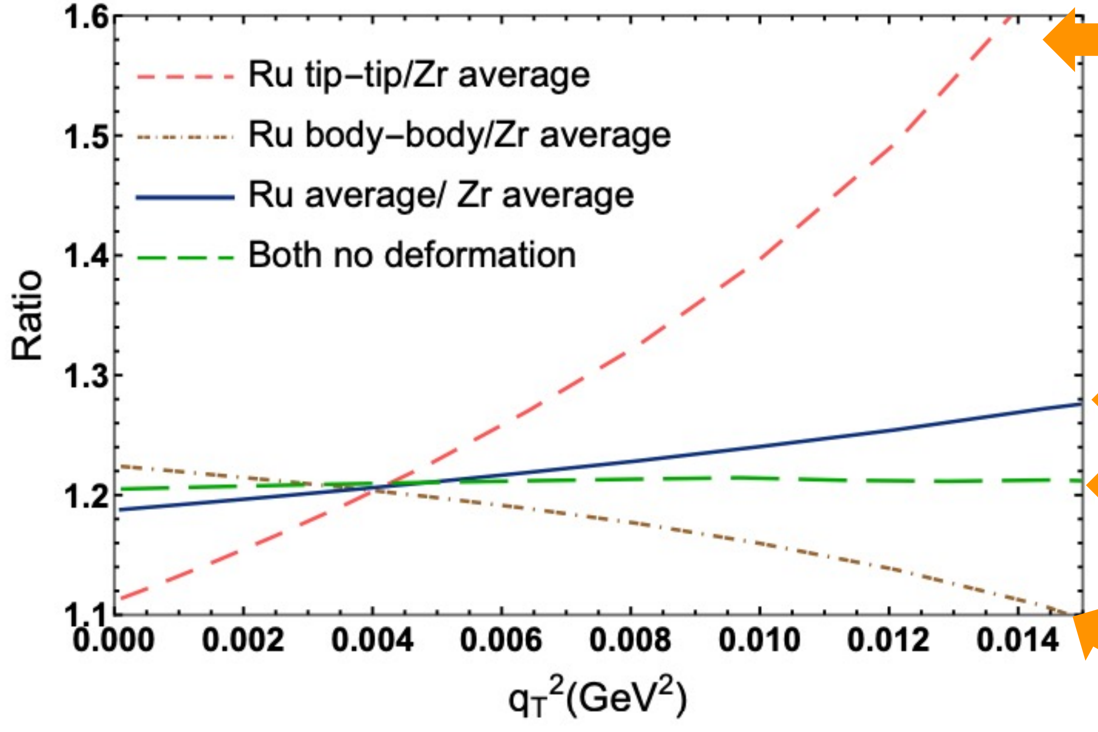


$$
\begin{aligned}
v_4\{\Psi_4\} &= v_{4L} \\
&+\chi \boxed{\cos[4(\psi_2 - \psi_4)]} v_2^2\{\psi_2\}
\end{aligned}
$$

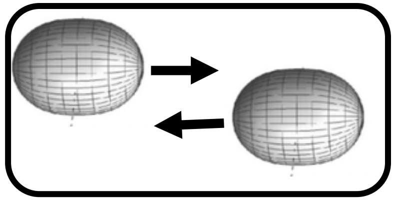
Summary

Summary (I)

- **Nuclear deformation effect is the key to understand photon-nuclear interaction in isobaric collisions.**



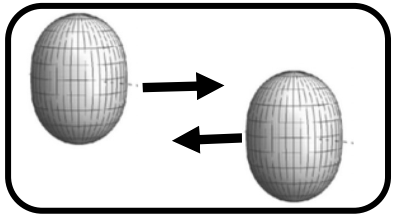
Lin, Hu, Xu, SP, Wang, PRD (2024)



Tip-tip collisions

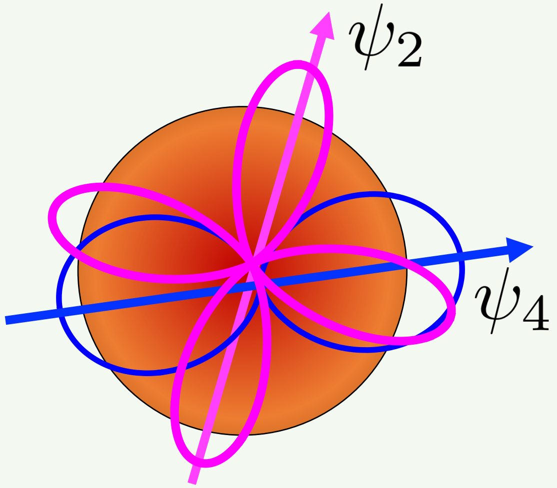
With deformation

Without deformation



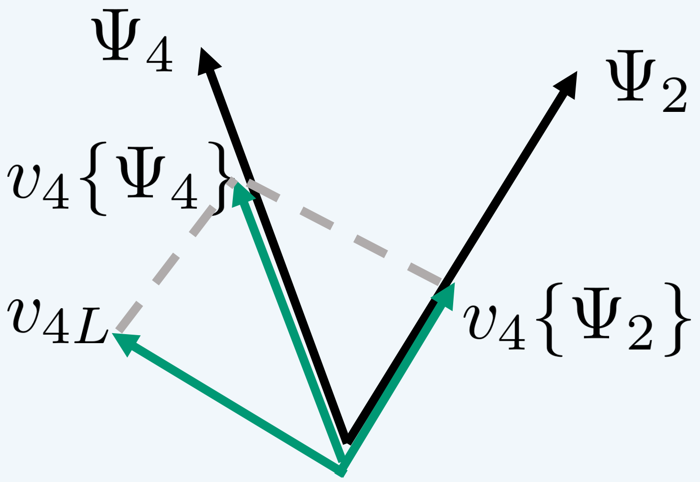
Body-body collisions

Summary (II)



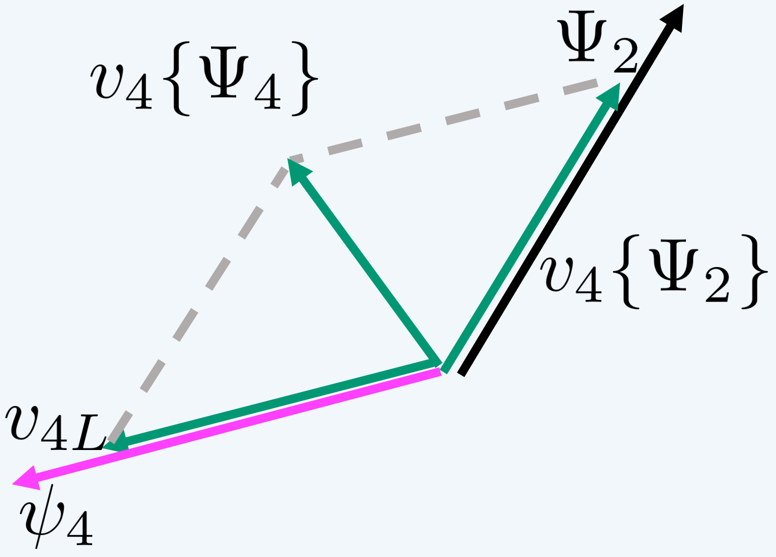
perturbations given by initial conditions, e.g. deformations, fluctuations

Commonly used in experiments:



$$v_4\{\Psi_4\}^2 = v_{4L}^2 + v_4\{\Psi_2\}^2 \approx v_{4L}^2 + \boxed{\chi v_2^2\{\psi_2\}}$$

Our results in Gubser flow:



$$v_4\{\Psi_4\} = v_{4L} + \boxed{\chi \cos 4(\psi_2 - \psi_4)} v_2^2\{\psi_2\}$$

Nonlinear response coefficient is related to the initial participant planes.

Thank you for listening!

Any comments and suggestions are welcome!