Mesons (π, K, η) under external electromagnetic fields

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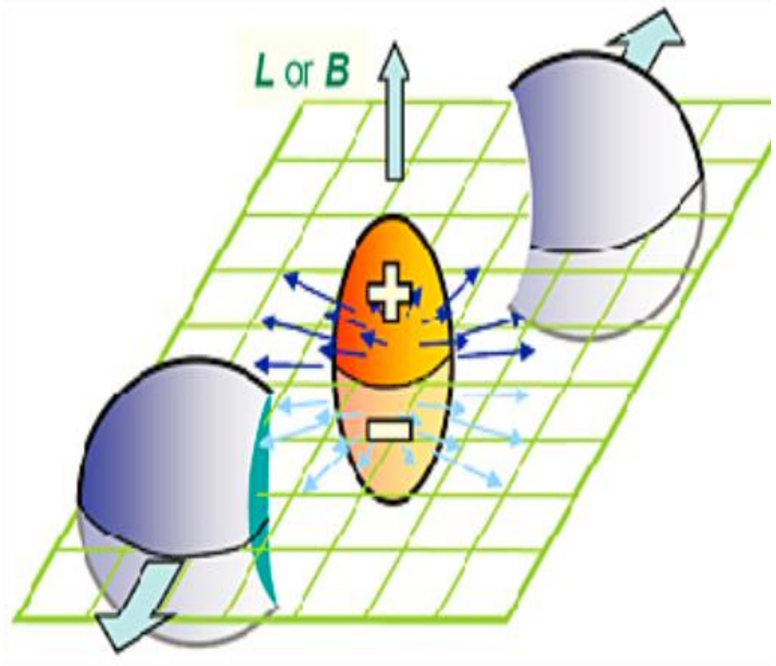
Outline

1 Background

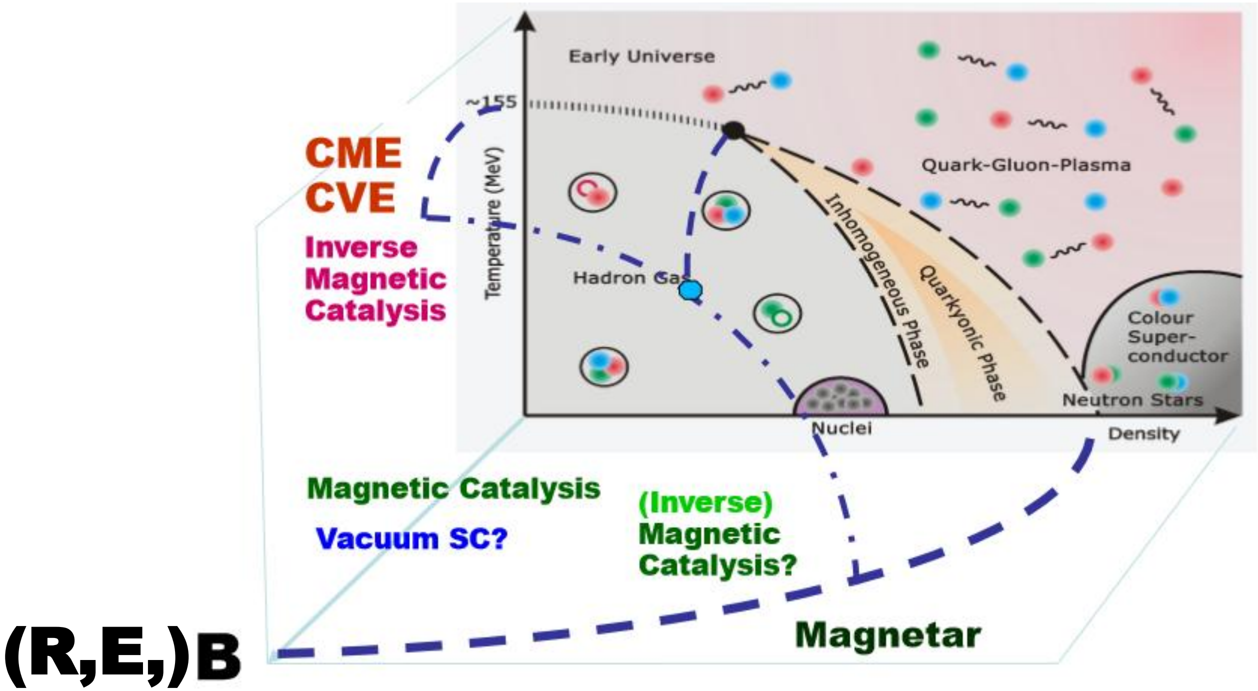
HIC: LHC, RHIC, FAIR,NICA, …



T, density,
E, B, R

$$B, E \sim \gamma \frac{Z\alpha_{EM}}{R_A^2} \sim 10^{18\sim20}\,\text{Gauss}$$

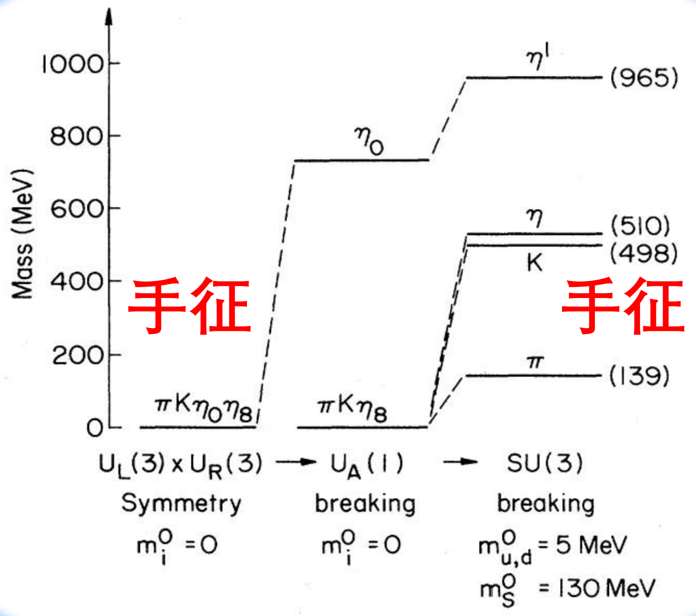
QCD相图

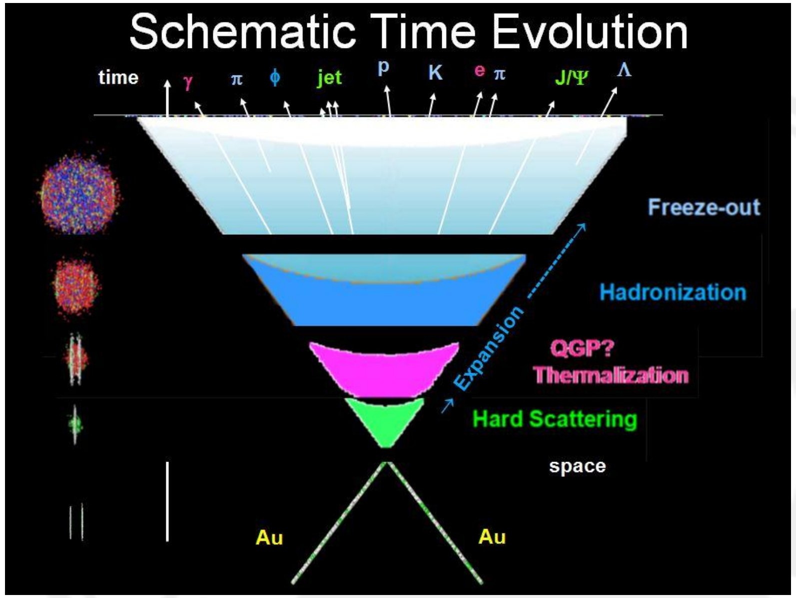


1 强相互作用相变与介子紧密相关

- 朗道相变理论： 连续相变对应于对称性自发破缺/恢复。
- Goldstone定理：连续对称性自发破缺，若不存在长程力，则一定存在零质量的Goldstone玻色子。

※ 利用介子性质研究QCD相变





2 mesons @ eB

2 Mass spectra of meson @ eB 手征有效理论

(P)NJL model beyond mean field

$\mathbf{B} = (0, 0, B)$

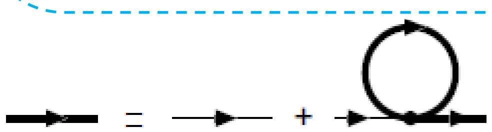
2-flavor $\boxed{\pi}$

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\nu D^\nu - m_0\right)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] - \mathcal{U}(\Phi, \bar{\Phi}),$$

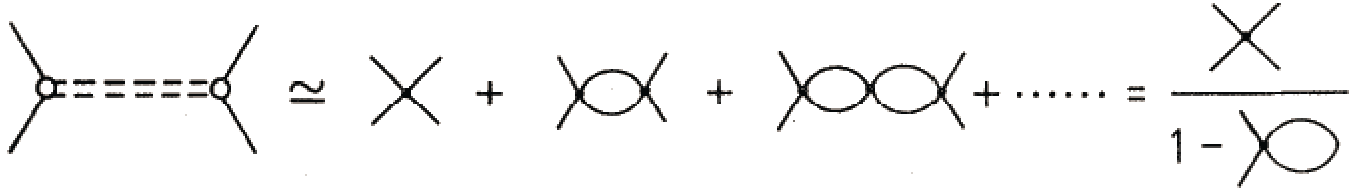
3-flavor $\boxed{K, \pi, \eta, \eta'}$

$$\begin{aligned}
\mathcal{L} &= \bar{\psi}\left(i\gamma^\mu D_\mu - \hat{m}_0\right)\psi + \mathcal{L}_4 + \mathcal{L}_6 - \mathcal{U}(\Phi, \bar{\Phi}), \\
\mathcal{L}_4 &= G\sum_{\alpha=0}^{8}\left[(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2\right], \\
\mathcal{L}_6 &= -K\left[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi\right], \\
\mathcal{U}(\Phi, \bar{\Phi}) &= T^4\left[-\frac{b_2(t)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\bar{\Phi}^3 + \Phi^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2\right].
\end{aligned}$$

(1)Quarks： mean field



(2)**Mesons**: RPA resummation (quantum fluctuation)



S.J. Mao and Y.X. Wang, PRD96, 034004(2017); L.Y. Li and S.J. Mao, PRD108, 054001(2023), arXiv: 2511.14150; J.Mei, T.Xia, S.J. Mao, PRD107,074018(2023);

quark propagator under external magnetic field
---- Ritus propagator

$$\mathbf{B} = (0, 0, B)$$

$$S_f(x,y) = i\sum_{n=0}^{\infty}\int \frac{d\tilde{p}}{(2\pi)^3} e^{-i\tilde{p}\cdot(x-y)} P_n(x_1, p_2) D_f(\bar{p}) P_n(y_1, p_2),$$

g_n 是磁场中Dirac方程的解

$$P_n(z,q) = \frac{1}{2}\left[g_n^{s_f}(z,q) + I_n g_{n-1}^{s_f}(z,q)\right] + \frac{i s_f}{2}\left[g_n^{s_f}(z,q) - I_n g_{n-1}^{s_f}(z,q)\right]\gamma^1\gamma^2,$$

$$D_f^{-1}(\bar{p}) = \gamma\cdot\bar{p} - m, \quad \bar{p} = (p_0, 0, -s_f\sqrt{2|Q_f B|n}, p_3)$$

Landau level

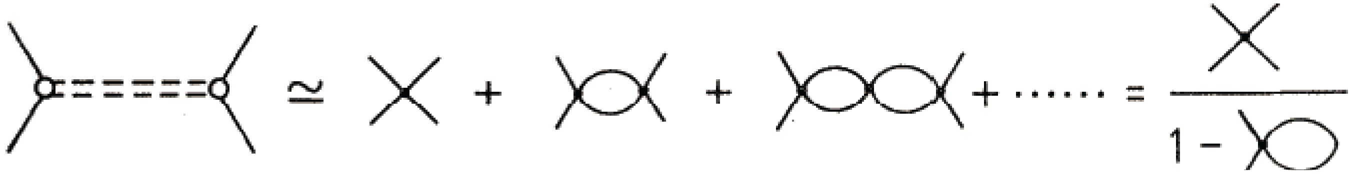
$$\tilde{p} = (p_0, 0, p_2, p_3) \qquad I_n = 1 - \delta_{n0}$$

$$g_n^{s_f}(z,q) = \phi_n(z - s_f q/|Q_f B|)$$

$$\phi_n(z) = (2^n n!\sqrt{\pi}|Q_f B|^{-1/2})^{-1/2} \times e^{-z^2|Q_f B|/2} H_n(z/|Q_f B|^{-1/2})$$

refs: Ann. Phys. 69, 555 (1972); Nucl. Phys. B747, 266 (2006);Ann. Phys. 295, 33 (2002) .

Mesons: RPA resummation



meson propagator:

$$\mathcal{D}_M(x,z)=2G\delta(x-z)+\int d^4y\,2G\Pi_M(x,y)\mathcal{D}_M(y,z),$$

$$\Pi_M(x,y)=i\mathrm{Tr}[\Gamma_M^* S(x,y)\Gamma_M S(y,x)]$$

$$\boxed{\begin{aligned}\mathcal{D}_M(\bar{k}) &= \int d^4x\,d^4y\,F_k^*(x)\mathcal{D}_M(x,y)F_k(y),\\ \Pi_M(\bar{k}) &= \int d^4x\,d^4y\,F_k^*(x)\Pi_M(x,y)F_k(y)\end{aligned}}$$

F_k 是磁场中Klein-Gordon方程的解

with the neutral meson vertex

$$\Gamma_M=\begin{cases} i\gamma_5\lambda_0, & M=\eta_0\\ i\gamma_5\lambda_3, & M=\pi_0\\ i\gamma_5\lambda_8, & M=\eta_8\\ i\gamma_5(\lambda_6\pm i\lambda_7)/\sqrt{2}, & M=K_0,\bar{K}_0\end{cases},$$

and charged meson vertex

$$\Gamma_M=\begin{cases} i\gamma_5(\lambda_1\pm i\lambda_2)/\sqrt{2}, M=\pi^\pm\\ i\gamma_5(\lambda_4\pm i\lambda_5)/\sqrt{2}, M=K^\pm\end{cases}.$$

spectral function

$$\begin{aligned}\rho_M(\omega,\mathbf{0}) &= -2\mathrm{Im}\mathcal{D}_{\pi_0}(\omega+i\epsilon,\mathbf{0})\\ &= -2\mathrm{Im}\left(\frac{2G}{1-2G\Pi_M(\omega+i\epsilon,\mathbf{0})}\right).\end{aligned}$$

pole mass:

$$1-G\Pi_M(m_M,\mathbf{0})=0,$$

- **neutral mesons** K_0, π_0, η, η'

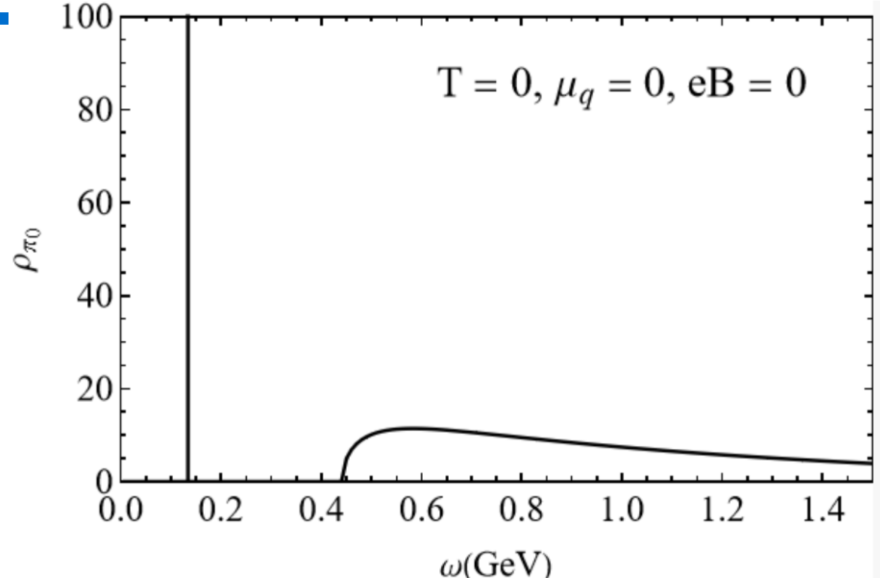
2.1

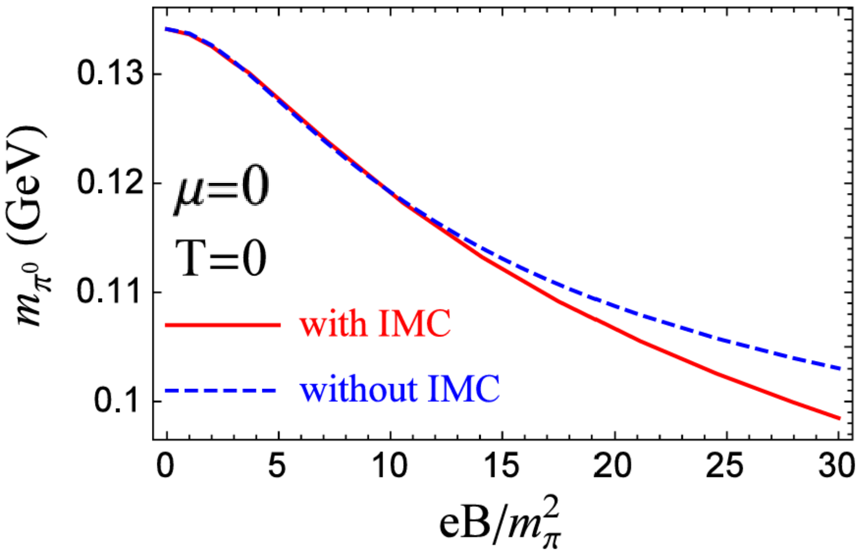
Spectral function of π_0 @ eB

T=0 μ=0

Zhou, **Mao**, arXiv:2025.XXXXX;Li, **Mao**, PRD 108, 054001 (2023)

eB=0

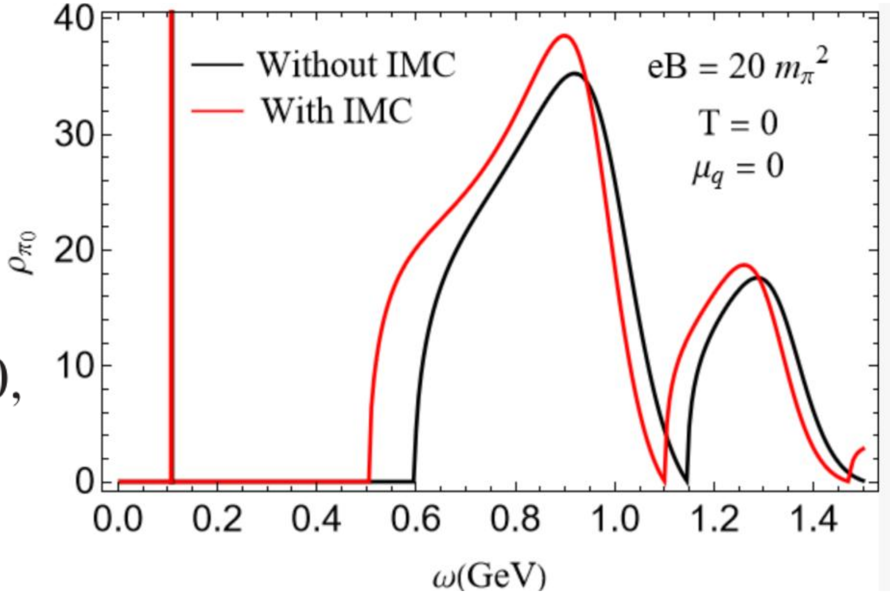


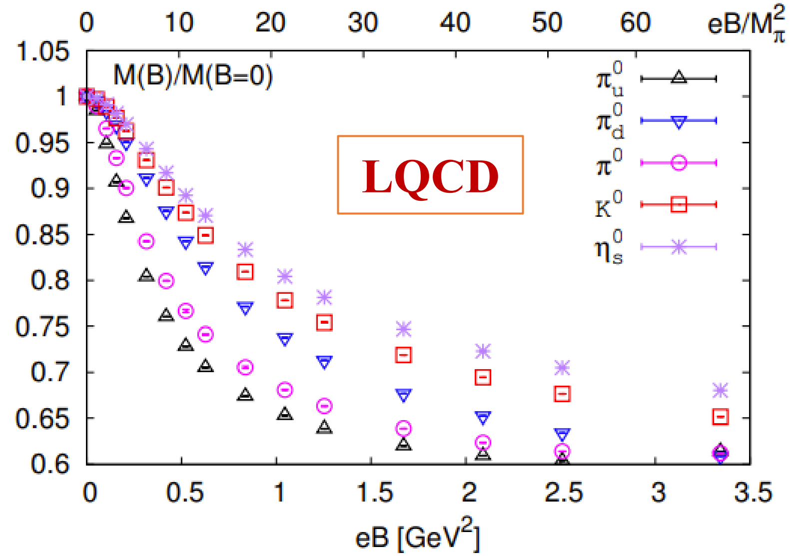


finite eB (Landau levels)

pole mass:

$$1 - G\Pi_M(m_M, \mathbf{0}) = 0,$$



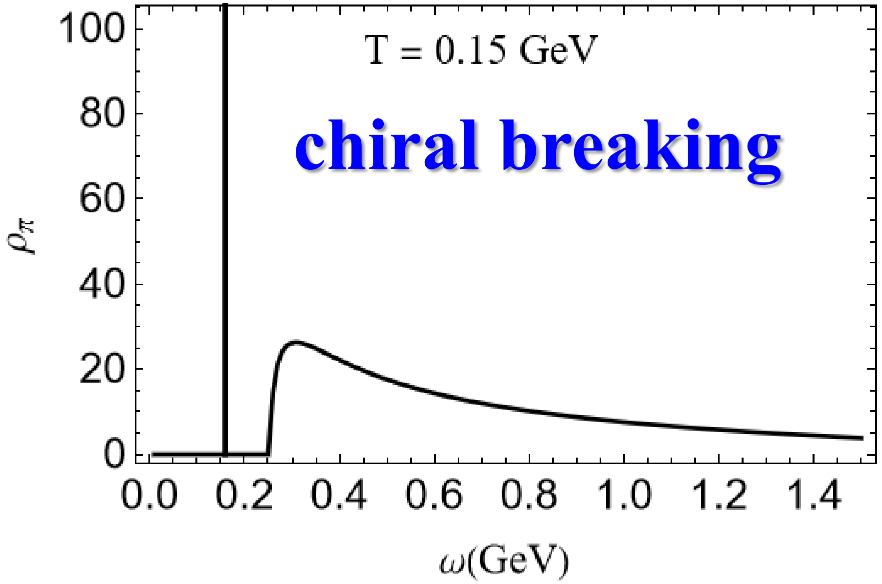


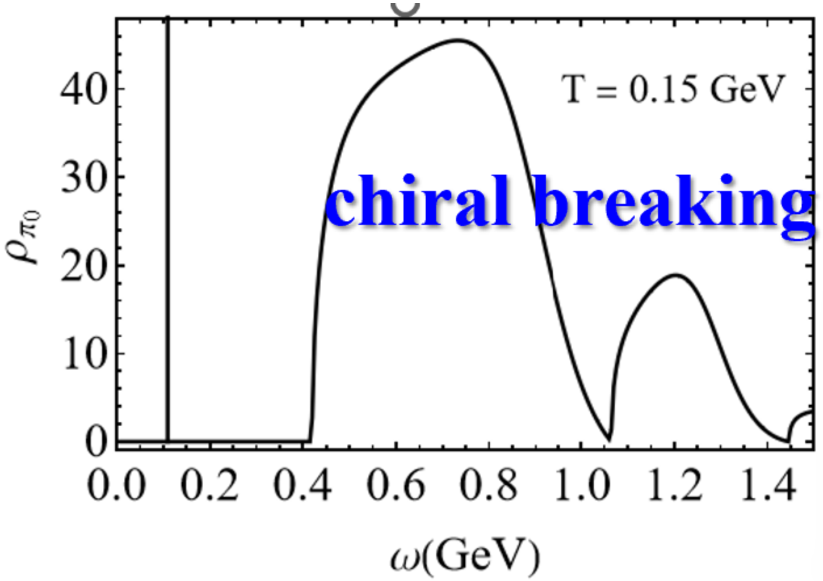
2.1

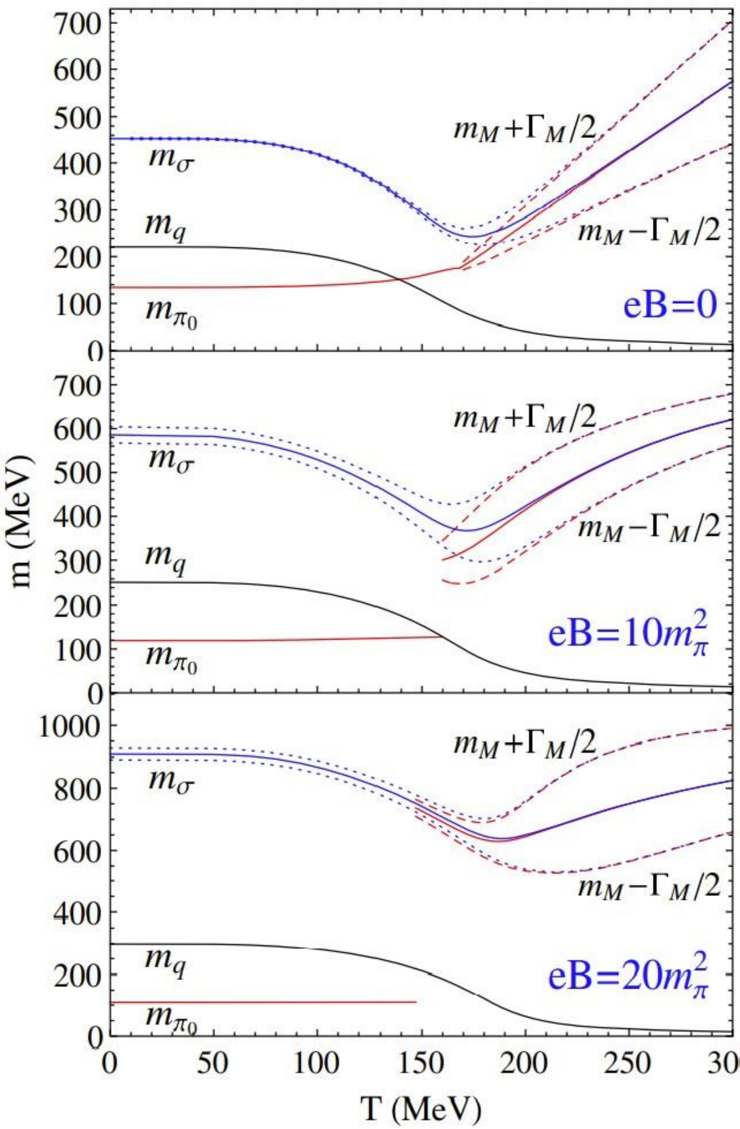
Spectral function of π_0 @ eB-T $\boxed{\mu=0}$

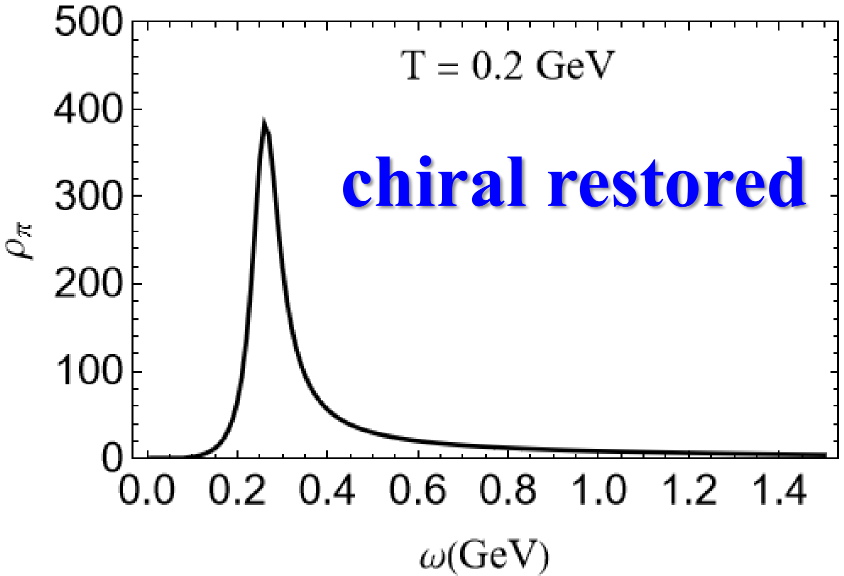
Zhou, **Mao**, arXiv:2025.XXXXX;
Wang, **Mao**, PRD 96, 034004 (2017)

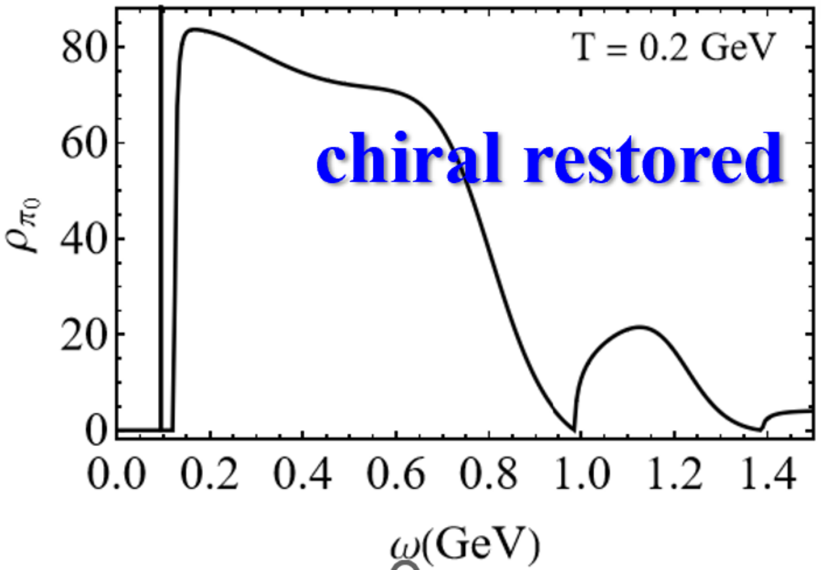
eB=0 | finite eB (Landau levels) | 磁场引起质量跳变@Mott 相变



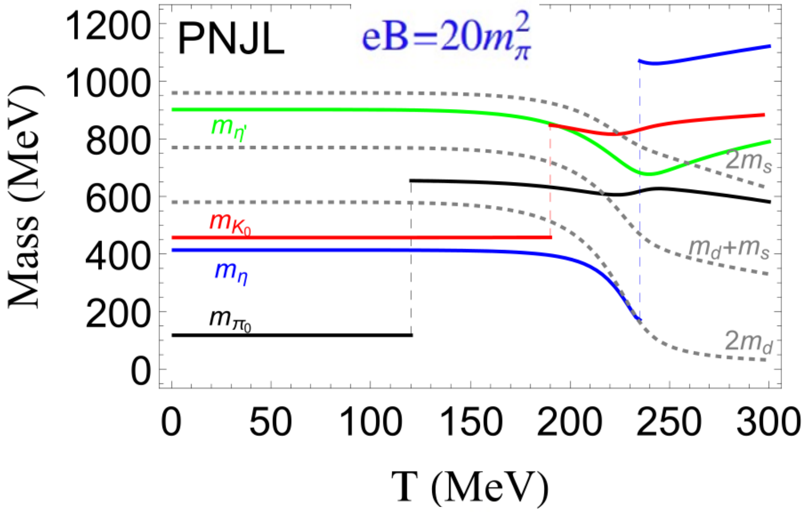


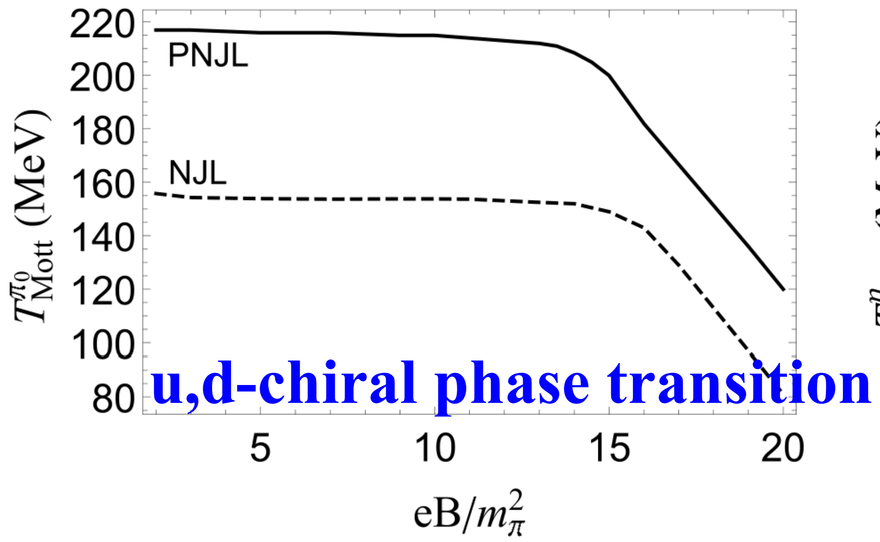


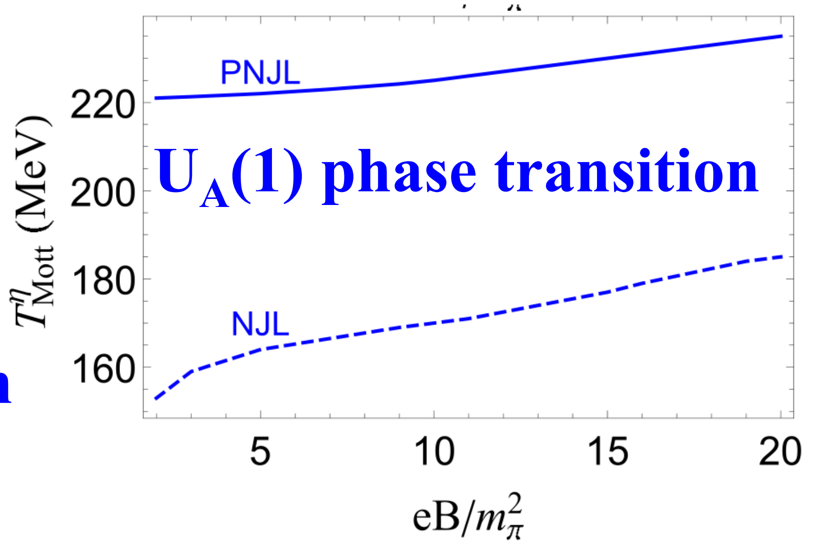


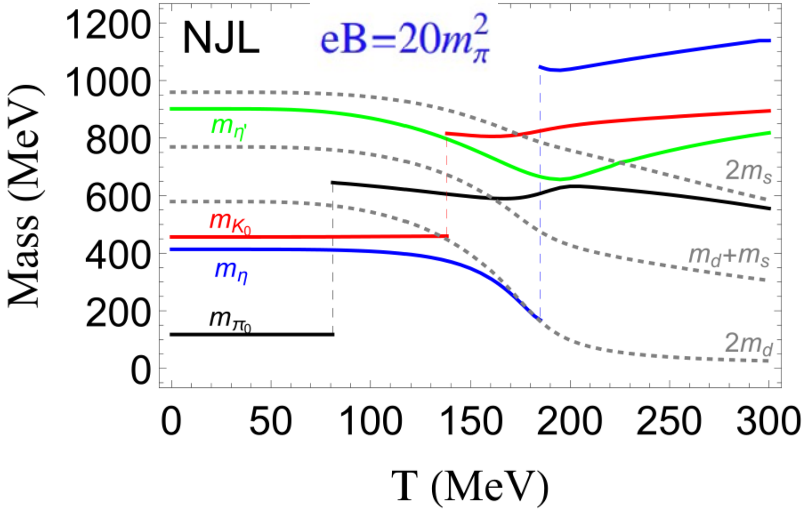


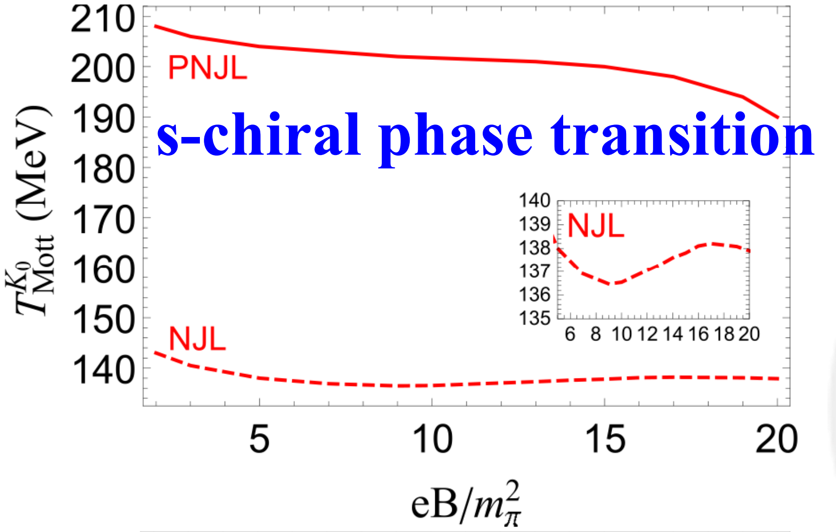
2.1 K_0, π_0, η, η' mesons @ eB-T $\mu=0$











非单调的介子谱
和Mott相变温度?
不好理解

Mei,Xia,Mao, PRD 107, 074018 (2023), Li, Mao, arXiv: 2511.14150

2.1 Spectral function of π_0 @ eB- μ

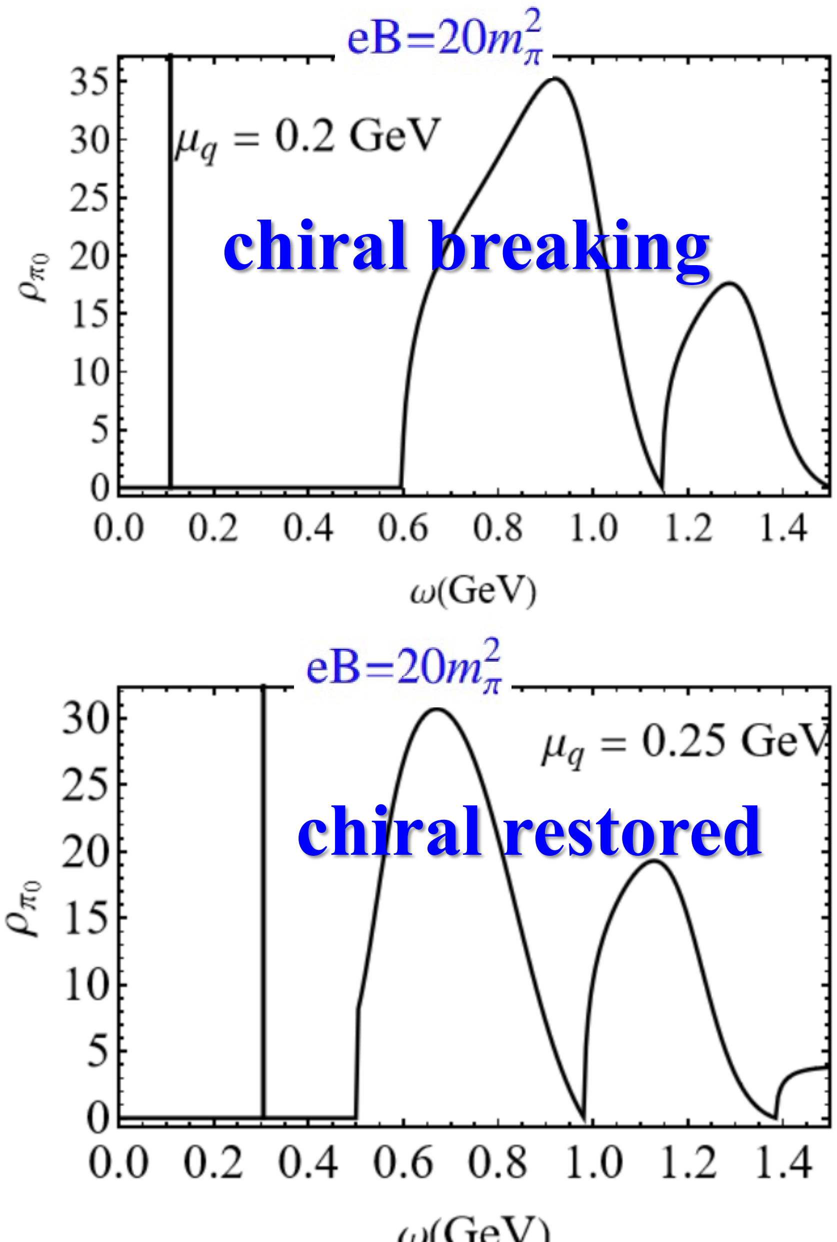
T=0

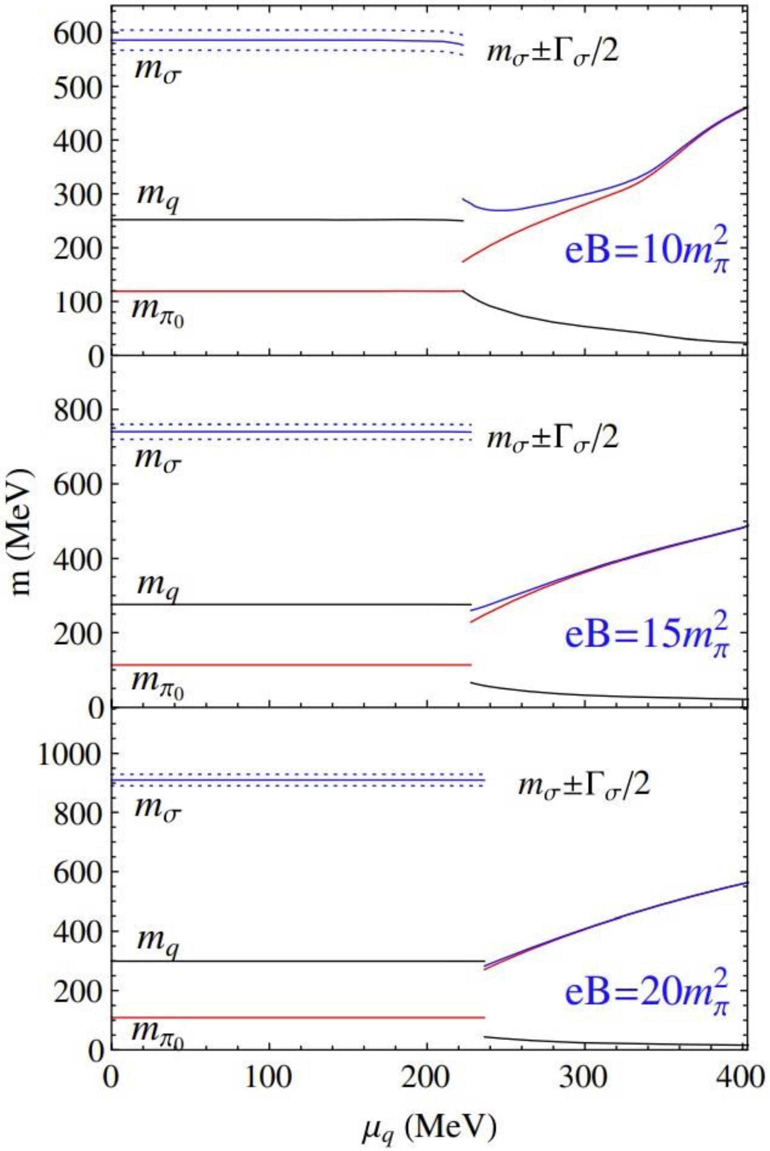
Zhou, **Mao**, arXiv:2025.XXXXX;
Wang, **Mao**, PRD 96, 034004 (2017)

一级相变引起质量跳变

finite eB (Landau levels)

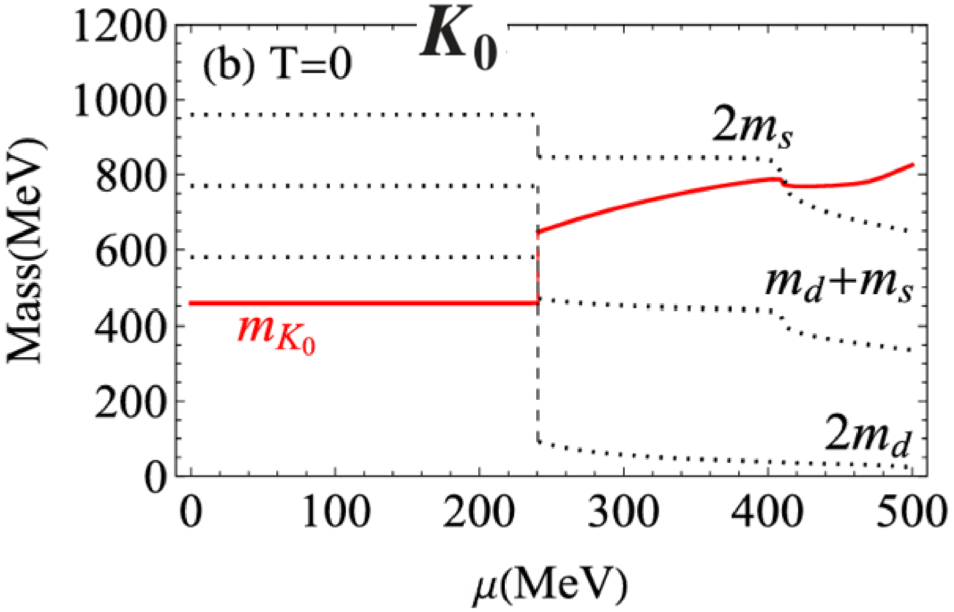
Pauli Blocking

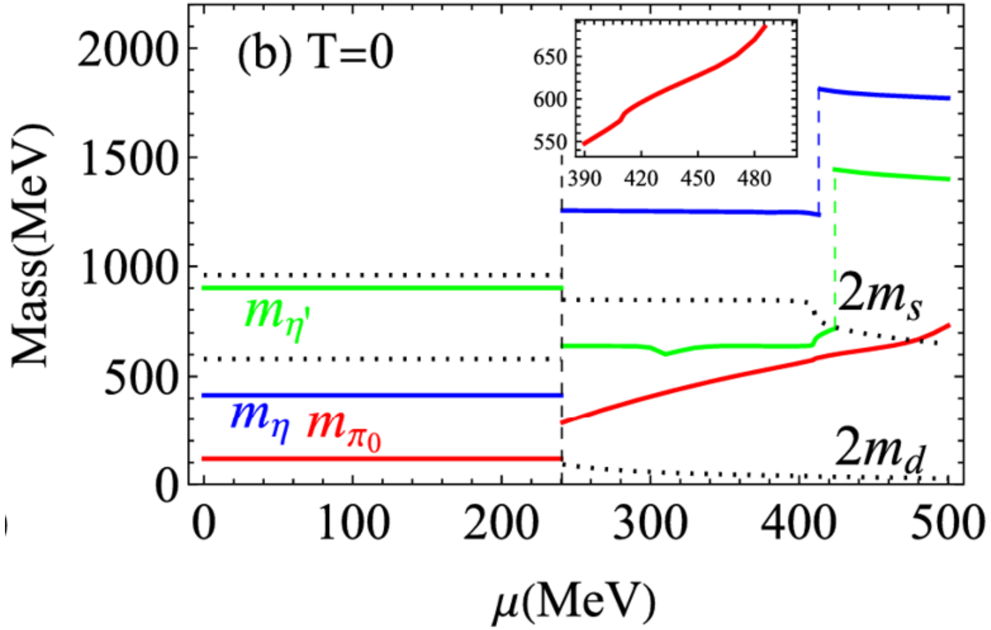




K_0, π_0, η, η' mesons @ eB- μ

Mei,Xia, Mao, PRD 107, 074018 (2023)



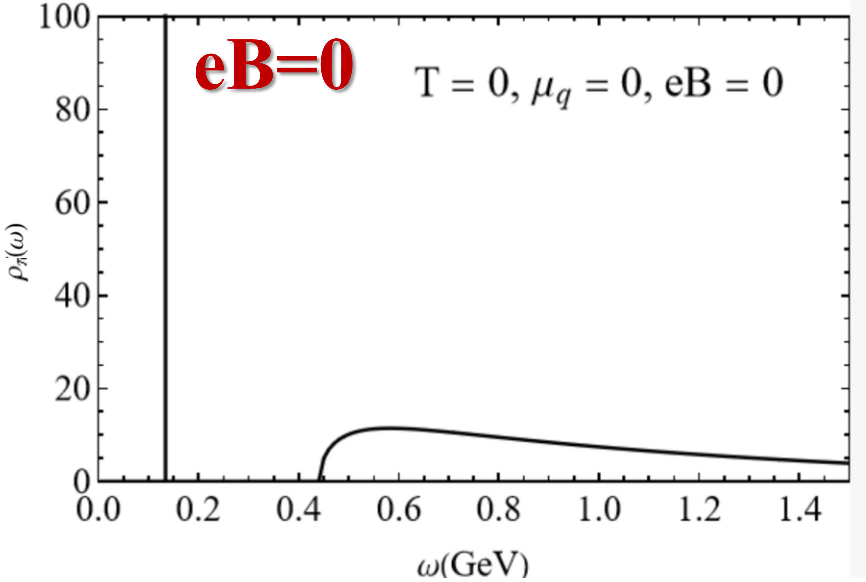


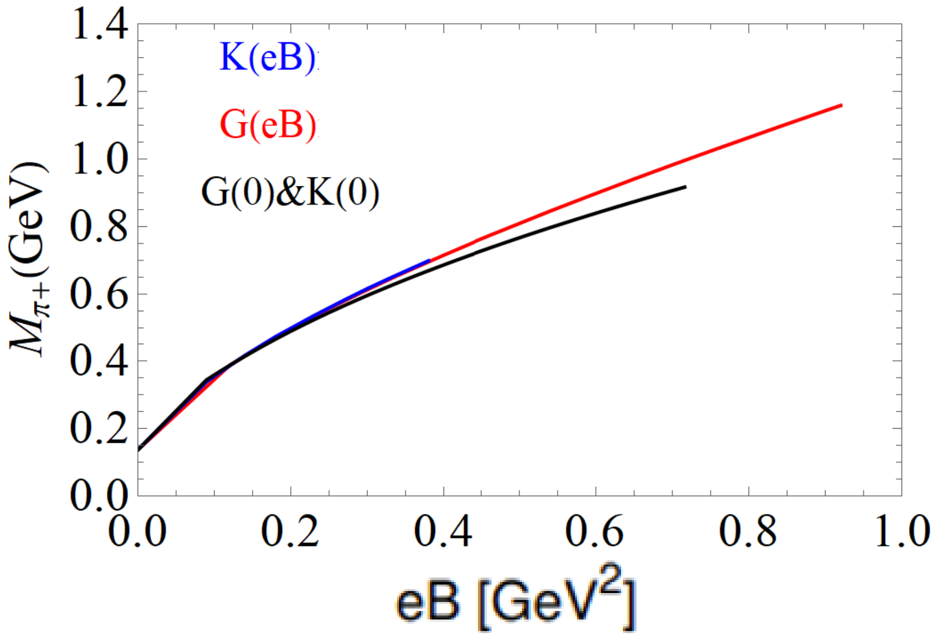
一级相变引起质量跳变 + Landau levels 引起的质量跳变

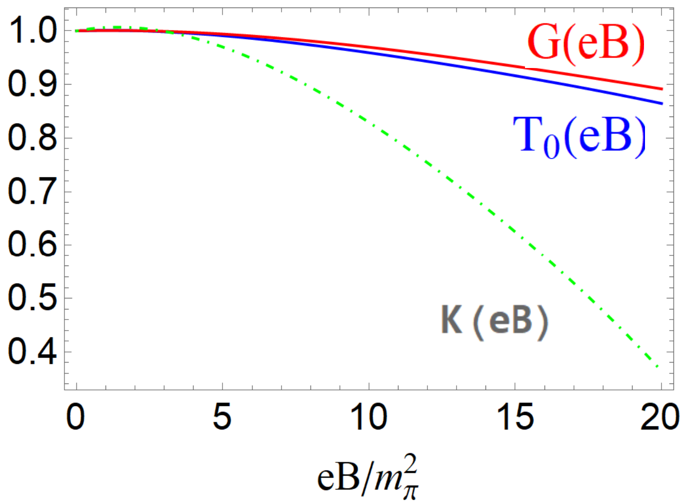
- **charged mesons** π^+, K^+

2.2

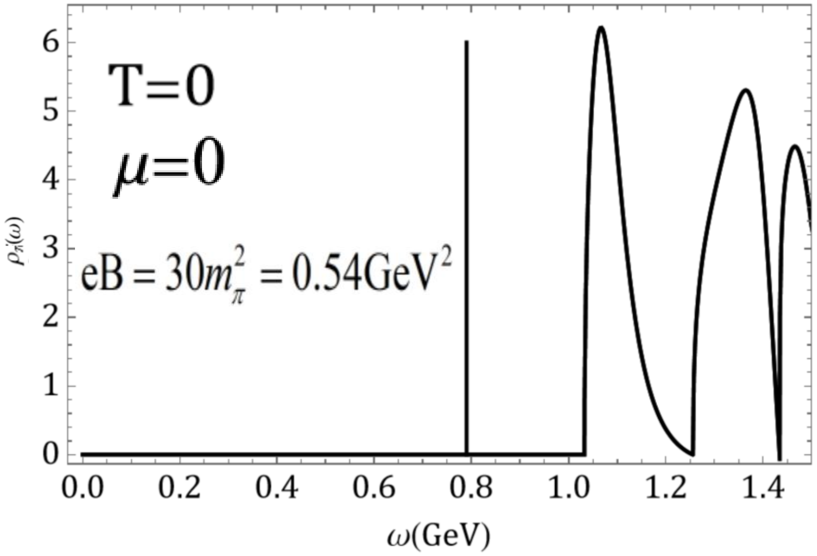
Spectral function of π_+ @ eB $|Q_u| \neq |Q_d|$

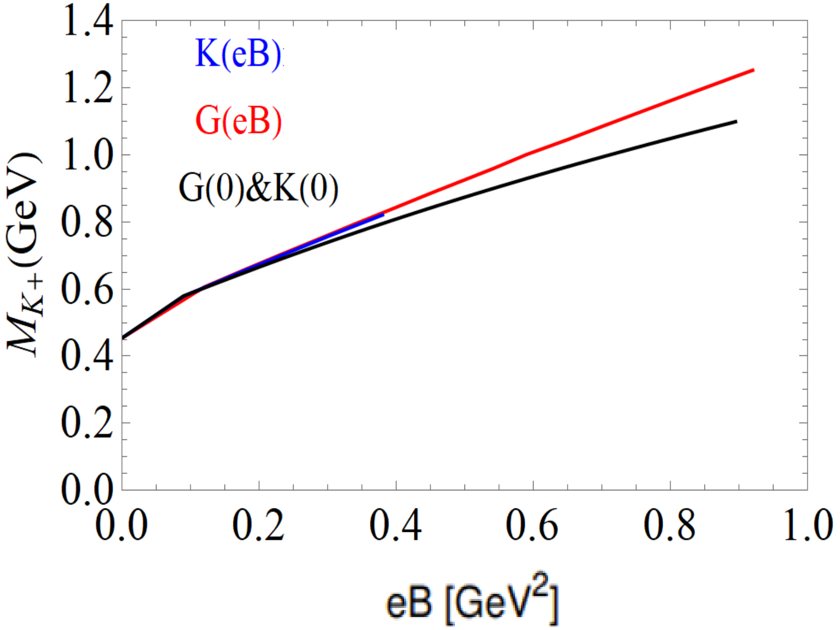


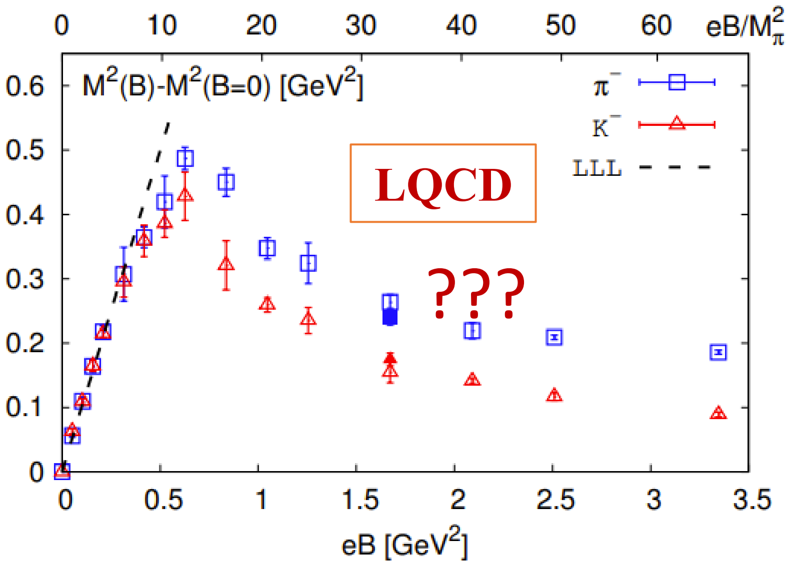




finite eB (Landau levels)





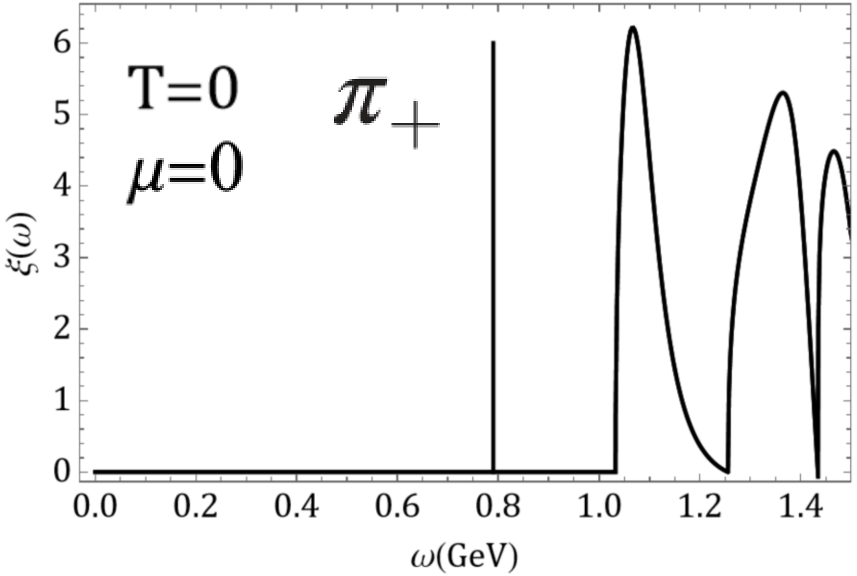


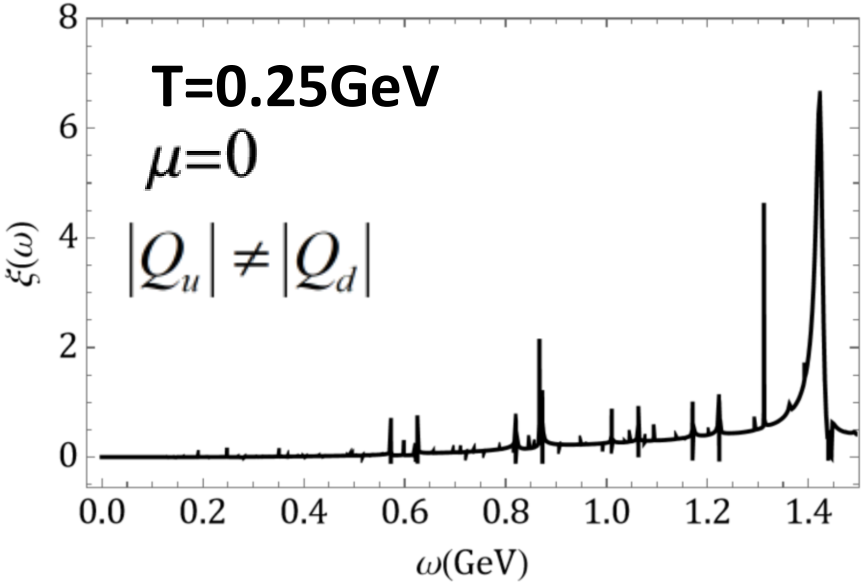
2.2

charged pion and kaon@ eB-T

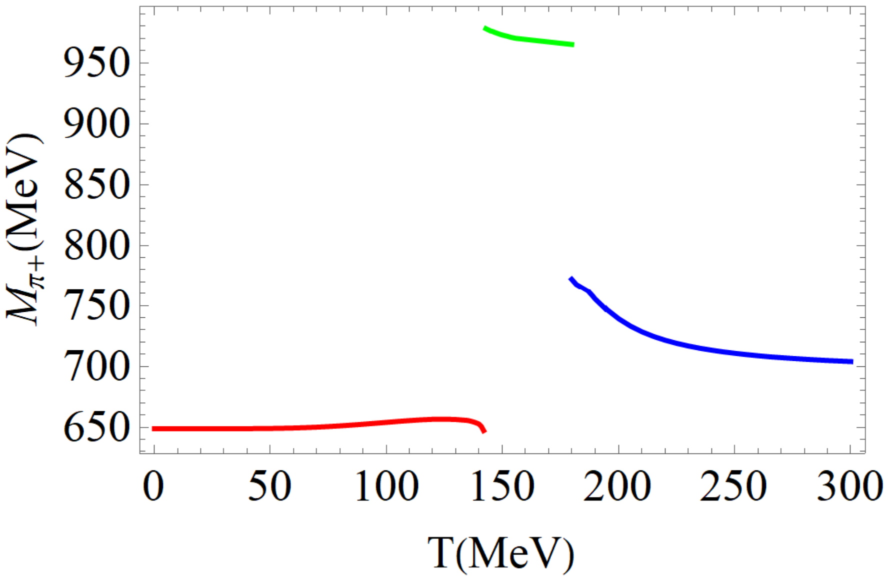
$\mu=0$

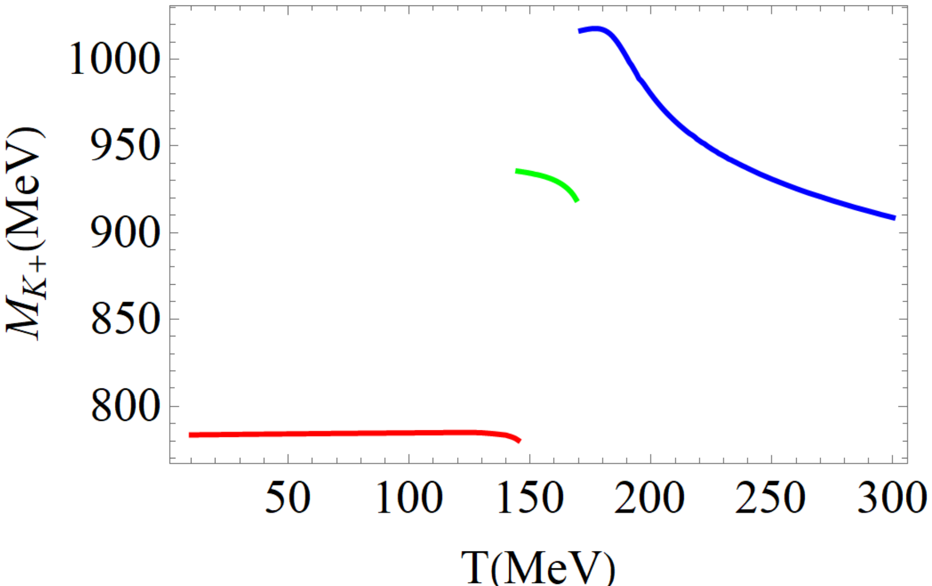
spectral function





pole mass: Landau levels 引起的质量跳变

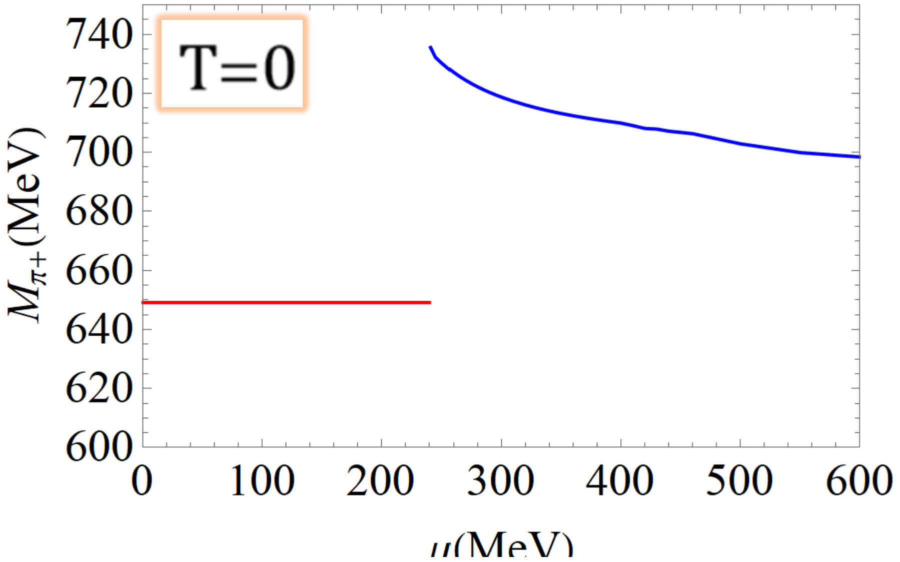


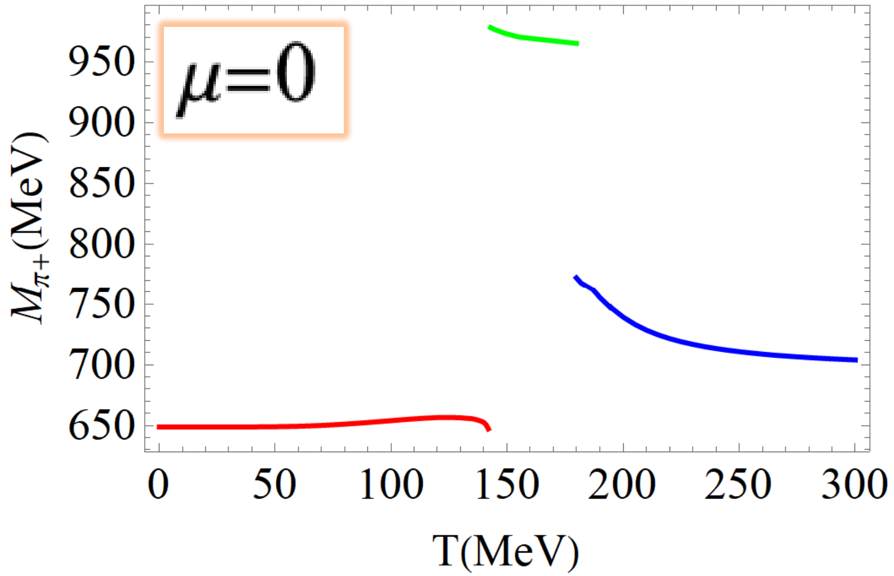


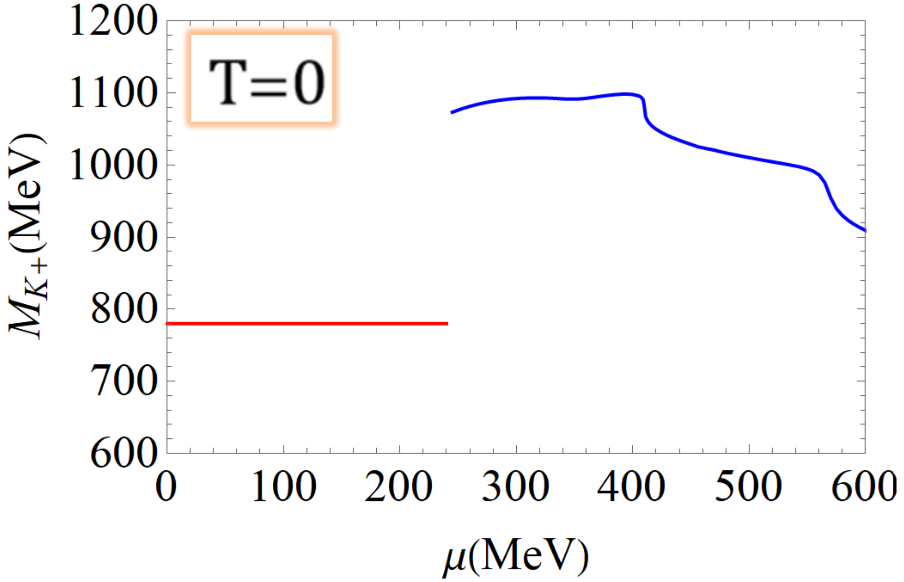
charged pion and kaon@ eB- μ pole mass

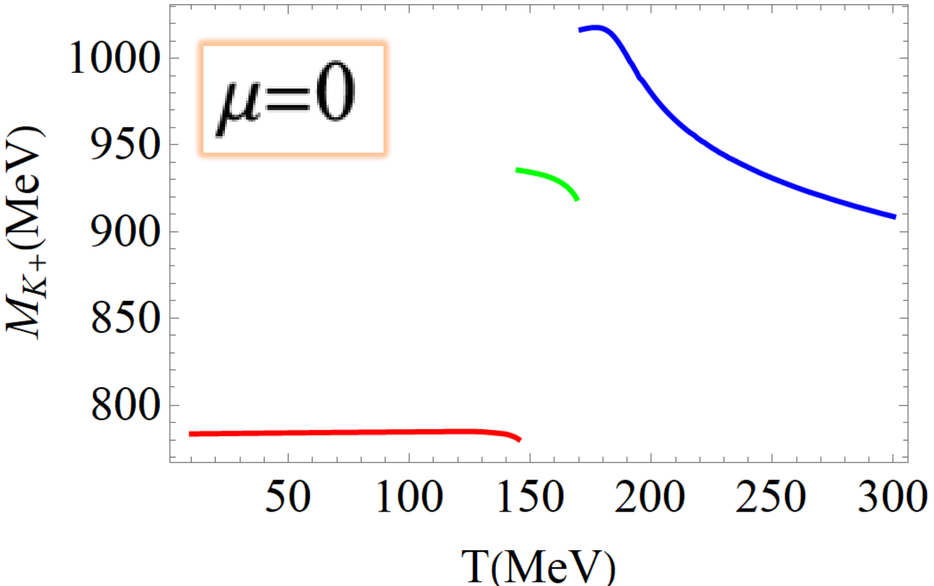
一级相变引起质量跳变

Landau levels 引起的质量跳变





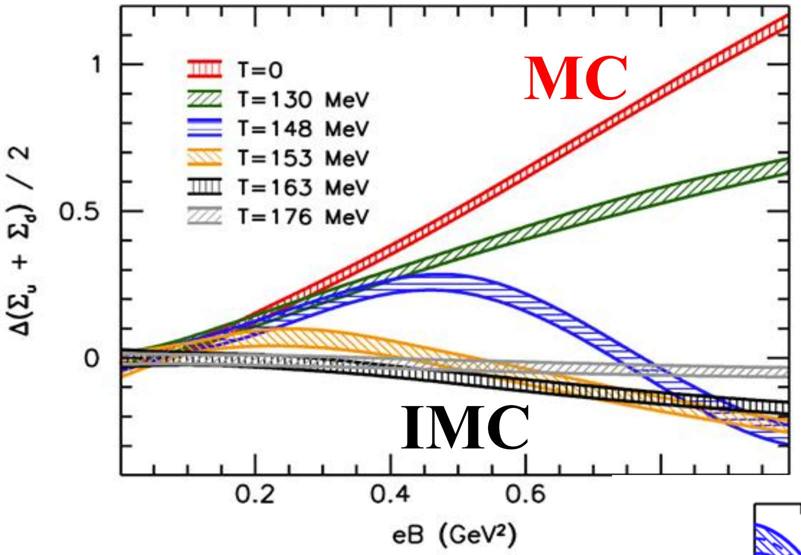


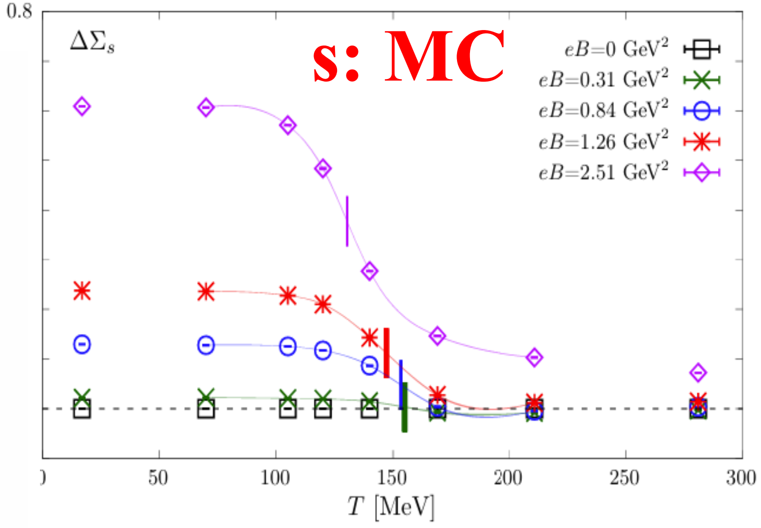


3 mesons and IMC

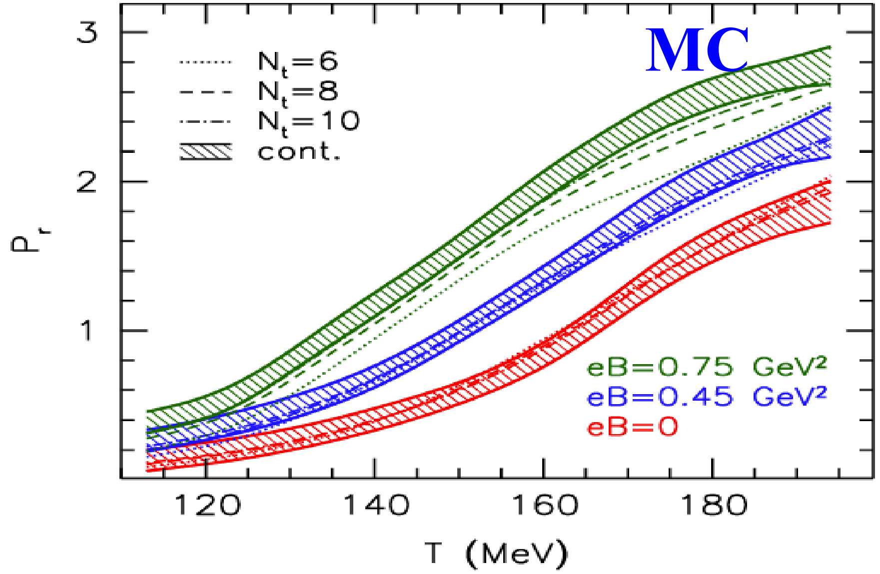
3.1 Inverse Magnetic Catalysis (IMC)

chiral condensate



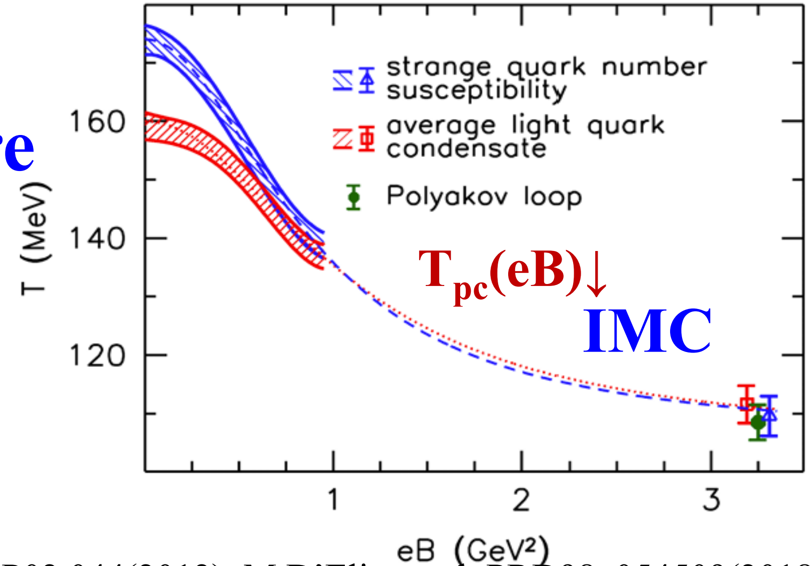


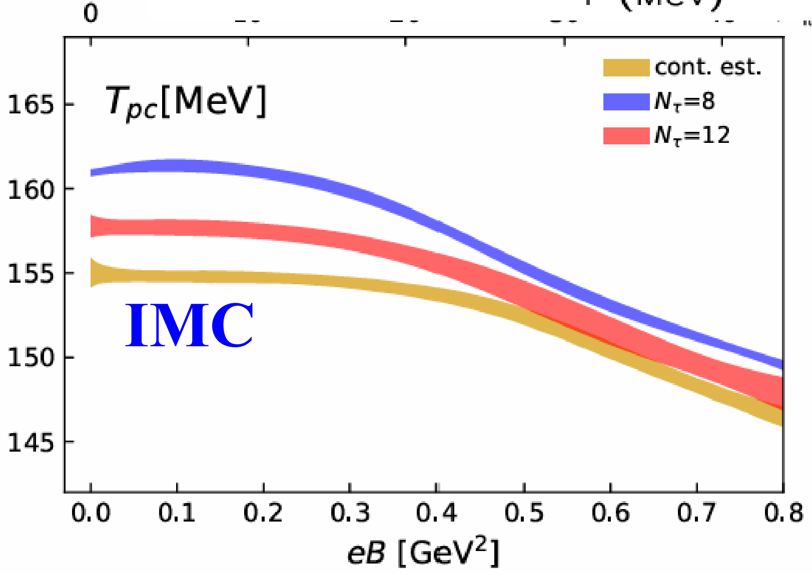
Polyakov loop



critical temperature

$T_{pc}(eB) \downarrow$





G. Bali, et.al, PRD86, 071502(2012);JHEP02,044(2012); M.D'Elia, et.al, PRD98, 054509(2018);H.T. Ding, et.al, PRD105, 034514(2022); PRD111, 114522 (2025).

3.2 Analytical efforts for IMC ---- Beyond mean field

Quarks

(1) magnetic inhibition: feedback from neutral pion to quarks

Fukushima et al., PRL110, 031601 (2013); Mao, PLB758,195(2016); PRD94,036007(2016); PRD97,011501(2018); Mei, Mao, Huang et al., PRD110, 034024(2024).

(2) thermo-magnetic effect: fluctuations @(P)QM

Kamikado et al. , JHEP 03,009(2014); Ayala et al.PRD90,036001(2014); PRD89,116017(2014); PRD92,096011(2015); EPJA 57, 234(2021).

(3) weakening (eB dependence) of coupling between quarks

Farias et al. , PRC 90, 025203 (2014); Ferreira et al. , PRD89,116011(2014); PRD89, 036006(2014); PRD89,016002(2014); PRD89, 056013(2014); Mueller et al. , PRD91,116010(2015); Braun et al. , PLB755, 265(2016); Mao,arXiv:2406.13531.

(4) quark anomalous magnetic moment:

Chaudhuri et al. , PRD99, 116025 (2019); EPJA56, 213 (2020); PRD103, 096021 (2021); Mao et al. , PRD102, 114035 (2020); PRD106, 034018 (2022); Ghosh, PRD103, 116008 (2021); Xu, Chao, Huang, PRD103,076015(2021); Feng, PRD**107**, 076004(2023); Tavares, PRD109, 016011 (2024) .

Gluons

(5) eB-dependent interaction between Polyakov loop and quarks

Fraga, PLB731, 154(2014); Ferreira, PRD89, 016002(2014); Mao, PRD110, 054002(2024).

(6) sphalerons & instantons :chirality imbalance, $U_A(1)$ anomaly

Chao, Huang et al. , PRD88, 054009 (2013); Fukushima PRD81, 114031 (2010); Mao et.al JPS Conf. Proc. 20, 011009 (2017);Zhou, Mao, arXiv:2025.XXXXX.

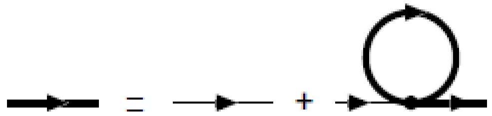
3.2 feeddown from pion G(eB)

NJL model beyond mean field

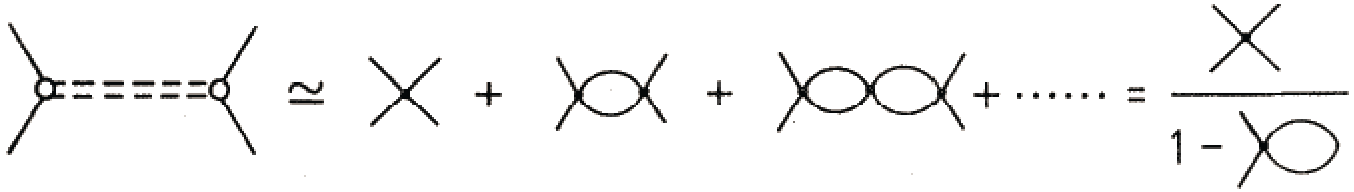
$$\mathcal{L}=\bar{\psi}\left(i\gamma_\nu D^\nu-m_0\right)\psi+\frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2+\left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] \qquad \mathbf{B}=(0,0,B)$$

idea: strong interaction & strong correlation

(1)Quarks： mean field



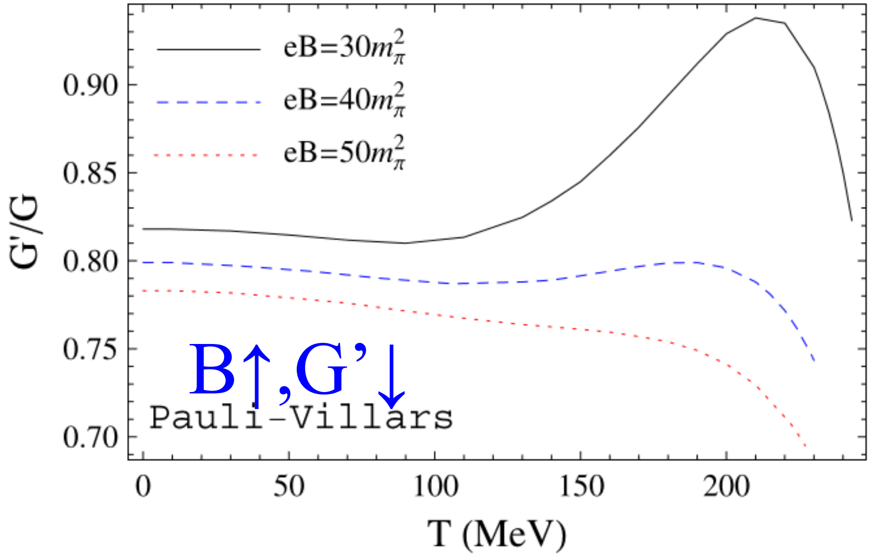
(2)Mesons: RPA resummation (quantum fluctuation)

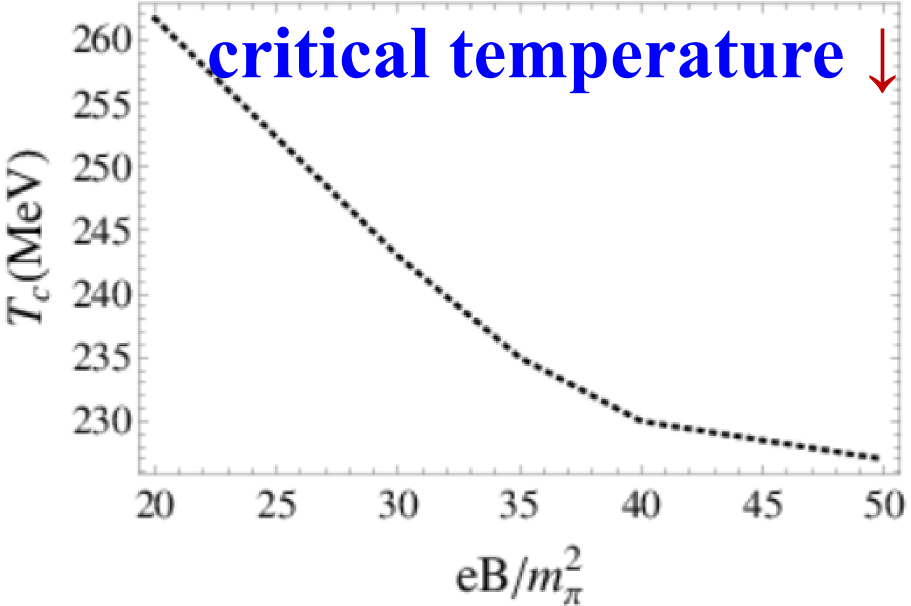


(3)Q-M system： $\Omega=\Omega_q+\Omega_M\rightarrow\Omega(G^{'}),\ \boxed{G^{'}(B,T)}$

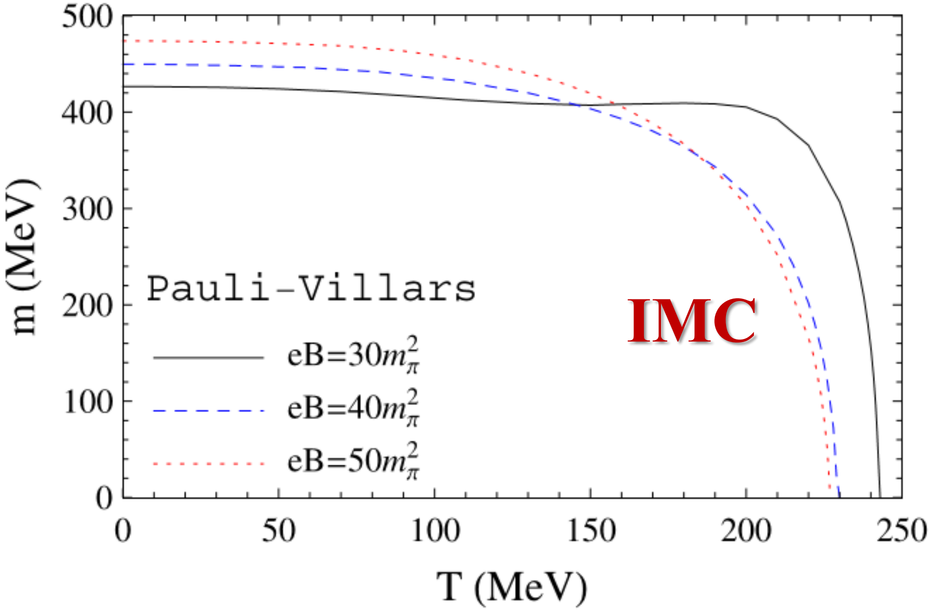
feed-down from mesons to quarks

3.2 chiral limit π_0 eB VS G





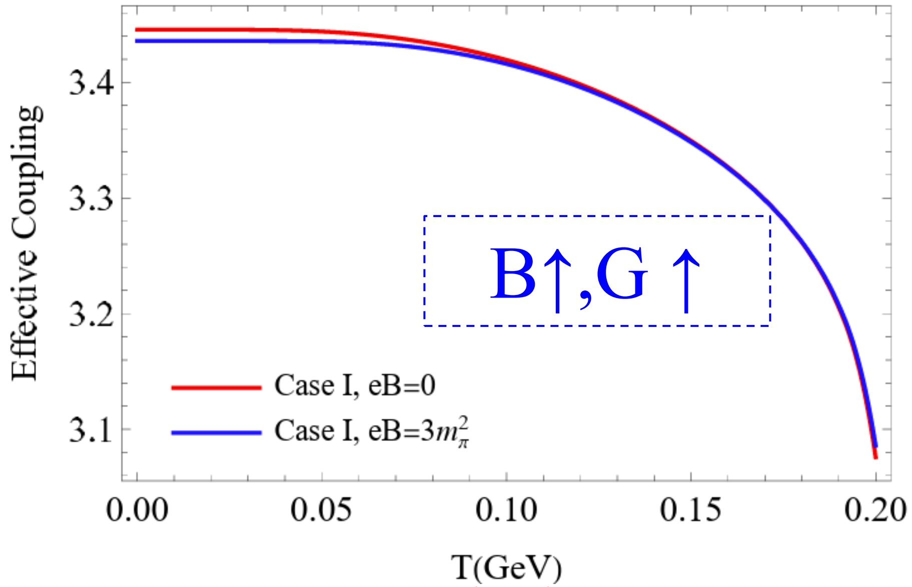
order parameters



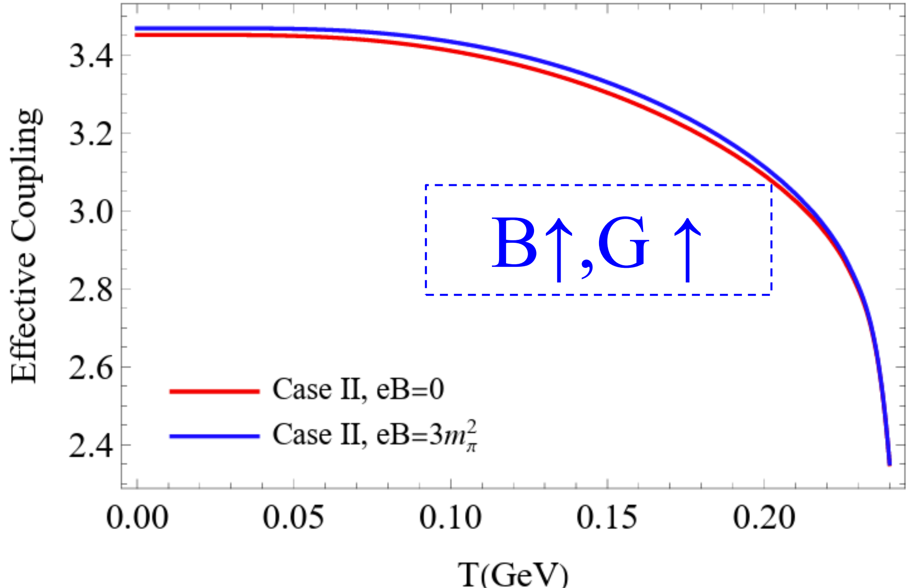
Phys. Lett. B758, 195-199 (2016);
Phys. Rev. D94, 036007 (2016);
Phys. Rev. D97, 011501(R) (2018);

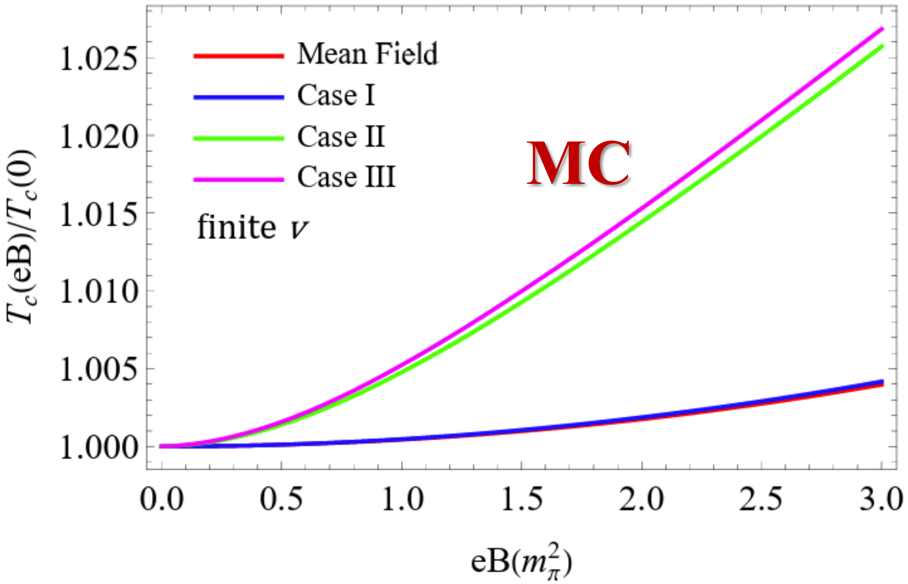
3.2 Physical case eB VS G

J.Mei, W.Rui, S.J.Mao,M.Huang,K.Xu, PRD110, 034024(2024)



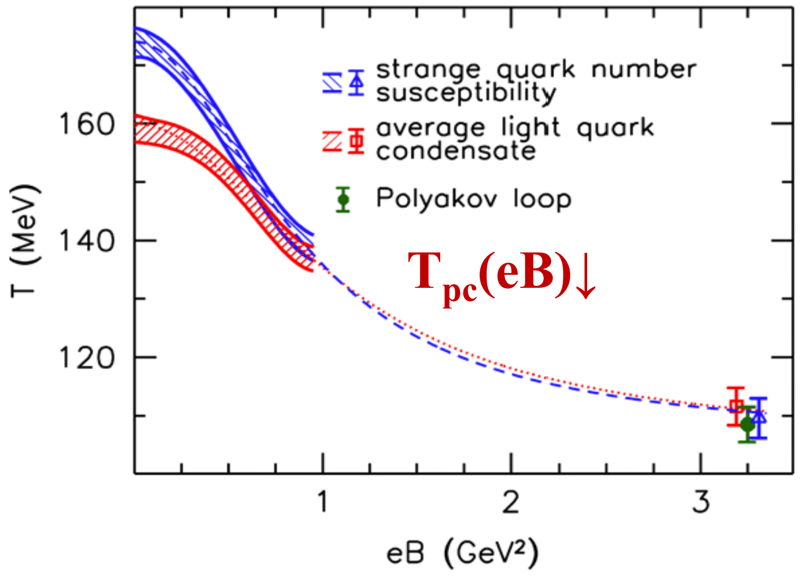
Case	Included mesons
0	None
I	π_0 only
II	$\pi_\pm$ only
III	π_0, $\pi_\pm$

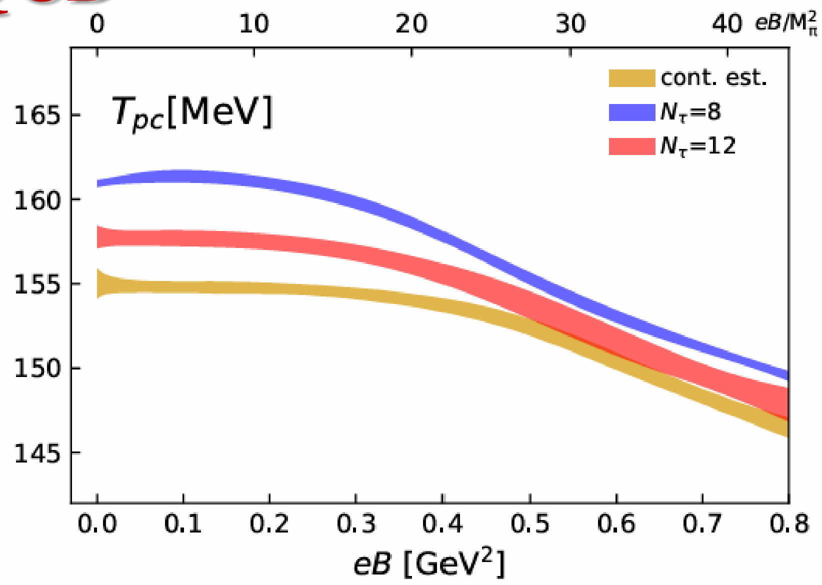




3.2

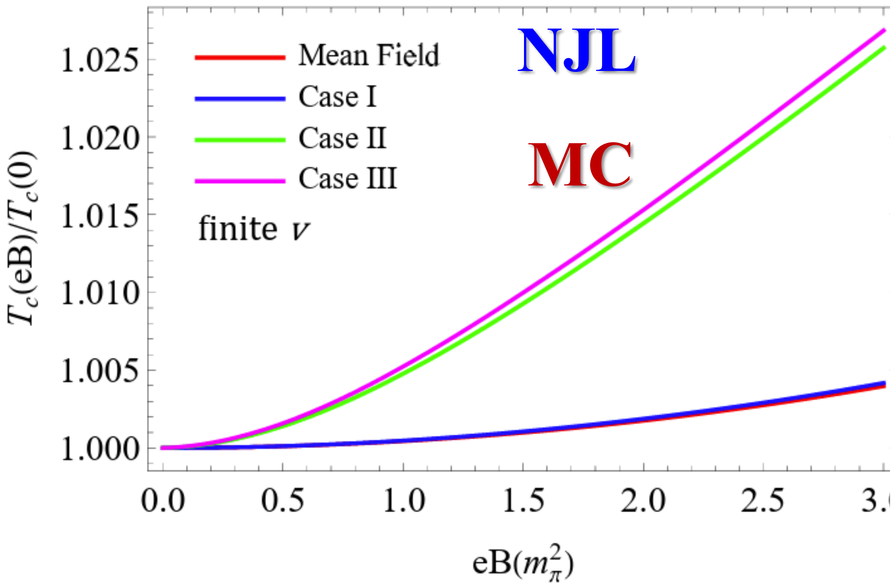
LQCD

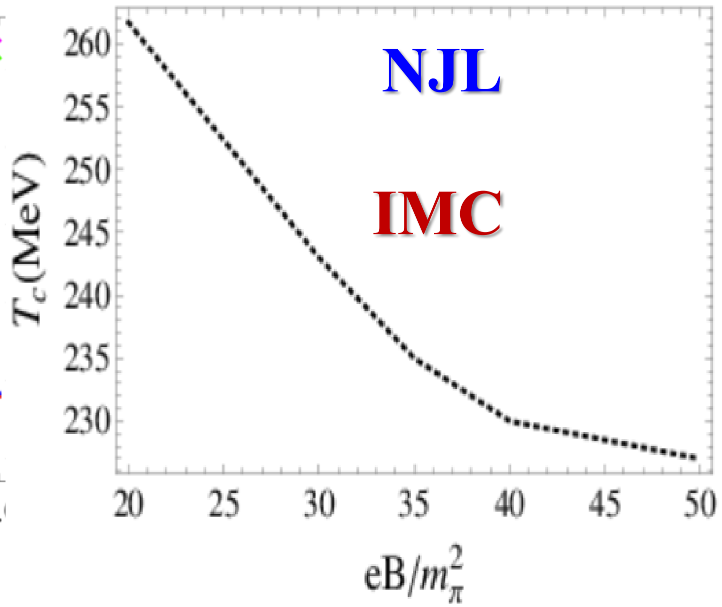


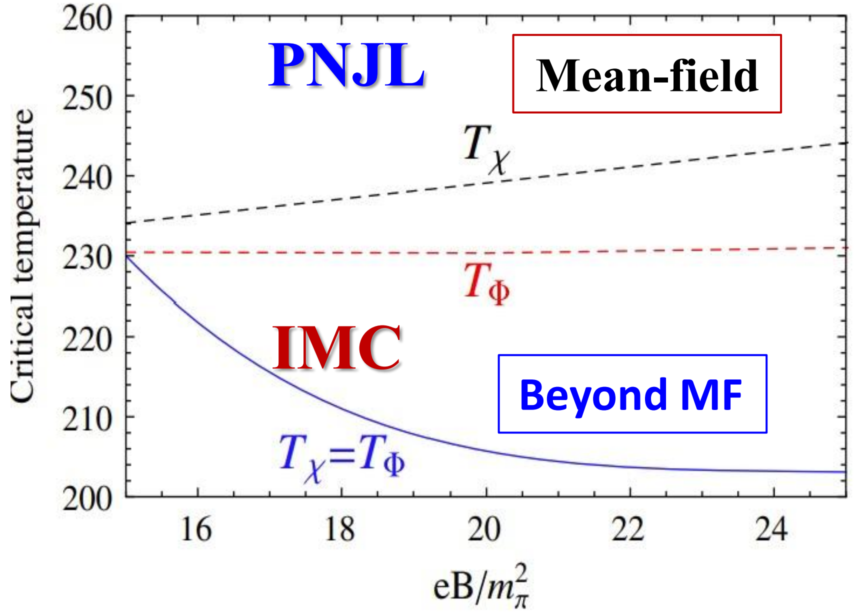


(P)NJL model beyond mean field

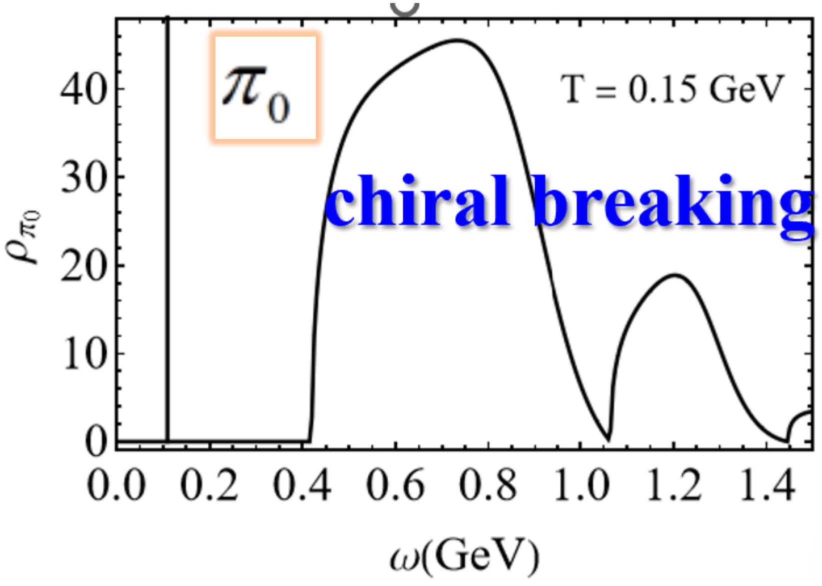
medium eB case, & 3-flavor case, under consideration

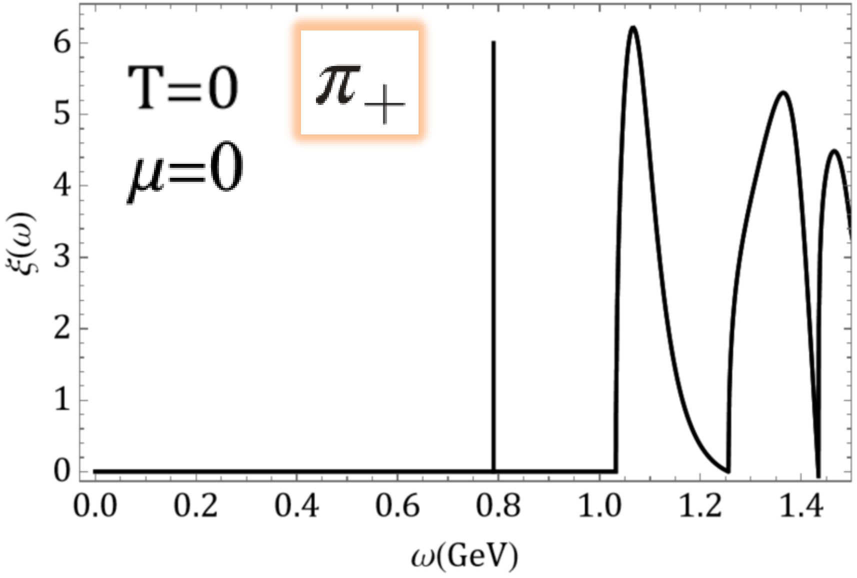


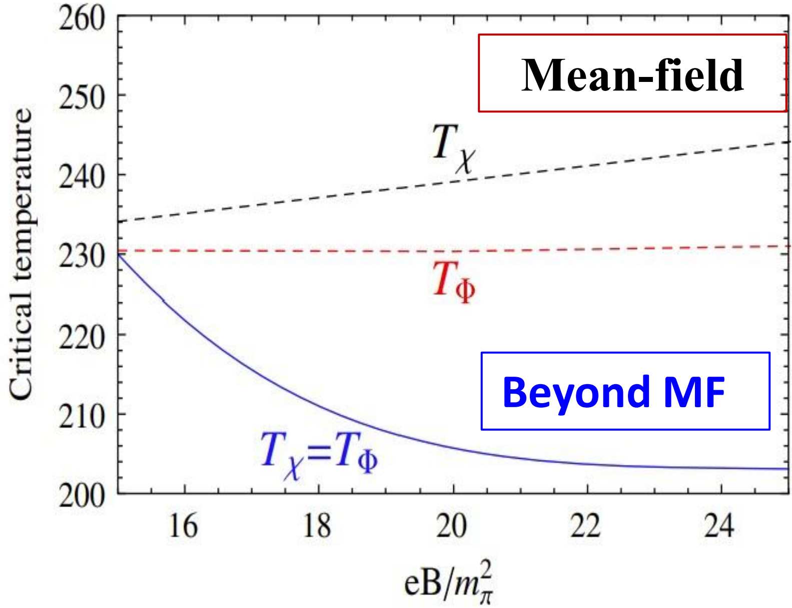


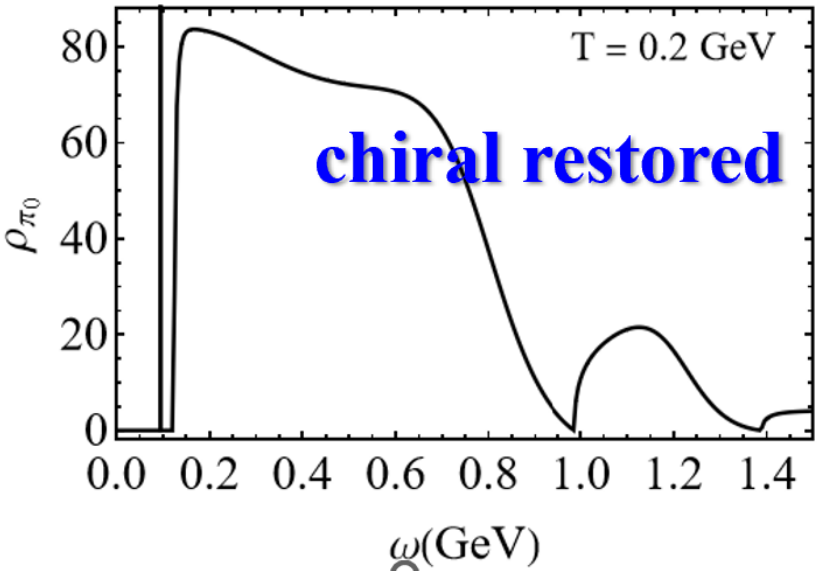


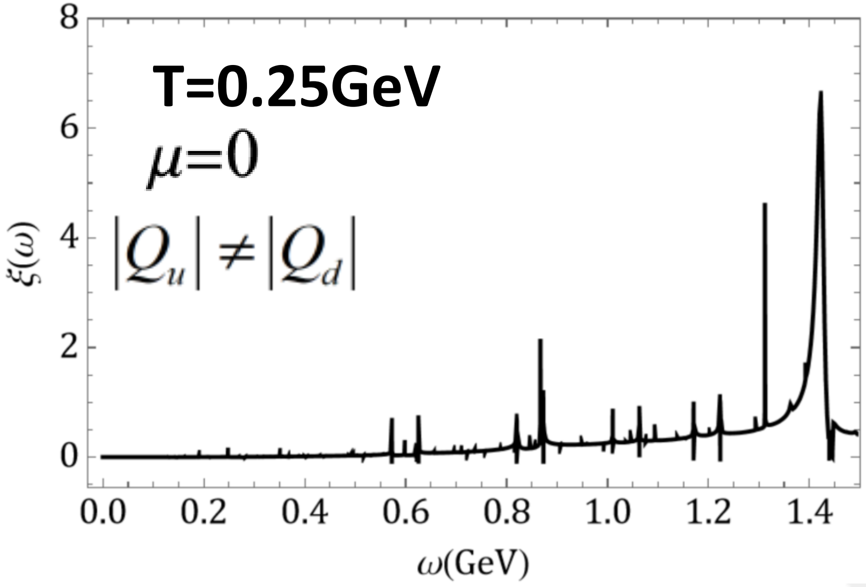
4 summary and outlook

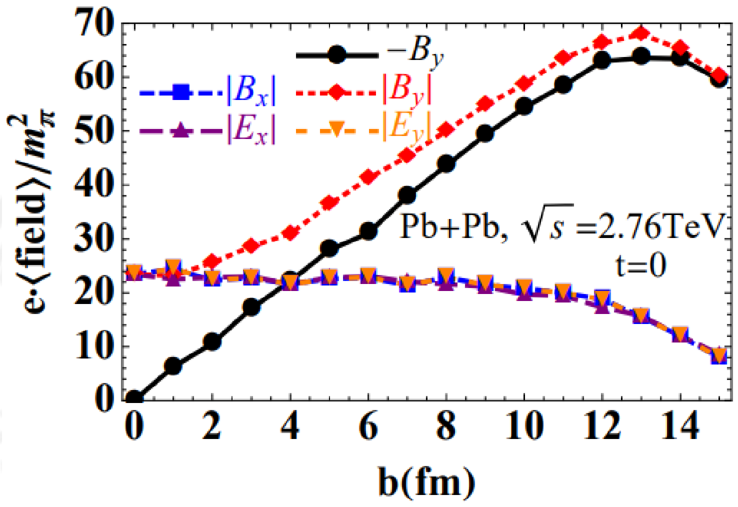




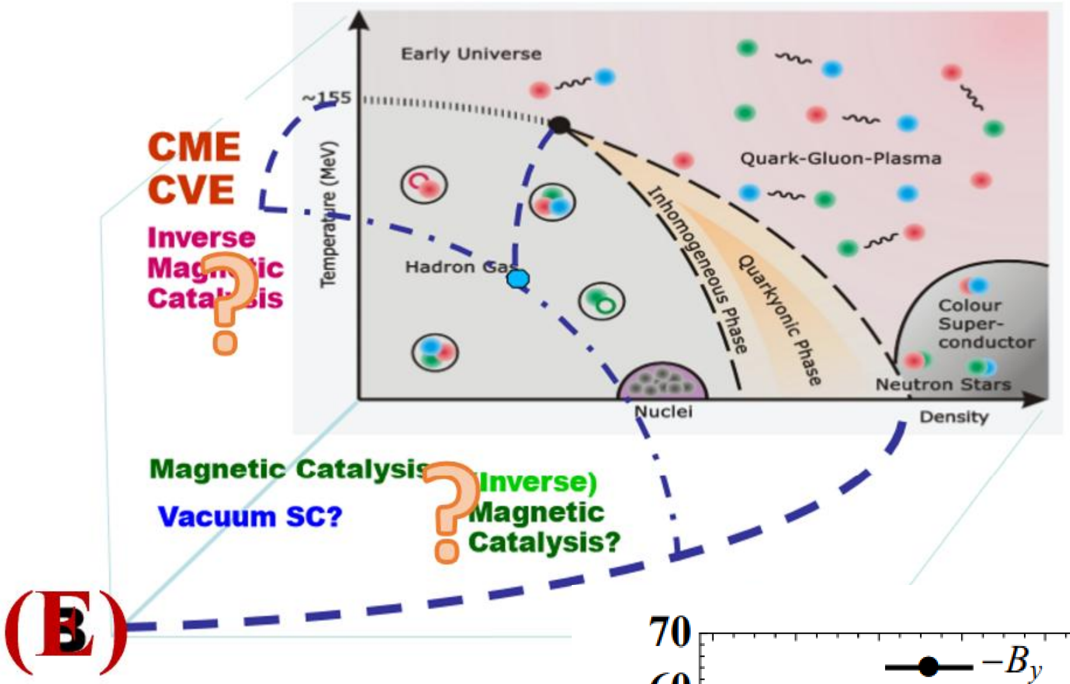


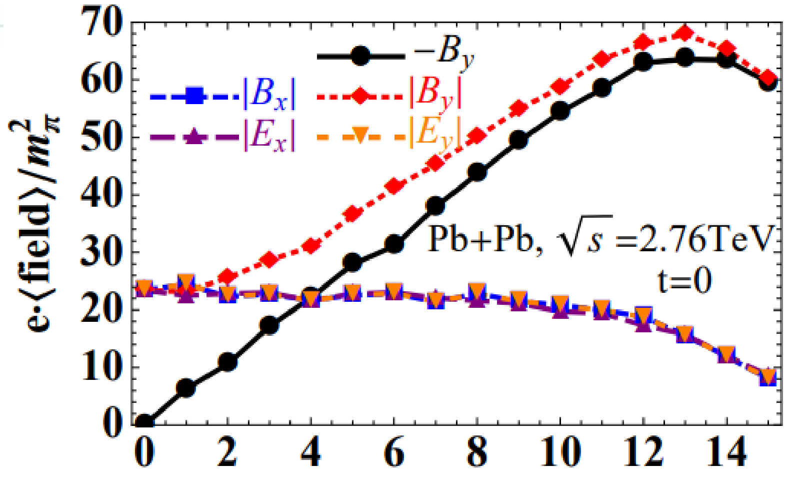




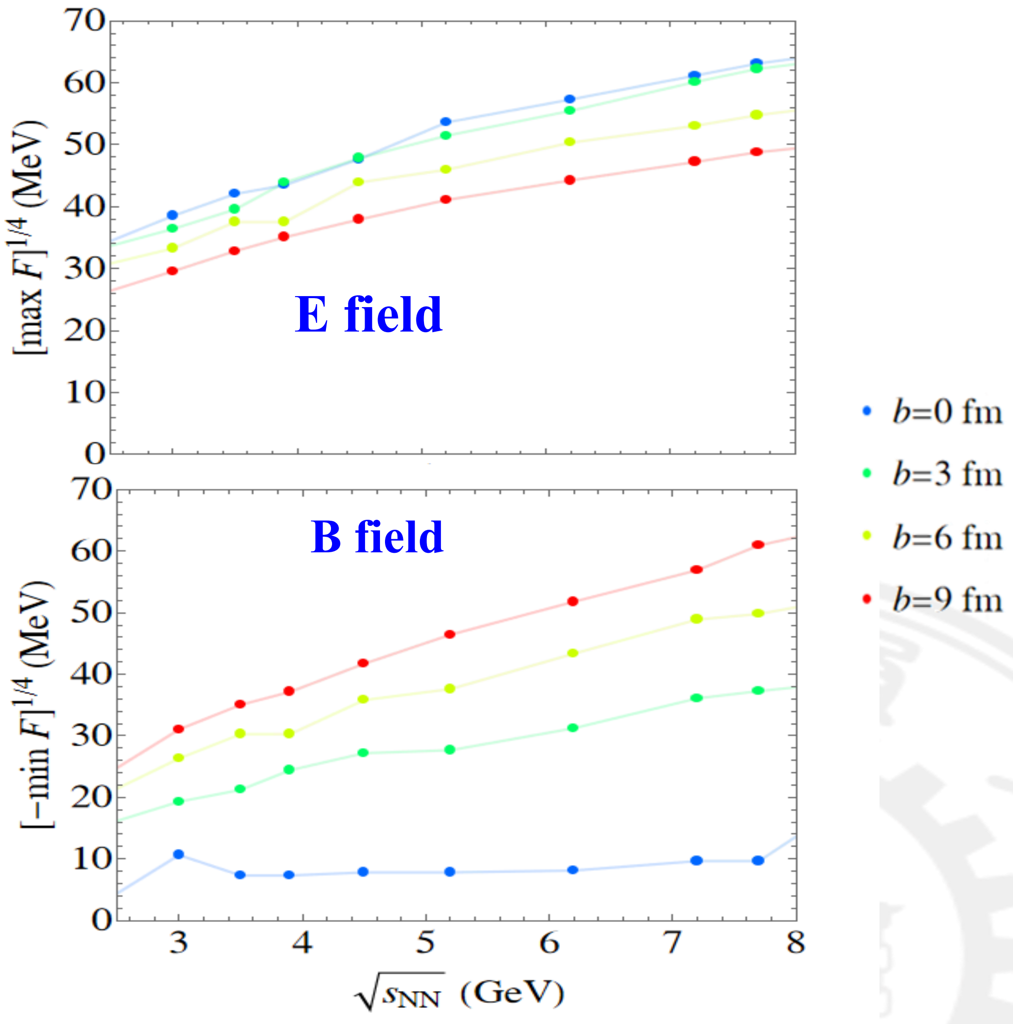


4.1 Electric Catalysis (EC)





Deng&Huang, PRC85, 044907(2012)



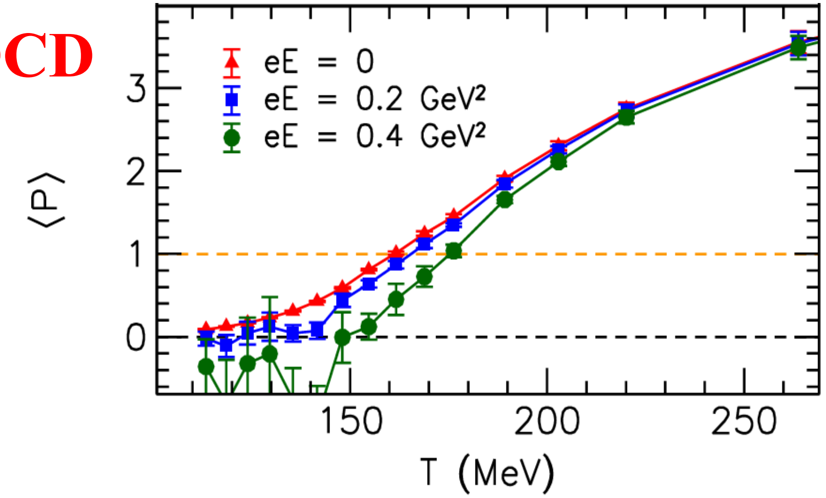
Taya,et.al,2402.17136; 2409.07685

4.1 Electric Catalysis (EC)

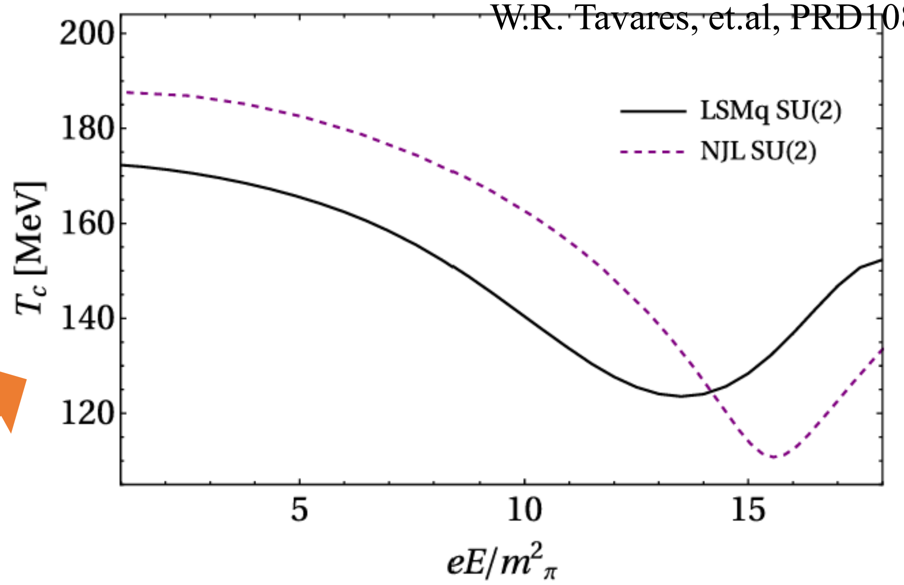
Analytical results in mean field

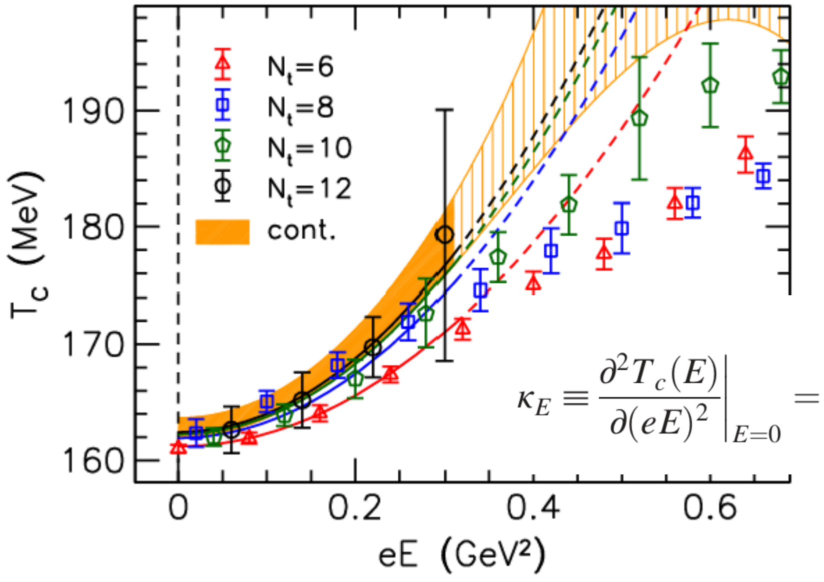
G.Q. Cao, X.G. Huang, PRD93, 016007(2016);
W.R. Tavares, et.al, PRD108, 016017(2023).

LQCD



???

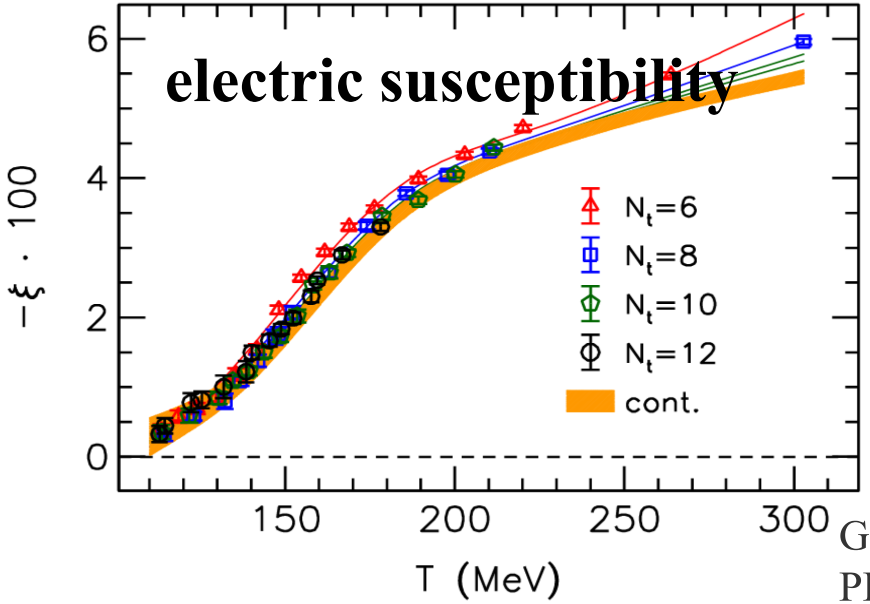




$$\kappa_E \equiv \frac{\partial^2 T_c(E)}{\partial (eE)^2}\bigg|_{E=0} = 0.37(9)\ \mathrm{GeV}^{-3}$$

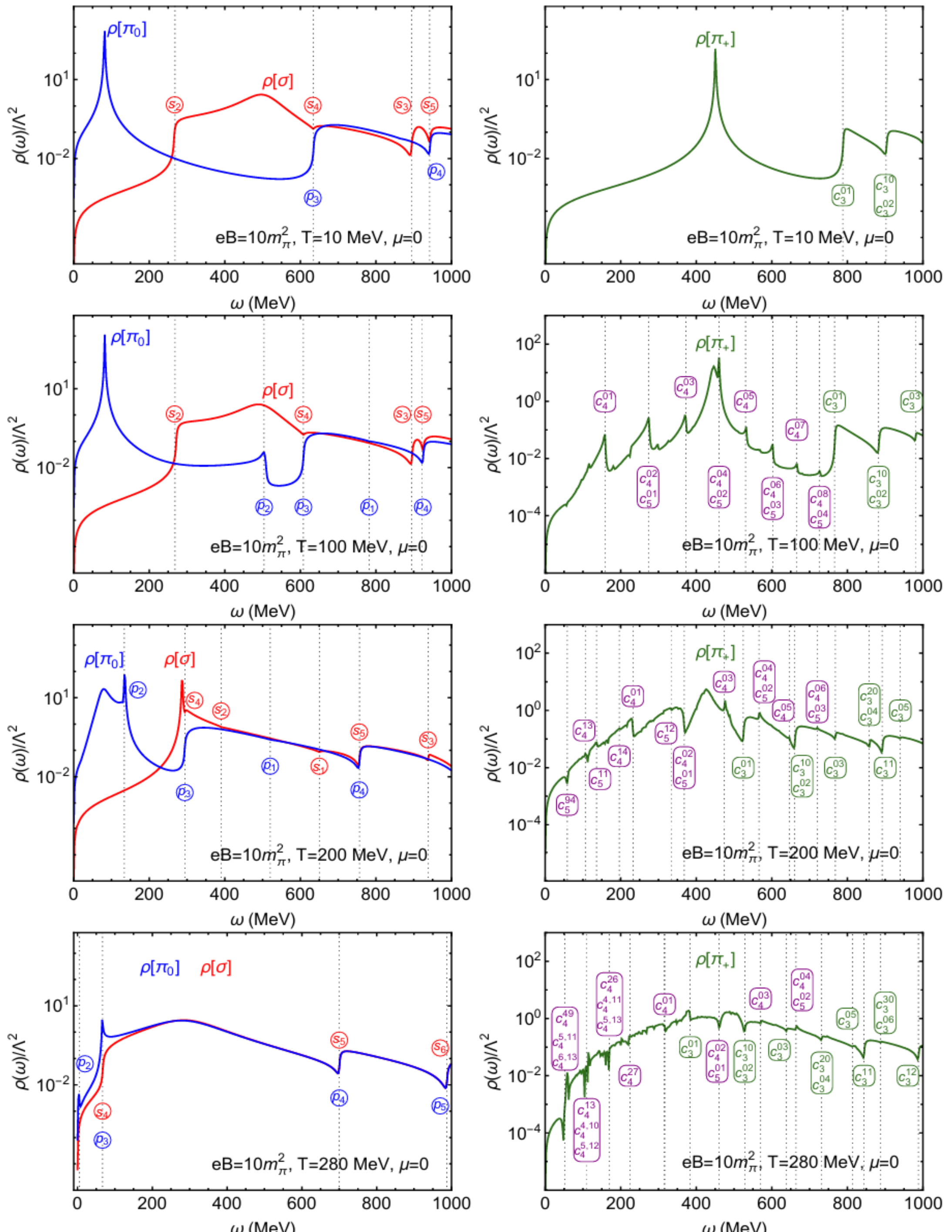
G.Endroedi and G. Marko, PRD109, 034506(2024).

electric susceptibility



G.Endroedi, G. Marko, PRD109, 034506(2024

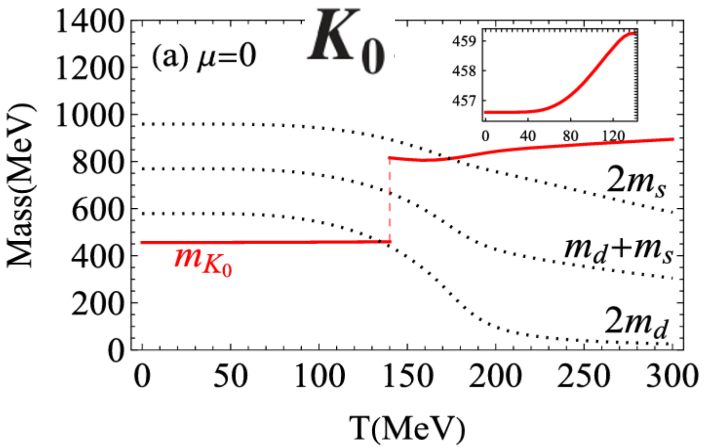
Back Up

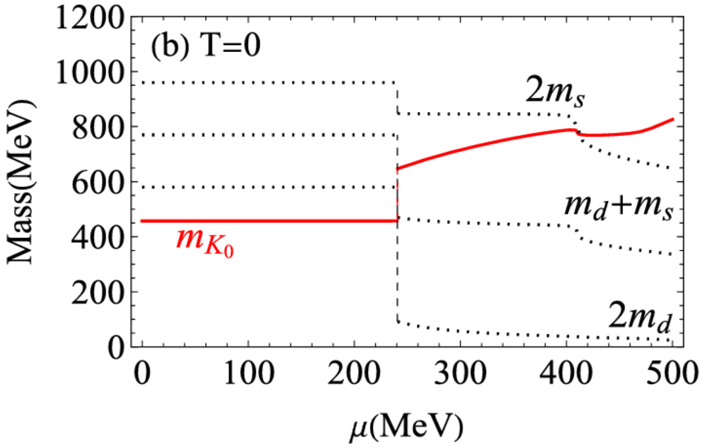


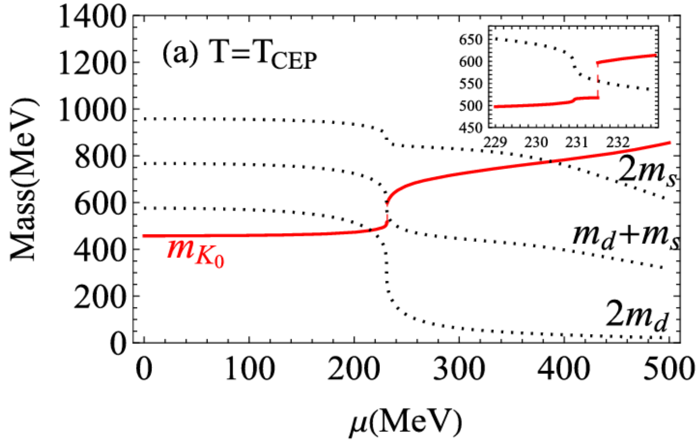
QM model with FRG,
from Ziyue Wang

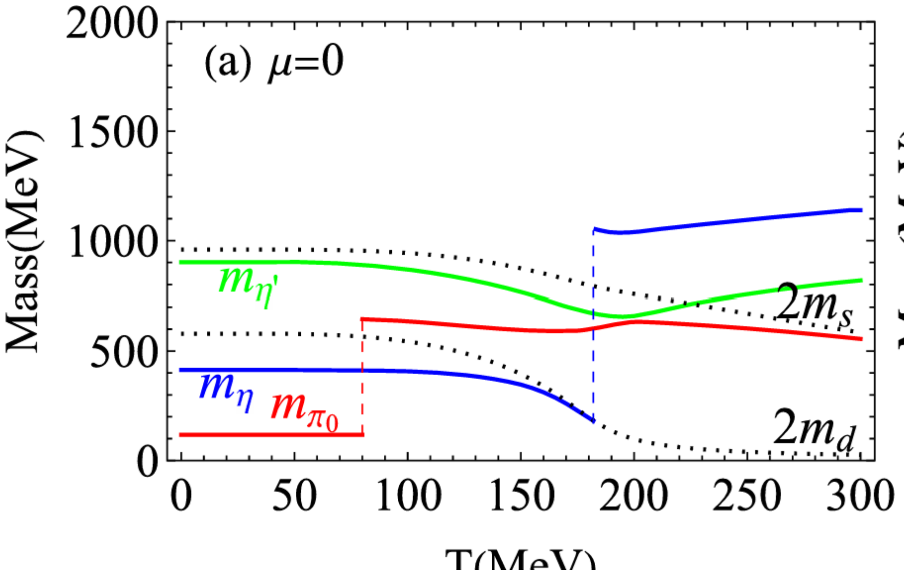
K_0, π_0, η, η' mesons @ T, μ, eB

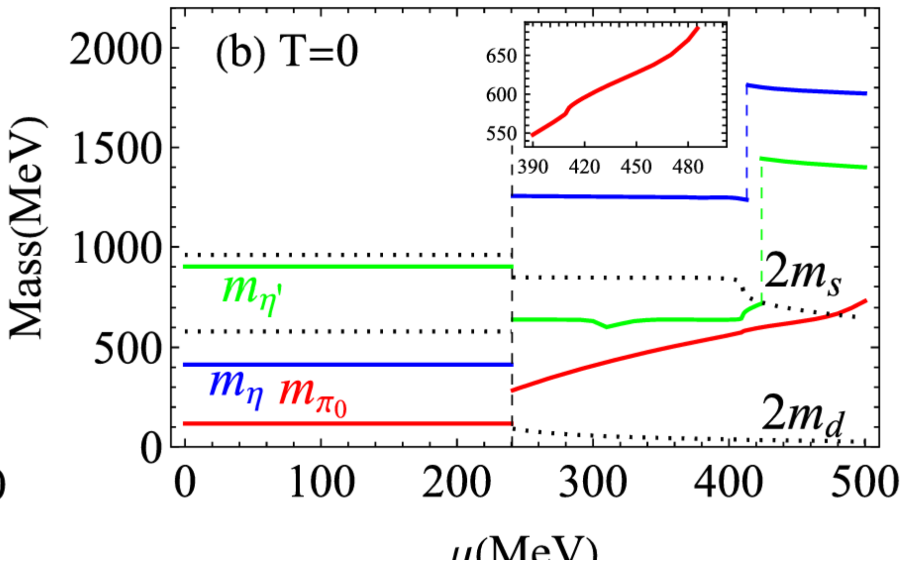
Mei,Xia, Mao, PRD 107, 074018 (2023)

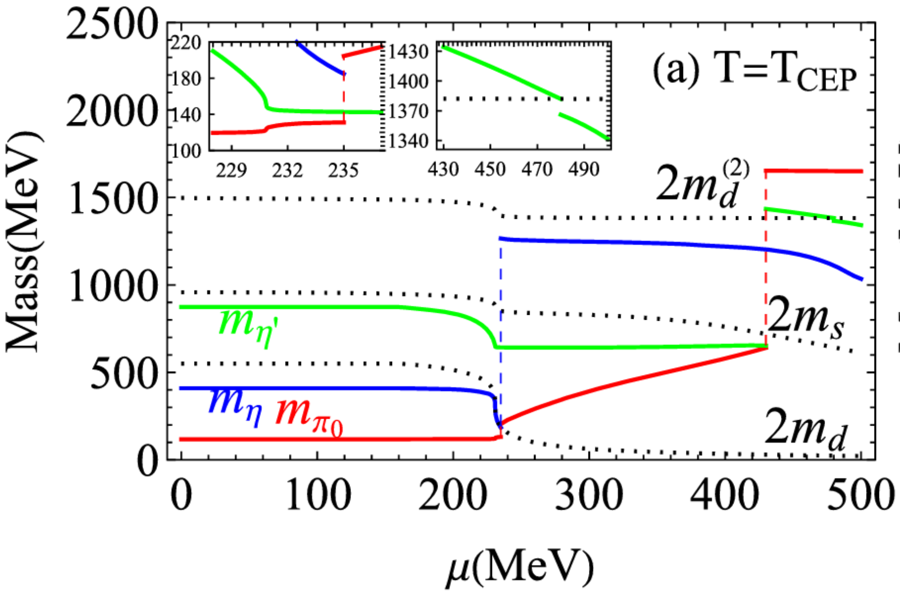


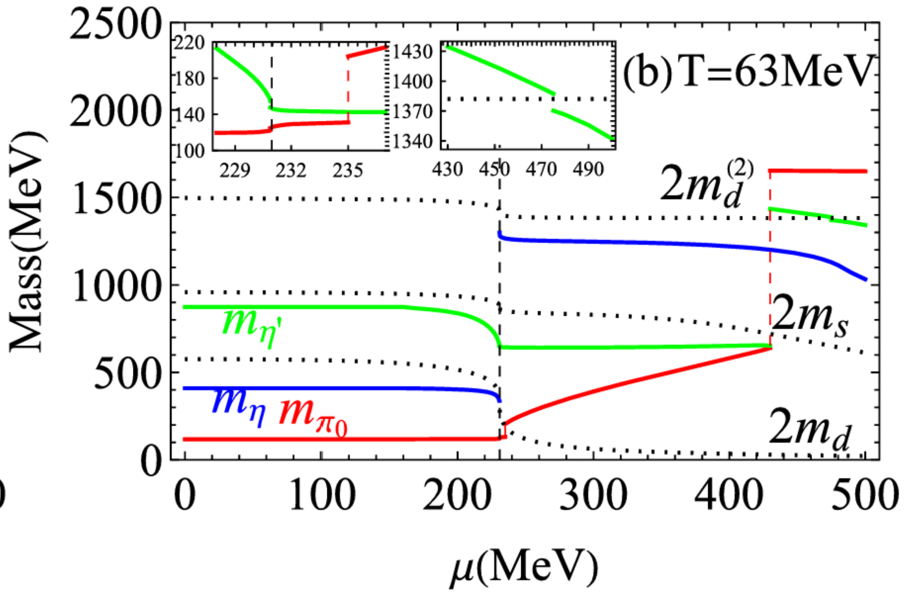




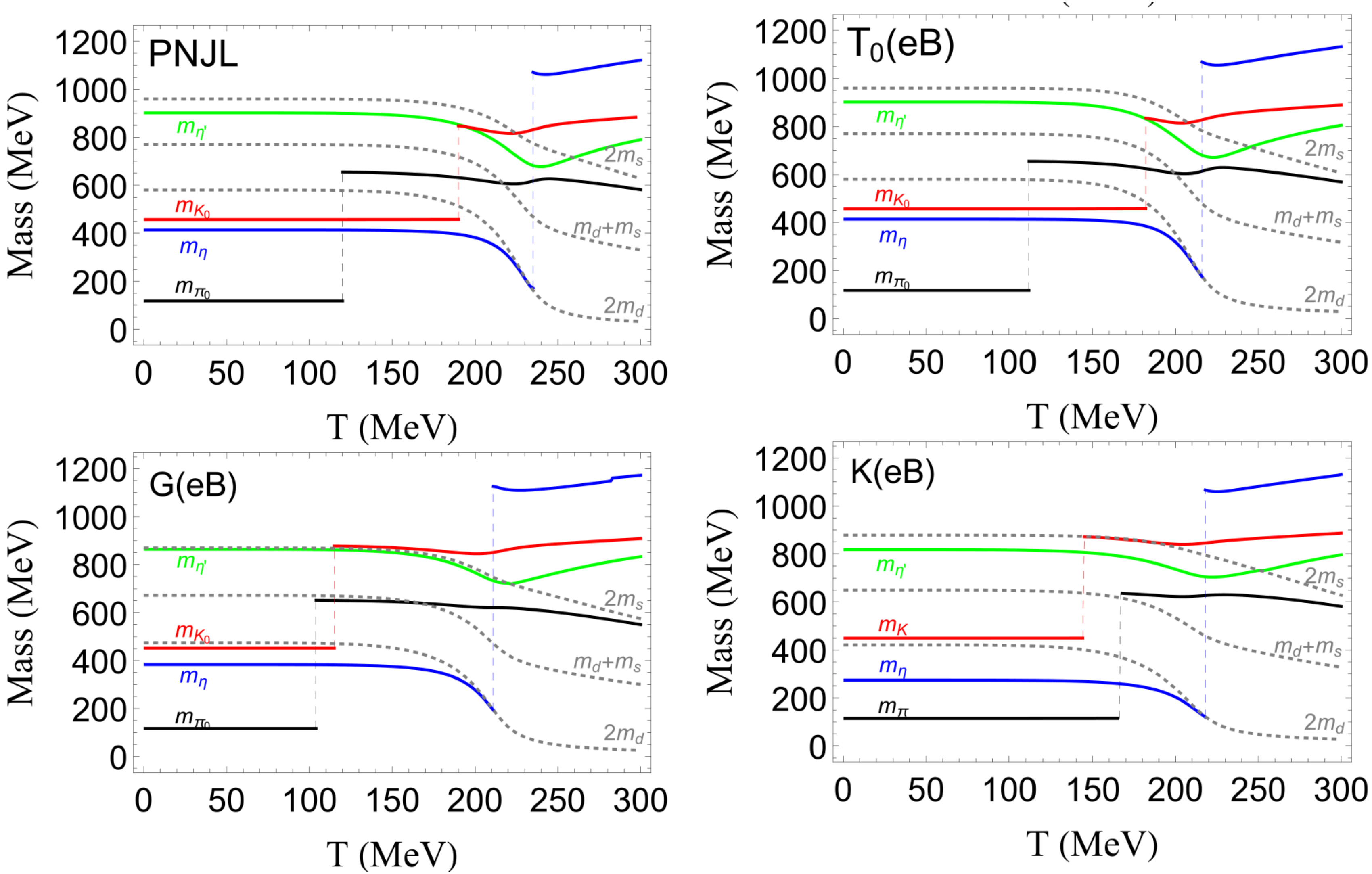


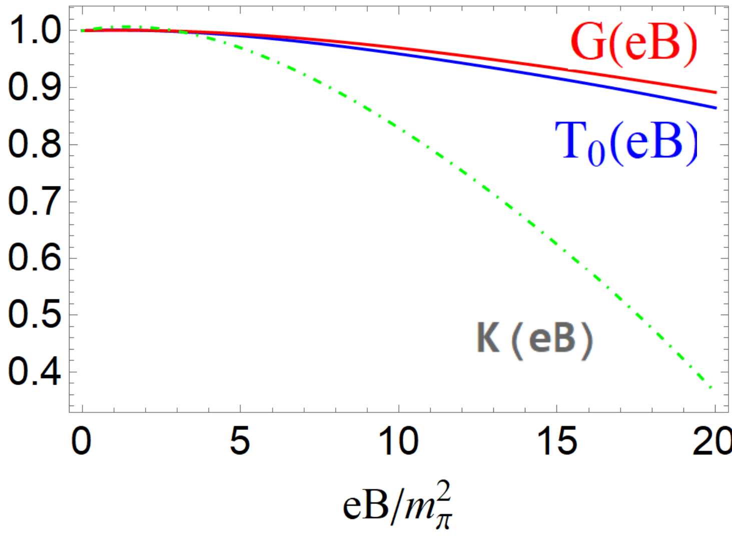






2.1 K_0, π_0, η, η' with IMC $\mu=0$





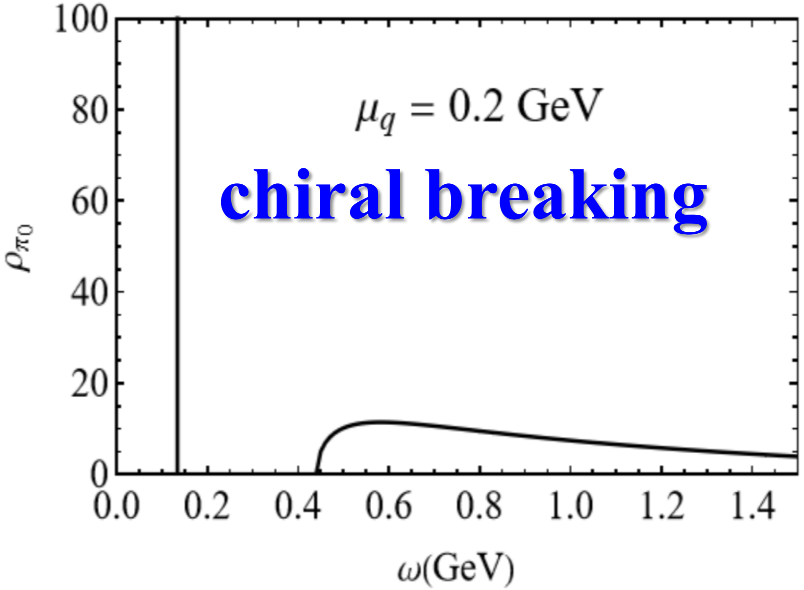
➢ **IMC leads to no qualitative difference**

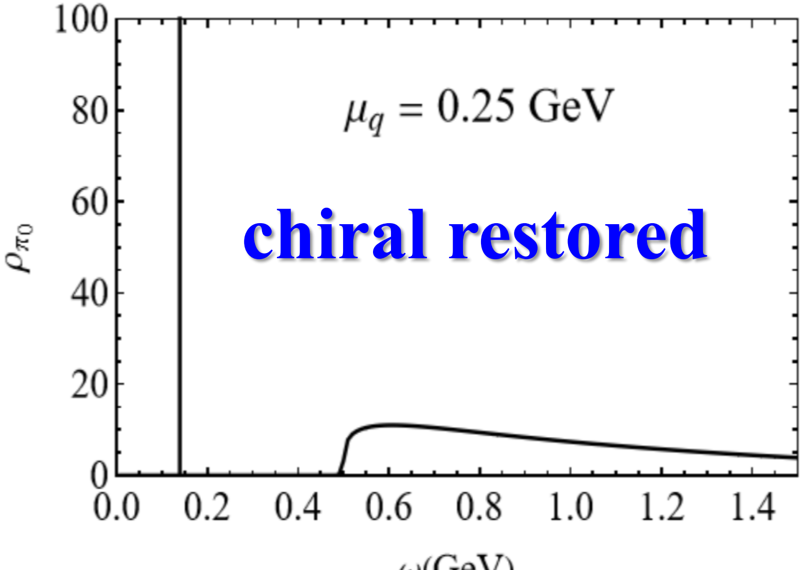
2.1 Spectral function of π_0 @ eB- μ

T=0

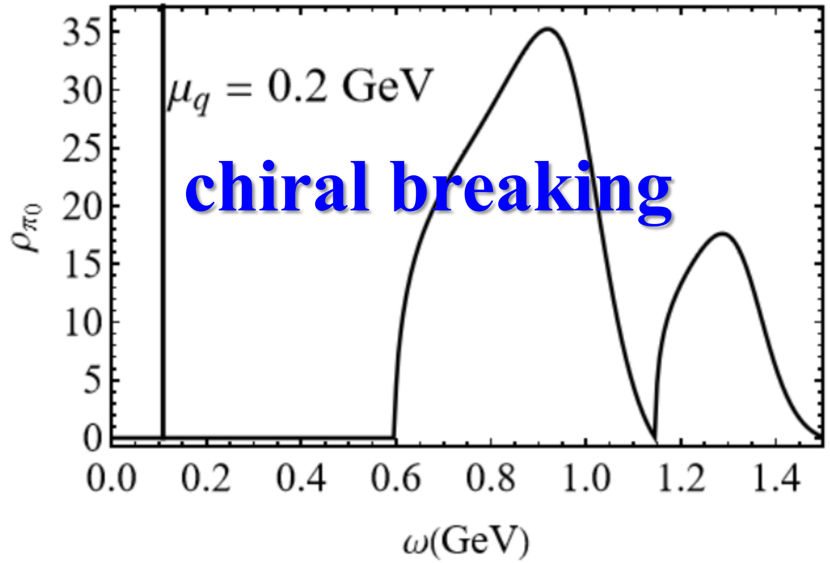
Zhou, **Mao**, arXiv:2025.XXXXX;
Wang, **Mao**, PRD 96, 034004 (2017)

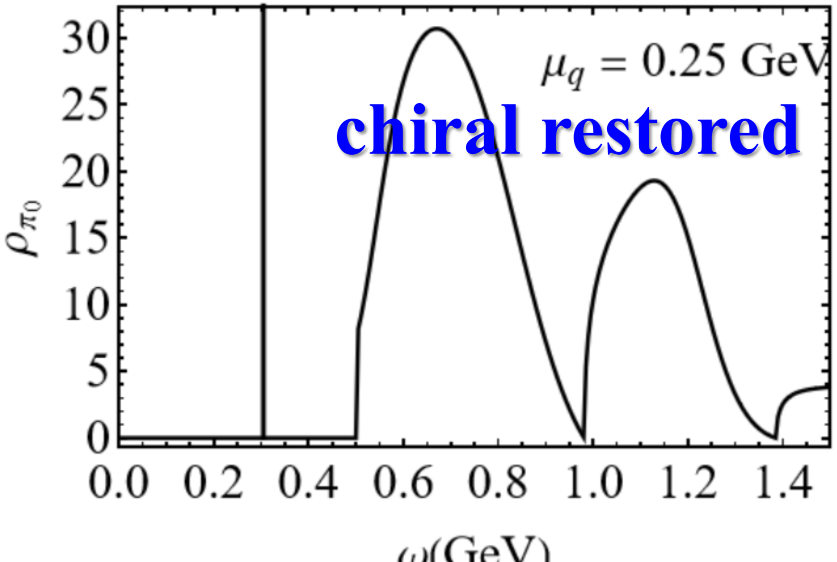
eB=0





finite eB (Landau levels)





一级相变引起质量跳变