A unitary coupled-channel approach to J/ψ radiative decay to pseudoscalar pairs

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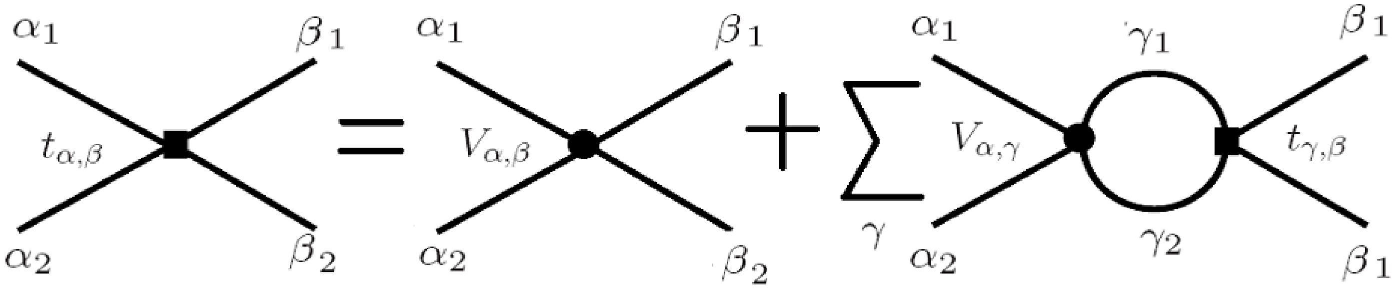
Outline

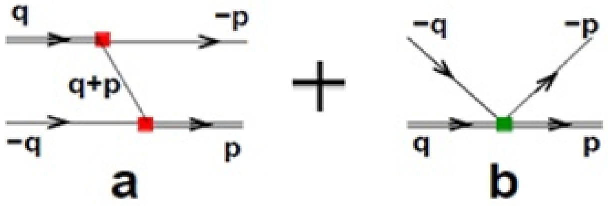
- Background
- Coupled-Channel model for $J/\psi \to \gamma PP$
- Toy model
- Summary

Background

- **Coupled Channel Approach: It describes interactions between different channels or pathways that a quantum system can take. Different possible states or interactions (channels) are considered simultaneously.**

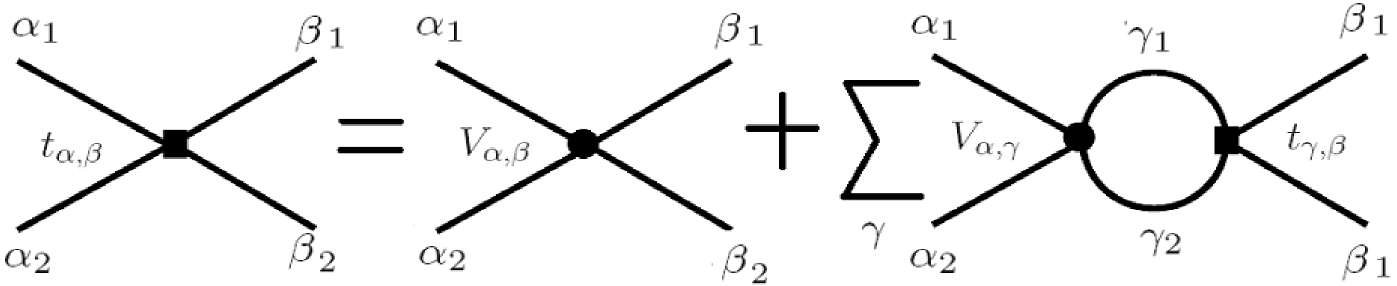


$$T=V+VGT$$



Background

- **Why Coupled Channel Approach?**
- **Non-perturbative effect / Loop contribution:** $\frac{1}{1-x} \sim 1 + x + x^2 + \cdots$
- **Unitary: |S|=1**
- **Unified: Connection between different processes, conjoint analysis**
- **Resonance description:** **BW:** $\frac{1}{s-m^2+i\Gamma m}$ VS Single channel : $\frac{1}{E-m_0-Re(\Sigma(E))+i\Gamma(E)/2}$
- **……**



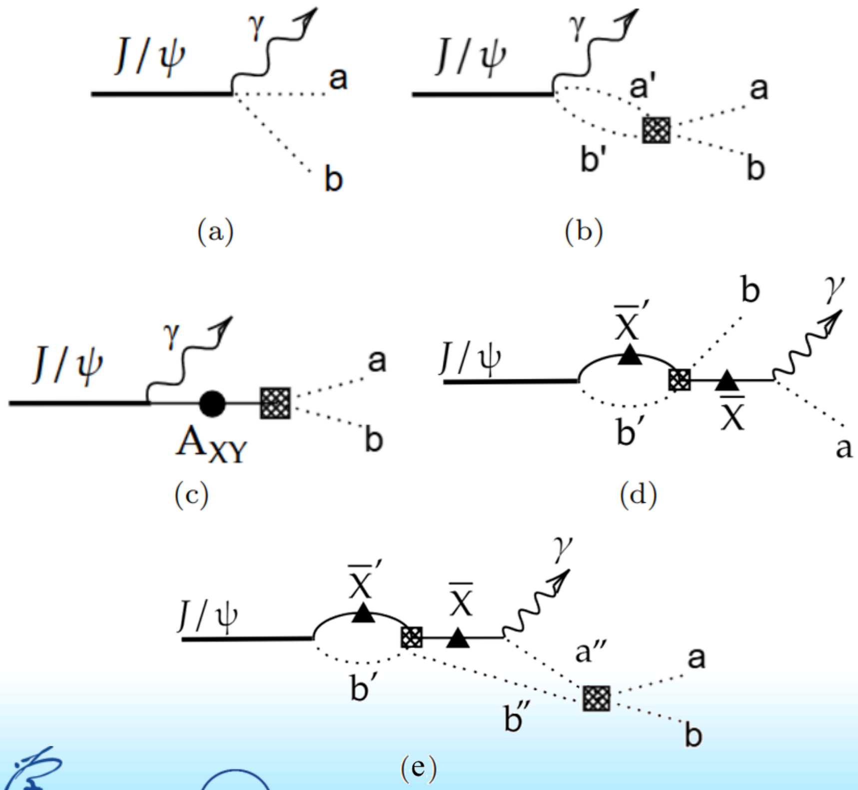
T=V+VGT

Coupled-Channel model for $J/\psi \to \gamma PP$

$J/\psi \to \gamma PP$: Three body final state, but electromagnetic interaction is much smaller than strong interaction, thus γP re-scattering is negligible.

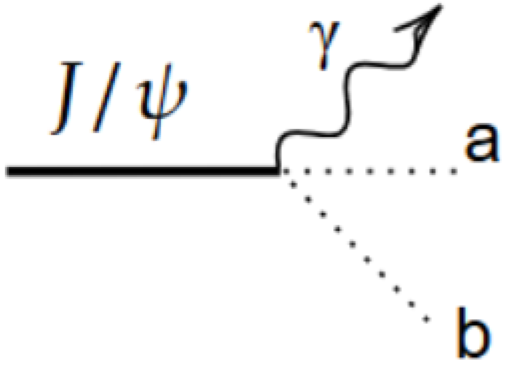
$J/\psi \to \gamma PP$:

a. bare coupling
b. PP re-scattering
c. $X \to PP$
d. $\bar{X}P$ re-scattering and then $\bar{X} \to \gamma P$
e. d plus PP re-scattering



Coupled-Channel model for $J/\psi \to \gamma PP$

$J/\psi \to \gamma PP$: a. bare coupling



(a)

$$\mathcal{M}^{(a)}_{J/\psi\to\ \gamma\alpha} = \sum_{L_1,L_2,L_1^z,L_2^z,S,S^z} \langle 1S^z_\gamma, L_2 L_2^z | S S^z\rangle \times \langle S S^z, L_1 L_1^z | 1 S^z_{J/\psi}\rangle \times \boxed{u^{L_1 L_2 S}_{J/\psi\to\ \gamma\alpha}} Y_{L_1 L_1^z}(-\hat{q}_\gamma) \boxed{f^{L_1}_{(J/\psi,\gamma)}(q_\gamma)} \times Y_{L_2 L_2^z}(\hat{q}_\alpha) \boxed{f^{L2}_\alpha(q_\alpha)},$$

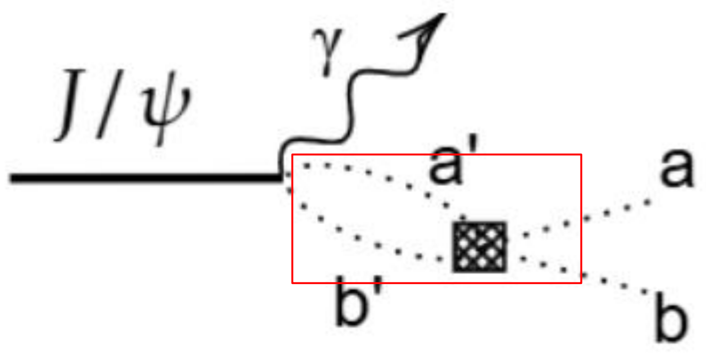
Lorentz structure

$$\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu_{J/\psi}\epsilon^{*\nu}_\gamma P^\beta_{(J/\psi)}\tilde{t}^{(1)\alpha}$$

$$f^L_\alpha(k) = \frac{(1+\frac{k^2}{\Lambda^2})^{-2-L/2}\cdot(k/m_0)^L}{\sqrt{E_{\alpha_1}(k)E_{\alpha_2}(k)}}$$

Coupled-Channel model for $J/\psi \to \gamma PP$

$J/\psi \to \gamma PP$: b. PP re-scattering



(b)

$$\mathcal{M}^{(b)}_{J/\psi\to\ \gamma\alpha} = \sum_{L_1,L_2,L_1^z,L_2^z,S,S^z} Y_{L_1L_1^z}(-\hat{q}_\gamma) f^{L_1}_{(J/\psi,\gamma)}(q_\gamma)$$
$$\times \langle 1S^z_\gamma, L_2L_2^z|SS^z\rangle\langle SS^z, L_1L_1^z|1S^z_{J/\psi}\rangle$$
$$\times \sum_\beta u^{L_1L_2S}_{J/\psi\to\ \gamma\beta} \boxed{M^{L_2}_{\beta\beta}(E)\tilde{t}^{L_2}_{\beta\alpha}(E) f^{L_2}_\alpha(q_\alpha)} Y_{L_2L_2^z}(\hat{q}_\alpha)$$

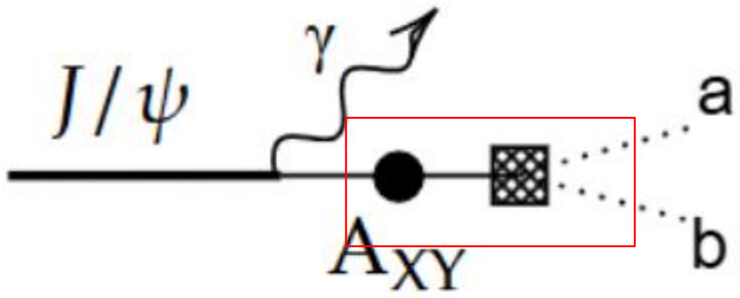
$$t^L_{\alpha\beta}(k,k';E) \quad = V^L_{\alpha\beta}(k,k') + \sum_\gamma \int dq\, q^2 \frac{V^L_{\alpha\gamma}(k,q)t^L_{\gamma\beta}(q,k';E)}{E-\omega_\gamma(q)+i\epsilon}$$

$$t^L_{\alpha\beta}(k,k',E) = f^L_\alpha(k)\tilde{t}^L_{\alpha\beta}(E) f^L_\beta(k'),$$
$$\tilde{t}^L_{\alpha\beta}(E) = \sum_\gamma \left(I - v^L M(E)\right)^{-1}_{\alpha\gamma} v^L_{\gamma\beta}.$$

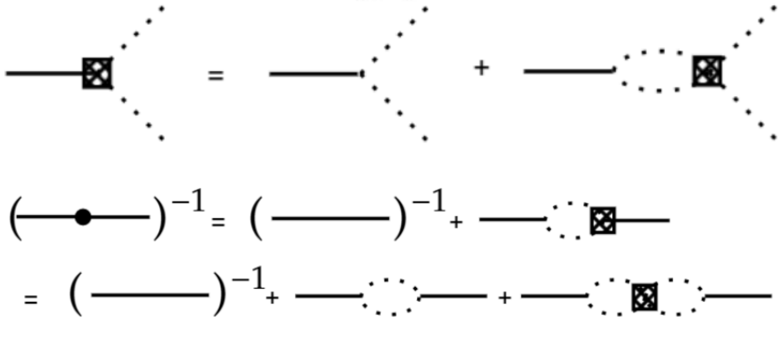
$$M_{\alpha\alpha}(E) = \int \frac{q^2 dq\, |f_\alpha(q)|^2}{E - w_\alpha(q) + i\epsilon}$$

Coupled-Channel model for $J/\psi \to \gamma PP$

$J/\psi \to \gamma PP$: c. $X \to PP$



(c)



$$\mathcal{M}^{(c)}_{J/\psi\to\ \gamma\alpha} = \sum_{L_1,L_2,L_1^z,L_2^z,S,S^z} Y_{L_1L_1^z}(-\hat{q}_\gamma) f^{L_1}_{(J/\psi,\gamma)}(q_\gamma)$$
$$\times \langle 1S^z_\gamma, L_2L_2^z|SS^z\rangle\langle SS^z, L_1L_1^z|1S^z_{J/\psi}\rangle$$
$$\times \sum_{XY} \boxed{u^{L_1L_2S}_{J/\psi\to\ \gamma X}} \boxed{A_{XY}(E)\mathcal{G}^{L_2}_{Y\alpha}(q_\alpha, E)} Y_{L_2L_2^z}(\hat{q}_\alpha)$$

Additional Vertex for $J/\psi \to \gamma\ X$

$$\mathcal{G}^L_{X\alpha}(k;E) = G^L_{X\alpha}(k) + \sum_\gamma g_{X\gamma}(E)\tilde{t}^L_{\gamma\alpha}(E) f_\alpha(k)$$

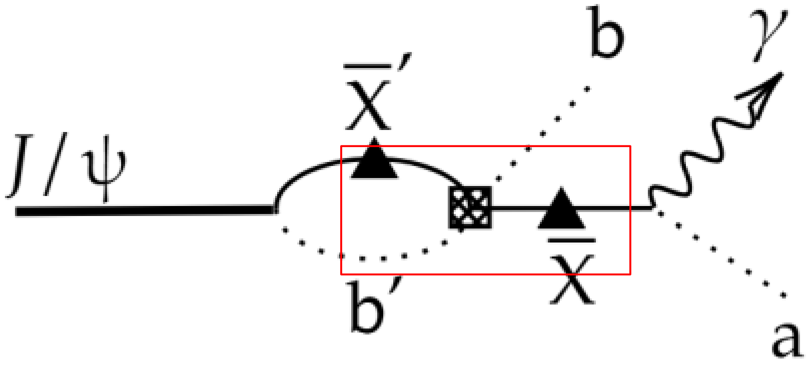
$$(A^{-1})_{XY}(E) = \delta_{XY}(E - m_X) - \Sigma^0_{XY}(E) - \Sigma^I_{XY}(E)$$

$$\Sigma^0_{XY}(E) = \sum_\gamma \int dq q^2 \frac{G_{X\gamma}(q)G_{Y\gamma}(q)}{E - \omega_\gamma(k) + i\epsilon}, \quad \Sigma^{\mathrm{I}}_{XY}(E) = \sum_{\alpha,\beta} g_{X\alpha}(E)\tilde{t}^L_{\alpha\beta}(E) g_{Y\beta}(E)$$

Coupled-Channel model for $J/\psi \to \gamma PP$

Additional Vertex for $J/\psi \to \bar{Y}c$

$J/\psi \to \gamma PP$: d. $\bar{X}P$ re-scattering and then $\bar{X} \to \gamma P$



(d)

$$\mathcal{M}^{(d)}_{J/\psi\to\gamma\alpha} = \sum_{L_1,L_1^z}\sum_{\bar{Y}c,S^z_{\bar{Y}}} Y^*_{L_1L_1^z}(\hat{q}_c)\langle S_{\bar{Y}}S^z_{\bar{Y}},L_1L_1^z|1S^z_{J/\psi}\rangle$$

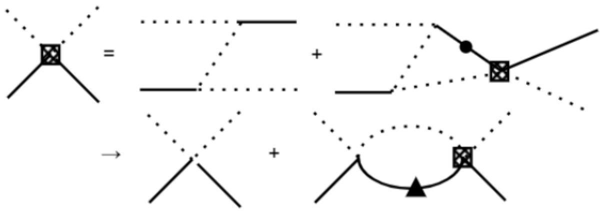
$$\times \boxed{u^{L_1}_{J/\psi\to\bar{Y}c}}\bar{M}_{\bar{Y}c}(m_{J/\psi})$$

$$\times \sum_{L_2,L_2^z}\sum_{\bar{X},S^z_{\bar{X}}}\langle S_{\bar{X}}S^z_{\bar{X}},L_2L_2^z|1S^z_{J/\psi}\rangle Y_{L_2L_2^z}(\hat{q}_b)$$

$$\times \bar{t}_{\bar{Y}c\bar{X}b}(m_{J/\psi})f^{L_1}_{\bar{X}b}(q_b)$$

$$\times \sum_{L_3L_3^z}\langle 1S^z_\gamma,L_3L_3^z|S_{\bar{X}}S^z_{\bar{X}}\rangle Y_{L_3L_3^z}(-\hat{q}_\gamma)$$

$$\times \frac{f^{L_3}_{\bar{X}\to\gamma a}(q_a)}{E(q_a)-m_{\bar{X}}+i\,\Gamma/2},$$

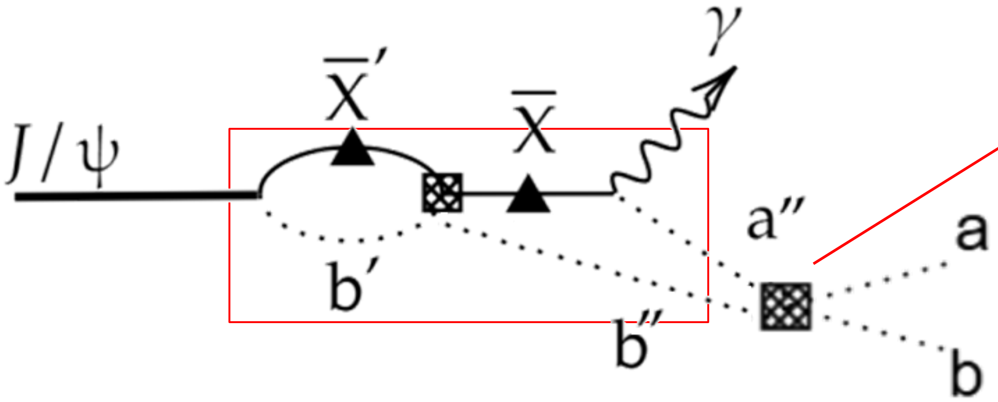


$$\bar{t} = (1-\bar{v}\bar{M})^{-1}\bar{v},$$

$$\bar{M}_{(aX)} = \int dq\,q^2\frac{|f_{(aX)}(q)|^2}{E-\omega_{(aX)}(q)+i\,\Gamma/2},$$

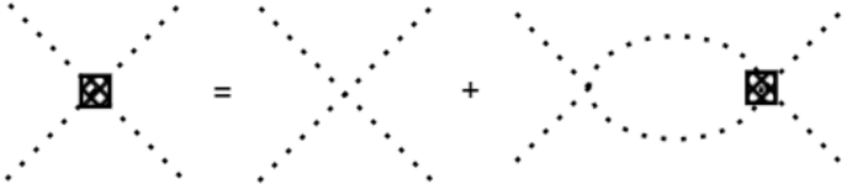
Coupled-Channel model for $J/\psi \to \gamma PP$

$J/\psi \to \gamma PP$: e. d plus PP re-scattering



(e)

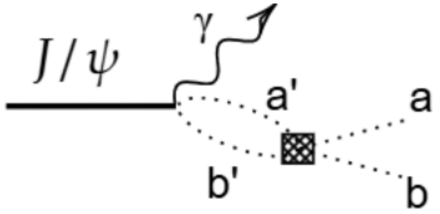
Full Amplitude



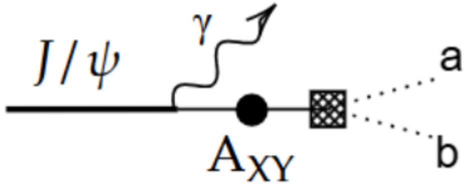
$$\tilde{T}^L_{\alpha\beta}(k,k';E) = \tilde{V}^L_{\alpha\beta}(k,k';E) + \sum_{\gamma} \int dq\, q^2 \frac{\tilde{V}^L_{\alpha\gamma}(k,q;E)\tilde{T}^L_{\gamma\beta}(q,k';E)}{E-\omega_\gamma(q)+i\epsilon}$$

$$\tilde{V}^L_{\alpha\beta}(k,k',E) = V^L_{\alpha\beta}(k,k') + \sum_X \frac{G^L_{X\alpha}(k)G^L_{X\beta}(k')}{E-m_X+i\epsilon}$$

$$\tilde{T}^L = t^L + T^L$$



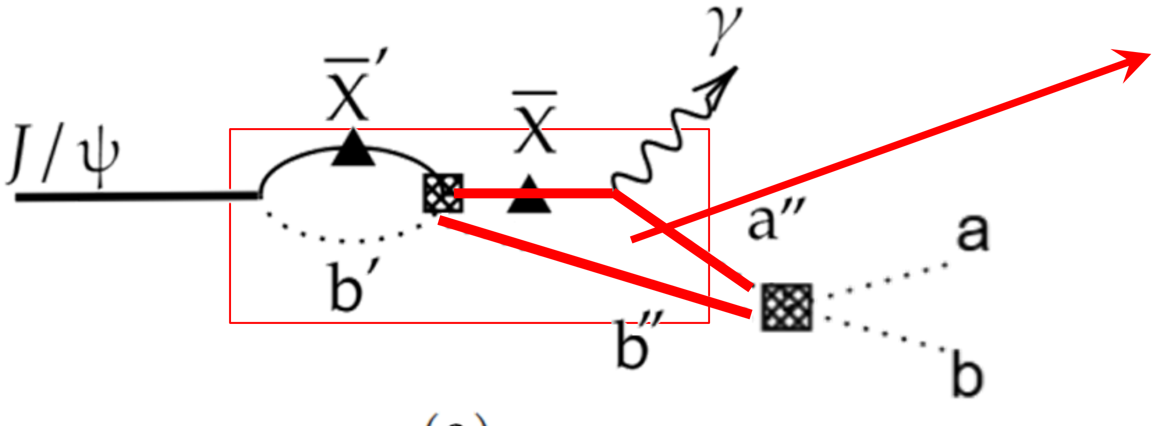
(b)



(c)

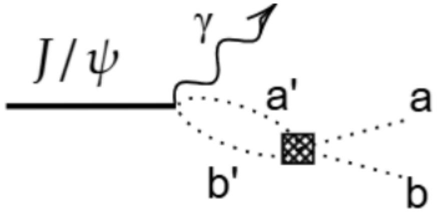
Coupled-Channel model for $J/\psi \to \gamma PP$

$J/\psi \to \gamma PP$: e. d plus PP re-scattering



(e)

Triangle Singularity



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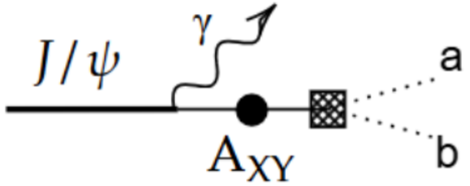


(c)

We should consider it additionally, here we do not express explicitly.

Toy model

$J/\psi \to \gamma K_S K_S$ and $J/\psi \to \gamma \pi^0 \pi^0$

Two 0^+ bare states, $f_0 \sim$ m = 1.221 GeV and $f_0' \sim$ m = 1.451 GeV
One 1^+ bare state for γK_S channel, $K_1 \sim m = 1.403$ GeV and $\Gamma = 0.173$ GeV

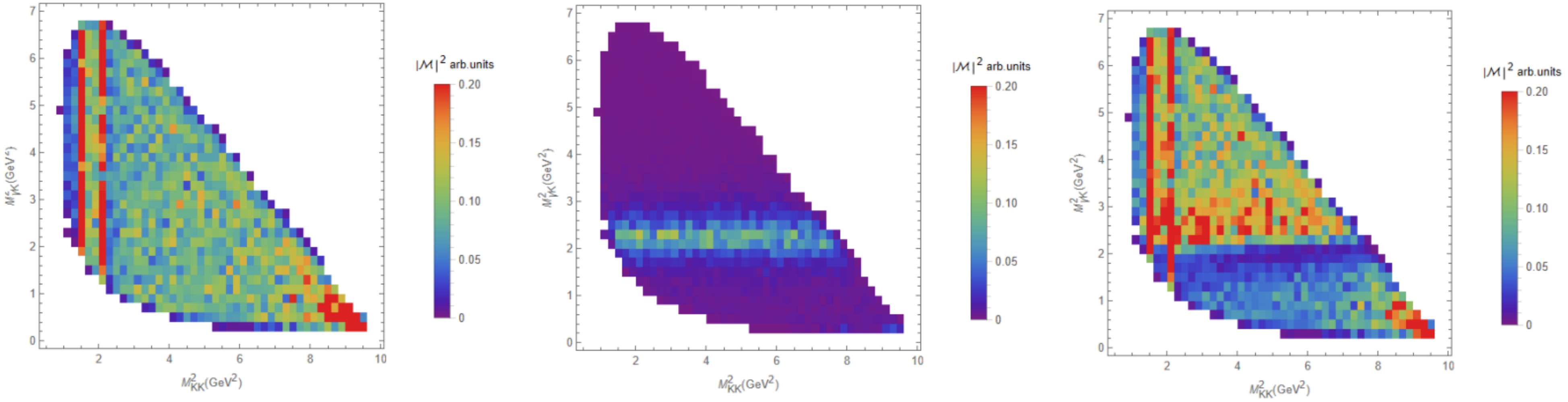
$R\ \{L\}$	$f_0\ \{0\}$	$f_0'\ \{1\}$
m_R	1.221	1.457
$g_{(R,KK)}$	0.810	-0.710
$g_{(R,\pi\pi)}$	-0.740	0.910
$u_{(J/\psi,\gamma R)}$	0.300	0.500
γPP		
$v_{KK,KK}$	0.980	-
$v_{KK,\pi\pi}$	0.870	-
$v_{\pi\pi,\pi\pi}$	0.980	-
$u_{(J/\psi,\gamma KK)}$	0.800	-
$u_{(J/\psi,\gamma\pi\pi)}$	0.800	-

R $\{L\}$	$K_1\ \{0\}$	
m_R	1.403	-
Γ_R	0.174	-
$\bar{u}_{(J/\psi,KR)}$	1.400	-
$\bar{v}_{(KR,KR)}$	1.000	-
$\bar{g}_{(R,\gamma K)}$	1.000	-

R$\{L\}$	M_{pole}(GeV)	RS
$f_0\{0\}$	$1.225 - 0.010\ i$	(uu)
$f_0'\{0\}$	$1.457 - 0.007\ i$	(uu)
$K_1\{0\}$	$1.403 - 0.087\ i$	-

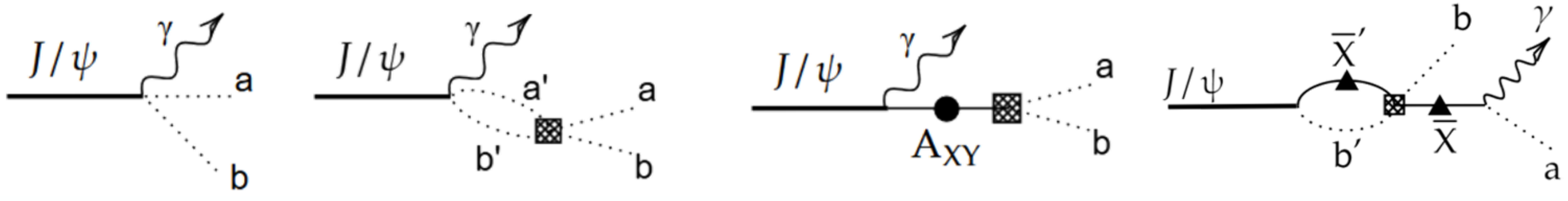
Toy model

R{L}	M_{pole}(GeV)	RS
$f_0\{0\}$	$1.225 - 0.010\ i$	(uu)
$f_0'\{0\}$	$1.457 - 0.007\ i$	(uu)
$K_1\{0\}$	$1.403 - 0.087\ i$	-



(a) (b) (c)

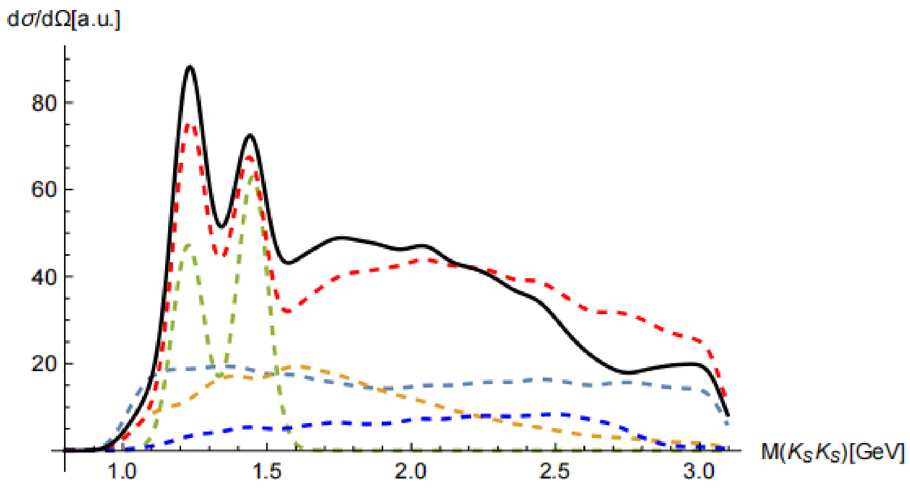
FIG. 5: The Dalitz plot on $M^2(KK)$ and $M^2(\gamma K)$for different amplitudes' square.(a) Tree-level, PP-rescattering and Bare states;(b) XP-rescattering; (c) All amplitudes.



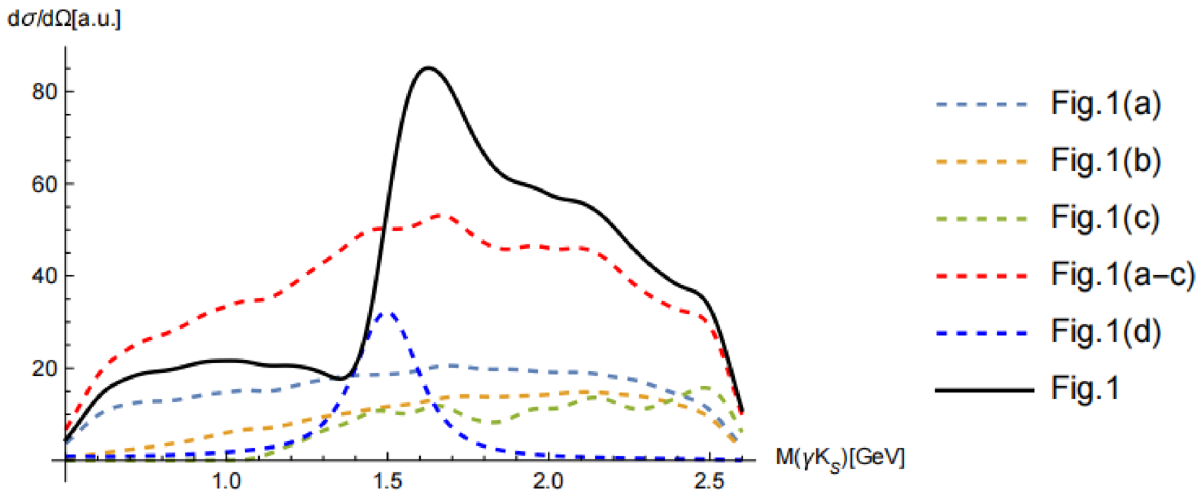
(a) (b) (c) (d)

Toy model

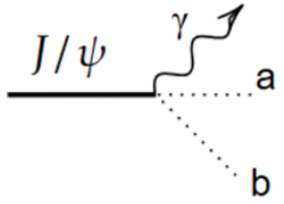
R{L}	M_{pole}(GeV)	RS
$f_0\{0\}$	$1.225 - 0.010\ i$	(uu)
$f'_0\{0\}$	$1.457 - 0.007\ i$	(uu)
$K_1\{0\}$	$1.403 - 0.087\ i$	-



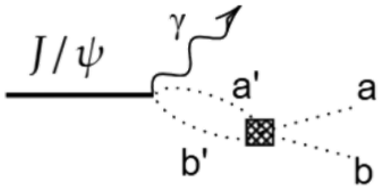
(a)



(b)



(a)



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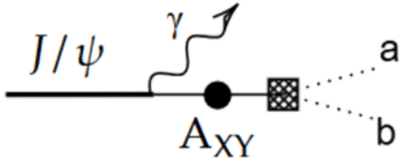
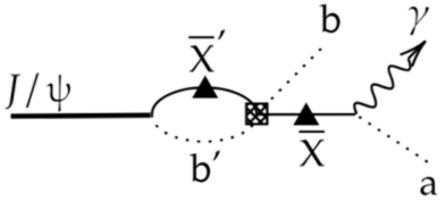


(c)



(d)

Summary

- We give a coupled-channel approach for $J/\psi \to \gamma PP$
- We also give numerical calculation based on a toy model
- Such formalism will be used in the BESIII analysis in future.

Thanks very much!