

NRQCD predictions of double J/ψ hadroproduction beyond LO

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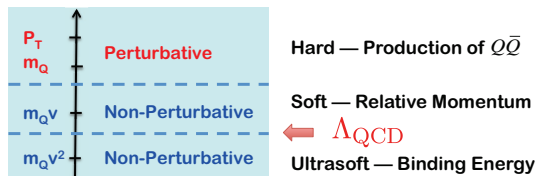
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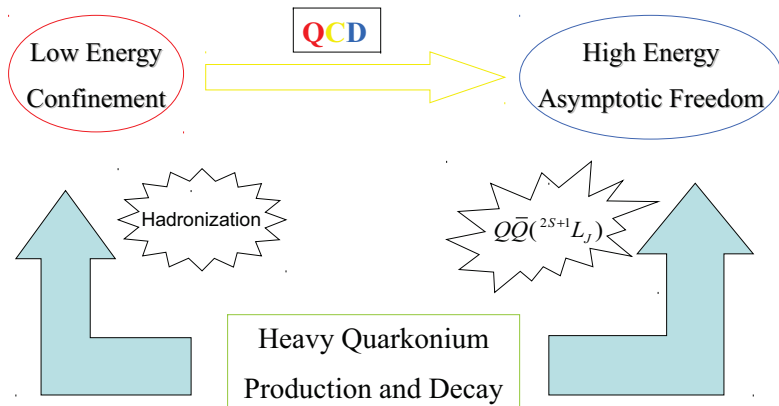
What is heavy quarkonium?

- Heavy quarkonium is one of the simplest QCD bound state constituted by heavy quark pair $Q\bar{Q}$ ($\Lambda_{QCD} \ll m_Q$).
- It is labeled by the spectroscopic notation $n^{2S+1}L_J$.
 - Its parity $P = (-1)^{L+1}$.
 - Its charge conjugation parity $C = (-1)^{L+S}$.
- There are 3 typical energy scales besides Λ_{QCD} :



- It is an approximately non-relativistic system:
 - Charmonium: $v_c^2 \approx 0.23 \ll 1$
 - Bottomonium: $v_b^2 \approx 0.08 \ll 1$

Quarkonium production—The ideal laboratory to probe QCD



J/ψ and Υ are ideal candidates for their large leptonic branching functions!

Factorization of heavy quarkonium production

- The general factorization formalism:

$$\sigma_{(A+B \rightarrow \text{quarkonium} + X)} = \sum_n \int \sigma_{A+B \rightarrow (Q\bar{Q})_n + X} \times f[(Q\bar{Q})_n \rightarrow \text{quarkonium}]$$

- The models in market include color-singlet model (CSM), color evaporation model (CEM), and nonrelativistic QCD (NRQCD) factorization.

Color Singlet Model

- Short Distance Part: The produced $Q\bar{Q}$ pair shares **the same quantum number** as the heavy meson.
- Long Distance Part: The non-relativistic wave function or its derivatives at origin.
- Non-factorizable in production and decay: S-wave relativistic corrections, P-wave at v^2 LO.

NRQCD Factorization formalism

NRQCD Factorization

- Short Distance Part: The produced $Q\bar{Q}$ pair can be **any possible quantum numbers**, even in **color octet**.
- Long Distance Part: The relative size of contribution from each Fock state obeys velocity scaling rules.
- The infrared divergence is absorbed into the NLO QCD corrections to the long distance matrix elements.

The challenges to NRQCD:

- ① The long-standing J/ψ polarization puzzle.
- ② The universality of the NRQCD LDMEs for J/ψ production up to QCD NLO.
- ③ The success of CSM to account for J/ψ production in e^+e^- annihilation, and η_c meson hadroproduction at the LHC.

New features in quarkonium pair hadroproduction

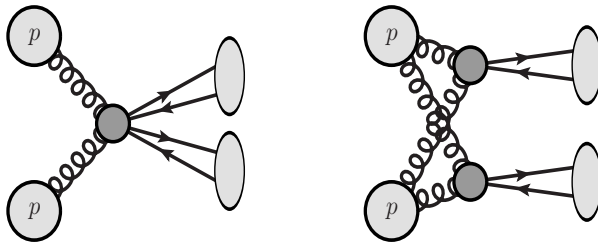
- ① Kinematically, it can be viewed as a gluon-gluon fusion type “Drell-Yan” process.
- ② To test NRQCD factorization or other models, and to provide crucial constraint on the long-distance matrix elements. (Barger et al. 1996)
- ③ Help to improve the NRQCD factorization formalism for double P -wave production. (He et al. 2018)
- ④ To extract σ_{eff} for DPS process. (Kom, et al. 2011)

Other benefits

- To help to understand the new fully heavy tetraquark states ($Q\bar{Q}Q\bar{Q}$). (LHCb Collaboration 2020)

Production mechanism of double quarkonium states

- Multi-final states can be produced through single parton scattering (SPS) or double parton scattering (DPS).



Schematical representation of SPS (left) and DPS (right) for a proton-proton collision. (Figure from Phys. Rev. D 95, 034029)

- The key parameter in DPS is σ_{eff} , which is supposed to be universal.

Experimental Status of hadroproduction

Table: The summary experimental measurements

Experiment	$\sqrt{s}$	states	kinematic condition
D0	1.96 TeV	$J/\psi + J/\psi$	$p_T^{J/\psi} > 4.0\text{GeV}, \eta^{J/\psi} < 2$
LHCb	7 TeV	$J/\psi + J/\psi$	$P_T^{J/\psi} < 10\text{GeV}$ $2.0 < y^{J/\psi} < 4.5$
	13 TeV	$J/\psi + J/\psi$ $J/\psi + \psi(2S)$	$P_T^{J/\psi} < 10(14)\text{GeV}$ $2.0 < y^{J/\psi} < 4.5$
CMS	7 TeV	$J/\psi + J/\psi$	$p_T^{J/\psi} > 4.5\text{GeV}, y^{J/\psi} < 2.2$
ATLAS	8 TeV	$J/\psi + J/\psi$	$p_T^{J/\psi} > 8.5\text{GeV}, y^{J/\psi} < 2.1$
ALICE	13 TeV	$J/\psi + J/\psi$	$p_T^{J/\psi} > 0\text{GeV}, 2.5 < y^{J/\psi} < 4.0$

Rich observables

- $\sigma, \frac{d\sigma}{dp_T^{\psi}}, \frac{d\sigma}{dy^{\psi}}, \frac{d\sigma}{dp_T^{\psi\psi}}, \frac{d\sigma}{dy^{\psi\psi}}, \frac{d\sigma}{dm^{\psi\psi}}, \frac{d\sigma}{d|\Delta y^{\psi\psi}|}, \frac{d\sigma}{d|\Delta\phi^{\psi\psi}|}, \frac{d\sigma}{dA_T^{\psi\psi}}.$

Theoretical study of quarkonium pair hadroproduction

- The J/ψ pair production was first proposed by Barger, et al. in 1996, in which the $2(c\bar{c}(^3S_1^{[8]}))$ contribution was studied.
- In 2002, Qiao found that the $2(c\bar{c}(^3S_1^{[1]}))$ channel contributes predominately to the total cross section.
- Baranov calculated the J/ψ pair production in k_T factorization approach in 2011.
- The NLO QCD corrections to the $2(c\bar{c}(^3S_1^{[1]}))$ channel was obtained by Sun et al. in 2014 and was updated in 2023.
- The complete LO NRQCD calculation including χ_c feed down were obtained by He and Kniehl in 2015.
- In 2019, He et al. studied the J/ψ pair production in the parton reggeization approach (PRA) with high-energy resummation.
- In 2023, the NLO QCD corrections to $2(c\bar{c}(^1S_0^{[8]}))$ channel was obtained by Sun.

NRQCD predictions @ D0 1.96 TeV

- The fiducial cross sections reported by D0 Collaborations:

$$\sigma_{D0, \text{fid}}^{\text{SPS}} = (70 \pm 6 \pm 22) \text{ fb}, \sigma_{D0, \text{fid}}^{\text{DPS}} = (59 \pm 6 \pm 22) \text{ fb},$$

- The CSM predictions in LO (Qiao and Sun 2013) , k_T factorization (Baranov 2013) and NLO* calculations (Lansberg and Shao 2013):

$$\sigma_{D0, \text{fid}}^{\text{SPS}} = 51.9 \text{ fb}, \sigma_{D0, \text{fid}}^{\text{SPS}, k_T} = 55.1_{-15.6}^{+28.5} {}_{-17.0}^{+31.0} \text{ fb}, \sigma_{D0, \text{fid}}^{\text{SPS}, \text{NLO}^*} = 90_{-50}^{+180} \text{ fb}.$$

- LO NRQCD predictions (He et al. 2021):

$$\sigma_{D0, \text{fid}}^{\text{SPS}, \text{NRQCD}} = 86.1_{-34.0}^{+59.7} \text{ fb},$$

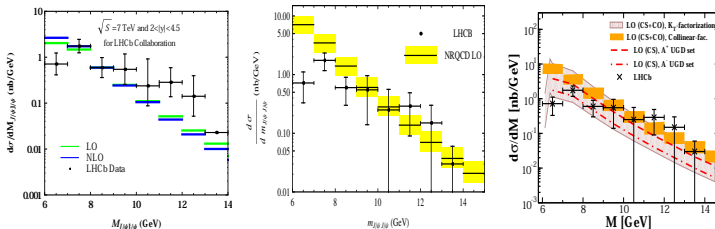
The D0 conclusion depended on the SPS inputs. When complete NRQCD calculation is taken into account their conclusion might change.

NRQCD predictions @ LHCb 7 TeV

- Total cross section (nb): CSM predictions at QCD NLO (Sun et al. 2016) and complete LO NRQCD (He, Kniehl 2015)

LHCb	LO CSM	NLO CSM	LO NRQCD
$5.1 \pm 1.0 \pm 1.1$	4.56 ± 1.13	$5.41^{+2.73}_{-1.14}$	$13.2^{+5.2}_{-4.1}$

- The invariant mass spectrum: NLO CSM, NRQCD LO, and k_T (Baranov, Rezaeian 2016)



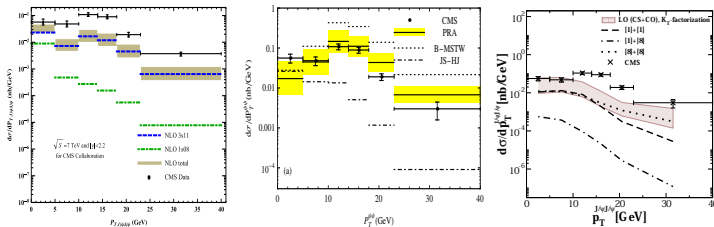
The theoretical predictions near threshold region strongly depend on the choice of m_c .

NRQCD predictions @ CMS 7 TeV I

- Total cross section (nb): NLO CSM (Sun et al. 2023), complete LO NRQCD (He, Kniehl 2015), PRA (He et al. 2019)

CMS	LO CSM	NLO CSM	LO NRQCD	PRA
1.49 ± 0.07	0.048	0.18	$0.15^{+0.08}_{-0.05}$	$1.68^{+1.32}_{-0.78}$

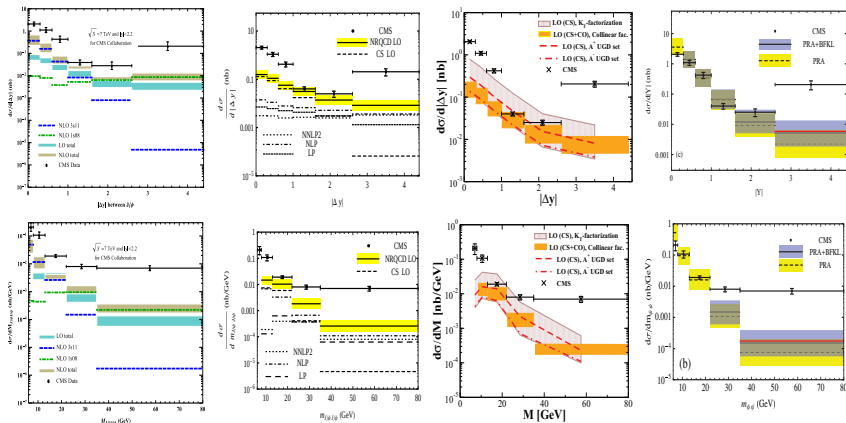
- The J/ψ pair p_T distribution including prediction of k_T factorization (Baranov and Rezaeian 2016):



Different way of generating the k_T dependent PDF could lead to complete different predictions.

NRQCD predictions @ CMS 7 TeV II

- $|\Delta y|$ and $m_{\psi\psi}$ distributions: NLO CSM, LO NRQCD, k_T and PRA



T-channel gluon exchange effect is significant and BKFL resummation can further increase fixed order calculations by a factor of 2.

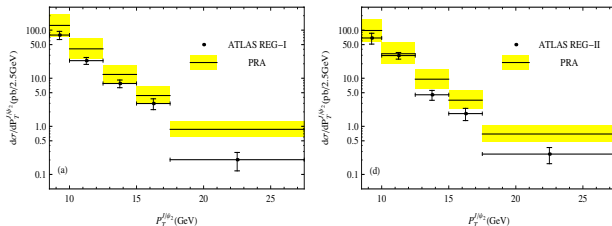
NRQCD predictions @ ATLAS 8 TeV I

- Total cross section: PRA (He et al. 2019)

$$\sigma(pp \rightarrow J/\psi J/\psi + X) = \begin{cases} 82.2 \pm 8.3 \text{ (stat)} \pm 6.3 \text{ (syst)} \pm 0.9 \text{ (BF)} \pm 1.6 \text{ (lumi)} \text{ pb, for } |y| < 1.05, \\ 78.3 \pm 9.2 \text{ (stat)} \pm 6.6 \text{ (syst)} \pm 0.9 \text{ (BF)} \pm 1.5 \text{ (lumi)} \text{ pb, for } 1.05 \leq |y| < 2.1. \end{cases}$$

$$\sigma_{\text{ATLAS}}^{\text{PRA}} = \begin{cases} 133.6^{+89.6}_{-52.2} \text{ pb, for } |y(J/\psi_2)| < 1.05 \\ 105.2^{+73.8}_{-41.6} \text{ pb, for } 1.05 < |y(J/\psi_2)| < 2.1 \end{cases}$$

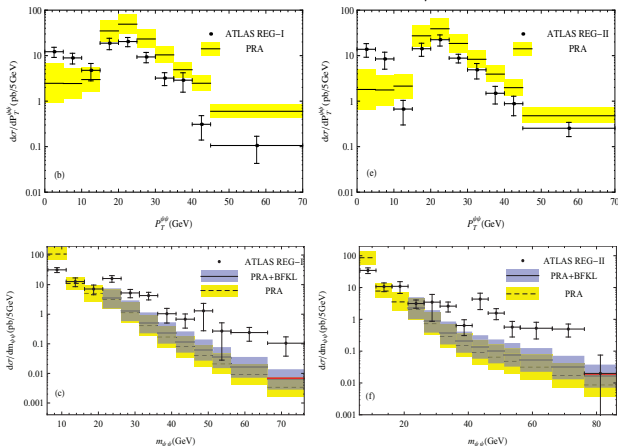
- p_T distribution of single J/ψ



PRA predictions agree fairly well with ATLAS measurements.

NRQCD predictions @ ATLAS 8 TeV II

- p_T and invariant mass distributions of the J/ψ pair:



SPS can not account for the invariant mass distribution in the upper bins.

Why shall we re-calculate the NLO QCD corrections?

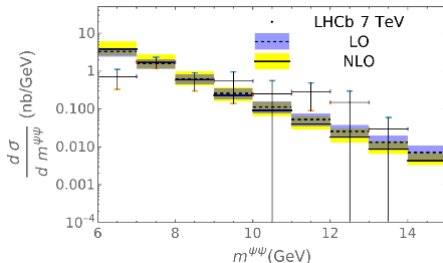
- Theoretically, the NLO QCD corrections become much more complicated when bound states are involved.
- In single J/ψ production, almost all channels were evaluated by more than one group.
- We calculated the NLO QCD corrections to the CS $gg \rightarrow 2c\bar{c}(^3S_1^{[1]})$.
- We found agreement upon adjusting inputs and eliminating a minor inconsistency in the paper of Sun et al. .
- We presented an up-to-date comparison with the latest LHC measurements.

CS NLO(α_s) @ LHCb 7 TeV

- Total cross section:

$$\sigma_{\text{LHCb}}^{\text{Tot}} = (5.1 \pm 1.0 \pm 1.1) \text{ nb},$$
$$\sigma_{\text{CS}}^{\text{LO}} = 6.02_{-1.59}^{+1.56} \text{ nb}, \sigma_{\text{CS}}^{\text{NLO}} = 6.46_{-2.01}^{+3.99} \text{ nb}.$$

- The K-factor of total cross section is about 1.07.
- The invariant mass spectrum of the J/ψ pair:



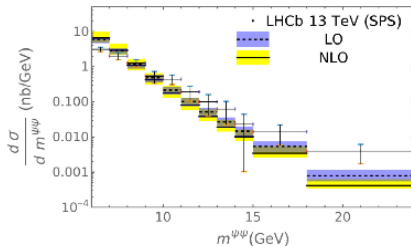
- The NLO QCD corrections near threshold region are positive, and become negative as $m^{\psi\psi}$ increases.

CS NLO(α_s) @ LHCb 13 TeV I

- Total cross section:

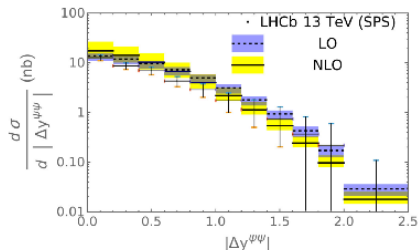
$$\sigma_{\text{LHCb}}^{\text{Tot}} = (16.36 \pm 0.28 \pm 0.88) \text{ nb}, \quad \sigma_{\text{LHCb}}^{\text{SPS}} = (7.9 \pm 1.2 \pm 1.1) \text{ nb},$$
$$\sigma_{\text{CS}}^{\text{LO}} = 10.9^{+1.4}_{-2.5} \text{ nb}, \quad \sigma_{\text{CS}}^{\text{NLO}} = 11.4^{+5.8}_{-3.2} \text{ nb}.$$

- The K-factor of total cross section is only about 1.05.
- The invariant mass spectrum of the J/ψ pair:



- When $m^{\psi\psi} > 15\text{GeV}$, the CS contribution tends to underestimate the LHCb measurements leaving room for CO contribution.

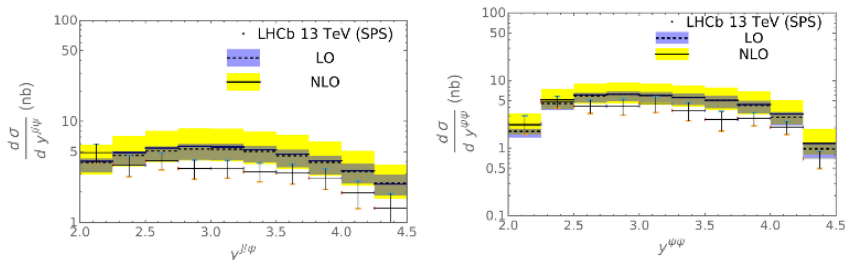
- The $|\Delta y^{\psi\psi}|$ distribution of the J/ψ pair:



- The NLO QCD corrections barely shift the LO predictions for $|\Delta y^{\psi\psi}| < 1$ region.
- When $|\Delta y^{\psi\psi}| > 1$, the NLO QCD corrections result in a reduction from 30% to 40%.
- Unlike the invariant mass spectrum, in the last bin there is no room for CO contribution.

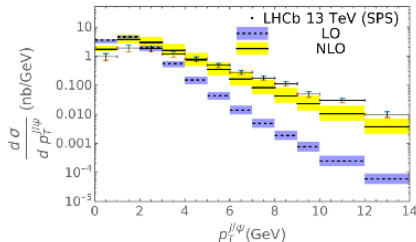
CS NLO(α_s) @ LHCb 13 TeV III

- The rapidity distributions of single J/ψ (left) and J/ψ pair (right):



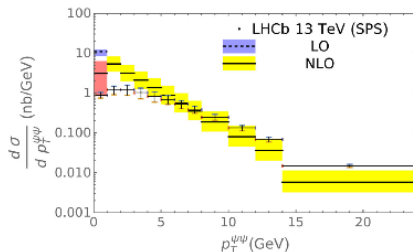
- The tiny effect of NLO QCD corrections are almost uniform in the whole region of $2.0 < y^\psi(y^{\psi\psi}) < 4.5$.
- In most bins, the central values of CS predictions lie slightly above the experimental data.
- CS predictions can reasonably well describe the data within errors.

- The p_T distribution of single J/ψ :



- The LO prediction is about 3.7 times larger than LHCb measurements in the first bin, while falls off by more than 2 orders of magnitude in the last bin.
- The NLO QCD corrections bring about -51% decrease and 61 times increase in the first and last bins.
- The NLO QCD corrections greatly moderate the conflict!

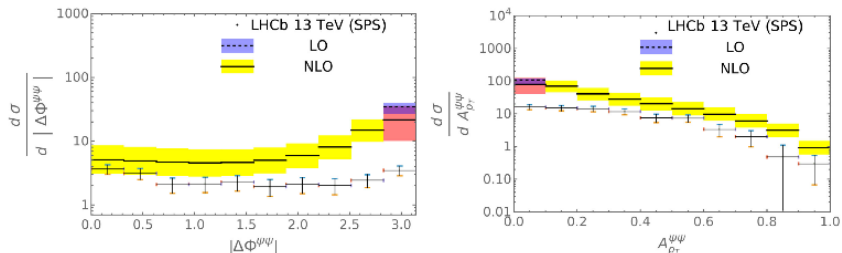
- The p_T distribution of J/ψ pair:



- In collinear parton model (CPM), only the $2 \rightarrow 3$ real corrections contribute when $p_T^{\psi\psi} \neq 0$.
- In small $p_T^{\psi\psi}$ region, perturbation theory is spoiled due to soft, hard-collinear gluons, and quarks radiation.
- Yet, as the value of $p_T^{\psi\psi}$ becomes larger, the NLO prediction falls off faster than experimental data.

CS NLO(α_s) @ LHCb 13 TeV VI

- The $|\Delta\Phi^{\psi\psi}|$ (left) and $A_{p_T}^{\psi\psi}$ (right) distributions of J/ψ pair:



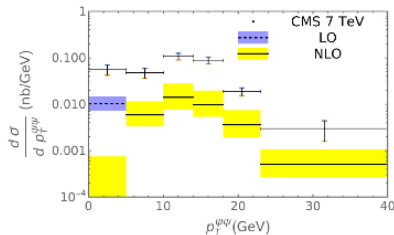
- At LO, $|\Delta\Phi^{\psi\psi}| = \pi$, $A_{p_T}^{\psi\psi} = 0$, so near these end point regions, perturbative results are not reliable too.
- The NLO predictions tends to agree with experimental data away from the end point regions.
- Small p_T resummation approach can improve the results obtained in fixed-order computation.

CS NLO(α_s) @ CMS 7 TeV I

- Total cross section:

$$\sigma_{\text{CMS}} = (1.49 \pm 0.07 \pm 0.13) \text{ nb},$$
$$\sigma_{\text{CS}}^{\text{LO}} = 0.052^{+0.020}_{-0.016} \text{ nb}, \sigma_{\text{CS}}^{\text{NLO}} = 0.147^{+0.136}_{-0.061} \text{ nb}.$$

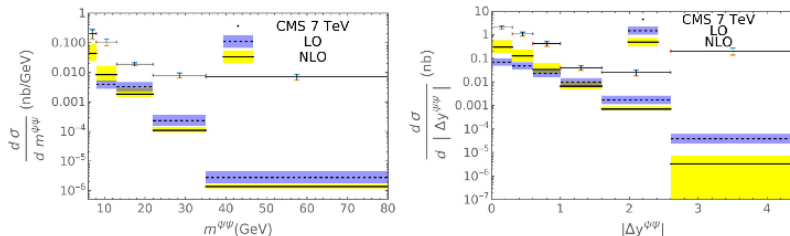
- The CS predictions at QCD NLO are about 10 times smaller than CMS data, even though the K-factor is about 2.8.
- The p_T spectrum of the J/ψ pair:



- The CSM predictions lie below the CMS data in the whole $p_T^{\psi\psi}$ range.

CS NLO(α_s) @ CMS 7 TeV II

- The $m^{\psi\psi}$ (left) and $|\Delta y^{\psi\psi}|$ (right) distributions of J/ψ pair:



- The CS contribution at QCD NLO can describe none of the data in experimental bins too.
- The NLO QCD corrections improve theoretical predictions in $m^{\psi\psi} < 13$ GeV and $|\Delta y^{\psi\psi}| < 1$ regions.
- In the residual bins, NLO QCD corrections turn to be negative and lead to 50% and 90% reduction in the last bin, respectively.

- Total cross section:

$$\sigma(pp \rightarrow J/\psi J/\psi + X) = \begin{cases} 82.2 \pm 8.3 \text{ (stat)} \pm 6.3 \text{ (syst)} \pm 0.9 \text{ (BF)} \pm 1.6 \text{ (lumi)} \text{ pb, for } |y| < 1.05, \\ 78.3 \pm 9.2 \text{ (stat)} \pm 6.6 \text{ (syst)} \pm 0.9 \text{ (BF)} \pm 1.5 \text{ (lumi)} \text{ pb, for } 1.05 \leq |y| < 2.1. \end{cases}$$

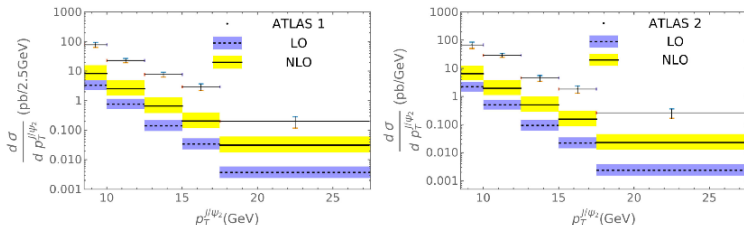
$$\sigma_{\text{CS}}^{\text{LO}} = \begin{cases} 2.97_{-0.96}^{+1.42} \text{ pb, for } |y(J/\psi_2)| < 1.05 \\ 1.98_{-0.65}^{+0.97} \text{ pb, for } 1.05 < |y(J/\psi_2)| < 2.1 \end{cases}$$

$$\sigma_{\text{CS}}^{\text{NLO}} = \begin{cases} 8.61_{-3.56}^{+7.87} \text{ pb, for } |y(J/\psi_2)| < 1.05 \\ 6.64_{-2.82}^{+6.36} \text{ pb, for } 1.05 < |y(J/\psi_2)| < 2.1 \end{cases}$$

- The K-factor are about 2.9 and 3.4 in central and non-central regions.
- However, the same as CMS case, CS contributions are still about 1 order of magnitude smaller than ATLAS measurements.

CS NLO(α_s) @ ATLAS 8 TeV II

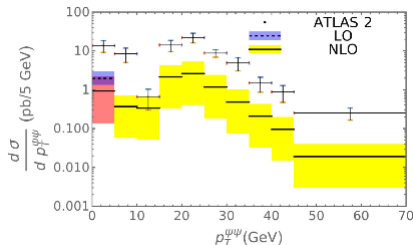
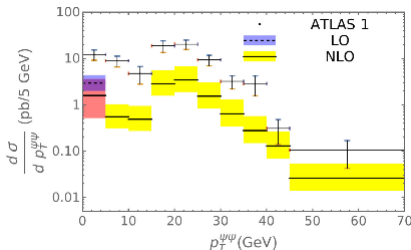
- The p_T distribution of the soft J/ψ in central (left) and forward (right) y^{ψ_2} regions:



- The LO predictions systematically underestimate the ATLAS measurements by up to 2 orders of magnitude.
- The K-factors range from 2.5 and 2.9 in the first bins to 8.5 and 9.8 in the last bins.

CS NLO(α_s) @ ATLAS 8 TeV III

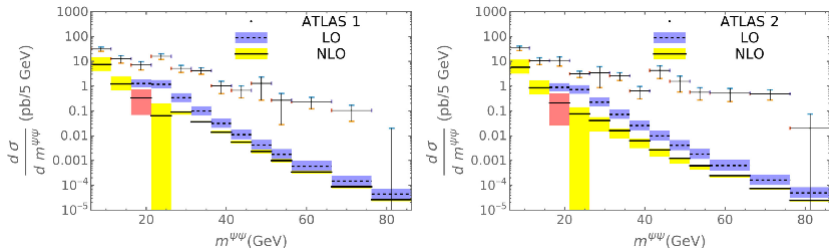
- The p_T distribution of J/ψ pair in central (left) and forward (right) y^{ψ_2} regions:



- The CS predictions at QCD NLO nicely reproduce the shapes of $p_T^{\psi\psi}$ distribution similar to CMS case.
- Incidentally, there is agreement within errors in the last two and third bins of the central and forward y^{ψ_2} regions, respectively.

CS NLO(α_s) @ ATLAS 8 TeV IV

- The J/ψ pair invariant mass distributions in central (left) and forward (right) y^{ψ_2} regions:



- The LO predictions show up from the third bin due to kinematics cut, and that is why the NLO results are negative in the third bin.
- From the fourth bins onward, the NLO QCD corrections are negative, ranging from -95% (-90%) in the fourth bin to -40% (-50%) in the last bins of central (forward) y^{ψ_2} region.

Why do we need the relativistic corrections?

- Theoretically, $v_c^2 \approx \alpha_s(2m_c)$ for charmonium.
- The NLO v^2 corrections played an important role to resolve the large discrepancy of double charmonium (He et al. 2007) and single J/ψ (He et al. 2010, Jia 2010) production in e^+e^- .
- For prompt J/ψ photo- and hadroproduction, the relativistic corrections have considerable influence on both yield and polarization. (He and Kniehl 2014,2015)
- In relativistic quark model calculation, the total cross sections of double J/ψ hadroproduction lie significantly below LHCb measurements. (Martynenko and Trunin 2012)
- Near threshold region, the NRQCD predictions strongly depended on the choice of m_c . (Li et al. 2013)

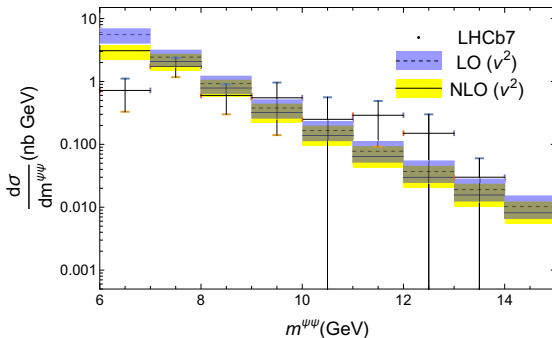
CS NLO(v^2) @ LHCb 7 TeV I

- NLO v^2 corrections render 32% reduction to prompt cross section:

$$\sigma_{\text{LHCb}}^{\text{Tot}} = (5.1 \pm 1.0 \pm 1.1) \text{ nb},$$

$$\sigma_{\text{CS}}^{\text{LO}} = 9.60^{+2.57}_{-2.73} \text{ nb}, \quad \sigma_{\text{CS}}^{\text{NLO}(v^2)} = 6.55^{+1.92}_{-1.89} \text{ nb}$$

- The J/ψ pair invariant mass distribution:



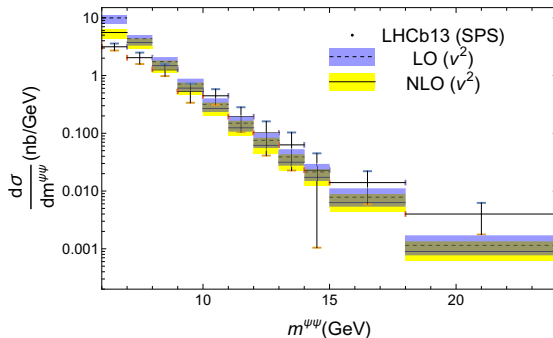
- The NLO v^2 contribution is -34% in direct production and -60% in $\psi(2S)$ feed-down, which lead to a total 45% reduction in the first bin.

CS NLO(v^2) @ LHCb 13 TeV I

- NLO total cross section: Conclusion similar to 7 TeV

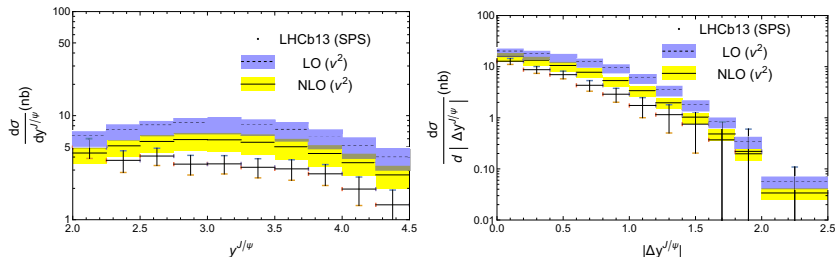
$$\sigma_{\text{LHCb}}^{\text{Tot}} = (16.36 \pm 0.28 \pm 0.88) \text{ nb}, \quad \sigma_{\text{LHCb}}^{\text{SPS}} = (7.9 \pm 1.2 \pm 1.1) \text{ nb},$$
$$\sigma_{\text{CS}}^{\text{LO}} = 17.43_{-3.98}^{+2.44} \text{ nb}, \quad \sigma_{\text{CS}}^{\text{NLO}(v^2)} = 12.02_{-2.81}^{+1.98} \text{ nb}$$

- The J/ψ pair invariant mass distribution:



- The v^2 corrections are all negative, they are about -44% in the first bin and -20% in the last bin.

- The $y^{J/\psi}$ (left) and $|\Delta y^{\psi\psi}|$ (right) distributions:



- The NLO v^2 corrections are constantly about -31% in all $y^{J/\psi}$ bins.
- The NLO v^2 corrections result in a reduction from 20% in the first bin to 40% in the last bin for $|\Delta y^{\psi\psi}|$ distribution.

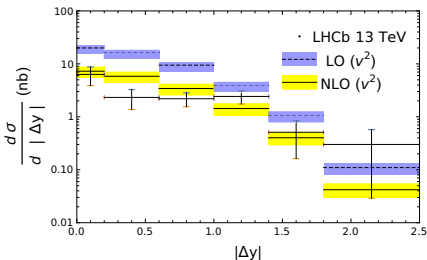
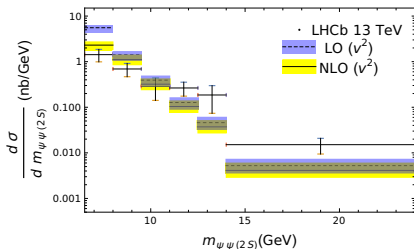
CS NLO(v^2) @ LHCb 13 TeV III

- NLO v^2 corrections leads to -65% reduction to $J/\psi + \psi(2S)$ total cross section.

$$\sigma_{\text{LHCb}}^{\text{Tot}} = (4.49 \pm 0.71 \pm 0.26) \text{ nb},$$

$$\sigma_{\text{CS}}^{\text{LO}} = 16.23_{-3.67}^{+2.17} \text{ nb}, \quad \sigma_{\text{CS}}^{\text{NLO}(v^2)} = 5.93_{-1.44}^{+1.21} \text{ nb}$$

- The distributions of J/ψ and $\psi(2S)$ invariant mass (left) and $|\Delta y|$ (right):



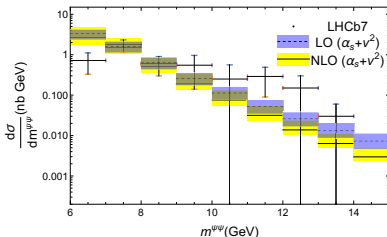
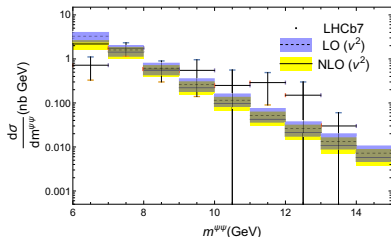
CS NLO($\alpha_s + v^2$) @ LHCb 7 TeV

- NLO $\alpha_s + v^2$ corrections leads to -16% reduction to direct double J/ψ total cross section.

$$\sigma_{\text{LHCb}}^{\text{Tot}} = (5.1 \pm 1.0 \pm 1.1) \text{ nb},$$

$$\sigma_{\text{CS}}^{\text{LO}} = 6.02^{+1.56}_{-1.59} \text{ nb}, \quad \sigma_{\text{CS}}^{\text{NLO}(\alpha_s + v^2)} = 5.04^{+3.70}_{-1.62} \text{ nb}$$

- The NLO v^2 (left) and $\alpha_s + v^2$ (left) corrections to invariant mass spectrum:



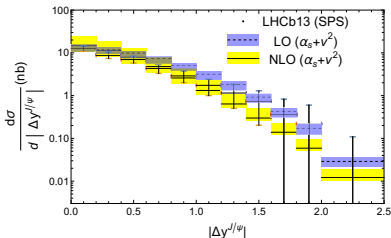
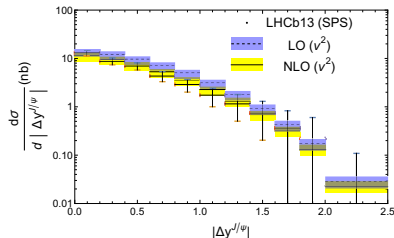
CS NLO($\alpha_s + v^2$) @ LHCb 13 TeV I

- NLO $\alpha_s + v^2$ corrections leads to -18% reduction to direct double J/ψ total cross section.

$$\sigma_{\text{LHCb}}^{\text{SPS}} = (7.9 \pm 1.2 \pm 1.1) \text{ nb},$$

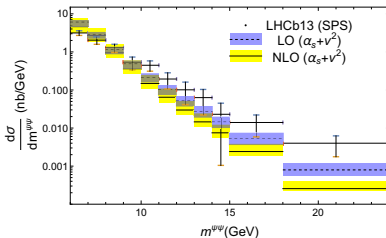
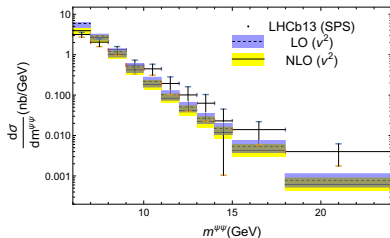
$$\sigma_{\text{CS}}^{\text{LO}} = 10.9^{+1.4}_{-2.5} \text{ nb}, \quad \sigma_{\text{CS}}^{\text{NLO}(\alpha_s+v^2)} = 8.9^{+5.6}_{-2.7} \text{ nb}$$

- The NLO v^2 (left) and $\alpha_s + v^2$ (left) corrections to $|\Delta y^{\psi\psi}|$ distribution:



CS NLO($\alpha_s + v^2$) @ LHCb 13 TeV II

- The NLO v^2 (left) and $\alpha_s + v^2$ (left) corrections to invariant mass spectrum:



- The NLO α_s and v^2 joint corrections are negative in the whole region of J/ψ pair invariant spectrum.
- In the last bin the corrections can reach -68% leaving more room for CO contribution.

Conclusion and summary

- 1 The NLO QCD and relativistic corrections are important to understand the mechanism of J/ψ pair hadroproduction.
- 2 In LHCb case, the CS contribution can account for most of the experimental measurements although some resummation techniques are needed to improve theoretical predictions.
- 3 In CMS and ATLAS cases, the CS contribution are far below the experimental measurements, which indicates the CO contribution may play an important role to fill the large discrepancies.
- 4 Much more theoretical effort are still needed to explain the experimental measurements.

Thank you!

Back up

Setup of Numerical predictions

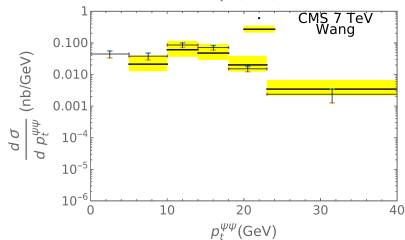
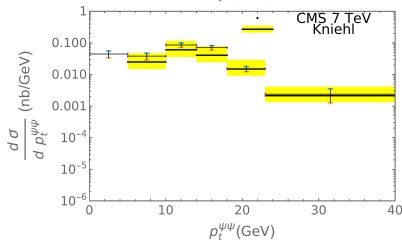
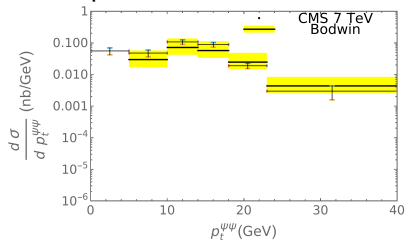
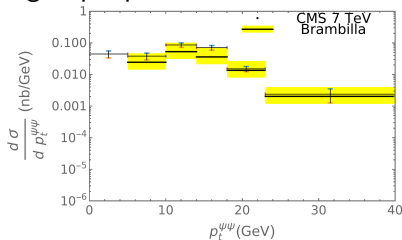
- We choose LO CTEQ6L PDFs, and one-loop running of α_s ,
 $\mu_r = \mu_f = (\sqrt{(p_T^{\psi_1})^2 + 4m_c^2} + \sqrt{(p_T^{\psi_2})^2 + 4m_c^2})/2$.
- We by hand remove 20% contribution from $\psi(2S)$ feeddown.
- We investigate predictions of 4 group with the LDMEs listed below.

Table: The LDMEs in unit of GeV^3 for S -wave and GeV^5 for P -wave

	Brambilla	Bodwin	Kniehl	Wang
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle$	1.16	1.16	1.32	1.16
$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$	-0.00204	0.099	0.0304	0.097
$\langle \mathcal{O}^{J/\psi}(^3P_J^{[8]}) \rangle$	0.0429	0.011	-0.0091	-0.0214
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$	0.0106	0.011	0.0016	-0.0046

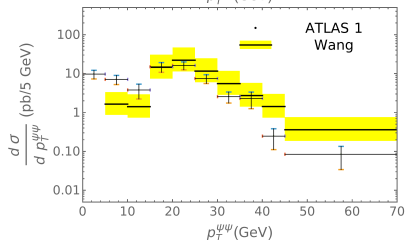
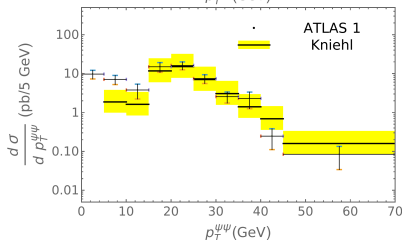
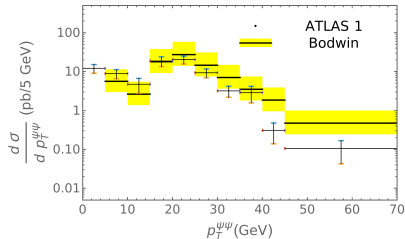
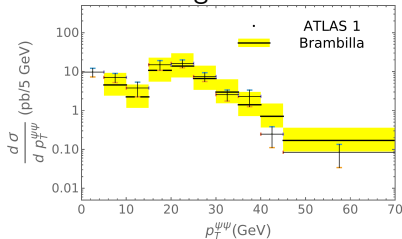
Numerical predictions for 7 TeV CMS

- All groups' predictions can describe the experimental data.



Numerical predictions for 8 TeV ATLAS central

- All groups' predictions can describe the experimental data except for Bodwin and Wang's.



Numerical predictions for 8 TeV ATLAS forward

- All groups' predictions can describe the experimental data.

