

Wilson Loops One-Point Function in ABJM Theory

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Outline

- Introduction
- Wilson-loop one-point functions
- Holographic computations
- Comparison with results from localization
- Conclusion and outlook

Introduction

- Defects are everywhere in QFTs: **line operators, boundary, domain wall, relation to (higher-form) symmetry, ...**
- Wilson(-'t Hooft) loop operators in gauge theory are related to **confinement, precise definition of the gauge theories, ...**
- In supersymmetric gauge theory, it is natural to consider Wilson loops (WLs) **preserving part of the supersymmetries.**

BPS Wilson loops

Various tools are used to study the BPS WLs,

- Weak coupling: Feynman diagrams.
- Strong coupling: holographic dual, lattice (?).
- General coupling: supersymmetric localization, integrability, conformal bootstrap (defect CFT).

BPS Wilson loops in AdS_5/CFT_4

- The half-BPS circular WL in $\mathcal{N} = 4$ SYM in the fundamental representation is dual to a half-BPS fundamental string solution in $AdS_5 \times S^5$.

[Berenstein, Corrado, Fischler, Maldacena, 98][Drukker, Gross, Ooguri, 99]

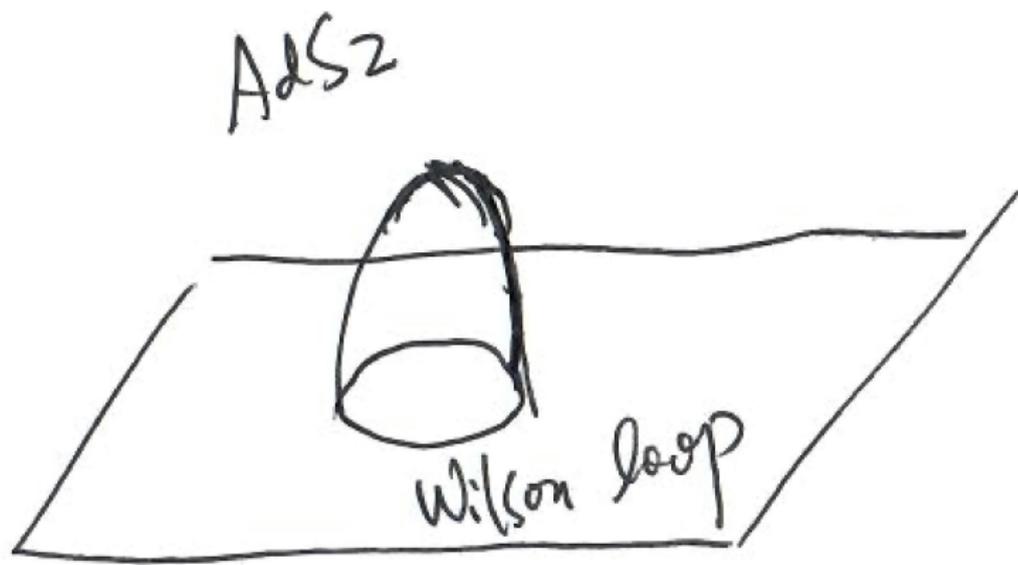
- String theory predicts that the vev of this WL is

$$\langle W[C] \rangle \sim e^{\sqrt{\lambda}},$$

in the large N , large $\lambda := g_{\text{YM}}^2 N$ limit.

BPS Wilson loops in $AdS_5 \times S^5$

The dual description is in term of a classical string solution in $AdS_5 \times S^5$. For the case at hand, its worldsheet is a minimal surface in AdS_5 .



BPS Wilson loops in AdS_5/CFT_4

- Based on results from one-loop perturbation calculations in the field theory side, [Erickson, Semenoff, Zarembo, 00] conjectured that this vev can be computed using the Gaussian matrix model (Later confirmed by localization [Pestun, 07]).
- The field theory result in the large N limit is

$$\langle W[C] \rangle = \frac{2}{\sqrt{\lambda}} I_1(\lambda).$$

- In the large λ limit, the above results gives

$$\langle W[C] \rangle \sim \frac{e^{\sqrt{\lambda}}}{\sqrt{\frac{\pi}{2}} \lambda^{3/4}}.$$

- This is one of the earliest precision checks of AdS/CFT duality beyond non-renormalization theorem.

ABJM theory

- ABJM theory is a three-dimensional ($3d$) $\mathcal{N} = 6$ $U(N) \times U(N)$ Chern-Simons-matter theory.
- The fields in the theory include

Fields	representation of $U(N) \times U(N)$	representation of $SU(4)$ R-symmetry group
A_μ	(adjoint , 1)	1
\hat{A}_μ	(1 , adjoint)	1
Y^I	(\square , $\bar{\square}$)	4
ψ_I	(\square , $\bar{\square}$)	$\bar{\mathbf{4}}$

ABJM theory

- ABJM theory is the low energy effective theory of N $M2$ -branes at the tip of $\mathbb{C}^4/\mathbb{Z}_k$ orbifold singularity.
- It admits two kinds of large N limits,

limit	holographic dual
$N, k \rightarrow \infty$, with $\lambda = N/k$ fixed	type IIA string theory on $AdS_4 \times \mathbb{C}P^3$
$N \rightarrow \infty$, with k fixed	M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$

BPS Wilson loops in ABJM theory

WLs	gravity duals
Half-BPS WLs [Drukker, Trancanelli, 09]	$AdS_2(\subset AdS_4) \times \text{point}(\in \mathbb{CP}^3)$
Bosonic 1/6-BPS WLs [Drukker, Plefka, Young][Chen, JW] [Rey, Suyama, Yamaguchi] 08	$F1$ -string smearing on a $\mathbb{CP}^1 \subset \mathbb{CP}^3$
Fermionic 1/6-BPS WLs [Ouyang, JW , Zhang, 15]	$F1$ -string with a complicated boundary conditions in \mathbb{CP}^3 [Correa, Giraldo-Silva, 19]

Half-BPS Wilson loops

- We consider the half-BPS circular Wilson loop [Drukker, Trancanelli, 09] along the circle $C = \{x^\mu = (R \cos \tau, R \sin \tau, 0) | \tau \in [0, 2\pi]\}$,

$$W[C] = \text{Tr} \mathcal{P} \exp \left(-i \oint_C d\tau \mathcal{L}_{1/2}^C(\tau) \right).$$

- Here the superconnection $\mathcal{L}_{1/2}^C$ is given by

$$\mathcal{L}_{1/2}^C = \begin{pmatrix} \mathcal{A}^C & \bar{f}_1^C \\ f_2^C & \hat{\mathcal{A}}^C \end{pmatrix}.$$

Half-BPS Wilson loops

- The components explicitly read:

$$\mathcal{A}^C = A_\mu \dot{x}^\mu - i \frac{2\pi}{k} (2\alpha_I \bar{\alpha}^J - \delta_I^J) |\dot{x}| Y^I \bar{Y}_J,$$

$$\hat{\mathcal{A}}^C = \hat{A}_\mu \dot{x}^\mu - i \frac{2\pi}{k} (2\alpha_I \bar{\alpha}^J - \delta_I^J) |\dot{x}| \bar{Y}_J Y^I,$$

$$\bar{f}_1^C = -\sqrt{\frac{2\pi}{k}} \bar{\alpha}^I \bar{\zeta} \psi_I |\dot{x}|,$$

$$f_2^C = \sqrt{\frac{2\pi}{k}} \bar{\psi}^I \eta \alpha_I |\dot{x}|,$$

Half-BPS Wilson loops

- As usual

$$\dot{x}^\mu = dx^\mu/d\tau.$$

- α_I are complex parameters satisfying

$$\sum_{I=1}^4 \alpha_I \bar{\alpha}^I = 1,$$

with $\bar{\alpha}^I = (\alpha_I)^*$.

- The spinors $\bar{\zeta}^\alpha$ and η_α are

$$\bar{\zeta}^\alpha = \left(e^{i\tau/2}, e^{-i\tau/2} \right), \quad \eta_\alpha = \begin{pmatrix} e^{-i\tau/2} \\ e^{i\tau/2} \end{pmatrix}.$$

1/3-BPS chiral primary operators

- The 1/3-BPS chiral primary operator (CPO)

$$\mathcal{O}^A = (C^A)_{I_1 \dots I_L}^{J_1 \dots J_L} \text{Tr}(Y^{I_1} \bar{Y}_{J_1} \dots Y^{I_L} \bar{Y}_{J_L}),$$

- Here C^A is a symmetric traceless tensor, satisfying

$$(C^A)_{I_1 \dots I_L}^{J_1 \dots J_L} = (C^A)_{(I_1 \dots I_L)}^{(J_1 \dots J_L)}$$

and

$$\delta_{J_1}^{I_1} (C^A)_{I_1 \dots I_L}^{J_1 \dots J_L} = 0.$$

Correlation functions

- We define the normalization factor $\mathcal{N}_{\mathcal{O}}$ using the two point function,

$$\langle \mathcal{O}^A(x) \mathcal{O}^{B\dagger}(y) \rangle = \frac{\delta^{AB} \mathcal{N}_{\mathcal{O}^A}}{|x - y|^{2\Delta_A}},$$

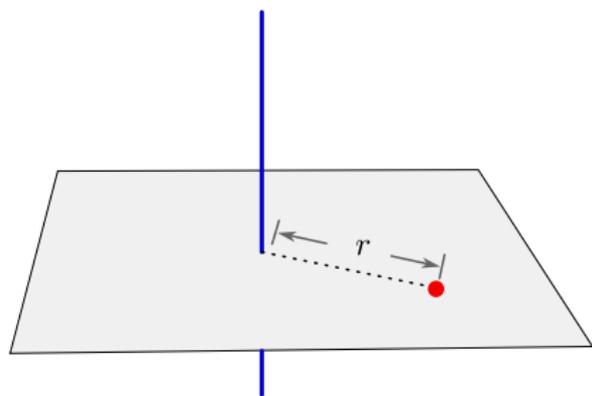
with Δ_A being the conformal dimension of \mathcal{O}^A .

- For the circular Wilson loop with contour $x(\tau) = (R \cos \tau, R \sin \tau, 0)$ and a local operator at a generic point (y_1, y_2, y_3) , conformal symmetry fixes the correlation function to be of the form
[\[Berenstein et al., 98\]](#)[\[Alday, Tseytlin, 11\]](#)

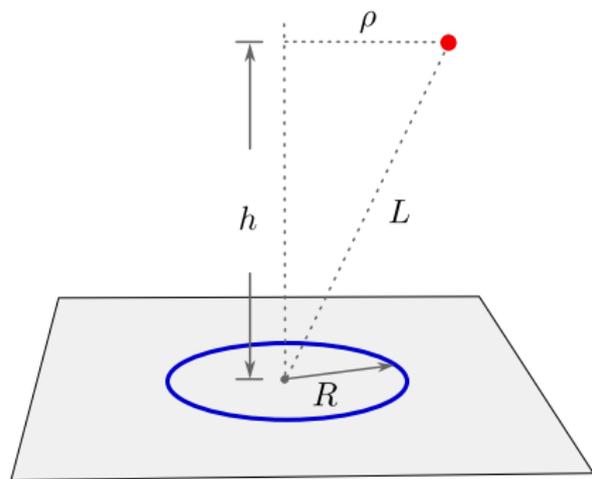
$$\frac{\langle W[C] \mathcal{O}(y) \rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}} \langle W[C] \rangle} = C_{\mathcal{O}} \frac{R^{\Delta}}{((h^2 + \rho^2 - R^2)^2 + 4R^2 h^2)^{\Delta/2}},$$

where $\rho = \sqrt{y_1^2 + y_2^2}$, $h = |y_3|$ and $C_{\mathcal{O}}$ is a function of k and N .

Correlation function



(a) Wilson line



(b) Wilson loop

Figure: (a) Wilson line one-point function. (b) Wilson loop one-point function.

Correlation functions

- In the OPE limit $L := \sqrt{\rho^2 + h^2} \gg R$, we have

$$\frac{\langle W[C]\mathcal{O}(y) \rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[C] \rangle} = C_{\mathcal{O}} \frac{R^{\Delta}}{L^{2\Delta}}.$$

- For the Wilson line one-point function of a CPO, we have,

$$\frac{\langle W[L]\mathcal{O}(r) \rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[L] \rangle} = \frac{B_{\mathcal{O}}}{\sqrt{\mathcal{N}_{\mathcal{O}}}} \frac{1}{r^{\Delta}}.$$

where r is the distance between the local operator.

- The coefficients $B_{\mathcal{O}}$ and $C_{\mathcal{O}}$ are related by [\[Lewkowycz, Maldacena, 13\]](#)

$$\frac{B_{\mathcal{O}}}{\sqrt{\mathcal{N}_{\mathcal{O}}}} = \frac{C_{\mathcal{O}}}{2^{\Delta}}.$$

Holographic dual of ABJM theory

- ABJM with large N and finite k is dual to M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$.
- The eleven-dimensional supergravity background consists of the metric

$$ds^2 = g_{MN}dX^M dX^N = ds_{AdS_4}^2 + 4ds_{S^7/\mathbb{Z}_k}^2,$$

and the four-form flux

$$F_4 = 3\Omega_{AdS_4}.$$

- we have set the radius of the AdS_4 to 1.
- From the AdS/CFT dictionary, the eleven-dimensional Planck length is given by

$$l_p = \left(\frac{2}{\pi^2 k N} \right)^{1/6}.$$

Holographic dual

- The metric on the unit (Euclidean) AdS_4 in Poincaré coordinates is

$$ds^2_{AdS_4} = \frac{1}{z^2}(dz^2 + \delta_{ij}dx^i dx^j),$$

where $i, j = 1, 2, 3$.

- To describe the metric on S^7/\mathbb{Z}_k , we parameterize the space using four complex coordinates z^I , ($I = 1, \dots, 4$) subject to the constraint $\sum_{I=1}^4 |z^I|^2 = 1$. Explicitly,

$$z^1 = \cos \xi \cos \frac{\theta_1}{2} \exp \left[i \left(\zeta + \frac{\psi + \varphi_1}{2} \right) \right],$$

$$z^2 = \cos \xi \sin \frac{\theta_1}{2} \exp \left[i \left(\zeta + \frac{\psi - \varphi_1}{2} \right) \right],$$

$$z^3 = \sin \xi \cos \frac{\theta_2}{2} \exp \left[i \left(\zeta + \frac{-\psi + \varphi_2}{2} \right) \right],$$

$$z^4 = \sin \xi \sin \frac{\theta_2}{2} \exp \left[i \left(\zeta + \frac{-\psi - \varphi_2}{2} \right) \right],$$

Holographic dual

- We should impose the angular identification $\zeta \sim \zeta + \frac{2\pi}{k}$ due to the \mathbb{Z}_k quotient.
- The metric on S^7/\mathbb{Z}_k , induced from the flat metric $ds_{\mathbb{C}^4} = \sum_{i=1}^4 dz^I d\bar{z}_I$, takes the form

$$ds_{S^7/\mathbb{Z}_k} = (d\zeta + A)^2 + ds_{\mathbb{C}P^3}.$$

- The one-form A is given by

$$A = \frac{1}{2} \cos(2\xi) d\psi + \frac{1}{2} \cos^2 \xi \cos \theta_1 d\varphi_1 + \frac{1}{2} \sin^2 \xi \cos \theta_2 d\varphi_2,$$

- the Fubini-Study metric on $\mathbb{C}P^3$ is

$$\begin{aligned} ds_{\mathbb{C}P^3} &= d\xi^2 + \frac{1}{4} \cos^2 \xi (d\theta_1^2 + \sin^2 \theta_1 d\varphi_1^2) \\ &+ \frac{1}{4} \sin^2 \xi (d\theta_2^2 + \sin^2 \theta_2 d\varphi_2^2) \\ &+ \cos^2 \xi \sin^2 \xi (d\psi + \frac{1}{2} \cos \theta_1 d\varphi_1 - \frac{1}{2} \cos \theta_2 d\varphi_2). \end{aligned}$$

Holographic dual

In Poincaré coordinates, the four-form flux F_4 and its corresponding three-form potential c_3 are

$$F_4 = \frac{3}{z^4} dz \wedge dx^1 \wedge dx^2 \wedge dx^3 ,$$

$$c_3 = -\frac{1}{z^3} dx^1 \wedge dx^2 \wedge dx^3 .$$

M-theory dual of the WL

- The WL in the fundamental representation (of $U(N|N)$) is dual to a probe membrane in $AdS_4 \times S^7/\mathbb{Z}_k$.
- The membrane action is

$$S_{M2} = T_{M2} \left(\int d^3\sigma \sqrt{\det \tilde{g}} - P[c_3] \right)$$

- $\tilde{g}_{ab} = \partial_a X^M \partial_b X^N g_{MN}$ is the induced metric on the M2-brane worldvolume and $P[c_3]$ is the pullback of c_3 to the membrane worldvolume.
- T_{M2} is the tension of the M2-brane

$$T_{M2} = \frac{1}{4\pi^2 l_p^3}.$$

M-theory dual of the WL

- Denoting the worldvolume coordinates of this M2-brane as $(\sigma^0, \sigma^1, \sigma^2)$, the embedding of this brane is

$$\begin{aligned} z &= R \sin \sigma^1, & x_1 &= R \cos \sigma^1 \cos \sigma_0, & x_2 &= R \cos \sigma^1 \sin \sigma^0, \\ x_3 &= 0, & \zeta &= \sigma^2 + \zeta^0, & \xi &= \xi^0, & \theta_1 &= \theta_1^0, & \varphi_1 &= \varphi_1^0, & \theta_2 &= \theta_2^0, \\ \varphi_2 &= \varphi_2^0, & \psi &= \psi^0, \end{aligned}$$

where $\zeta^0, \xi^0, \theta_1^0, \varphi_1^0, \theta_2^0, \varphi_2^0, \psi^0$ are constants and the worldvolume coordinate ranges are

$$0 \leq \sigma^0 \leq 2\pi, \quad 0 \leq \sigma^1 \leq \frac{\pi}{2}, \quad 0 \leq \sigma^2 \leq \frac{2\pi}{k},$$

- The worldvolume of this M2-brane has the topology $AdS_2 \times S^1$ with AdS_2 in AdS_4 and S^1 along the ζ -cycle which is the M-theory cycle.

M-theory dual of the WL

- The parameters α_I defining the Wilson line in the boundary theory are related to the brane embedding via [Lietti, Mauri, Penati, Zhang, 17]

$$\alpha_I = \bar{z}_I(\xi^0, \theta_1^0, \varphi_1^0, \theta_2^0, \varphi_2^0, \zeta^0).$$

- This provided important ingredients of the precise holographic dual of the WL, and was not mentioned in [Drukker, Trancanelli, 09].

M-theory dual of the CPO

- The chiral primary operators \mathcal{O}^A are dual to fluctuations of both the metric and the three-form potential in the eleven-dimensional background:

$$\begin{aligned}G_{MN} &= g_{MN} + \delta g_{MN}^A \mathbf{Y}^A, \\C_{MNP} &= c_{MNP} + \delta c_{MNP}^A \mathbf{Y}^A.\end{aligned}$$

- Here $\mathbf{Y}^A = (C^A)_{I_1 \dots I_L}^{J_1 \dots J_L} \alpha_{J_1} \dots \alpha_{J_L} \bar{\alpha}^{I_1} \dots \bar{\alpha}^{I_L}$ are spherical harmonics of S^7/\mathbb{Z}_k with radius 1.

M-theory dual of the CPO

- The explicit forms of the fluctuations are [Biran, et al., 84][Castellani, et al. 84][Bastianelli, Zucchini, 99]:

$$\delta g_{mn}^A = \frac{4}{J+2} \left[\nabla_m \nabla_n + \frac{J(J+6)}{8} g_{mn} \right] s^A - \frac{7J}{6} g_{mn} s^A,$$

$$\delta g_{\alpha\beta}^A = \frac{1}{3} J g_{\alpha\beta} s^A,$$

$$\delta c_{mnp}^A = 2\epsilon_{mnpq} \nabla^q s^A,$$

where $J = 2\Delta = 2L$.

- The scalar field $s^A(x, z)$ depends only on the coordinates (x, z) of AdS_4 and is determined by the boundary source $s_0^A(y)$

$$s^A(x, z) = \int d^3y G_\Delta(y; x, z) s_0^A(y),$$

M-theory dual of the CPO

- $G_{\Delta}(y; x, z)$ is the boundary-to-bulk propagator

$$G_{\Delta}(y; x, z) = c \left(\frac{z}{z^2 + |x - y|^2} \right)^{\Delta},$$

with the constant c being

$$c = 2^{\Delta-1} \pi l_p^4 \sqrt{k} \frac{\Delta + 1}{\Delta} \sqrt{2\Delta + 1}.$$

The OPE limit

- In the OPE limit $L \gg R$, we can approximate

$$\begin{aligned}G_{\Delta}(y; x, z) &\approx c \frac{z^{\Delta}}{\tilde{L}^{2\Delta}}, \\ \partial_z s^A &\approx \frac{\Delta}{z} s^A, \\ \partial_{\mu} \partial_{\nu} s^A &\approx \delta_{\mu}^z \delta_{\nu}^z \frac{\Delta(\Delta-1)}{z^2} s^A,\end{aligned}$$

- The metric fluctuation simplifies to:

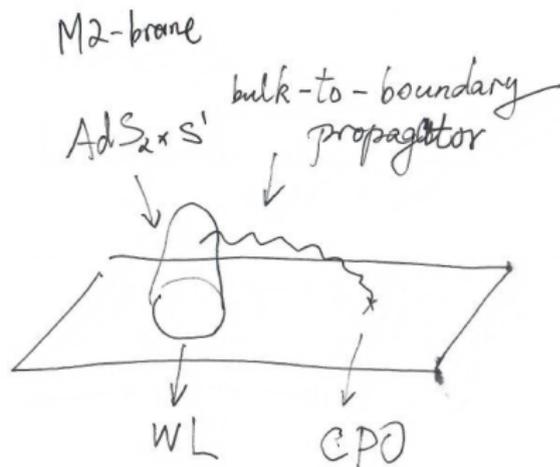
$$\delta g_{mn}^A = -\frac{2}{3} J g_{mn} s^A + \frac{J}{z^2} \delta_m^z \delta_n^z s^A.$$

Holographic WL 1-pt function

- The WL one-point correlation function

$$\frac{\langle W[C] \mathcal{O}^A(y) \rangle}{\langle W[C] \rangle \sqrt{\mathcal{N}_0}}$$

is computed from the variation of the M2-brane action due to background field fluctuations.



Holographic WL 1-pt function

The Wess-Zumino term does not contribute, so the action variation is

$$\delta S_{\text{M2}} = T_{\text{M2}} \int d^3\sigma \sqrt{\det \tilde{g}_{ab}} \frac{1}{2} \tilde{g}^{ab} \partial_a X^M \partial_b X^N \delta g_{MN}^A \mathbf{Y}^A,$$

where $\tilde{g}_{ab} = \partial_a X^M \partial_b X^N g_{MN}$ is the induced metric on the worldvolume.

Holographic WL 1-pt function

- The action variation is

$$\begin{aligned}\delta S_{\text{M2}} &= T_{\text{M2}} \int d^3\sigma \left(2 \frac{\cos \sigma_1}{\sin^2 \sigma_1} \right) \frac{1}{2} (-J \sin^2 \sigma) s^A \mathbf{Y}^A \\ &= -JT_{\text{M2}} \int d^3\sigma \cos \sigma_1 s^A \mathbf{Y}^A.\end{aligned}$$

- Using the AdS/CFT dictionary, the correlation function reads

$$\begin{aligned}\frac{\langle W[C] \mathcal{O}^A(y) \rangle}{\langle W[C] \rangle \sqrt{\mathcal{N}_{\mathcal{O}}}} &= -\frac{\delta S_{\text{DBI}}}{\delta s_0^A(y)} = JT_{\text{M2}} \int d^3\sigma \cos \sigma_1 G_{\Delta}(y; x, z) \mathbf{Y}^A \\ &= \frac{2^{\Delta+\frac{1}{4}} \sqrt{\pi(2\Delta+1)} R^{\Delta}}{k\lambda^{\frac{1}{4}}} \frac{R^{\Delta}}{L^{2\Delta}} \mathbf{Y}^A,\end{aligned}$$

Holographic WL 1-pt function

- We define the OPE coefficients $C_{\mathcal{O}^A}$ as

$$\frac{\langle W[C] \mathcal{O}^A(y) \rangle}{\langle W[C] \rangle \sqrt{\mathcal{N}_{\mathcal{O}}}} = C_{\mathcal{O}^A} \frac{R^\Delta}{L^{2\Delta}} \mathbf{Y}^A.$$

- The holographic prediction is

$$C_{\mathcal{O}^A} = \frac{2^{\Delta+\frac{1}{4}} \sqrt{\pi(2\Delta+1)}}{k\lambda^{\frac{1}{4}}}.$$

Localization

- The correlation function of a half-BPS Wilson line and a CPO with $\Delta = 1$ can be computed using the supersymmetric localization. [\[Guerrini, 23\]](#)
- The half-BPS Wilson line is placed along the x^3 -axis at $(x^1, x^2, x^3) = (0, 0, s)$.
- The polarization vector α_I in the definition of the Wilson loop is chosen to be $\alpha_I = (1, 0, 0, 0)$.

Localization

- The 1/3-BPS operator is inserted at $(r \cos \tau, r \sin \tau, 0)$, and its polarization vector are chosen as

$$n_I = \frac{1}{\sqrt{2}} \left(e^{-\frac{i}{2}\tau}, 0, e^{\frac{i}{2}\tau}, 0 \right), \quad \bar{n}^I = \frac{1}{\sqrt{2}} \left(e^{\frac{i}{2}\tau}, 0, -e^{-\frac{i}{2}\tau}, 0 \right).$$

- Then 1/3-BPS operator with $\Delta = 1$ in the along the circle is

$$\mathcal{O}(\tau) = \text{Tr}(n_I(\tau)Y^I(\tau)\bar{n}^J(\tau)\bar{Y}_J(\tau)).$$

- This special choice of polarization vector makes the local operator in the **twisted-translated frame**. [Drukker, Plefka, 09][Basso, Komatsu, Vieira, 15][Pereira, 17][Yang, Jiang, Komatsu, JW, 21]

Two point functions

- The conformal symmetry and R-symmetry give strong constraints on the two-point function of \mathcal{O} ,

$$\langle \mathcal{O}(\tau_1) \mathcal{O}(\tau_2) \rangle = \mathcal{N}_{\mathcal{O}} d_{12} d_{21} .$$

- Here d_{ij} is defined as

$$d_{ij} := \frac{n_i \cdot \bar{n}_j}{|x_i - x_j|} .$$

- For the operators in the twisted-translated frame, we have

$$d_{12} = \frac{n(\tau_1) \cdot \bar{n}(\tau_2)}{|x(\tau_1) - x(\tau_2)|} = \frac{i \sin \frac{\tau_1 - \tau_2}{2}}{2r |\sin \frac{\tau_1 - \tau_2}{2}|} ,$$

leading to

$$d_{12} d_{21} = \frac{1}{4r^2} ,$$

Two point functions

- $\mathcal{N}_{\mathcal{O}}$ is the normalization constant.
- In ABJM, it is a non-trivial function of the coupling constant $1/k!$
- For arbitrary finite N and k , the integrated two-point function is given by [Agmon, et al., 17][Binder, et al., 19]

$$\left\langle \int d\tau_1 \mathcal{O}(\tau_1) \int d\tau_2 \mathcal{O}(\tau_2) \right\rangle = -\frac{1}{16\pi^2 r^4} \frac{1}{Z} \frac{\partial^2 Z[m]}{\partial m^2} \Bigg|_{m=0},$$

where $Z[m]$ is the partition function of the mass-deformed ABJM theory where m is the mass-deformation parameter.

Localization

- Using localization [Kapustin, Willett, Yaakov, 09][Jafferis, 10][Hama, Hosomichi, Lee, 10], $Z[m]$ can be expressed as

$$Z[m] = \frac{1}{(N!)^2 (2\pi)^{2N}} \int d\lambda_i d\mu_i \exp \left[\frac{ik}{4\pi} \left(\sum \lambda_i^2 - \sum \mu_i^2 \right) \right] \\ \times \frac{\prod_{i \neq j}^N 2 \sinh \frac{\lambda_i - \lambda_j}{2} \prod_{i \neq j}^N 2 \sinh \frac{\mu_i - \mu_j}{2}}{\prod_{i,j} 2 \cosh \frac{\lambda_i - \mu_j}{2} 2 \cosh \left(\frac{\mu_j - \lambda_i}{2} - \pi m r \right)}.$$

Localization

- By using the Fermi gas approach, all order perturbative $1/N$ corrections to $Z[m]$ can be resummed. The result is given in terms of the Airy function [Marino, Putrov, 11][Guerrini, 23]

$$Z^{\text{pert.}}[m] = e^A C^{-1/3} \text{Ai}[C^{-1/3}(N - B)],$$

where A, B, C are functions of k, r, m . Here we do not need the explicit expressions for A and B . As for C , we have

$$C = \frac{2}{\pi^2 k (1 + 4m^2 r^2)}.$$

Localization

- the large N limit of the integrated two-point function reads

$$\begin{aligned} \left\langle \int d\tau_1 \mathcal{O}(\tau_1) \int d\tau_2 \mathcal{O}(\tau_2) \right\rangle &= -\frac{1}{16\pi^2 r^4} \frac{1}{Z[m]} \frac{\partial^2 Z[m]}{\partial m^2} \Bigg|_{m=0} \\ &= \frac{\sqrt{2k} N^{3/2}}{12\pi r^2}. \end{aligned}$$

- Here we have used the asymptotic expansion of the Airy function

$$\text{Ai}(x) \sim \exp\left(-\frac{2}{3}x^{3/2}\right),$$

where an irrelevant factor $(1/x)^{1/4}/(2\sqrt{\pi})$ has been omitted.

Localization

- Let us recall that,

$$\langle \mathcal{O}(\tau_1) \mathcal{O}(\tau_2) \rangle = \mathcal{N}_{\mathcal{O}} d_{12} d_{21} .$$

- Performing the τ -integrals on both sides of this equation yields

$$\left\langle \int d\tau_1 \mathcal{O}(\tau_1) \int d\tau_2 \mathcal{O}(\tau_2) \right\rangle = \frac{\pi^2 \mathcal{N}_{\mathcal{O}}}{r^2} .$$

Comparing this with the localization results fixes the normalization

$$\mathcal{N}_{\mathcal{O}} = \frac{\sqrt{2k} N^{3/2}}{12\pi^3} .$$

in the large N limit.

Localization

- The correlation function of the Wilson line $W[L]$ and \mathcal{O} is given by [Guerrini, 23]

$$\frac{\langle W[L]\mathcal{O}(0) \rangle}{\langle W[L] \rangle} = \frac{\mathcal{B}}{r},$$

where \mathcal{B} is the Bremsstrahlung function.

- Its large N expansion (with finite k) takes the form [Beccaria, Tseytlin, 25]

$$\mathcal{B}(k, N)|_{N \gg 1} = \frac{1}{2\pi} \sqrt{\frac{N}{2k}} - \frac{1}{2\pi k} \cot \frac{2\pi}{k} + \mathcal{O}(N^{-1/2}).$$

Localization

- Including the normalization factor, we have

$$\frac{\langle W[L]\mathcal{O}(0) \rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[L] \rangle} = \frac{\mathcal{B}}{\sqrt{\mathcal{N}_{\mathcal{O}}}r},$$

The coefficient of $1/r$ is

$$\frac{\mathcal{B}}{\sqrt{\mathcal{N}_{\mathcal{O}}}} = \frac{\sqrt{3\pi}}{2^{3/4}k\lambda^{1/4}},$$

Comparison

- The holographic computation of the Wilson loop OPE coefficient for the CPO gives

$$C_{\mathcal{O}} = \frac{2^{5/4} \sqrt{3\pi}}{k\lambda^{1/4}},$$

by setting $\Delta = 1$. The holographic correlation function for a normalized CPO of dimension $\Delta = 1$ with the Wilson line takes the form

$$\frac{\langle W[L]\mathcal{O} \rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[L] \rangle} = C_{\mathcal{O}} \mathbf{Y}^A \frac{1}{2r},$$

where r is the distance from the CPO to the line, and \mathbf{Y}^A is the spherical harmonic factor.

Comparison

- For the Wilson line considered here, the polarization vector is $\alpha_I = (1, 0, 0, 0)$, and the spherical harmonic evaluates to

$$\mathbf{Y}^A = \bar{n}^I n_J \alpha_I \bar{\alpha}^J = 1/2.$$

- Thus, the holographic prediction for $\Delta = 1$ is

$$\frac{\langle W[L] \mathcal{O} \rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}} \langle W[L] \rangle} = \frac{C_{\mathcal{O}} \mathbf{Y}^A}{2r} = \frac{\sqrt{3\pi}}{2^{3/4} k \lambda^{1/4} r},$$

which exactly matches the localization result.

Conclusion

- Correlation functions of BPS Wilson loops and local operators are important observables in supersymmetric gauge theories.
- We computed $\langle W[C]\mathcal{O} \rangle$ for a half-BPS Wilson loop and a CPO by using AdS/CFT duality.
- The results coincide with localization prediction for $\Delta = 1$.
- We also computed $\langle W[L]T_{\mu\nu} \rangle$ for a half-BPS Wilson line holographically.
- The results involving the Bremsstrahlung function $\mathcal{B}(k, N)$ and confirmed by the previous field theory results.

Outlook: holography

- It is valuable to compute **holographical** WL one-point functions for WLs in higher-dimensional representation and/or with less supersymmetries.
- For the $F1$ -string smearing in a $\mathbb{CP}^1 \subset \mathbb{CP}^3$, I am wondering whether the correct prescription is simply to **average** over this \mathbb{CP}^1 .
- How to treat the $F1$ -string complicated boundary condition in \mathbb{CP}^3 ?

Outlook: localization and bootstrap

- A localization formula for $\langle W[C]\mathcal{O} \rangle$ with a **generic** CPO \mathcal{O} would be highly desirable.
- It is interesting to study more ABJM (integrated) correlation functions of WLs, single-trace BPS operator or giant gravitons from localization results in **mass-deformed** ABJM theory.
- WL two-point functions encode **defect operator spectra**, We should try to compute them using superconformal bootstrap.

Outlook: Integrability

- **Integrability** can be applied to compute the WL one-point functions for non-BPS operators in the planar limit.
- For WLs that correspond to integrable boundary states, the OPE coefficients can be computed using the **worldsheet g -function approach** [Jiang, Komastu, Vescovi, 19].
- Steps have been taken at weak coupling and the result showed that half-BPS and bosonic $1/6$ -BPS WLs give integrable boundary states. [Jiang, Wu, Yang, 23]
- A **finite-coupling** analysis remains an open challenge.

Thanks for your Attention !