Squared Amplitudes in SYM and ABJM

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Based on [2410.09859] [2503.15593] [2508.03813],

With **Jacob Bourjaily**, **Song He** (何颂), **Canxin Shi** (施灿欣), **Yao-Qi Zhang** (张耀奇).

Others (in SYM) and we (in ABJM) discovered:

Permutation symmetry (loops≈legs) in inclusive $\mathrm{d}\sigma(2 \to n)$,

which further motivates the study of:

Amplitude / correlator duality in planar SYM(!) and ABJM(?).

SYM: Superamplitudes

$$S_{\mathrm{SYM}} = \frac{1}{g_{\mathrm{YM}}^2} \int \mathrm{d}^4x \,\mathrm{tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\phi)^2 + \frac{i}{2}\bar{\psi}\not{D}\psi + \mathcal{L}_{\mathrm{int}}\right]$$

4d integrable QFT: planar $SU(N_c)$ $\mathcal{N}$=4 superconformal Yang-Mills theory

$$|\Phi(\tilde{\eta})\rangle = |{+1}\rangle + (\tilde{\eta})^1|{+\tfrac{1}{2}}\rangle + (\tilde{\eta})^2|0\rangle + (\tilde{\eta})^3|{-\tfrac{1}{2}}\rangle + (\tilde{\eta})^4|{-1}\rangle$$

⇕ fermionic Fourier transform:
chiral superspace ←→ antichiral superspace

$$|\overline{\Phi}(\eta)\rangle = (\eta)^4|{+1}\rangle + (\eta)^3|{+\tfrac{1}{2}}\rangle + (\eta)^2|0\rangle + (\eta)^1|{-\tfrac{1}{2}}\rangle + |{-1}\rangle$$

State sum: $\int \mathrm{d}\tilde{\eta}\mathrm{d}\eta \, F(\eta)e^{\eta\tilde{\eta}}G(\tilde{\eta}) = \int \mathrm{d}\tilde{\eta}\left[\int \mathrm{d}\eta \, F(\eta)e^{\eta\tilde{\eta}}\right]G(\tilde{\eta})$

SYM: Differential Cross Sections

MHV $\qquad\qquad\qquad$ $\overline{\text{MHV}}$

$$\mathcal{A}(\lambda,\tilde\lambda,\tilde\eta) = \frac{\delta^8(\lambda\tilde\eta)}{\langle 12\rangle\cdots\langle n1\rangle} + \cdots + \frac{\delta^8(\lambda\tilde\eta)}{\langle 12\rangle\cdots\langle n1\rangle} R_{n,n-4}(\lambda,\tilde\lambda,\tilde\eta)$$

$$\Updownarrow$$

$$\mathcal{A}(\tilde\lambda,\lambda,\eta) = \frac{\delta^8(\tilde\lambda\eta)}{[12]\,\cdots\,[n1]} R_{n,n-4}(\tilde\lambda,\lambda,\eta) + \cdots + \frac{\delta^8(\tilde\lambda\eta)}{[12]\,\cdots\,[n1]}$$

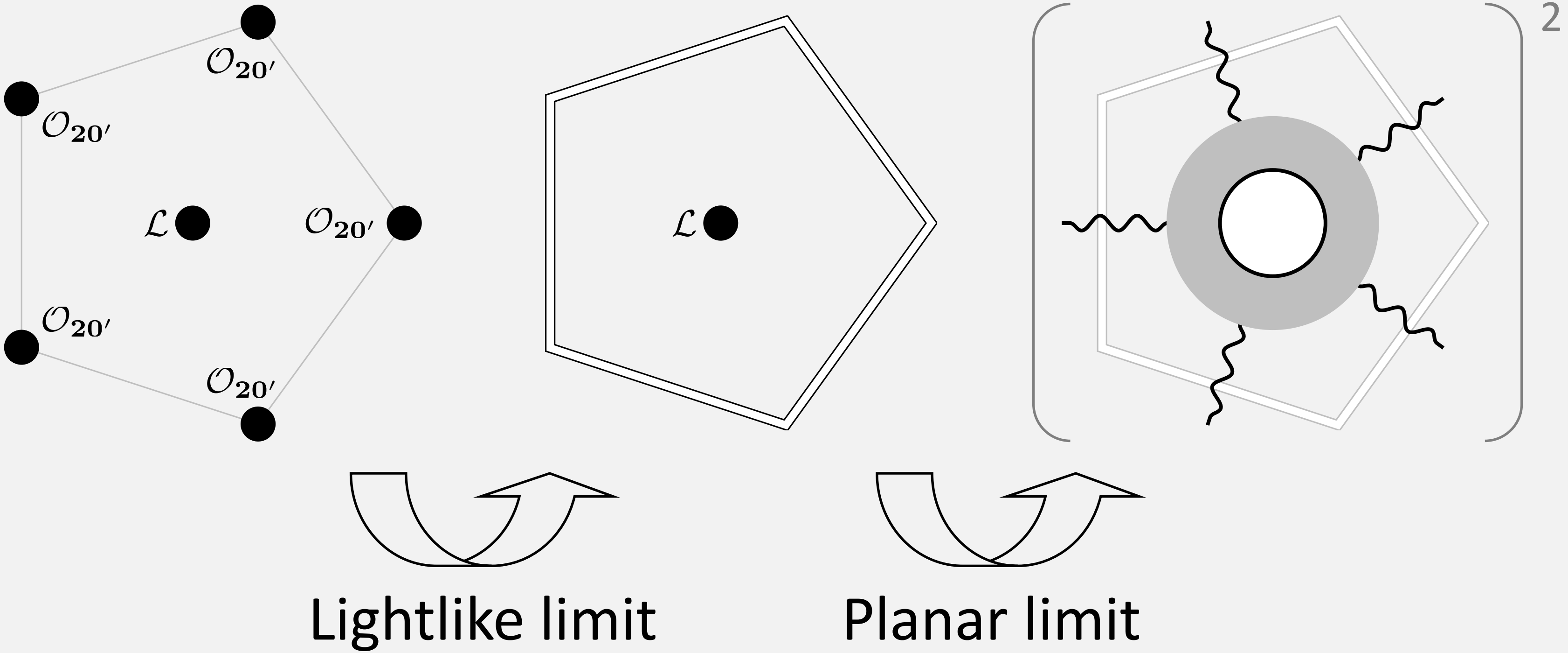
$$\sum_{3\cdots n} |{}_{\text{out}}\langle n\cdots 3|21\rangle_{\text{in}}|^2 \propto \int d\tilde\xi_F d\tilde\eta_F \mathcal{A}(\lambda,\tilde\lambda,\tilde\xi)\delta(\tilde\xi_F-\tilde\eta_F)\mathcal{A}(\lambda,\tilde\lambda,\tilde\eta)$$

$$= s_{12}^4\delta(\tilde\xi_I-\tilde\eta_I)\int d\tilde\eta d\chi \frac{\mathcal{A}(\tilde\lambda,\lambda,\chi)}{\delta^8(\tilde\lambda\chi)} e^{-\tilde\eta\chi}\frac{\mathcal{A}(\lambda,\tilde\lambda,\eta)}{\delta^8(\lambda\tilde\eta)}$$

$= 2\times M(\lambda,\tilde\lambda)$; eq.(2) of [2508.03813]

Independent of 1,2 helicities
due to N=4 supersymmetry.
See e.g. [Chicherin et al., 2512.23791].

Correlator / Wilson-Loop / Amplitude



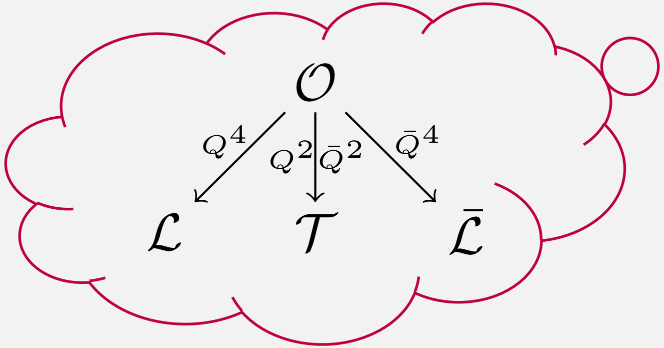
[Alday et al., 1007.3243]

Lagrangian Insertion & Loop Integrand

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle^{(\ell)} \sim \int \mathrm{d}^4 x_5 \cdots \mathrm{d}^4 x_{4+\ell} \langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \mathcal{L}_5 \cdots \mathcal{L}_{4+\ell} \rangle^{(0)}$$

$$\mathcal{O}_i \equiv \mathcal{O}(x_i)$$

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \mathcal{L}_5 \cdots \mathcal{L}_{4+\ell} \rangle^{(0)} = 2 x_{13}^4 x_{24}^4 \frac{N_c^2 - 1}{(4\pi^2)^{4+\ell}} \boxed{F_{4+\ell}(x_1, \cdots, x_{4+\ell})}$$

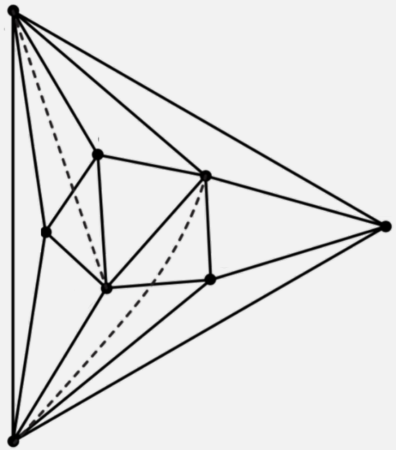


permutation invariant

conformal weight = 4

rational with simple poles

SYM: Planar (4-)f-Graphs



$$\equiv \frac{x_{16}^2 x_{37}^2}{x_{12}^2 x_{13}^2 x_{14}^2 x_{15}^2 x_{17}^2 x_{23}^2 x_{27}^2 x_{28}^2 x_{34}^2 x_{36}^2 x_{38}^2 x_{45}^2 x_{46}^2 x_{56}^2 x_{57}^2 x_{67}^2 x_{68}^2 x_{78}^2}$$

+ inequivalent permutations

SYM: Loops ≈ Legs (!)

dual loop momenta $\quad N = n + L$

$$M_n^{(L)}(x_1, \cdots, x_n, x_{n+1}, \cdots, x_{n+L}) = \lim_{x_{12}^2, \cdots, x_{n1}^2 \to 0} (x_{12}^2 \cdots x_{n1}^2) F_N(x_1, \cdots, x_N)$$

$p_i = x_{(i+1) \bmod n} - x_i$

Loops ≈ legs in squared amplitudes ← Lagrangian-inserted correlators of operators in the same multiplet

[Eden et al., 1201.5329] ↓ (ℓ = 6)

[Boujaily et al., 1609.00007] ↓ (ℓ = 10)

[2503.15593] ↓ (ℓ = 12)

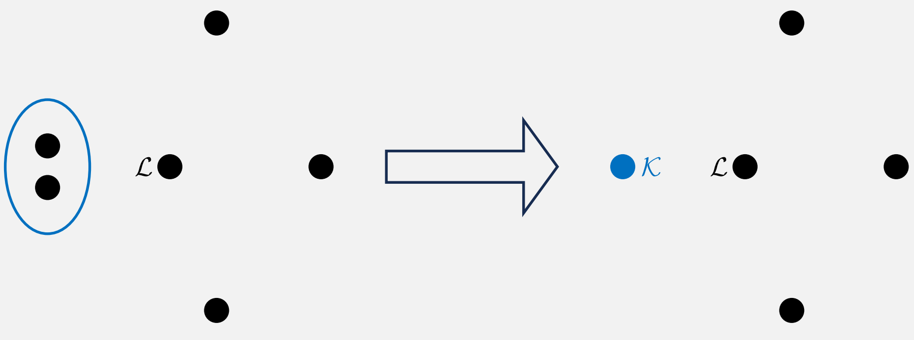
$\ell =$	1	2	3	4	5	6	7	8	9	10	11	12
recursed cells:	1	10	146	2,684	56,914	1,329,324	33,291,164	878,836,728	24,175,924,094	687,444,432,396	20,086,271,785,340	600,384,612,445,304
f-graphs:	1	1	1	3	7	36	220	2,707	42,979	898,353	22,024,902	619,981,403
(contributing:)	1	1	1	3	7	26	127	1,060	10,525	136,433	2,048,262	35,503,735

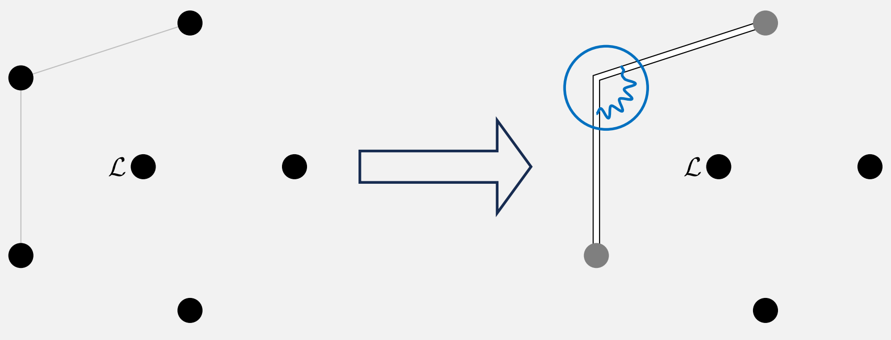
↑ (ℓ = 3) [Eden et al., 1108.3557]

↑ (ℓ = 8) [Boujaily et al., 1512.07912]

↑ (ℓ = 11) [2410.09859]

SYM: OPE Limit and Cusp Limit



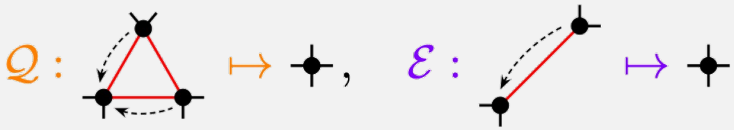


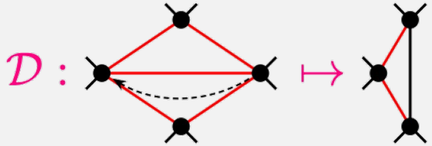
div. from $x_{N+1} \to x_{1,2}$

$$\frac{\int \mathrm{d}^4 x_{N+1} F_{N+1}}{F_N} \sim \frac{\gamma_{\mathcal{K}}^{(1)}}{2} \log x_{12}^2$$

div. from $x_{N+1} \to x_2$

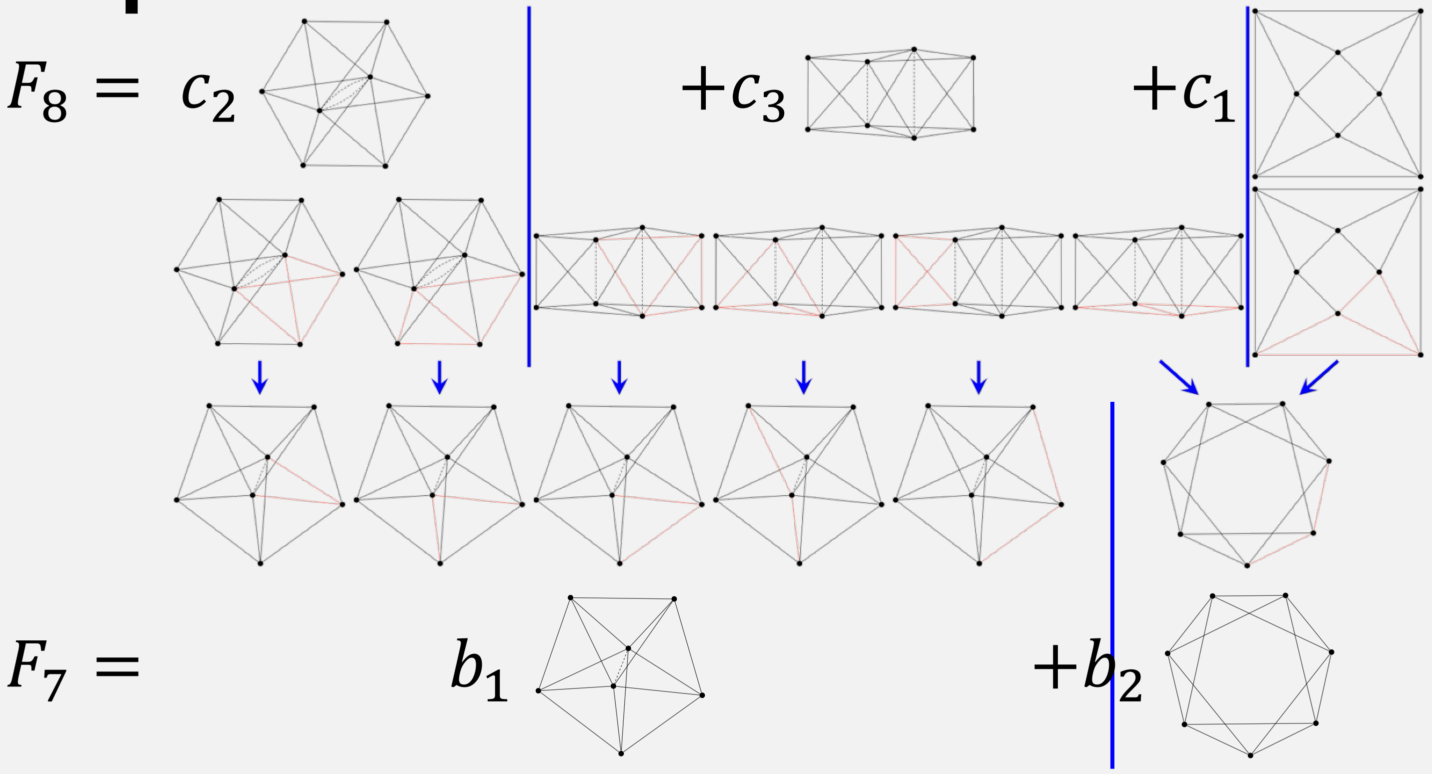
$$\frac{\int \mathrm{d}^4 x_{N+1} F_{N+1}}{F_N} \sim -\frac{\Gamma_{\text{cusp}}^{(1)}}{4} \log x_{12}^2 \log x_{23}^2$$





stronger version of amplitude soft limit

Examples



$c_2 = b_1$ $c_3 = b_1$ $c_3 + c_1 = b_2 (= 0)$

ABJM: Superamplitudes [Aharony et al., 0806.1218]

$$S_{\text{ABJM}} = \frac{k}{2\pi}\left[\int \operatorname{tr}\left(A\wedge \mathrm{d}A - \hat{A}\wedge \mathrm{d}\hat{A}\right) + \int \mathrm{d}^3x\,\left(-|D\phi|^2 + i\bar{\psi}\not{D}\psi + \mathcal{L}_{\text{int}}\right)\right]$$

3d integrable QFT: planar $U(N_c)\times U(N_c)$ $\mathcal{N}$=6 superconformal Chern-Simons-matter theory

$$|\Phi(\eta)\rangle = \quad |\phi_4\rangle + (\eta)^1|\psi^{1,2,3}\rangle + (\eta)^2|\phi_{1,2,3}\rangle + (\eta)^3|\psi^4\rangle \in (\mathbf{N}_c,\bar{\mathbf{N}}_c)$$
$$|\bar{\Psi}(\eta)\rangle = \quad |\bar{\psi}_4\rangle + (\eta)^1|\bar{\phi}^{1,2,3}\rangle + (\eta)^2|\bar{\psi}_{1,2,3}\rangle + (\eta)^3|\bar{\phi}^4\rangle \in (\bar{\mathbf{N}}_c,\mathbf{N}_c)$$

fermionic Fourier transform:
3 superspace ←→ **3*** superspace

$$|\Psi(\bar{\eta})\rangle = (\bar{\eta})^3|\phi_4\rangle + (\bar{\eta})^2|\psi^{1,2,3}\rangle + (\bar{\eta})^1|\phi_{1,2,3}\rangle + \quad |\psi^4\rangle \in (\mathbf{N}_c,\bar{\mathbf{N}}_c)$$
$$|\bar{\Phi}(\bar{\eta})\rangle = (\bar{\eta})^3|\bar{\psi}_4\rangle + (\bar{\eta})^2|\bar{\phi}^{1,2,3}\rangle + (\bar{\eta})^1|\bar{\psi}_{1,2,3}\rangle + \quad |\bar{\phi}^4\rangle \in (\bar{\mathbf{N}}_c,\mathbf{N}_c)$$

$$[\mathcal{A}(\Phi\bar{\Psi}\cdots;+k)]^* = \mathcal{A}(\bar{\Phi}\Psi\cdots;-k) \Leftrightarrow \mathcal{A}_{\text{shift}}(\bar{\Psi}\Phi\cdots;+k)$$

ABJM: Differential Cross Sections

$$\mathcal{A}_{\text{shift}}(\lambda,\eta;+k) = \delta^8(\lambda\eta)\left[A^{(0)}_{\text{shift}}(\lambda,\eta) + k^{-1}A^{(1)}_{\text{shift}}(\lambda,\eta) + k^{-2}A^{(2)}_{\text{shift}}(\lambda,\eta) + \cdots\right]$$

$$\Updownarrow$$

$$\mathcal{A}(\lambda,\bar{\eta};-k) = \delta^8(\lambda\bar{\eta})\left[\; A^{(0)}(\lambda,\bar{\eta}) - k^{-1}\; A^{(1)}(\lambda,\bar{\eta}) + k^{-2}\; A^{(2)}(\lambda,\bar{\eta}) + \cdots\right]$$

$$\sum_{3\cdots n} \left|{}_{\text{out}}\langle n\cdots 3|21\rangle_{\text{in}}\right|^2 \propto \int \mathrm{d}\xi_F \mathrm{d}\eta_F \mathcal{A}_{\text{shift}}(\lambda,\xi;+k)\delta(\xi_F - \eta_F)\mathcal{A}(\lambda,\eta;+k)$$

Independent of 1,2 helicities due to N=6 supersymmetry.

$$= s_{12}^3\delta(\xi_I - \eta_I)\int \mathrm{d}\eta \mathrm{d}\bar{\chi}\frac{\mathcal{A}(\lambda,\bar{\chi};-k)}{\delta^6(\lambda\bar{\chi})}e^{-\eta\bar{\chi}}\frac{\mathcal{A}(\lambda,\eta;+k)}{\delta^6(\lambda\eta)}$$

$= 2\times M(\lambda;+k)$; eq.(10) of [2508.03813]

ABJM: Loops ≈ Legs (?) checked up to F_{10}

dual loop momenta $\quad N = n + L$

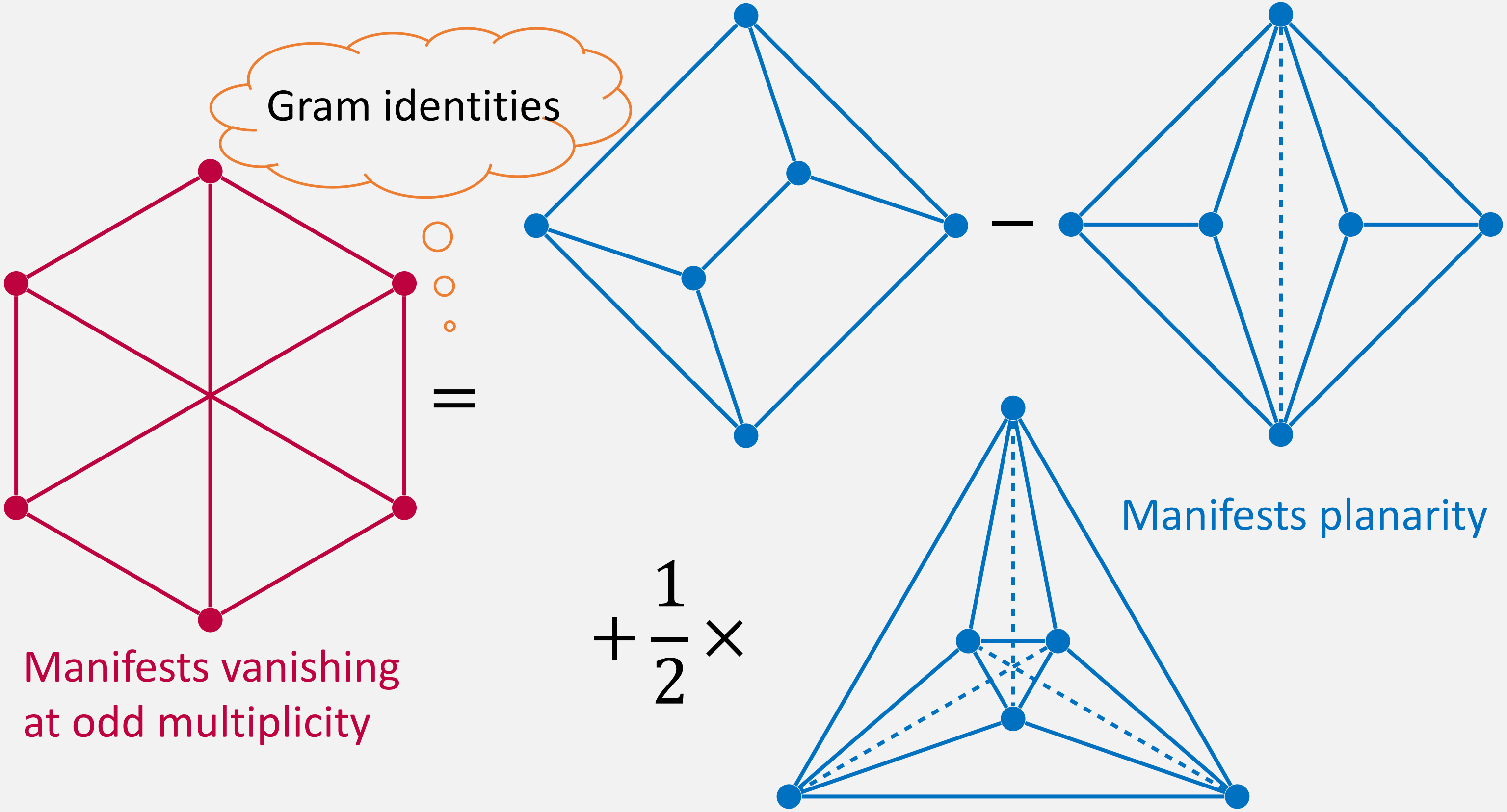
$$M_n^{(L)}(x_1, \cdots, x_n, x_{n+1}, \cdots, x_{n+L}) = \lim_{x_{12}^2, \cdots, x_{n1}^2 \to 0} (x_{12}^2 \cdots x_{n1}^2) F_N(x_1, \cdots, x_N)$$

$p_i = x_{(i+1) \bmod n} - x_i$

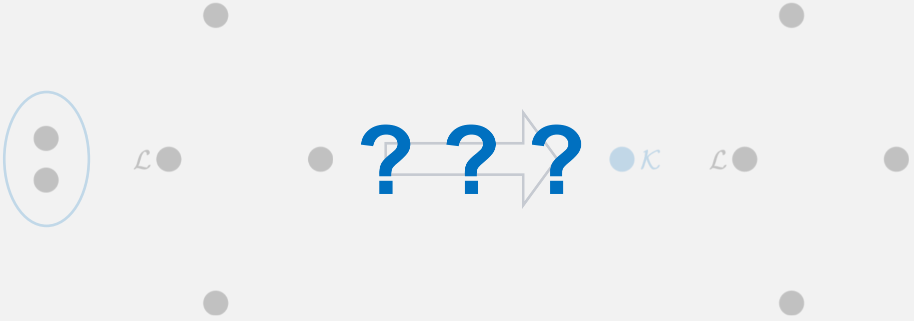
Loops ≈ legs in squared amplitudes → Lagrangian-inserted correlators of operators in the same multiplet???

ABJM: Planar∩Bipartite (3-)f-Graphs

$F_6 = 2\times$

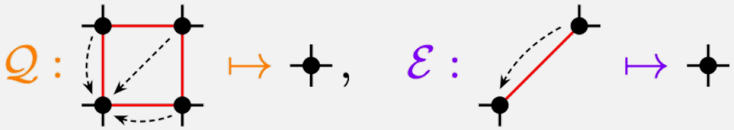


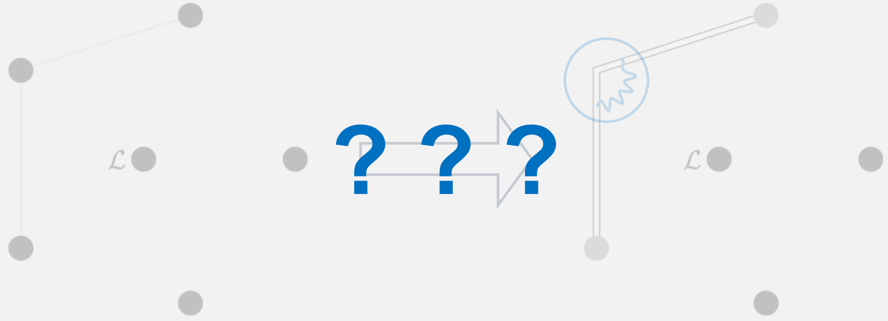
ABJM: OPE Limit and Cusp Limit



div. from $x_{N+1,N+2} \to x_{1,2}$

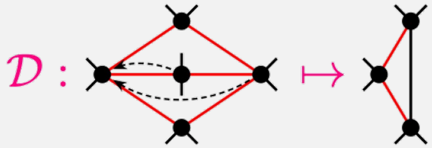
$$\frac{\int d^3x_{N+1} d^3x_{N+2} F_{N+2}}{F_N}$$





div. from $x_{N+1,N+2} \to x_2$

$$\frac{\int d^3x_{N+1} d^3x_{N+2} F_{N+2}}{F_N}$$



stronger version of amplitude double soft limit

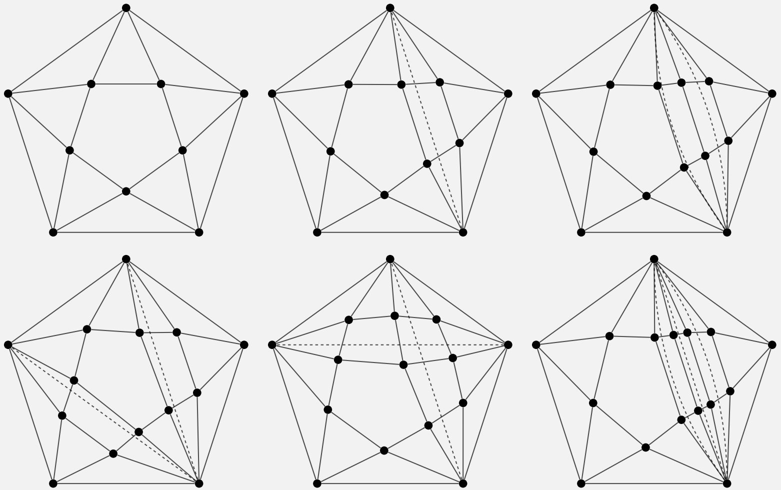
Outlook

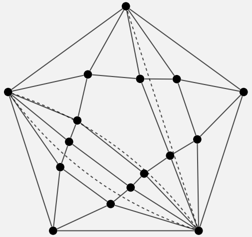
- “Correlator” in weakly-coupled ABJM, reproduce SYM success?
- Exploit permutation symmetry in higher-point SYM correlators
- Squared amplitudes in other theories, on-shell methods
- Consequences for observables such as energy correlators
- Odd-dimensional Feynman integrals

Back-up Slides

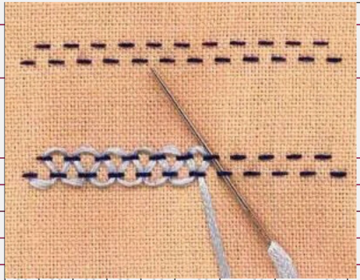
Polygonal Fishnets

$$\text{coef} = \frac{\prod \mathcal{C}_{m(f)}}{\mathcal{C}_{\max m(f)}}$$





$$\mathcal{C}_3 = +1,\ \mathcal{C}_4 = -1,\ \mathcal{C}_5 = +2,$$
$$\mathcal{C}_6 = -5,\ \mathcal{C}_7 = +14,\ \mathcal{C}_8 = -42.$$



	1	2	3	4	5	6	7	8	9	10	11	12
+14									0	1	1	9
+5									0	1	15	194
+17/4									0	0	0	2
+4									0	1	8	88
+15/4									0	0	0	2
+7/2									0	0	0	3
+13/4									0	0	0	3
+3									0	0	3	33
+11/4									0	0	1	30
+5/2									0	3	24	338
+9/4									0	1	13	280
+2	0	0	0	0	0	1	1	8	40	306	2,631	26,524
+7/4	0	0	0	0	0	0	0	0	0	5	75	1,355
+3/2	0	0	0	0	0	0	0	0	1	61	1,368	23,703
+5/4	0	0	0	0	0	0	0	0	0	8	240	5,796
+1	1	1	1	2	5	15	70	472	4,013	39,649	422,353	4,715,081
+3/4	0	0	0	0	0	0	0	0	0	278	9,830	247,553
+1/2	0	0	0	0	0	0	0	78	1,289	25,603	448,236	7,496,410
+1/4	0	0	0	0	0	0	0	0	0	2,717	141,343	4,766,337
0	0	0	0	0	0	10	93	1,647	32,454	761,920	19,976,640	584,477,668
−1/4	0	0	0	0	0	0	0	0	0	2,714	141,393	4,769,077
−1/2	0	0	0	0	0	0	0	63	1,240	25,030	444,432	7,467,462
−3/4	0	0	0	0	0	0	0	0	0	281	9,834	247,461
−1	0	0	0	1	2	10	56	434	3,906	39,300	421,022	4,709,533
−5/4	0	0	0	0	0	0	0	0	0	10	240	5,852
−3/2	0	0	0	0	0	0	0	3	21	268	2,896	34,040
−7/4	0	0	0	0	0	0	0	0	0	0	15	657
−2	0	0	0	0	0	0	0	1	14	184	2,181	25,182
−9/4	0	0	0	0	0	0	0	0	0	3	41	544
−5/2	0	0	0	0	0	0	0	0	0	1	21	359
−11/4	0	0	0	0	0	0	0	0	0	0	1	41
−3	0	0	0	0	0	0	0	0	0	0	3	52
−7/2	0	0	0	0	0	0	0	0	0	0	0	5
−4	0	0	0	0	0	0	0	0	0	0	0	8
−9/2	0	0	0	0	0	0	0	0	0	0	0	3
−5	0	0	0	0	0	0	0	1	1	8	42	328
−14	0	0	0	0	0	0	0	0	0	0	0	1
−42	0	0	0	0	0	0	0	0	0	0	0	1
	1	1	1	3	7	26	127	1,060	10,525	136,433	2,048,262	35,503,735
	1	1	1	3	7	36	220	2,707	42,979	898,353	22,024,902	619,981,403

+57/8	1
+33/8	3
+27/8	1
+25/8	5
+23/8	5
+21/8	6
+19/8	12
+17/8	6
+15/8	14
+13/8	70
+11/8	78
+9/8	314
+7/8	2985
+5/8	6618
+3/8	41362
+1/8	428464
−1/8	428118
−3/8	41252
−5/8	6630
−7/8	2938
−9/8	285
−11/8	70
−13/8	71
−15/8	20
−17/8	29
−19/8	8
−21/8	9
−13/4	1
−27/8	2
−15/4	2
−17/4	8
−10	1