Seeing through the confinement screen

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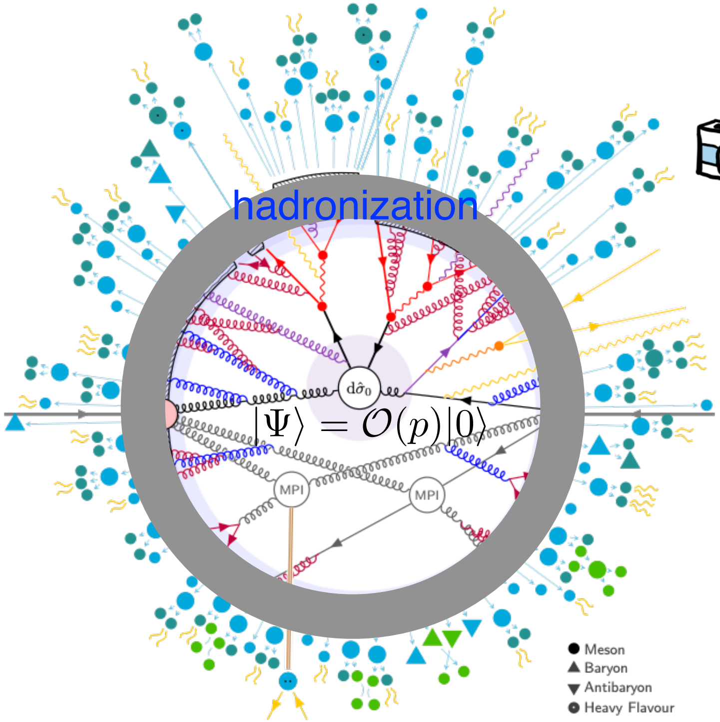
Motivation

- Can we predict the number of particles in a high-energy QCD process?



📷 $\Rightarrow \langle \mathrm{N}^h \rangle_Q$

$$p^2 = Q^2 \gg \Lambda^2_{\mathrm{QCD}}$$

- Particle number itself encodes non-perturbative physics, but its scale evolutoin can be predicted using perturbation theory.

$$\frac{d}{d\log Q} \log\langle \mathrm{N}^h \rangle_Q =?$$

Hadron multiplicties

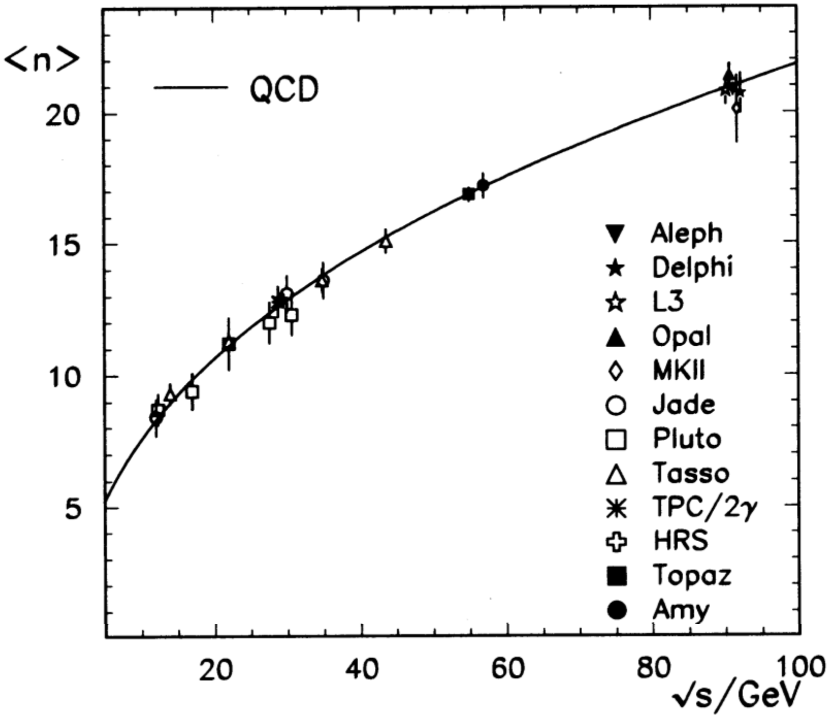
- An old problem: hadron multiplicities

  [Bassetto, Ciafaloni, Marchesini '80; Amati, Bassetto, Ciafaloni, Marchesini, Veneziano '80; Furmanski, Petronzio, Pokorski '79; Konishi '79; Mueller '81]

- Perturbative expansion breaks down; requires resummation:

$$\frac{d}{d\log Q}\log\langle \mathbb{N}^h\rangle_Q = \sqrt{\frac{2C_A}{\pi}\alpha_s} + O(\alpha_s)$$

[Schmelling '95]



- Today: a new framework that can obtain (and generalize) this result.

Running and matching of detectors/light-ray operators

Hadron detectors

- Energy detector (ANEC): [Sterman ’75]

$$\mathcal{E}(\widehat{n}) = \lim_{r\to\infty} r^{d-2}\int_0^\infty dt\; n^i T^0{}_i(t, r\widehat{n}) \qquad \mathcal{E}(\widehat{n})|\vec{p}\rangle = \omega_{\vec{p}}\,\delta^{(2)}(\widehat{p}-\widehat{n})|\vec{p}\rangle$$

- Hadron number detector:

$$z = (1, \vec{n}),\;\; z^2 = 0$$

$$\mathbb{N}^h(z) \propto \sum_h \int \underbrace{dE\, E^{d-3}}_{\text{phase space measure}} a_h^\dagger(p) a_h(p)\Big|_{p=Ez}$$

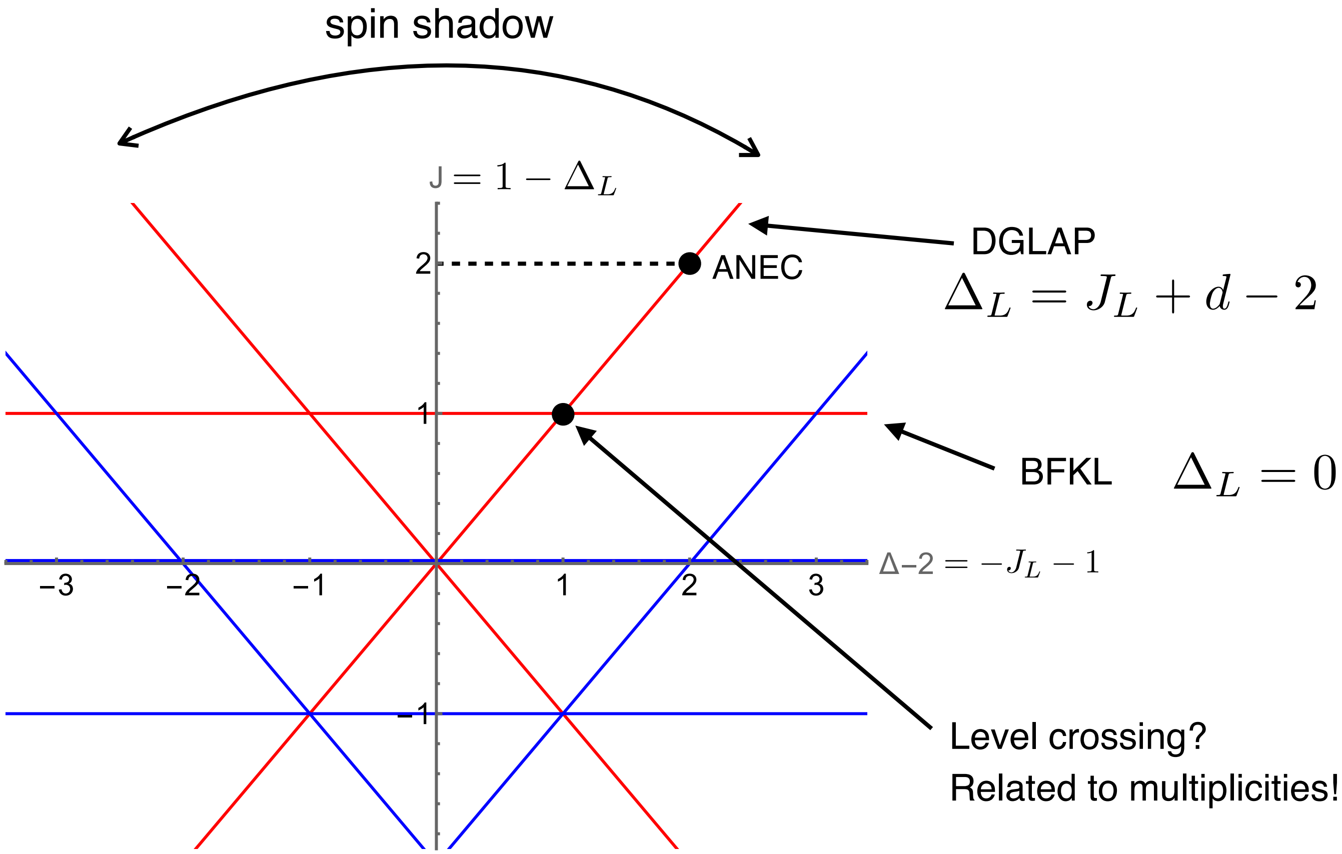
sum over particles

- Assuming massless particles; can also define detectors for massive particles

- Each detector has a scaling dimension Δ_L and a Lorentz boost weight J_L. The hadron number detector has $J_L = 2 - d$.

Detectors in perturbative QCD

- Chew-Frautschi plot:



DGLAP
$\Delta_L = J_L + d - 2$

BFKL $\Delta_L = 0$

Level crossing?
Related to multiplicities!

- What is the relation between QCD detectors and hadron multiplicities?

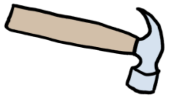
- What is the space of QCD detectors at one-loop?

Light-ray matching hypothesis

- Measurements at infinity in the theory of hadron can be **matched** onto a linear combination of detectors/light-ray operators in perturbative QCD.

$$\mathcal{D}^{\mathrm{IR}}(z) = \sum_i C_i \mathcal{D}_i^{\mathrm{UV}}(z)$$

local operator v.s. detector matching


: measurements at a point

Precisely described by local operators in the UV

$$\langle \mathcal{O}^{\mathrm{UV}}(x_1)\ldots\mathcal{O}^{\mathrm{UV}}(x_n)\rangle$$

$$\Big\downarrow \text{ long distances}$$

$$\mathcal{O}^{UV}(x) = \sum_i C_i \mathcal{O}_i^{IR}(x)$$


: measurements at infinity

Precisely described by detectors in the IR

$$\langle\Psi|\mathcal{D}^{\mathrm{IR}}(z_1)\ldots\mathcal{D}^{\mathrm{IR}}(z_n)|\Psi\rangle$$

$$\Big\downarrow \text{ high-energy states}$$

$$\mathcal{D}^{\mathrm{IR}}(z) = \sum_i C_i \mathcal{D}_i^{\mathrm{UV}}(z)$$

- Detectors/operators on both sides must transform in the same representation under symmetries preserved along the RG flow.

Confinement as a Lorentz-invariant filter

See Hao's talk for more details

- IR detectors with definite Lorentz spin should be matched onto UV detectors with the same Lorentz spin:

$$\mathcal{D}^{\mathrm{IR}}_{J_L}(z) = \sum_i C_i(J_L,\mu)\mathcal{D}^{\mathrm{UV}}_{J_L,i}(z,\mu)$$

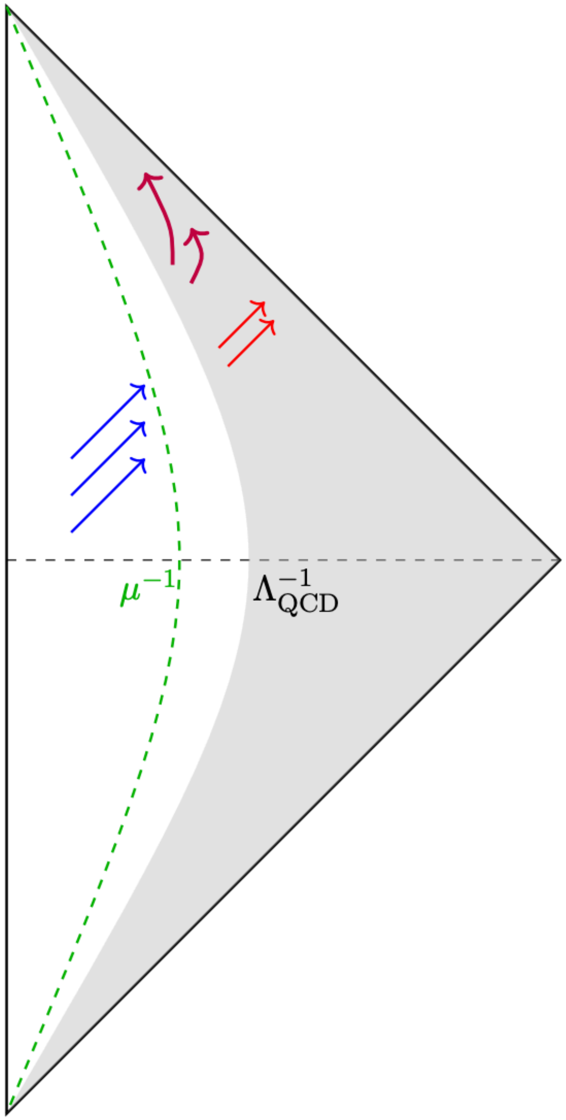
$$\langle \mathbb{N}^h \rangle = \sum_i C_i(J_L,\mu)\langle \mathcal{D}^{\mathrm{UV}}_{J_L,i}(z,\mu)\rangle\Big|_{J_L=2-d}$$

Dimensional analysis: $\langle \mathcal{D}^{\mathrm{UV}}_{J_L,i}\rangle_Q \sim Q^{-\Delta_{L,i}}$

$$C_i(J_L) \sim (\Lambda_{\mathrm{QCD}})^{\Delta_{L,i}}$$

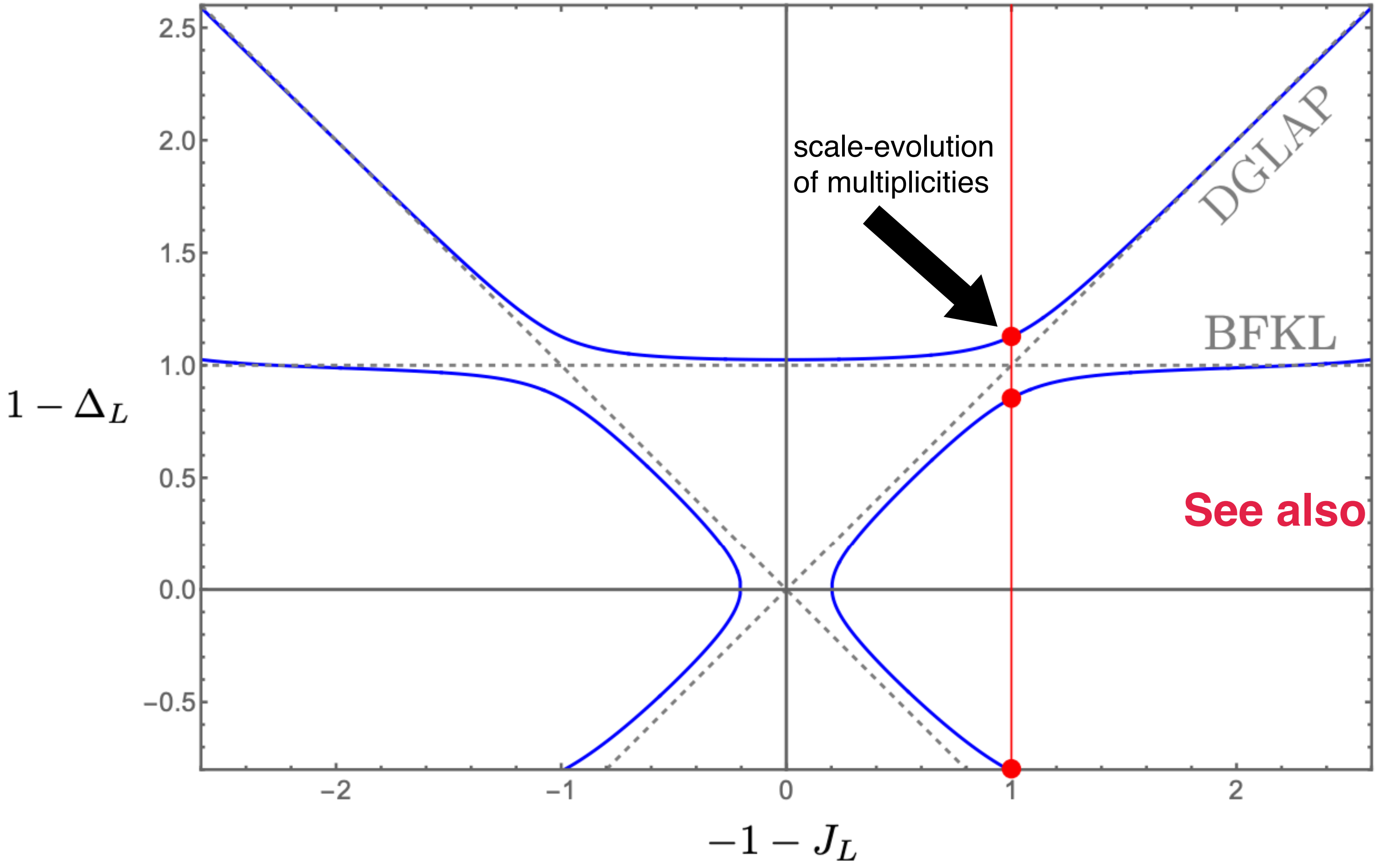
- At high-energy, QCD detector with the smallest Δ_L controls the scale-evolution of hadron multiplicities.

$$\frac{d}{d\log Q}\log\langle \mathbb{N}^h\rangle_Q = -\Delta_{L,i_{\min}}(J_L,\alpha_s(Q))\Big|_{J_L=2-d} + \ldots$$



Hadron multiplicities from DGLAP/BFKL mixing

- Chew-Frautschi plot of pure Yang-Mills theory at one loop:



scale-evolution of multiplicities

See also Nikolay's talk

Light-ray operators and detectors in CFT

- Light-ray operators [Kravchuk, Simmons-Duffin '18] are analytic continuation of null integral (light transform) of local operators.

$$\mathbf{L}[\mathcal{O}](x,z) = \int_{-\infty}^{\infty} d\alpha (-\alpha)^{-\Delta-J} \mathcal{O}(x - z/\alpha, z)$$

$$z^2 = 0$$

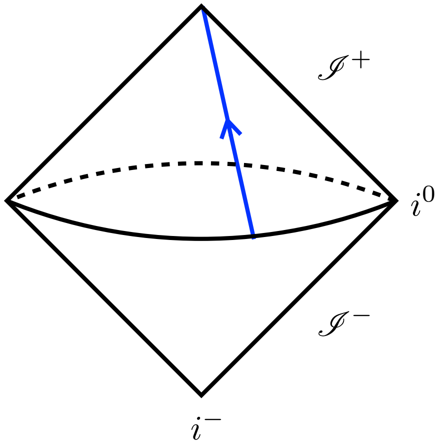
$$\mathcal{O}(x,z) = \mathcal{O}^{\mu_1 \dots \mu_J}(x) z_{\mu_1} \dots z_{\mu_J}$$

- $\mathbf{L}[\mathcal{O}]$ has quantum numbers $(\Delta_L, J_L) = (1-J, 1-\Delta)$

- Choosing $x = \infty$ gives an integral along future null infinity, and $z = (1,\vec{n})$ is the embedding space coordinate of the celestial sphere S^{d-2}. This is a "detector operator" $\mathcal{D}_{J_L}(z)$.

- Each detector has a "spin shadow" $\mathbf{S}_J : J_L \to 2-d-J_L$.

$$\mathbf{S}_J[\mathcal{D}_{J_L}](z) = \int D^{d-2} z' (-2z \cdot z')^{2-d-J_L} \mathcal{D}_{J_L}(z')$$



DGLAP detectors in free theory

- In the free theory, detectors can also be defined by how they act on particles.
- DGLAP detector in free theory:

(assuming pure Yang-Mills for now)

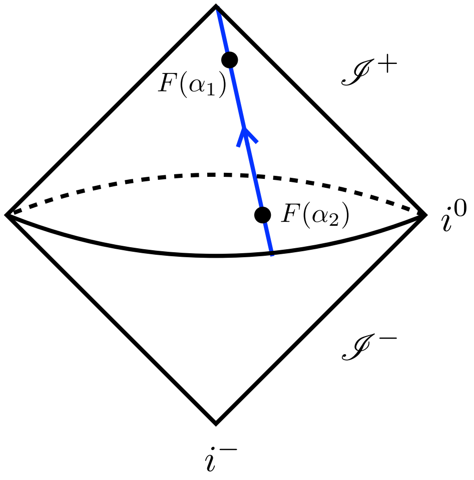
$$\mathcal{D}_{J_L}^{\mathrm{DGLAP}}(z) \propto \delta^{ab} \sum_{\lambda} \int dE\, E^{-J_L-1} \left[a^{\dagger}_{\lambda,a}(p) a_{\lambda,b}(p) \right] \Big|_{p=Ez}$$

- A detector that measures flux of E^{2-d-J_L}. $J_L = 1-d$ gives the ANEC operator.
- Can be written as a bilocal integral along a null direction z at future null infinity.

$$\mathcal{D}_{J_L}^{\mathrm{DGLAP}}(z) \propto \int d\alpha_1 d\alpha_2 |\alpha_1 - \alpha_2|^{d-2+J_L} F_a(\alpha_1, z) F^a(\alpha_2, z)$$

$$J_L = 1 - d - n \Rightarrow \mathcal{D}_{J_L}^{\mathrm{DGLAP}} \propto \mathbf{L}[F \partial^n F]$$

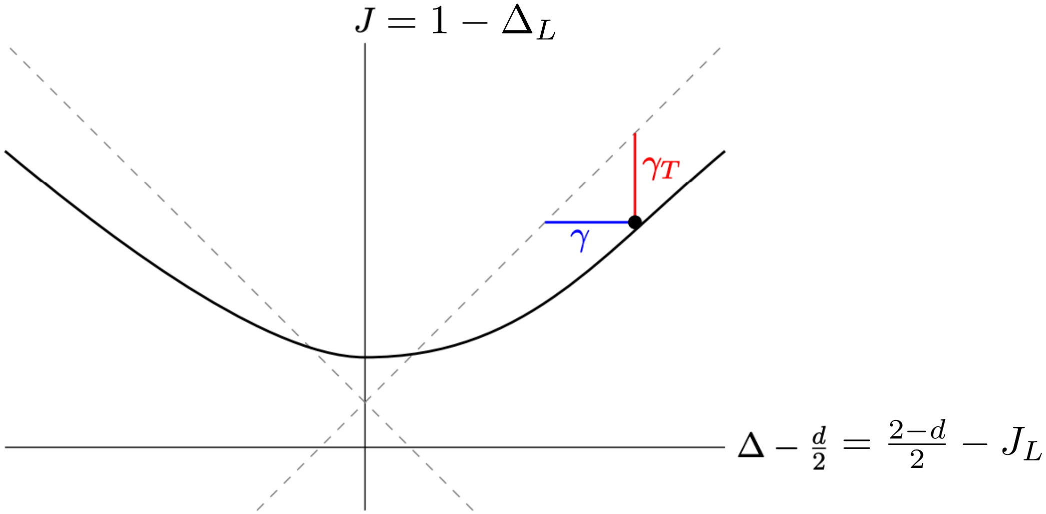
light transform of twist-2
local operators



Reciprocity relation

[Caron-Huot, Kologlu, Kravchuk, Meltzer, Simmons-Duffin ’22]

- Local operators: J fixed, Δ corrected (spacelike anomalous dimension)
- Detectors: J_L fixed, Δ_L corrected (timelike anomalous dimension)



- Reciprocity relation: [Gribov, Lipatov ’72; Basso,Korchemsky ’06; Dokshitzer, Marchesini ’06]

$$\gamma_T(J) = \gamma(J - \gamma_T(J)) \qquad \gamma(J) = \gamma_T(J + \gamma(J))$$

- Instead of J, we should use J_L as the quantum number for the anomalous dimensions of detectors.

DGLAP detectors

- DGLAP detector in free theory:

(assuming pure Yang-Mills for now)

$$\mathcal{D}^{\mathrm{DGLAP}}_{J_L}(z) \propto \delta^{ab} \sum_{\lambda} \int dE\, E^{-J_L-1} \left[a^{\dagger}_{\lambda,a}(p) a_{\lambda,b}(p)\right] \Big|_{p=Ez}$$

- Well-defined for all $J_L \in \mathbb{C}$. **(See Hao's talk later)**

- Become not IR safe after turning on interactions ⇒ need to be renormalized

- For generic J_L, running of the DGLAP detector is controlled by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation.

$$\gamma_{\mathrm{DGLAP}}(J_L) = 4C_A \left(\psi(-J_L) + \gamma_E - \frac{1}{(J_L+2)(J_L+1)} - \frac{1}{J_L(J_L-1)}\right) - \beta_0$$

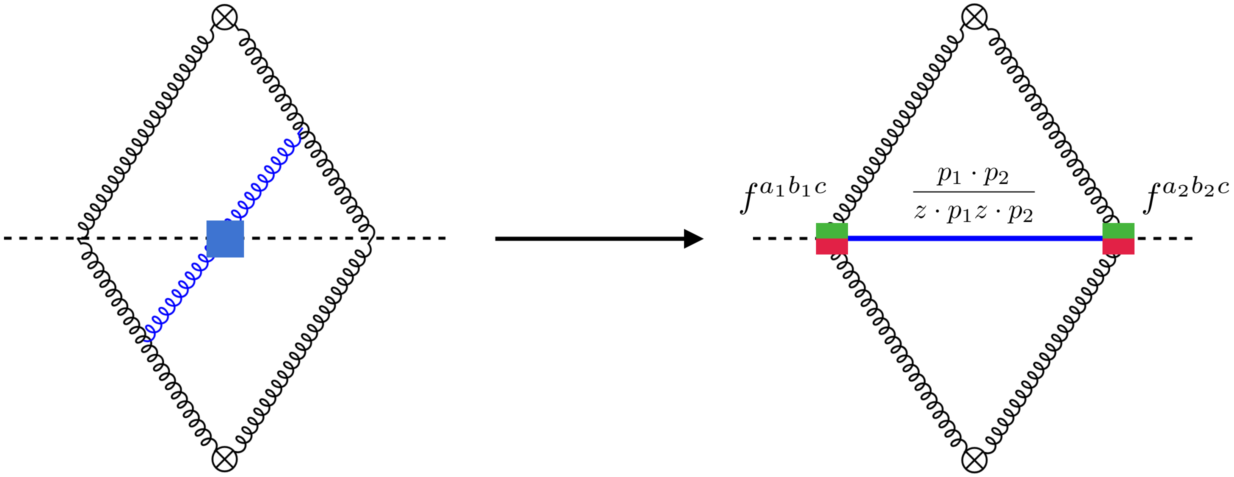
- Multiiplicities correspond to $J_L = -2 \;\; \Rightarrow \gamma_{\mathrm{DGLAP}}(J_L) \sim \dfrac{2C_A}{\pi(J_L+2)}$

BFKL detector from the soft theorem

- The pole at $J_L = -2$ comes from the leading soft theorem.

$$\langle \mathcal{D}^{\mathrm{DGLAP}}_{J_L}(z)\rangle^{1-\mathrm{loop}} = \int d\mathrm{PS}_{n-1}\int d\beta\,\beta^{-J_L-1}|\mathcal{F}_{\mathrm{tree}}(\beta z, p_2, \ldots, p_n)|^2$$

$$\sim \int d\mathrm{PS}_{n-1}\int \underbrace{d\beta\,\beta^{-J_L-3}}_{\frac{1}{J_L+2}} \sum_{i,j=2}^{n} \underbrace{\frac{p_i\cdot p_j}{z\cdot p_i z\cdot p_j} f^{a_ib_ic}f^{a_jb_jc}\left(\mathcal{F}^{a_2\ldots b_j\ldots a_n}_{\mathrm{tree}}\right)^{*}\mathcal{F}^{a_2\ldots b_i\ldots a_n}_{\mathrm{tree}}}_{\text{tree-level matrix element of BFKL detector}}$$



BFKL detector in free theory

- The BFKL detector is a product of two "color-interference" detectors smeared over the celestial sphere:

project onto a single Lorentz irrep

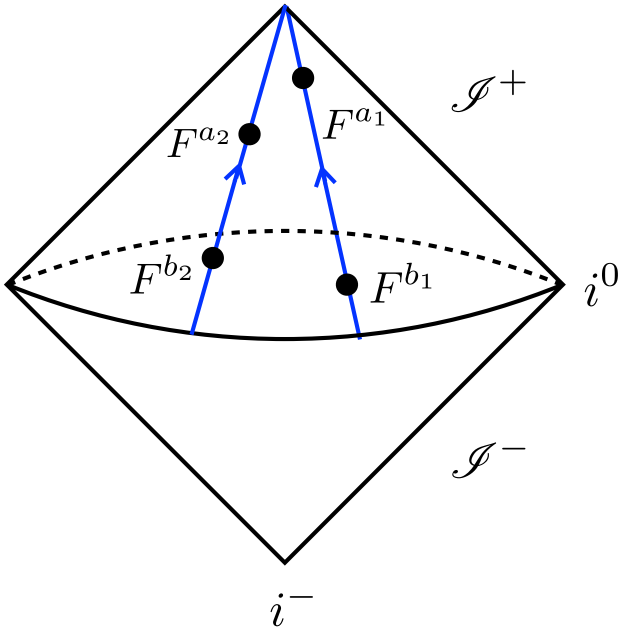
$$\mathcal{D}^{\mathrm{BFKL}}_{J_L}(z) = \int D^{d-2}z_1 D^{d-2}z_2\, K_{J_L}(z_1, z_2, z) : \mathcal{N}^c(z_1)\mathcal{N}^c(z_2) :$$

color correlations

$$\mathcal{N}^c(z) \propto i f^{abc} \sum_\lambda \int \frac{E^{d-2}dE}{2E} \left[a^\dagger_{\lambda,a}(p) a_{\lambda,b}(p)\right]\Big|_{p=Ez}$$

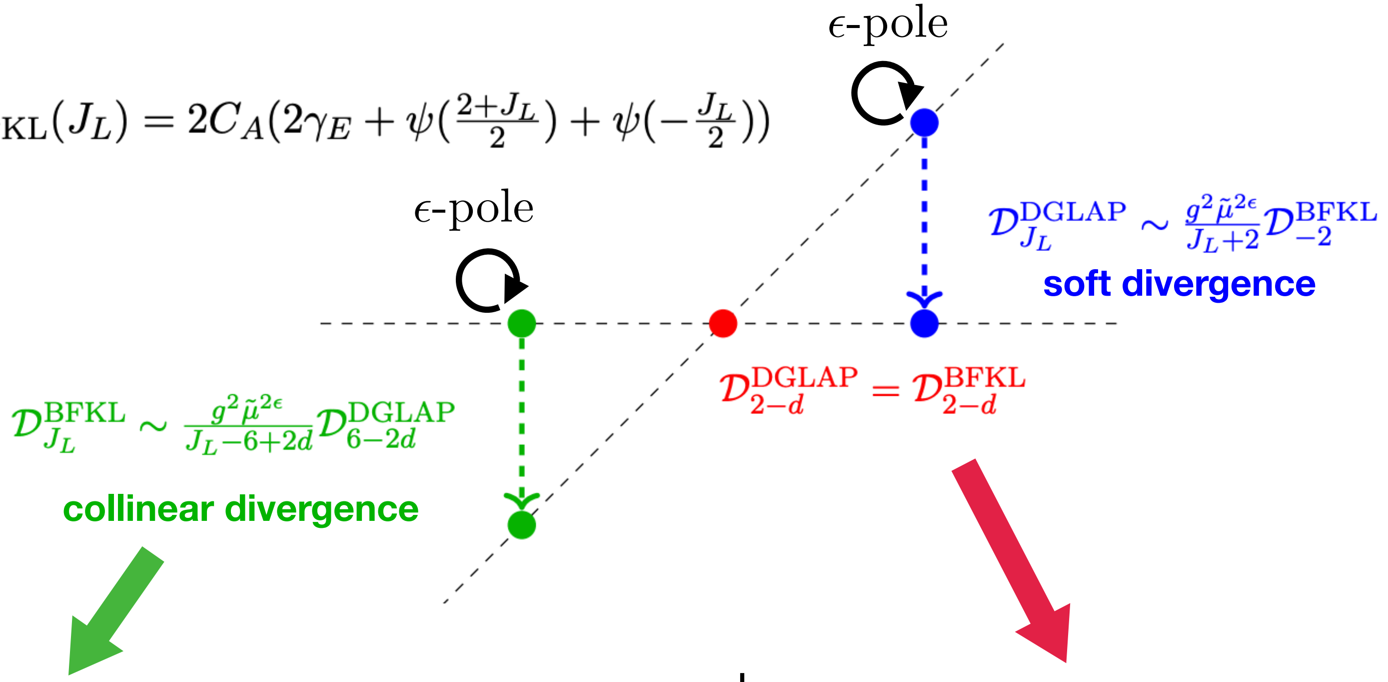
$$K_{J_L} \propto \left(\frac{-2z_1 \cdot z_2}{(-2z_1 \cdot z)(-2z_2 \cdot z)}\right)^{-\frac{J_L}{2}}$$

- A Wilson loop at future null infinity decorated by four field strengths.
- $\Delta_L = 0$ for all values of $J_L \Rightarrow$ a horizontal trajectory at $J = 1$ on the C-F plot.
- Running is controlled by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation.



DGLAP/BFKL mixing

$$\gamma_{\rm BFKL}(J_L) = 2C_A(2\gamma_E + \psi(\tfrac{2+J_L}{2}) + \psi(-\tfrac{J_L}{2}))$$



ϵ-pole

ϵ-pole

$$\mathcal{D}^{\rm DGLAP}_{J_L} \sim \frac{g^2\tilde{\mu}^{2\epsilon}}{J_L+2}\mathcal{D}^{\rm BFKL}_{-2}$$

soft divergence

$$\mathcal{D}^{\rm DGLAP}_{2-d} = \mathcal{D}^{\rm BFKL}_{2-d}$$

$$\mathcal{D}^{\rm BFKL}_{J_L} \sim \frac{g^2\tilde{\mu}^{2\epsilon}}{J_L-6+2d}\mathcal{D}^{\rm DGLAP}_{6-2d}$$

collinear divergence

light-ray OPE:

$$\mathcal{N}^c(z_1)\mathcal{N}^c(z_2) \sim \frac{g^2\tilde{\mu}^{2\epsilon}}{2z_1\cdot z_2}\mathcal{D}^{\rm DGLAP}_{6-2d}(z_2)$$

$$\langle\mathcal{D}^{\rm BFKL}_{J_L}(z)\rangle \sim \int d\theta\,\theta^{J_L-5+2d}\frac{g^2\tilde{\mu}^{2\epsilon}}{\theta^2}\langle\mathcal{D}^{\rm DGLAP}_{6-2d}(z)\rangle$$

$$\sim \frac{g^2\tilde{\mu}^{2\epsilon}}{J_L-6+2d}\langle\mathcal{D}^{\rm DGLAP}_{6-2d}(z)\rangle$$

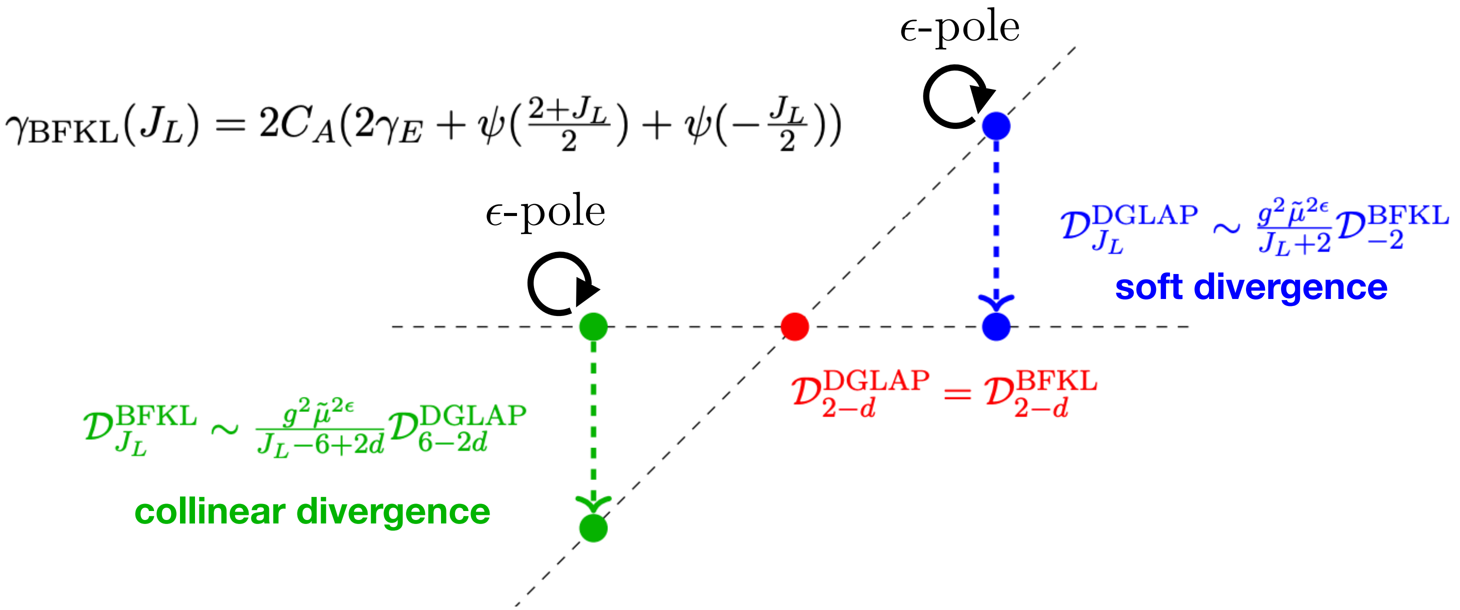
$$\mathcal{D}^{\rm BFKL}_{2-d}(z) = \frac{\mathrm{vol}(S^{d-3})}{2} :\mathcal{N}^c(z)\mathcal{G}^c:$$

color charge operator:

$$\mathcal{G}^c = \int D^{d-2}z'\,\mathcal{N}^c(z')$$

$$\Rightarrow \mathcal{D}^{\rm BFKL}_{2-d}(z) = -\frac{\mathrm{vol}(S^{d-3})}{2}C_A\mathcal{D}^{\rm DGLAP}_{2-d}(z)$$

DGLAP/BFKL mixing



$$\gamma_{\mathrm{BFKL}}(J_L) = 2C_A(2\gamma_E + \psi(\tfrac{2+J_L}{2}) + \psi(-\tfrac{J_L}{2}))$$

$$\mathcal{D}^{\mathrm{DGLAP}}_{J_L} \sim \frac{g^2\tilde{\mu}^{2\epsilon}}{J_L+2}\mathcal{D}^{\mathrm{BFKL}}_{-2}$$

soft divergence

$$\mathcal{D}^{\mathrm{DGLAP}}_{2-d} = \mathcal{D}^{\mathrm{BFKL}}_{2-d}$$

$$\mathcal{D}^{\mathrm{BFKL}}_{J_L} \sim \frac{g^2\tilde{\mu}^{2\epsilon}}{J_L-6+2d}\mathcal{D}^{\mathrm{DGLAP}}_{6-2d}$$

collinear divergence

- A non-degenerate multiplet at the intersection:

$$\mathbb{D}_{J_L} = U_1 \begin{pmatrix} \mu^{J_L+2-2\epsilon}\mathcal{D}^{\mathrm{DGLAP}}_{J_L,g} \\ \mathcal{D}^{\mathrm{BFKL}}_{J_L,g} \end{pmatrix}, \quad U_1 = \begin{pmatrix} -\frac{C_A\pi^{1-\epsilon}}{\Gamma(1-\epsilon)} & 0 \\ \frac{\frac{C_A\pi^{1-\epsilon}}{\Gamma(1-\epsilon)}}{J_L+2-2\epsilon} & \frac{1}{J_L+2-2\epsilon} \end{pmatrix}$$

- Renormalized detector: $[\mathbb{D}_{J_L}]_R \equiv \mathcal{Z}^{-1}_{J_L}\mathbb{D}_{J_L}$

should remove both ϵ-poles and J_L-poles

Renormalization of DGLAP/BFKL detectors

- Dilatation matrix of the multiplet:

$$D = \begin{pmatrix} -J_L - 2 & 0 \\ \boxed{1} & 0 \end{pmatrix} + \frac{\alpha_s}{2\pi} \begin{pmatrix} -\gamma_1(J_L) & 4C_A \\ -\gamma_{21}(J_L) & -\gamma_2(J_L) \end{pmatrix} + O(\alpha_s^2)$$

a log multiplet!

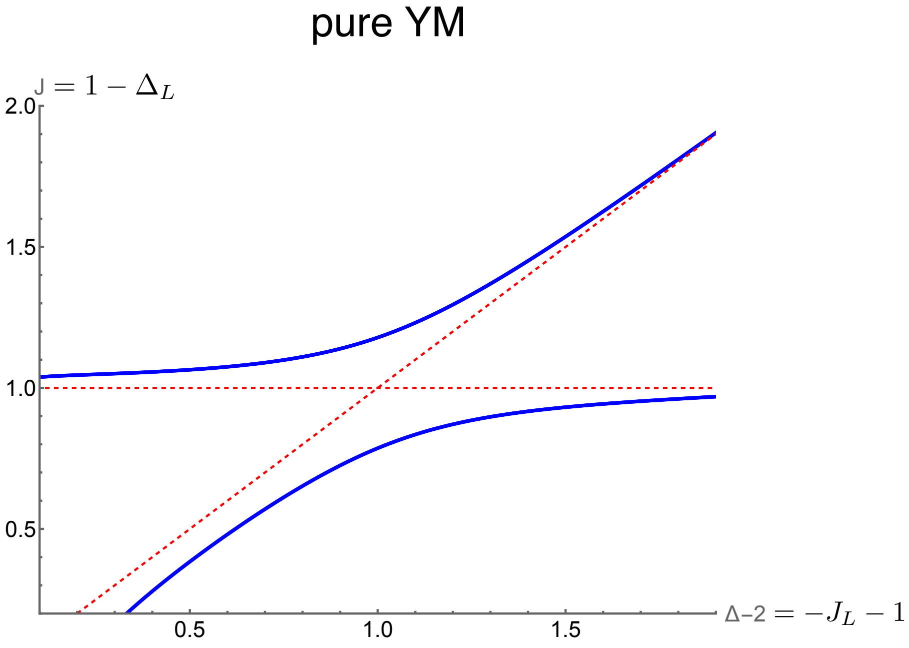
- Why $\sqrt{\alpha_s}$?

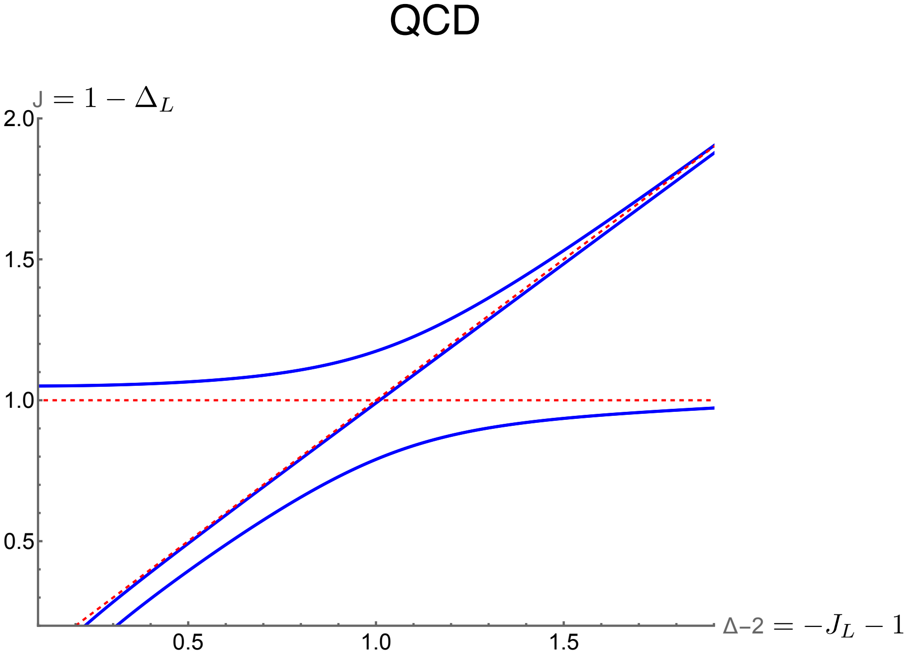
$$\begin{tikzcd}[column sep=tiny] & \Delta_L^+ = \frac{1}{2}\left(J_L + 2 + \sqrt{(J_L+2)^2 + \frac{8C_A}{\pi}\alpha_s + \dots}\right) + \dots \arrow[dl, "\alpha_s \to 0"] \arrow[dr, "J_L \to -2"] & \\ J_L + 2 + \alpha_s \underbrace{\left(\frac{2C_A}{\pi\,(J_L+2)} + \dots\right)}_{\gamma_T^{\mathrm{DGLAP}}(J_L)} + O(\alpha_s^2) & & \sqrt{\frac{2C_A}{\pi}\alpha_s} + O(\alpha_s) \end{tikzcd}$$

Renormalization of DGLAP/BFKL detectors

$$D = \begin{pmatrix} -J_L - 2 & 0 \\ 1 & 0 \end{pmatrix} + \frac{\alpha_s}{2\pi} \begin{pmatrix} -\gamma_1(J_L) & 4C_A \\ -\gamma_{21}(J_L) & -\gamma_2(J_L) \end{pmatrix} + O(\alpha_s^2)$$

$$\mathrm{Det}(D + \Delta_L) = 0$$


pure YM


QCD

Mapping out the C-F plot of QCD

- Define a more general hadron detector that measures massive particles and have general Lorentz spin:

$$\mathbb{N}_{J_L}(z) \propto \sum_h \int d^d p\, \delta(p^2 - m^2)(2p \cdot z)^{J_L} a_h^\dagger(p) a_h(p)$$

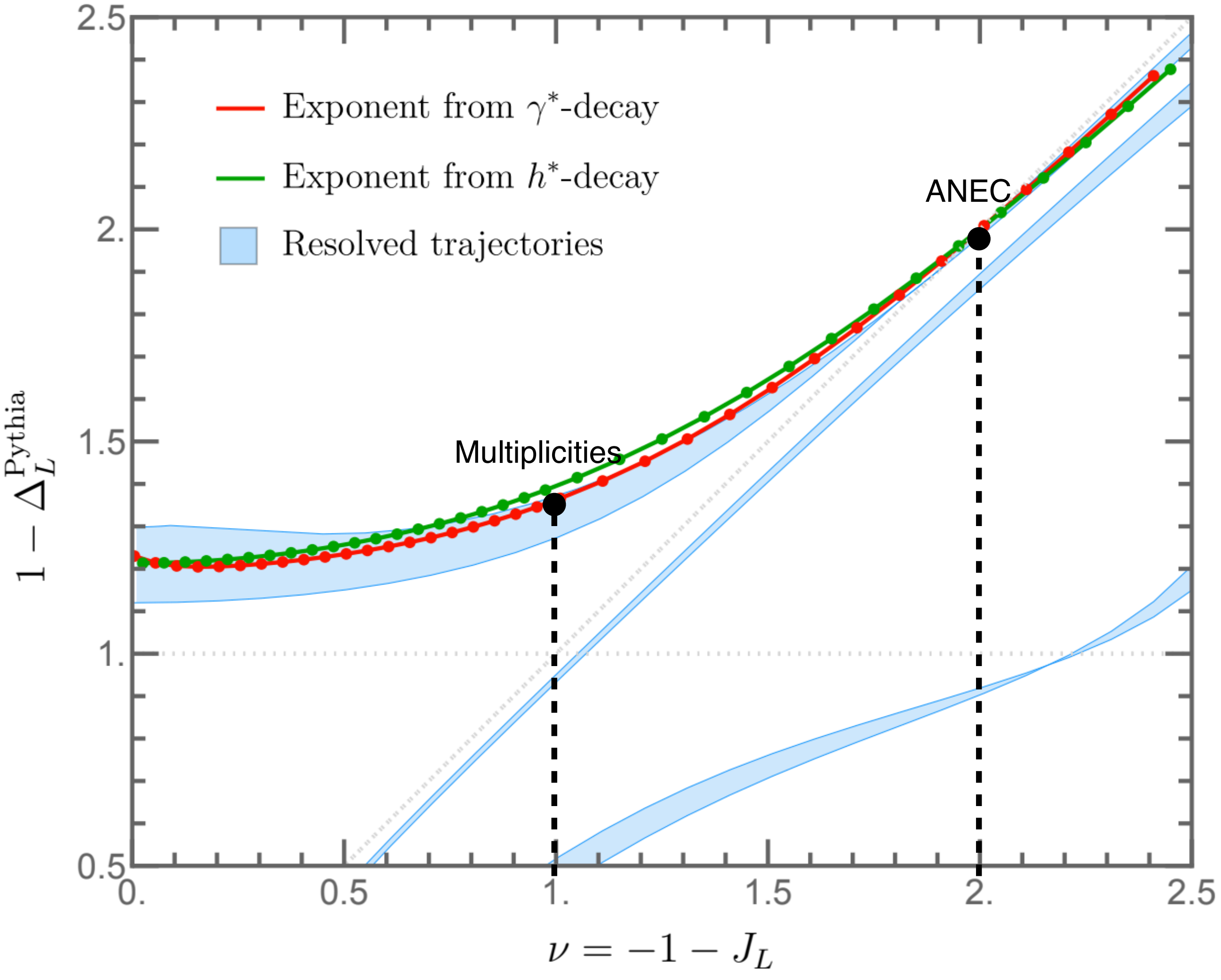
$$\underbrace{\frac{d}{d\log Q} \log\langle \mathbb{N}_{J_L}(z)\rangle_Q}_{\Big\downarrow} = -\Delta_{L,i_{\min}}(J_L, \alpha_s(Q)) + \ldots$$

Can be extracted from Pythia simulation/experiments

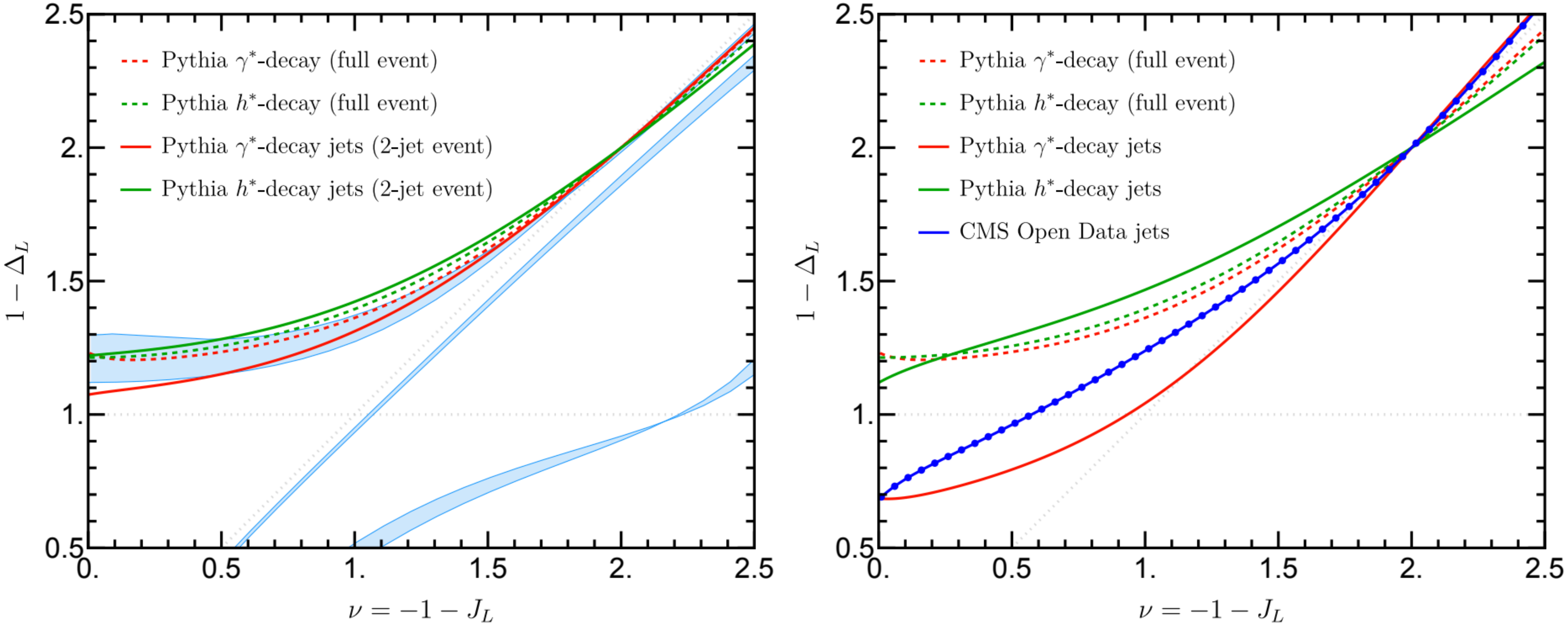
- By varying J_L, we can see the shape of the leading trajectory of QCD in data!

Comparison with Pythia simulation

$$\frac{d}{d\log Q}\log\langle\mathbb{N}_{J_L}(\vec{n})\rangle_Q = -\Delta_{L,i_{\min}}(J_L,\alpha_s(Q))+\dots.$$



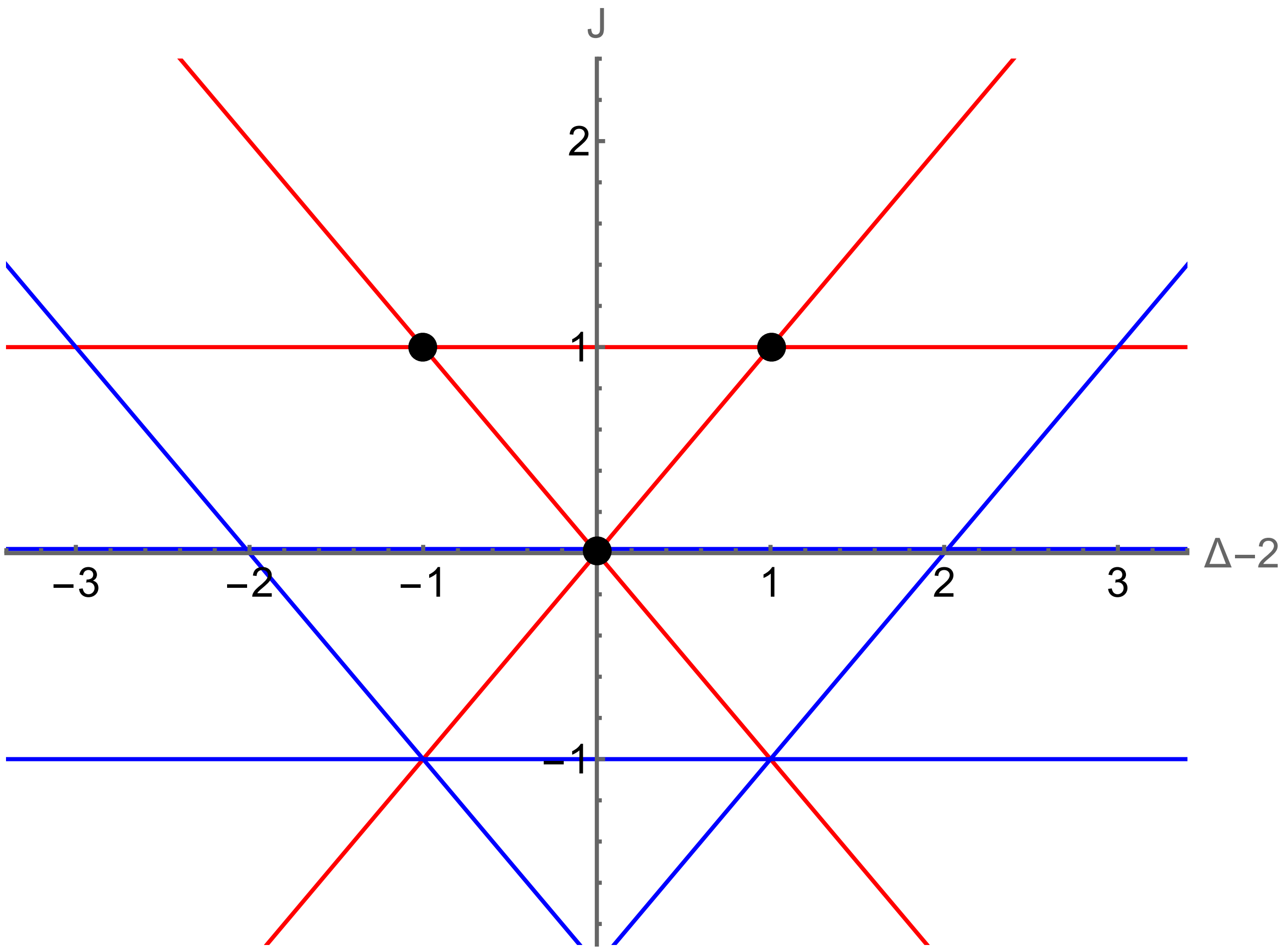
Comparison with jet algorithm and CMS open data



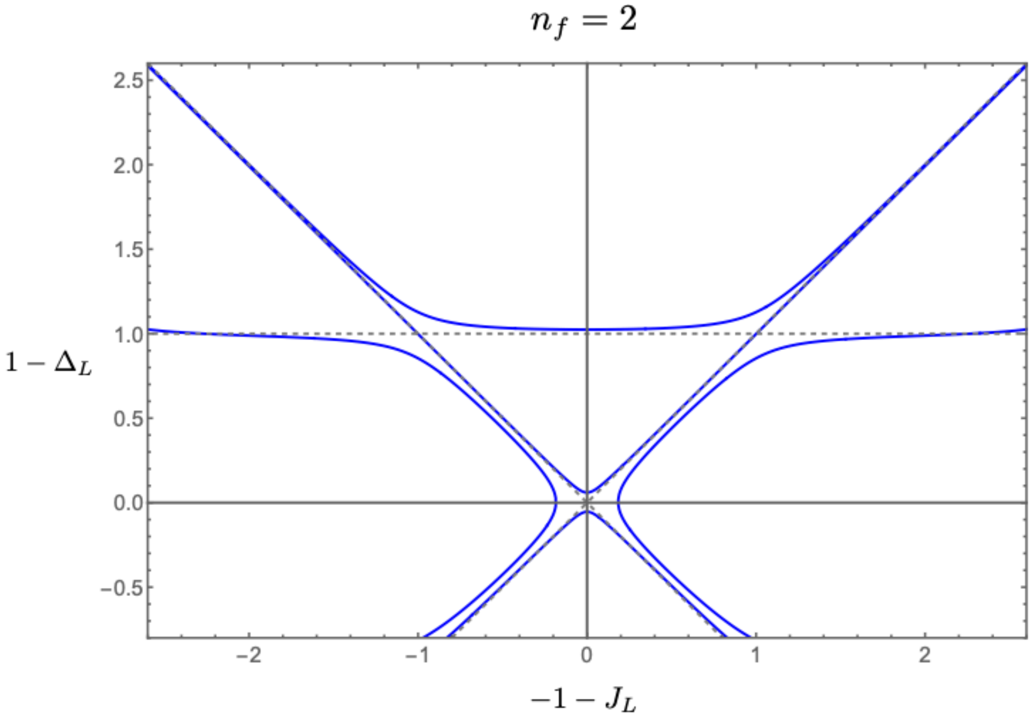
- Jet algorithms break Lorentz invariance, but seem to only lead to mild modifications.
- Analytical results don’t agree with CMS open data due to the additional energy dependence from the quark/gluon fraction.

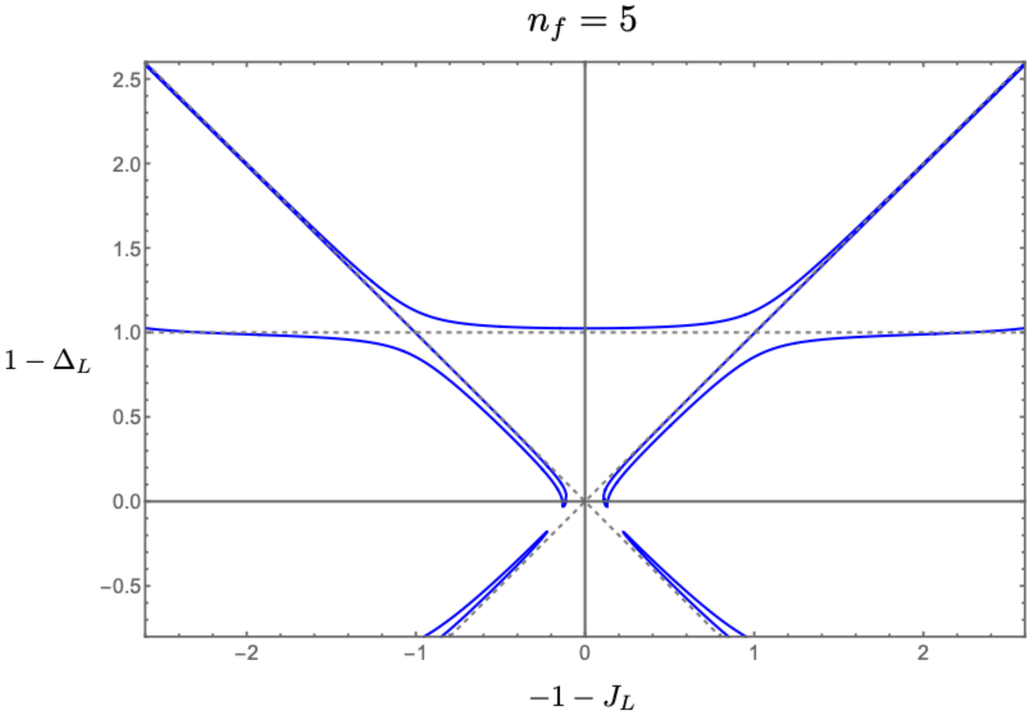
Subleading trajectories

- The DGLAP anomalous dimension also has poles at $J = 0, -1, -2, \ldots.$
- The pole at $J = 0$ (or $J_L = -1$) is due to the mixing between DGLAP and its spin shadow (subleading soft theorem [Casali '14]). The two detectors also become exactly the same at the intersection.



C-F plot including subleading trajectories

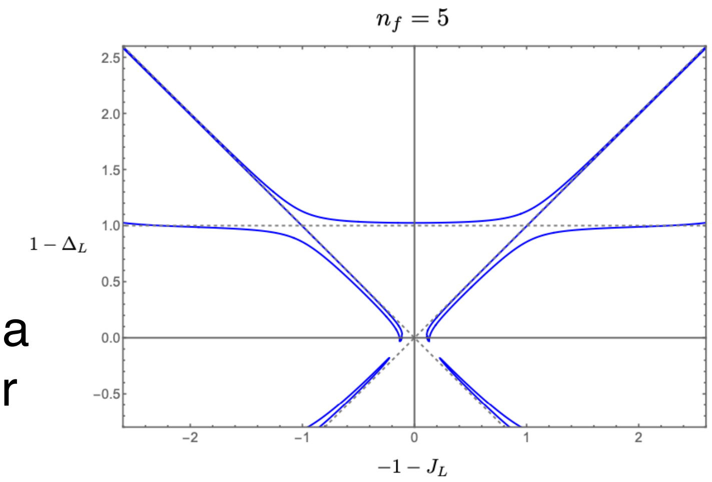




Summary and outlook

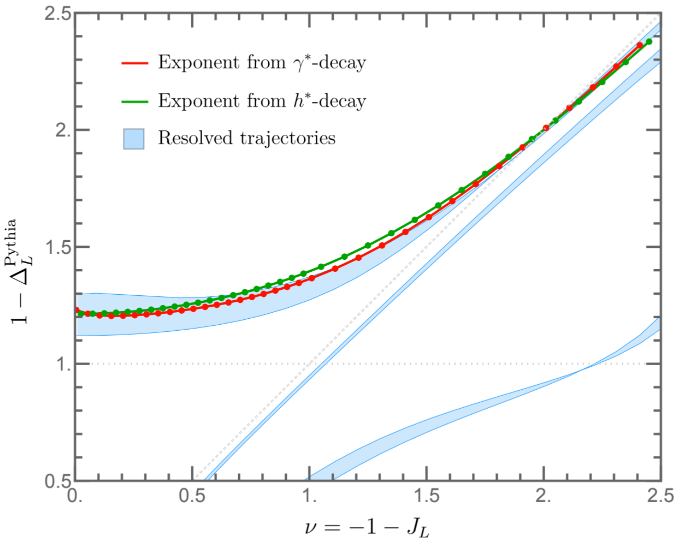
Summary:

- Explored the Chew-Frautschi plot of QCD at one-loop, and resolved the mixing between DGLAP/BFKL and DGLAP/shadow DGLAP trajectories.
- Argued that hadron multiplicities can be matched onto a linear combination of QCD detectors. The prediction for the Q-evolution agrees well with Pythia simulation.



Future Directions:

- Improve the comparison with CMS open data.
- Subleading trajectories and other intersections? Generalized BFKL detectors?
- What other measurements can be matched onto QCD detectors? **See Hao's talk!**



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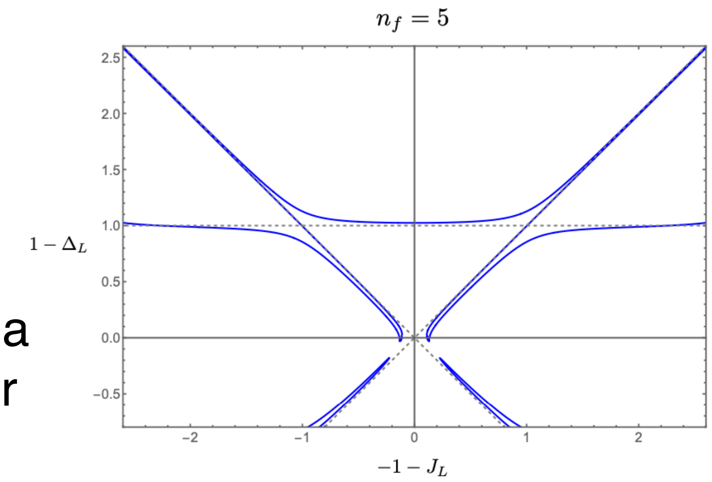

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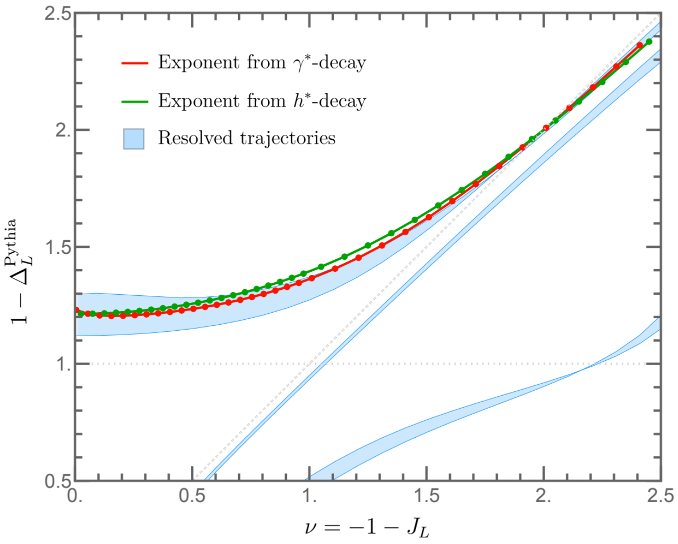

Thank you!