Conservative binary dynamics beyond Einstein

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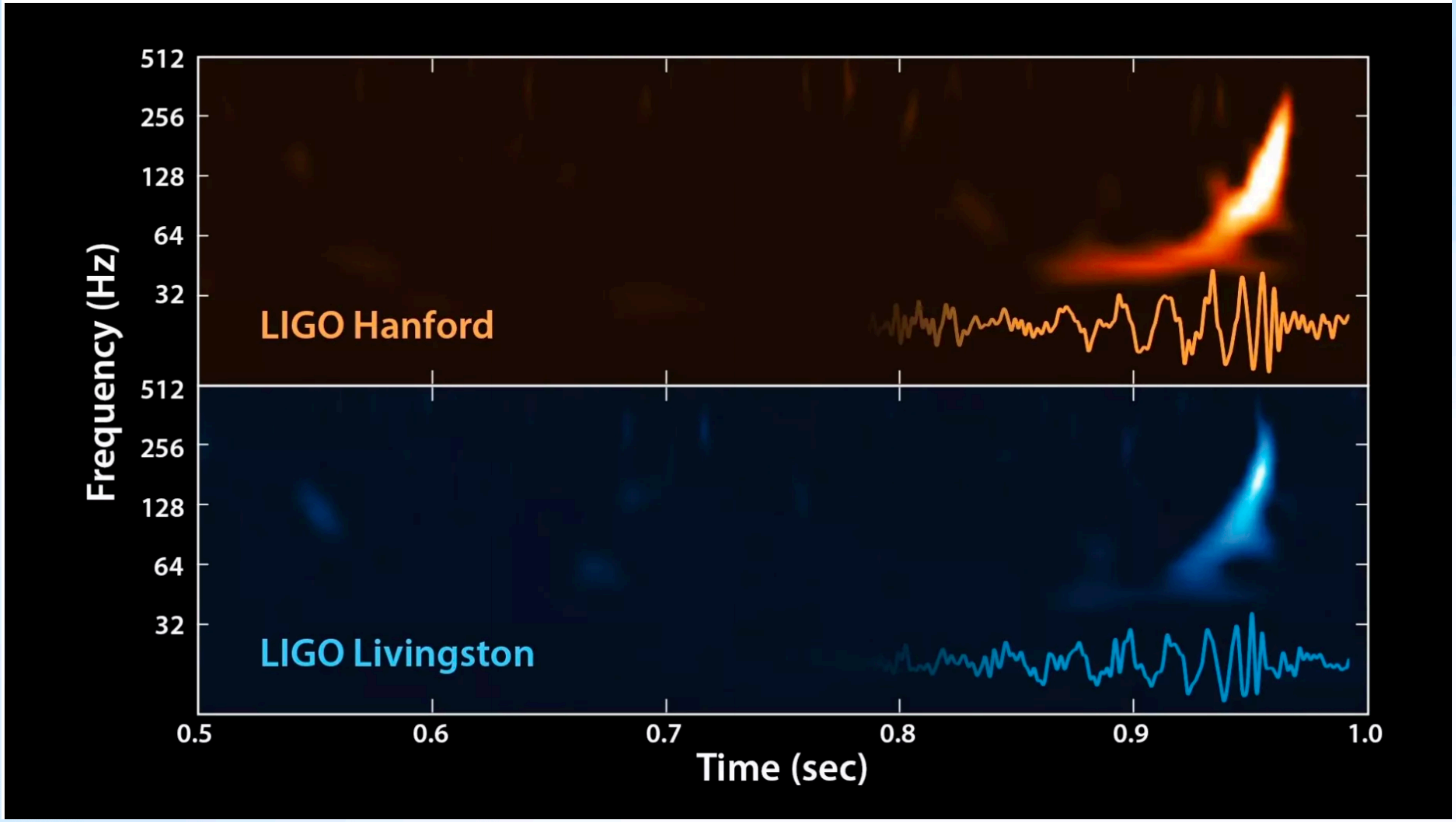
Contents

- Worldline EFT approach to gravitational waves observables
- The analytical results of Einstein-scalar-Gauss-Bonnet theory

  up to 3rd Post-Minkowskian order

  based on a recent work with Gabriel Almeida, Yuchen Du, Zhengwen Liu

Gravitational wave discovery



The first discovery of gravitational waves

Motivation for high-precision computation

- Build accurate templates of waveform
- Precisely test general relativity
- Search for deviations and new physical signals beyond general relativity

EFT for gravity

- Post-Minkowskian (PM) Expansion in G

3PM
Bern, Cheung, Roiban, Shen, Solon, Zeng (2019)
Kälin, Liu, Porto (2020)
. . .

4PM
Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng (2022)
Dlapa, kälin, Liu, Neef, Porto (2022)
Damgaard, Hansen, Plante, Vanhove (2023)
Jakobsen, Mogull, Plefka, Sauer, Xu (2023)
. . .

5PM (partial)
Driesse, Jakobsen, Mogull, Plefka, Sauer (2024)
Bern, Herrmann, Roiban, Ruf, Smirnov (2025)
Dlapa, Kälin, Liu, Porto (2025)

- Post-Newtonian (PN) Expansion in v

3PN S. Foffa, R. Sturani(2011)

4PN Foffa, Sturani(2019), Foffa, Porto, Rothstein, Sturani(2019)

5PN Almeida, Foffa, Sturani(2021,2023), Bl¨umlein, Maier, Marquard, Scha¨fer(2022)
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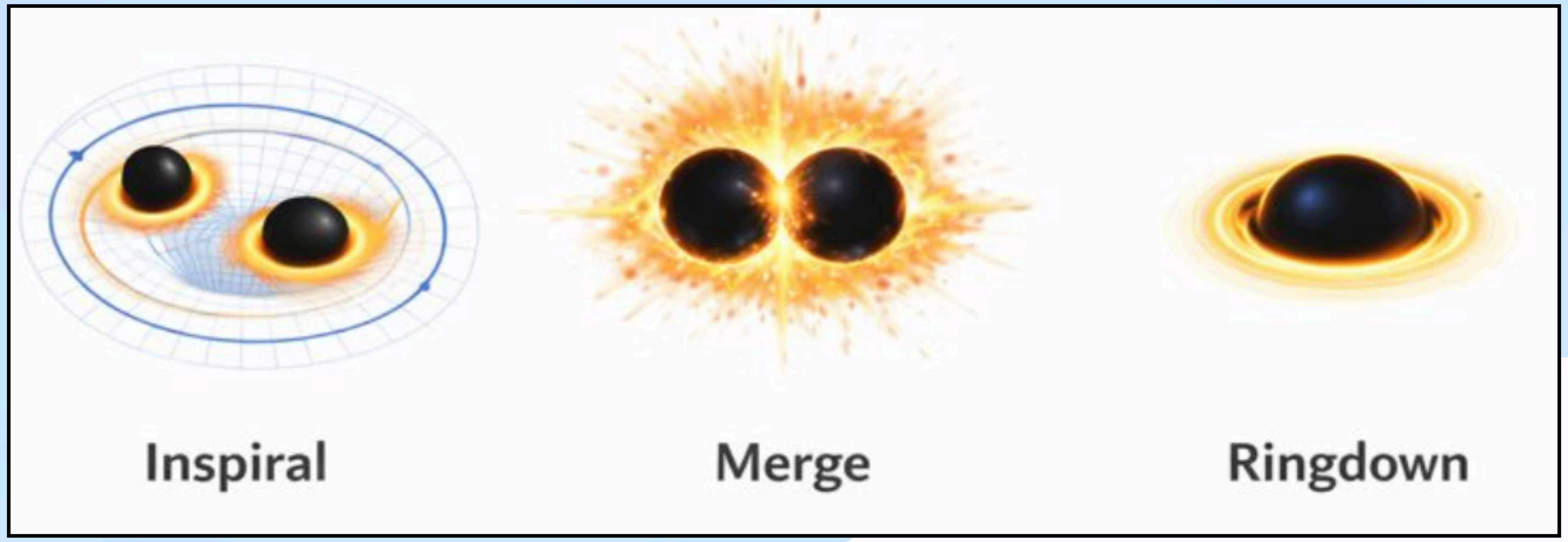
Investigation on gravity beyond GR remains limited

Gravitational two-body problem



PN/PM expansion — Numerical Relativity — Black hole perturbation

Inspiral: The interaction is weak. The separation is large.

- The separation of the two bodies is much larger than their sizes.

$$S_{\rm WL} = -\sum_{i=1}^{2} \frac{m_i}{2} \int d\tau\, g_{\mu\nu} \dot{x}_i^{\mu}(\tau) \dot{x}_i^{\nu}(\tau)$$

Point-particle

- The Einstein-Hilbert action

$$S_{\rm EH} = -\frac{1}{16\pi G} \int d^D x \sqrt{-g} R$$

Effective field theory

- Effective action

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Weak-field expansion

$$e^{iS_{\text{eff}}(x_i(\tau_i))} = \int Dh_{\mu\nu} e^{iS_{\text{EH}} + iS_{\text{WL}}}$$

Goldberger, Rothstein hep-th/0409156

- Perturbation

$$S_{\text{eff}} = \int d\tau (L_0 + GL_1 + G^2 L_2 + \cdots)$$

$$L_0 = -\sum_{i=1}^{2} \frac{m_u}{2} \eta_{\mu\nu} \dot{x}_i^\mu \dot{x}_i^\nu$$

Källin, Porto 2006.01184

- Classical equation of motion

$$\delta S_{eff} = 0 \longrightarrow m_i \ddot{x}_i^\mu(\tau_i) = -\eta^{\mu\nu} \sum_{n=1}^{\infty} G^n \left(\frac{\partial L_n}{\partial x_i^\nu} - \frac{\partial}{\partial \tau_i} \frac{\partial L_n}{\partial x_i^\nu} \right)$$

$$x_i^\mu(\tau_i) = b_i^\mu + u_i^\nu \tau_i + \sum_{i=1}^{\infty} G^n \delta_n x_i^\mu(\tau_i)$$

- Physical observables

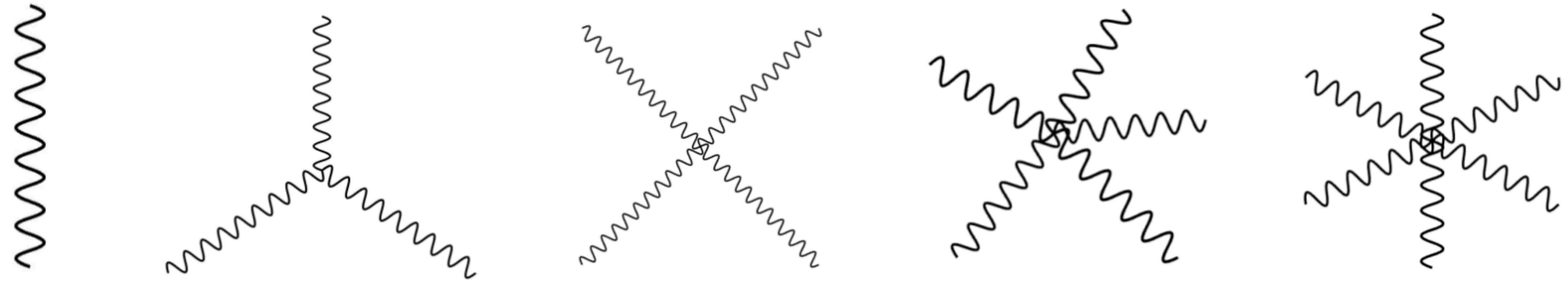
$$\Delta p_i^\mu = p_i^\mu(+\infty) - p_i^\mu(-\infty) = \int_{-\infty}^{\infty} d\tau_i \; m_i \ddot{x}_i^\mu(\tau_i) = -\eta^{\mu\nu} \sum_{n=1}^{\infty} G^n \int_{-\infty}^{\infty} d\tau_i \frac{\partial L_n}{\partial x_i^\nu(\tau_i)}$$

Worldline Effective field theory

- Worldlines as classical sources

$$= -\frac{i\kappa m}{2}\int d\tau e^{ik\cdot x(\tau)}\dot{x}^{\mu}(\tau)\dot{x}^{\nu}(\tau)$$

- GR



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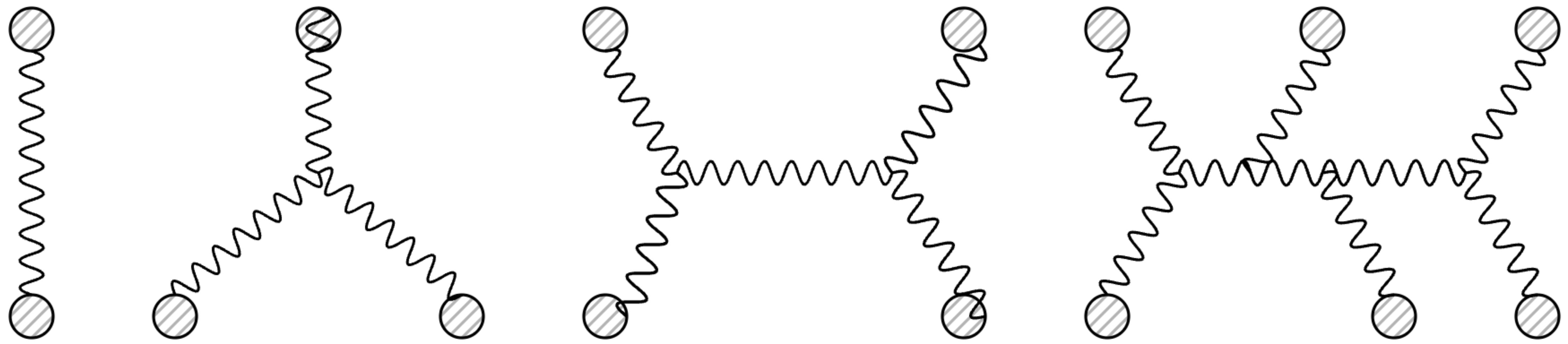
- Extract classical physics by saddle-point approximation in path integral



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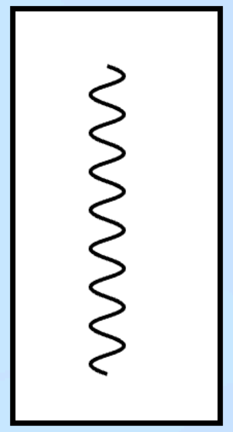
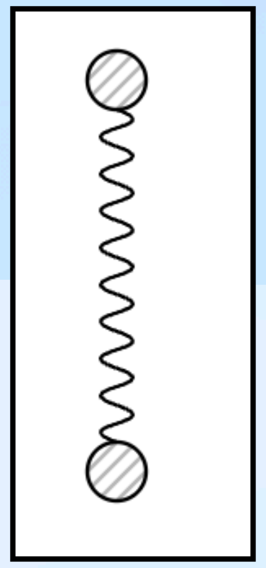
1PM example

- Propagator



$$D_{\mu\nu,\alpha\beta}(x_1,x_2) = \int \frac{d^Dq}{(2\pi)^D}\frac{-ie^{iq\cdot(x_1(\tau_1)-x_2(\tau_2))}}{-q^2-i\epsilon}\left(\frac{1}{2}\eta_{\mu\alpha}\eta_{\nu\beta}+\frac{1}{2}\eta_{\mu\beta}\eta_{\nu\alpha}-\frac{1}{D-2}\eta_{\mu\nu}\eta_{\alpha\beta}\right)$$

- Effective action



$$S_1 = -4\pi G\int d\tau_1 d\tau_2 \frac{d^Dq}{(2\pi)^D}\frac{-e^{iq\cdot(x_1-x_2)}}{-q^2-i\epsilon}\left(2(\dot{x}_1\cdot\dot{x}_2)^2-\dot{x}_1^2\dot{x}_2^2\right)$$

- Equation of motion

$$m_1\ddot{x}_1^{\mu}(\tau_1) = -\eta^{\mu\nu}\frac{\delta S_1}{\delta x_1^{\nu}(\tau)} = 4\pi G\int d\tau_2\frac{d^Dq}{(2\pi)^D}\frac{-iq^{\mu}e^{iq\cdot(x_1(\tau_1)-x_2(\tau_2))}}{-q^2-i\epsilon}\left(2(\dot{x}_1(\tau_1)\cdot\dot{x}_2(\tau_2))^2-\dot{x}_1^2(\tau_1)\dot{x}_2^2(\tau_2)\right)$$

$$x_i^{\mu} = b_i^{\mu}+u_i^{\mu}\tau_i$$

0PM approximation of trajectory

$$\gamma = u_1\cdot u_2$$

One simple scale

1PM example

Kälin, Porto (2020)

- Equation of motion

$$x_i^\mu = b_i^\mu + u_i^\mu \tau_i$$

$$\gamma = u_1 \cdot u_2$$

0PM approximation of trajectory

$$m_1 \ddot{x}_1^\mu(\tau_1) = -\eta^{\mu\nu} \frac{\delta S_1}{\delta x_1^\nu(\tau)} = 4\pi G \int d\tau_2 \frac{d^D q}{(2\pi)^D} \frac{-iq^\mu e^{iq\cdot(x_1(\tau_1)-x_2(\tau_2))}}{-q^2 - i\epsilon} \Big(2(\dot{x}_1(\tau_1)\cdot\dot{x}_2(\tau_2))^2 - \dot{x}_1^2(\tau_1)\dot{x}_2^2(\tau_2)\Big)$$

- Impulse at 1PM

$$m_1 \ddot{x}_1^\mu(\tau) = 4\pi G \int d\tau_2 \frac{d^D q}{(2\pi)^D} \frac{-iq^\mu e^{iq\cdot(b_1 + u_1\tau_1 - b_2 - u_2\tau_2)}}{-q^2 - i\epsilon} (2\gamma^2 - 1)$$

$$b^\mu \equiv b_1^\mu - b_2^\mu$$

$$\Delta p_1^\mu|_{1\mathrm{PM}} = 4\pi G m_1 m_2 (2\gamma^2 - 1) \int \frac{d^D q}{(2\pi)^{D+2}} \frac{-iq^\mu \delta(q\cdot u_1)\delta(q\cdot u_2) e^{iq\cdot b}}{-q^2 - i\epsilon} = \frac{GM}{|b|} \frac{2\gamma^2 - 1}{\sqrt{\gamma^2 - 1}} \frac{\mu b^\mu}{|b|}$$

$$\mu \equiv \frac{m_1 m_2}{m_1 + m_2}, \quad M \equiv m_1 + m_2$$

1PM example

- Correction to $\dot{x}_1(t)$ and $x_1(t)$

$$\dot{x}_1^\mu(t) = u_1^\mu + \int_{-\infty}^{t} d\tau_1 \ddot{x}_1^\mu(\tau_1) = u_1^\mu + \frac{4\pi G(2\gamma^2-1)}{m_1} \int_{-\infty}^{t} d\tau_1 \int d\tau_2 \frac{d^D q}{(2\pi)^D} \frac{-iq^\mu e^{iq\cdot(b+u_1\tau_1-u_2\tau_2)}}{-q^2-i\epsilon}$$

$$x_1^\mu(t) = b_1^\mu + u_1^\mu \tau_1 + \int_{-\infty}^{t} d\tau_1 \dot{x}_1^\mu(\tau_1)$$

- Regulator

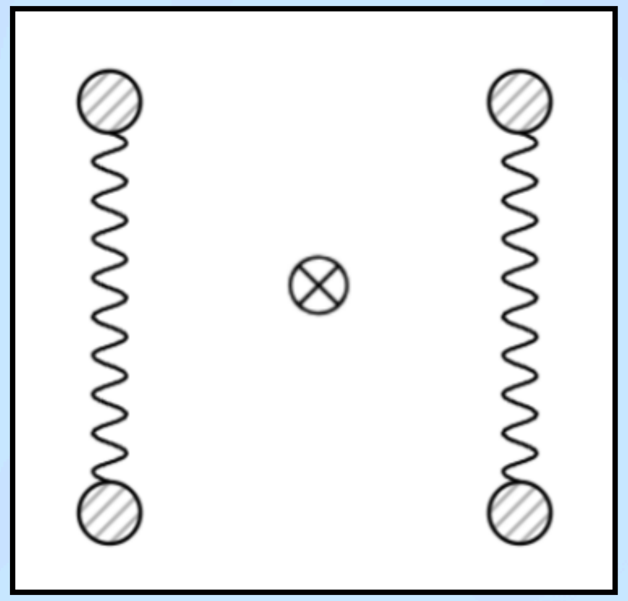
$$e^{iq\cdot u_1\tau}|_{\tau\to-\infty} \to e^{i(q\cdot u_1 - i\epsilon)\tau}|_{\tau\to-\infty} = 0$$
$$e^{-iq\cdot u_2\tau}|_{\tau\to-\infty} \to e^{i(q\cdot u_2 + i\epsilon)\tau}|_{\tau\to-\infty} = 0$$

→

$$\int_{-\infty}^{t} d\tau_1 e^{iq\cdot u_1\tau_1} \to \frac{1}{q\cdot u_1 - i\epsilon}$$
$$\int_{-\infty}^{t} d\tau_2 e^{-iq\cdot u_2\tau_2} \to \frac{1}{q\cdot u_2 + i\epsilon}$$

2PM example

- 1PM iteration contribution



$$x_i^\mu(\tau_i) = b_i^\mu + u_i^\mu \tau_i + G\ \delta x_i^\mu(\tau_i)$$

$$\dot{x}_i^\mu(\tau_i) = u_i^\mu + G\delta\dot{x}_i^\mu(\tau_i)$$

$$\ddot{x}_i^\mu(\tau_i) = G\delta\ddot{x}_i^\mu(\tau_i)$$

Corrections of kinematic variables

$$\Delta p_1^\mu \sim \int \frac{d^D q}{(2\pi)^D} \frac{\delta(q\cdot u_1)\delta(q\cdot u_2)e^{iq\cdot b}}{(-q^2 - i\epsilon)^{v_0}} \int \frac{d^D l}{(2\pi)^D} \frac{\delta(l\cdot u_1)}{(\pm l\cdot u_2 - i\epsilon)^{v_1}} \frac{1}{[-l^2 - i\epsilon]^{v_2}[-(q-l)^2 - i\epsilon]^{v_3}}$$

Different integral families

One-loop Feynman integral

General structure at N PM

·Impulse at N PM

$$\Delta P_1^\mu \sim \int d^D q \frac{\delta(q\cdot u_1)\delta(q\cdot u_2)e^{iq\cdot b}}{(-q^2-i\epsilon)^{v_0}} \prod_{i=1}^{N-1} \int d^D l_i \frac{\delta(l_i\cdot u_a)}{(\pm l_i\cdot u_b - i\epsilon)^{v_i}} \frac{1}{D_1^{k_1} D_2^{k_2} D_3^{k_3}}$$

- One delta-function $\delta(l_i \cdot u_a)$ for each loop
- Single scale γ of multi-loop integrals at any order. $q \cdot u_i = 0, \quad u_i^2 = 1, \quad u_1 \cdot u_2 = \gamma$
- Could be solved by modern technique in Feynman integrals!

Modern QFT Framework

Drape, Kalin, Liu, Porto (2023)

Physical observables

$$O = \sum_i c_i I_i$$

Integration-By-Parts Reduction

Reduced to Master Integrals (MI)

$$\int d^D l \cdots \frac{\partial}{\partial l^\mu} \frac{v^\mu}{D_1^{\nu_1} \cdots D_N^{\nu_N}} = 0$$

MI = $\{ f_1, f_2, \cdots f_n \}$

Differential equations of the MI

Evaluating MI by differential equations

$$\frac{d\vec{f}(x,\epsilon)}{dx} = A(x,\epsilon)\vec{f}(x,\epsilon)$$

$$\vec{g} = T\vec{f}$$

Transform to the canonical basis

Henn (2013)

$$\frac{d\vec{g}(x,\epsilon)}{dx} = \epsilon M(x)\vec{g}(x,\epsilon)$$

Factorize ϵ

Integrate iteratively

$$g(x,\epsilon) = \sum_n \epsilon^n g^n(x)$$

$$g^{(n)}(x) = \int_{x_0}^{x} M(x')g^{(n-1)}(x') + g^{(n)}(x_0)$$

PN limit as boundary condition: v->0

Analytical expression

Einstein-scalar-Gauss-Bonnet

$$S = S_{\mathrm{EH}} + S_{\phi} + S_{\mathrm{WL}}$$

$$S_{\mathrm{EH}} = -\frac{1}{16\pi G}\int d^D x\sqrt{-g}R$$

Einstein-Hilbert action

$$S_{\phi} = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\left(\frac{1}{4}g^{\mu\nu}\nabla^{\mu}\phi\nabla^{\nu}\phi + \alpha f(\phi)\mathcal{G}\right)$$

Scalar-Tensor action and Gauss-Bonnet coupling action
(Scalar fields with Gauss-Bonnet term)

$$S_{\mathrm{WL}} = -\frac{1}{2}\sum_{i=1}^{2}\int d\tau_i(g_{\mu\nu}\dot{x}_i^{\mu}\dot{x}_i^{\nu} + 1)m_i(\phi)$$

Worldlines of point particles (black holes)

Gauss-Bonnet invariant

$$\mathcal{G} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$$

Only combination of quadratic curvature terms yielding second-order equation of motion.

- Allowing for BH solutions different from GR, evading no-hair theorem.
  Sotiriou, Zhou (2014)
- Well-motivated by low-energy limit of heterotic string theory.
  Gross, Sloan(1987)
- Discussions and considerations in cosmology and astronomy

Post-Minkowski expansion

Weak field expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Scalar field expansion around $\phi_0(x)$

$$\phi = \phi_0 + \Phi$$

Mass expansion

$$m_i(\phi) = m_i(1 + c_{i1}\Phi + \frac{c_{i2}}{2!}\Phi^2 + \frac{c_{i3}}{3!}\Phi^3 + \cdots)$$

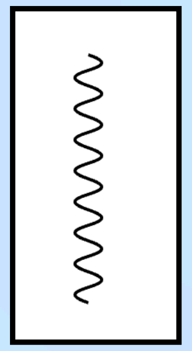
$f(\phi)$ expansion

$$f(\phi) = f_0 + f_1\Phi + \frac{f_2}{2!}\Phi^2 + \frac{f_3}{3!}\Phi^3 + \cdots$$

$$e^{iS_{\rm eff}} = \int D\Phi \; Dh_{\mu\nu} \; e^{i(S_{\rm EH}+S_\Phi+S_{WL})}$$

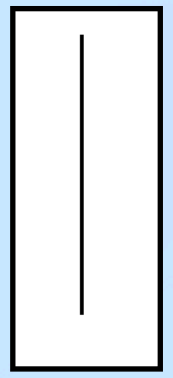
ESGB

- Graviton



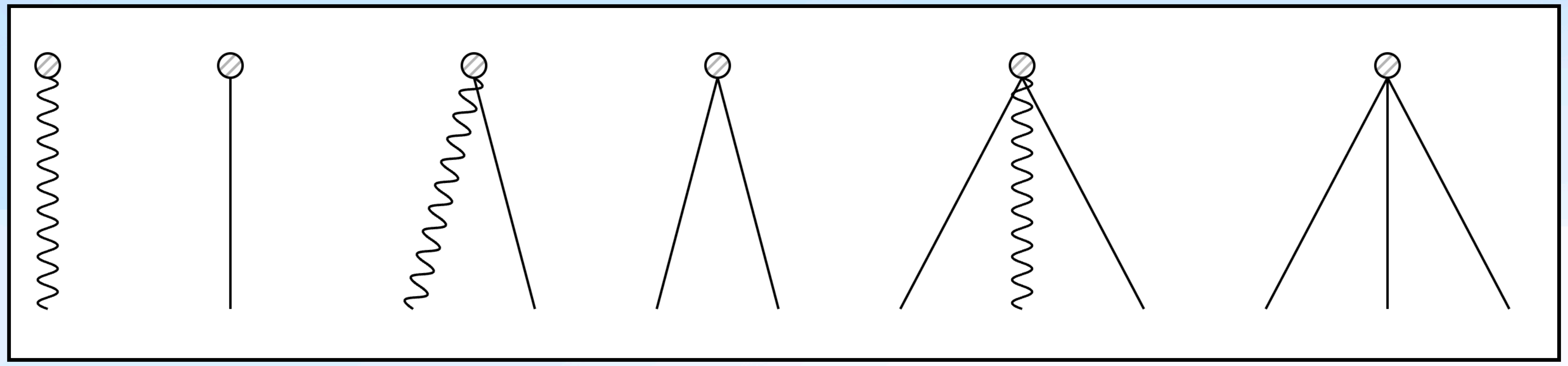
$$D_{\mu\nu,\alpha\beta}(x_1,x_2) = \int \frac{d^D q}{(2\pi)^D} \frac{-ie^{iq\cdot(x_1(\tau_1)-x_2(\tau_2))}}{-q^2 - i\epsilon} \left(\frac{1}{2}\eta_{\mu\alpha}\eta_{\nu\beta} + \frac{1}{2}\eta_{\mu\beta}\eta_{\nu\alpha} - \frac{1}{D-2}\eta_{\mu\nu}\eta_{\alpha\beta}\right)$$

- Scalar field



$$D(x_1,x_2) = \int \frac{d^D q}{(2\pi)^D} \frac{-ie^{iq\cdot(x_1(\tau_1)-x_2(\tau_2))}}{-q^2 - i\epsilon}$$

- Source



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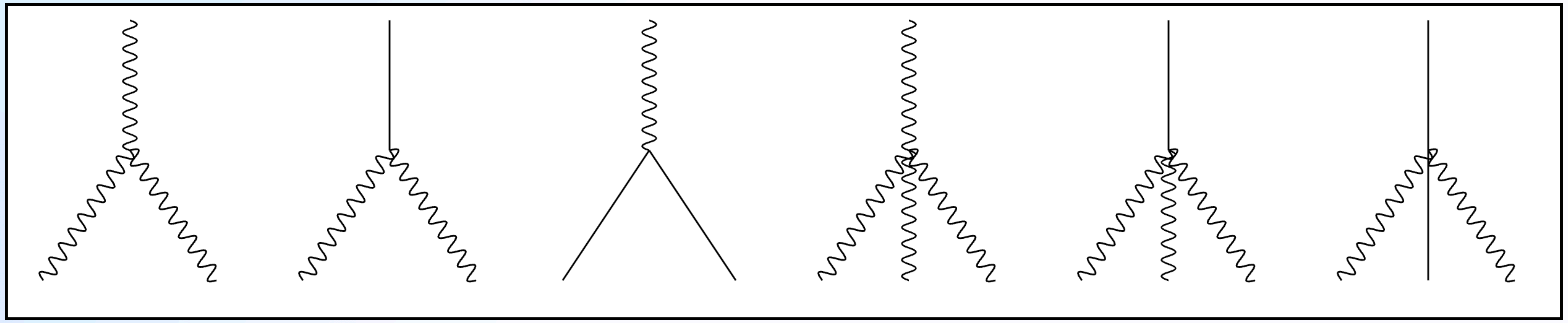
- Vertex



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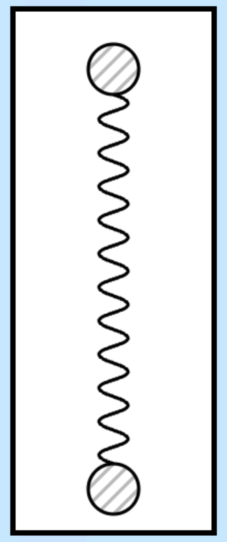
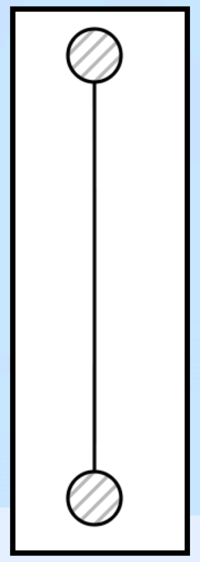
GR | Gauss-Bonnet | Scalar-Tensor | GR | Gauss-Bonnet | Scalar-Tensor Gauss-Bonnet

1PM

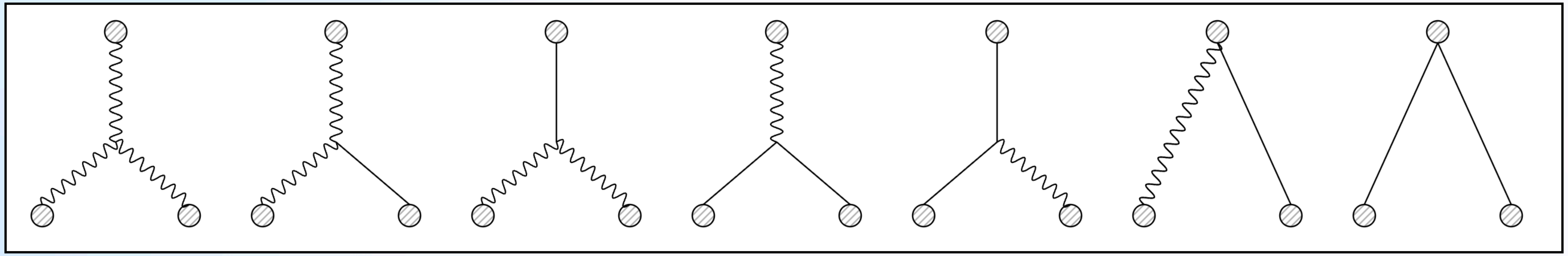


$$8\pi G m_1 m_2 \int d\tau_1 d\tau_2 \frac{d^D q}{(2\pi)^D} \frac{e^{iq\cdot(x_1(\tau_1)-x_2(\tau_2))}}{-q^2 - i\epsilon} \Big((\dot{x}_1(\tau_1)\cdot\dot{x}_2(\tau_2))^2 - \frac{1}{D-2}\dot{x}_1^2(\tau_1)\dot{x}_2^2(\tau_2) \Big)$$



$$8\pi G m_1 m_2 \int d\tau_1 d\tau_2 \frac{d^D q}{(2\pi)^D} \frac{e^{iq\cdot(x_1(\tau_1)-x-2(\tau_2))}}{-q^2 - i\epsilon} (1+\dot{x}_1^2(\tau_1))(1+\dot{x}_2^2(\tau_2))$$

2PM



1PM and 2PM results

- 1PM

$$\Delta p_1^\mu|_{1PM} = -\frac{GM}{|b|}\frac{2\mu}{\sqrt{\gamma^2-1}}(2\gamma^2 - 1 - 8c_{11}c_{21})\frac{b^\mu}{|b|}$$

- 2PM

$$\Delta p_1^\mu|_{2PM} = \left(\frac{GM}{|b|}\right)^2\left(C_b\frac{b^\mu}{|b|} + C_1 u_1^\mu + C_2 u_2^\mu\right)$$

$$C_b = \frac{\mu\pi}{4\sqrt{\gamma^2-1}}\Big(3 - 15\gamma^2 - 64c_{11}c_{21} + 128c_{11}^2c_{21}^2$$
$$+8\frac{m_1}{M}c_{11}^2(\gamma^2 - 1 + 16c_{22}) + 8\frac{m_2}{M}c_{21}^2(\gamma^2 - 1 + 16c_{12})\Big)$$
$$+\frac{12\alpha\mu\pi}{|b|^2 M\sqrt{\gamma^2-1}}\Big((c_{11} - c_{21})(m_1 - m_2) - 3(c_{11}m_1 + c_{21}m_2)\gamma^2\Big)$$
$$C_1 = -\frac{2\nu}{(\gamma^2-1)^2}(m_2 + m_1\gamma)(2\gamma^2 - 1 - 8c_{11}c_{21})^2$$
$$C_2 = \frac{2\nu}{(\gamma^2-1)^2}(m_1 + m_2\gamma)(2\gamma^2 - 1 + 8c_{11}c_{21})^2$$

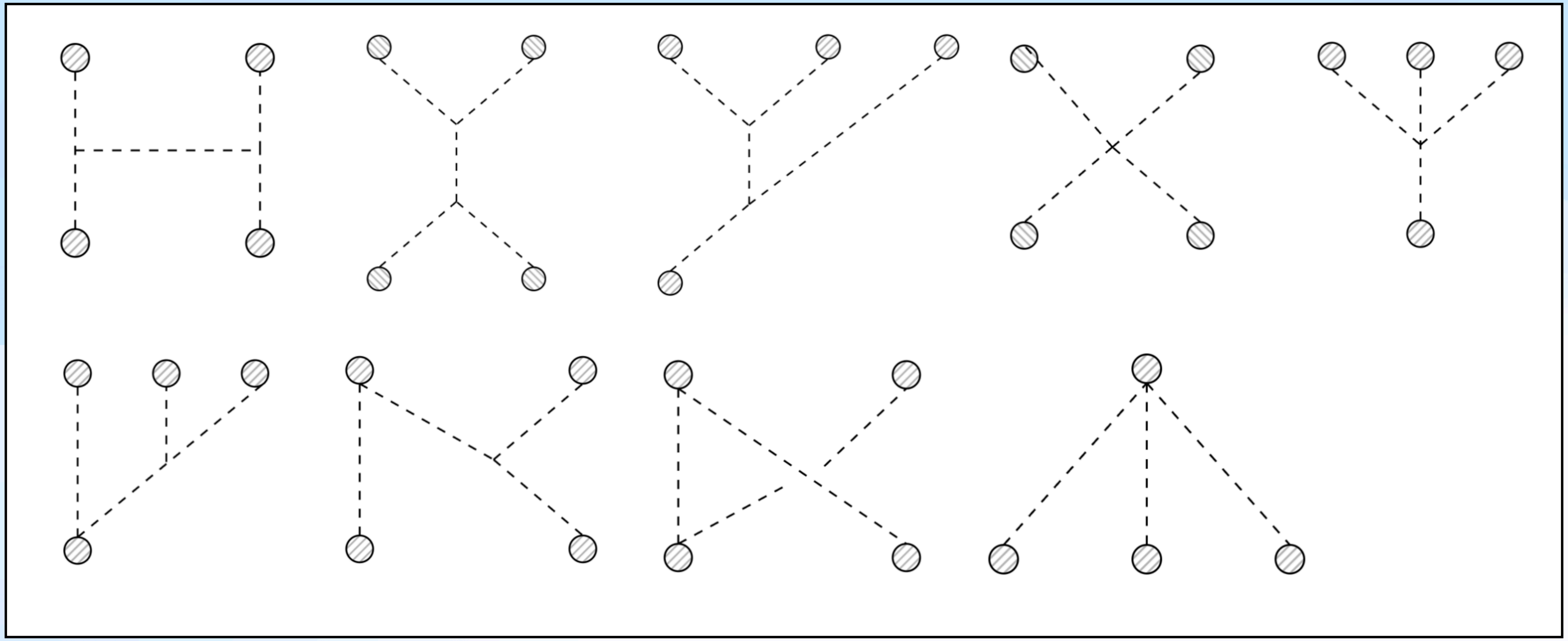
Scalar-Tensor

Gauss-Bonnet

$$\mu \equiv \frac{m_1 m_2}{m_1 + m_2}, \quad M \equiv m_1 + m_2, \quad \nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2}$$

3PM

- Topologies of diagrams



3PM

- Four families of integrals in 3 PM

$$I^{(1)}_{1,1,1,k_1,\cdots,k_9} = \int \frac{d^D l_1 d^D l_2}{(2\pi)^{2D}} \frac{\delta(l_1\cdot u_1)\delta(l_2\cdot u_1)}{(l_1\cdot u_2 - i\epsilon)^{k_3}(l_2\cdot u_2 - i\epsilon)^{k_4} D_1^{k_5}\cdots D_5^{k_9}}$$

$$I^{(2)}_{1,1,1,k_1,\cdots,k_9} = \int \frac{d^D l_1 d^D l_2}{(2\pi)^{2D}} \frac{\delta(l_1\cdot u_1)\delta(l_2\cdot u_1)}{(l_1\cdot u_2 - i\epsilon)^{k_3}(-l_2\cdot u_2 - i\epsilon)^{k_4} D_1^{k_5}\cdots D_5^{k_9}}$$

$$I^{(3)}_{1,1,1,k_1,\cdots,k_9} = \int \frac{d^D l_1 d^D l_2}{(2\pi)^{2D}} \frac{\delta(l_1\cdot u_1)\delta(l_2\cdot u_2)}{(l_1\cdot u_2 - i\epsilon)^{k_3}(l_2\cdot u_1 - i\epsilon)^{k_4} D_1^{k_5}\cdots D_5^{k_9}}$$

$$I^{(4)}_{1,1,1,k_1,\cdots,k_9} = \int \frac{d^D l_1 d^D l_2}{(2\pi)^{2D}} \frac{\delta(l_1\cdot u_1)\delta(l_2\cdot u_2)}{(l_1\cdot u_2 - i\epsilon)^{k_3}(-l_2\cdot u_1 - i\epsilon)^{k_4} D_1^{k_5}\cdots D_5^{k_9}}$$

- Propagators

$$D_1 = -l_1^2 - i\epsilon,\quad D_2 = -l_2^2 - i\epsilon,\quad D_3 = -(q-l_1-l_2)^2 - i\epsilon,\quad D_4 = -(q-l_1)^2 - i\epsilon,\quad D_5 = -(q-l_2)^2 - i\epsilon$$

3PM

Evaluated directly by Feynman parametrization

- Master integrals in S family

$$\{I^{(1)}_{110011100},\ I^{(1)}_{110111100},\ I^{(1)}_{111111100}\}$$

Dlapa, Kälin, Liu, Porto (2023)

- Master integrals in H family

$$\{\ I_{110010110},\ I_{110011011},\ I_{110011100},\ I_{110011111},\ I_{110011200},\ I_{110011211},\ I_{110021100},\ I_{111111100}, I_{110111100}\ \}$$

Transform to canonical basis

$$g_1 = \frac{\epsilon^4}{4} I_{110011011}$$

$$g_2 = \frac{\epsilon^4}{4}\sqrt{\gamma^2-1}\, I_{110011100}$$

$$g_3 = \frac{\epsilon^3}{4}\sqrt{\gamma^2-1}\, I_{110011200}$$

$$g_4 = -\frac{3\epsilon^4(\gamma^2-1)}{4\gamma} I_{110011100} + \frac{\epsilon^3(\gamma^2-1)}{8\gamma} I_{110011200} + \frac{\epsilon^2(2\epsilon+1)}{8\gamma} I_{110021100}$$

$$g_5 = \frac{\epsilon^2(2\epsilon-1)(4\epsilon-1)}{4\sqrt{\gamma^2-1}} I_{110010110}$$

$$g_6 = \frac{\epsilon^4}{4}\sqrt{\gamma^2-1}\, I_{110011111}$$

$$\begin{aligned} g_7 = {} & \frac{\epsilon^2(2\epsilon-1)(4\epsilon-1)\left(2\gamma^2(1+\epsilon)-4\epsilon-1\right)}{4\gamma(\gamma^2-1)(\epsilon+1)} I_{110010110} - \frac{\epsilon^5}{\gamma(2\epsilon+1)} I_{110011011} \\ & - \frac{3\epsilon^4(8\epsilon^2+7\epsilon+1)(\gamma^2-1)}{4\gamma(\epsilon+1)(2\epsilon+1)} I_{110011100} + \frac{\epsilon^3(2\gamma^2\epsilon+2\epsilon+1)}{8\gamma} I_{110011111} \\ & + \frac{\epsilon^2(32\epsilon^3+34\epsilon^2+9\epsilon+1)}{8\gamma(\epsilon+1)(2\epsilon+1)} I_{110021100} + \frac{\epsilon^3(\epsilon+1)(\gamma^2-1)}{8\gamma(2\epsilon+1)} I_{110011211} \\ & - \frac{\epsilon^3(14\gamma^2\epsilon^2+7\gamma^2\epsilon-\gamma^2-6\epsilon^2-7\epsilon-1)}{8\gamma(\epsilon+1)(2\epsilon+1)} I_{110011200} \end{aligned}$$

$$g_8 = \frac{\epsilon^4}{8}(\gamma^2-1) I_{111111100}, \quad g_9 = -\sqrt{\gamma^2-1}\, I_{110111100}$$

3PM

- Canonical differential equations

$$\frac{d}{dx}\vec{g}(x,\epsilon) = \epsilon\left(\frac{M_0}{x} + \frac{M_1}{x-1} + \frac{M_{-1}}{x+1}\right)\vec{g}(x,\epsilon)$$

$$x = \gamma - \sqrt{\gamma^2 - 1}$$

$$M_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -12 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 4 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & -12 & 8 & 0 & 8 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$M_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & -4 & -2 & 0 & 0 & 0 & 0 & 0 \\ -4 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & -4 & -2 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Expanded in ϵ

$$\vec{g}(x,\epsilon) = \sum_n \epsilon^n \vec{g}^{(n)}(x) \longrightarrow \vec{g}^{(n)}(x) = \int_{x_0}^{x} dt\left(\frac{M_0}{x} + \frac{M_{-1}}{x-1} + \frac{M_1}{x+1}\right)\vec{g}^{(n-1)}(t) + \vec{g}^{(n)}(x_0)$$

- Multiple polylogarithms

$$G(a_1, \cdots a_n; z) = \int_0^z \frac{dt}{t-a_1} G(a_2, \cdots; t), \quad G(;z) \equiv 1$$

- PN limit as the boundary condition: $v \to 0$

PN limit as the boundary condition: $v \to 0$

Dlapa, Kälin, Liu, Porto (2023)

- Going to the rest frame of particle 1: $u_1 = (1,0,0,0),\ u_2 = (\gamma,0,0,\gamma v)$

- Expansion in potential region: $l_i \sim (v,1)|\vec{q}|$ $\quad\delta(l_1 \cdot u_1)\delta(l_2 \cdot u_2) = \delta(l_1^0)\delta(\gamma v l_2^0 - \gamma v l_2^z)$

  Contribute to conservative dynamics

$$I_{11\alpha_1\alpha_2\beta_1\cdots\beta_5}\Big|_{v->0}^{(\text{potential})} = -v^{-\alpha_1-\alpha_2}\int_{\vec{l}_1,\vec{l}_2} \frac{1}{(\mp\vec{l}_1\cdot\vec{u}_1 - i\epsilon)^{\alpha_1}(\mp\vec{l}_2\cdot\vec{u}_2 - i\epsilon)^{\alpha_2}}\frac{1}{\vec{D}_1^{\beta_1}\cdots\vec{D}_5^{\beta_5}} + O(v^{-\alpha_1-\alpha_2+1})$$

$$\vec{D}_1 = \vec{l}_1^2 - i\epsilon,\quad \vec{D}_2 = \vec{l}_2^2 - i\epsilon,\quad \vec{D}_3 = (\vec{l}_1 + \vec{l}_2 - \vec{q})^2 - i\epsilon,\quad \vec{D}_4 = (\vec{l}_1 - \vec{q})^2 - i\epsilon,\quad \vec{D}_5 = (\vec{l}_2 - \vec{q})^2 - i\epsilon$$

- Integration-By-Parts for the boundary integrals in 3 dimension

$$\vec{g}(x_0 = 1) = \begin{pmatrix} g_1|_{v\to 0} \\ \vdots \\ g_9|_{v\to 0} \end{pmatrix} = \sum \epsilon^n \vec{g}^{(n)}(x_0 = 1)$$

3PM

$$\Delta p_1^\mu|_{3\text{PM}} = \left(\frac{GM}{|b|}\right)^3 \left(C_b \frac{b^\mu}{|b|} + C_1 u_1^\mu + C_2 u_2^\mu\right)$$

$$\begin{aligned}
C_b = & \frac{1}{3x^2(x^2-1)^5} \frac{4\mu}{M^2} \Big((m_1^2+m_2^2)[12(1-2x^2\cdots) \\
& -8c_{11}c_{21}x^2(5-26x^2+\cdots) + \frac{\alpha f_1}{|b|^2}(\cdots) + \frac{\alpha f_2}{|b|^2}(\cdots)]\Big) \\
& -\frac{1}{3x^2(x^2-1)^5} \frac{4\mu m_1^2 m_2^2}{M^2} \Big((-5+55x^2+\cdots) + 6(x^2-1)^3(1-8x^2+\cdots)\log(x) \\
& +12c_{11}c_{21}(\cdots) + \cdots + \frac{\alpha f_1}{|b|^2}(\cdots)\Big) \\
C_1 = & \frac{1}{2x(x^2-1)^4} \frac{\mu}{M^2}(m_1 + 2m_2 x + m_1^2)(1 + 16c_{11}c_{21}x^2 + \cdots) \\
& \times(m_1(-256c_{11}c_{21}x^2+\cdots) + m_2(-256c_{11}c_{21}x^2+\cdots)) \\
& -\frac{\alpha f_1}{|b|^2}(1+16c_{11}c_{21}x^2+\cdots)(\cdots) \\
C_2 = & C_1|_{1\leftrightarrow 2}
\end{aligned}$$

Scalar-Tensor

Gauss-Bonnet

Conclusion and future work

Conclusion:

- A brief introduction to worldline EFT approach.
- We for the first time calculated the two-body dynamics in ESGB up to 3PM.

Future work:

- High order computations beyond General relativity
- Finite-size & spin effects

Thank you for listening!