Duality and Safety of Charge Correlators in QCD from Effective Field Theory

Gherardo Vita

CERN

New Frontiers of Quantum Field and Gravity - PKU, 13 January 2026

Based on:

"On the Edge of Safety: Charge-Charge Correlation in the Back-to-Back Limit"
Monni, **GV**, Xu, Zhu [2508.00977]

"Correlation Function/Wilson Loop Duality in Gauge Theory from Effective Field Theory"
Chen, Monni, Pang, **GV**, Zhu [2510.07377]

Correlation Functions

- An ambitious program is to rethink observables for particle physics at colliders in terms of **QFT correlation functions of local operators**

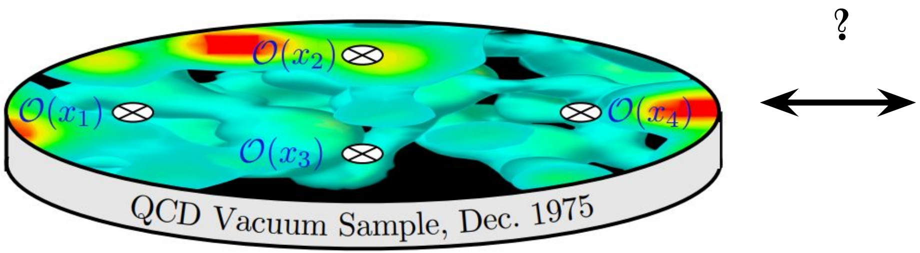
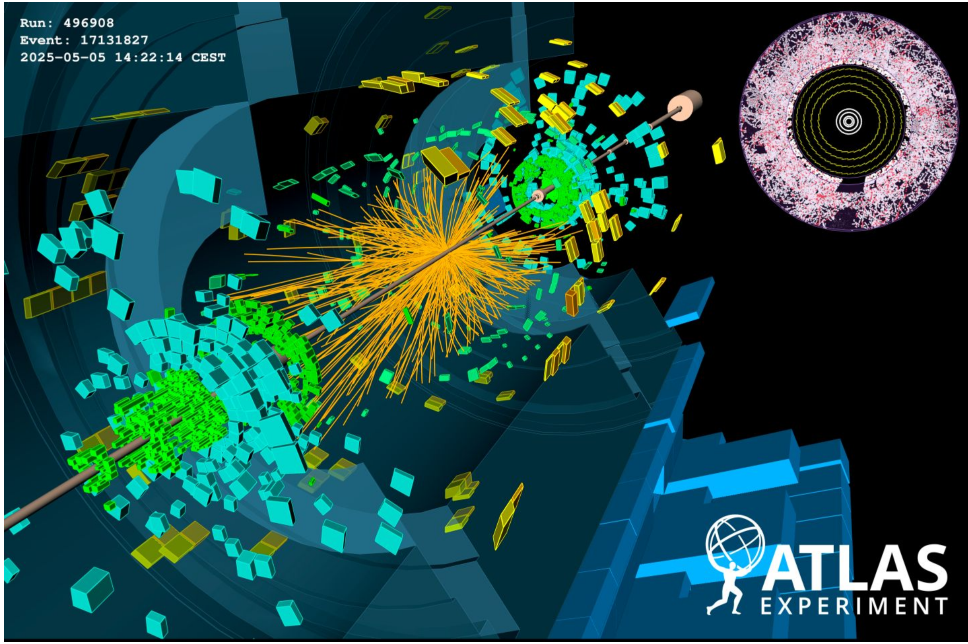


Illustration from [Bossi et al. ‘25]



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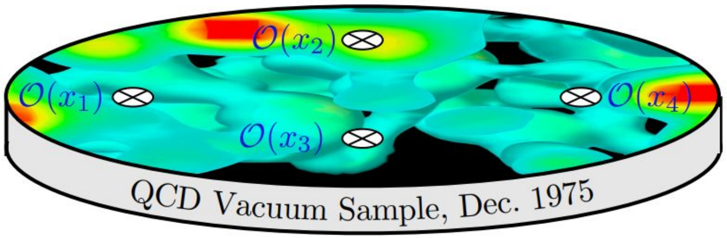

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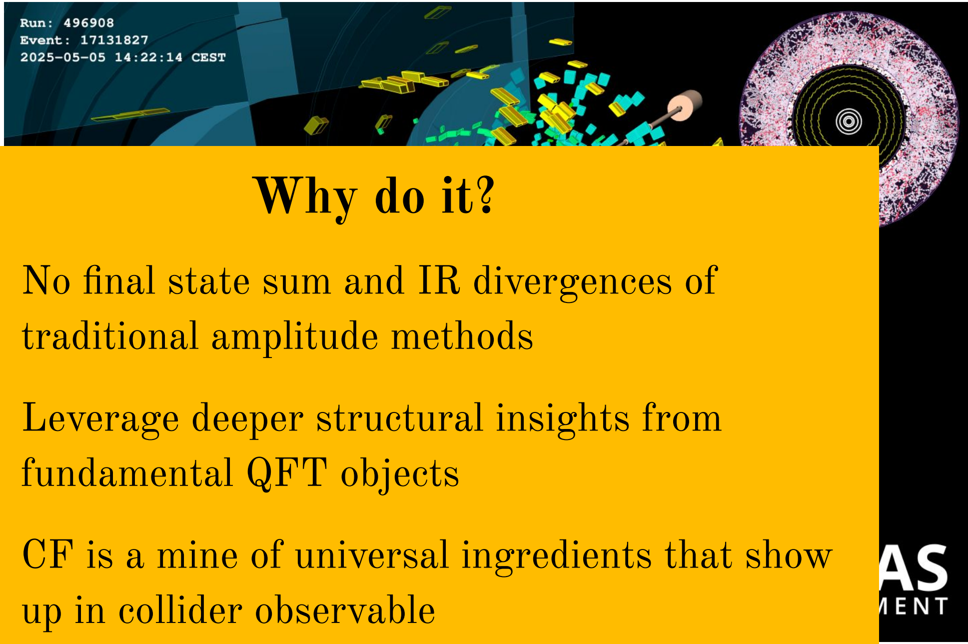


Why do it?

↪ No final state sum and IR divergences of traditional amplitude methods

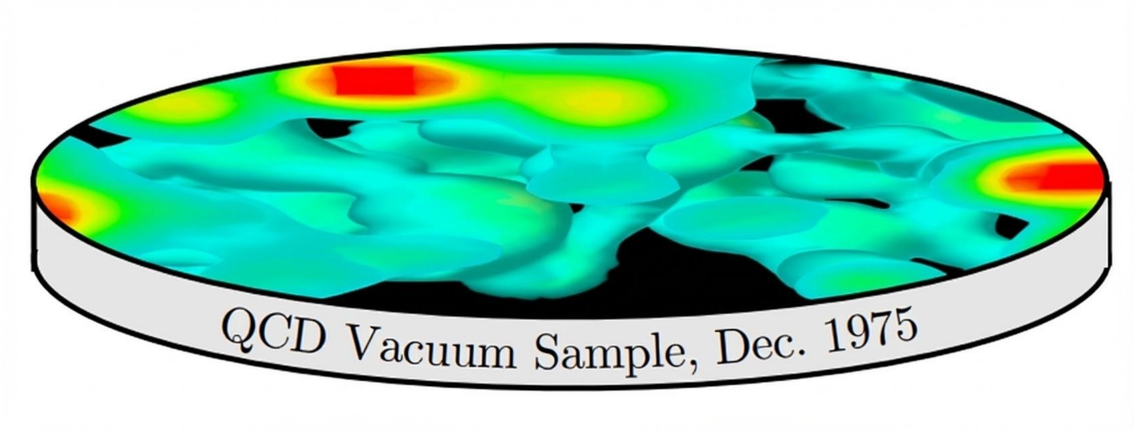
↪ Leverage deeper structural insights from fundamental QFT objects

↪ CF is a mine of universal ingredients that show up in collider observable

↪ It's fun!

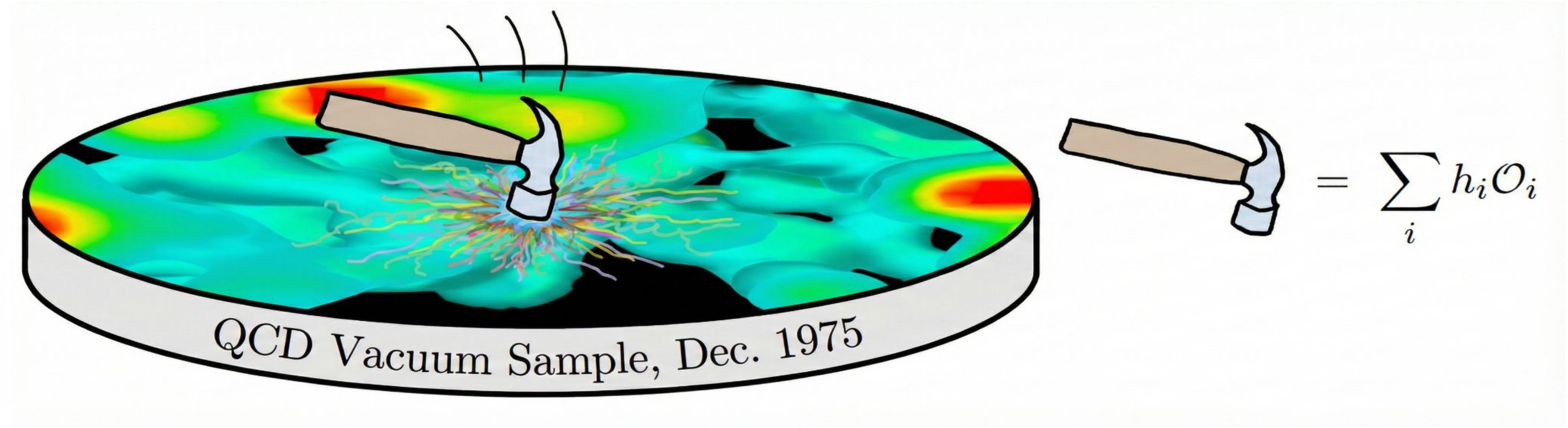
Correlation Functions

- Start with the QCD vacuum



Correlation Functions

- Start with the QCD vacuum
- Excite it via insertion of local operators of the theory



Correlation Functions

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- We'll consider **electromagnetic currents**

$$J^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$$

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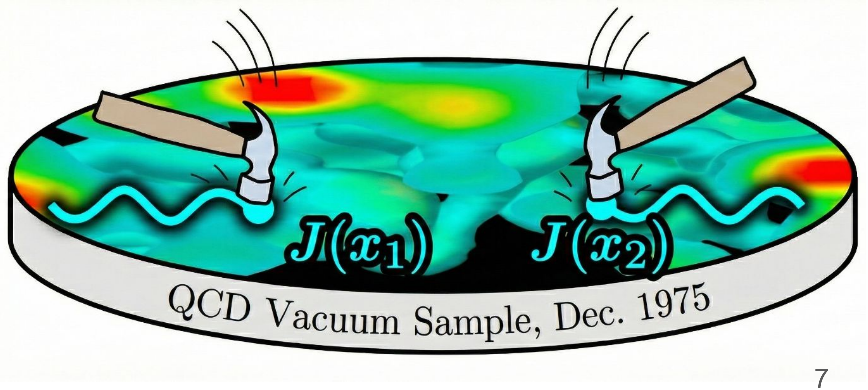
- **2-point correlation function** $$\langle\Omega|J(0)J(x)|\Omega\rangle$$



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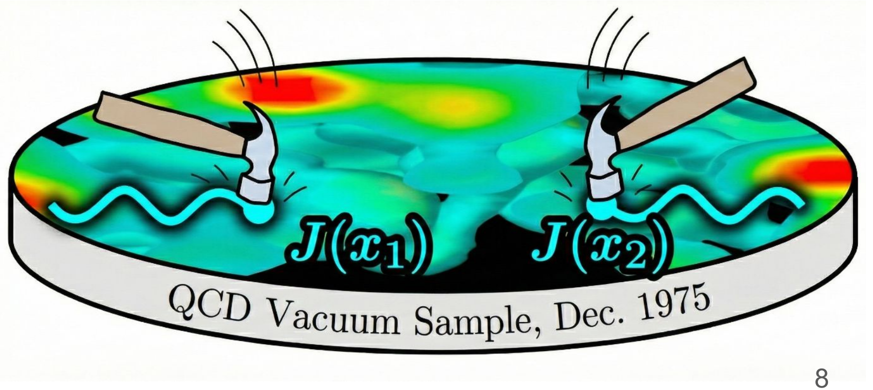
Adler Function [Adler (1974)], *R-ratio, Hadronic Vacuum Polarization*

↪ Measures total cross section in e^+e^-

↪ Very well known object, many loops known

↪ Used to extract the anomalous dimension of the coupling, $\beta[\alpha_s]$, to 5 loops



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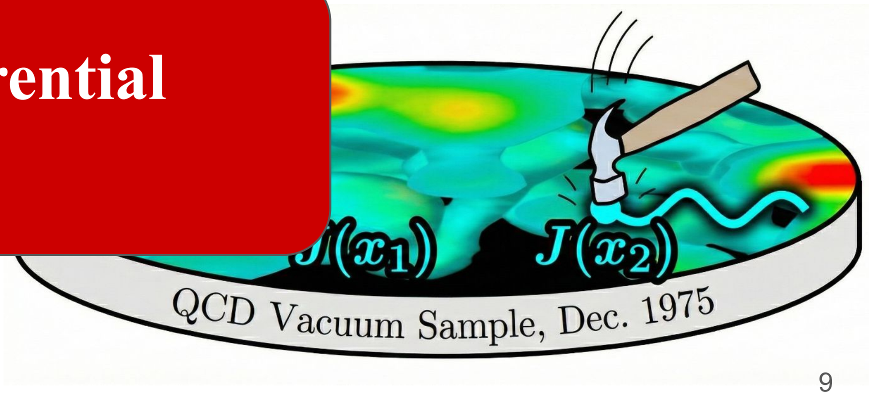
↪ C

↪ M

↪ V

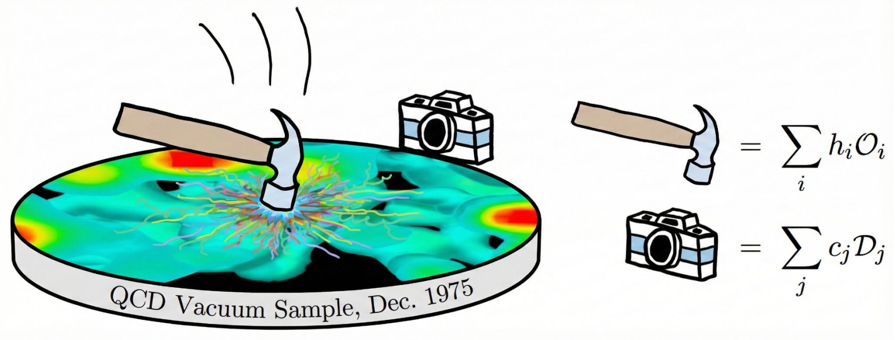
Ok, but what about differential observables?

↪ Used to extract the anomalous dimension of the coupling, $\beta[\alpha_S]$, to 5 loops



What is a detector?

- As we want to ask more fine grained questions to our vacuum sample, we need to add detectors and understand detectors in QFT



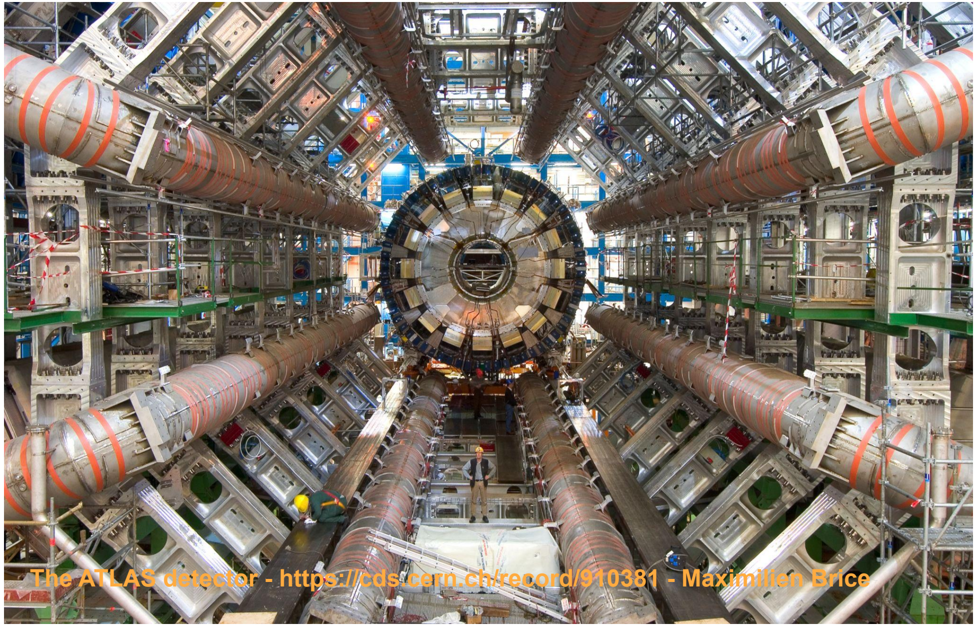


Illustration inspired by [Simon Caron-Huot, Kologlu,Kravchuk, Meltzer, Simmons-Duffin ‘22]

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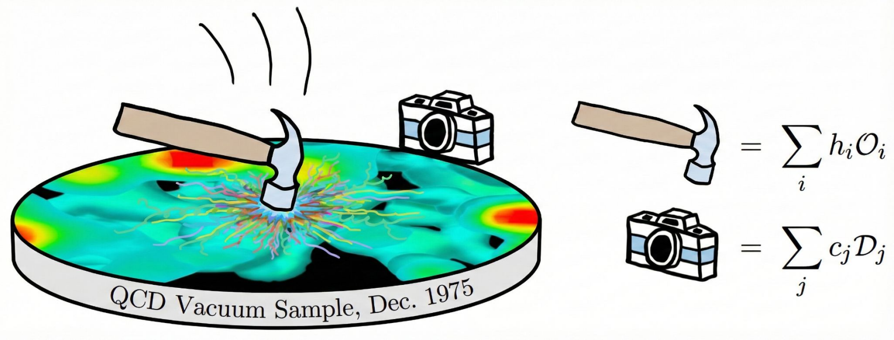

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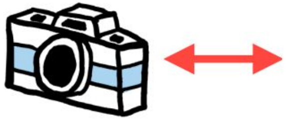
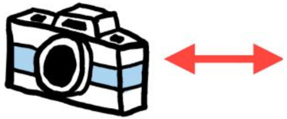
Energy detector



$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} \mathrm{d}t\, n_i T^{i0}(t, r\vec{n})$$

Charge detector



$$\mathcal{Q}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} \mathrm{d}t\, n_i J^{i}(t, r\vec{n})$$

[Hofman, Maldacena], [Korchemsky, Sterman], [Ore, Sterman],
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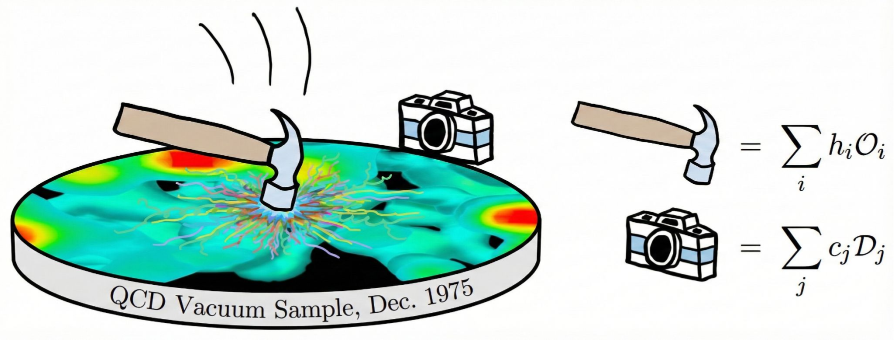

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Energy detector

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Local Operator

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Correlation Function for Differential Observables

- We can construct detector operators via non-local operations on local operators in the theory

- Construct collider observables as correlation functions of detector operators

"Collider correlators"

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Correlation Function for Differential Observables

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"Collider correlators"

- Construct them as transforms of **multipoint correlation functions** of local operators

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$$\langle\Omega|J(0)\Phi(x_2)\Phi(x_3)J(x)|\Omega\rangle$$

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From Correlation Functions to Collider Correlators

Multipoint Correlation Functions of Local Operators

Standard QFT quantity encoding information on the underlying theory

Promote local operators to detector operators

Collider Correlators

Asymptotic Observable Measurable in Experiments at Colliders

Outlook: from Correlation Functions to Collider Correlators

Multipoint Correlation Functions of Local Operators

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Asymptotic Observable Measurable in Experiments at Colliders

Example

4 point correlation function of EM charges

$$\langle \Omega | J^{\mu}(x_1) J^{\nu}(x_2) J^{\rho}(x_3) J^{\sigma}(x_4) | \Omega \rangle$$

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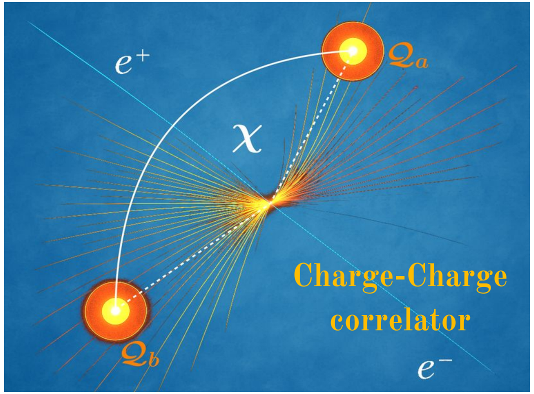

From Correlation Functions to Collider Observables

$$\langle\Omega|J(0)\Phi(x_2)\Phi(x_3)J(x)|\Omega\rangle \longrightarrow \sigma(v) \sim \sum_X \mathcal{F}_v(X)\left|\mathcal{M}_{e^+e^-\to X}\right|^2$$

- Very powerful tool in **conformal theories** (CFTs) since CF are very constrained
- Significant work in N=4

  [Hofman, Maldacena '08] [Alday, Eden, Korchemsky, Maldacena, Sokatchev '10]
  [Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov '13]
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  [Korchemsky '19] [Henn, Sokatchev, Yan, Zhiboedov '19] [many more ...]
- Spectacular series of results

  ↪ Correlation function/Wilson Loop/Amplitude Dualities

  ↪ OPE data from integrability to constrain amplitudes

  ↪ Ingredients of resummation of EEC

  ↪ Many more...

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Practical example

3 loop determination of **4 point correlation function** in N=4 SYM → **EEC** analytically to NNLO in N=4 SYM

[Drummond, Duhr, Eden, Heslop, Pennington, Smirnov] (1303.6909)

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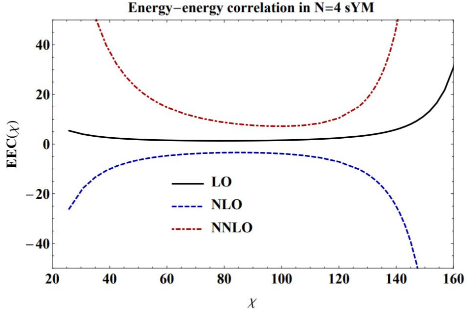

$$\langle 0|O(x_1)O(x_2)O(x_3)O(x_4)|0\rangle = \frac{\mathcal{G}(z,\bar z)}{(x_{13}^2 x_{24}^2)^2}$$

$$\mathrm{EEC}(\zeta) = \frac{1}{4\pi^3\zeta^2}\lim_{\epsilon\to 0}\int_0^1 d\bar z\int_0^{\bar z} dt\,\frac{1}{t(\zeta-\bar z)+(1-\zeta)\bar z} \times \mathrm{dDisc}_{\bar z=1}\mathrm{Disc}_{z=0}\left[(z-\bar z)\mathcal{G}_\epsilon(z,\bar z)\right]$$

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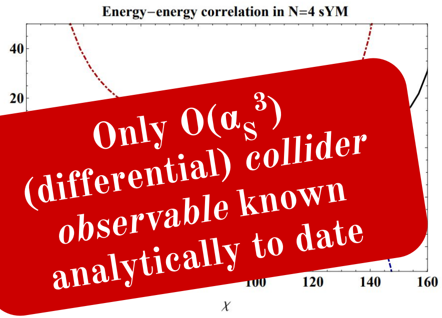

Only O(α_s^3) (differential) *collider observable* known analytically to date

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Can we extend this program to QCD?

Multipoint Correlation Functions in QCD

In full QCD we know
very little beyond two point
functions

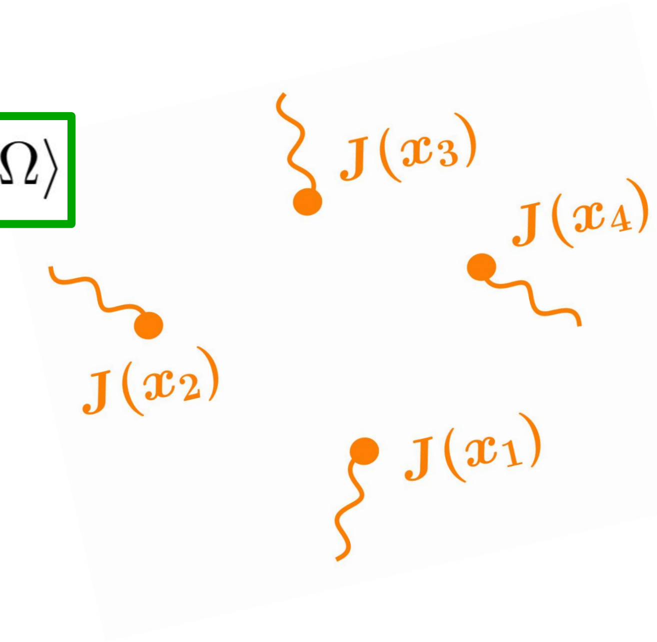
Multipoint Correlation Functions in QCD

- Obvious challenge for QCD correlators is complexity of computations and reduced symmetries (no supersymmetry, no conformal symmetry)

- **4 point function**

$$G(x_1, x_2, x_3, x_4) = \langle \Omega | J(x_1) J(x_2) J(x_3) J(x_4) | \Omega \rangle$$

↪ Depends (non-trivially) on 2 variables in a CFT, but 6 variables in full QCD



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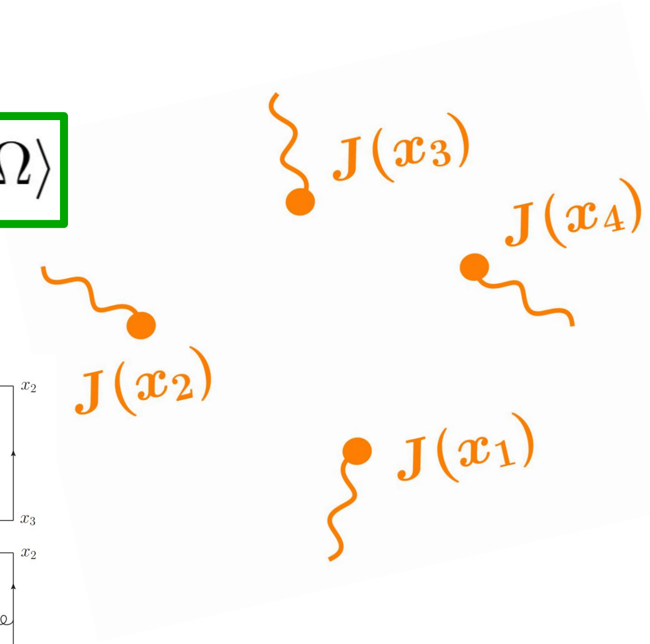
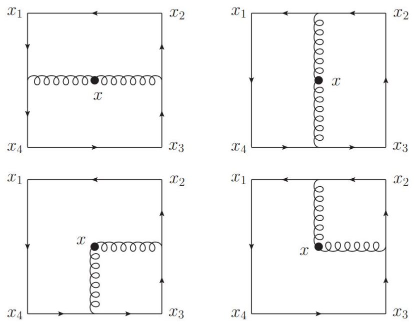


- First results beyond N=4 is QCD at first order [Chicherin, Henn, Sokatchev, Yan ‘20]
  => no running coupling, still conformal



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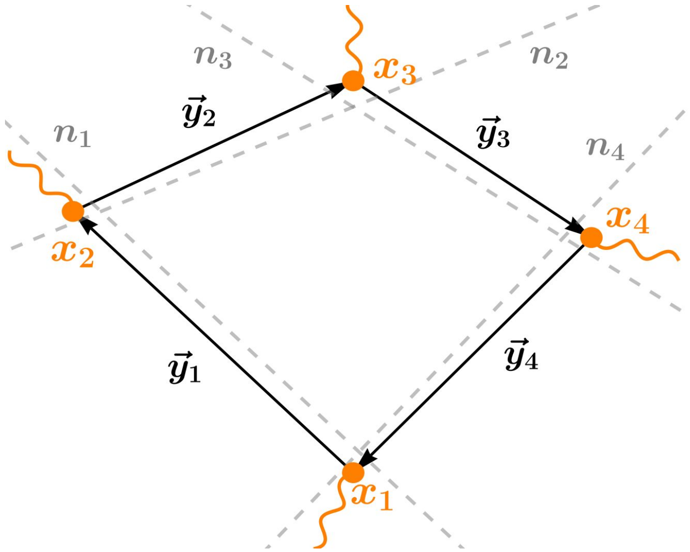

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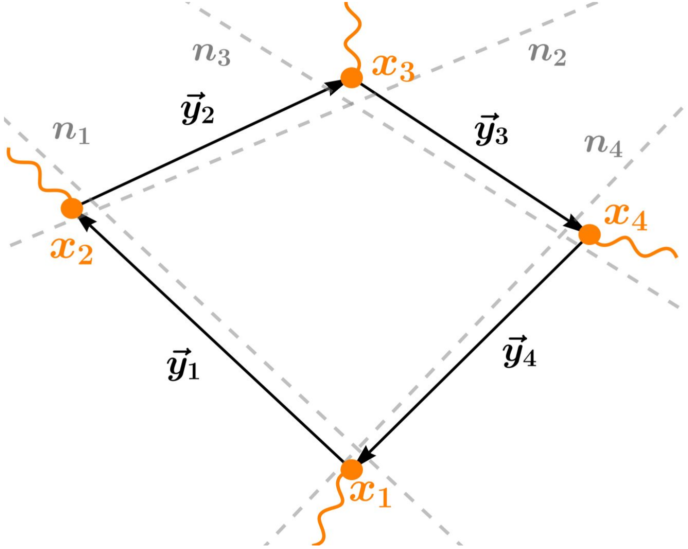
- We consider the **Sequential Lightcone Limit (SLC)**

$$(x_i - x_{i+1})^2 \ll (x_i - x_{i+2})^2$$

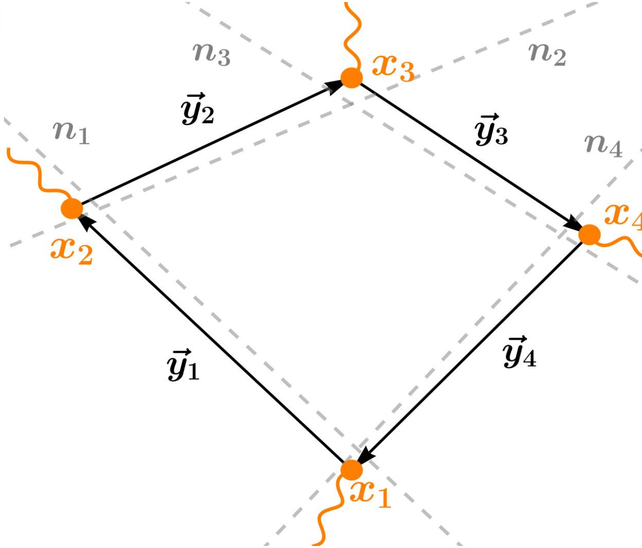


- Consecutive separations become light-like. Light-like directions identified by n_i^μ

- Limit where:
  - ↪ CF develops **lightcone singularities**
  - ↪ Manifestation of Correlation Function/Wilson Loop duality in N=4, [Alday, Eden, Korchemsky, Maldacena, Sokatchev '10]
  - ↪ Connected to back-to-back asymptotic of double point correlation observables [Korchemsky '19] [Chen '23]

EFT Description of SLC Limit

- Hierarchy of Scales $(x_i - x_{i+1})^2 \ll (x_i - x_{i+2})^2$



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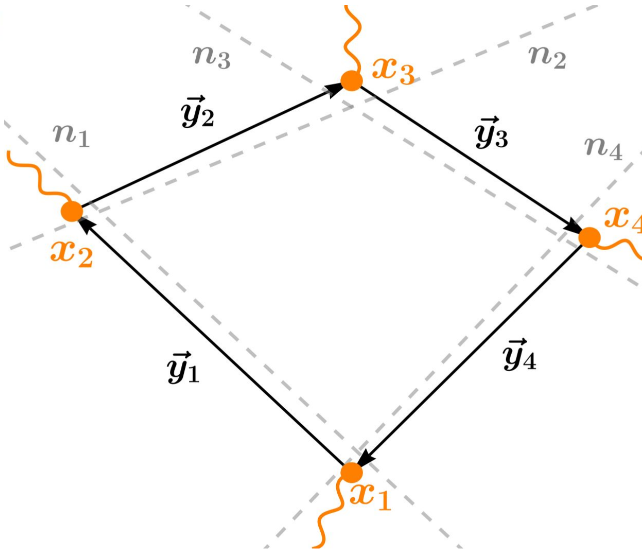
  Use EFT to treat this limit



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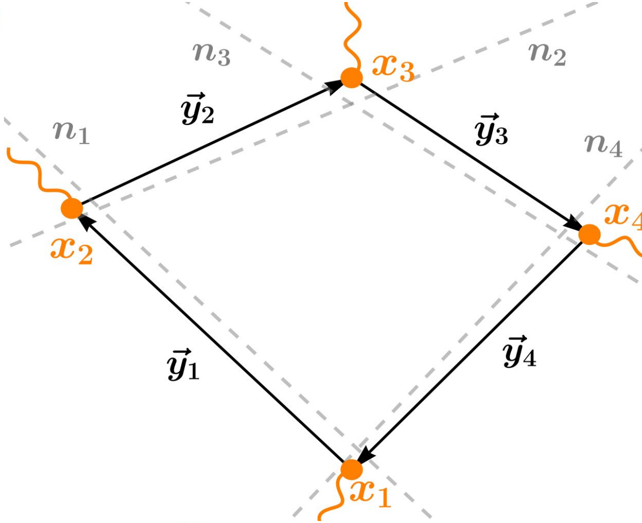

SLC is a power expansion around lightcone directions

$$y_i^2 \equiv (x_{i+1} - x_i)^2 \qquad \frac{y_i^+}{y_i^-} \sim \lambda^2 \ll 1$$

$$y_i^\mu \sim L(\lambda^2, 1, \lambda)\,, \quad |y_i^2| \sim L^2\lambda^2$$

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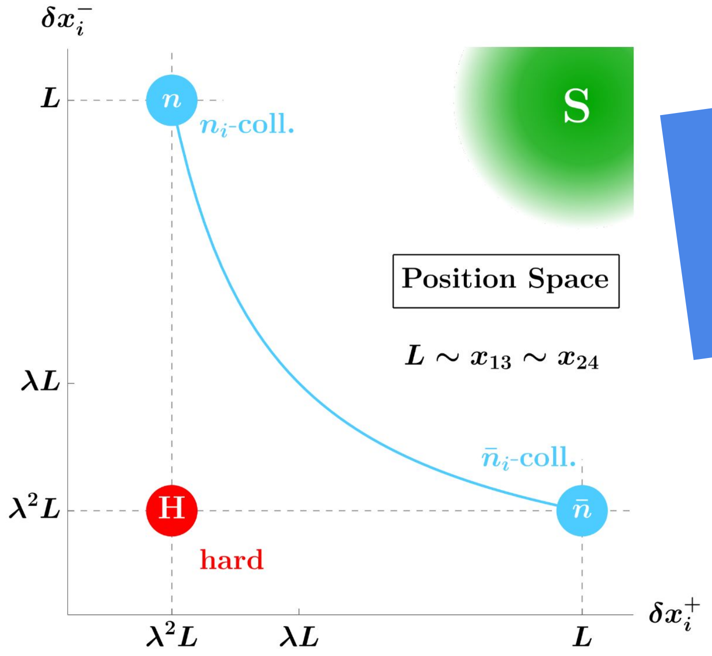
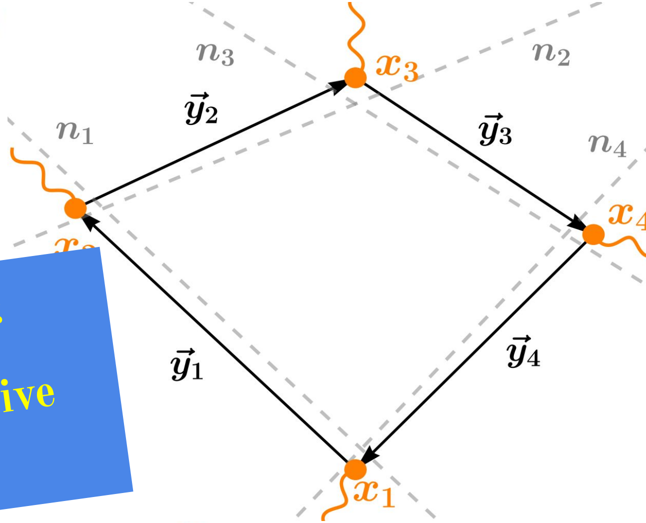


SLC is a power

EFT for the problem is:

Soft and Collinear Effective Theory (SCET)



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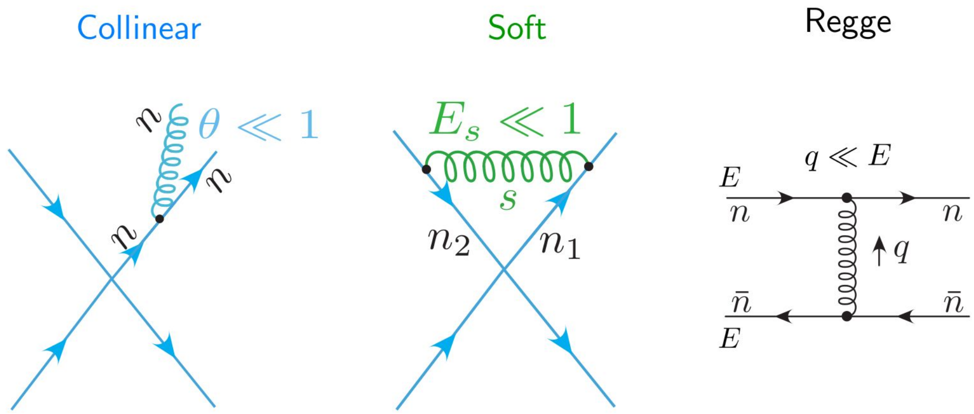
Effective Field Theories for QCD at Colliders

EFTs are powerful theoretical tools to separate physics happening at different scales

Collider physics is driven by QCD radiation in the presence of large kinematic hierarchies

- ➢ Infrared divergences
- ➢ Large logarithms
- ➢ Dominant contribution to cross sections
- ➢ Factorization (Breaking)



Soft and Collinear Effective Theory (SCET) is the **EFT** capturing physics in these limits

[Bauer, Fleming, Pirjol, Stewart '01]

Effective Field Theories for QCD at Colliders

Systematic power expansion of QCD limits at **Lagrangian** level

$$\mathcal{L}_{\mathrm{QCD}} \to \overbrace{\sum_i \left(\mathcal{L}^{(i)}_{\mathrm{hard}} + \sum_{n-\mathrm{coll}} \mathcal{L}^{(i)}_{n} + \mathcal{L}^{(i)}_{\mathrm{soft}}\right) + \mathcal{L}^{(0)}_{\mathrm{Regge/Glauber}}}^{\mathcal{L}_{\mathrm{SCET}}}$$

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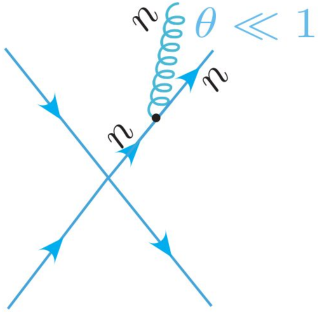
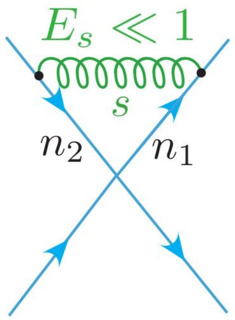
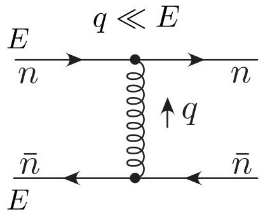
Collinear



Soft



Regge



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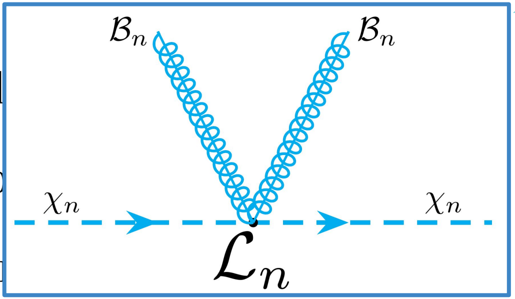
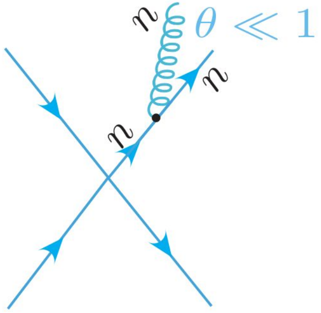
Systematic power expansion of QCD limits at **Lagrangian** level

$$\mathcal{L}_{\rm QCD} \to \underbrace{\sum_i \Big(\mathcal{L}^{(i)}_{\rm hard} + \sum_{n-{\rm coll}} \mathcal{L}^{(i)}_{n} + \mathcal{L}^{(i)}_{\rm soft} \Big) + \mathcal{L}^{(0)}_{\rm Regge/Glauber}}_{\mathcal{L}_{\rm SCET}}$$

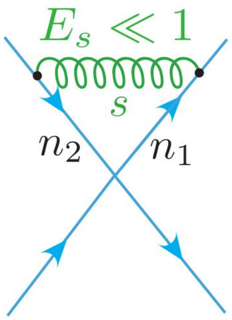


- ➢ Infrared
- ➢ Large lo
- ➢ Domina
  cross sections
- ➢ Factorization (Breaking)

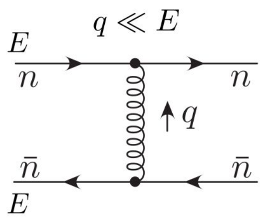
Collinear



Soft



Regge



Soft and Collinear Effective Theory (SCET) is the **EFT** capturing physics in these limits

[Bauer, Fleming, Pirjol, Stewart '01]

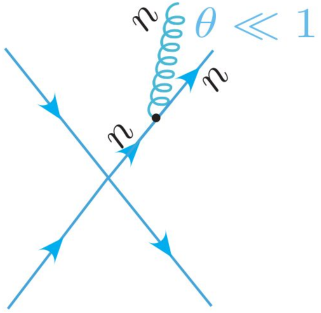
Effective Field Theories for QCD at Colliders

SCET is the natural **EFT** for the Lightlike Dynamics of Gauge Theories

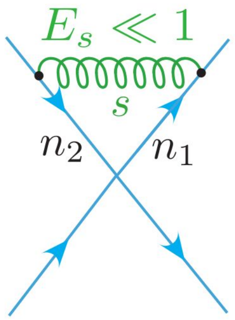
$$\mathcal{L}_{\mathrm{SCET}} \rightarrow \sum_i \left(\mathcal{L}^{(i)}_{\mathrm{hard}} + \sum_{n-\mathrm{coll}} \mathcal{L}^{(i)}_{n} + \mathcal{L}^{(i)}_{\mathrm{soft}} \right) + \mathcal{L}^{(0)}_{\mathrm{Regge/Glauber}}$$

- ➢ Infrared divergences
- ➢ Large logarithms
- ➢ Dominant contribution to cross sections
- ➢ Factorization (Breaking)

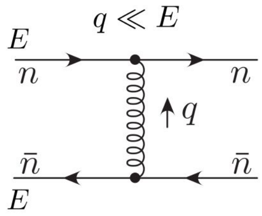
Collinear



Soft



Regge



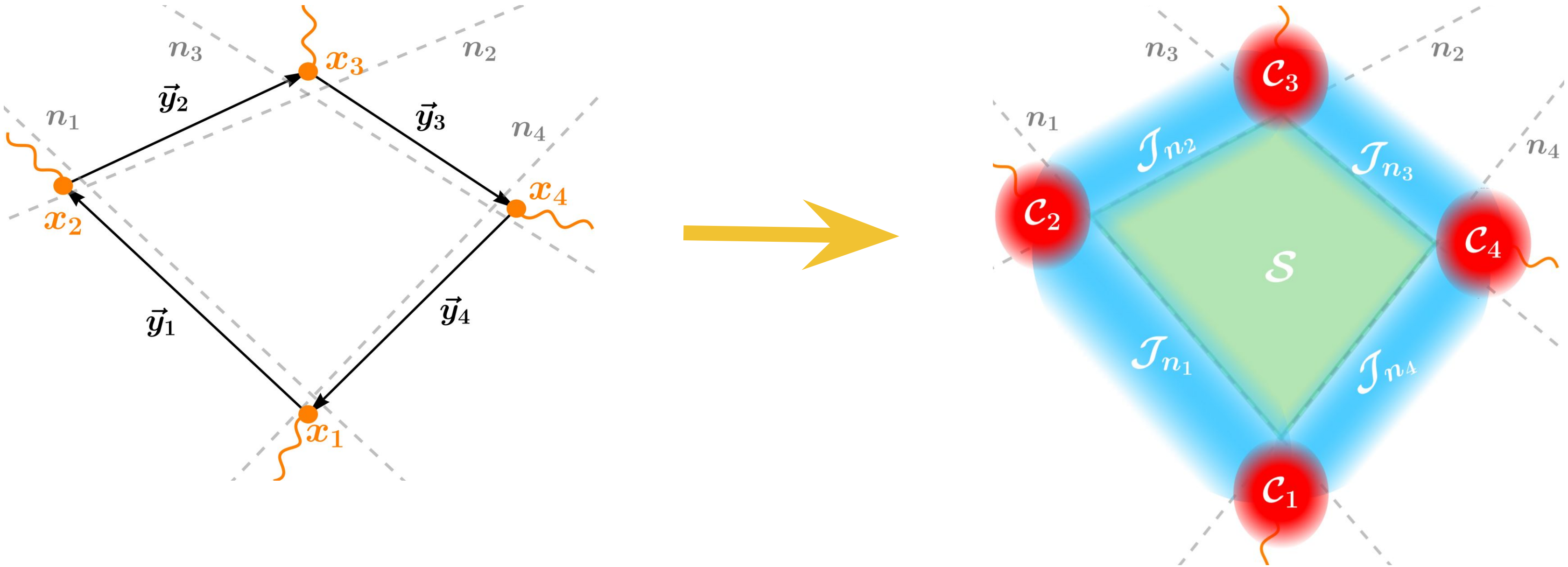
Soft and Collinear Effective Theory (SCET) is the **EFT** capturing physics in these limits

[Bauer, Fleming, Pirjol, Stewart '01]

Sequential Light Cone Limit from SCET

- Arrive at factorization for n-point correlation functions in the SLC

$$G(x_1, x_2, x_3, x_4) \sim \mathcal{W}(\{x\}) \prod_{i=1}^{4} \int_0^\infty \mathrm{d}\omega_i \, \mathcal{C}_i(\omega_i \omega_{i+1}) \mathcal{J}_{n_i}(\omega_i / y_i^-) e^{-\omega_i \frac{y_i^2}{2 y_i^-}}$$



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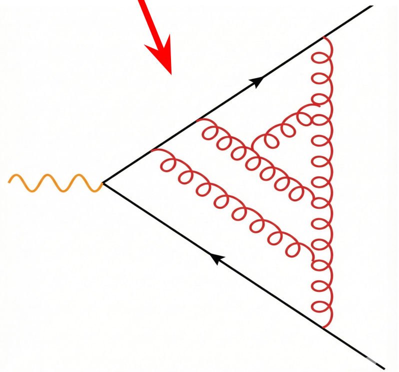
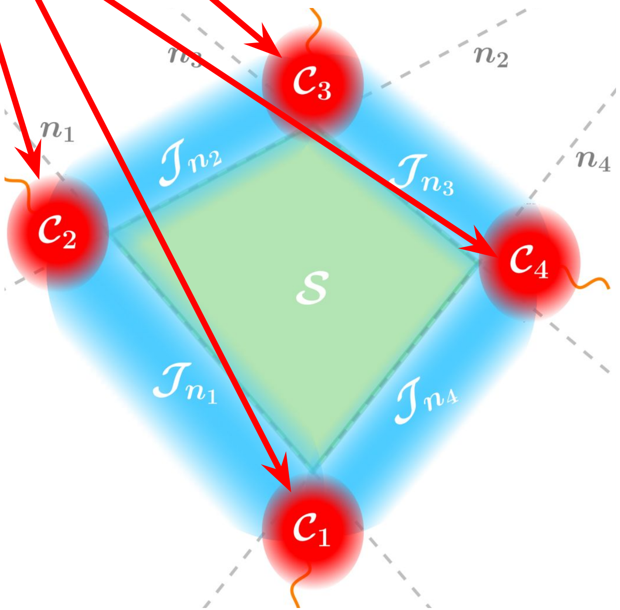
Hard Wilson Coefficient (quark form factor)

$$j^\mu(x_i) \to C_i(\omega_i\omega_{i+1})\bar{\chi}_{n_i}(x_i)\gamma_\perp^\mu \chi_{n_{i-1}}(x_i)$$



Virtual corrections from integrating out hard modes in electromagnetic current matching



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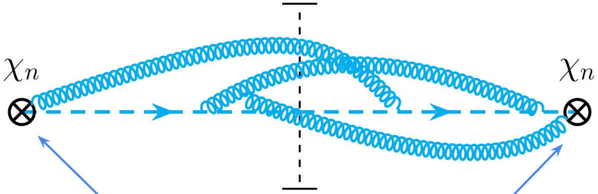
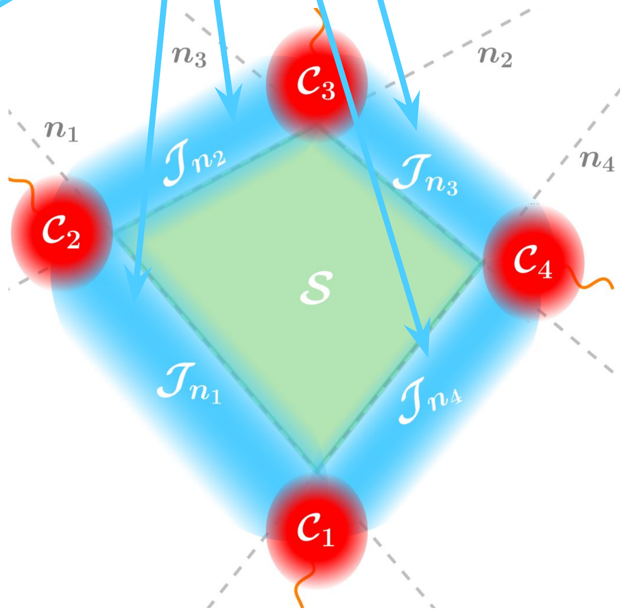
Propagators of Collinear Modes (Thrust Jet Functions)

$$J_n \sim \langle \mathbf{T} \bar{\chi}_n(x) \frac{\bar{n}\!\!\!/}{4\, N_c} \chi_n(0) \rangle$$



SCET gauge-invariant collinear fermion operator



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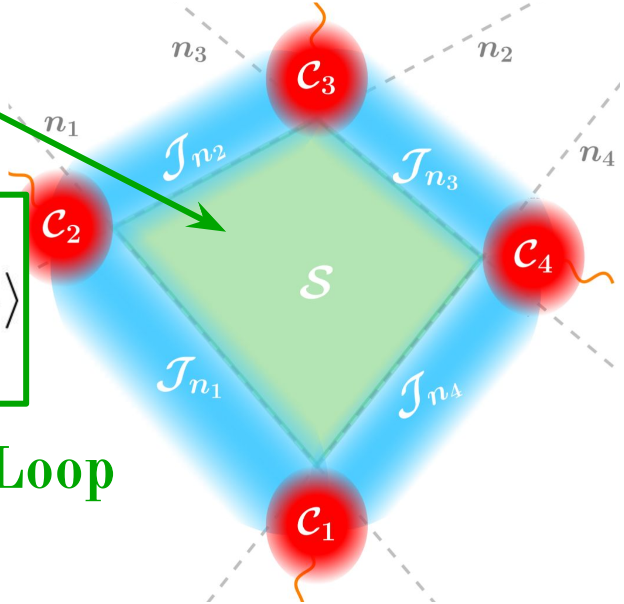
$$\mathcal{W}(\{x\}) = \langle \mathcal{P} \exp\left\{ ig \int_{\Gamma(\{x\})} \mathrm{d}z^{\mu} A^a_{s\,\mu}(z) T^a \right\} \rangle$$

Null polygonal Wilson Loop



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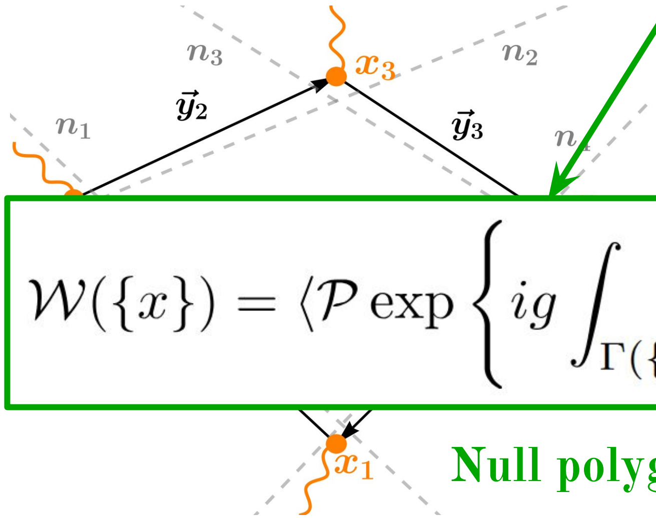
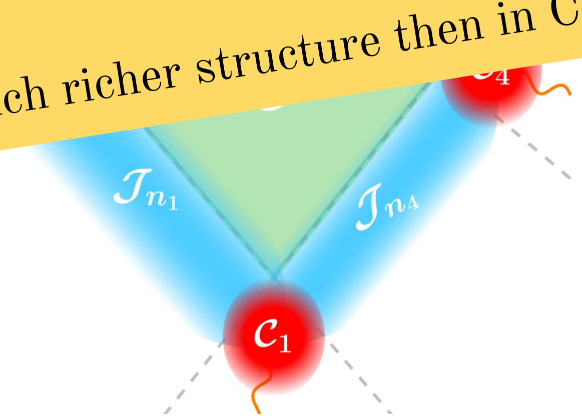


$$\mathcal{W}(\{x\}) = \langle \mathcal{P} \exp \left\{ ig \int_{\Gamma(\{x\})} \mathrm{d}z^\mu A^a_{s\,\mu}(z) \ldots \right\}$$

Null polygonal Wilson Loop

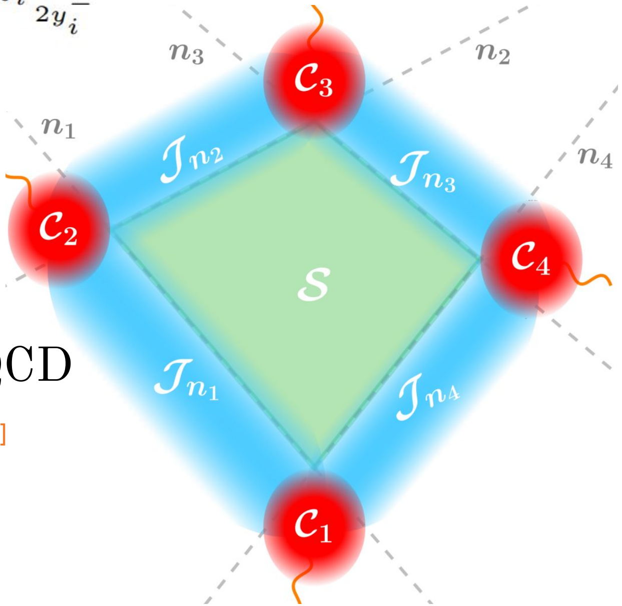
Established Correlation Function / Wilson Loop **duality** in QCD!

Much richer structure then in CFTs



The Four Point Correlation Function in QCD

$$G(x_1,x_2,x_3,x_4) \sim \mathcal{W}(\{x\}) \prod_{i=1}^{4} \int_0^\infty \mathrm{d}\omega_i\, \mathcal{C}_i(\omega_i \omega_{i+1}) \mathcal{J}_{n_i}(\omega_i / y_i^-) e^{-\omega_i \frac{y_i^2}{2 y_i^-}}$$



- **Wilson coefficient:** known to 4 loops [Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser '22]
- **Jet functions:** known to 3 loops from thrust [Brüser, Liu, Stahlhofen '18]
- **Null polygonal Wilson Loop:** only 2 loop logs known in QCD [Korchemskaya, Korchemsky '92]
  - ↪ We calculated the 2 loop constants
  - ↪ Derived the 3 loop logs from RGE

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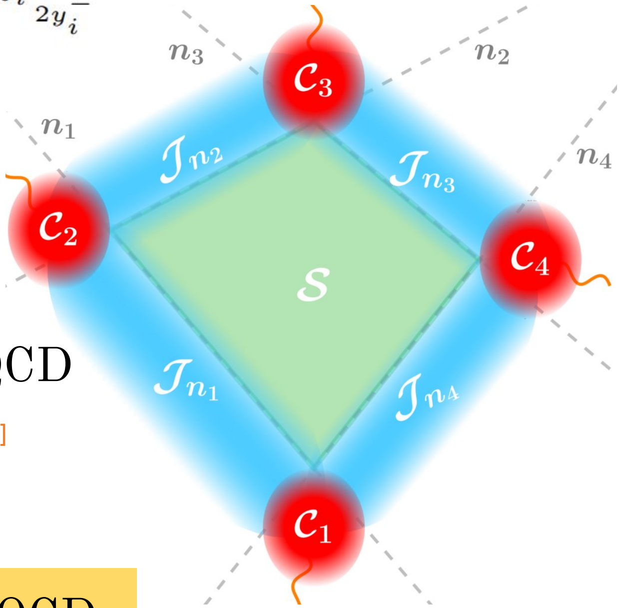

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We determined the SLC of the 4 point function in QCD to **N3LL (2 loops + logs at 3 loops)**

=> huge jump forward from just $O(\alpha_S)$

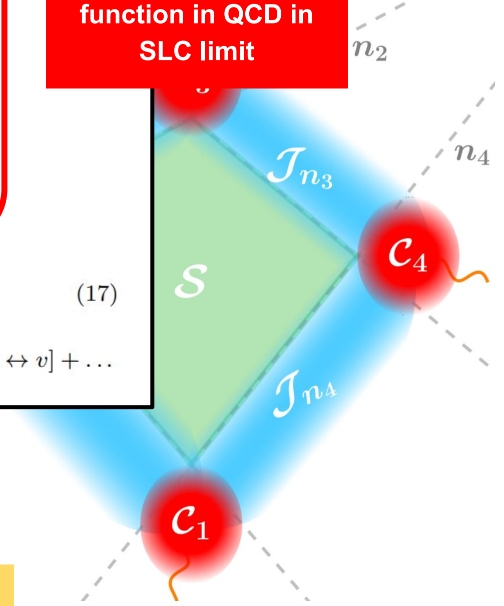
The Four Point Correlation Function in QCD

$$
\begin{aligned}
R_2 = C_F^2 \Big[& -28\zeta_2 - 48\zeta_3 + 116\zeta_4 + L_u\,(8 - 2L_{uv})\,L_v + 2L_u^2 L_v^2 + \left(4\zeta_2 - \frac{7}{2}\right) L_{uv}^2 + \left(4\zeta_2 - 24\zeta_3 + \frac{5}{2}\right) L_{uv} \Big] \\
& + C_F C_A \left[\frac{68\zeta_2}{9} + \frac{28\zeta_3}{3} - 16\zeta_4 + L_u L_v \left(4\zeta_2 - \frac{11 L_{uv}}{6} - \frac{235}{18}\right) + \frac{11 L_{uv}^2}{12} + \left(-\frac{112\zeta_2}{3} + 12\zeta_3 + \frac{221}{18}\right) L_{uv} \right] \\
& + C_F n_f \left[-\frac{8\zeta_2}{9} + \frac{8\zeta_3}{3} + L_u \left(\frac{L_{uv}}{3} + \frac{17}{9}\right) L_v + \left(\frac{16\zeta_2}{3} - \frac{13}{9}\right) L_{uv} - \frac{L_{uv}^2}{6} \right] + 16\left(C_F^2 - \frac{1}{2} C_F C_A\right) \\
& + \frac{1}{2}\beta_0 R_1 \ln \frac{x_{13}^2 x_{24}^2 \mu^4}{16} + C_F \beta_0 \left[r_2(L_{uv}, x) + r_2\left(L_{uv}, \frac{1}{x}\right) + \frac{L_{13}^2}{4} + \frac{L_{24}^2}{4} + \ln(x) L_{13} L_{24} - \frac{1}{2}\left(L_v L_{13}^2 + L_u L_{24}^2\right) \right]
\end{aligned}
$$

$$
\begin{aligned}
R_3 = & \, C_A C_F^2 \left(-\frac{4}{9}\pi^4 L_u^2 - \frac{2}{45}\pi^4 L_u L_v - \frac{2}{3}\pi^2 L_u^2 L_v^2 - \frac{56}{3}\pi^2 L_u \zeta_3 - 24 L_u^2 L_v \zeta_3 - 120 L_u \zeta_5 \right) + C_A^2 C_F \left(-\frac{11}{45}\pi^4 L_u L_v - 40 L_u \zeta_5 \right) \\
& + C_F^3 \left(-\frac{4}{9}\pi^4 L_u^2 - \frac{26}{15}\pi^4 L_u L_v - \frac{4}{3}\pi^2 L_u^3 L_v - \frac{2}{3}\pi^2 L_u^2 L_v^2 - \frac{2}{3} L_u^3 L_v^3 + 32\pi^2 L_u \zeta_3 - \frac{16}{3} L_u^3 \zeta_3 + 48 L_u^2 L_v \zeta_3 + 240 L_u \zeta_5 \right) + [u \leftrightarrow v] + \ldots
\end{aligned}
\qquad (17)
$$

[Korchemskaya, Korchemsky '92]

Two loop correlation function in QCD in SLC limit



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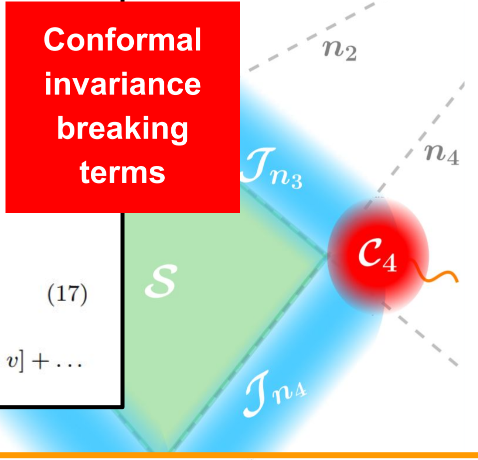
$$+ C_FC_A\left[\frac{68\zeta_2}{9} + \frac{28\zeta_3}{3} - 16\zeta_4 + L_uL_v\left(4\zeta_2 - \frac{11L_{uv}}{6} - \frac{235}{18}\right) + \frac{11L_{uv}^2}{12} + \left(-\frac{112\zeta_2}{3} + 12\zeta_3 + \frac{221}{18}\right)L_{uv}\right]$$
$$+ C_Fn_f\left[-\frac{8\zeta_2}{9} + \frac{8\zeta_3}{3} + L_u\left(\frac{L_{uv}}{3} + \frac{17}{9}\right)L_v + \left(\frac{16\zeta_2}{3} - \frac{13}{9}\right)L_{uv} - \frac{L_{uv}^2}{6}\right] + 16\left(C_F^2 - \frac{1}{2}C_FC_A\right)$$
$$+ \frac{1}{2}\beta_0R_1\ln\frac{x_{13}^2x_{24}^2\mu^4}{16} + C_F\beta_0\left[r_2(L_{uv},x) + r_2\left(L_{uv},\frac{1}{x}\right) + \frac{L_{13}^2}{4} + \frac{L_{24}^2}{4} + \ln(x)L_{13}L_{24} - \frac{1}{2}\left(L_vL_{13}^2 + L_uL_{24}^2\right)\right],$$

$$R_3 = C_AC_F^2\left(-\frac{4}{9}\pi^4L_u^2 - \frac{2}{45}\pi^4L_uL_v - \frac{2}{3}\pi^2L_u^2L_v^2 - \frac{56}{3}\pi^2L_u\zeta_3 - 24L_u^2L_v\zeta_3 - 120L_u\zeta_5\right) + C_A^2C_F\left(-\frac{11}{45}\pi^4L_uL_v - 40L_u\zeta_5\right) \tag{17}$$
$$+ C_F^3\left(-\frac{4}{9}\pi^4L_u^2 - \frac{26}{15}\pi^4L_uL_v - \frac{4}{3}\pi^2L_u^3L_v - \frac{2}{3}\pi^2L_u^2L_v^2 - \frac{2}{3}L_u^3L_v^3 + 32\pi^2L_u\zeta_3 - \frac{16}{3}L_u^3\zeta_3 + 48L_u^2L_v\zeta_3 + 240L_u\zeta_5\right) + [u \leftrightarrow v] + \ldots$$

Conformal invariance breaking terms



↪ We calculated the 2 loop constants

↪ Derived the 3 loop logs from RGE

$$r_2(L_{uv},x) = 2\mathrm{Li}_3(-x) - 2\ln(x)\mathrm{Li}_2(-x) - \frac{1}{4}L_{uv}\ln^2(x) - \frac{1}{2}(6\zeta_2 + \ln^2(x))\ln\frac{(1+x)^2}{x} + \frac{1}{4}\ln^2(x)\,. \tag{18}$$

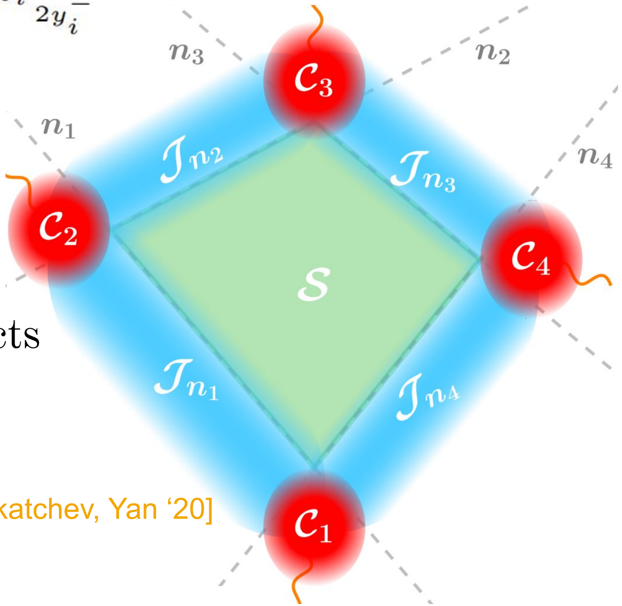
We determined the SLC of the

to **N3LL (2 loops + logs at 3 loops)**

=> huge jump forward from just $O(\alpha_s)$

The Four Point CF in QCD: Checks

$$G(x_1,x_2,x_3,x_4) \sim \mathcal{W}(\{x\}) \prod_{i=1}^{4} \int_0^\infty \mathrm{d}\omega_i\, \mathcal{C}_i(\omega_i\omega_{i+1}) \mathcal{J}_{n_i}(\omega_i/y_i^-) e^{-\omega_i \frac{y_i^2}{2y_i^-}}$$



- Several checks of the theoretical framework
  - ↪ Cancellation of poles (bare factorization)
  - ↪ Self consistency of results when employing renormalized objects
  - ↪ 1 loop result in terms of only **conformal cross ratios**
  - ↪ 1 loop result **matches** asymptotic expansion of [Chicherin, Henn, Sokatchev, Yan '20]
  - ↪ Verified that our CF satisfies the **anomalous conformal Ward identities**

$$K^\mu \langle j_1 \dots j_4 \rangle = -\frac{\beta(g)}{g} \int d^4x 2x^\mu \langle g \frac{\partial}{\partial g} \mathcal{L}(x) j_1 \dots j_4 \rangle$$

  - ↪ N=4 limit of our CF **matches N=4 result to 3 loops!**

Outlook: from Correlation Functions to Collider Correlators

Multipoint Correlation Functions of Local Operators

Standard QFT quantity encoding information on the underlying theory

Promote local operators to detector operators

Collider Correlators

Asymptotic Observable Measurable in Experiments at Colliders

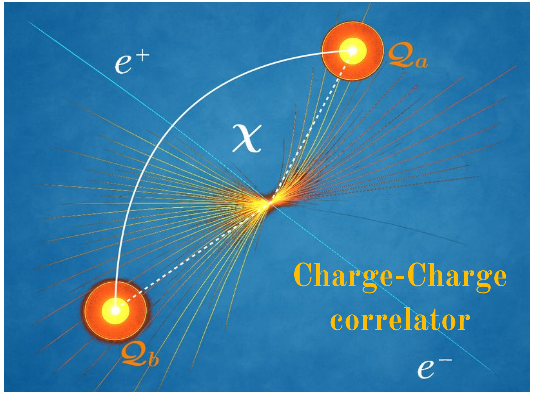
Example

4 point correlation function of EM charges

$$\langle\Omega|J^\mu(x_1)J^\nu(x_2)J^\rho(x_3)J^\sigma(x_4)|\Omega\rangle$$

Charge detector

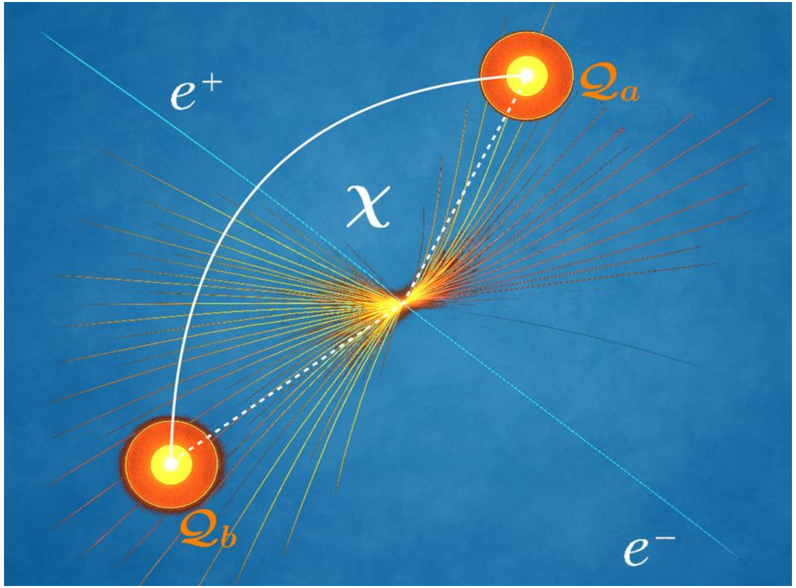
$$\mathcal{Q}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} \mathrm{d}t\, n_i J^i(t, r\vec{n})$$



Charge-Charge Correlation

- A collider observable that one can naturally define for study charge dynamics is

$$\mathrm{QQC}(\chi) = \sum_{a,b} \int \mathrm{d}\sigma_{e^+e^- \to a,b+X} \frac{Q_a Q_b}{\sigma_{\mathrm{tot}}} \delta(\cos\chi - \hat{n}_a \cdot \hat{n}_b)$$



- Formally defined as the Light Transform of Charge Detector Operators

$$\mathrm{QQC}(\zeta) = \int d^4x\, e^{iq\cdot x} \left[\prod_{i=2,4} \int_{-\infty}^{\infty} d(n_i \cdot x_i) \lim_{\bar{n}_i \cdot x_i \to \infty} \frac{(\bar{n}_i \cdot x_i)^2}{8} \bar{n}_i^{\mu_i} \right] \langle \Omega | J_\mu(x) J_{\mu_2}(x_2) J_{\mu_4}(x_4) J^\mu(0) | \Omega \rangle \,.$$

$$\mathcal{Q}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} \mathrm{d}t\, n_i J^i(t, r\vec{n})$$

Charge-Charge Correlation

- Similar structure to the Energy-Energy Correlation:
  - Weighted cross section
  - Function of angle. Usually expressed in terms of $z \equiv \frac{1}{2}(1 - \cos\chi)$

$\chi \to 0$
$z \to 0$
Collinear/Forward/ small angle Limit

$\chi \to \pi$
$z \to 1$
Back-to-Back Limit

- Leading Order result reads

$$F^{\text{QCD}}_{\text{QQC}}(\zeta) = 4\pi^2 (nn')^2 \sigma_0^{-1} \int d^4x\, e^{iqx} \langle J^\mu(x) \mathcal{Q}\mathcal{Q} J_\mu(0)\rangle$$
$$= \frac{C_F g^2}{4\pi^2} \frac{2\log(1-\zeta) + \zeta(2+\zeta)}{\zeta(1-\zeta)} + O(g^4)\,.$$

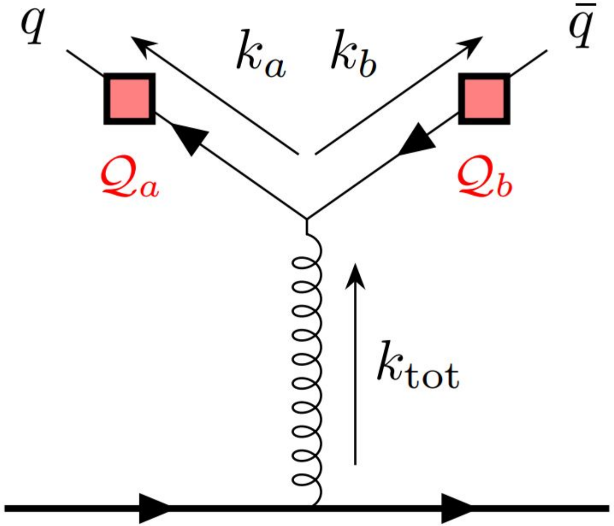
[Chicherin, Henn, Sokatchev, Yan ‘20]

Charge-Charge Correlation: Infrared Unsafety

$$\mathrm{QQC}(\chi) = \sum_{a,b} \int d\sigma_{e^+e^- \to a,b+X} \frac{Q_a Q_b}{\sigma_{\text{tot}}} \delta(\cos\chi - \hat{n}_a \cdot \hat{n}_b)$$

- Beyond LO the QQC is known to be **IRC unsafe**

- Problem stems from soft/collinear gluons (zero charge, no contribution to the observable) splitting into **quark pairs** (charged, now contributing to the obserbable)



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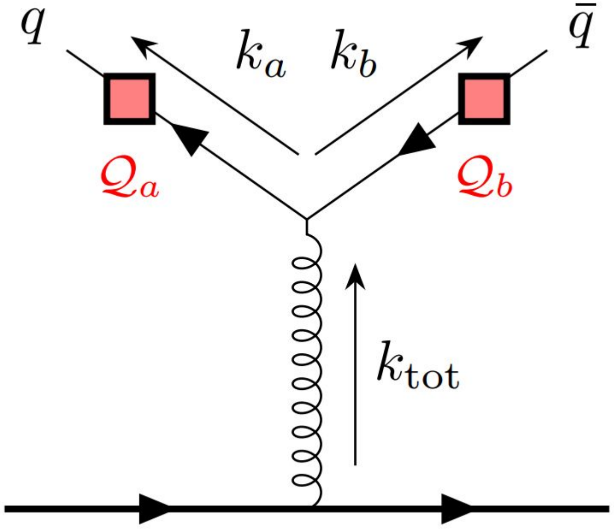
- Beyond LO the QQC is known to be **IRC unsafe**
- Problem stems from soft/collinear gluons (zero charge, no contribution to the observable) splitting into **quark pairs** (charged, now contributing to the obserbable)
- However, this correlation is **kinematically suppressed** as we take the quarks apart => **possibility to suppress effect in the back-to-back limit!**



$$|\mathcal{M}_{q\bar{q}}|^2 \sim e^{-3|\Delta\eta_{q\bar{q}}|}$$

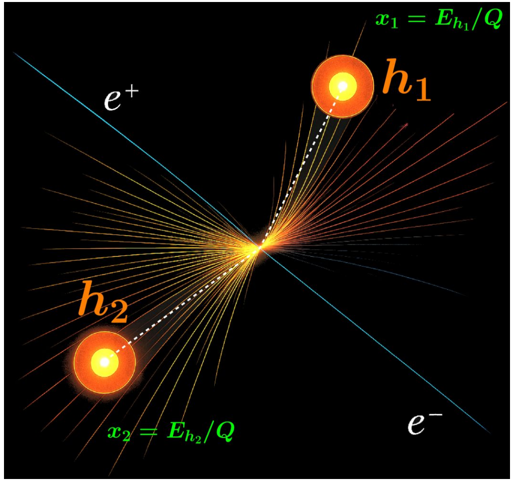
Charge-Charge Correlation: Leading Power Safety

- Hence we studied leading asymptotic behaviour of QQC in the back-to-back limit.

Charge-Charge Correlation: Leading Power Safety

- Hence we studied leading asymptotic behaviour of QQC in the back-to-back limit.
- QQC in back-to-back is integral over energies of **dihadron production in b2b**, summed over all hadrons weighted by **their charge**

$$\lim_{z\to 1}\frac{d\sigma^{e^+e^-}_{\rm QQC}}{dz} = \int_0^1 dx_a \int_0^1 dx_b \int d^2\vec{q}_T \delta\left(1 - z - \frac{\vec{q}_T^{\,2}}{Q^2}\right) \sum_{h_a,h_b} \underline{Q_{h_a}Q_{h_b}} \lim_{\vec{q}_T\to 0} \boxed{\frac{d\sigma_{e^+e^-\to h_a h_b}}{dx_a dx_b d^2\vec{q}_T}},$$



Charge-Charge Correlation: Leading Power Safety

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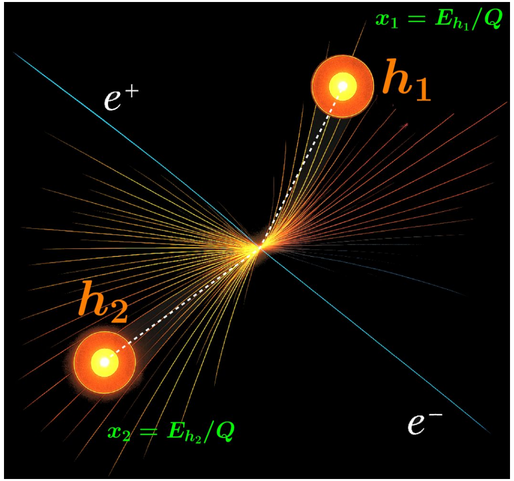

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Hard Function: $H_{q\bar{q}}(Q)$

TMD Fragmentation Functions: $D_{h_1/q}(x_1, b_T)$, $D_{h_2/\bar{q}}(x_2, b_T)$

TMD (qT) Soft Function: $S_q(b_T)$

If QQC is safe in this limit:

- Non perturbative functions will disappear
- Ingredients will not diverge

QQC - Leading Power Safety: Jet Function Magic

- This indeed happens, thanks to the following steps

Perturbative Matching Kernel

Longitudinal FF

⇛ OPE of TMD onto **longitudinal FF** $D_{h/q}(x, b_T) = \sum_i \int_x^1 \frac{\mathrm{d}t}{t} \mathcal{K}_{qi}(t, b_T)\, d_{h/i}\left(\frac{x}{t}\right)$

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$$\sum_{q'} Q_{q'} \int_0^1 \mathrm{d}x \left[\mathcal{K}_{qq'}(x, b_T) - \mathcal{K}_{q\bar{q}'}(x, b_T)\right]$$

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- This indeed happens, ... steps
  - ⇛ OPE of TMD onto ...

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QQC - Factorization in the back-to-back limit

- We derive the factorization theorem for the QQC in this limit

Hard Function

$$\frac{\mathrm{d}\sigma^{\mathrm{QQC}}}{\mathrm{d}z} = \frac{\hat{\sigma}_0}{8} \sum_{q,\bar{q}} H_{q\bar{q}}(Q,\mu) \int_0^\infty \mathrm{d}(b_T Q)^2 \, J_0\left(b_T Q\sqrt{1-z}\right) \times J_q^{\mathrm{QQC}}\left(b_T, \mu, \frac{\nu}{Q}\right) J_{\bar{q}}^{\mathrm{QQC}}\left(b_T, \mu, \frac{\nu}{Q}\right) S_q(b_T, \mu, \nu)$$

QQC Jet Functions

TMD (qT)
Soft Function

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known to 4 loops

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QQC Jet Functions

Novel Object!

$$J_q^{\mathrm{QQC}}(b_T) = \sum_{q'} Q_{q'} \int_0^1 dx \left[\mathcal{K}_{qq'}(x, b_T) - \mathcal{K}_{q\bar{q}'}(x, b_T)\right]$$

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We determined it to N3LO

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known to 3 loops

Calculated to 3 loops

QQC Jet Functions

TMD (qT) Soft Function

- Determined all ingredients for **N4LL resummation**!

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From unsafe to N4LL accurate!

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A new observable for charge dynamics in QFT!

From unsafe to N4LL accurate!

QQC - Analytic Checks

- Observable completely determined in pQCD. No need for track or frag. functions

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$$\frac{d\sigma^{QQC}}{dz} = \frac{\hat{\sigma}_0}{8} \sum_{q,\bar{q}} H_{q\bar{q}}(Q,\mu) \int_0^\infty d(b_T Q)^2 \, J_0\left(b_T Q\sqrt{1-z}\right) \times J_q^{QQC}\left(b_T, \mu, \frac{\nu}{Q}\right) J_{\bar{q}}^{QQC}\left(b_T, \mu, \frac{\nu}{Q}\right) S_q(b_T, \mu, \nu)$$

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Both of them verified to 3 loops ✅

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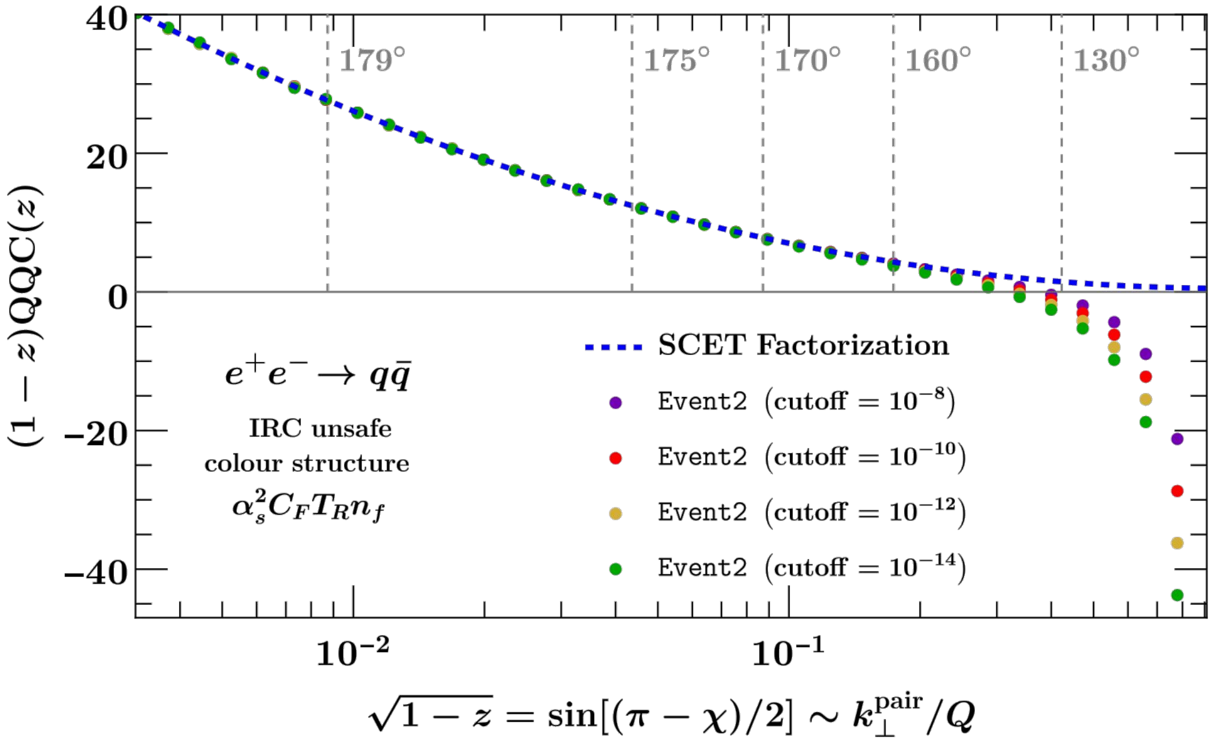
Both of them verified to 3 loops ✅

- In N=4 stress energy tensor and R-charge current are in same supermultiplet
  - ⇒ $\mathrm{QQC}_{\mathcal{N}=4}(z) = \mathrm{EEC}_{\mathcal{N}=4}(z)$ (up to trivial factors)
  - ⇒ In the back-to back limit we can obtain N=4 from QCD
  - ⇒ We took EEC at N4LL and QQC at N4LL and checked that they match in N=4

- Checked one loop expansion from full angle result of [Chicherin, Henn, Sokatchev, Yan '20]

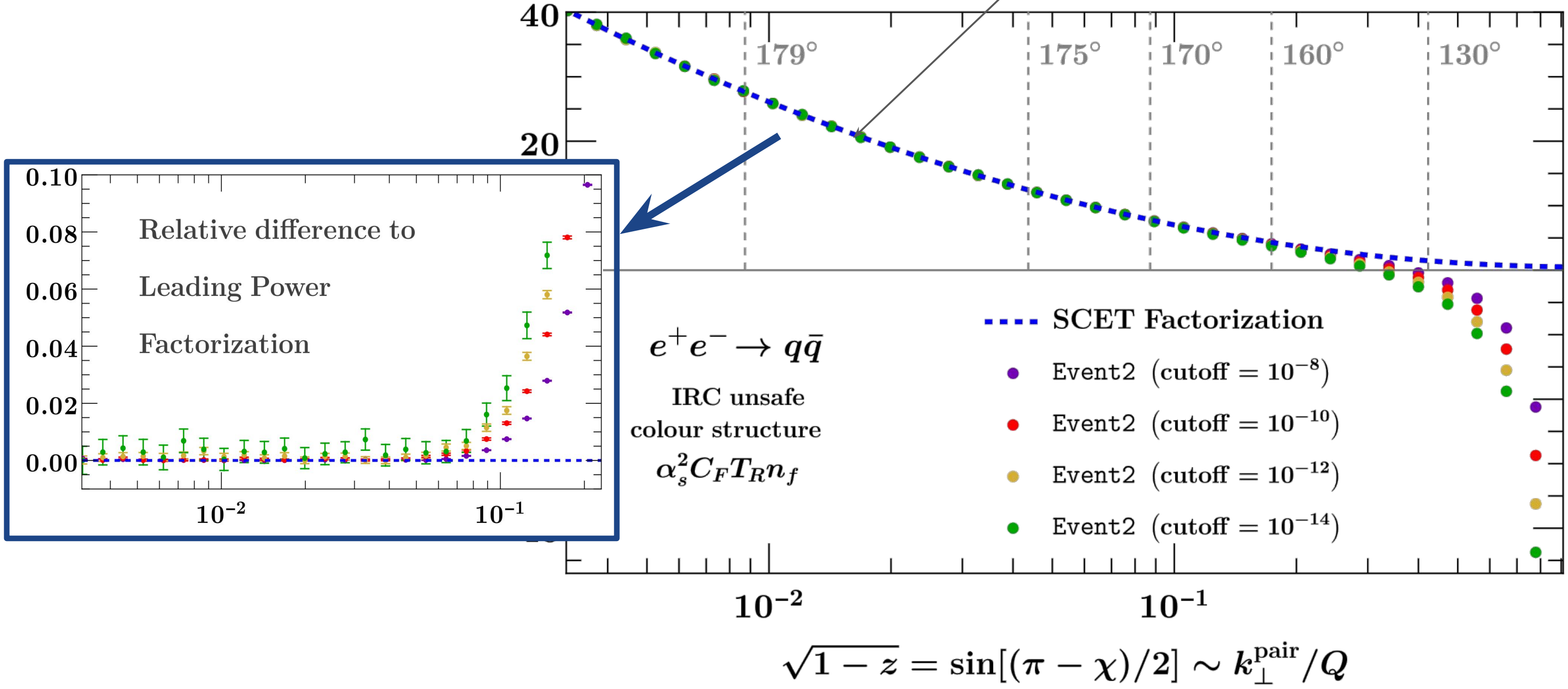
QQC - Numerical Check

- We took EVENT2 (parton level MC program, accurate to NNLO). Checked that the **numerical result against the perturbative expansion** of our factorization theorem



QQC - Numerical Check

Perfect agreement with SCET Factorization Theorem in the z→1 limit with no cut-off dependence



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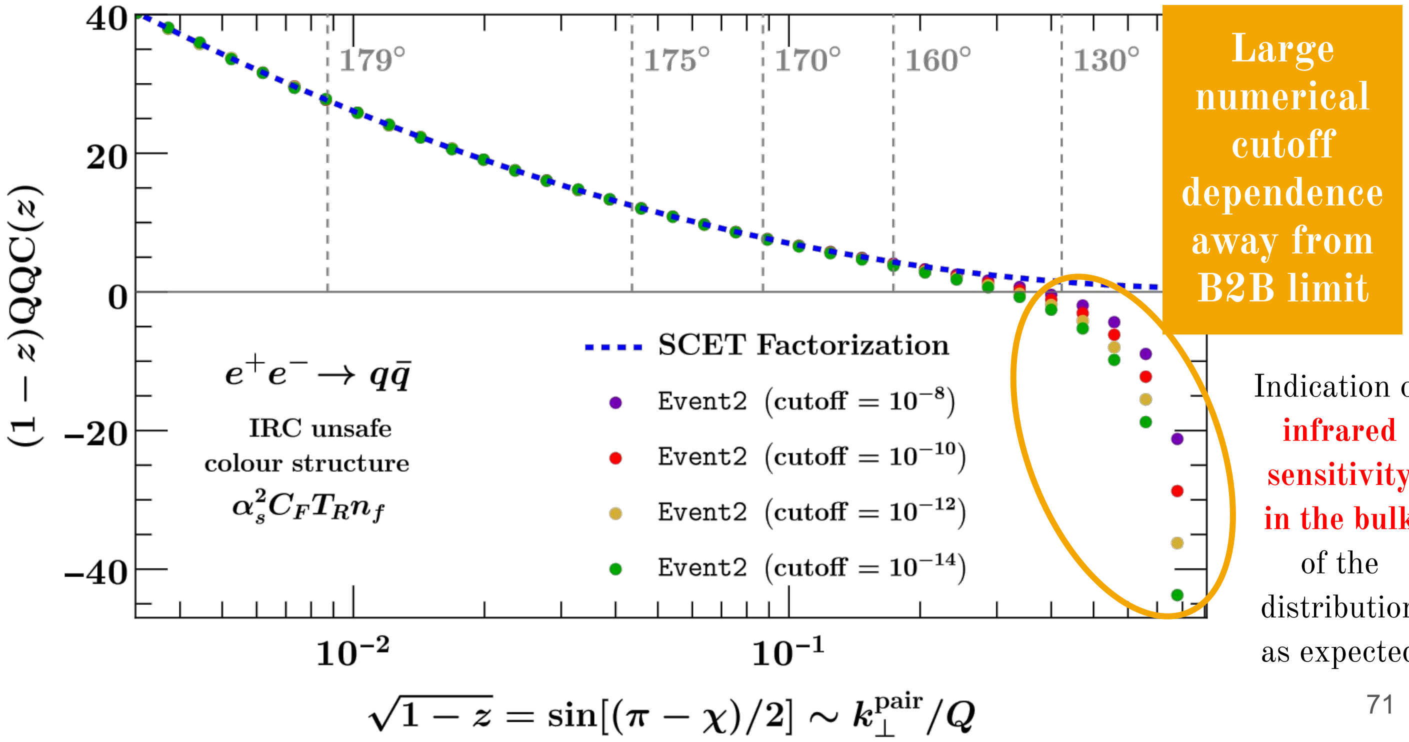
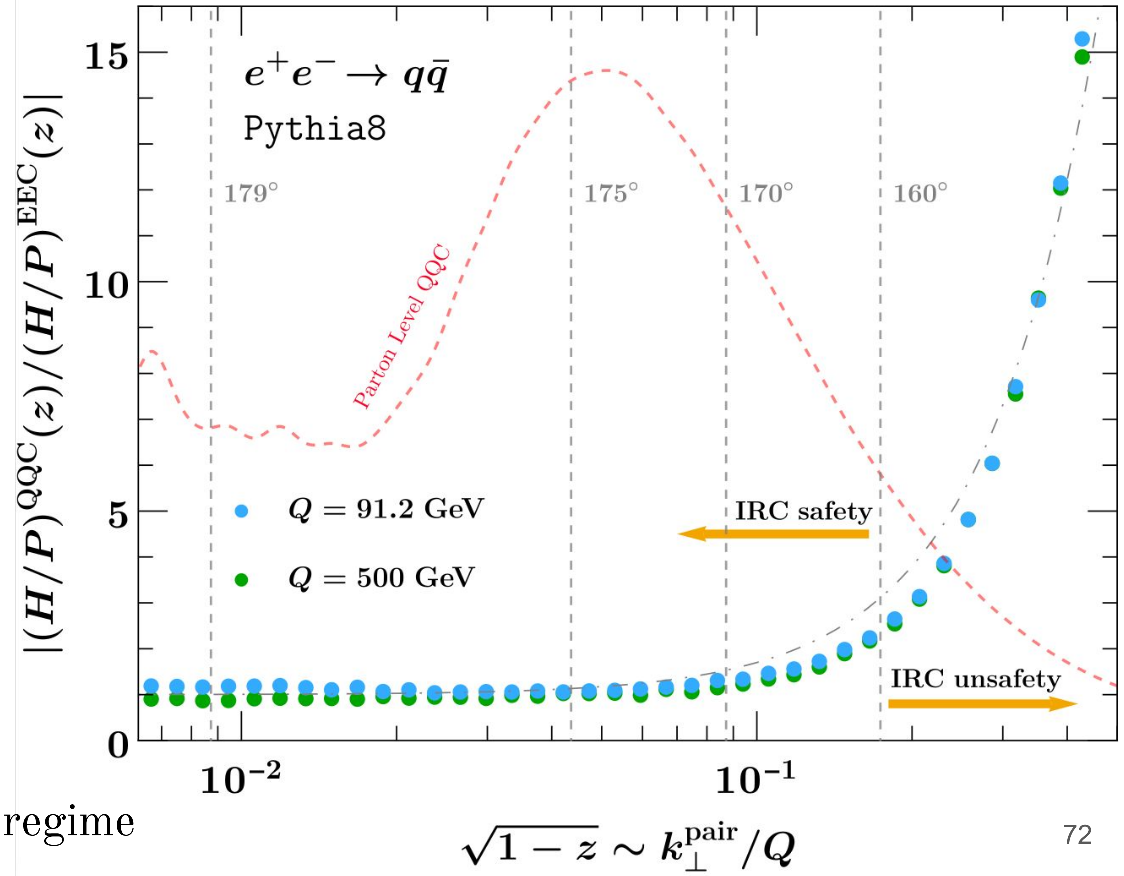


Indication of **infrared sensitivity in the bulk** of the distribution as expected

Hadronization Effects

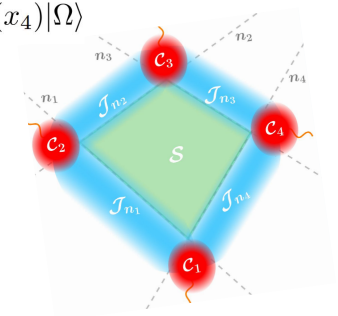
- Test with Parton Shower hadronization corrections of QQC w.r.t. EEC (reference for IRC safe observable with similar perturbative structure)
- **Additional confirmation** of suppression of non-perturbative effects in back-to-back region
- **IRC safe "region"** covers large fraction of back-to-back limit
- Potential for **precision probe** of this regime



Conclusions

➢ Opened a new window on **multipoint CFs** in QCD

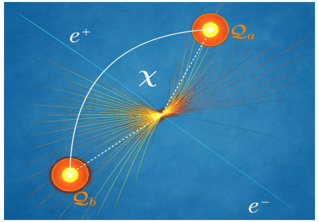
$$\langle \Omega | J^\mu(x_1) J^\nu(x_2) J^\rho(x_3) J^\sigma(x_4) | \Omega \rangle$$

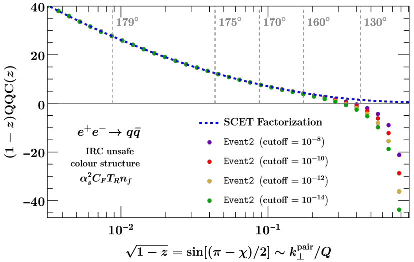


- ↪ Factorization in SLC limit of n-point CF of EM currents
- ↪ First demonstration of **CF/Wilson Loop duality** in QCD
- ↪ Established connection with resummation ingredients for collider via SCET
- ↪ Pushed to N3LL accuracy of 4 point CFs in QCD, giving first information beyond LO

➢ Explored **QQC observable** in the **back-to-back limit**

- ↪ Fact Thm in terms of finite pert objects (LP safety)
- ↪ Determined novel QQC jet functions to 3 loops and **resummation for QQC to N4LL**
- ↪ Started pheno study with MC for assessing impact of hadronization corrections away from b2b limit





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