Generalized Detector Correlators

Based on 2601.xxxxx

In Collaboration with Mark Gonzalez, Ian Moult

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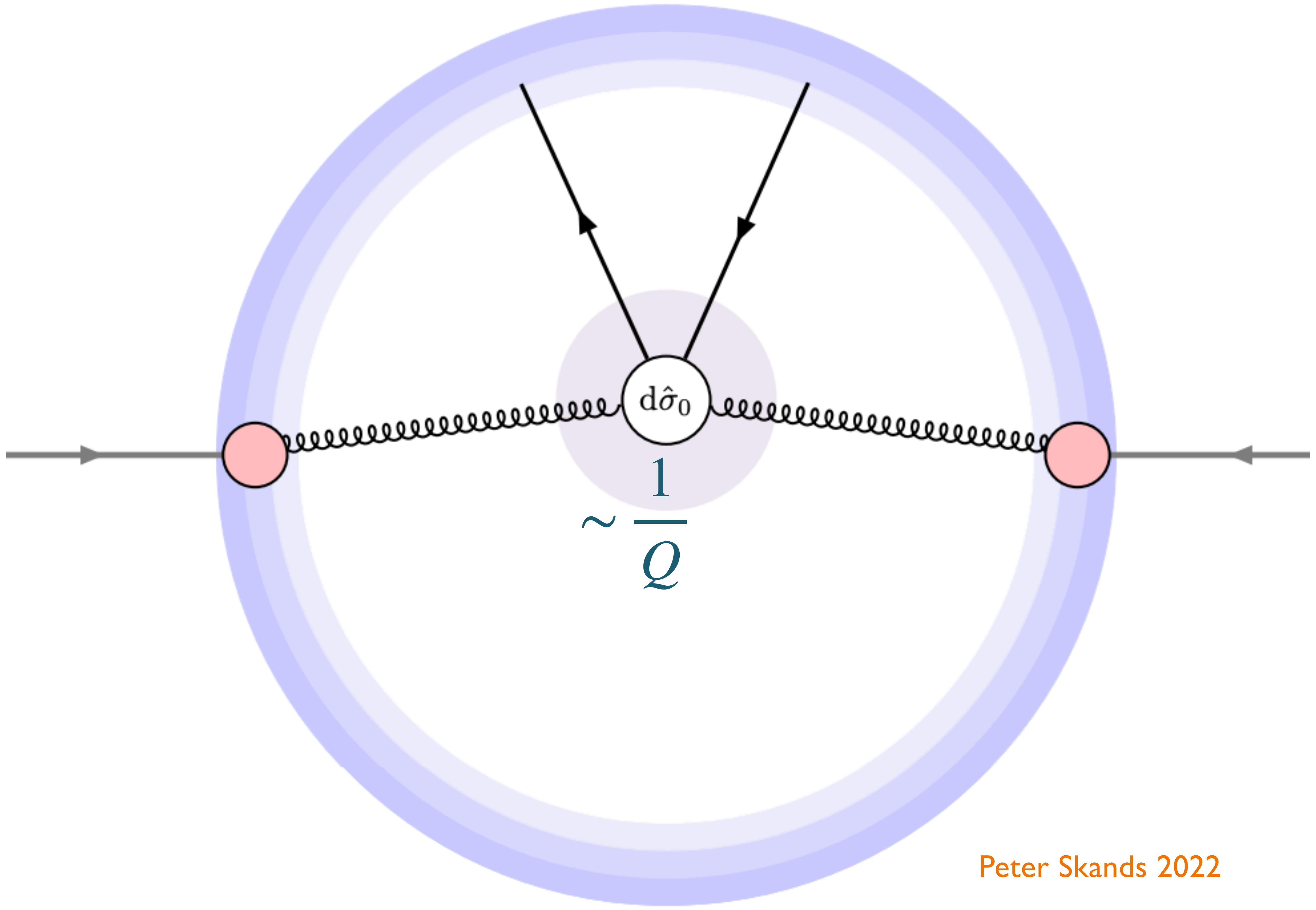
Yale University → Argonne

Yale

New Frontiers of Quantum Field and Gravity

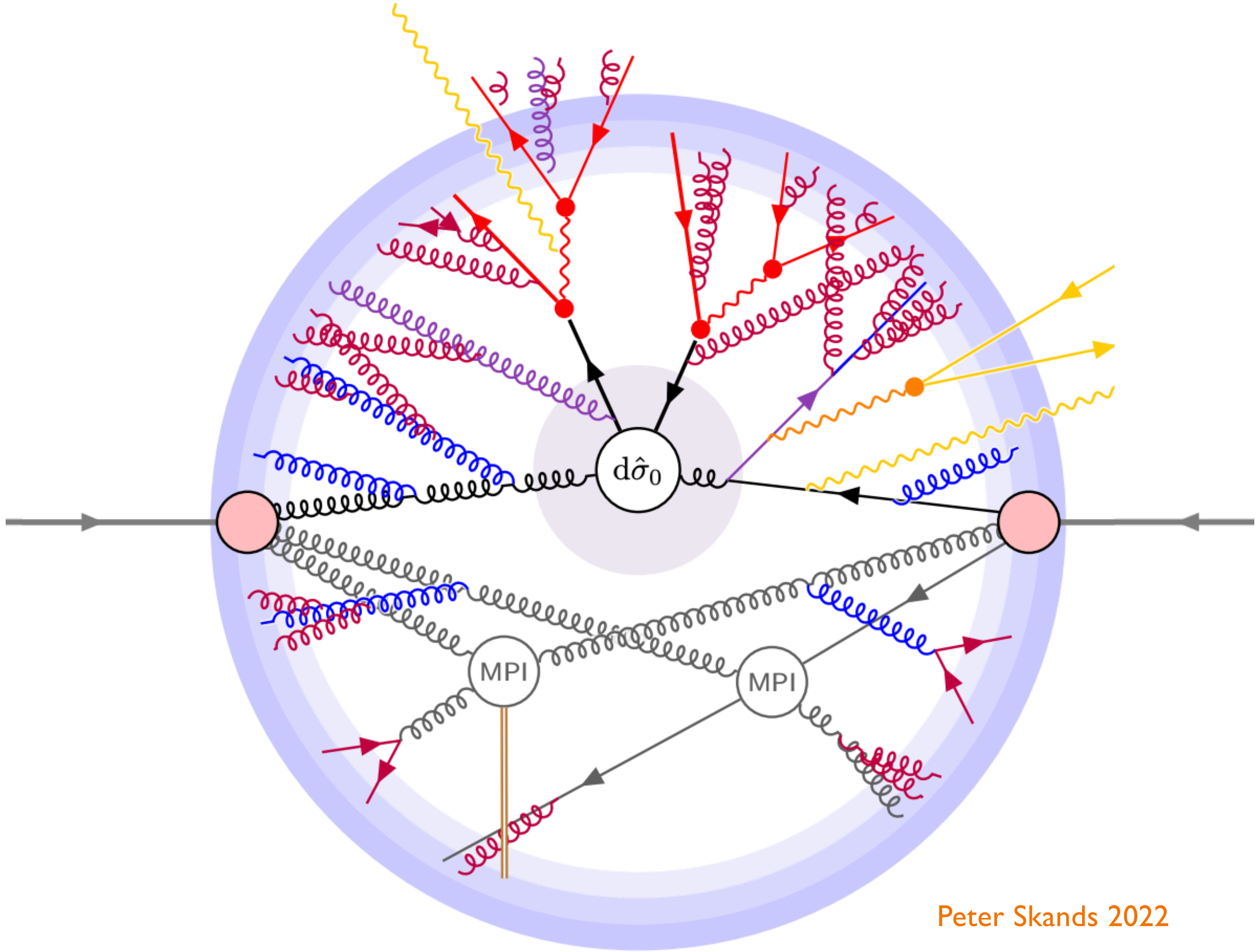
Jan 13th, 2026

HIGH-ENERGY SCATTERING AND HADRONIZATION



Hard scattering at distance $\sim \frac{1}{Q}$

HIGH-ENERGY SCATTERING AND HADRONIZATION

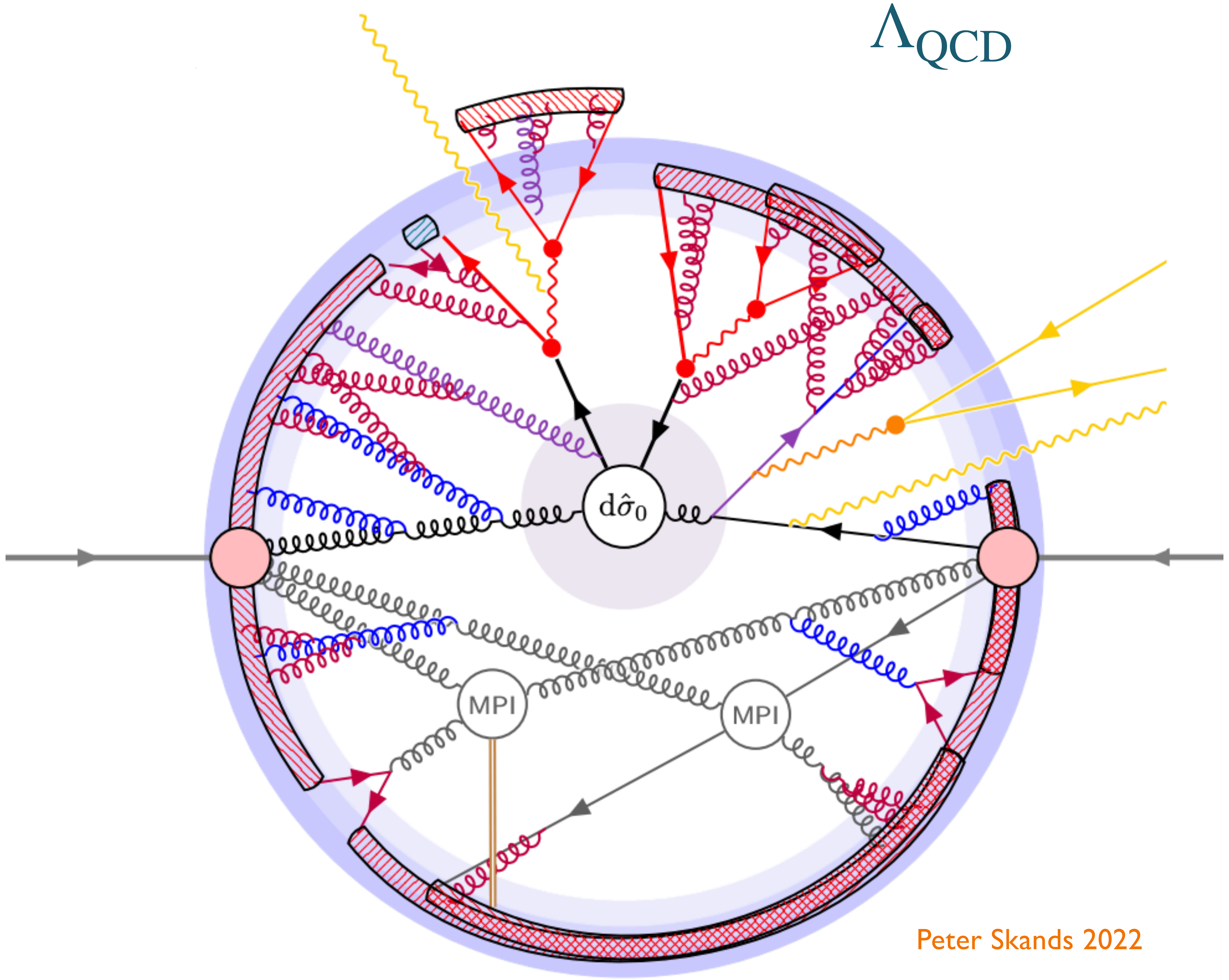


Peter Skands 2022

Radiative corrections and collinear / soft parton splittings

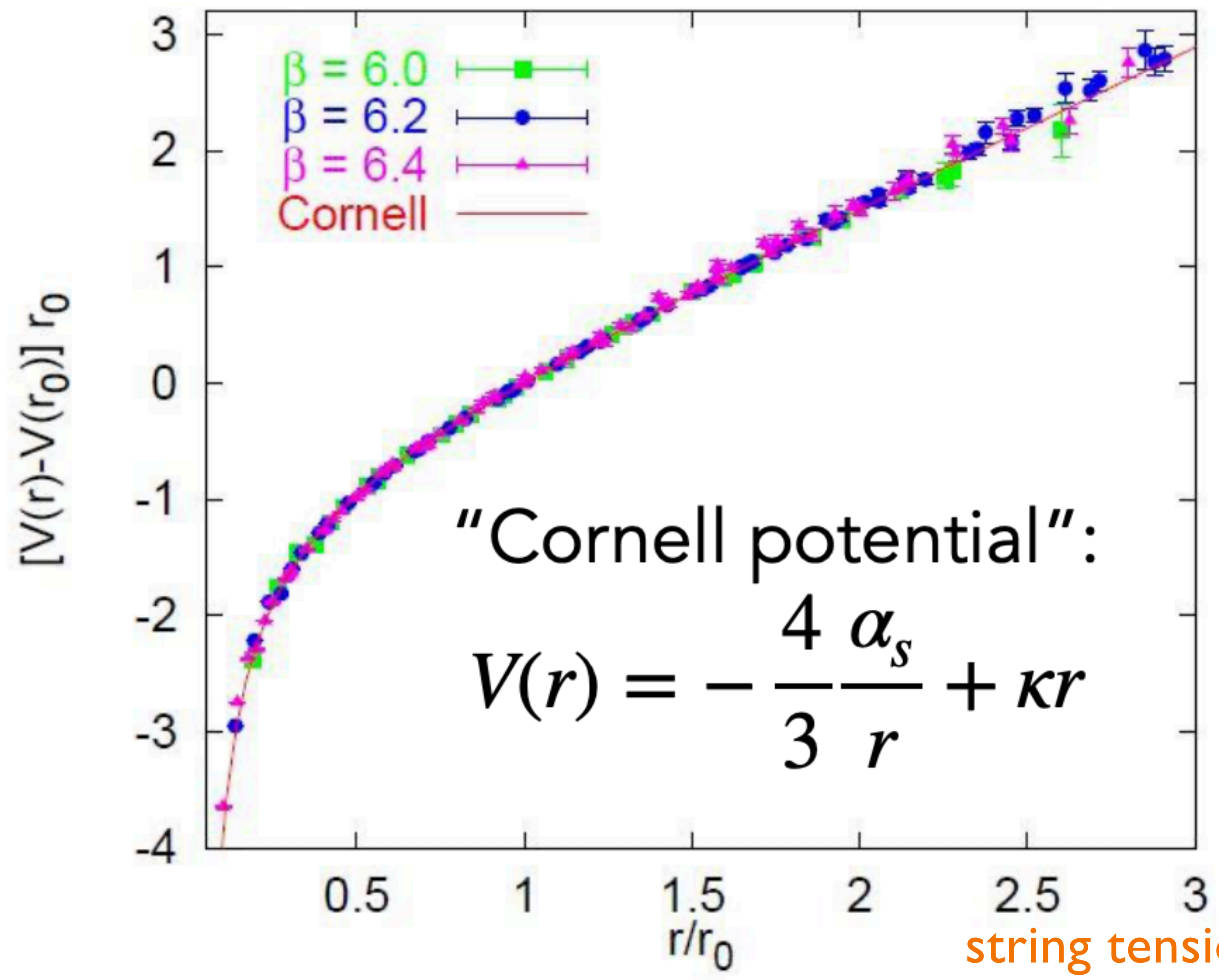
HIGH-ENERGY SCATTERING AND HADRONIZATION

Confinement at distance $\sim \frac{1}{\Lambda_{\text{QCD}}}$



Peter Skands 2022

Lund string model with
linear potential between "color partners"



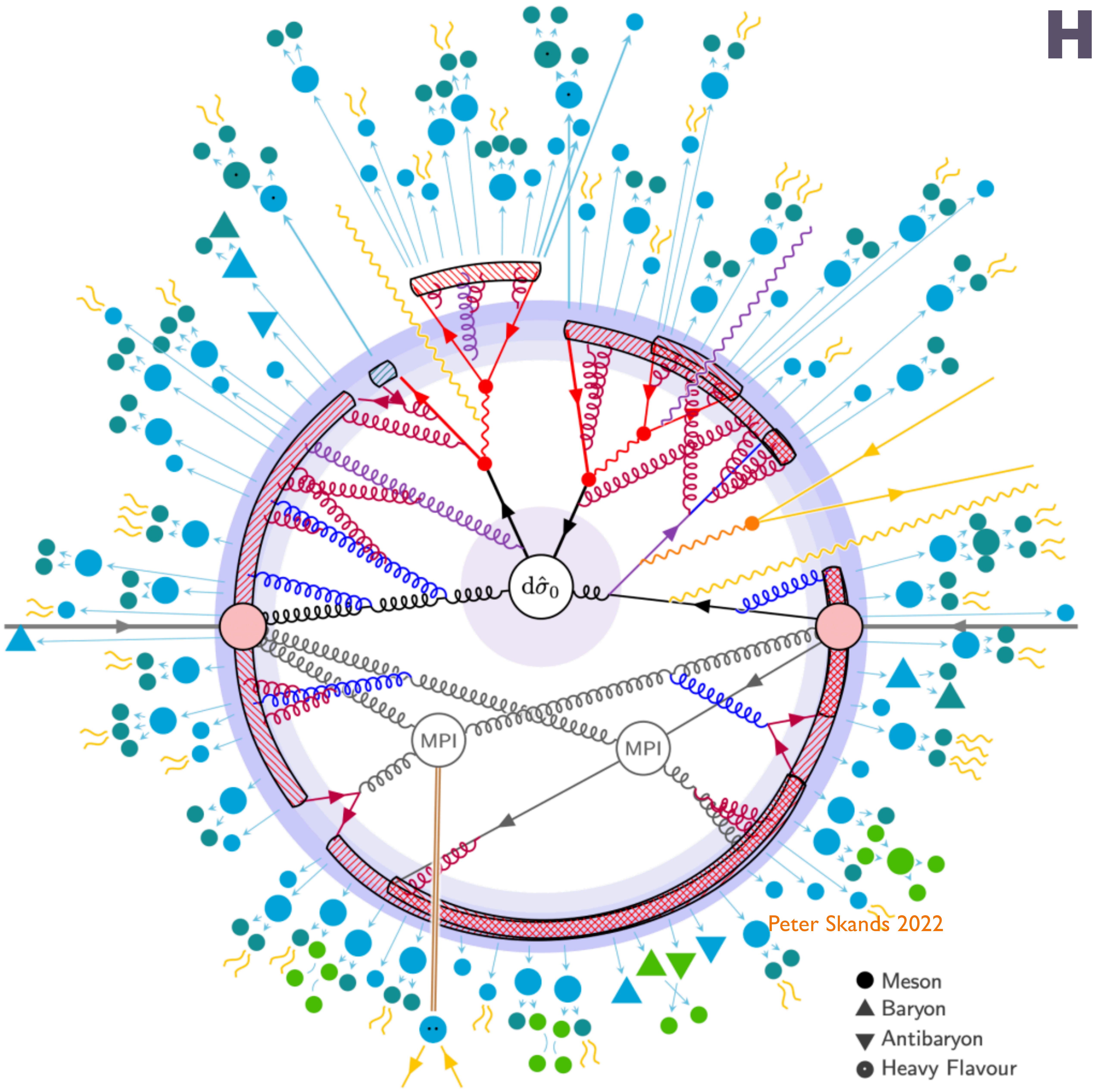
"Cornell potential":

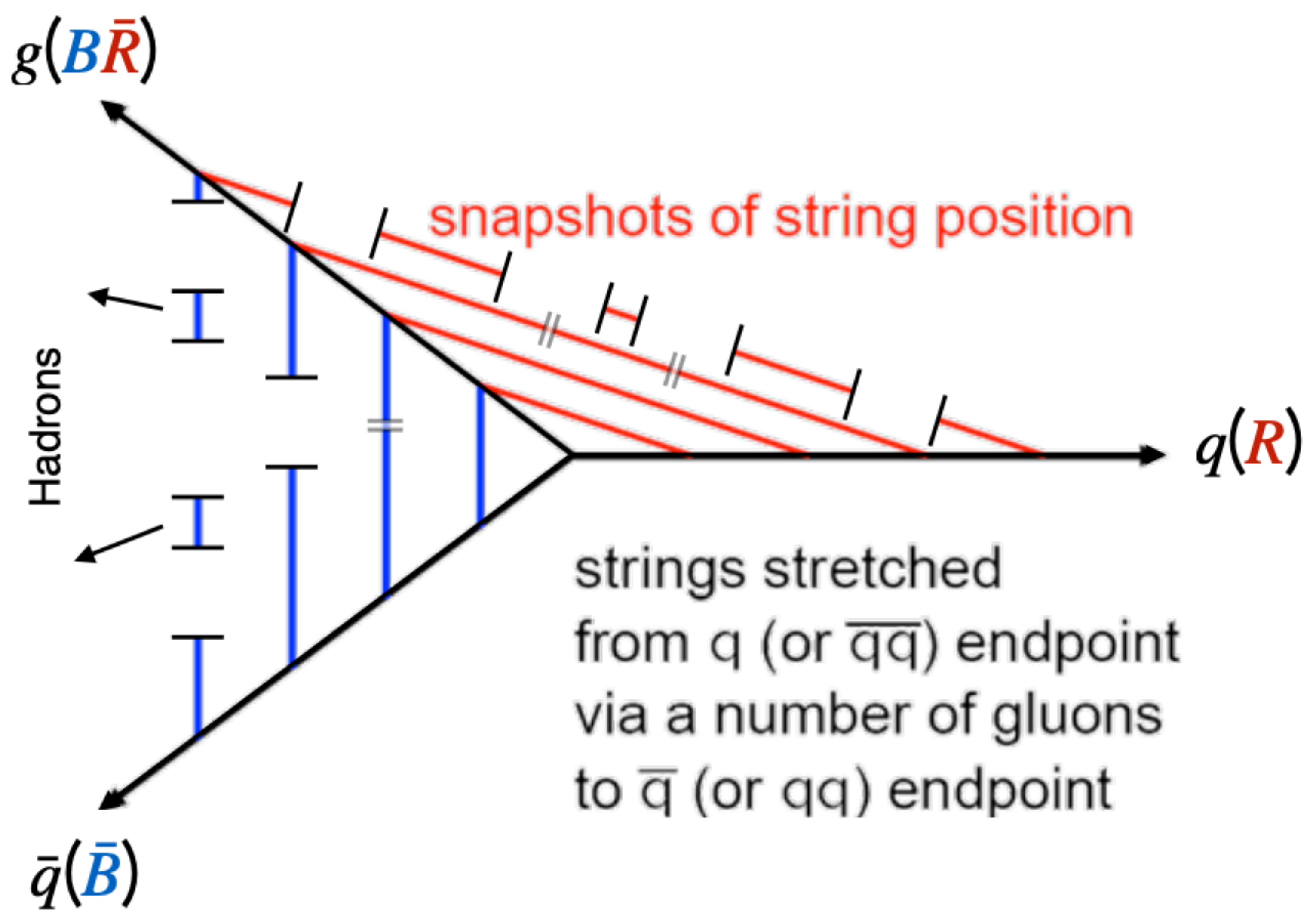
$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \kappa r$$

$r_0 = 0.5\text{fm}$

string tension $\kappa \sim 0.9\,\text{GeV/fm}$

HIGH-ENERGY SCATTERING AND HADRONIZATION

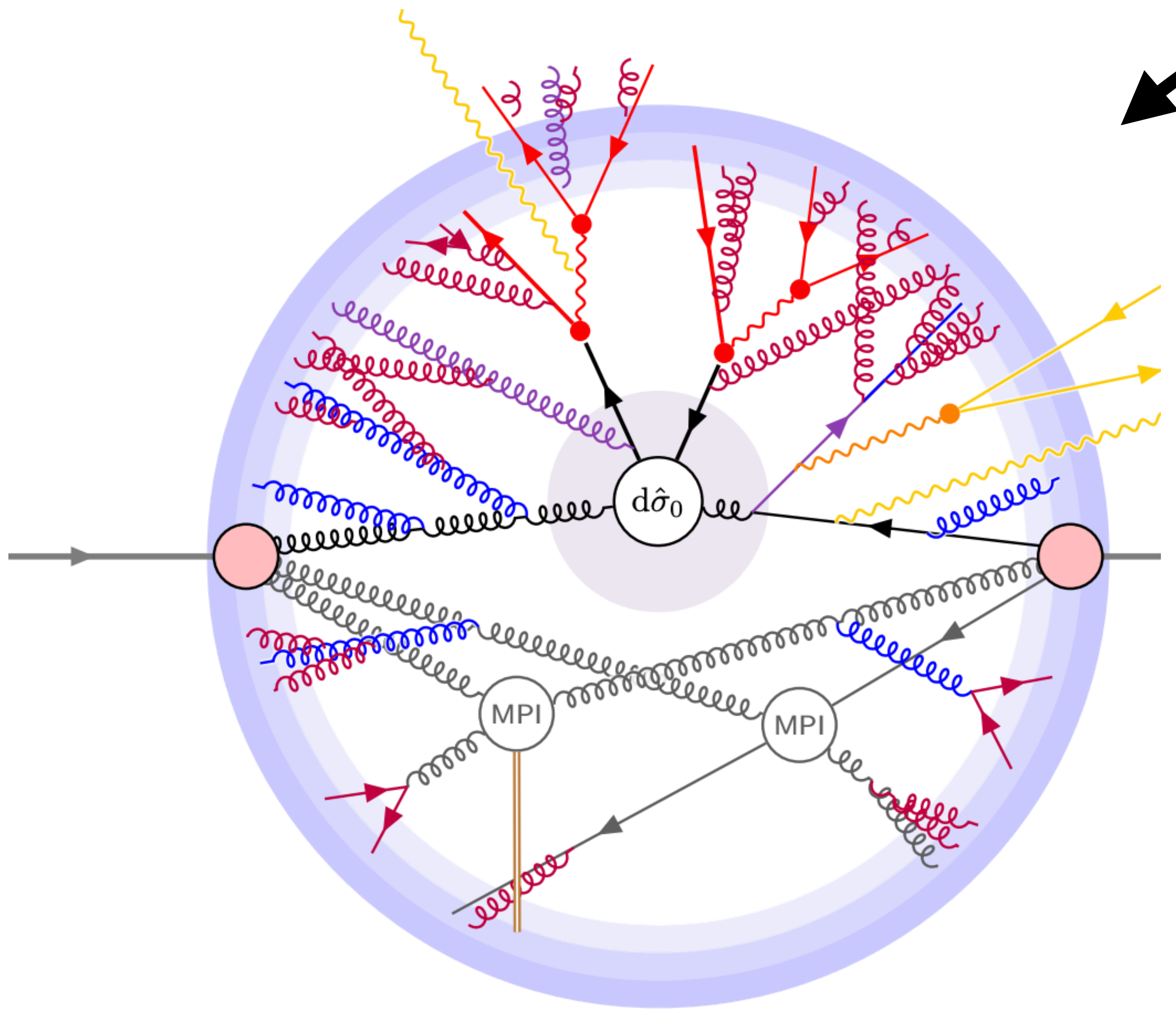




String breaking to hadrons dynamics

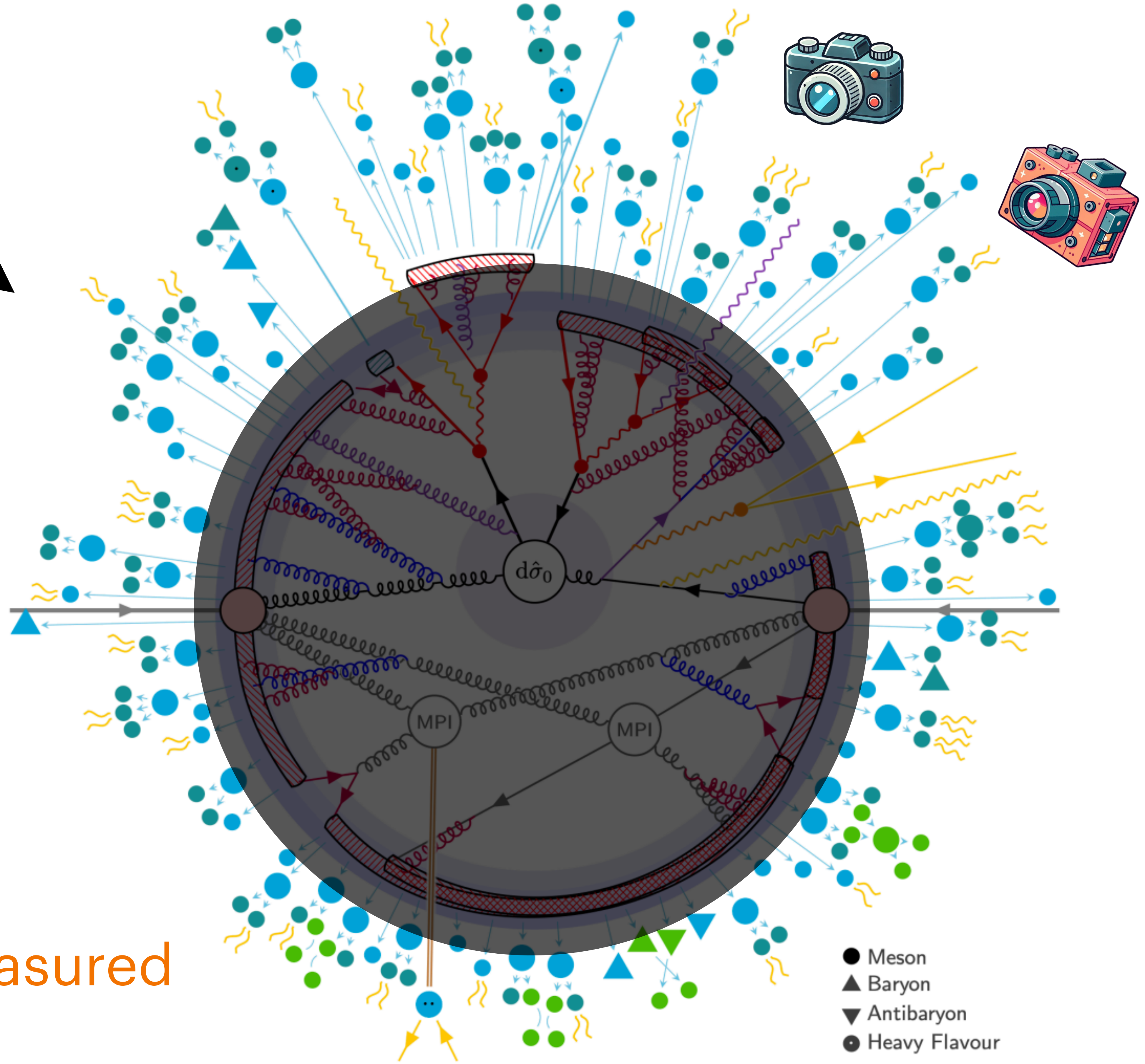
HIGH-ENERGY SCATTERING AND HADRONIZATION

Hadronization



can be calculated

Quarks and Gluons

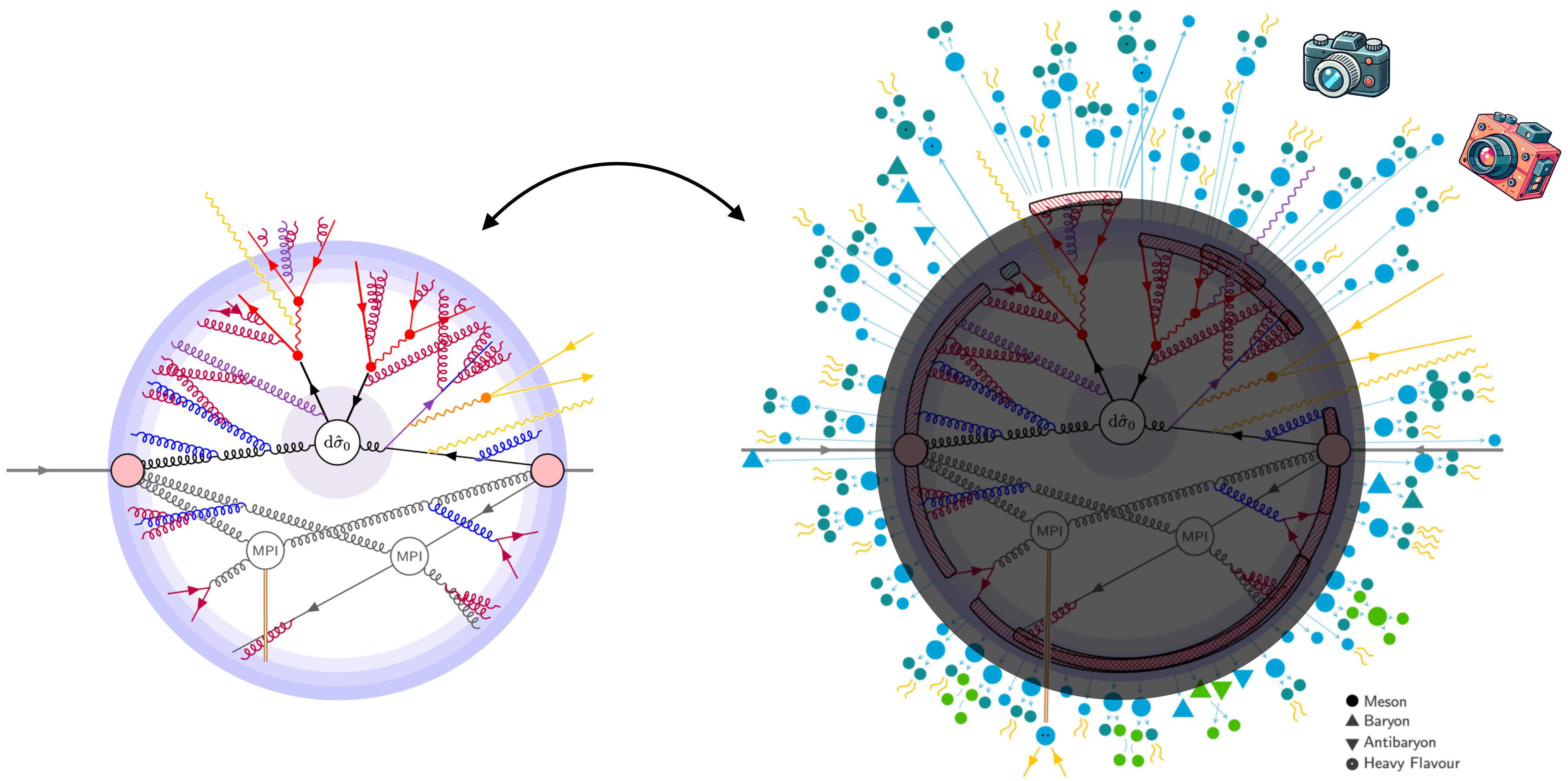


can be measured

Theory of Pions

$$\langle \Psi | \mathcal{D}_{\mathrm{IR}}(\vec{n}_1) \mathcal{D}_{\mathrm{IR}}(\vec{n}_2) \cdots \mathcal{D}_{\mathrm{IR}}(\vec{n}_N) | \Psi \rangle$$

HIGH-ENERGY SCATTERING AND HADRONIZATION



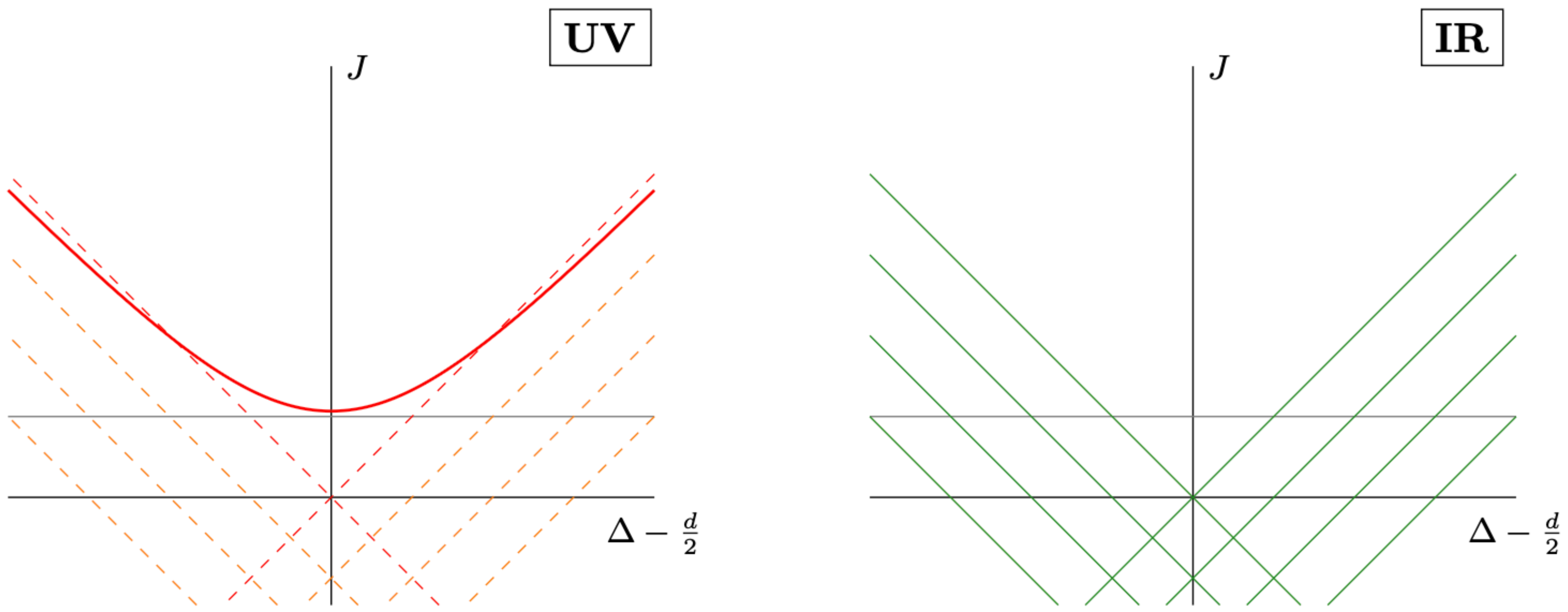
- What are interesting IR detector correlators?

$$\langle\Psi|\mathcal{D}_{\rm IR}(\vec{n}_1)\mathcal{D}_{\rm IR}(\vec{n}_2)\cdots\mathcal{D}_{\rm IR}(\vec{n}_N)|\Psi\rangle$$

Generalized Detector Correlators

- What is the matching procedure between the space of UV and IR detectors?

See also the two beautiful talks by Cyuan-Han and Hao yesterday



- What is the Light-Ray OPE structure of general correlators?

- What confinement dynamics can generalized correlators teach us about?

Outline.

- Generalized detectors

- Detector Functions : mapping between IR and UV detectors

- Operator Product Expansion (OPE) of generalized detectors

- Nonperturbative Power Corrections

- Applications

Outline.

- What are interesting IR detector correlators?
  - Generalized detectors
- What is the matching procedure between the space of UV and IR detectors?
  - Detector Functions : mapping between IR and UV detectors
- What is the Light-Ray OPE structure of general correlators?
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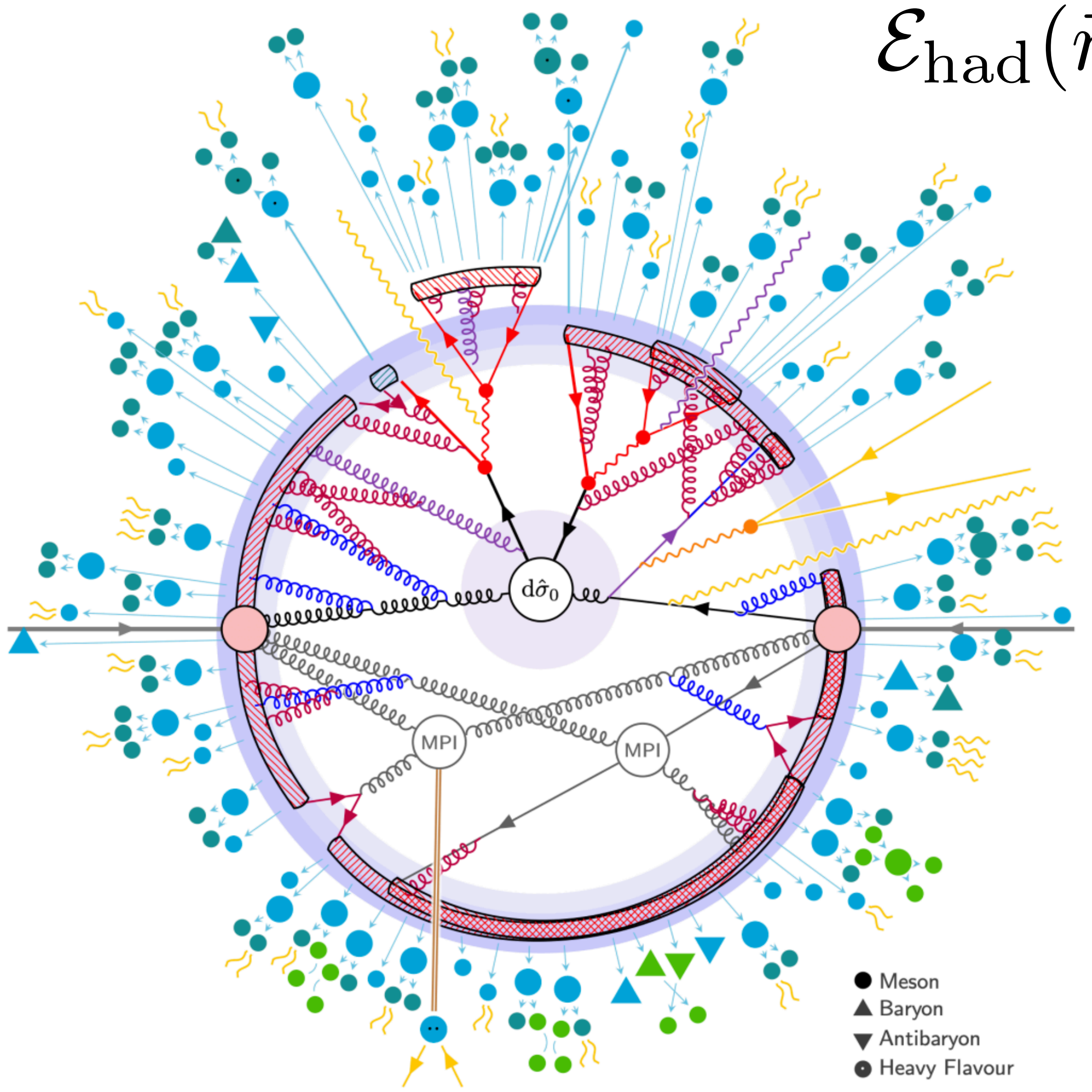
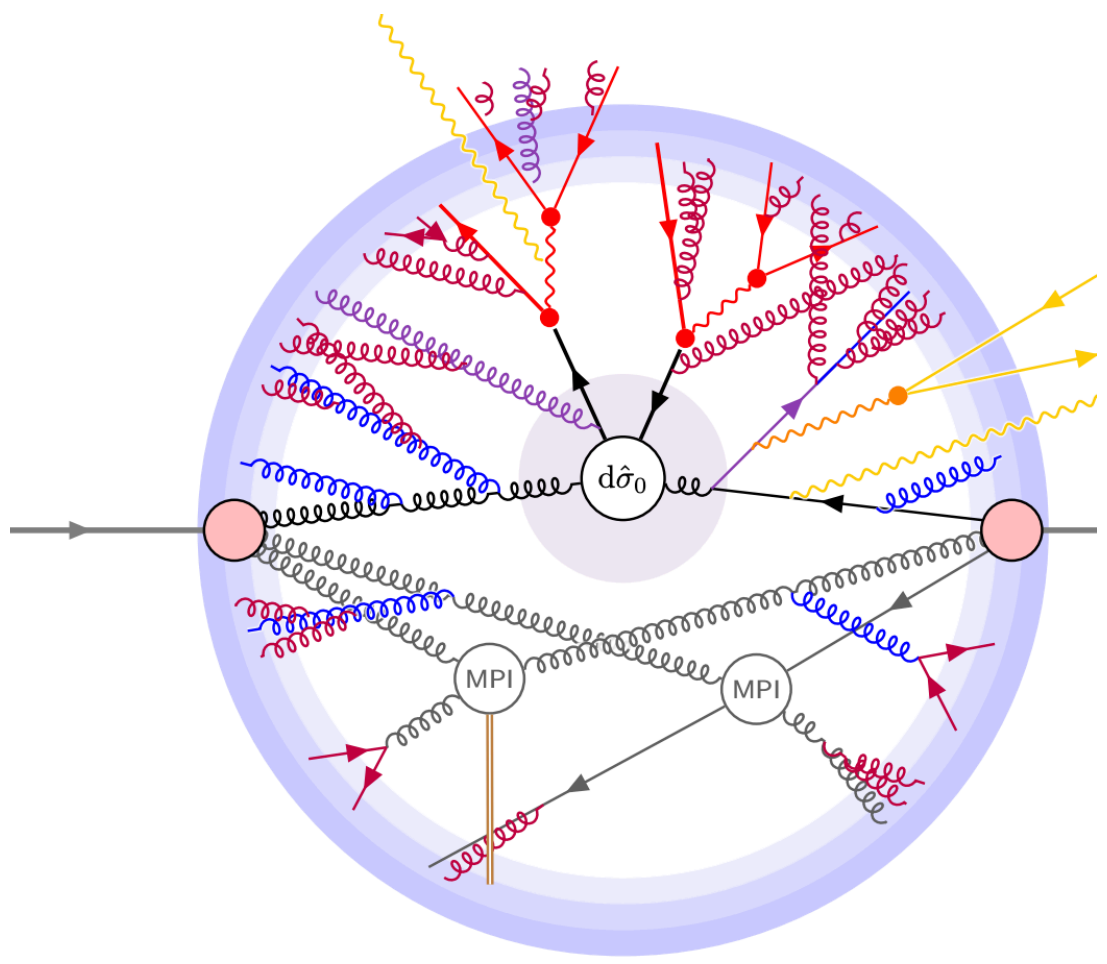
Outline.

- Generalized detectors

ANEC / ENERGY FLOW OPERATOR

$$\mathcal{E}_{\rm had}(\vec{n}) = \mathcal{E}_{\rm parton}(\vec{n})$$





Energy Flow Operators

$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\hat{n})$$

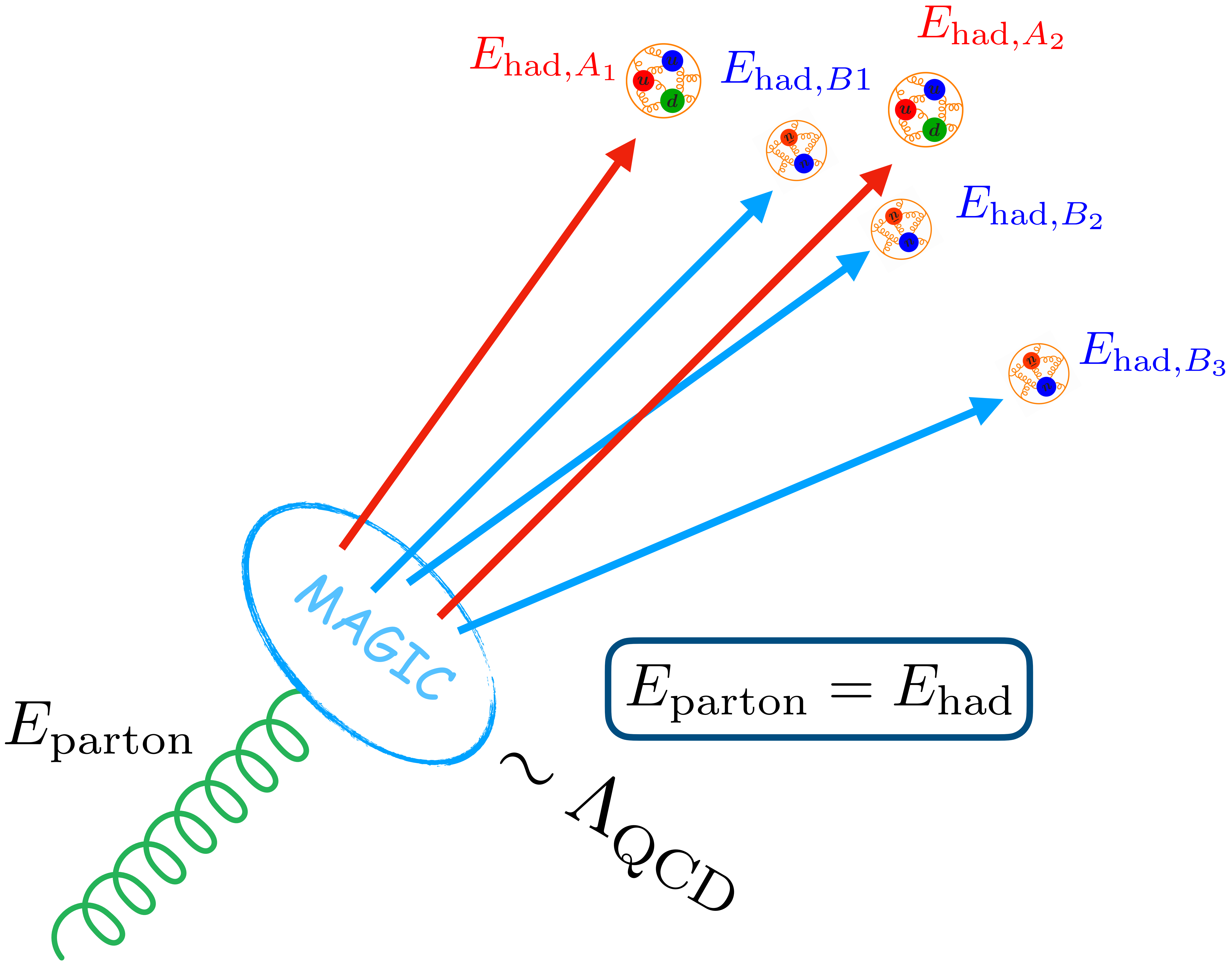
$$\mathcal{E}(\hat{n})|X\rangle = \sum_a E_a \delta^{(2)}\left(\Omega_{\vec{p}_a} - \Omega_{\hat{n}}\right)|X\rangle$$

Sterman `75
Basham, Brown, Ellis, Love, `78-79
Sveshnikov, Tkachov, `95
Korchemsky, Sterman, `01
Bauer, Fleming, Lee, Sterman `08

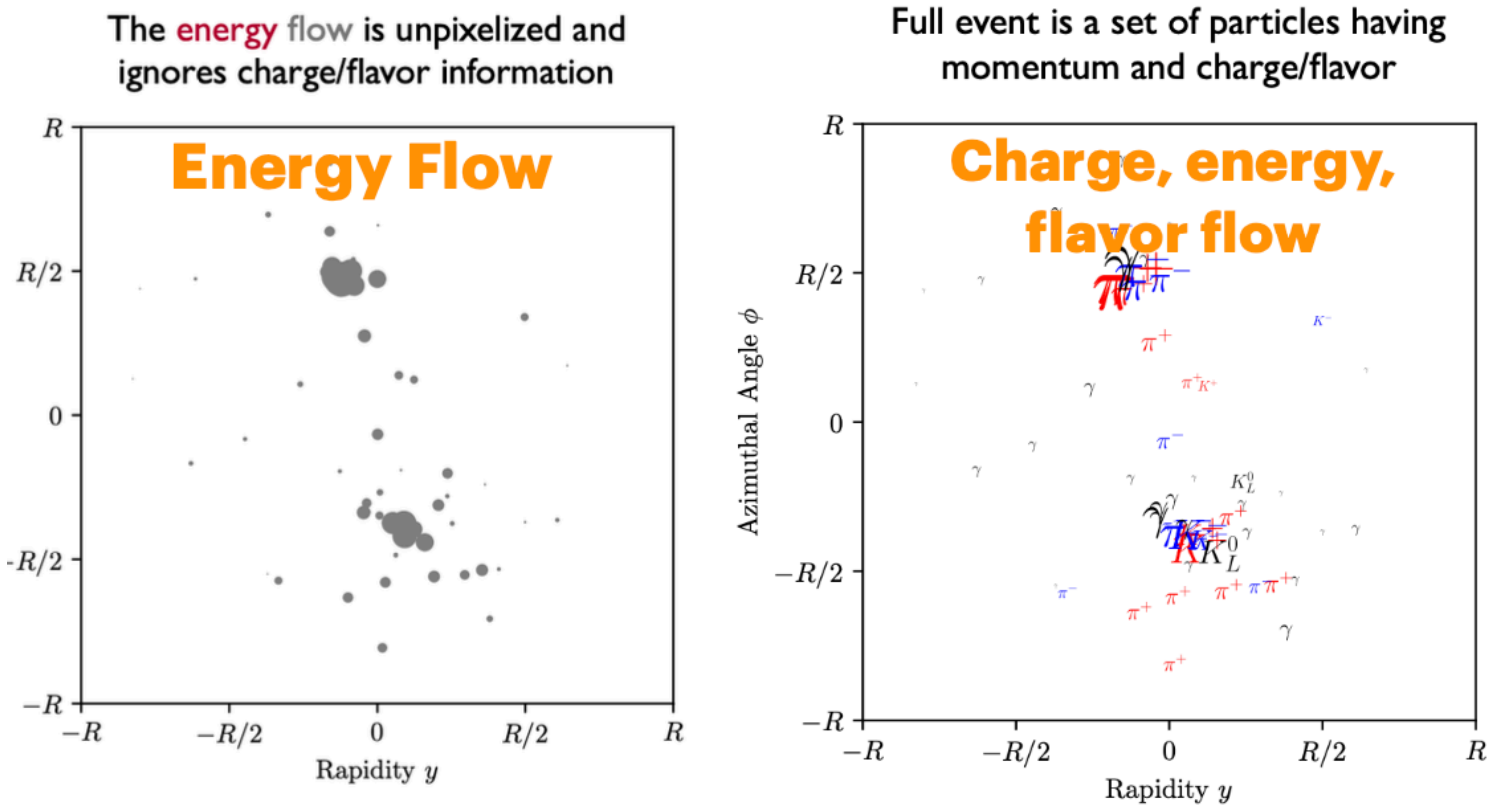
- Many formal and phenomenological studies of ANEC exists
- Energy flow operator forms a very natural object to organize IRC safe asymptotic observables

Chen, Moult, Zhang, Zhu `20

CONFINEMENT MAGIC AND DETECTORS



$$E_{\mathrm{parton}} = E_{\mathrm{had}}$$



$$E_{\mathrm{parton}} = E_{\mathrm{had}} \neq E_{\mathrm{had},A}\,,\; E_{\mathrm{had},B}$$

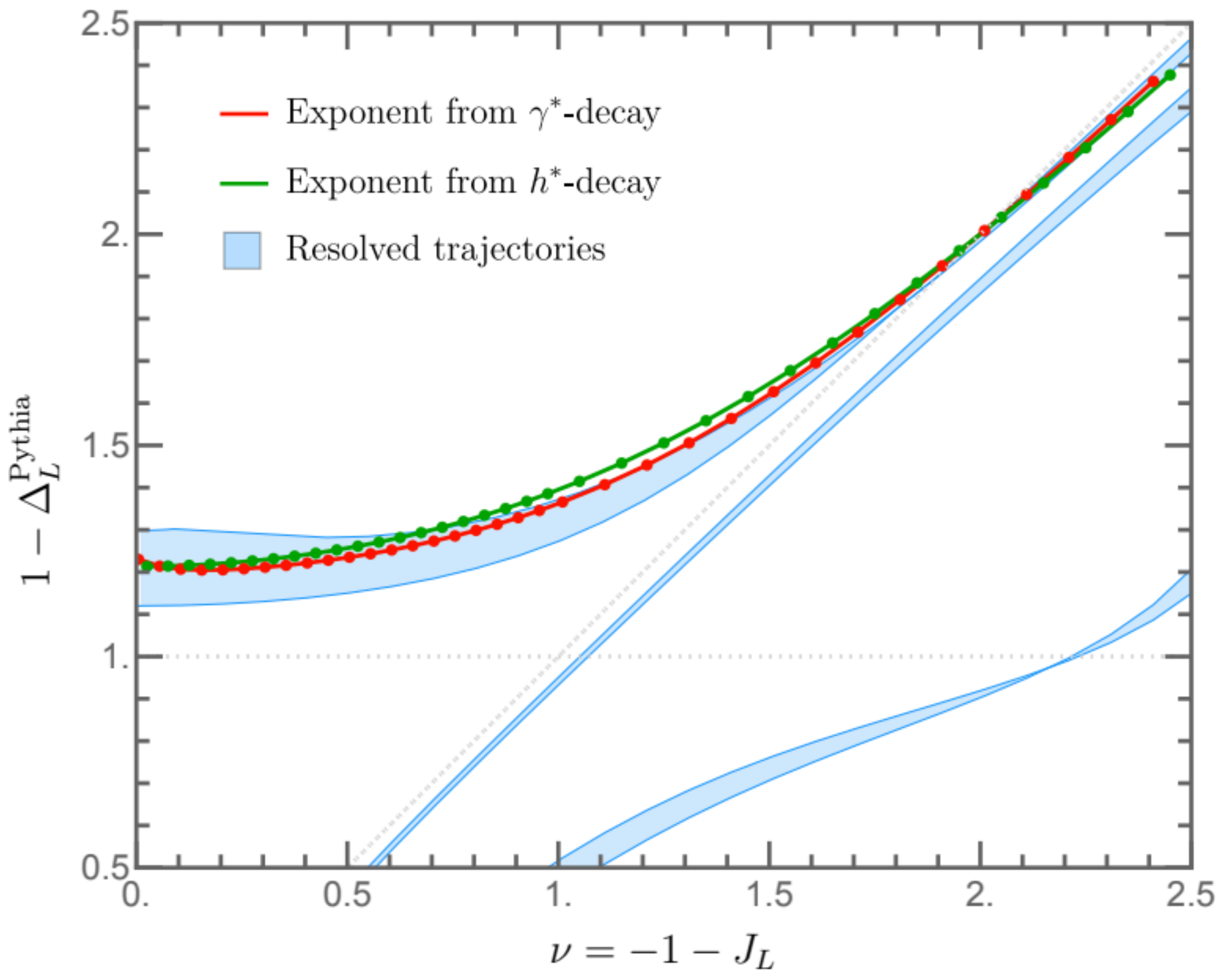
$$E^n_{\mathrm{parton}} \neq \sum_i E^n_{\mathrm{had,i}}, \quad n \neq 1$$

- QCD is gapped and there are many more general detectors which can probe the confinement magic!

SEEING THROUGH THE CONFINEMENT SCREEN

From Cyuan-Han and Hao's talk yesterday

Chang, Chen, Simmons-Duffin, Zhu `25



- Study of 1-point correlation function of general detector of the form

$$\langle \Psi | \mathbb{O}^R_{J_L}(\vec{n}) | \Psi \rangle$$

Cyuan-Han and Hao called it $\mathbb{N}_{J_L}(\vec{n})$

$$\mathbb{O}^R_{J_L}(\vec{n})|X\rangle = \sum_{i \in R \subset X} E_i^{(2-d-J_L)} |X\rangle$$

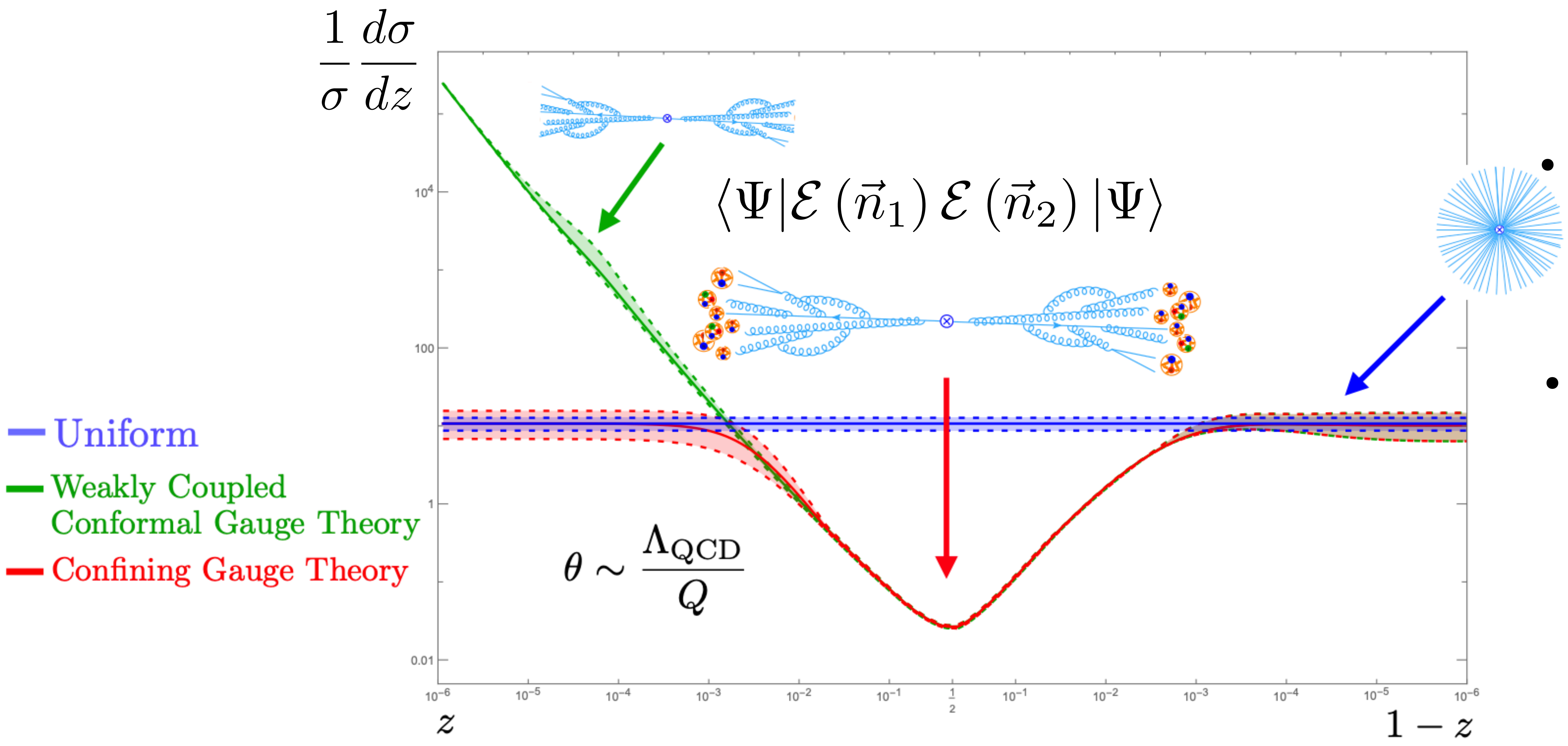
- Lorentz Spin J_L encodes Lorentz spin of the IR detector

- We want to study

$$\langle \Psi | \mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1) \cdots \mathbb{O}^{R_N}_{J_{L_N}}(\vec{n}_N) | \Psi \rangle$$

$\Longrightarrow$ This introduces angular scale, $\vec{n}_i \cdot \vec{n}_j = \dfrac{1 - \cos\theta_{ij}}{2}$

CORRELATORS OF ANEC / ENERGY FLOW OPERATOR



$$z = \frac{1-\cos\theta}{2}$$

Electron-Positron Alliance Collab `25

$$\mathcal{E}(\vec{n}) = \mathbb{O}_{-3}(\vec{n})$$

- Correlation introduces angular scale: $Q\sqrt{z}$
- Small angle regions:

confinement: $\Lambda_{\rm QCD} \sim Q\sqrt{z} \ll Q$

KL, Stewart `25
Chang, Chen, Liu, Simmons-Duffin, Yuan, Zhu `25

pre-confinement: $\Lambda_{\rm QCD} \ll Q\sqrt{z} \ll Q$

- Energy flow operator forms a very natural object to organize IRC safe asymptotic observables

Chen, Moult, Zhang, Zhu `20

- Leading contribution of energy correlators "insensitive" to IR dynamics in the pre-confinement region (Will discuss subleading contribution sensitive to $\Lambda_{\rm QCD}$ later)

CORRELATORS OF ANEC / ENERGY FLOW OPERATOR

- We will study multi-point general detector correlators of the form, with largest angle x_L projected

$$\langle\Psi|\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\cdots\mathbb{O}^{R_N}_{J_{L_N}}(\vec{n}_N)|\Psi\rangle_{x_L} \equiv \int \prod d\Omega_i \langle\Psi|\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\cdots\mathbb{O}^{R_N}_{J_{L_N}}(\vec{n}_N)|\Psi\rangle\, \delta(x_L - \max[z_{ij}])$$

in the pre-confinement region $\Lambda_{\mathrm{QCD}} \ll Q\sqrt{x_L} \ll Q$

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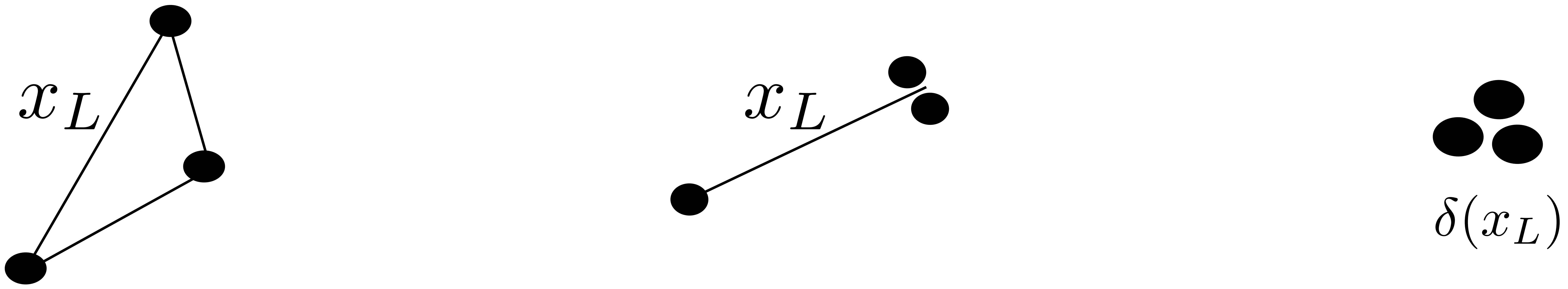
in the pre-confinement region $\Lambda_{\mathrm{QCD}} \ll Q\sqrt{x_L} \ll Q$

x_L measurement for N=3 detectors :



We can get contributions from various detector configurations.

CORRELATORS OF ANEC / ENERGY FLOW OPERATOR

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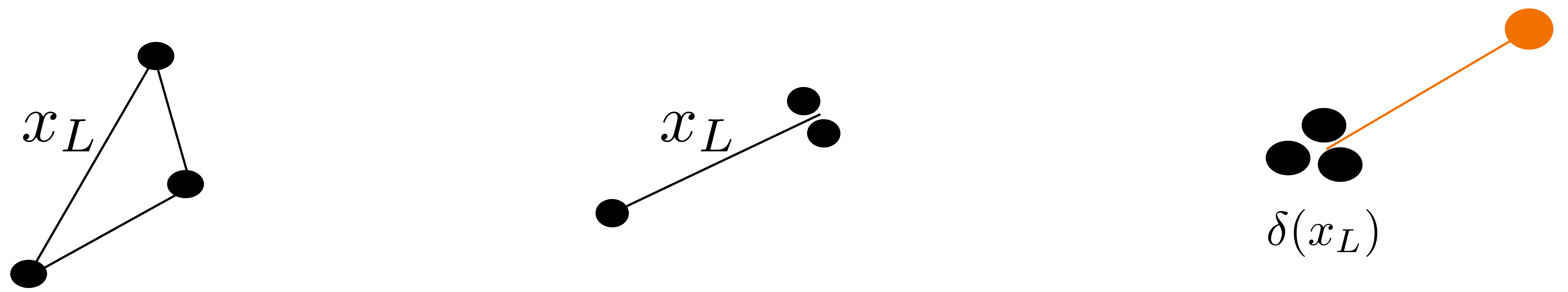
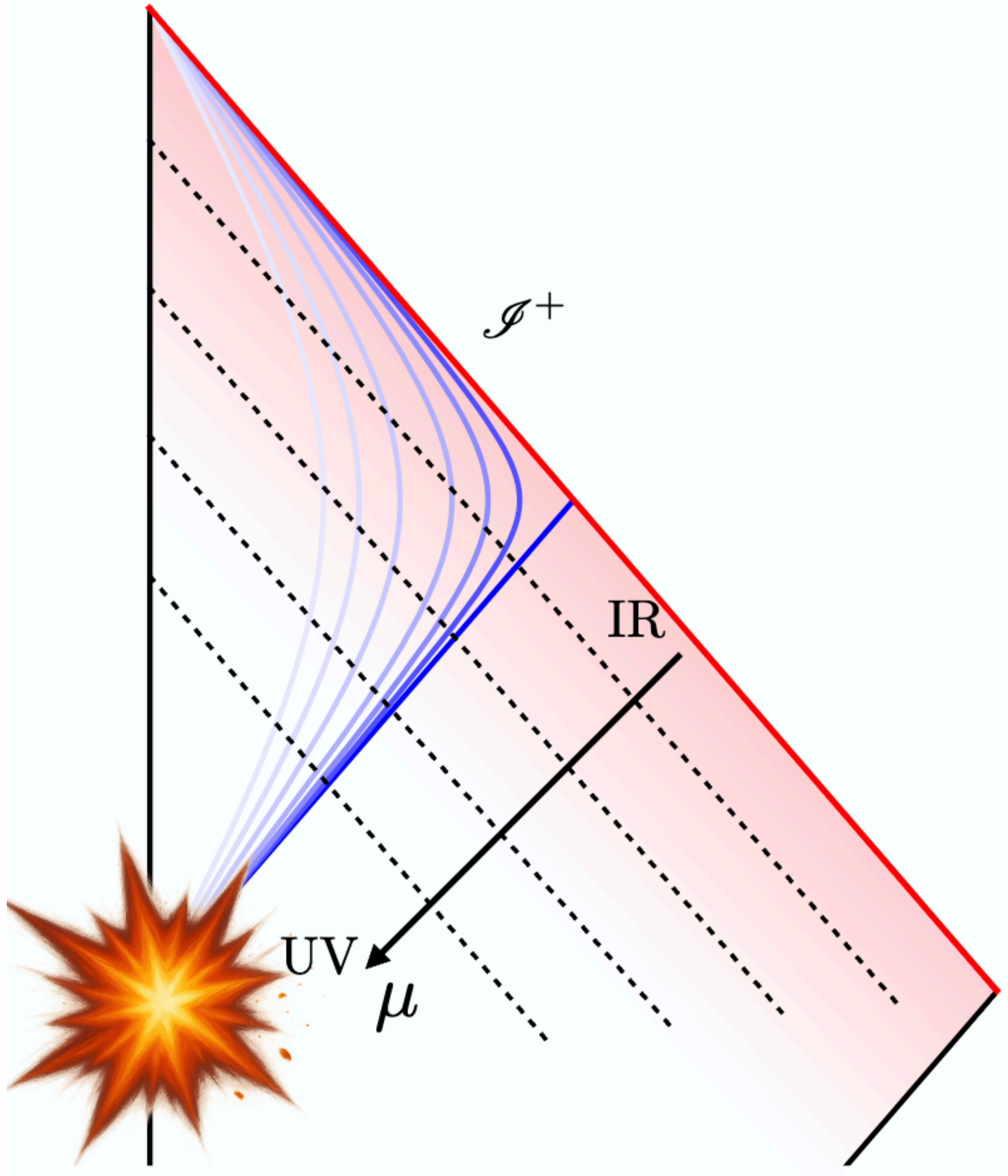


We can get contributions from various detector configurations.
Delta function becomes important when we include additional detector.

Outline.

- **Detector Functions** : mapping between IR and UV detectors

DETECTOR FUNCTIONS



- Detector functions are Wilson coefficients between IR and UV detectors

IR detector | Detector (matching) function | UV detector

$$\mathbb{O}^R_{J_L}(\vec{n}) \simeq \sum_k C^R_{J_L,k}(\mu)\,\mathbb{D}_{J_L,k}(\vec{n},\mu)$$

IR theories of pions

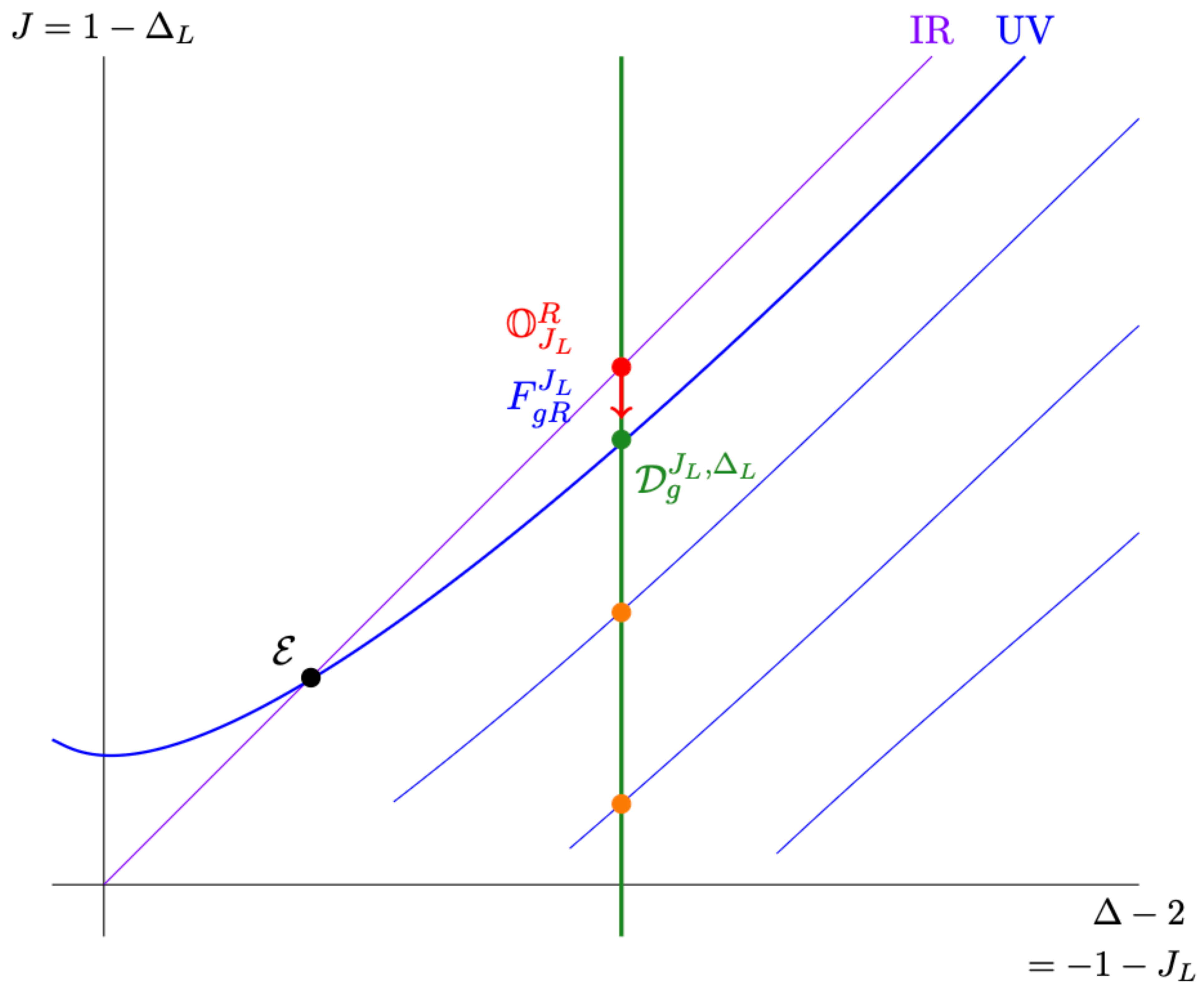
Quarks and gluons

(Cyuan-Han and Hao's talk)

- Lorentz Spin J_L encodes Lorentz spin of the IR detector

DETECTOR FUNCTIONS

- Detector functions are Wilson coefficients between IR and UV detectors



IR detector | Detector (matching) function | UV detector | *pure YM

$$\mathbb{O}^R_{J_L}(\vec{n}) = \sum_k C^R_{J_L,k}(\mu)\, \mathbb{D}_{J_L,k}(\vec{n},\mu)$$

$$= F^{J_L}_{gR}(\mu)\mathcal{D}_{J_L g}(\vec{n},\mu) + \cdots$$

twist-2 gluon DGLAP detector

- Lorentz Spin J_L encodes Lorentz spin of the IR detector

SINGLE HADRON FRAGMENTATION FUNCTIONS

- Detector functions are Wilson coefficients between IR and UV detectors. Physically, they transform gluon state to pion states

*pure YM

IR detector — Detector (matching) function — UV detector

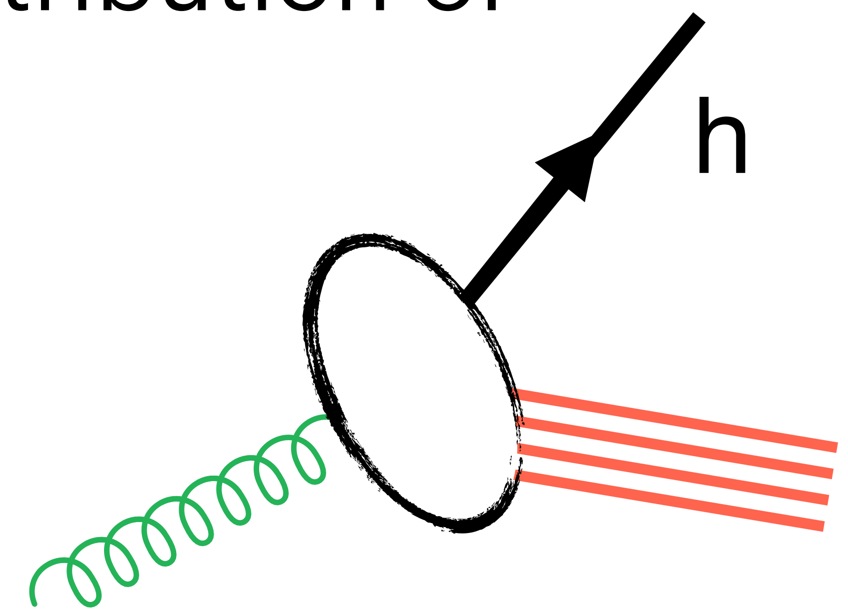
$$\mathbb{O}^{R}_{J_L}(\vec{n}) \simeq F^{J_L}_{gR}(\mu)\mathcal{D}_{J_L g}(\vec{n},\mu)$$

IR theories of pions — Quarks and gluons

- Single Hadron Fragmentation functions $D_{g\to h}(z)$ describes the energy fraction z distribution of a hadron h produced from g:

$$D_{g\to h}(z) = -\frac{1}{(d-2)\,(N_c^2-1)\,p_h^-}\int \frac{dy^+}{2(2\pi)} e^{ik^- y^+/2} \sum_X \langle 0|G^a_{-\lambda}\left(y^+,0,y_\perp\right)|Xh\rangle\langle Xh|G^{\lambda,a}_{-}(0)|0\rangle$$

$$\sim \langle g\,|\,\pi X\rangle\langle \pi X\,|\,g\rangle$$

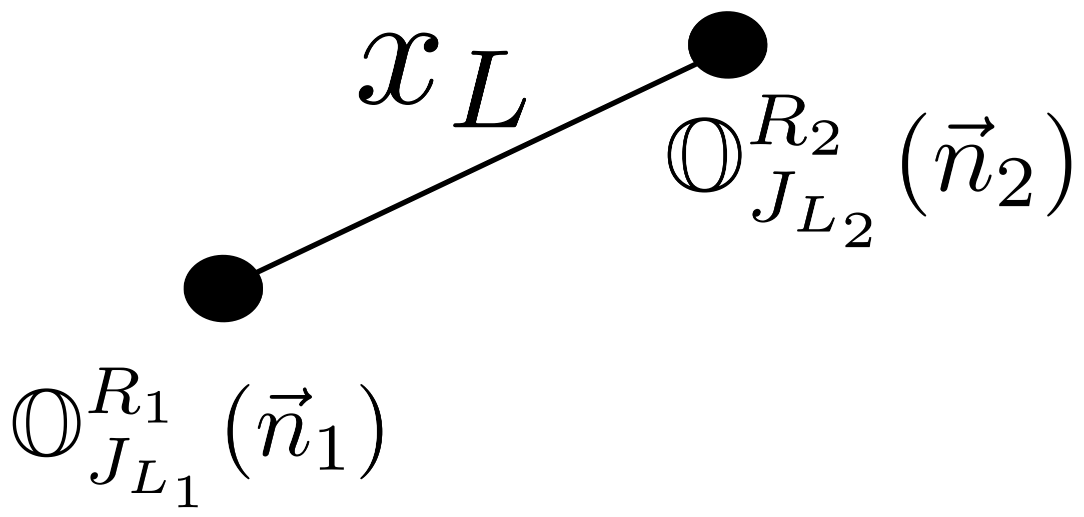


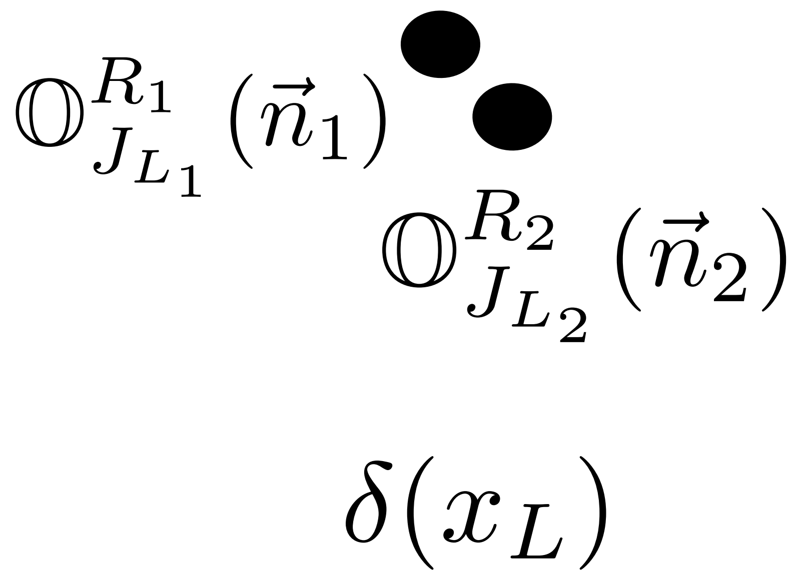
- Detector function for a single detector is a moment of such single hadron fragmentation function

$$F^{J_L}_{gR}(\mu) = \sum_{h\in R}\int_0^1 dz\, z^{2-d-J_L} D_{g\to h}(z,\mu)$$

PRODUCT OF IR DETECTOR OPERATORS

Now consider $\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2)$ in the region $\Lambda_{\rm QCD} \ll Q\sqrt{x_L} \ll Q$





1) Separately match each $\mathbb{O}^{R_i}_{J_{L_i}}(\vec{n}_i)$ to UV detectors, then perform the light-ray OPE

2) This is in the limit $\vec{n}_1 \to \vec{n}_2$, where from the UV detector perspective cannot resolve in the region we consider. Must match two detectors together.

PRODUCT OF IR DETECTOR OPERATORS

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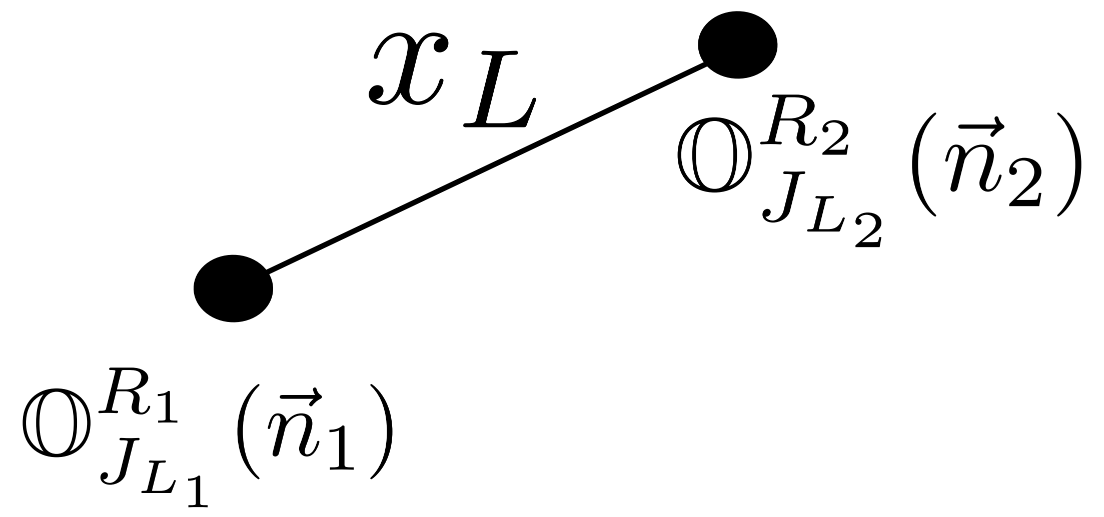

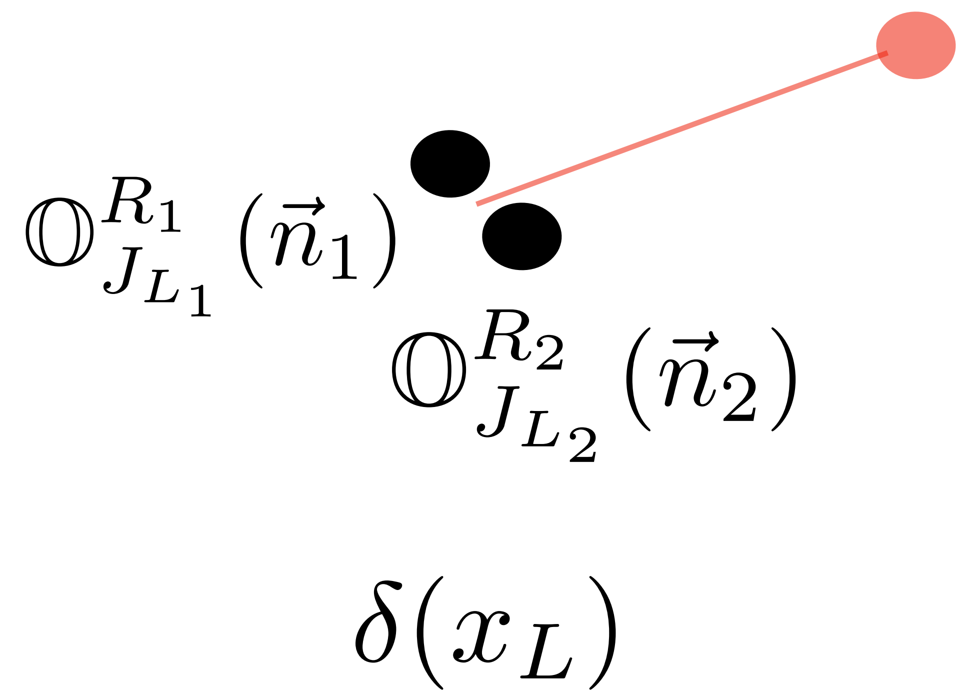

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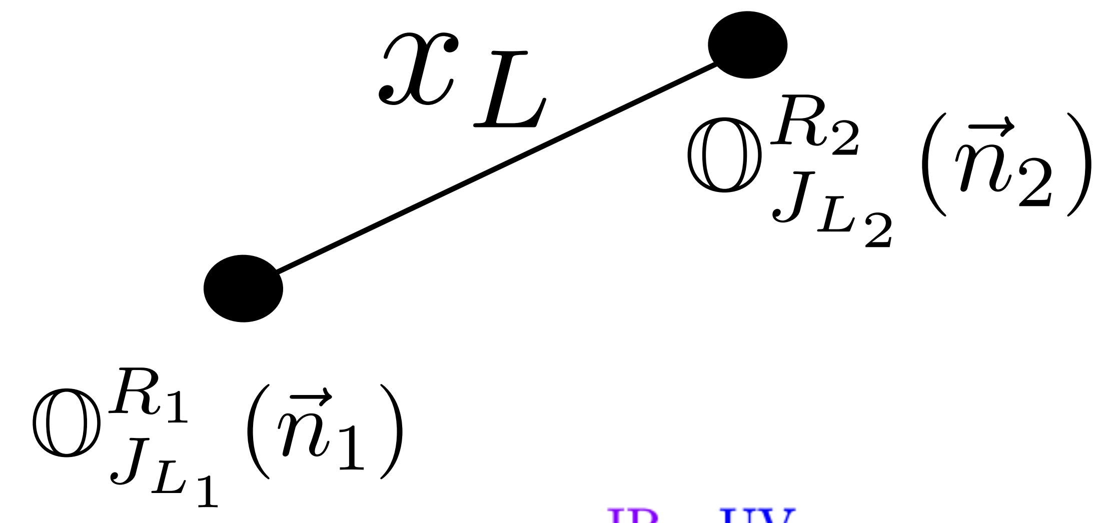
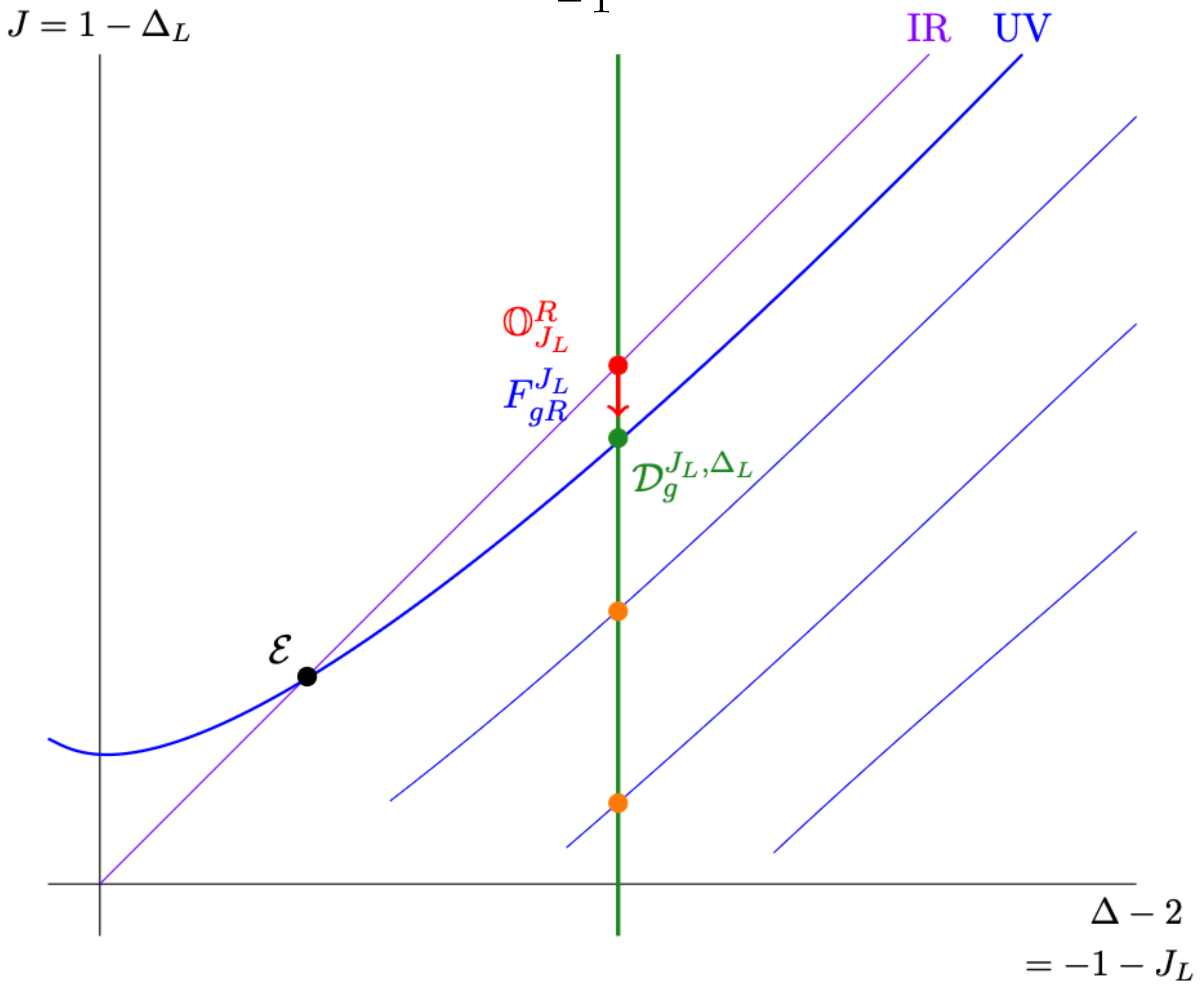
Such contact term becomes important at three-point.

PRODUCT OF IR DETECTOR OPERATORS

Now consider $\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2)$ in the region $\Lambda_{\mathrm{QCD}} \ll Q\sqrt{x_L} \ll Q$



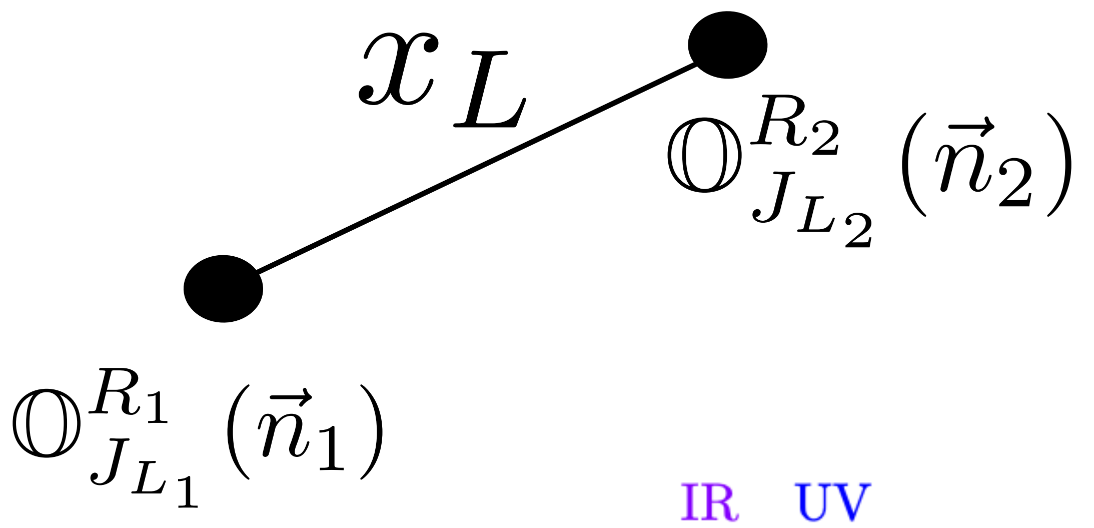


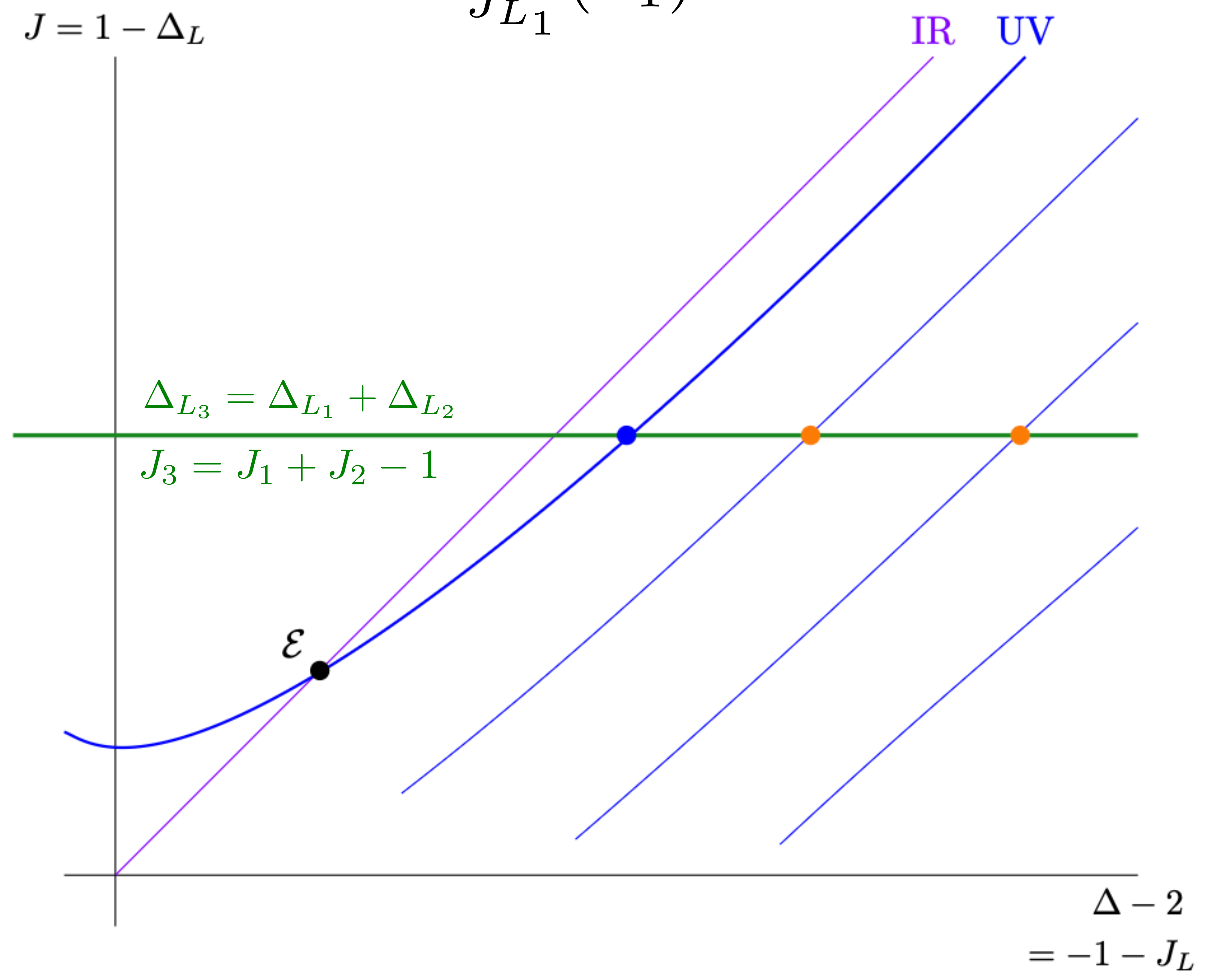
1) Separately match each $\mathbb{O}^{R_i}_{J_{L_i}}(\vec{n}_i)$ to UV detectors, then perform the light-ray OPE

$$\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\,\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2) = F^{J_{L_1}}_{gR_1}(\mu)F^{J_{L_2}}_{gR_2}(\mu)\mathcal{D}_{J_{L_1}g}(\vec{n}_1,\mu)\mathcal{D}_{J_{L_2}g}(\vec{n}_2,\mu) + \cdots$$

PRODUCT OF IR DETECTOR OPERATORS

Now consider $\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2)$ in the region $\Lambda_{\mathrm{QCD}} \ll Q\sqrt{x_L} \ll Q$





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$$= F^{J_{L_1}}_{gR_1}(\mu)F^{J_{L_2}}_{gR_2}(\mu)\sum_{J_{L_3}} C(J_{L_3},x_L,\mu)\mathbb{D}^{J_{L_3}}_{\Delta_{L_3}}(\vec{n},\mu) + \cdots$$

$$\approx F^{J_{L_1}}_{gR_1}(\mu)F^{J_{L_2}}_{gR_2}(\mu)C(J_{L_3},x_L,\mu)\mathcal{D}^{J_{L_3}}_{g}(\vec{n},\mu) + \cdots$$

PRODUCT OF IR DETECTOR OPERATORS

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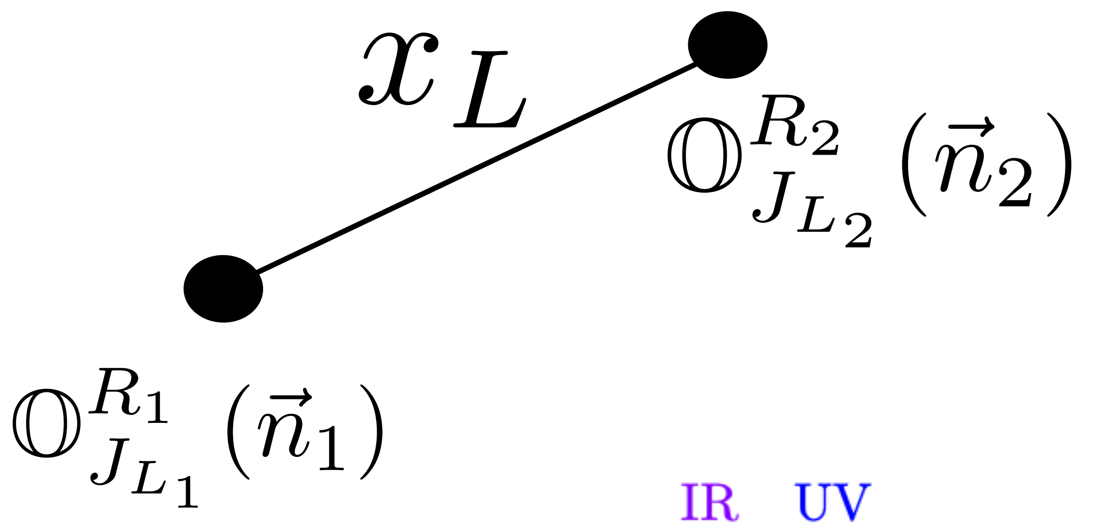

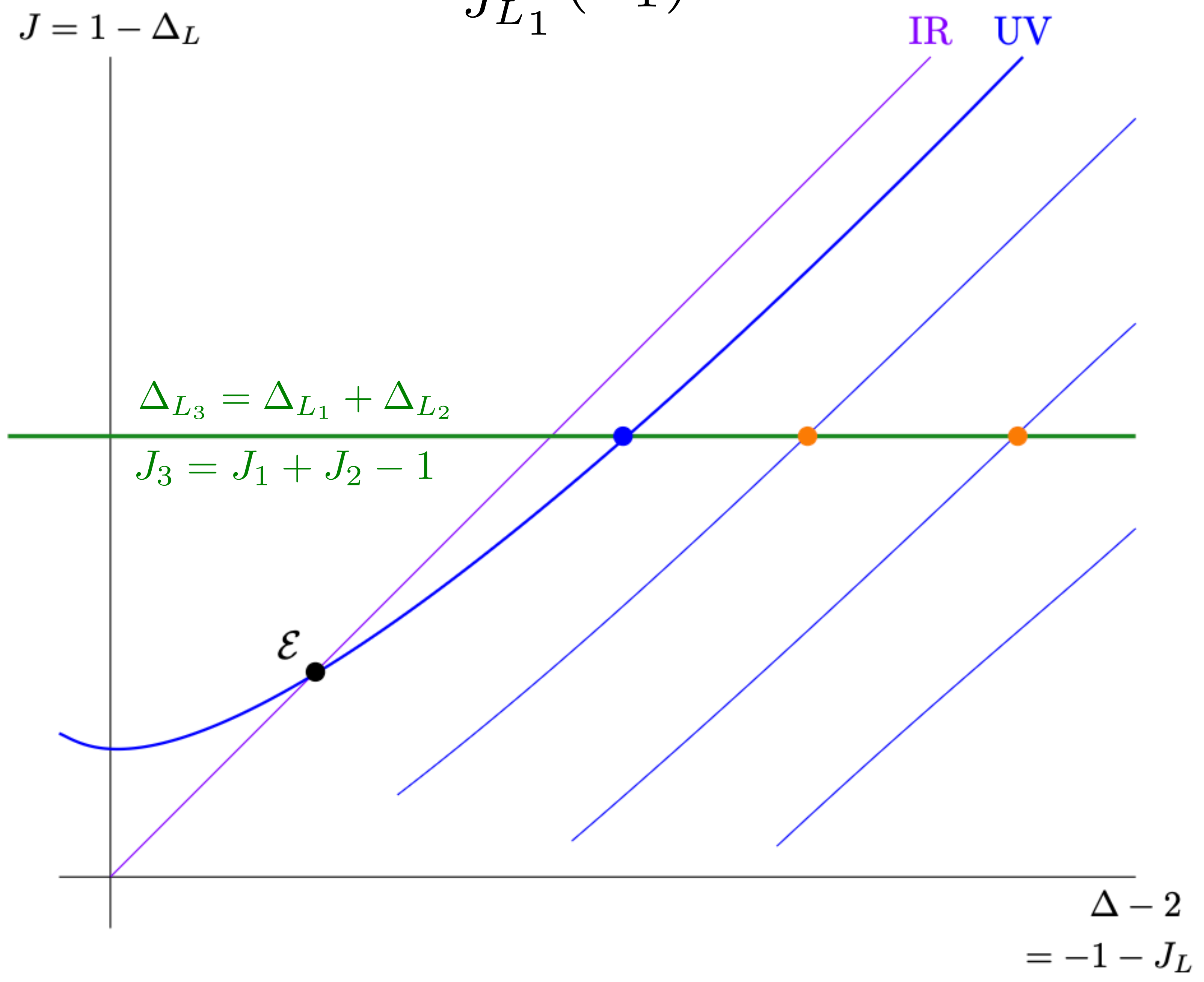

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$$= F^{J_{L_1}}_{gR_1}(\mu)F^{J_{L_2}}_{gR_2}(\mu)\sum_{J_{L_3}} C(J_{L_3},x_L,\mu)\mathbb{D}^{J_{L_3}}_{\Delta_{L_3}}(\vec{n},\mu) + \cdots$$

$$\approx F^{J_{L_1}}_{gR_1}(\mu)F^{J_{L_2}}_{gR_2}(\mu)C(J_{L_3},x_L,\mu)\mathcal{D}^{J_{L_3}}_{g}(\vec{n},\mu) + \cdots$$

Here $C \sim 1/x_L$.

From the selection rule, we find $J_3 = -3 - J_{L_1} - J_{L_2} - 2\gamma_T(-1 - J_{L_1}) - 2\gamma_T(-1 - J_{L_2})$

non-integer spin light-ray operator

PRODUCT OF IR DETECTOR OPERATORS

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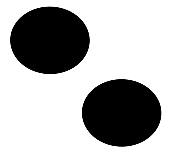
$\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)$  $\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2)$ $\delta(x_L)$

2) This is in the limit $\vec{n}_1 \to \vec{n}_2$, where from the UV detector perspective cannot resolve in the region we consider. Must match two detectors together.

$$\lim_{\vec{n}_1\to\vec{n}_2} \mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2) \sim \lim_{\vec{n}_1\to\vec{n}_2} \sum_{h_1\in R_1}\sum_{h_2\in R_2}\int dE_1 dE_2 E_1^{-J_{L_1}-1}E_2^{-J_{L_2}-1} a^\dagger_{h_1}a_{h_1}a^\dagger_{h_2}a_{h_2}$$

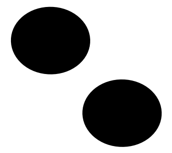
$$\sim \lim_{\vec{n}_1\to\vec{n}_2} \sum_{h_1\in R_1}\sum_{h_2\in R_2}\int dE_1 dE_2 E_1^{-J_{L_1}-1}E_2^{-J_{L_2}-1} a^\dagger_{h_1}a^\dagger_{h_2}a_{h_2}a_{h_1} + \delta^2(\vec{n}_1-\vec{n}_2)\delta_{R_1R_2}\sum_{h_1\in R_1}\int dE_1 E_1^{-J_{L_1}-J_{L_2}-3}a^\dagger_1 a_1$$

$$\sim \lim_{\vec{n}_1\to\vec{n}_2} :\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2): + \delta^2(\vec{n}_1-\vec{n}_2)\delta_{R_1R_2}\mathbb{O}^{R_1}_{J_{L_1}+J_{L_2}-2}(\vec{n}_1)$$

PRODUCT OF IR DETECTOR OPERATORS

Now consider $\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2)$ in the region $\Lambda_{\rm QCD} \ll Q\sqrt{x_L} \ll Q$

$\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)$



$\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2)$

$\delta(x_L)$

2) This is in the limit $\vec{n}_1 \to \vec{n}_2$, where from the UV detector perspective cannot resolve in the region we consider. Must match two detectors together.

$$\lim_{\vec{n}_1\to\vec{n}_2} \mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2) \sim \lim_{\vec{n}_1\to\vec{n}_2} \sum_{h_1\in R_1}\sum_{h_2\in R_2}\int dE_1 dE_2 E_1^{-J_{L_1}-1}E_2^{-J_{L_2}-1} a^\dagger_{h_1}a_{h_1}a^\dagger_{h_2}a_{h_2}$$

$$\sim \lim_{\vec{n}_1\to\vec{n}_2} \sum_{h_1\in R_1}\sum_{h_2\in R_2}\int dE_1 dE_2 E_1^{-J_{L_1}-1}E_2^{-J_{L_2}-1} a^\dagger_{h_1}a^\dagger_{h_2}a_{h_2}a_{h_1} + \delta^2(\vec{n}_1-\vec{n}_2)\delta_{R_1R_2}\sum_{h_1\in R_1}\int dE_1 E_1^{-J_{L_1}-J_{L_2}-3}a^\dagger_1 a_1$$

$$\sim \lim_{\vec{n}_1\to\vec{n}_2} :\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2): + \delta^2(\vec{n}_1-\vec{n}_2)\delta_{R_1R_2}\mathbb{O}^{R_1}_{J_{L_1}+J_{L_2}-2}(\vec{n}_1)$$

Measures two-particle state

Acts like a single-detector

PRODUCT OF IR DETECTOR OPERATORS

Now consider $\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2)$ in the region $\Lambda_{\rm QCD} \ll Q\sqrt{x_L} \ll Q$

$\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)$ $\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2)$ $\delta(x_L)$

2) This is in the limit $\vec{n}_1 \to \vec{n}_2$, where from the UV detector perspective cannot resolve in the region we consider. Must match two detectors together.

$$\lim_{\vec{n}_1\to\vec{n}_2}\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2) \sim \lim_{\vec{n}_1\to\vec{n}_2}\sum_{h_1\in R_1}\sum_{h_2\in R_2}\int dE_1 dE_2 E_1^{-J_{L_1}-1}E_2^{-J_{L_2}-1}a^\dagger_{h_1}a_{h_1}a^\dagger_{h_2}a_{h_2}$$

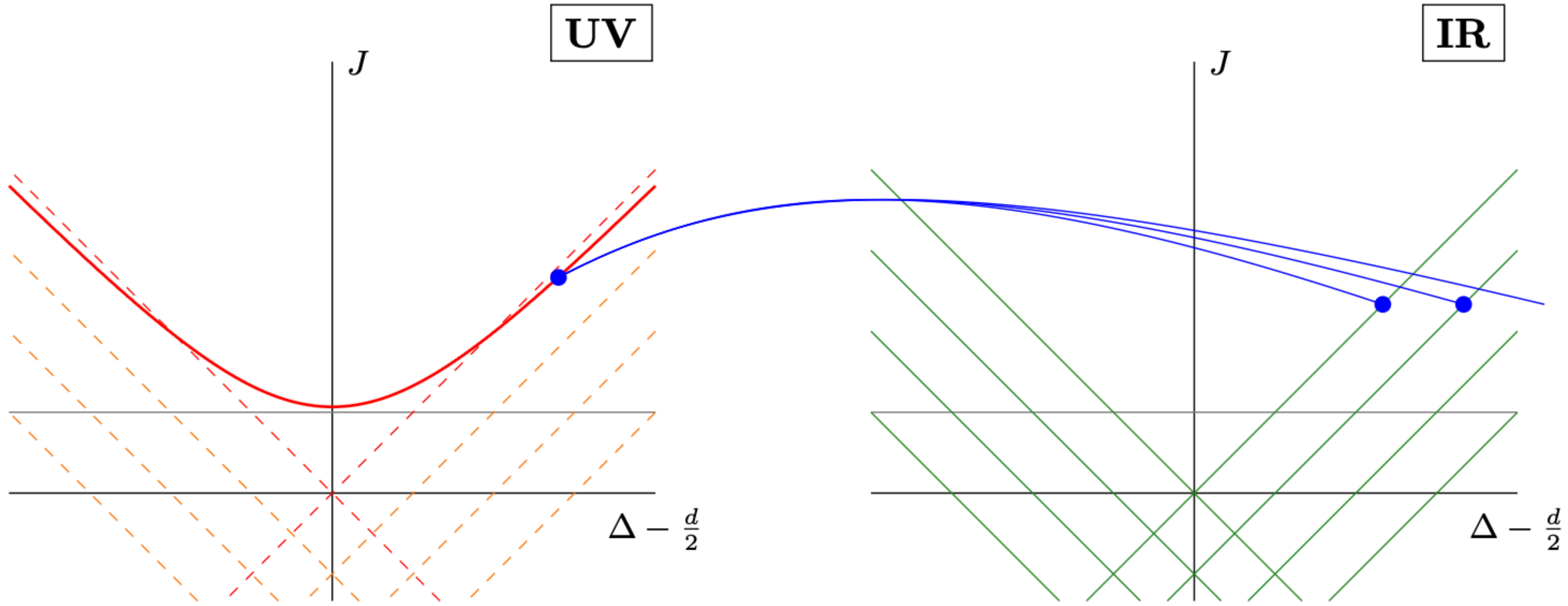
$$\sim \lim_{\vec{n}_1\to\vec{n}_2}\sum_{h_1\in R_1}\sum_{h_2\in R_2}\int dE_1 dE_2 E_1^{-J_{L_1}-1}E_2^{-J_{L_2}-1}a^\dagger_{h_1}a^\dagger_{h_2}a_{h_2}a_{h_1} + \delta^2(\vec{n}_1-\vec{n}_2)\delta_{R_1R_2}\sum_{h_1\in R_1}\int dE_1 E_1^{-J_{L_1}-J_{L_2}-3}a^\dagger_1 a_1$$

$$\sim \lim_{\vec{n}_1\to\vec{n}_2} :\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2): + \delta^2(\vec{n}_1-\vec{n}_2)\delta_{R_1R_2}\mathbb{O}^{R_1}_{J_{L_1}+J_{L_2}-2}(\vec{n}_1)$$

Measures two-particle state Acts like a single-detector

$$= \delta(x_L)\left(F^{J_{L_1},J_{L_2}}_{gR}(\mu)\mathcal{D}_g^{J_{L_1}+J_{L_2}-2} + F^{J_{L_1}+J_{L_2}-2}_{gR}(\mu)\mathcal{D}_g^{J_{L_1}+J_{L_2}-2}\right)$$

PRODUCT OF IR DETECTOR OPERATORS

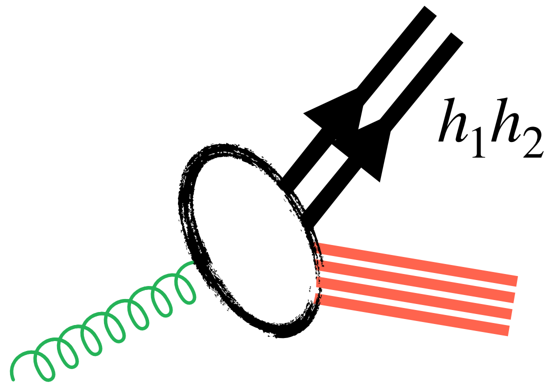


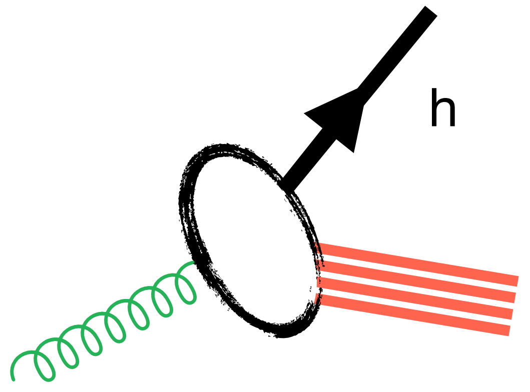
$$\lim_{\vec{n}_1\to\vec{n}_2} :\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2): +\delta^2(\vec{n}_1-\vec{n}_2)\delta_{R_1R_2}\mathbb{O}^{R_1}_{J_{L_1}+J_{L_2}-2}(\vec{n}_1)$$

Measures two-particle state

Acts like a single-detector

$$= \delta(x_L)\left(F_{gR}^{J_{L_1},J_{L_2}}(\mu)\mathcal{D}_g^{J_{L_1}+J_{L_2}-2} + F_{gR}^{J_{L_1}+J_{L_2}-2}(\mu)\mathcal{D}_g^{J_{L_1}+J_{L_2}-2}\right)$$

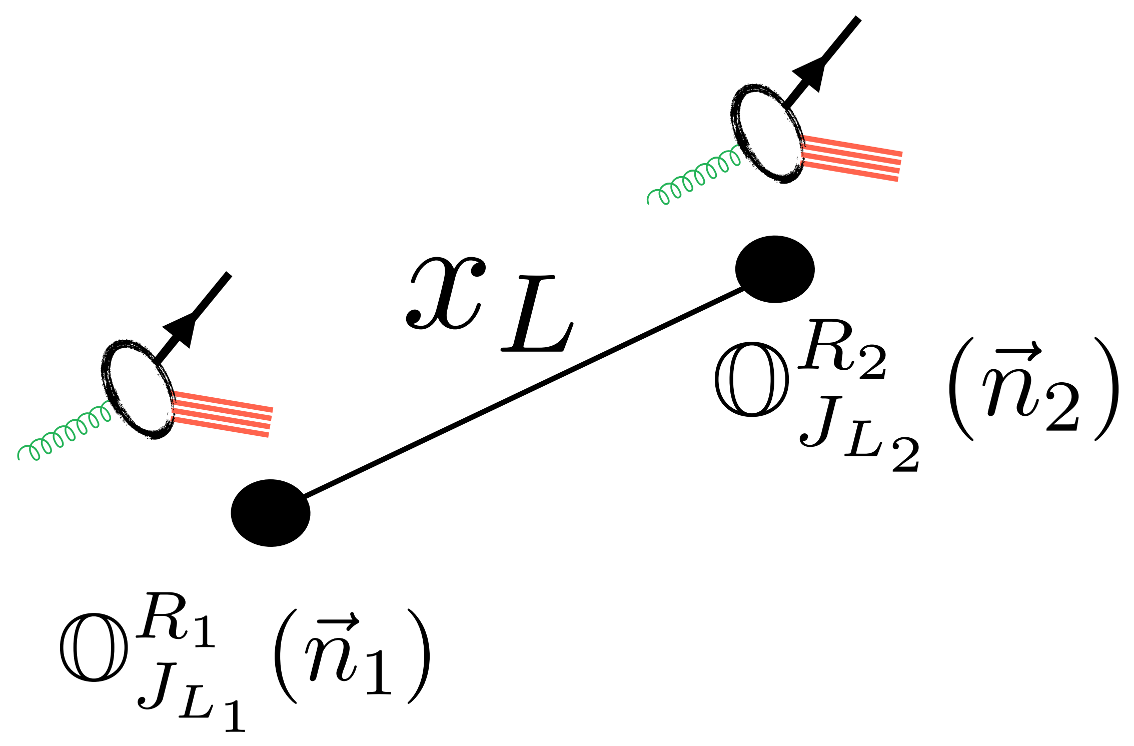


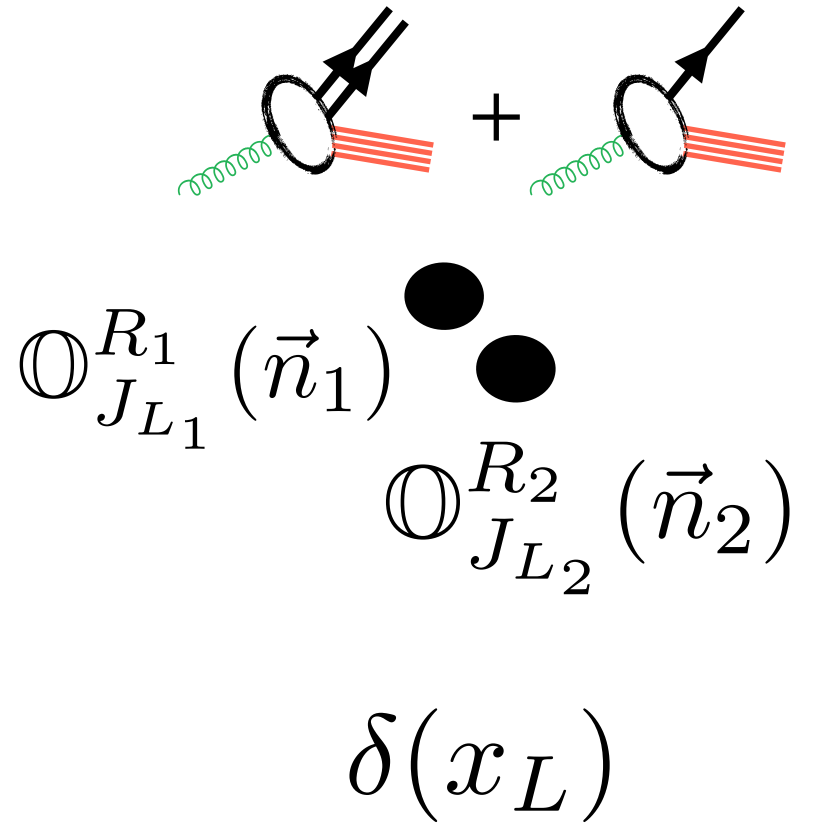


Outline.

- **Operator Product Expansion (OPE)**
of generalized detectors

LIGHT-RAY OPERATOR PRODUCT EXPANSION





$$\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\,\mathbb{O}^{R_2}_{J_{L_2}}(\vec{n}_2) \approx F^{J_{L_1}}_{gR_1}(\mu)F^{J_{L_2}}_{gR_2}(\mu)C(J_{L_3}, x_L, \mu)\mathcal{D}^{J_{L_3}}_g(\vec{n}, \mu) + \delta(x_L)\left(F^{J_{L_1}, J_{L_2}}_{gR}(\mu)\mathcal{D}^{J_{L_1}+J_{L_2}-2}_g + F^{J_{L_1}+J_{L_2}-2}_{gR}(\mu)\mathcal{D}^{J_{L_1}+J_{L_2}-2}_g\right)$$

↓

Scaling given by UV detector of spin

$$J_3 = -3 - J_{L_1} - J_{L_2} - 2\gamma_T(-1 - J_{L_1}) - 2\gamma_T(-1 - J_{L_2})$$

For $J_{L_1} = J_{L_2} = -3$ (energy flow),

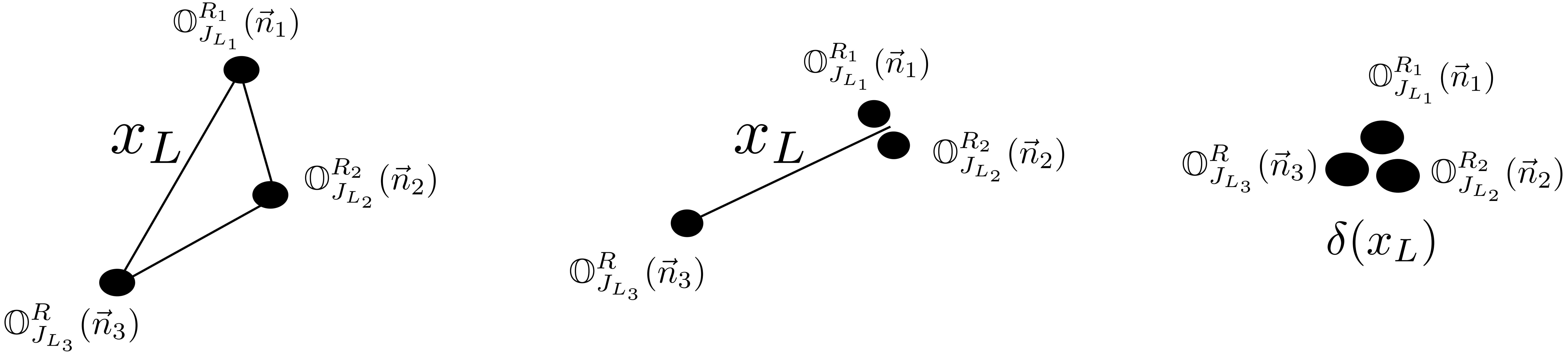
$F^{J_L}_g = 1, \gamma_T = 0$, reproducing well-known ANEC case.

↓

Not important for finite x_L measurement at two-point

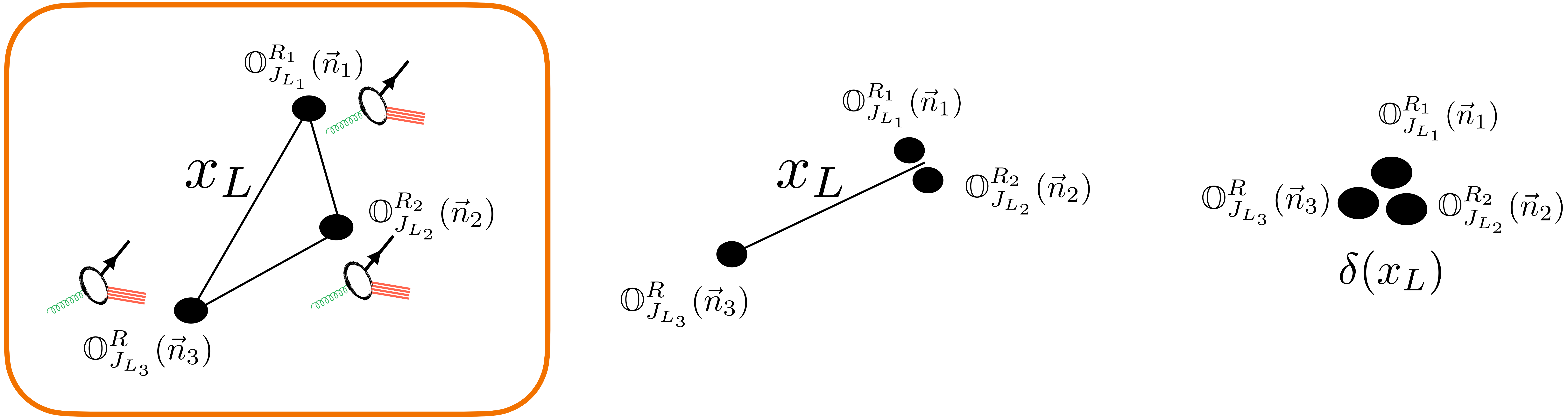
LIGHT-RAY OPERATOR PRODUCT EXPANSION

Now consider three detectors case:



LIGHT-RAY OPERATOR PRODUCT EXPANSION

Now consider three detectors case:

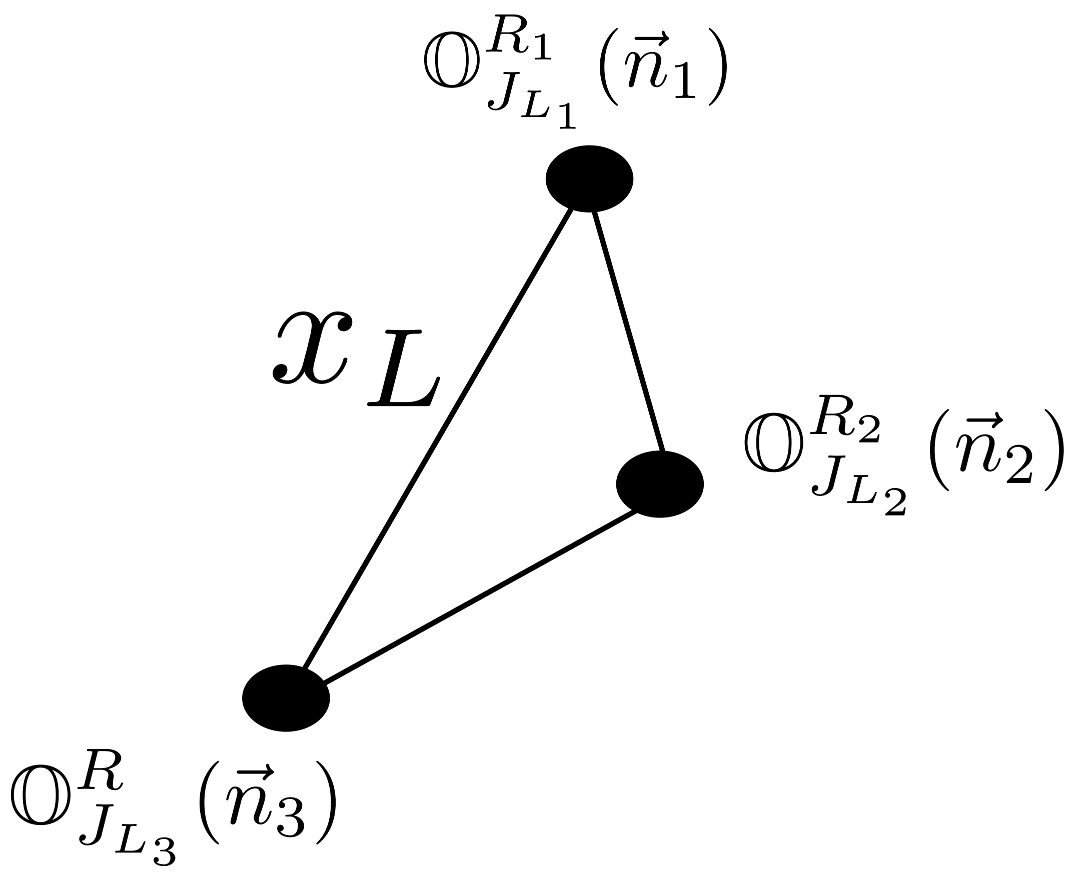


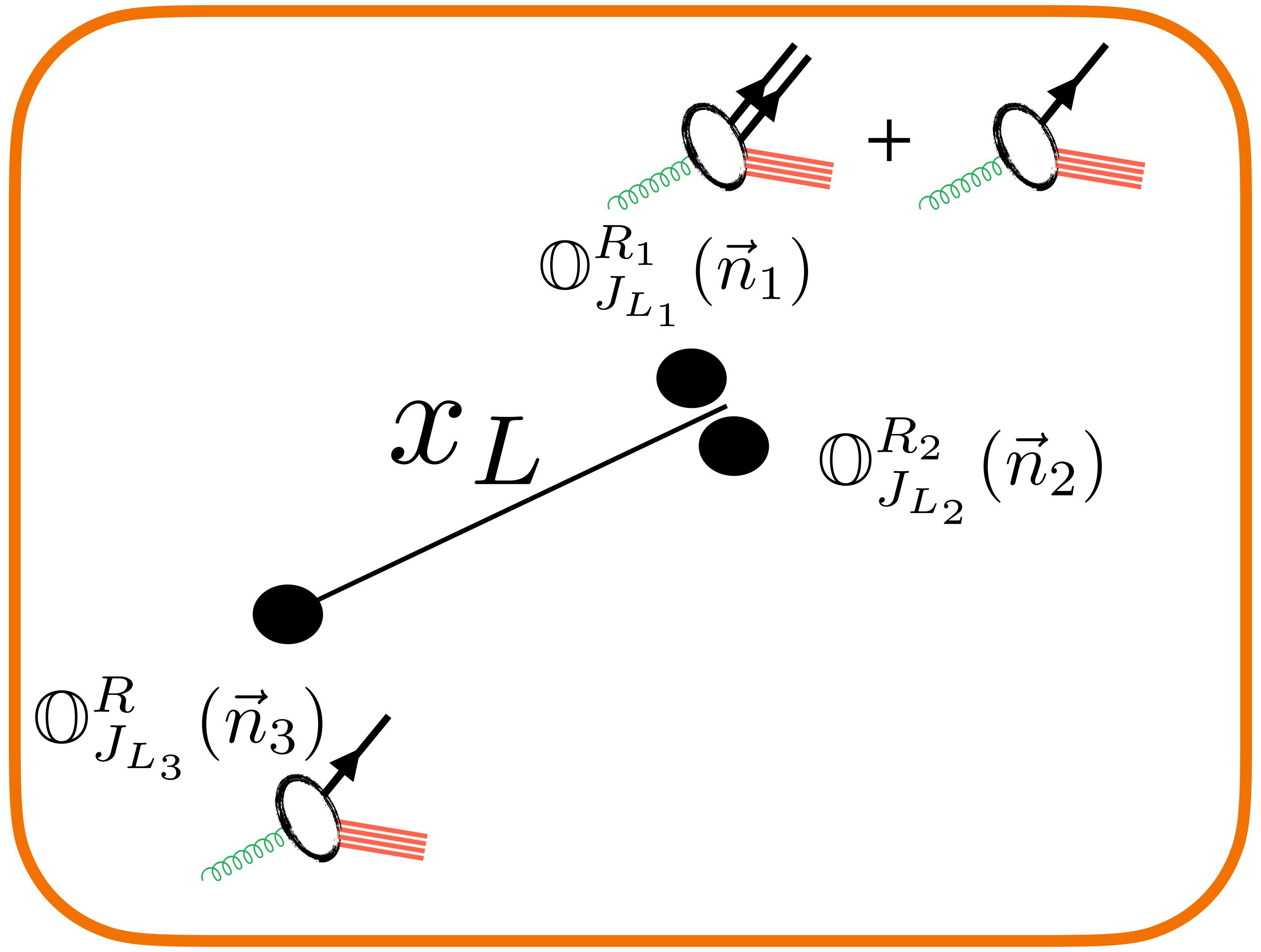
We match each IR detector separate then OPE the UV detectors.

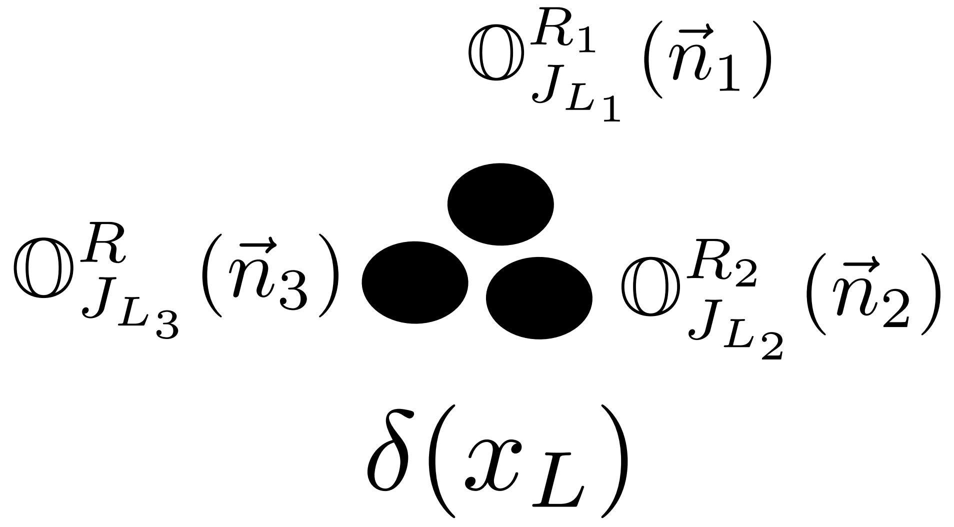
Fix the cross-ratio to fix the shape and consider how its size parameterized by x_L scales

LIGHT-RAY OPERATOR PRODUCT EXPANSION

Now consider three detectors case:



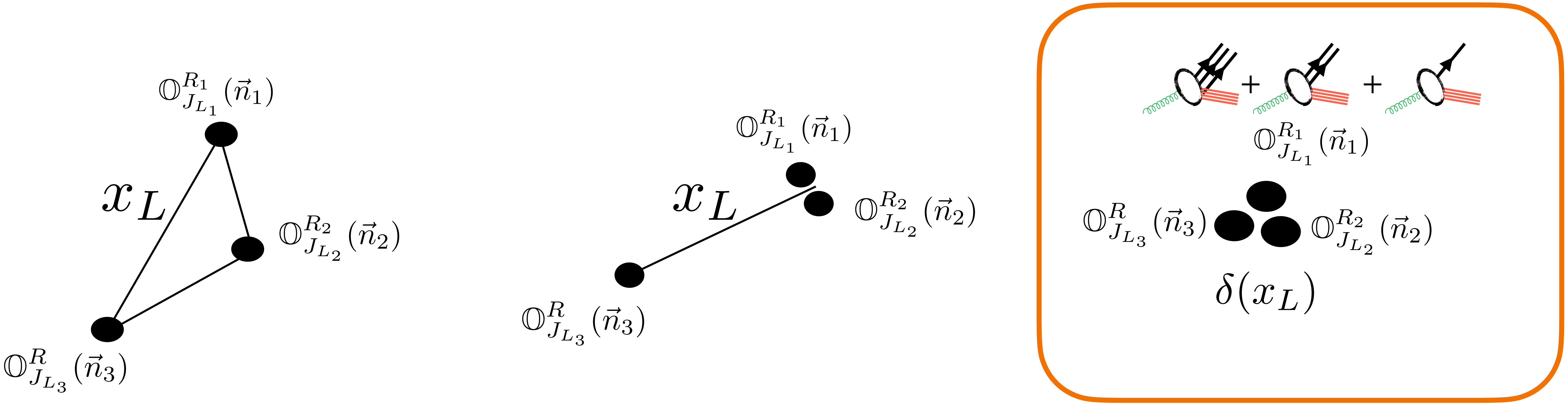




Take the contact term from the two-point case then take an additional detector separately matched.
Then OPE the UV detectors

LIGHT-RAY OPERATOR PRODUCT EXPANSION

Now consider three detectors case:

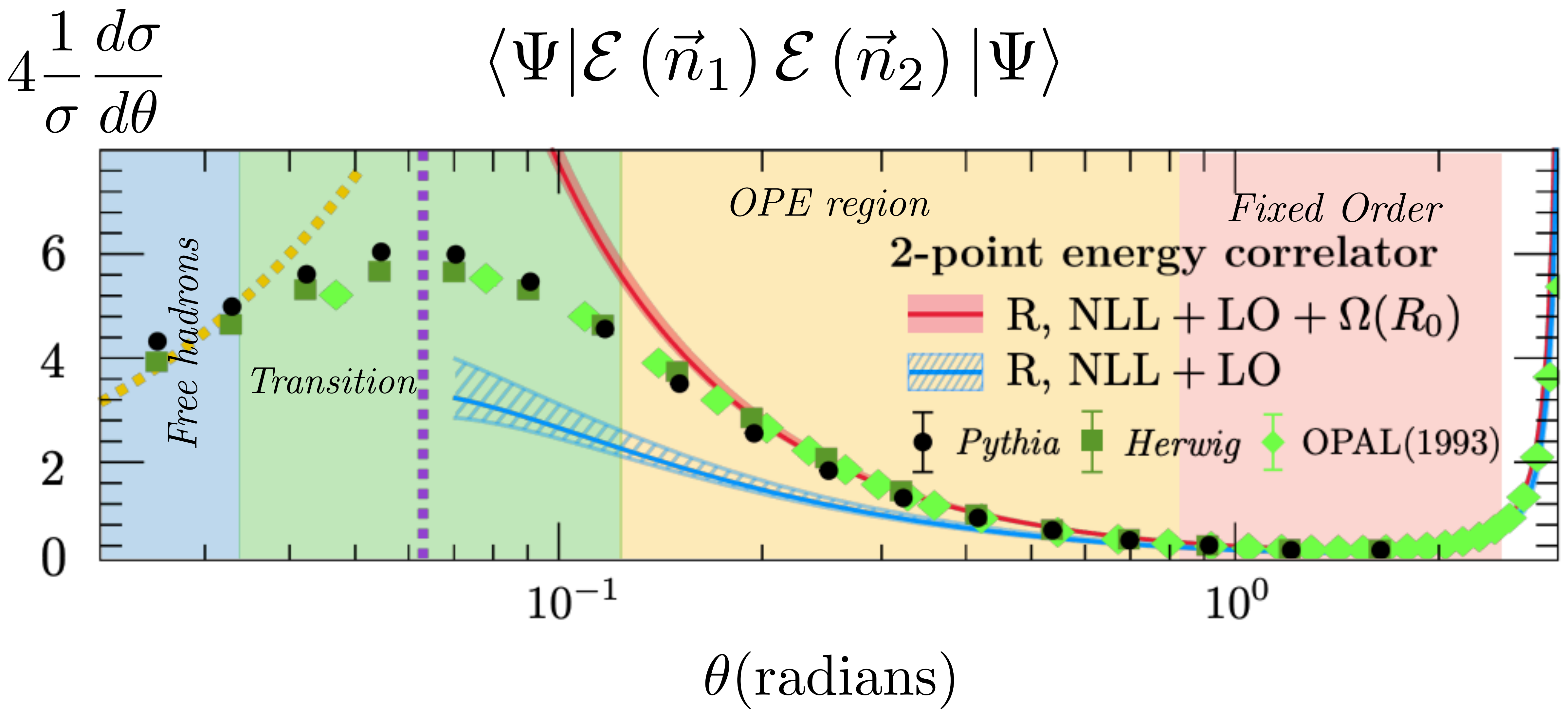


New contact term that becomes important contribution at four-point measurements.
Sensitive to tri-hadron fragmentation function as the matching between partonic and hadronic detectors!

Outline.

-Nonperturbative Power Corrections

NONPERTURBATIVE POWER CORRECTIONS



Perturbative, $1/x_L \sim 1/\theta^2$

Perturbative + NP

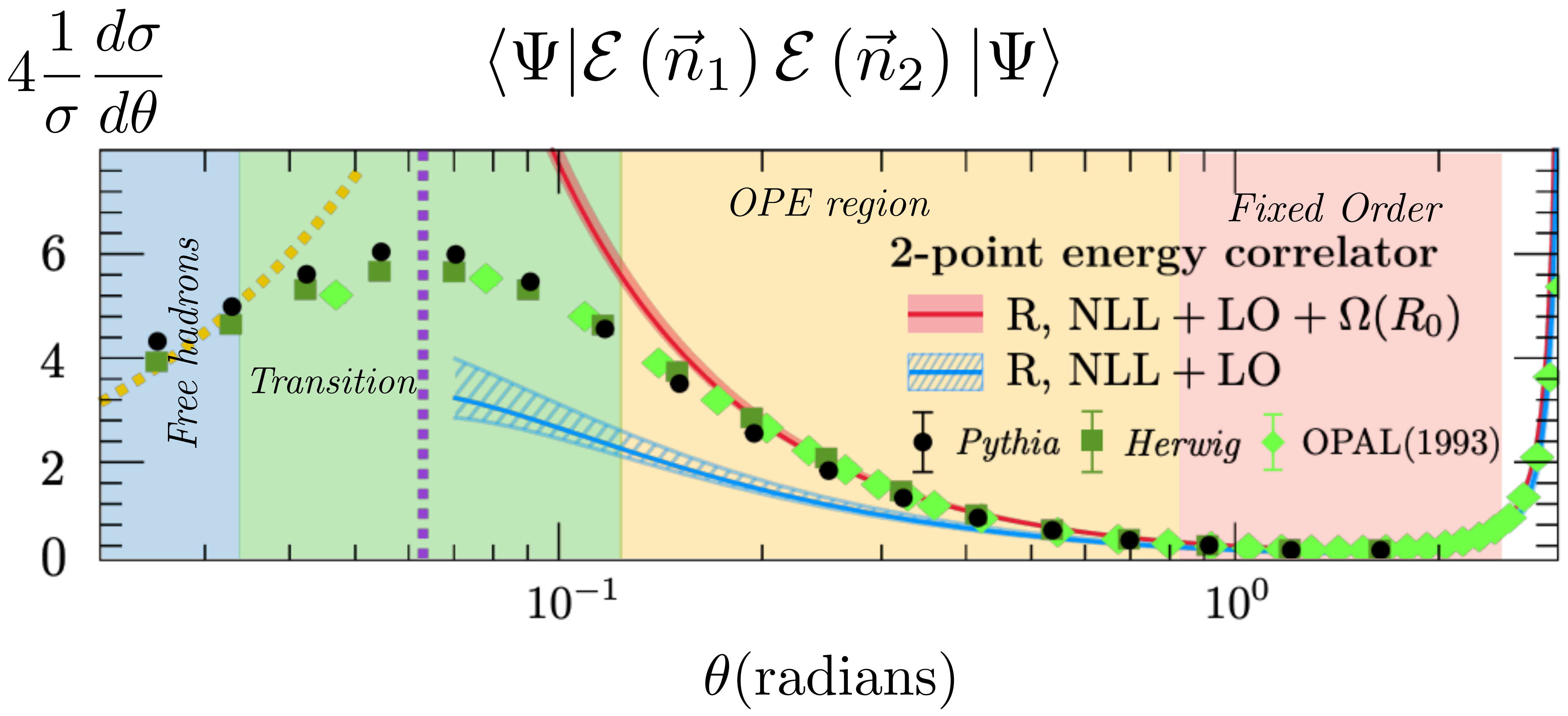
NP: $1/x_L^{3/2} \sim 1/\theta^3$

$$\lim_{n_1\to n_2} \mathcal{E}(n_1)\,\mathcal{E}(n_2) = \underbrace{C(x_L)}_{\sim 1/x_L}\mathbb{O}^{J_L\approx -4}(n_2) + \Lambda_{\text{QCD}}\underbrace{D(x_L)}_{\sim 1/x_L^{3/2}}\mathbb{O}^{J_L\approx -3}(n_2) + \cdots$$

Chen, Monni, Xu, Zhu `24
KL, Pathak, Stewart, Sun `24

In EEC, the leading nonperturbative power correction that goes as $\sim \Lambda_{\text{QCD}}$ introduces mass-dimension and has a different angular scaling $1/x_L^{\frac{3}{2}}$ relative to the leading perturbative contributions.

NONPERTURBATIVE POWER CORRECTIONS



Perturbative, $1/x_L \sim 1/\theta^2$

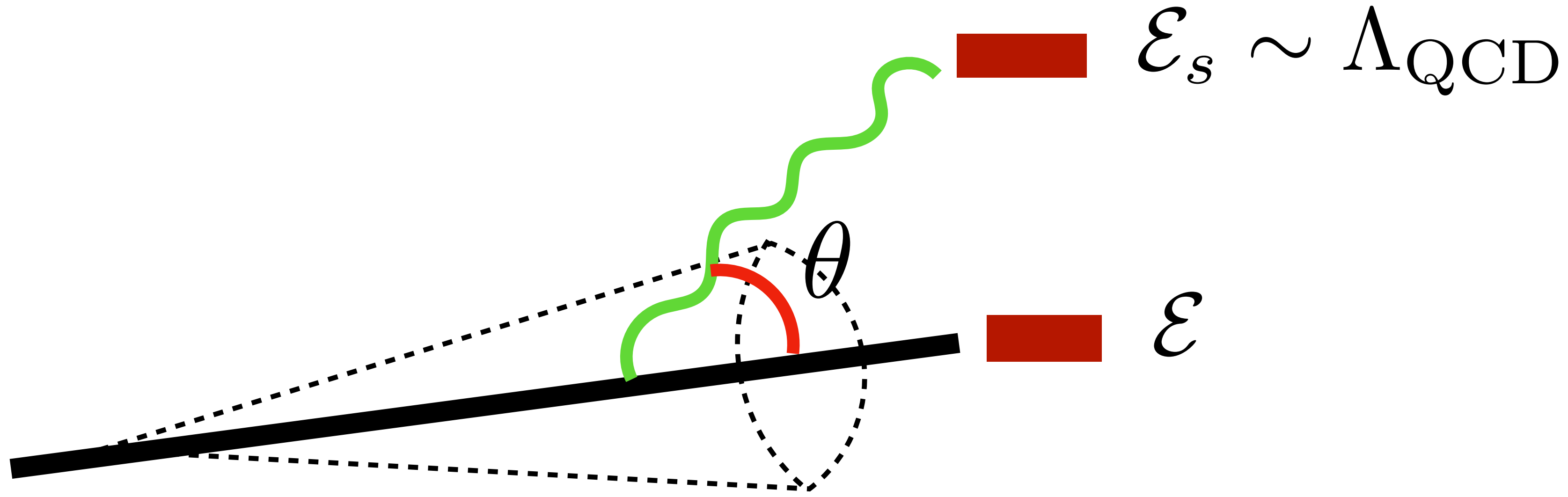
Perturbative + NP

NP: $1/x_L^{3/2} \sim 1/\theta^3$

$$\lim_{n_1 \to n_2} \mathcal{E}(n_1)\,\mathcal{E}(n_2) = C(x_L)\mathbb{O}^{J_L \approx -4}(n_2) + \Lambda_{\rm QCD}\, D(x_L)\,\mathbb{O}^{J_L \approx -3}(n_2) + \cdots$$

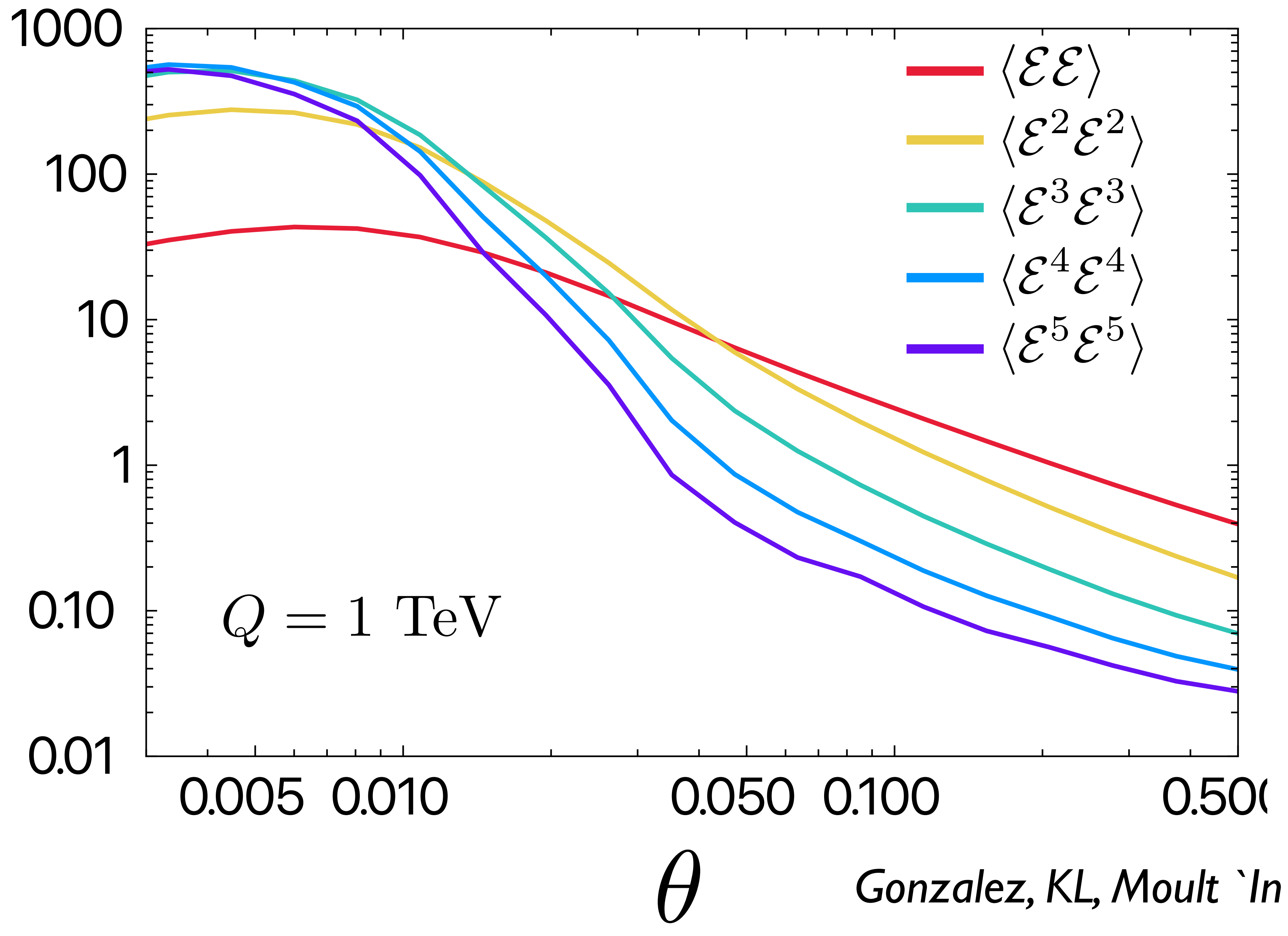
$\sim 1/x_L$ $\quad\quad$ $\sim 1/x_L^{3/2}$

Chen, Monni, Xu, Zhu `24
KL, Pathak, Stewart, Sun `24



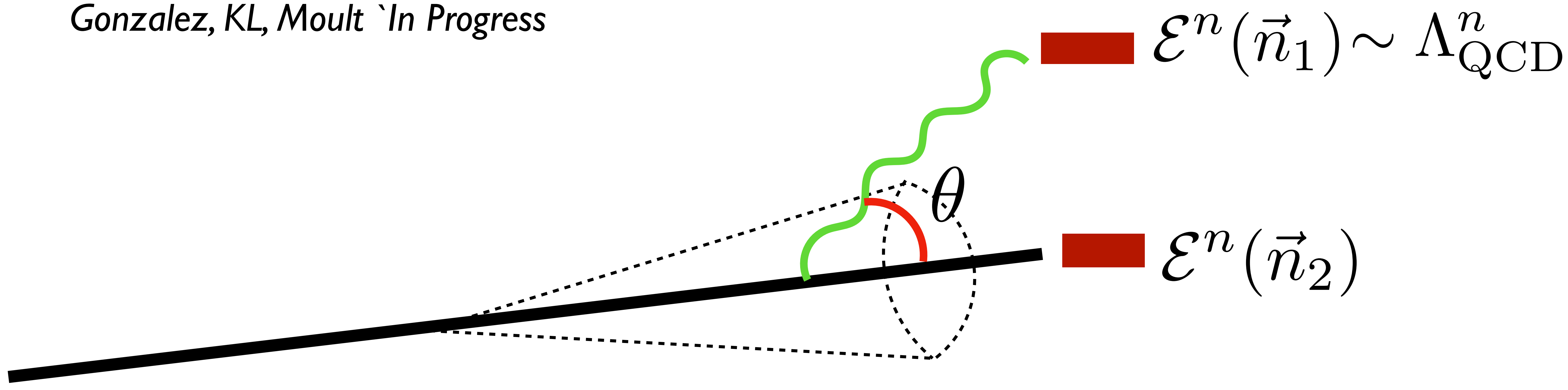
Physically, this comes from a configuration where soft emission gets energy weighted.

NONPERTURBATIVE POWER CORRECTIONS



Gonzalez, KL, Moult `In Progress

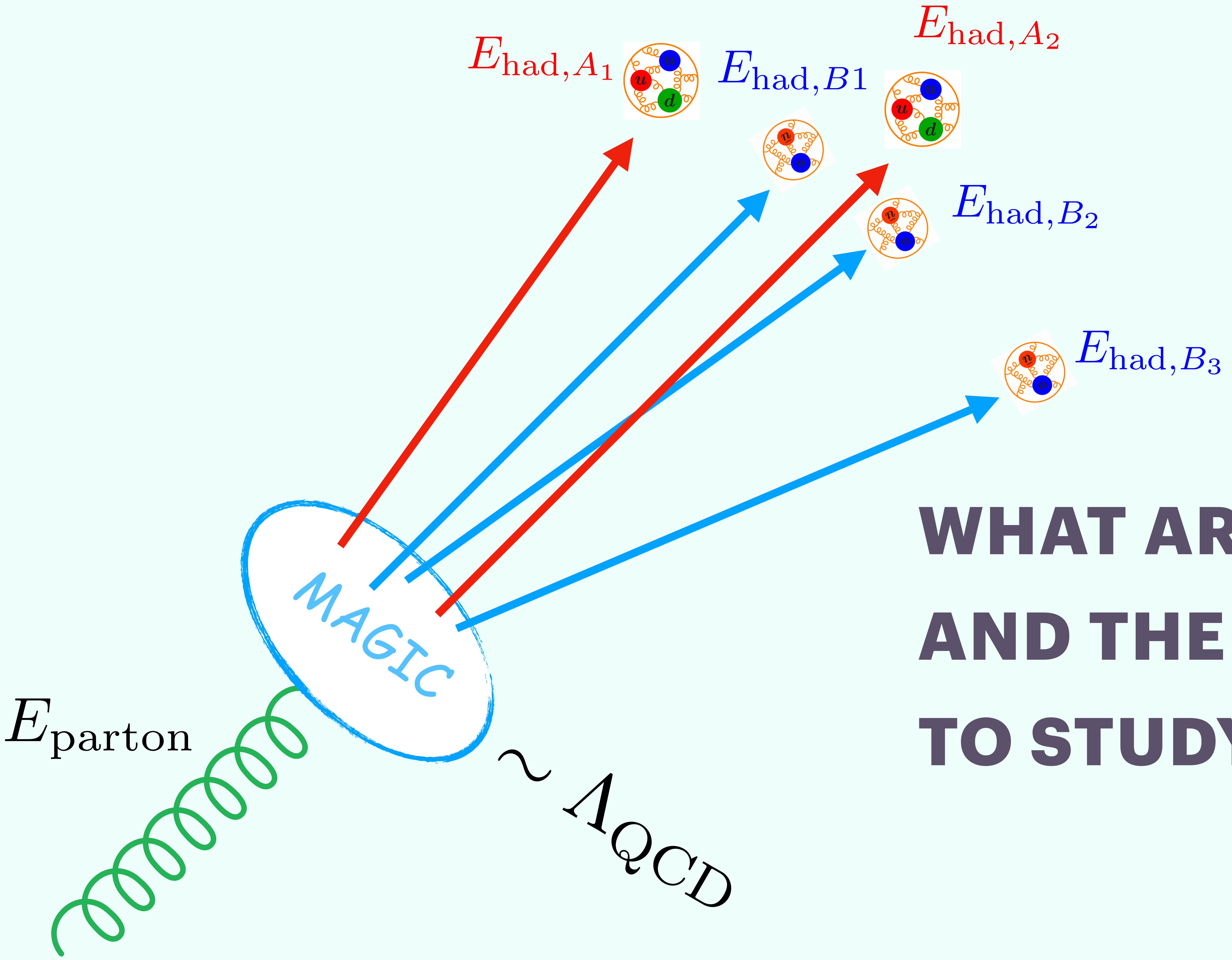
Now, with higher energy weighting, we expect to have sensitivity to the correction that goes as $\sim \Lambda_{\rm QCD}^n$, which gives *enhanced angular scaling* as $\sim \dfrac{1}{x_L^{-\frac{2+n}{2}}}$



Outline.

-**Applications**

Outline.



-Applications

WHAT ARE SOME INTERESTING DETECTORS AND THEIR CORRELATIONS $\langle \mathcal{E}_{R_1}^{n_1}(\vec{n}_1)\mathcal{E}_{R_2}^{n_2}(\vec{n}_2)\rangle$ **TO STUDY THE CONFINEMENT MAGIC?**

Outline.

Applications
of confinement magic and generalized detectors

1. $\langle \mathcal{E}^n(\vec{n}_1)\mathcal{E}^n(\vec{n}_2)\rangle$ *Gonzalez, KL, Moult*
Devereaux, Fan, Ke, KL, Moult `23

weighted energy detector

2. $\langle \mathcal{E}_{\rm trk}(\vec{n}_1)\mathcal{E}_{\rm trk}(\vec{n}_2)\rangle$ *KL, Li, Moult, Waalewijn*
Jaarsma, Li, Moult, Waalewijn, Zhu `23

charged energy detector

3. $\langle \mathcal{E}_+(\vec{n}_1)\mathcal{E}_-(\vec{n}_2)\rangle$ *KL, Moult `23*
KL, Moult, Song, Sterman

charged energy detector

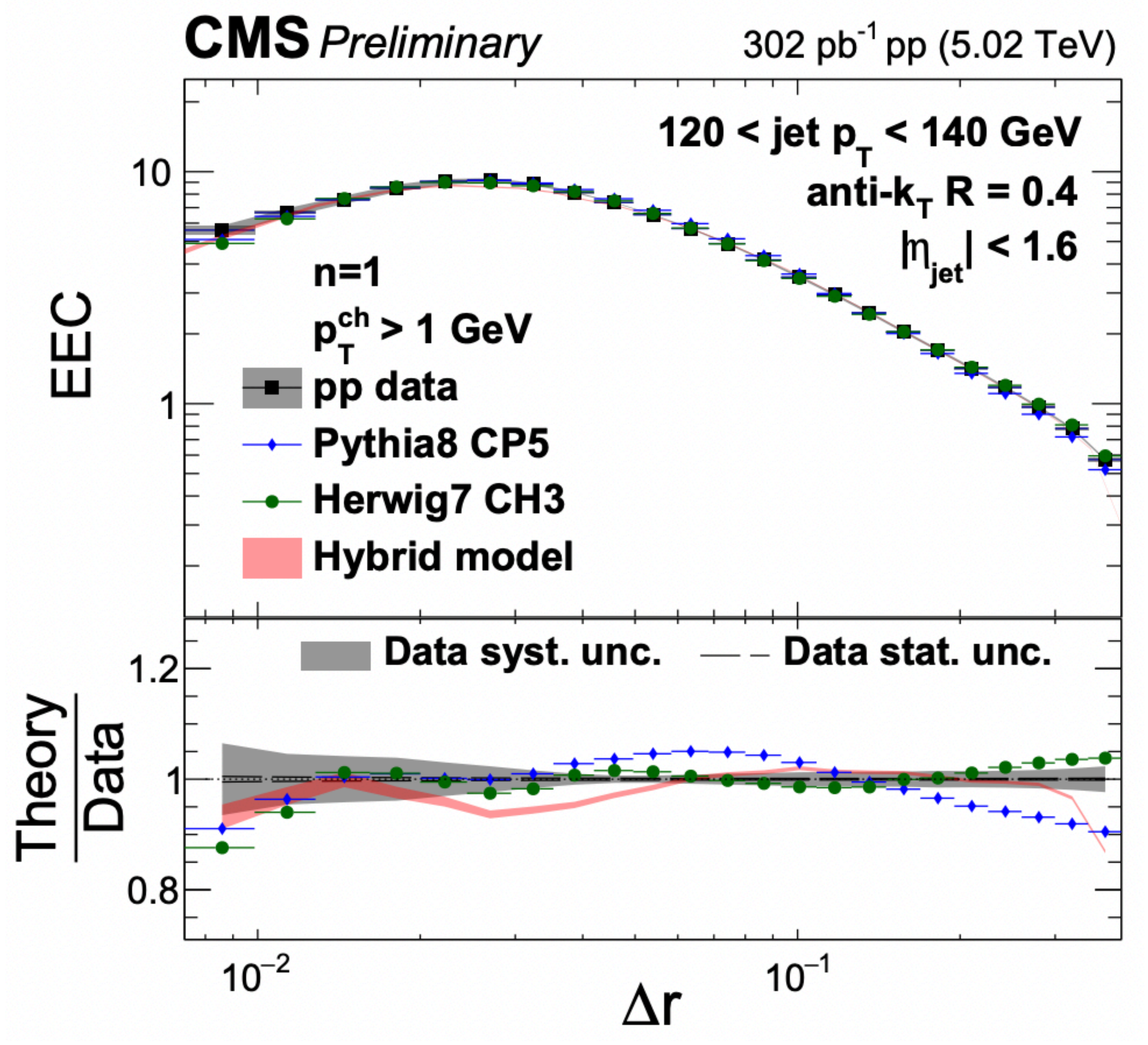
4. $\langle \mathcal{E}_H(\vec{n}_1)\mathcal{E}_{\bar{H}}(\vec{n}_2)\rangle$ *Barata, Brewer, KL, Silva `25*

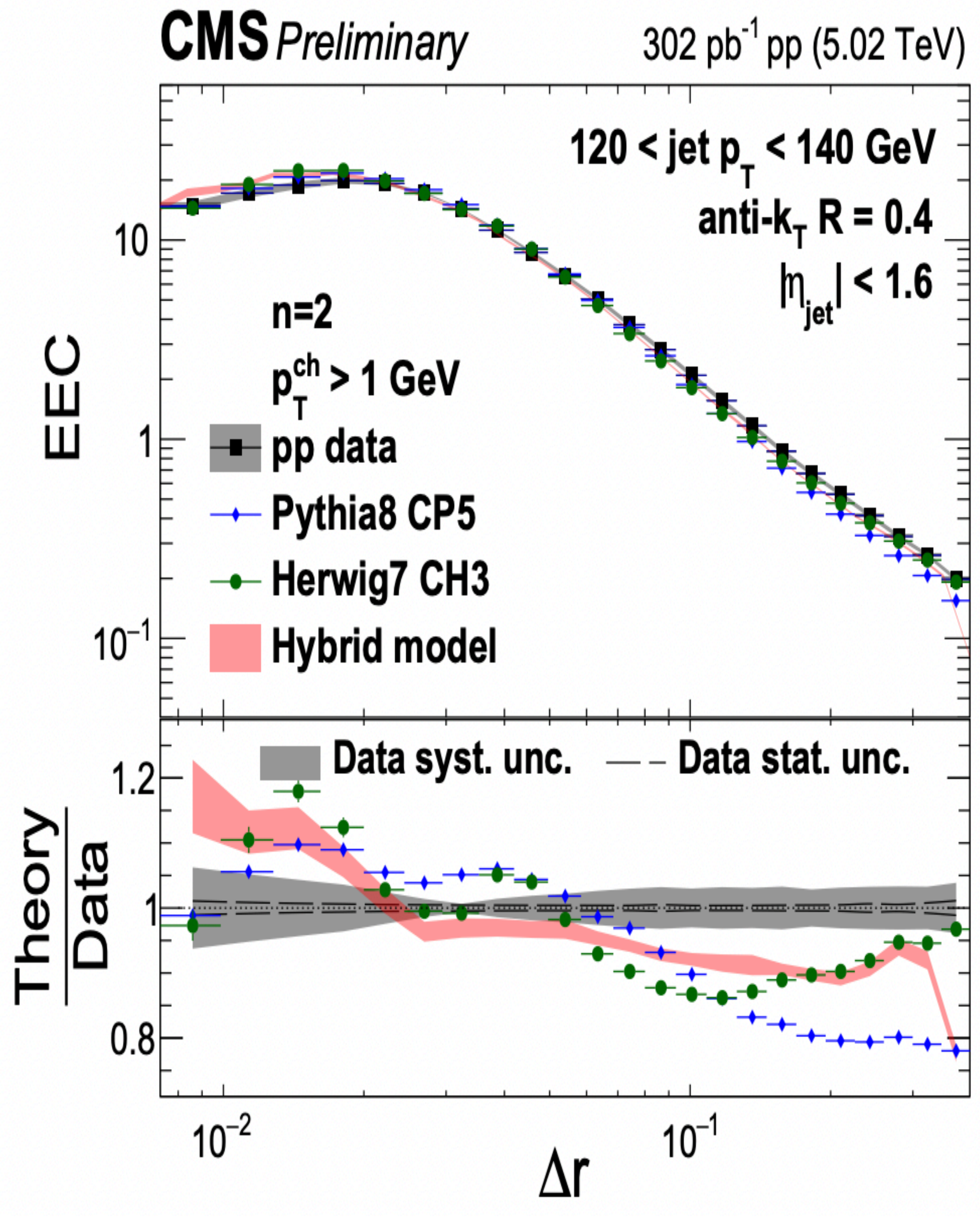
Heavy energy detector

NONPERTURBATIVE POWER CORRECTIONS IN 1. $\langle\mathcal{E}^n(\vec{n}_1)\mathcal{E}^n(\vec{n}_2)\rangle$

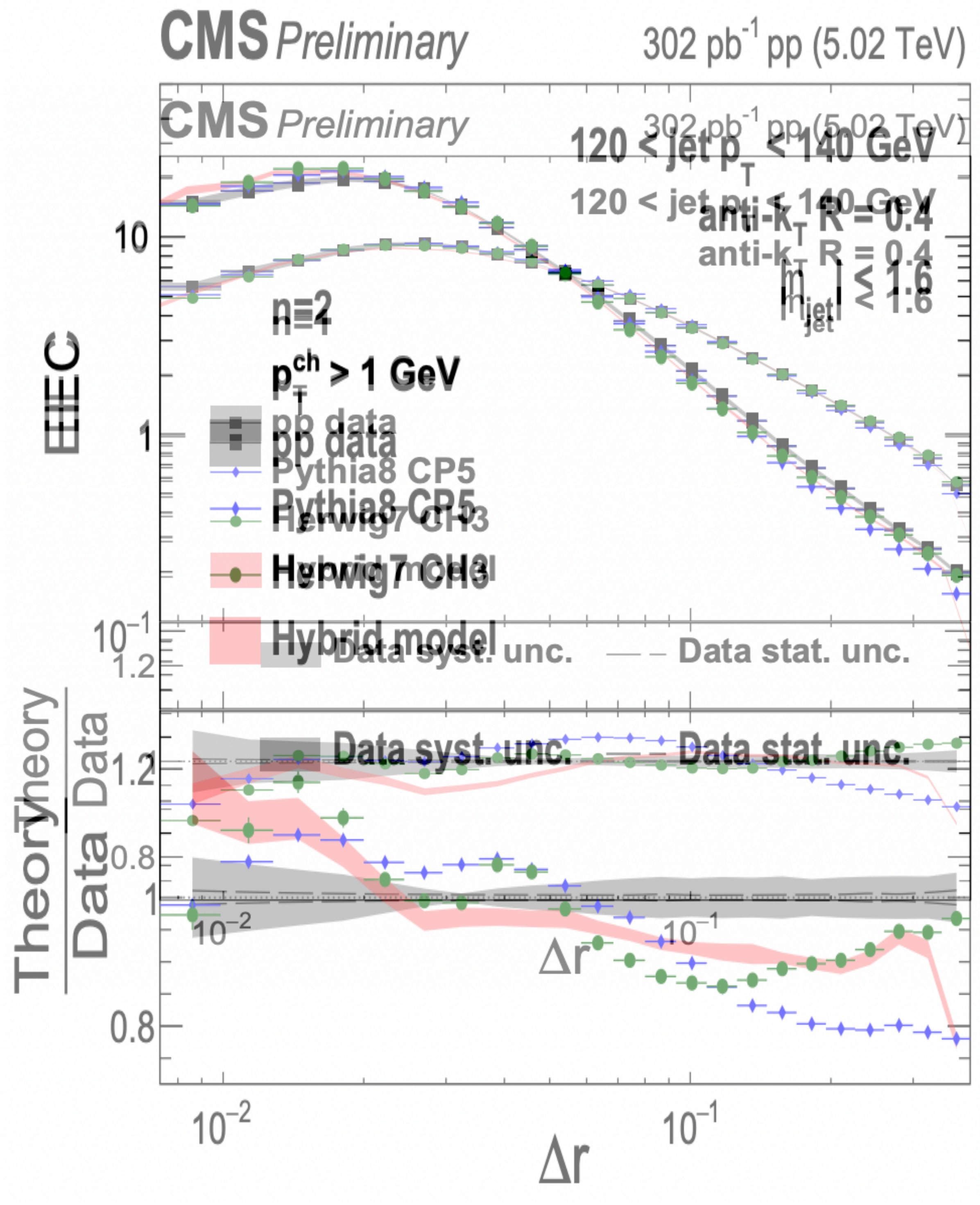
Gonzalez, KL, Moult

- **Same Power scaling modification clearly observed in experimental data**





$\Longrightarrow$

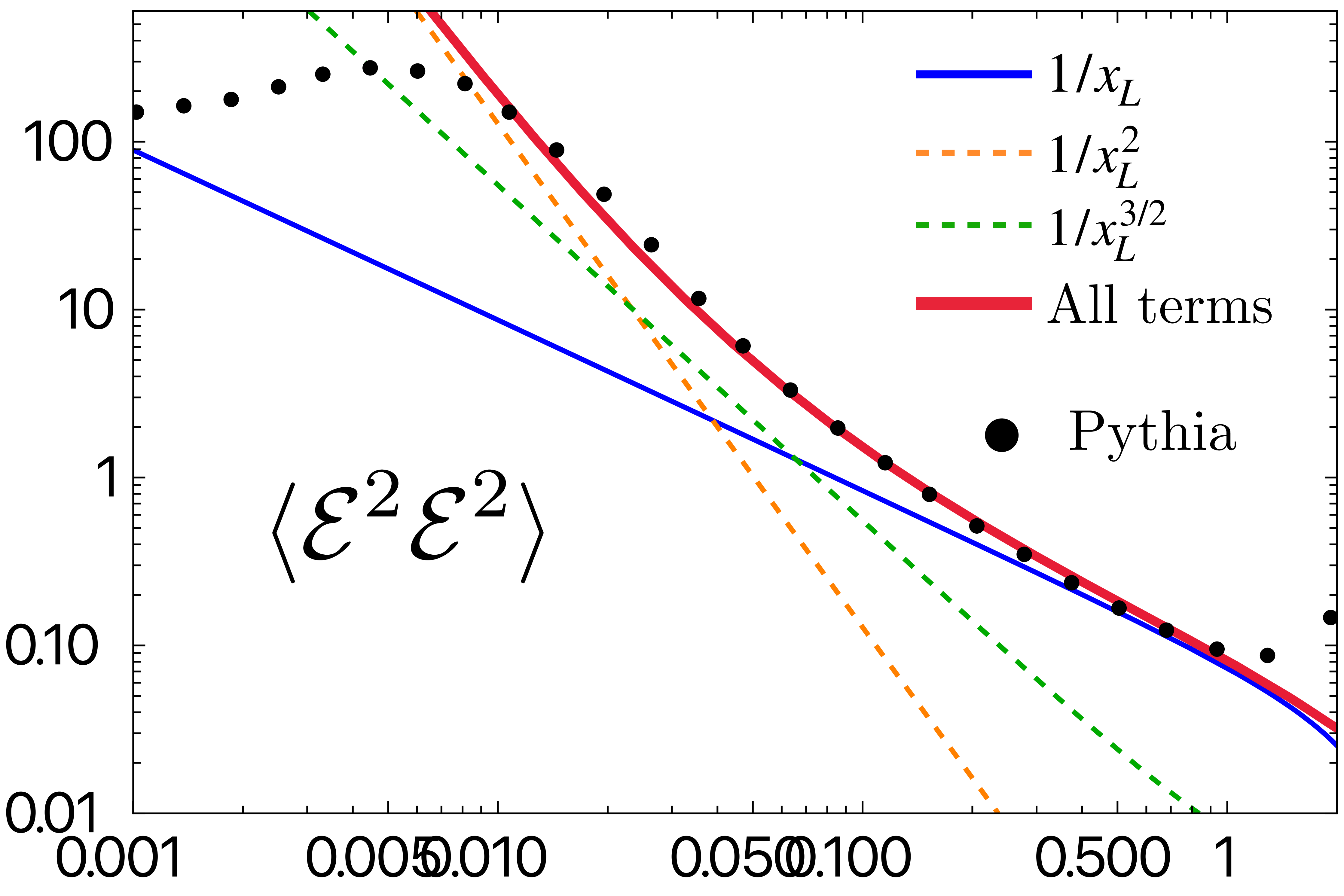


NONPERTURBATIVE POWER CORRECTIONS IN 1. $\langle \mathcal{E}^n(\vec{n}_1)\mathcal{E}^n(\vec{n}_2)\rangle$

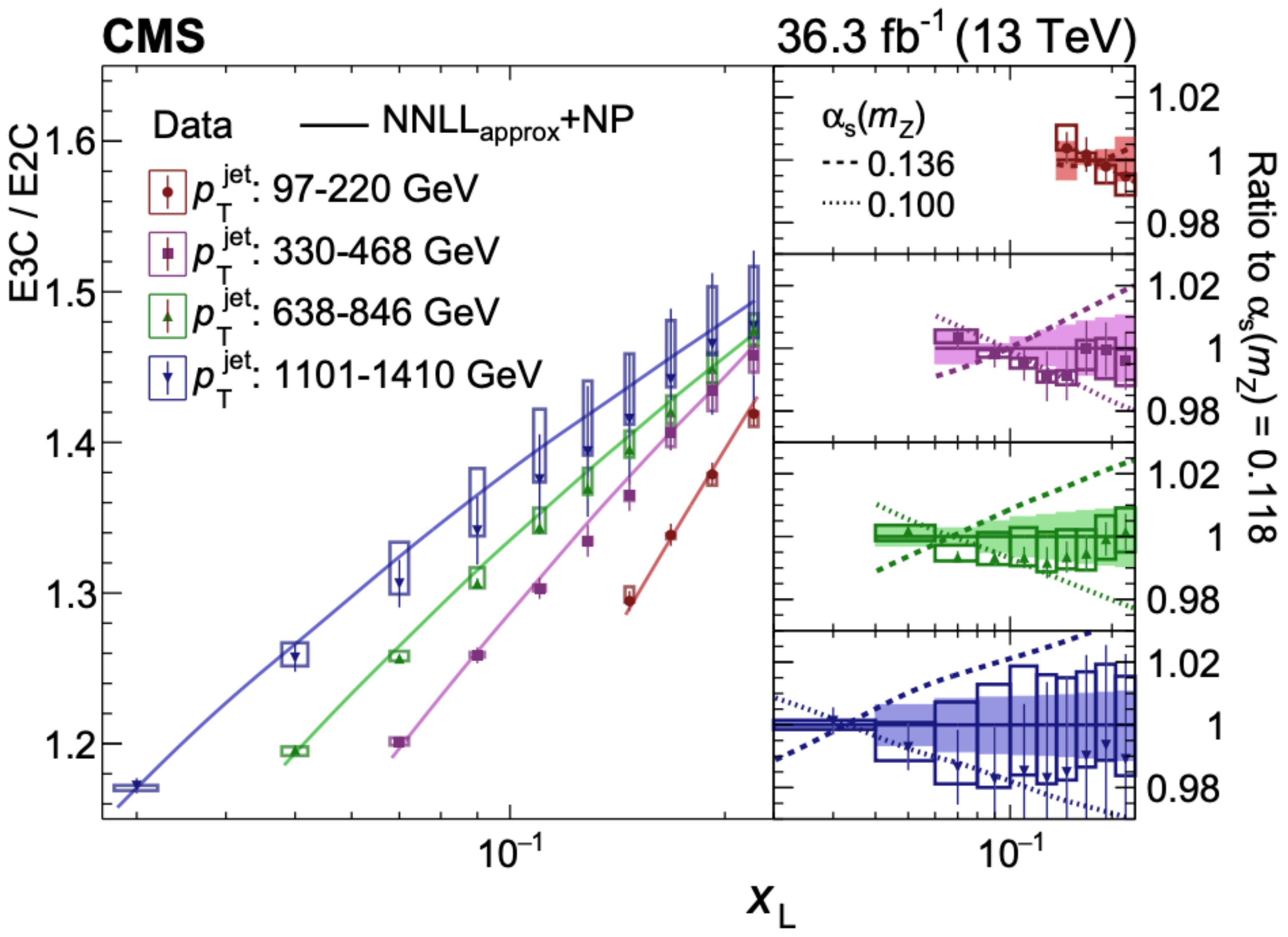
Gonzalez, KL, Moult

- Same Power scaling modification **clearly observed** in experimental MC

- The power scaling modification is consistent with the **full OPE structure**



2. $\langle \mathcal{E}_{\rm trk}(\vec{n}_1)\mathcal{E}_{\rm trk}(\vec{n}_2)\rangle$ PRECISION MEASUREMENTS

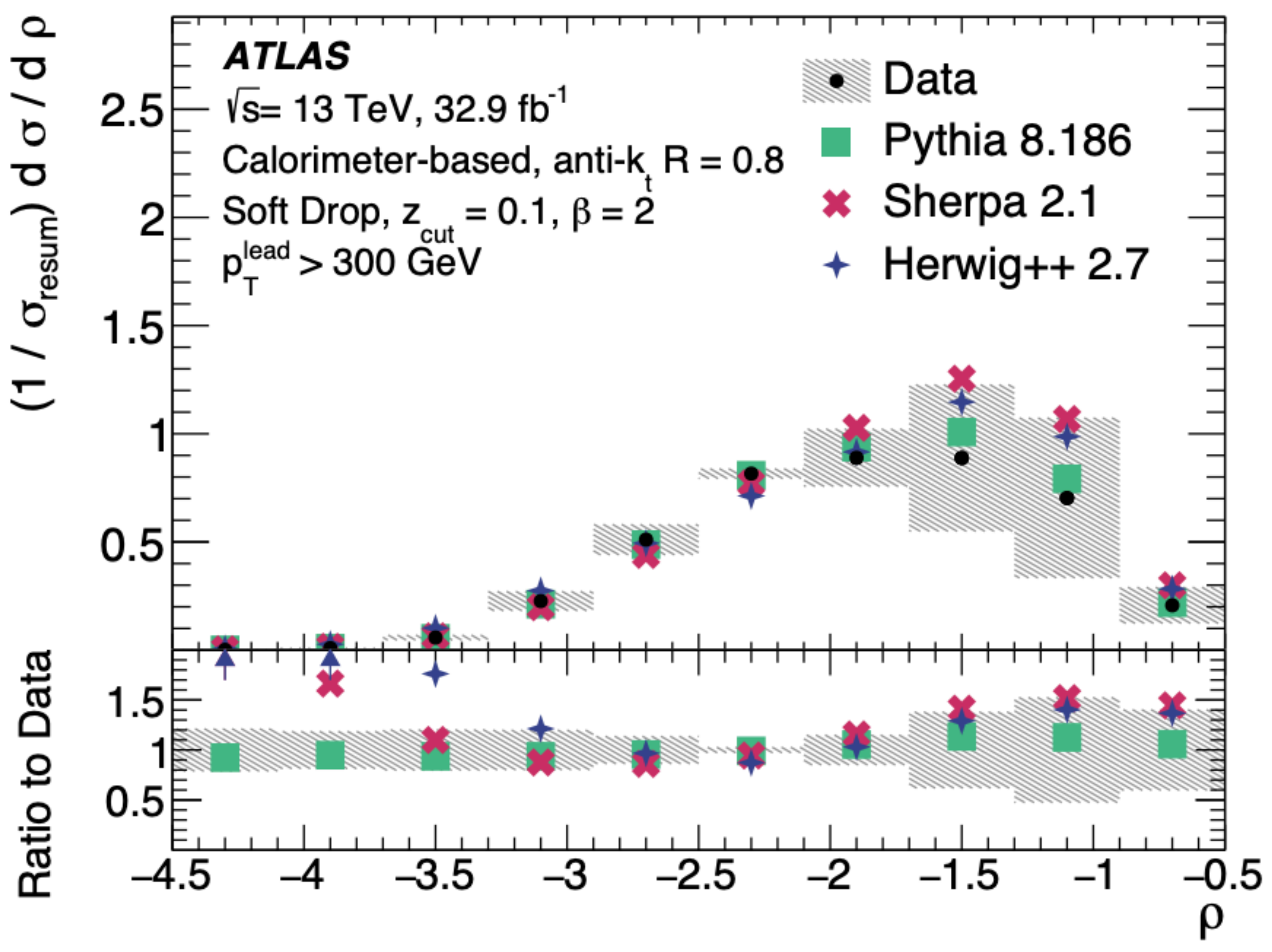


$$\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050}$$

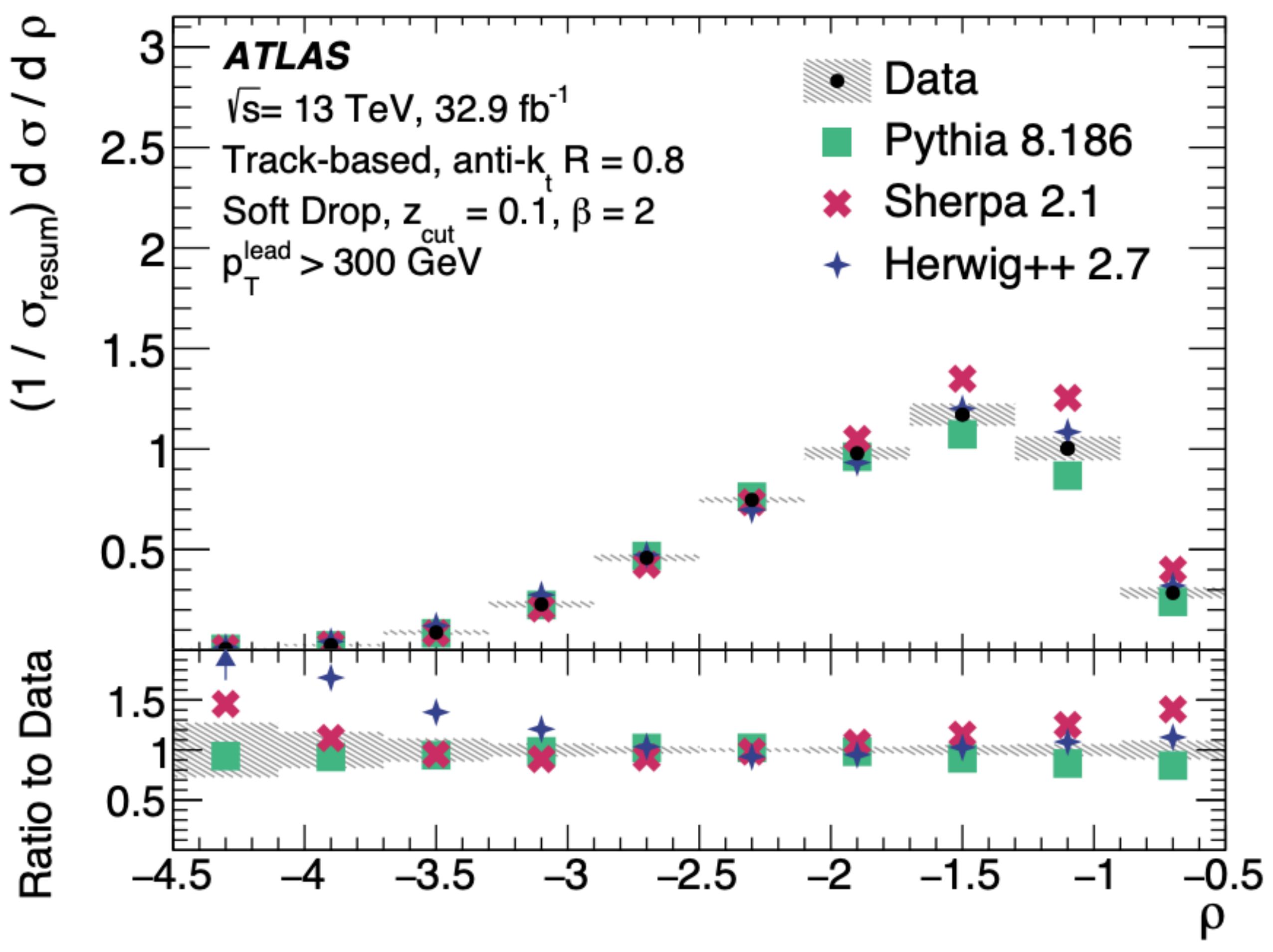
$$\frac{\langle \mathcal{E}_1\mathcal{E}_2\cdots\mathcal{E}_N\rangle}{\langle \mathcal{E}_1\mathcal{E}_2\rangle} \sim \frac{\langle \mathbb{O}^{[N+1]}\rangle}{\langle \mathbb{O}^{[3]}\rangle} \sim \theta_L^{\gamma(N+1)-\gamma(3)}$$

$$\gamma(N) \propto \alpha_s$$

- **Most precise jet substructure measurements of strong coupling constant with energy correlators**
- **Major experimental uncertainties come from measurements being performed at all particles level**

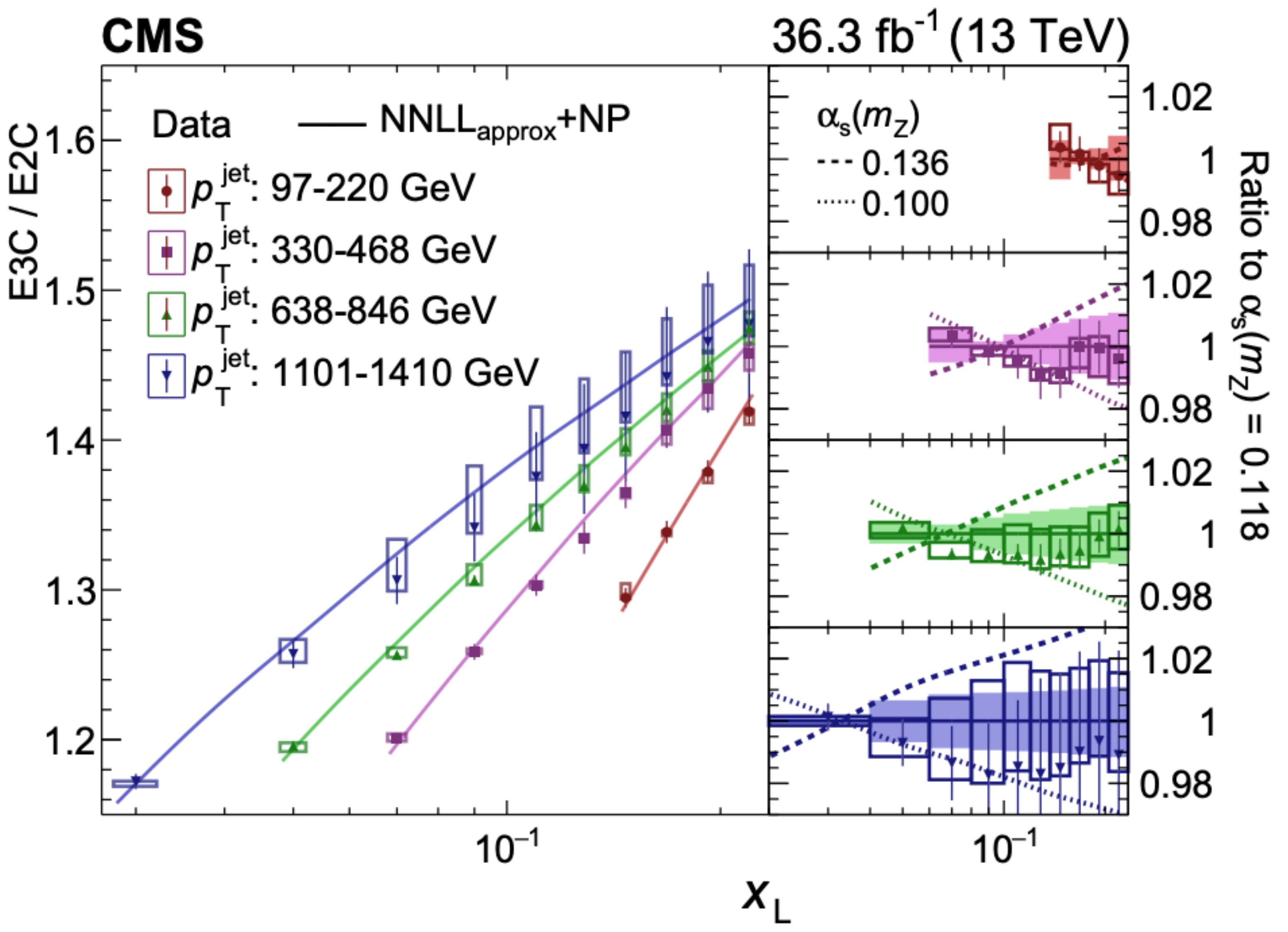


All particles



Tracks

2. $\langle \mathcal{E}_{\rm trk}(\vec{n}_1)\mathcal{E}_{\rm trk}(\vec{n}_2)\rangle$ PRECISION MEASUREMENTS



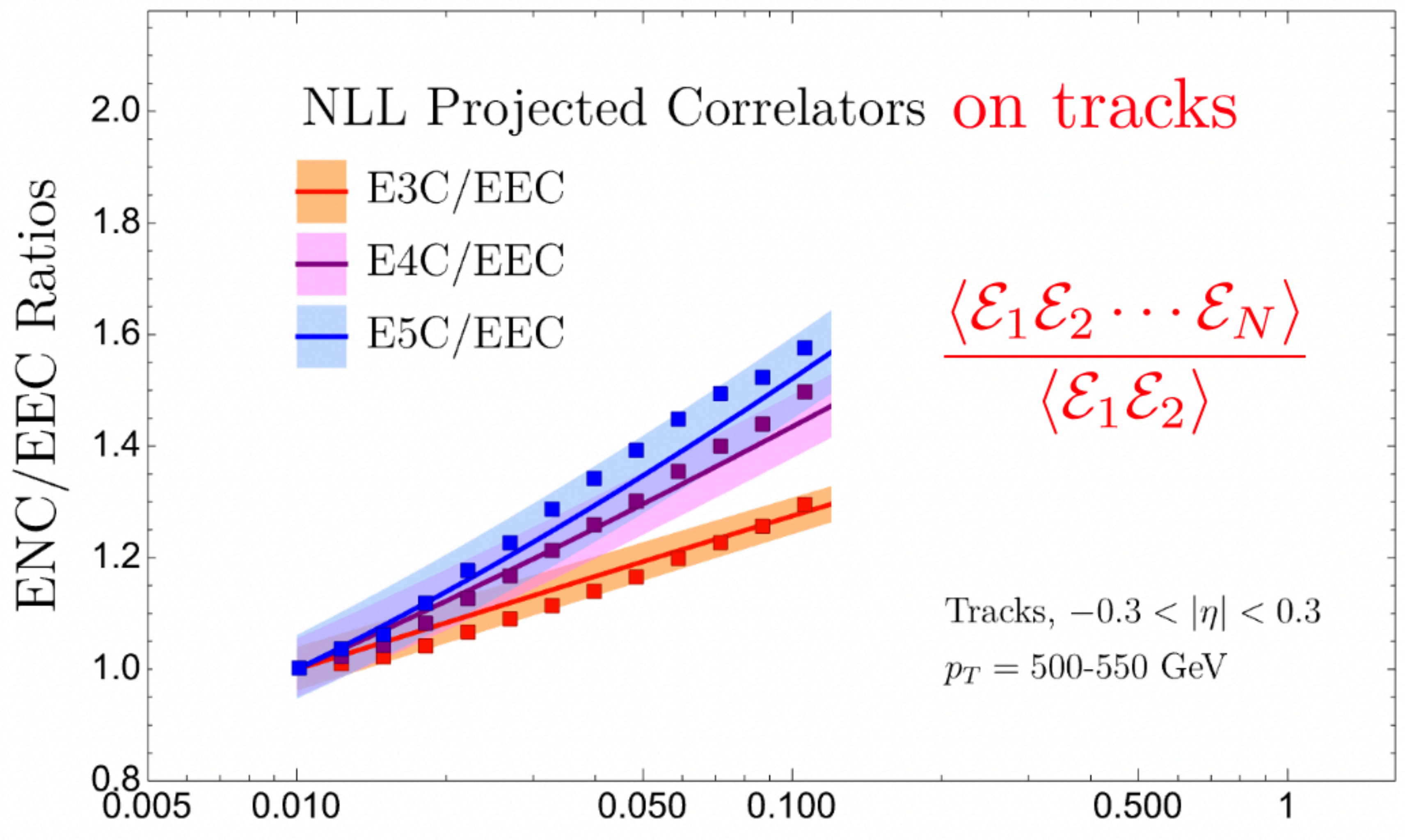
$$\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050}$$

$$\frac{\langle \mathcal{E}_1\mathcal{E}_2\cdots\mathcal{E}_N\rangle}{\langle \mathcal{E}_1\mathcal{E}_2\rangle} \sim \frac{\langle \mathbb{O}^{[N+1]}\rangle}{\langle \mathbb{O}^{[3]}\rangle} \sim \theta_L^{\gamma(N+1)-\gamma(3)}$$

$$\gamma(N) \propto \alpha_s$$

- **Most precise jet substructure measurements of strong coupling constant with energy correlators**
- **Major experimental uncertainties come from measurements being performed at all particles level**

Predictions for tracks in Energy Correlators



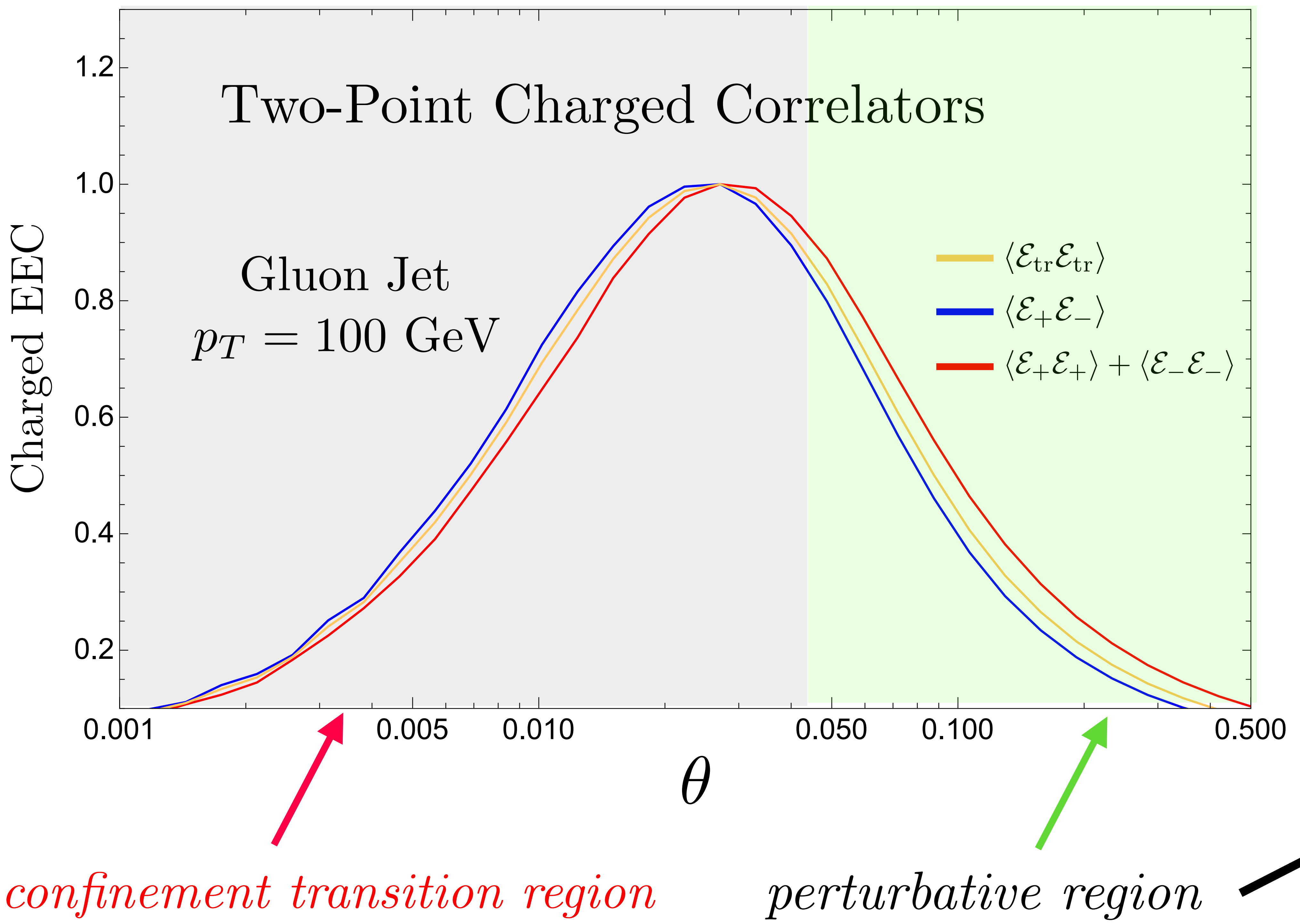
θ

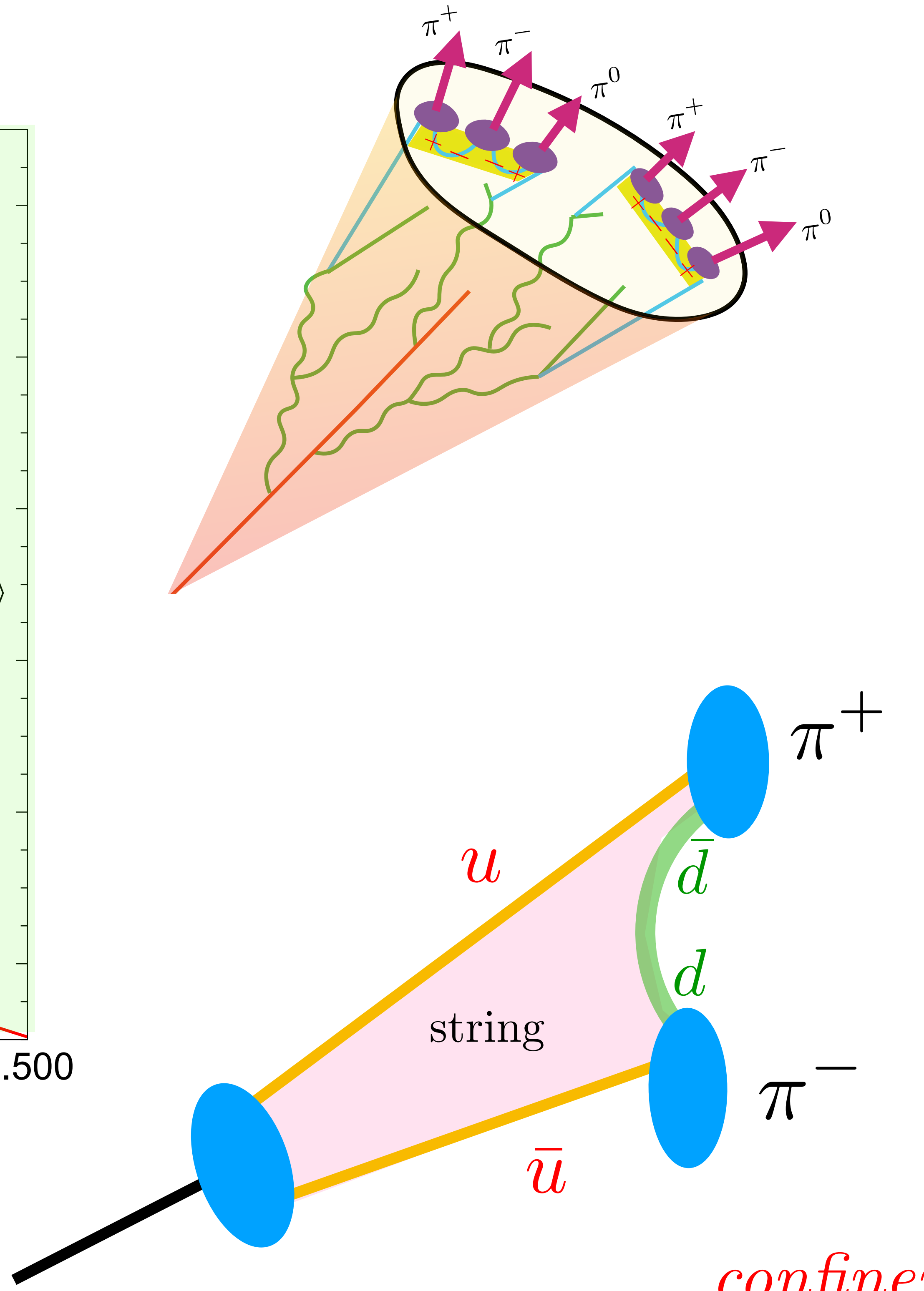
KL, Li, Moult, Waalewijn `In Progress

3. $\langle \mathcal{E}_+(\vec{n}_1)\mathcal{E}_-(\vec{n}_2)\rangle$ CONFINEMENT TRANSITION

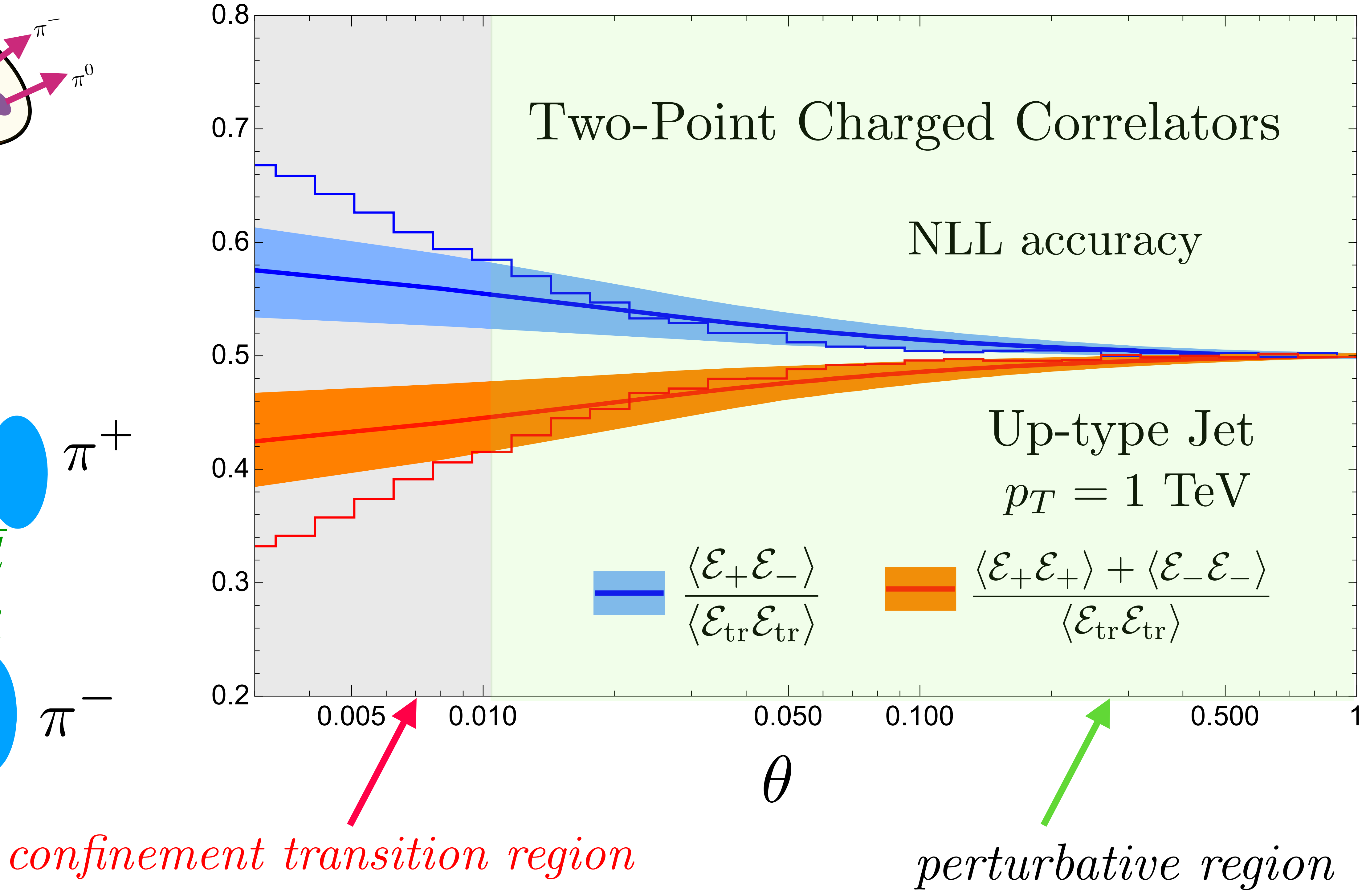
KL, Moult `23

- **Unlike-signed charged correlators are correlated more as the angle becomes smaller!**



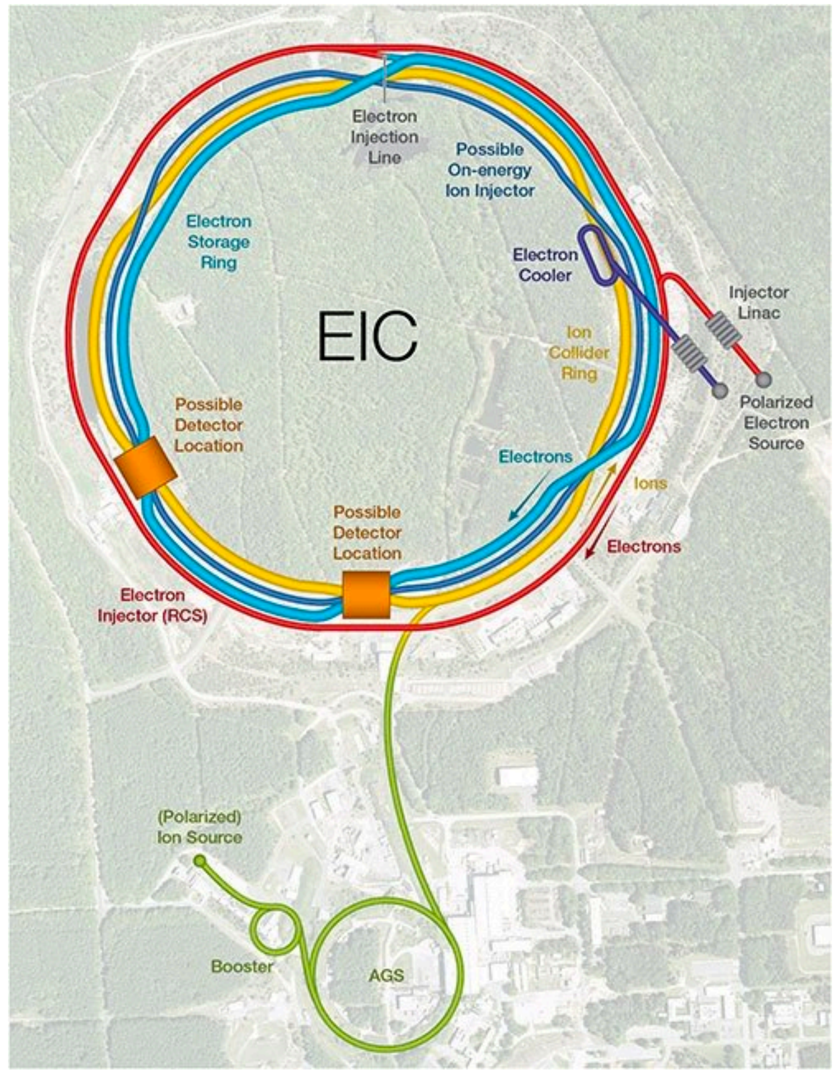


$\langle \mathcal{E}_+\mathcal{E}_-\rangle, \langle \mathcal{E}_+\mathcal{E}_+\rangle, \langle \mathcal{E}_-\mathcal{E}_-\rangle$

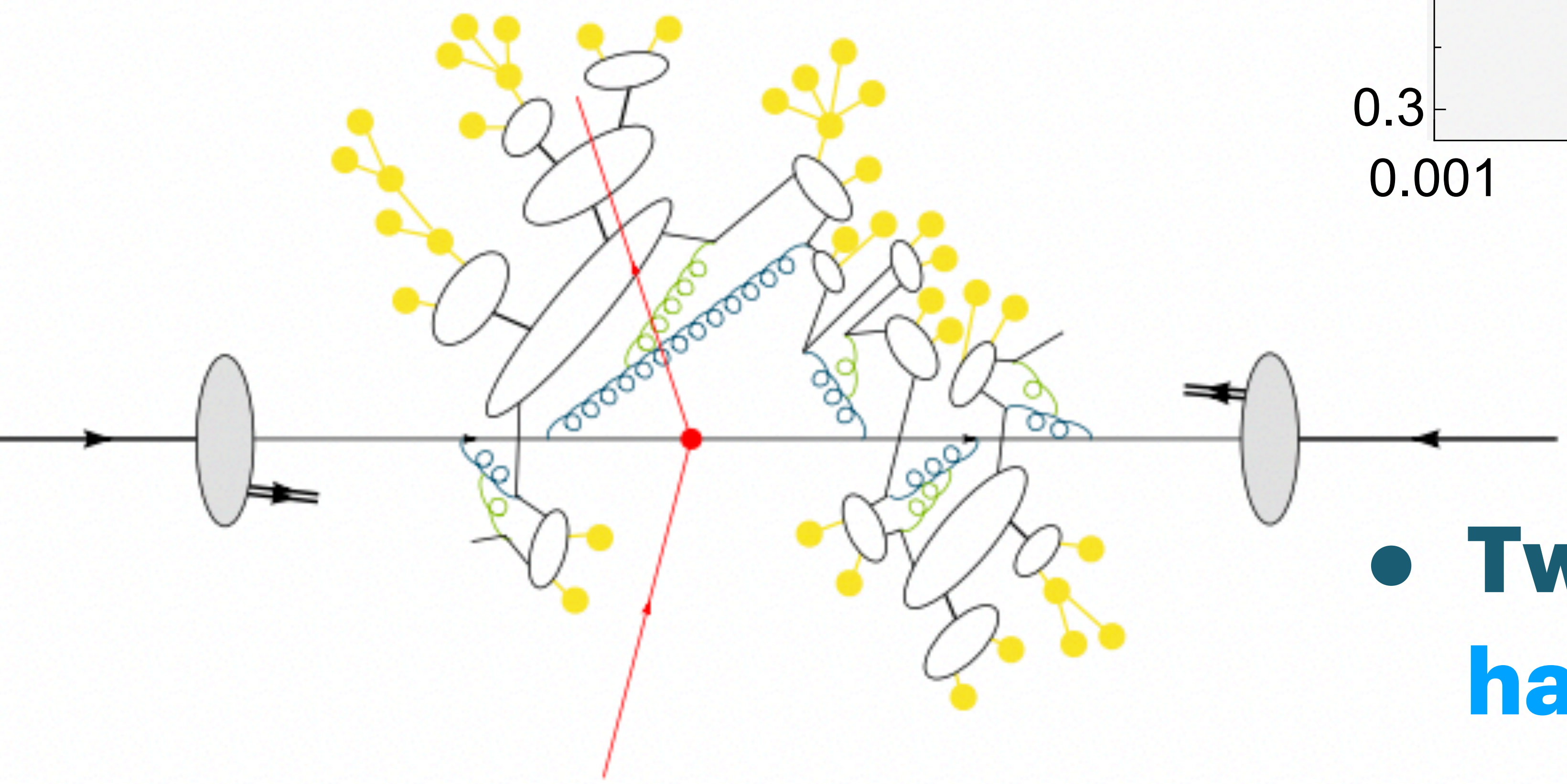


- **The correlation between unlike-signed hadron pair is expected to grow in string-like hadronization**

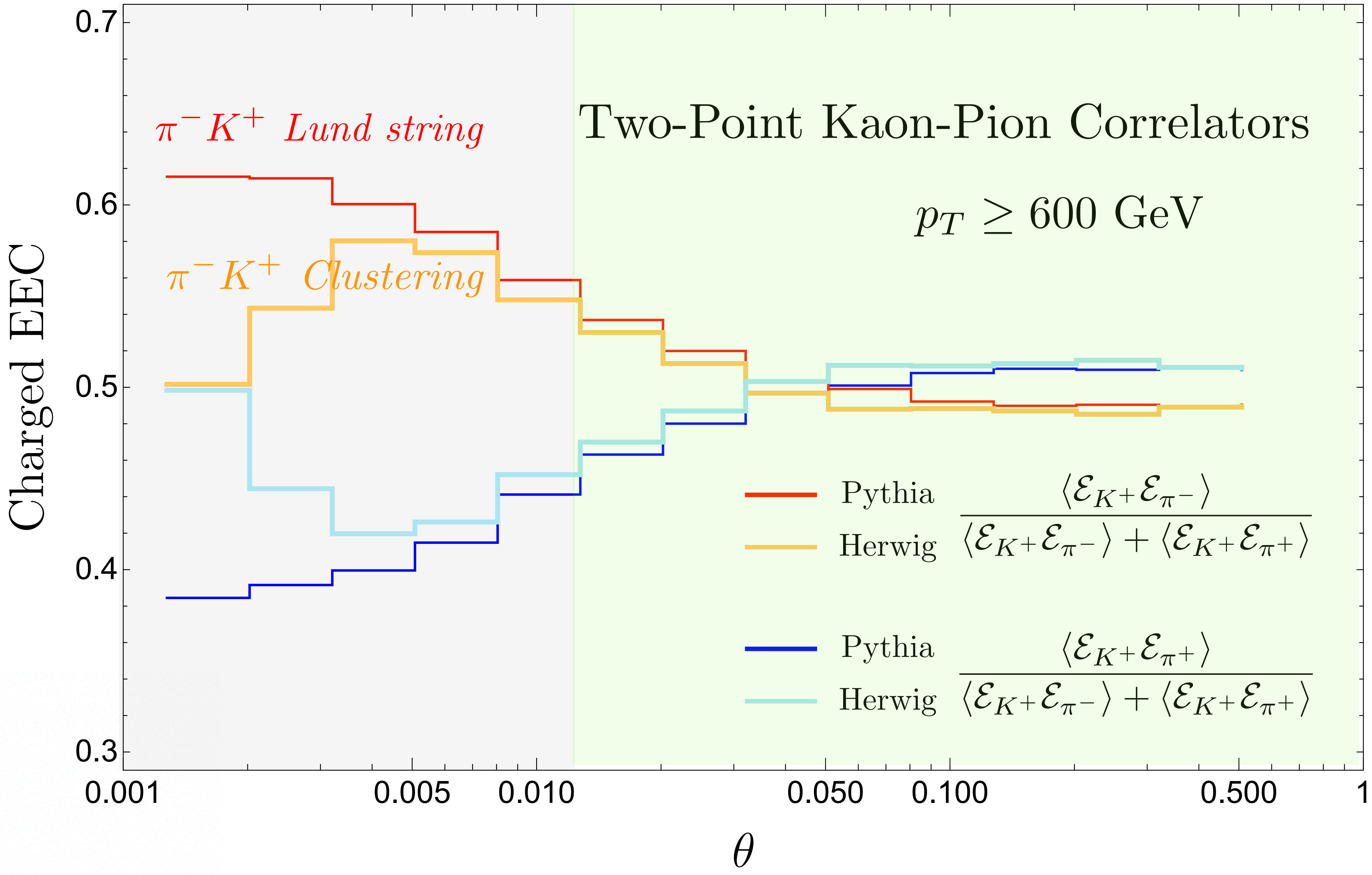
3. $\langle \mathcal{E}_{\pi^-}(\vec{n}_1)\mathcal{E}_{K^+}(\vec{n}_2)\rangle$ DISCRIMINATING HADRONIZATION MECHANISMS

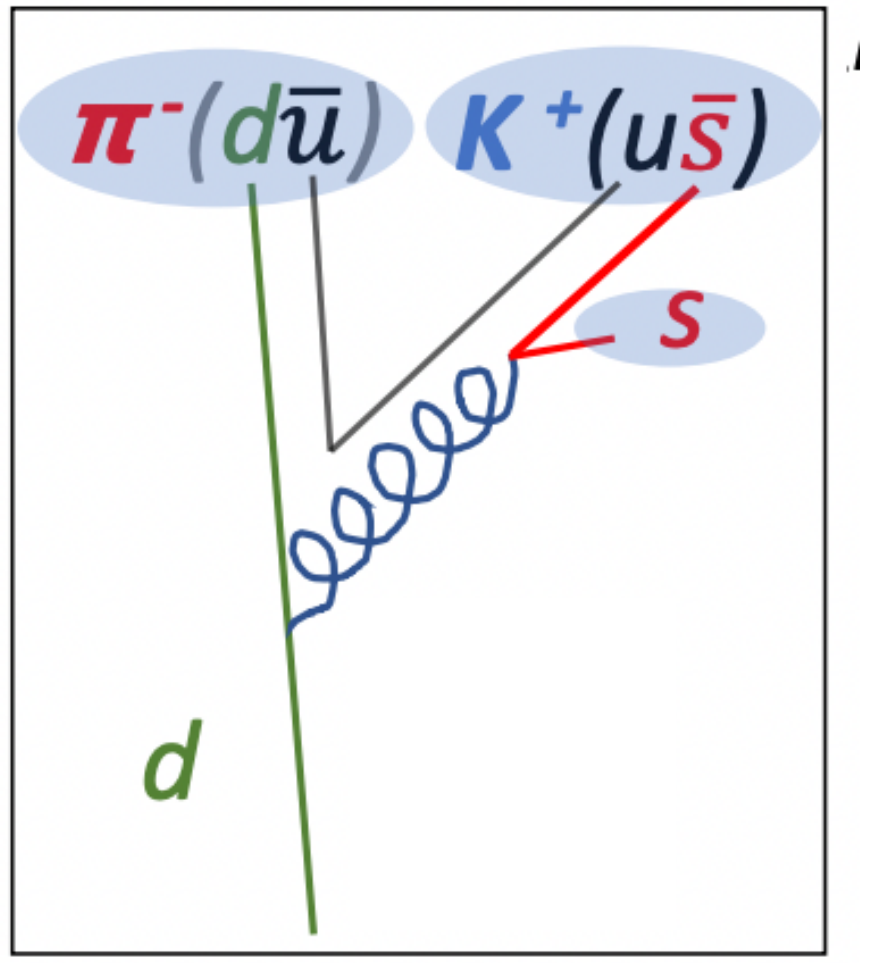


Clustering model (Herwig)



KL, Moult, Song, Sterman





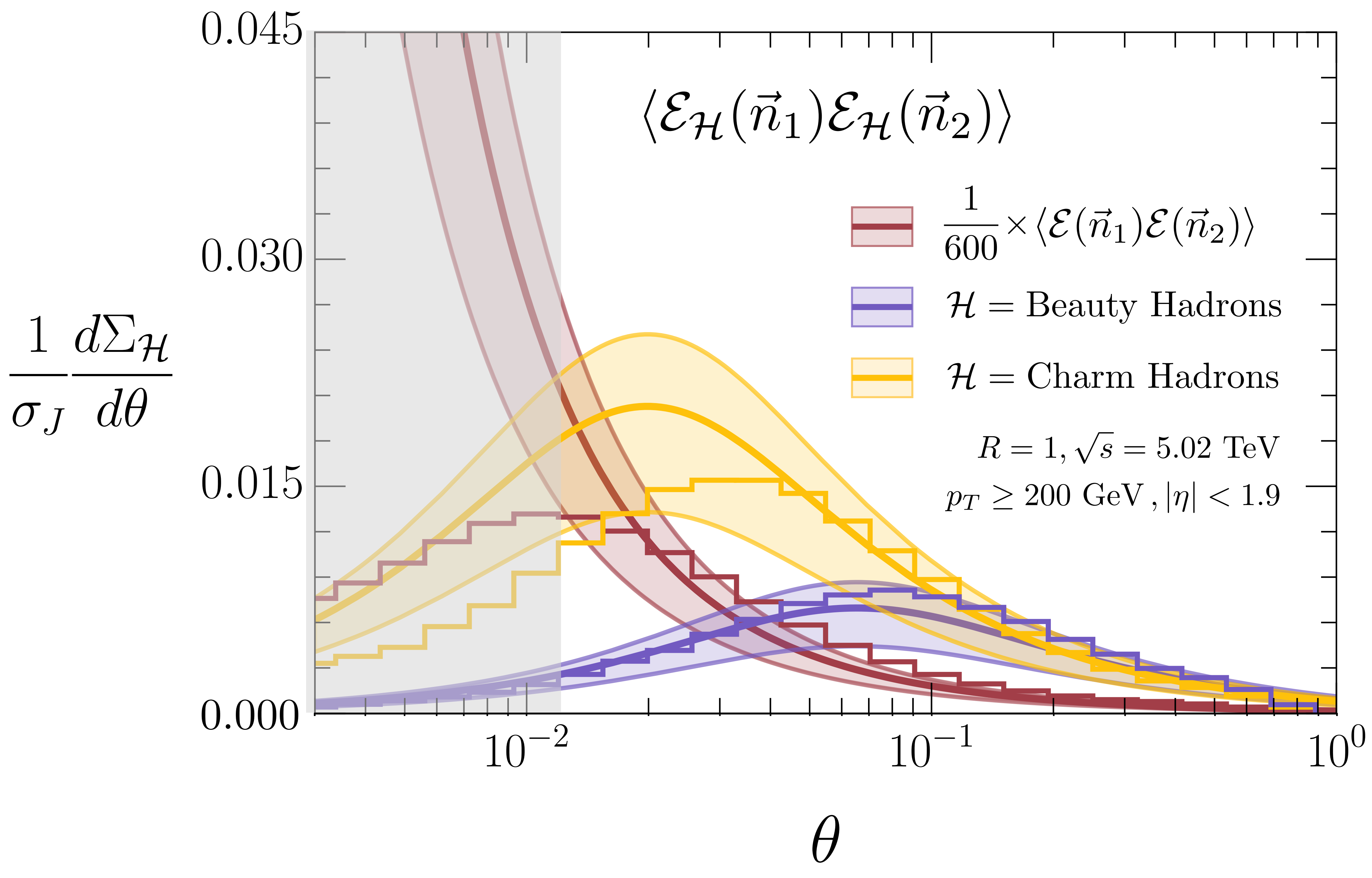
Lund string model (Pythia)

See also Chien, Deshpande, Mondal, Sterman

- **Two-point charged correlators already nontrivially probe the two hadronization mechanisms by eye, and pave the path to go even beyond!**

4. $\langle \mathcal{E}_H(\vec{n}_1)\mathcal{E}_{\bar{H}}(\vec{n}_2)\rangle$ PROFILING PARTONIC SPLITTING

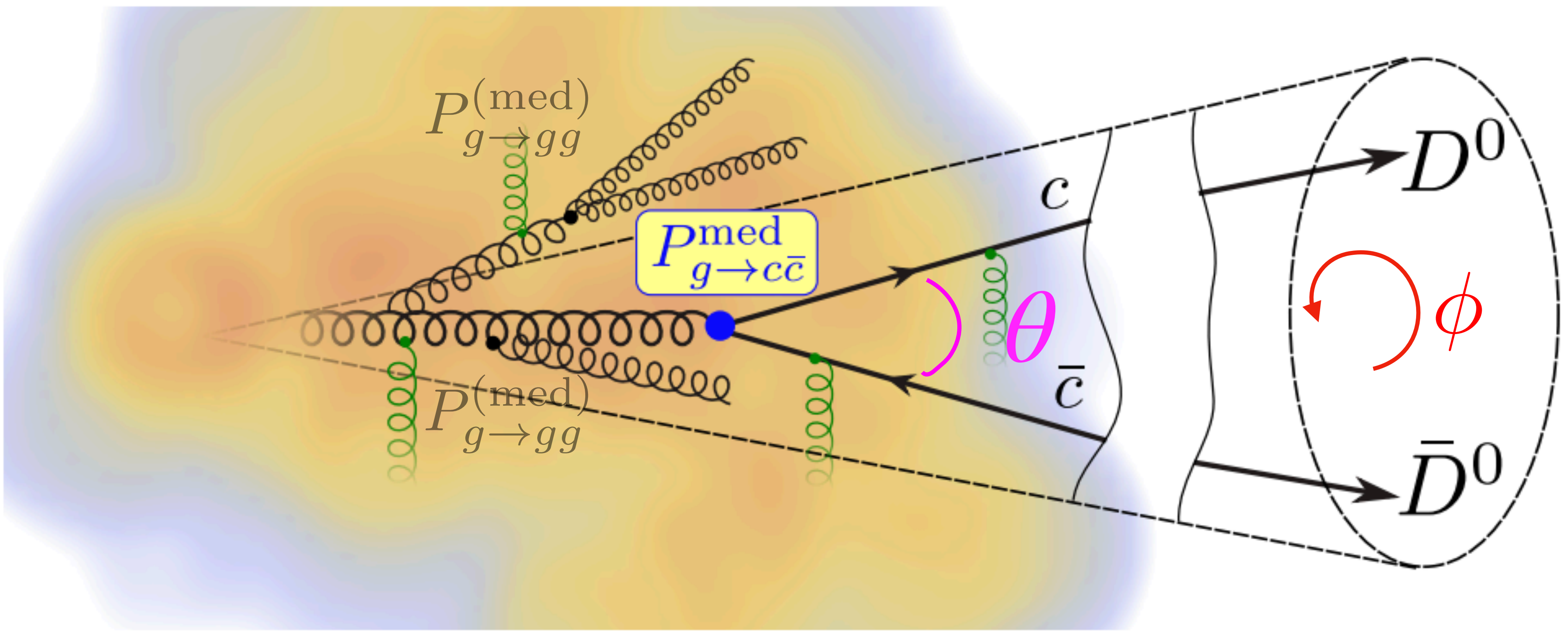
Barata, Brewer, KL, Silva `25

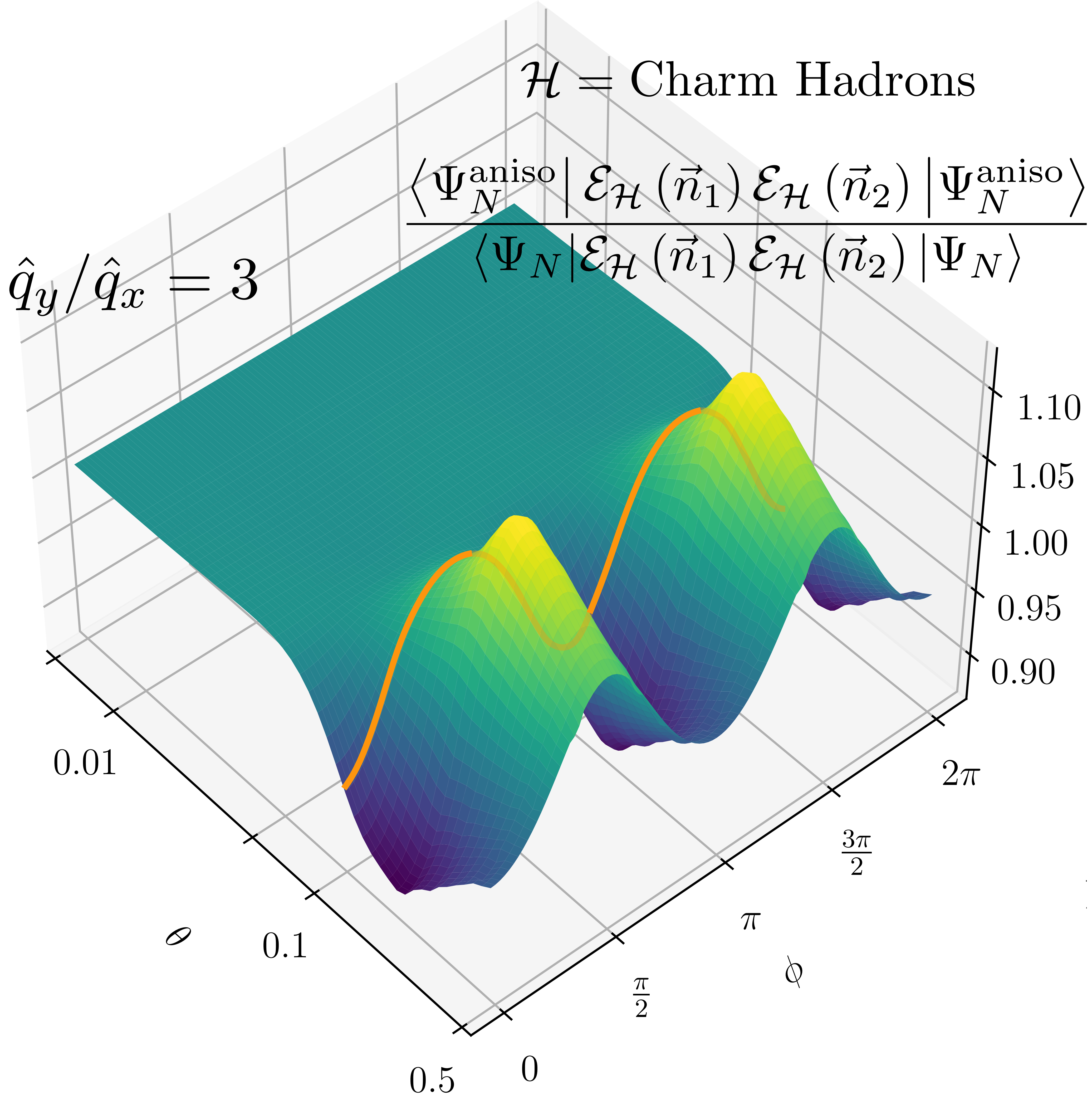


- **At leading order, enhanced sensitivity to $g \to Q\bar{Q}$ splitting.**
- **Mass effects clearly seen**

4. $\langle \mathcal{E}_H(\vec{n}_1)\mathcal{E}_{\bar{H}}(\vec{n}_2)\rangle$ PROFILING PARTONIC SPLITTING

Barata, Brewer, KL, Silva `25





- **Ability to clearly tag $g \to Q\bar{Q}$ splitting gives access to see how it is modified in thermal QGP**

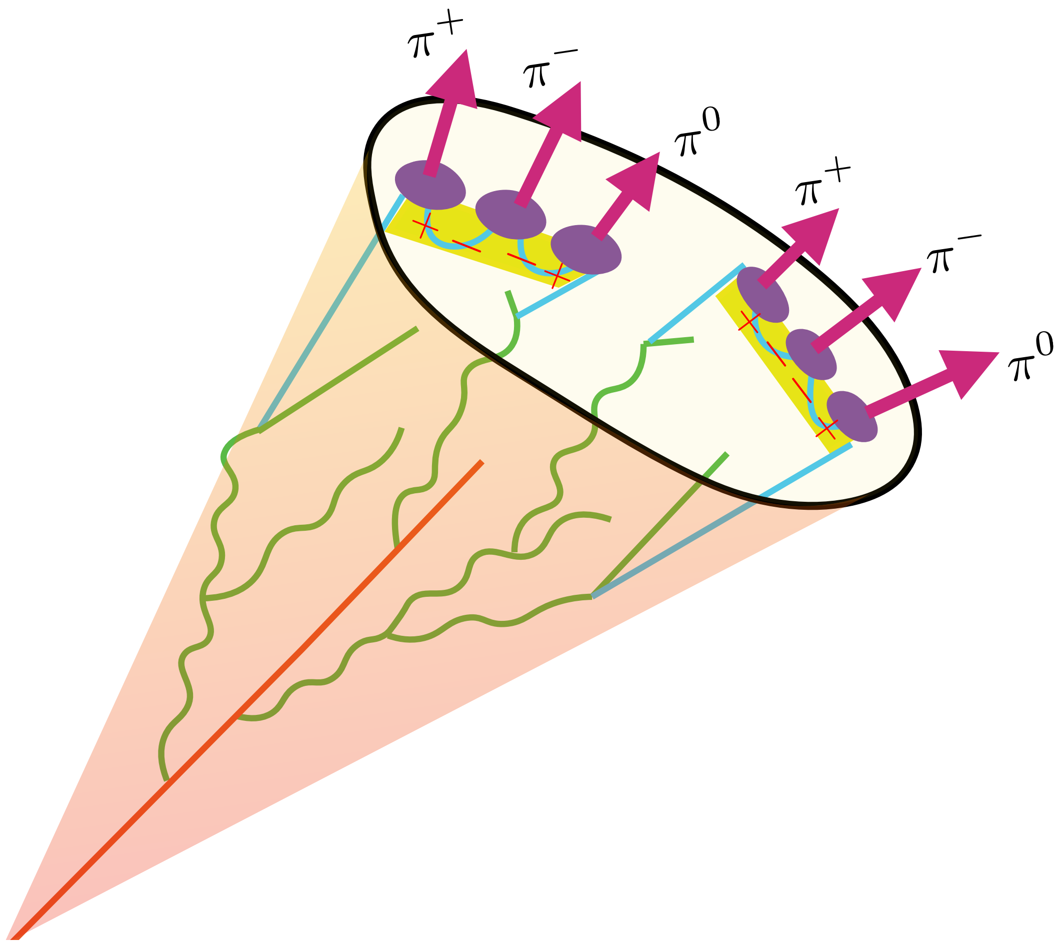
GENERALIZING ENERGY FLOW CORRELATIONS

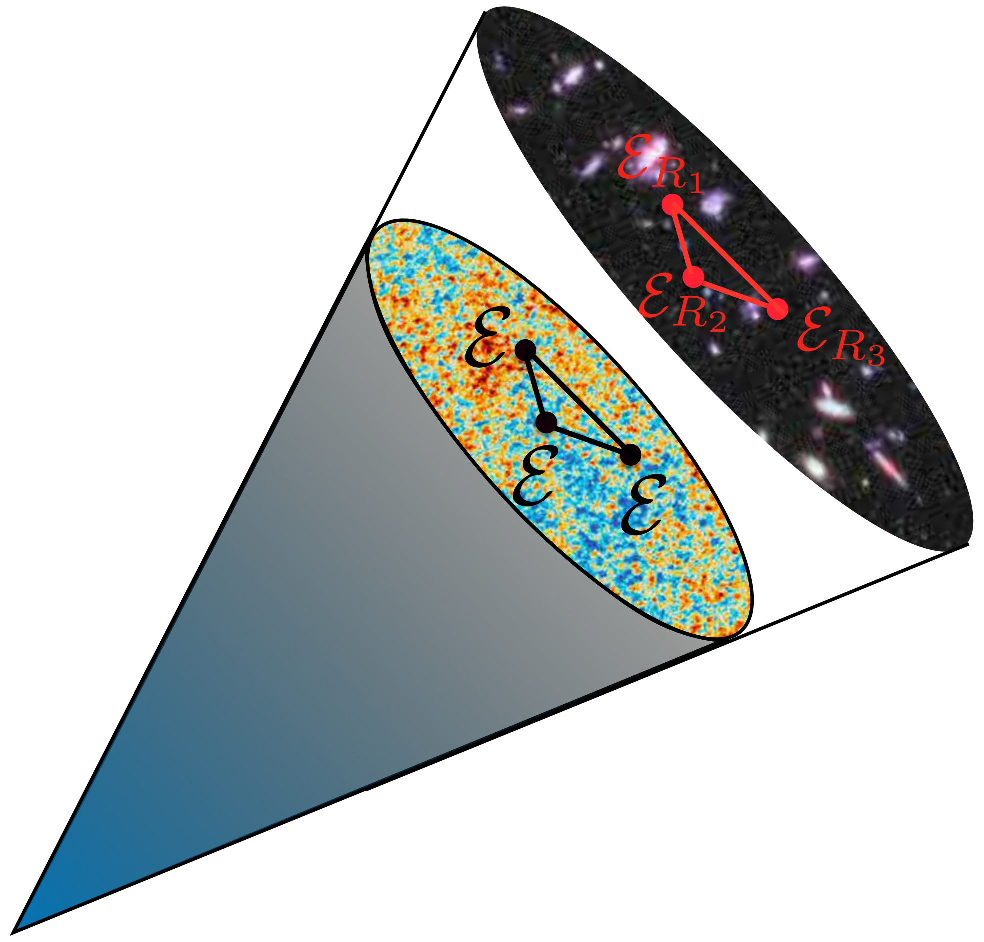
- **Writing down more general detectors allows us to systematically consider more general correlations!**

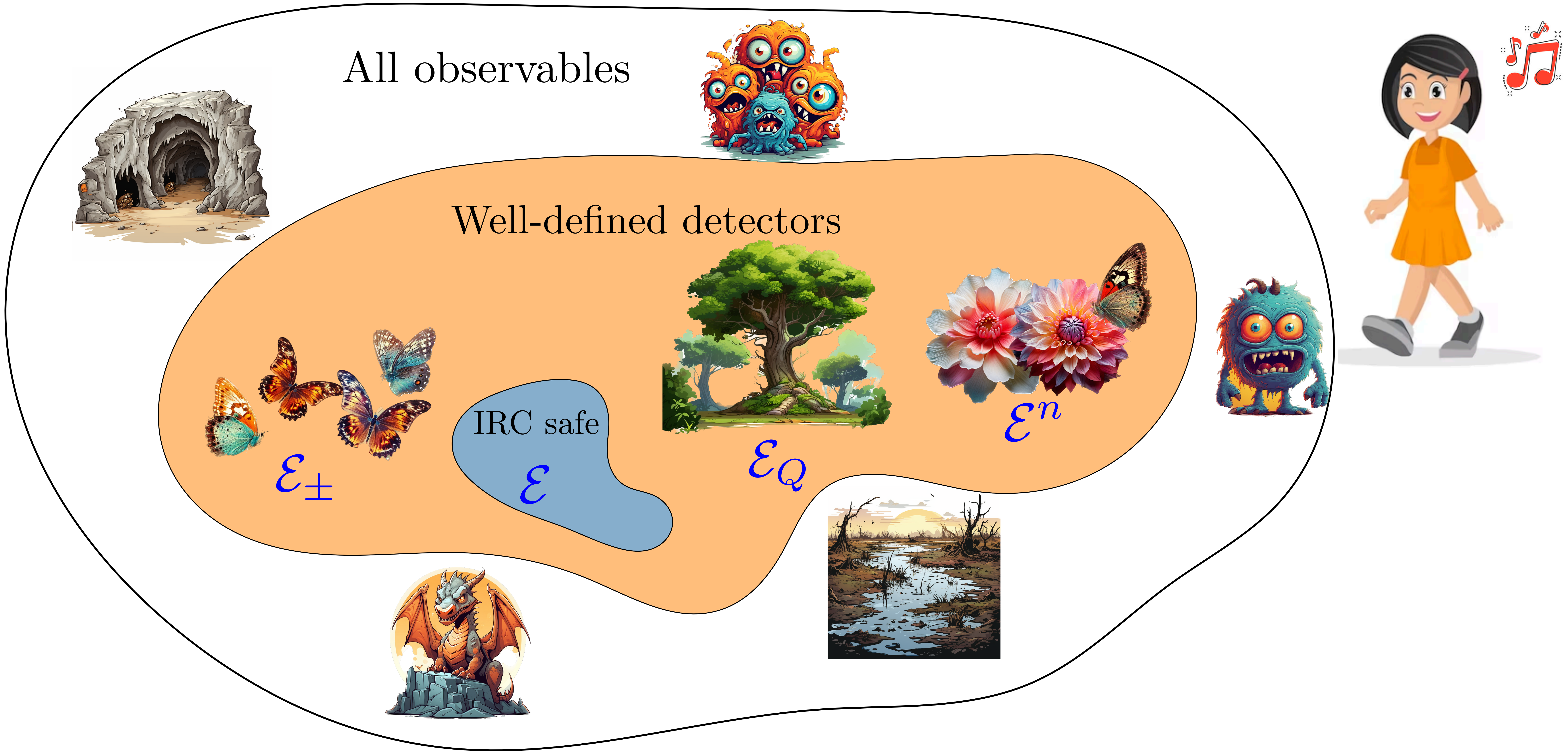
KL, Moult `23
Gonzalez, KL, Moult

$$\langle\Psi|\mathbb{O}^{R_1}_{J_{L_1}}(\vec{n}_1)\cdots\mathbb{O}^{R_N}_{J_{L_N}}(\vec{n}_N)|\Psi\rangle$$

- **By asking the right questions, one can let confinement do its magic!**







Let us explore the landscape of well-defined detectors and study its correlations!