Irrational CFT from coupled anyon chains?

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- c$<$1 minimal model
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- unitary
- no extended algebra
- discrete spectrum (unlike Liouville theory)
- isolated, or not inside a conformal manifold (unlike U(1) compact boson)

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This talk is about a lattice model which realises a candidate irrational CFT as a critical phase. The critical phase is reached without fine tuning!

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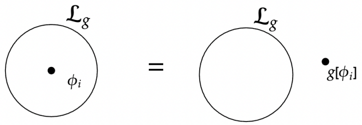
For the Ising model we have

$$\sigma \times \sigma \to 1 + \epsilon, \quad \epsilon \times \epsilon \to 1, \quad \sigma \times \epsilon \to \epsilon$$

Non-invertible symmetry

In 2D, symmetry operators are given by topological lines.



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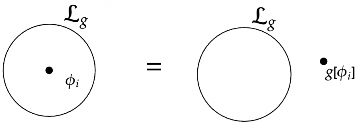
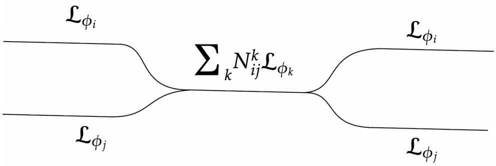


If g is an element of a matrix group, then $g[\phi_i] = R[g]_i^{\ j} \phi_j$.
Some topological lines of a 2D CFT are also are in one to one correspondence to the conformal primaries ϕ_i. The lines follow the same fusion rules given by the Verlindle's formula, they are called Verlindle lines.



Non-invertible symmetry

From the fusion rule of the 2D Ising CFT, we have

$$\mathcal{L}_\sigma \times \mathcal{L}_\sigma \to \mathcal{L}_1 + \mathcal{L}_\epsilon, \quad \mathcal{L}_\epsilon \times \mathcal{L}_\epsilon \to \mathcal{L}_1, \quad \mathcal{L}_\sigma \times \mathcal{L}_\epsilon \to \mathcal{L}_\epsilon$$

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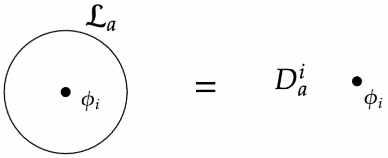
It is called non-invertible symmetry because they do not form a group structure. More than one object can appear after the fusion.

Tricritical Ising model and the Fibonacci symmetry

The action of the Verlindle line on the conformal primaries is



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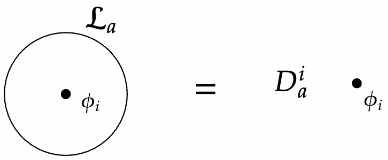

where $D_a^i = \frac{S_a^i}{S_0^i}$. For the tricritical Ising model, we have a topological line (which we denote as W), satisfying the fusion rule

$$W \times W = 1 + W$$

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The action of W on the conformal primaries are

	1	ϵ	σ	σ'	ϵ'	ϵ''
(r,s)	(1,1)	(1,2)	(2,3)	(2,1)	(3,2)	(1,4)
$(h,\bar{h})$	(0,0)	$(\frac{1}{10},\frac{1}{10})$	$(\frac{3}{80},\frac{3}{80})$	$(\frac{7}{16},\frac{7}{16})$	$(\frac{3}{5},\frac{3}{5})$	$(\frac{3}{2},\frac{3}{2})$
W	φ	$-\varphi^{-1}$	$-\varphi^{-1}$	φ	$-\varphi^{-1}$	φ

Here, $\varphi = \frac{1+\sqrt{5}}{2}$ is the golden ratio. We also have the Z_2 symmetry. We denote the Z_2 even/odd operators as ϵ/σ respectively.

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The tri-critical Ising model is stable if you preserve both the Fibonacci symmetry and the Z_2 symmetry. That is, there is no relevant operator invariant under both of the symmetries. If we were able to preserve these symmetries on the lattice level, we will get a lattice model realising the tricritical Ising model as a critical phase. (We will come back to this later.)

Coupled tricritical Ising model

We will consider N-copies of tricritical Ising model coupled together, that is

$$S = \sum_{a=1}^{N} S^{(a)}_{\mathrm{tri-Ising}} + K \int dx^2 \sum_{a \neq b} \sigma'^{(a)} \sigma'^{(b)}$$

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Here S_N is the permutation of the N-copies, and Z_2^{diag} is the diagonal Z_2 symmetry. One can see that the K term is the only relevant deformation that preserves such a symmetry. Remember that $\Delta_{\sigma'} = 7/8$.

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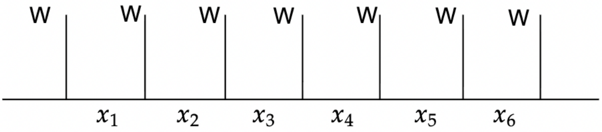
Anyon chain model

The anyon chain formulation allows us to preserve the non-invertible symmetry on the lattice level. [A. Feiguin, S. Trebst, A. W. W. Ludwig, M. Troyer, A. Kitaev, Z. Wang, M. H. Freedman, Phys. Rev. Lett. 98, 160409]. The Hilbert space is defined as a chain of "anyons",



The degree of freedom are defined on the bonds, it can be either 1 or W. Reading from left to right, the anyons have to fusion rules, such as $W \times W \to 1 + W$. Notice 1 and 1 can not be neighbouring!

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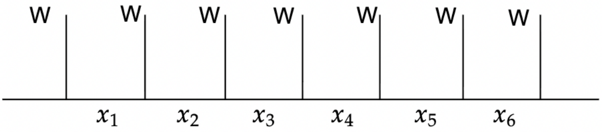

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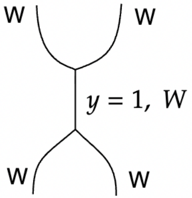
If one identify $|1\rangle = |+\rangle$ and $|W\rangle = |-\rangle$, the Hilbert space is just a spin chain with local constraints.

Anyon chain model

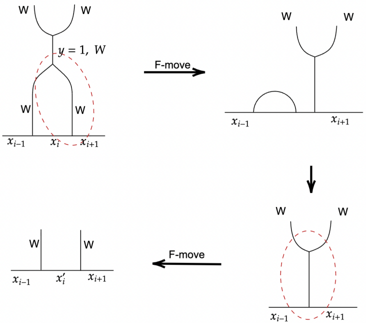
The Hamiltonian can also be written as anyon graphs:

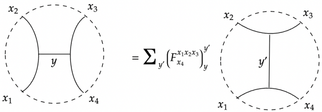
$$H_i(y) =$$



Anyon chain model

The action of the Hamiltonian can be calculated using the fusion rules:





so that

$$H(y)_i|x_{i-1}, x_i, x_{i+1}\rangle \to |x_{i-1}, x_i', x_{i+1}\rangle$$

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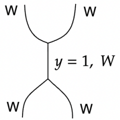
$H_i(y) =$ 

We can write the Hamiltonian as Pauli matrices

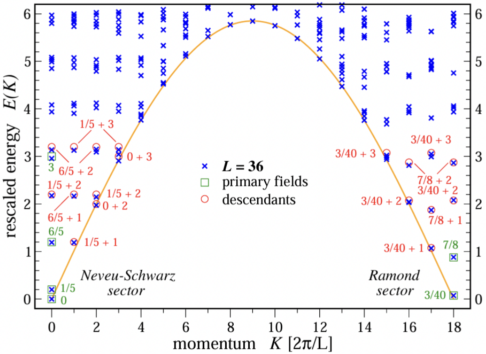
$$H = \sum_i H_i, \quad H_i = (n_{i-1} + n_{i+1} - 1) - n_{i-1} n_{i+1} (\varphi^{-3/2} \sigma_i^x - \varphi^{-3} n_i + 1 + \varphi)$$

Here $n_i = \frac{1}{2}(1 - \sigma_i^z)$. We can introduce an repulsive interaction to penalize neighbouring 1's,

$$H_{\mathrm{replusive}} = U \sum_i (1 - n_i)(1 - n_{i+1}).$$

Anyon chain model

One can study the model using exact diagonalization or DMRG [A. Feiguin, S. Trebst, A. W. W. Ludwig, M. Troyer, A. Kitaev, Z. Wang, M. H. Freedman, Phys. Rev. Lett. 98, 160409]. If $y = 1$ in $H_i(y)$, we have



The Z_2 even sector operators are located at $K = 0$, and the Z_2 odd sector operators are located at $K = \pi$. The Z_2 symmetry is given by lattice translation.

Identify the σ' operator

The σ' operator can be identified as

$$(-1)^i H_i$$

It is invariant under the Fib symmetry by construction. It is odd under the Z_2 symmetry because of the $(-1)^i$ factor. We can study the Hamiltonian

$$H = J \sum_i H_i + h \sum_i (-1)^i H_i .$$

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$$M_{4,5} + \int dx^2 \phi_{2,1}.$$

Recently [C. Copetti, L. Cordova, S. Komatsu, Phys.Rev.Lett. 133 (2024) 18, 181601] studied the S-matrix of this flow.

Two coupled anyon chains

We can consider two anyon chains coupled together

$$H = J\sum_i (H_i^{(1)} + H_i^{(2)}) + K\sum_i H_i^{(1)} H_i^{(2)}$$

The 2nd term breaks the translation translations into its diagonal subgroup. In the CFT limit, this means the

$$Z_2^{(1)} \times Z_2^{(2)} \to Z_2^{\mathrm{diag}}.$$

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Remember this is precisely the symmetry of the field theory ($N = 2$)

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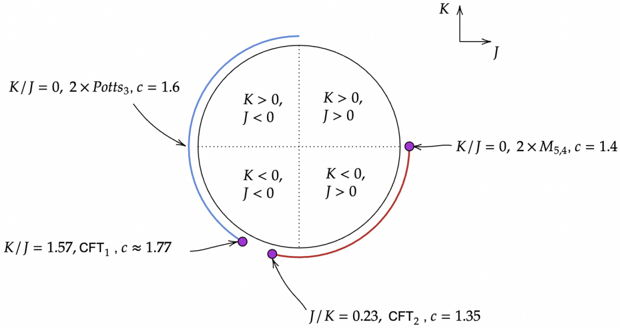
Two coupled anyon chains–phase diagram



Red: pseudo-critical phase. Blue: Critical phase. Uncovered: Gapped phase(s).

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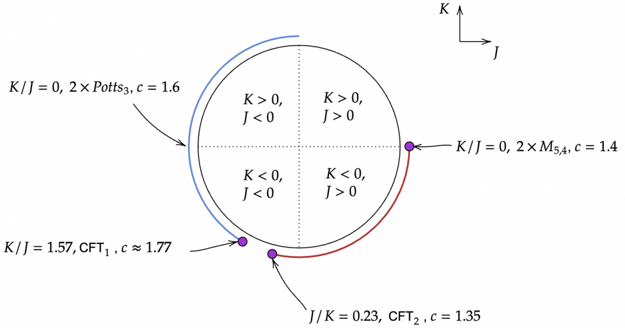

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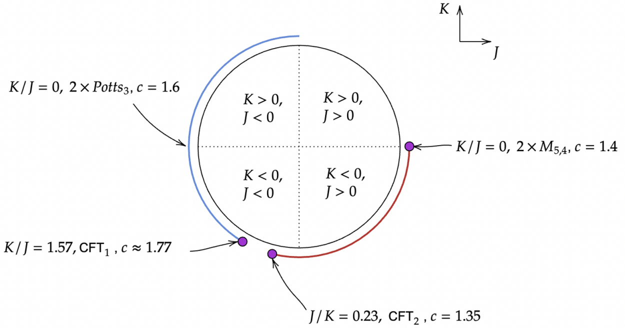

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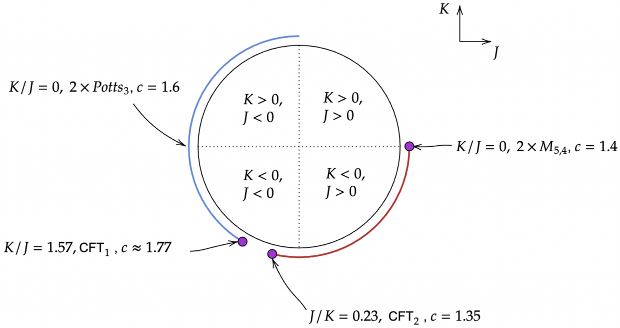

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When $N = 2$, we have

$$S = S^{(1)}_{\mathrm{tri-Ising}} + S^{(2)}_{\mathrm{tri-Ising}} + K \int dx^2 \sigma'^{(1)} \sigma'^{(2)} + \text{irrelevant operators.}$$

Positive K and negative K should have the same physics. Since we can redefine K by the flip $\sigma'^{(1)} \to -\sigma'^{(1)}$.

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Positive K and negative K should have the same physics. Since we can redefine K by the flip $\sigma'^{(1)} \to -\sigma'^{(1)}$. this is not the case for the lattice model.

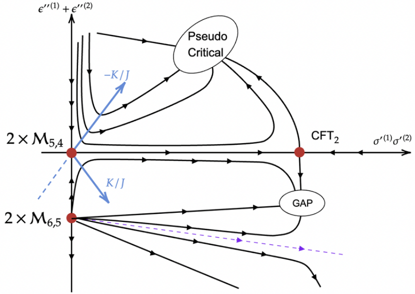
The lattice deformation

$$K \sum_i H_i^{(1)} H_i^{(2)},$$

does not correspond purely to $\sigma'^{(1)}\sigma'^{(2)}$, it also mixes with irrelevant operators such as $\epsilon'^{(1)} + \epsilon'^{(2)}$.

Two coupled anyon chains–phase diagram

This indicates that the irrelevant operators in are dangerously irrelevant operators.



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It has $c = 1.35$. The Fib symmetry comes from the fusion rule of $SU(2)_3$. The primaries are labeled by the j of $su(2)$, but with the important constraint $j < \frac{k}{2}$.

$$1 \times 1 \to 0 + 1.$$

Entanglement entropy

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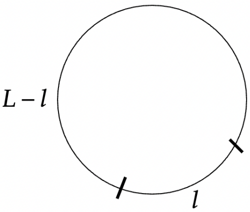

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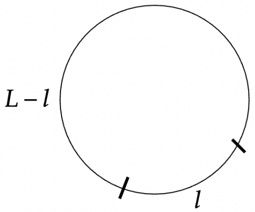

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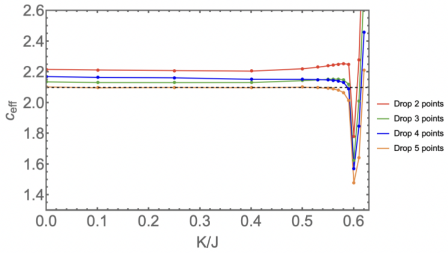
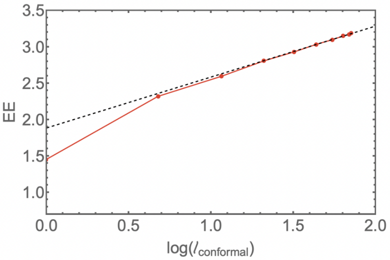
For CFT, it follows a nice formula

$$S = \frac{c}{3}\log(l_c), \qquad l_c = \frac{L}{\pi} sin(\frac{l}{L}\pi)$$

Three coupled anyon chains

We can further consider the model of three coupled anyon chains





A linear fit gives us

$$c = 2.10 \pm 0.03$$

The central charge is very close to the UV value.

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We have $c_{IR} \approx 2.088$. This is a very short flow!

Irrational CFT?

We can compare the CFT with coset models

$$\frac{g_k}{u(1)^{r_g}}, \quad \frac{g_k \oplus g_l}{g_{k+l}}.$$

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This is of course not a proof.

Perturbative limit: conformal perturbation

We can also understand the CFT from a perturbative limit. Consider N copies of the k-th minimal model

$$S = \sum_{a=1}^{N} M^{(a)}_{k,k+1} + K \int dx^2 \sum_{a \neq b} \phi^{(a)}_{(2,1)} \phi^{(b)}_{(2,1)}$$

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The $k = 5$ model is closed related to the coupled Potts model, whose study were pioneered in [Dotsenko, Jacobsen, Lewis, and Picco, Nucl. Phys. B 546 (1999) 505–557].

Perturbative limit: 3-ϵ

The CFT has a Landau-Ginzburg description in

$$\mathcal{L} = \sum_i \frac{1}{2}(\partial\phi_i)^2 + \lambda_1 \sum_i (\phi_i)^6 + \lambda_2 \sum_{i\neq j} (\phi_i)^3(\phi_j)^3$$

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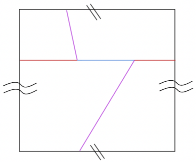
Remember the $4-\epsilon$ expansion is the standard technique to study the critical Ising model in 3D.

Future directions

1, Modular Bootstrap (WIP with R. Russo, M . Ferragatta and B. van Rees) considers the torus partition decorated by a web of topological lines.



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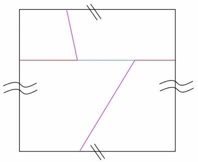

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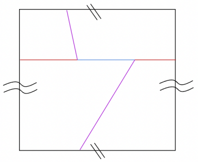
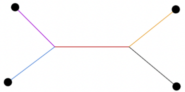


The crossing equation comes from the modular S transformation.
2, correlator bootstrap considers the four point function of twisted operators.



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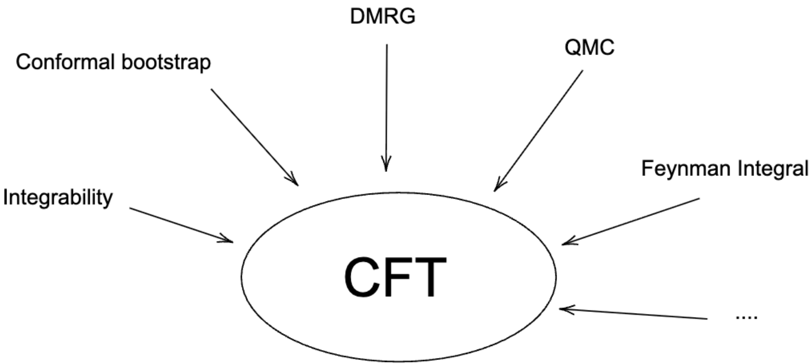
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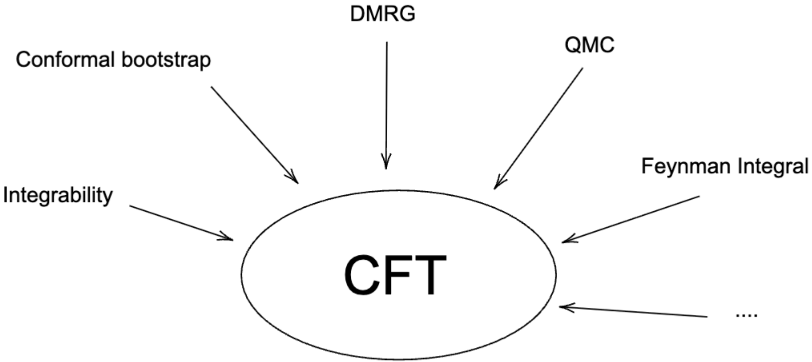
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Thank you!