New Frontiers of Quantum Fields and Gravity, Peking University, January 2026

Amplitudes and Hawking Radiation

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Motivation

1. Amplitudes compute classical gravitational waveforms

→ David, Giacomo yesterday

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2. Deeper understanding of classical backgrounds with amplitudes

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1. Amplitudes compute classical gravitational waveforms

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2. Deeper understanding of classical backgrounds with amplitudes

3. Seek new insight into QFT on a classical background

Outline

1. ABCs of quantum field theory in a background
2. Hawking's scattering process, thermal spectrum
3. Hawking meets the double copy

ABCs of QFT on a background

Aoude, Elkhidir, Ilderton, O'Connell, Rajeev
To appear soon!

ABCs of QFT on a background

Amplitudes, Bogoliubov, Crossing

Aoude, Elkhidir, Ilderton, O'Connell, Rajeev
To appear soon!

QFT on a background

Action

$$I = \int \mathrm{d}^4x \Big(\mathcal{L}_f + \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}m^2\phi^2 + \mathcal{O}(x)\phi^2(x) \Big) \, .$$

QFT on a background

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Interacting fields f
Background: $f = F + \delta f$
$F \neq 0$, $\langle \delta f \rangle = 0$

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Radiated particle
Often massless
Scalar for simplicity

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Interaction term
Assumed quadratic in ϕ
Could involve derivatives

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Examples: scalar field on time-dependent spacetime

pair-production from intense laser (fermionic)

QFT on a background

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Large background:

$$\begin{aligned} \mathcal{O}(x)\phi^2(x) &= P(f(x))\,\phi^2(x) \\ &= \Big(P(F(x)) + P'(F(x))\delta f(x) + \cdots \Big)\phi^2(x) \end{aligned}$$

QFT on a background

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Assumed negligible

QFT on a background

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Effective action: gaussian

$$I_{\rm eff} = \int \mathrm{d}^4x \left(\tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}m^2\phi^2 + P(F(x))\,\phi^2(x) \right).$$

QFT on a background

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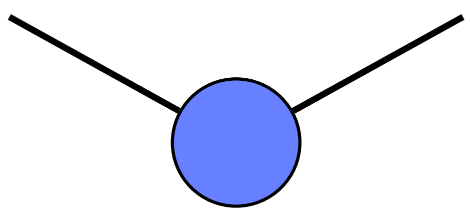
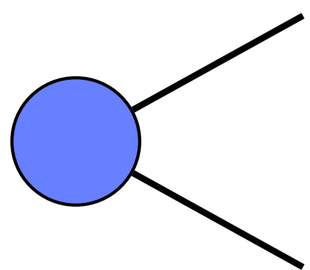
Spacetime dependence:
interesting dynamics!

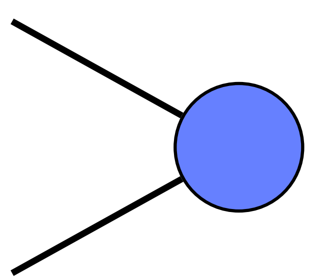
QFT on a background

$$I_{\text{eff}} = \int \mathrm{d}^4x \Big(\tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}m^2\phi^2 + P(F(x))\,\phi^2(x) \Big)\,.$$

Primitive processes:

$\mathcal{M}(p \to k) =$  $= \langle 0|a(k) S a^\dagger(p)|0\rangle$

$\mathcal{M}(0 \to k_1, k_2) =$  $= \langle 0|a(k_1)a(k_2)S|0\rangle$

$\mathcal{M}(k_1, k_2 \to 0) =$  $= \langle 0|S a^\dagger(k_1) a^\dagger(k_2)|0\rangle$

QFT on a background

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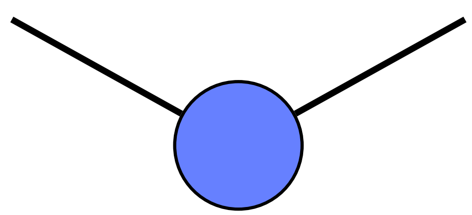
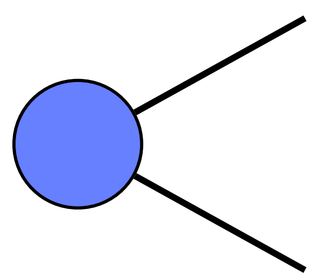
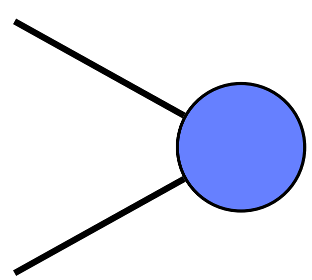
Primitive processes:

ϕ vacuum
on background

$\mathcal{M}(p \to k) =$  $= \langle 0|a(k)Sa^\dagger(p)|0\rangle$

$\mathcal{M}(0 \to k_1, k_2) =$  $= \langle 0|a(k_1)a(k_2)S|0\rangle$

$\mathcal{M}(k_1, k_2 \to 0) =$  $= \langle 0|Sa^\dagger(k_1)a^\dagger(k_2)|0\rangle$

QFT on a background

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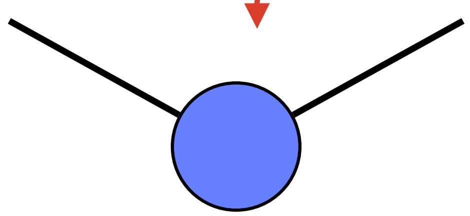
Primitive processes:

All orders

ϕ vacuum on background

$\mathcal{M}(p \to k) =$



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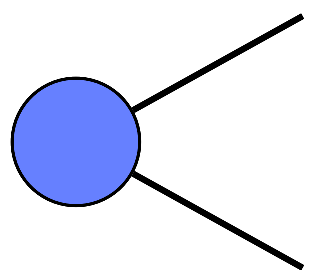
$\mathcal{M}(0 \to k_1, k_2) =$



$= \langle 0|a(k_1)a(k_2)S|0\rangle$

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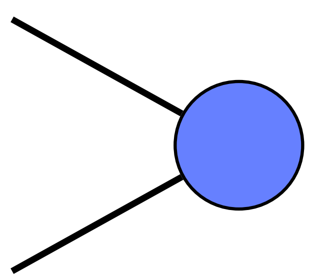

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QFT on a background

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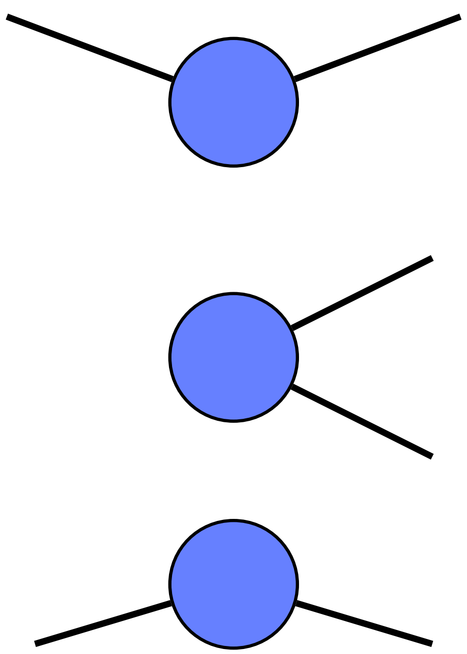
More complicated amplitudes built from primitives



$+\cdots$

QFT on a background

Non-trivial S matrix, but simple

$$S = \mathrm{T}\,\exp\left(i\int \mathrm{d}^4x\,\mathcal{O}(x) : \phi^2(x) :\right)$$
$$= \cdots U(t+\epsilon, t)U(t, t-\epsilon)\cdots$$

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For small enough ϵ

$$U(t+\epsilon,t) = \exp\left(i\int_t^{t+\epsilon} \mathrm{d}^4x\,\mathcal{O}(x) :\phi^2(x):\right)$$

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Quadratic in $a, a^\dagger$

BCH: S is an exponential of commutators

QFT on a background

$$S = e^{i\theta} e^{iN}$$

Commutators: quadratic in $a, a^\dagger$

$$N = \int \mathrm{d}\Phi(p_1, p_2)\Big(f_1(p_1, p_2) a^\dagger(p_1) a^\dagger(p_2) + f_1^*(p_1, p_2) a(p_1) a(p_2) + f_2(p_1, p_2) a^\dagger(p_1) a(p_2) \Big)$$

QFT on a background

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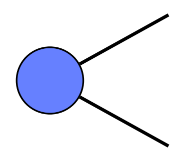

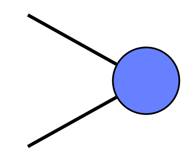

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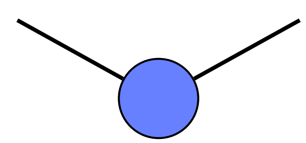
$$+ f_2(p_1, p_2) a^\dagger(p_1) a(p_2) \Big)$$



QFT on a background

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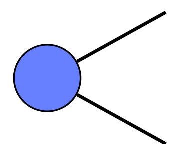

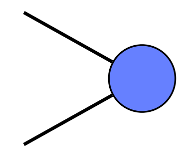

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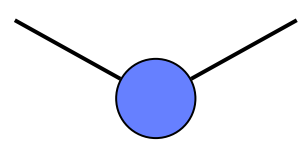
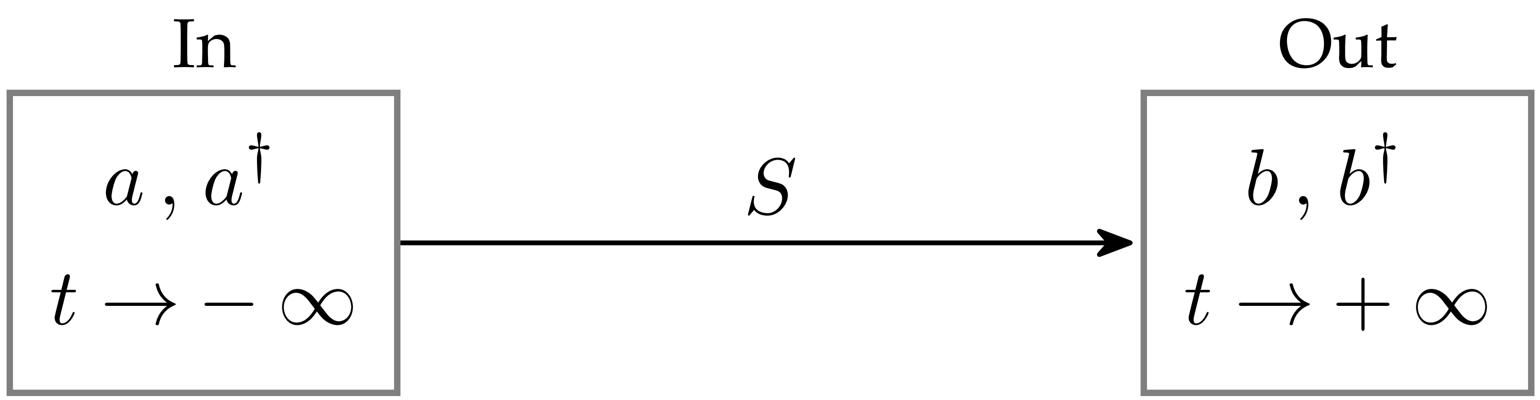


Consistent with primitive processes

Amplitudes built from f_i

QFT on a background

Past / future ladder operators

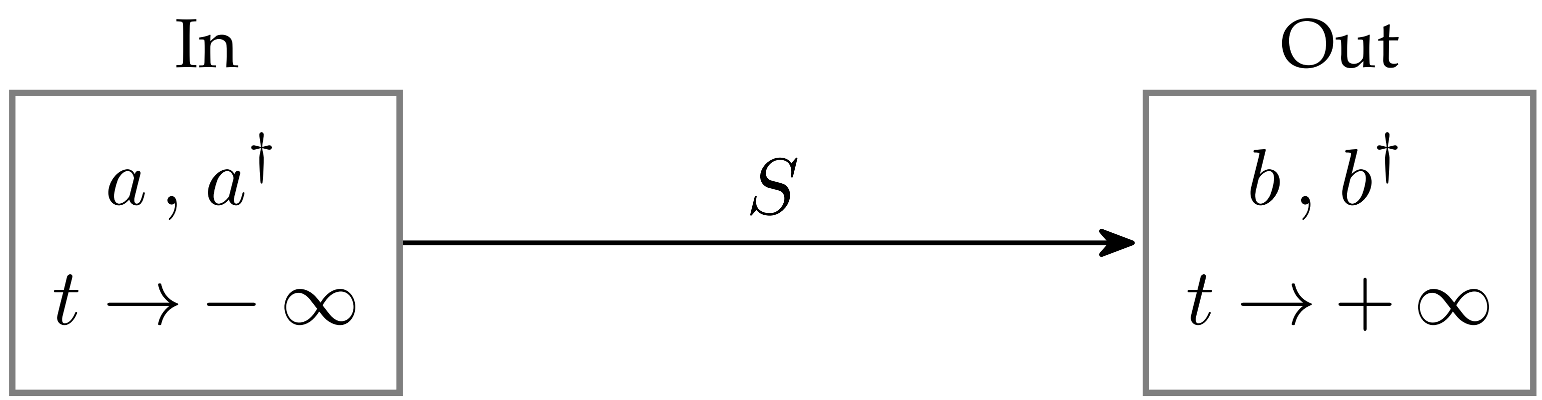


$$b(p) = S^\dagger a(p) S$$

$$b(p) = e^{-iN} a(p) e^{iN}$$

QFT on a background

Past / future ladder operators



$$b(p) = S^\dagger a(p) S$$

$$b(p) = e^{-iN} a(p) e^{iN}$$

Commutator $[a(p), N]$ is linear in $a, a^\dagger$:

$$b(p) = \int \mathrm{d}\Phi(k) \Big(\alpha(p,k) a(k) + \beta(p,k) a^\dagger(k) \Big)$$

QFT on a background

$$b(p) = \int \mathrm{d}\Phi(k)\Big(\alpha(p,k)a(k) + \beta(p,k)a^\dagger(k)\Big)$$

This is the Bogoliubov relation: α, β are Bogoliubov coefficients

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$$\alpha(p,k) = \langle 0|S^\dagger a(p) S a^\dagger(k)|0\rangle$$

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$$\alpha(p,k) = \langle 0|\overbrace{S^\dagger a(p) S}^{b(p)} a^\dagger(k)|0\rangle$$

$$\beta(p,k) = \langle 0|a(k) S^\dagger a(p) S|0\rangle$$

QFT on a background

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$$\mathcal{M}(k \to p) = \langle 0|a(p) S a^\dagger(k)|0\rangle$$

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QFT on a background

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Generalised amplitudes

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Amplitudes

QFT on a background

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Amplitudes

Amplitudes determined from generalised amplitudes (& vice versa)

QFT on a background

Crossing: α and β obtained from same retarded correlator

$$R(p_1, p_2) \equiv \int_{x_1, x_2} \mathrm{LSZ}(p_1, x_1)\mathrm{LSZ}(p_2, x_2) \langle 0|\mathrm{R}\{\phi(x_2)\phi(x_1)\}|0\rangle$$

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$$= \theta(x_2 - x_1)[\phi(x_2), \phi(x_1)]$$

QFT on a background

Crossing: α and β obtained from same retarded correlator

$$R(p_1, p_2) \equiv \int_{x_1, x_2} \mathrm{LSZ}(p_1, x_1) \boxed{\mathrm{LSZ}(p_2, x_2)} \langle 0 | \mathrm{R}\{\phi(x_2)\phi(x_1)\} | 0 \rangle$$

$$ie^{ip_2 \cdot x_2}(\partial_{x_2}^2 + m^2)$$

QFT on a background

Crossing: α and β obtained from same retarded correlator

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p_1, p_2 could be complex

QFT on a background

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When p_1, p_2 on-shell with positive energy

$$R(p_1, p_2) = \beta(p_2, p_1)$$

QFT on a background

Crossing: α and β obtained from same retarded correlator

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When p_1, p_2 on-shell with positive energy

$$R(p_1, p_2) = \beta(p_2, p_1)$$

When $p_1, -p_2$ on-shell with positive energy

$$R(p_1, p_2) = \delta_\Phi(p_1, p_2) - \alpha^*(p_2, p_1)$$

QFT on a background

Crossing: α and β obtained from same retarded correlator

$$R(p_1, p_2) \equiv \int_{x_1, x_2} \mathrm{LSZ}(p_1, x_1)\mathrm{LSZ}(p_2, x_2) \langle 0 | \mathrm{R}\{\phi(x_2)\phi(x_1)\} | 0 \rangle$$

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Cross into
one another

QFT on a background

Dynamics / observables given by (generalised) amplitudes:

$$S\,|0\rangle = \langle 0|S|0\rangle \exp\left(\frac{1}{2\,\langle 0|S|0\rangle}\int \mathrm{d}\Phi(p_1,p_2)a^\dagger(p_1)\mathcal{M}(0\to p_1p_2)a^\dagger(p_2)\right)|0\rangle$$

QFT on a background

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$$\langle 0|S^\dagger a^\dagger(k)a(k)S|0\rangle = \int \mathrm{d}\Phi(p)\,\beta^*(k,p)\beta(k,p)$$

QFT on a background

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$$\langle 0|S^\dagger a^\dagger(k)a(k)S|0\rangle = \int \mathrm{d}\Phi(p)\,\beta^*(k,p)\beta(k,p)$$

Sufficient to compute α, cross to get β: full information

Hawking Scattering

Hawking Scattering

Dynamical background metric “Vaidya”

$$g^{\mu\nu} = \eta^{\mu\nu} + \frac{2GM\Theta(t+r)}{r} k^\mu k^\nu$$

Hawking Scattering

Dynamical background metric "Vaidya"

$$g^{\mu\nu} = \eta^{\mu\nu} + \frac{2GM\Theta(t+r)}{r} k^\mu k^\nu$$

Kerr-Schild vector

$$k_\mu \mathrm{d}x^\mu = \mathrm{d}t + \mathrm{d}r$$

Hawking Scattering

Dynamical background metric "Vaidya"

Thin shell of radiation
falling in to origin

$$g^{\mu\nu} = \eta^{\mu\nu} + \frac{2GM\Theta(t+r)}{r} k^\mu k^\nu$$

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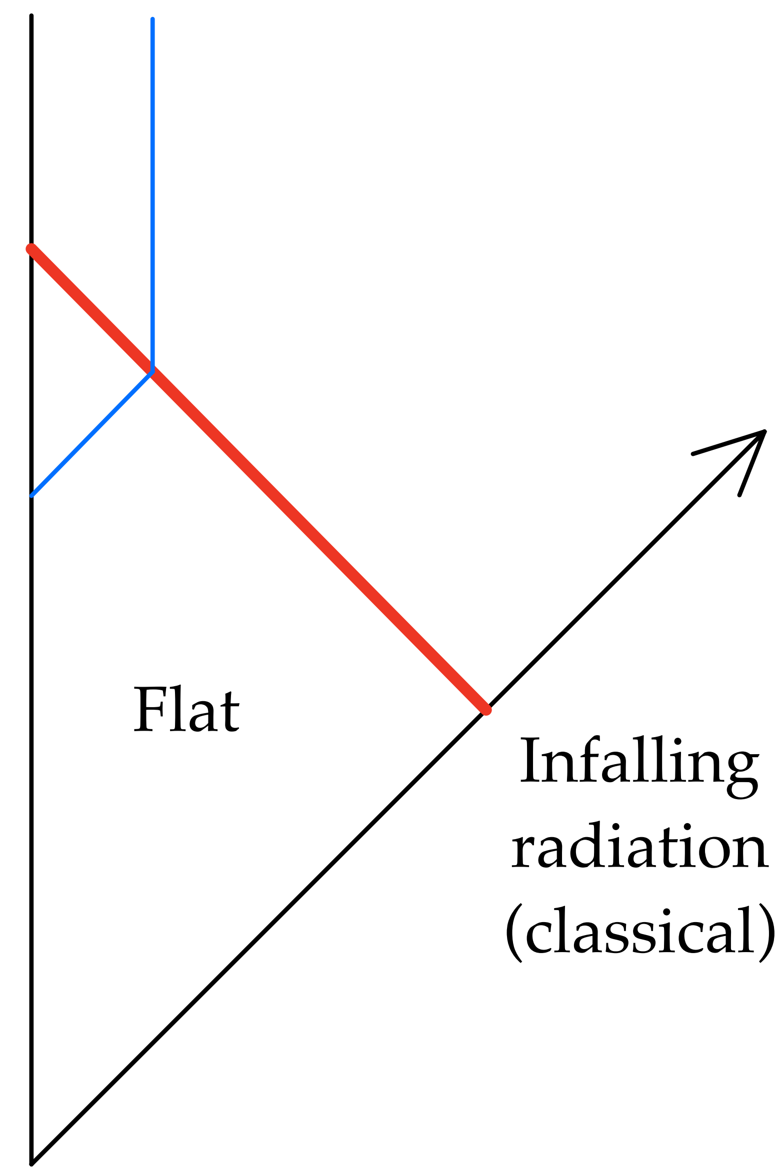

Hawking Scattering

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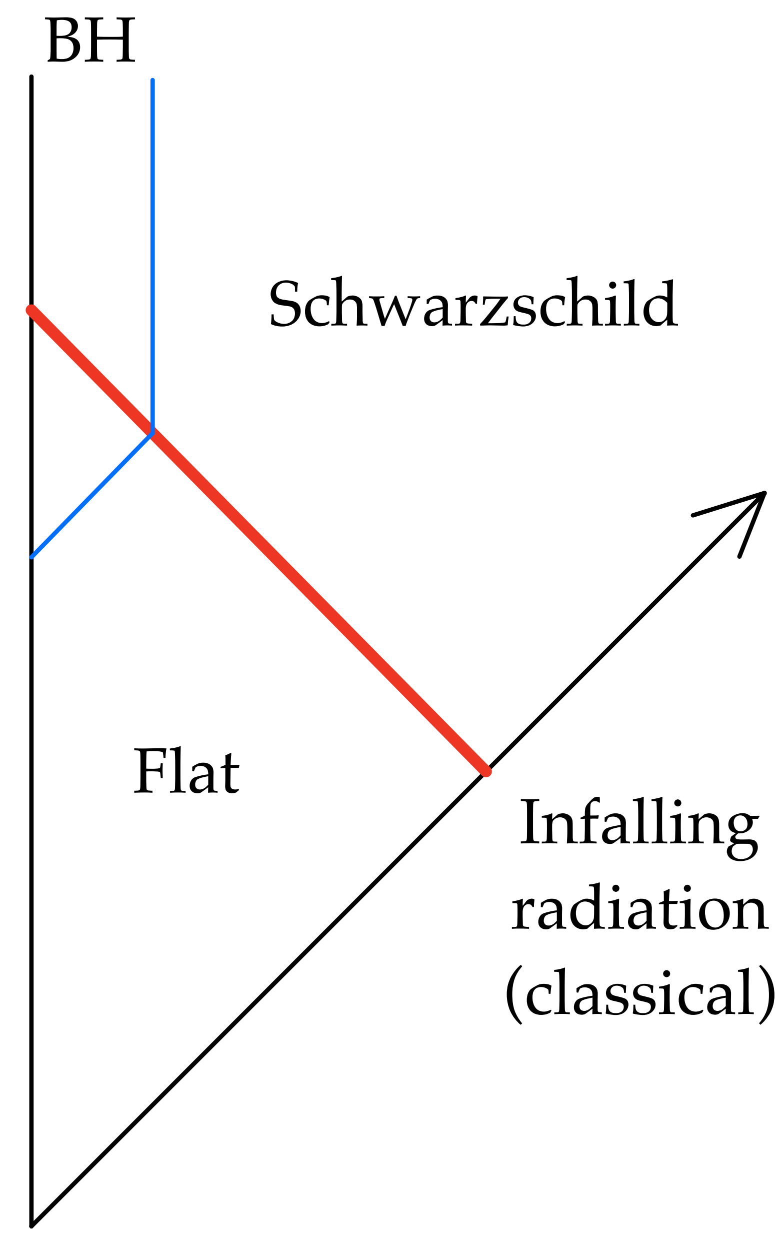
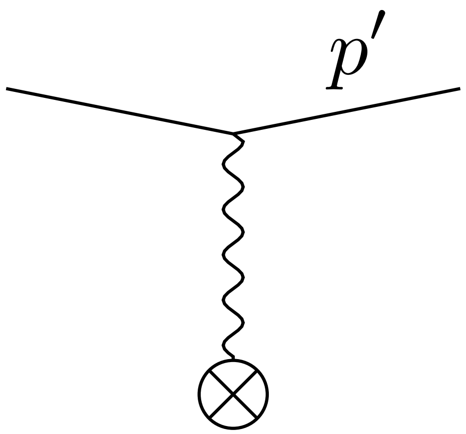
Kerr-Schild vector

$k_\mu \mathrm{d}x^\mu = \mathrm{d}t + \mathrm{d}r$



Hawking Scattering

Tree



$$I_{\text{int}} = \frac{1}{2}\int \mathrm{d}^4x \, h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

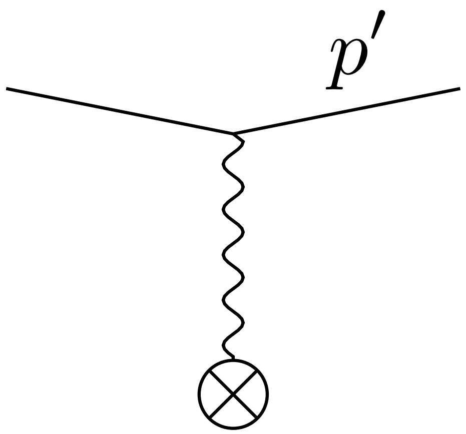
Compute at high energy (geometric optics):

$$\text{diagram} = \int \mathrm{d}v \, \varphi(v) e^{ip'\cdot b(v)} \big(-4GME' \log(-v/\mu)\big)$$

Aoude, Sergola, DOC

Hawking Scattering

Tree



$$I_{\rm int} = \frac{1}{2}\int \mathrm{d}^4x\, h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Compute at high energy (geometric optics):

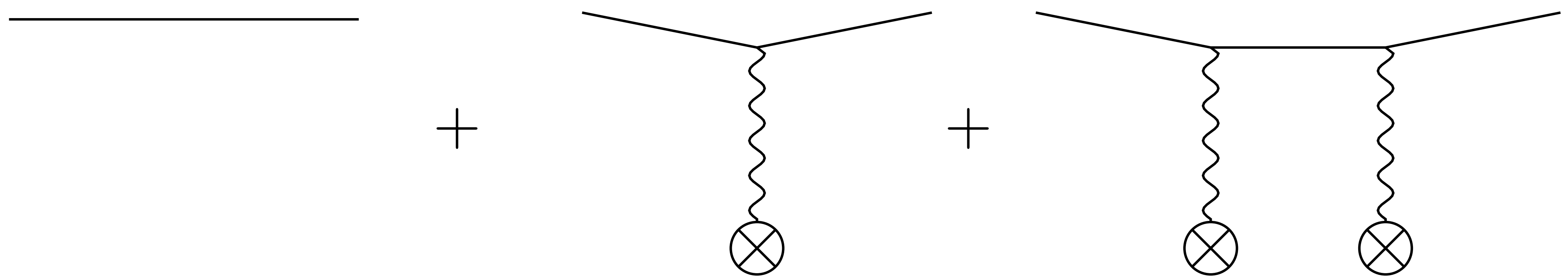
Initial state

$$\text{diagram} = \int \mathrm{d}v\, \varphi(v) e^{ip'\cdot b(v)}\big(-4GME'\log(-v/\mu)\big)$$

Aoude, Sergola, DOC

Hawking Scattering

Iterates & exponentiates

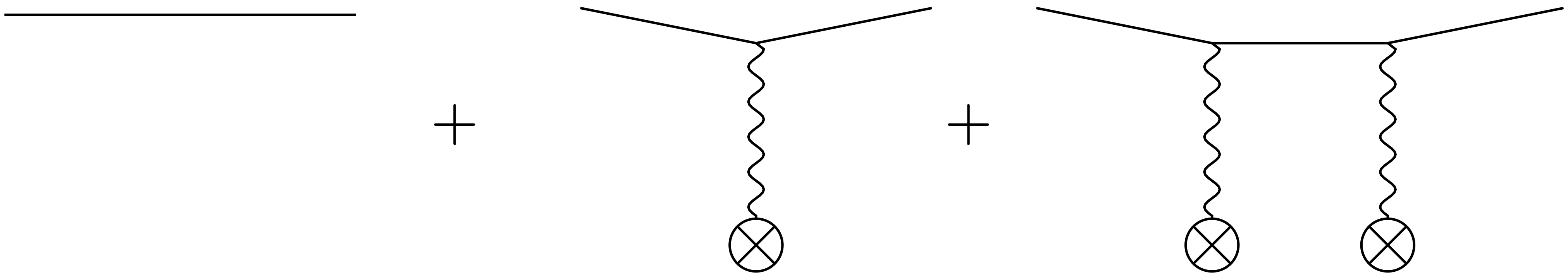


$$= \int \mathrm{d}v \, \varphi(v) e^{ip' \cdot b(v)} \exp\big(- 4iGME' \log(-v/\mu)\big)$$

Aoude, Sergola, DOC

Hawking Scattering

Iterates & exponentiates



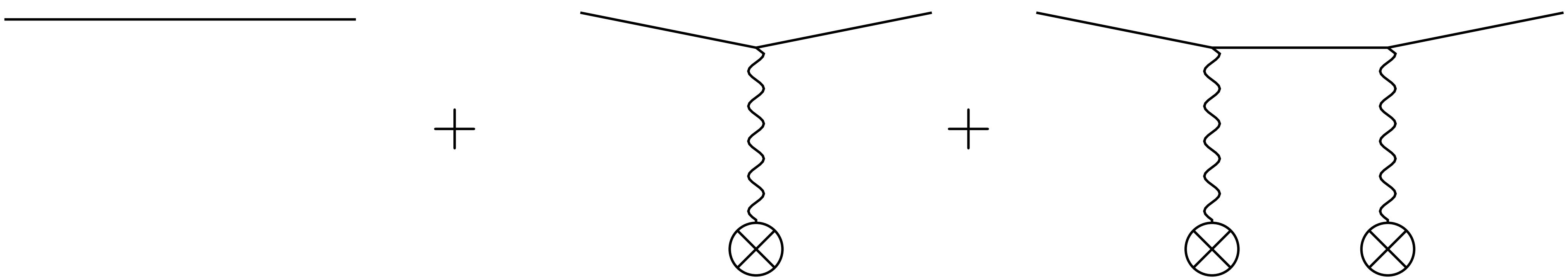
$$= \int \mathrm{d}v\, \varphi(v) e^{ip' \cdot b(v)} \exp\big(-4iGME' \log(-v/\mu)\big)$$

Spherical energy eigenstate $|\psi\rangle = \frac{1}{4\pi} \int \mathrm{d}\Omega \, |E_0, E_0 \mathbf{n}\rangle$

Aoude, Sergola, DOC

Hawking Scattering

Iterates & exponentiates



$$\alpha = \mathcal{N} \int \mathrm{d}v \, e^{i(E' - E_0)v} e^{-4iGM\,E' \log(-v/\mu)}$$

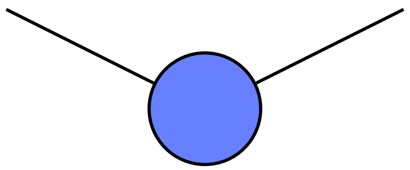
Spherical energy eigenstate $|\psi\rangle = \frac{1}{4\pi} \int \mathrm{d}\Omega \, |E_0, E_0 \mathbf{n}\rangle$

Aoude, Sergola, DOC

Pair Production

Crossing:

$$\alpha = \mathcal{N} \int \mathrm{d}v \, e^{i(E' - E_0)v} e^{-4iGM\, E' \log(-v/\mu)}$$



Pair Production

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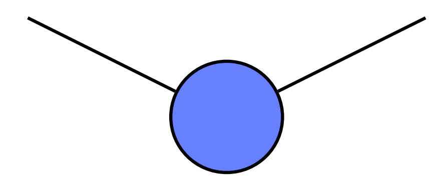
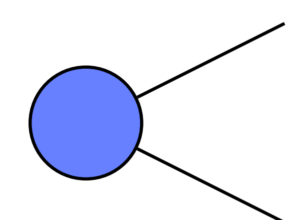


Crossing

$$\beta = \mathcal{N} \int \mathrm{d}v \, e^{i(E' + E_0)v} e^{-4iGM\,E' \log(-v/\mu)}$$



Pair Production

Count outgoing states

$$|\beta|^2 = (\text{factor}) \frac{1}{e^{8\pi GME'} - 1}$$

Thermal distribution

$$T = \frac{1}{8\pi GM}$$

Hawking meets the Double Copy

Aoude, Sergola, DOC
Ilderton, Lindved, Rajeev
Carrasco, Chen

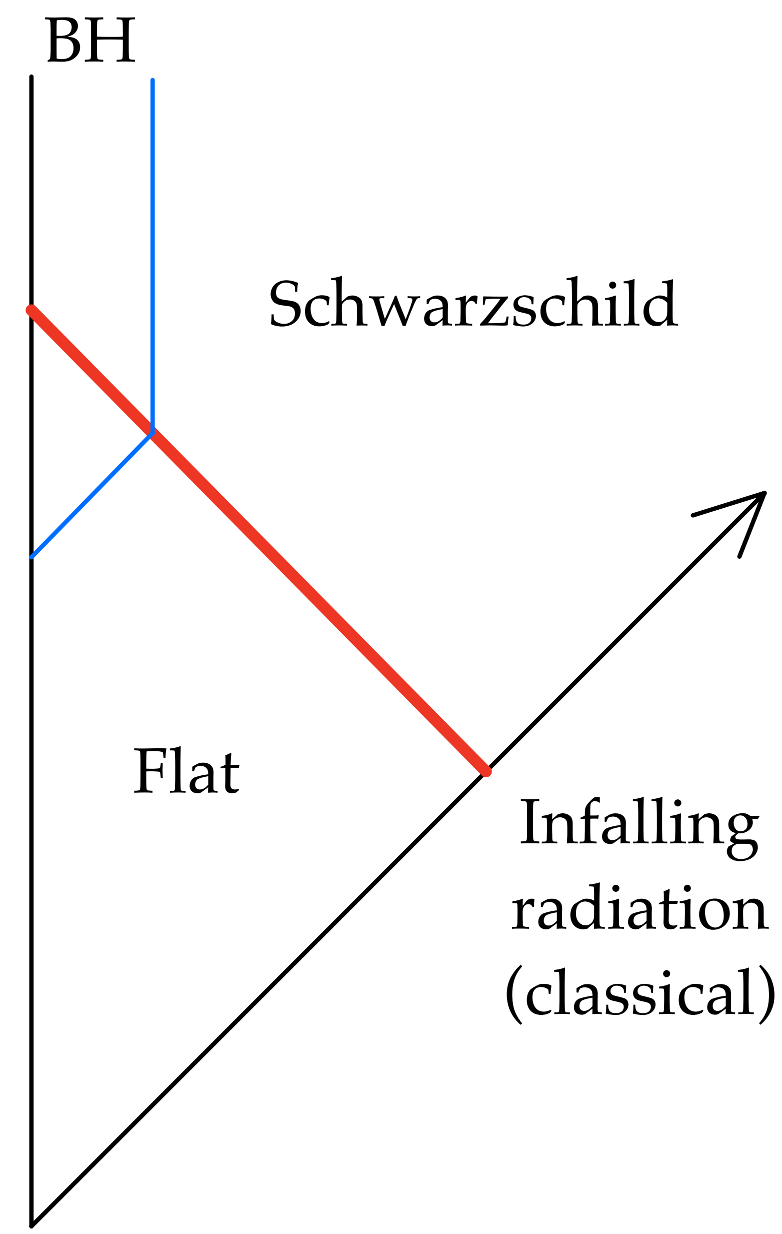
$\sqrt{\text{Vaidya}}$

Obvious Kerr-Schild single copy of Vaidya:

$$g^{\mu\nu} = \eta^{\mu\nu} + \frac{2GM\Theta(t+r)}{r} k^\mu k^\nu$$

Kerr-Schild vector

$$k_\mu \mathrm{d}x^\mu = \mathrm{d}t + \mathrm{d}r$$



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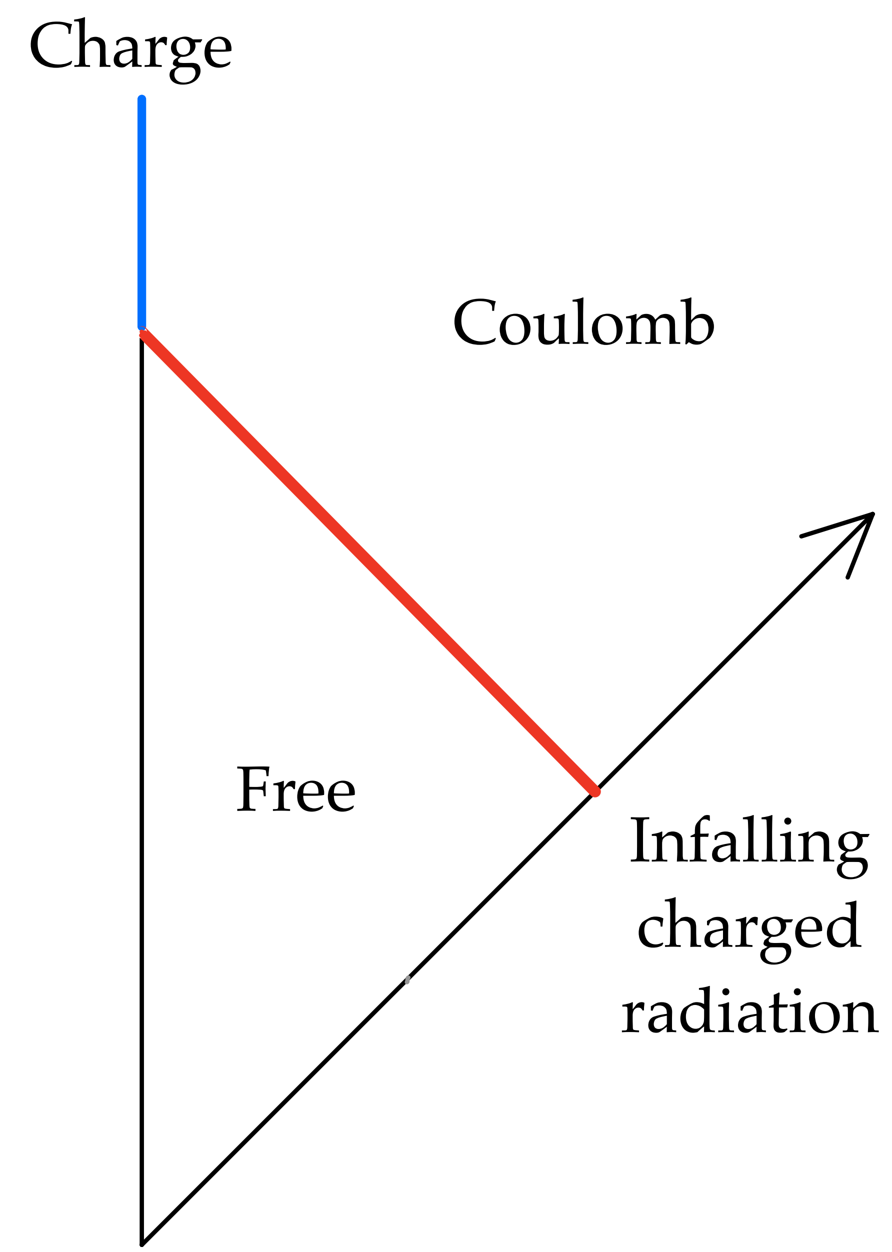
Kerr-Schild vector

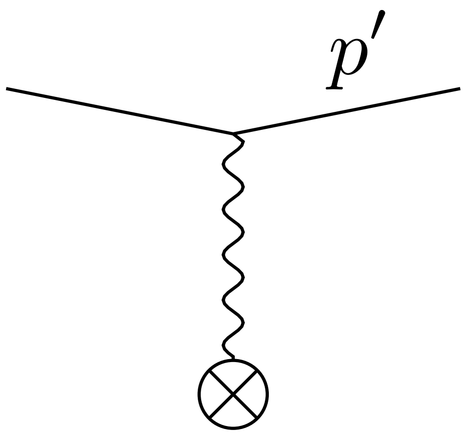
$$k_\mu \mathrm{d}x^\mu = \mathrm{d}t + \mathrm{d}r$$

$$A^\mu = \frac{Q\Theta(t+r)}{4\pi r} k^\mu$$



$\sqrt{\text{Hawking}}$ Scattering

Tree



$$I_{\text{int}} = iq \int \mathrm{d}^4 x\, A^\mu (\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger)$$

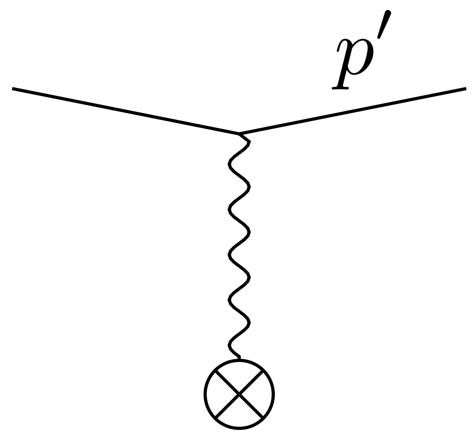
Compute as before:

$$\text{diagram} = \int \mathrm{d}v\, \varphi(v) e^{ip' \cdot b(v)} \big(+ 2\alpha \log(-v/\mu) \big) \qquad \alpha \equiv \frac{qQ}{4\pi}$$

Aoude, Sergola, DOC

$\sqrt{\text{Hawking}}$ Scattering

Tree



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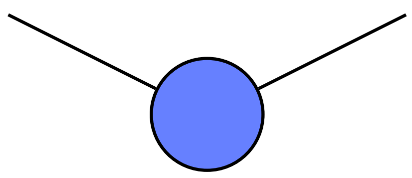
Compare gravity: $\int \mathrm{d}v\, \varphi(v) e^{ip'\cdot b(v)} \big(- 4GME' \log(-v/\mu)\big)$

Aoude, Sergola, DOC

$\sqrt{\text{Hawking}}$ Scattering

Exponentiation as before:

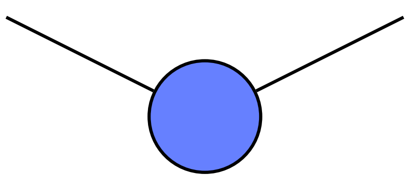
$$\alpha = \mathcal{N} \int \mathrm{d}v \, e^{i(E' - E_0)v} e^{2i\alpha \log(-v/\mu)}$$



$\sqrt{\text{Hawking}}$ Scattering

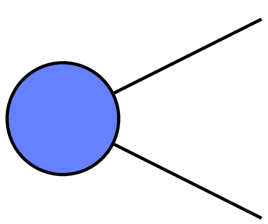
Exponentiation as before:

$$\alpha = \mathcal{N} \int \mathrm{d}v\, e^{i(E' - E_0)v} e^{2i\alpha \log(-v/\mu)}$$



Crossing

$$\beta = \mathcal{N} \int \mathrm{d}v\, e^{i(E' + E_0)v} e^{2i\alpha \log(-v/\mu)}$$



Comparison

Interesting to compare:

$$\beta = \mathcal{N} \int \mathrm{d}v \, e^{i(E'+E_0)v} e^{+2i\alpha \log(-v/\mu)}$$ Single copy

$$\beta = \mathcal{N} \int \mathrm{d}v \, e^{i(E'+E_0)v} e^{-4iGM\, E' \log(-v/\mu)}$$ Double copy

Comparison

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$$\beta = \mathcal{N} \int \mathrm{d}v \, e^{i(E'+E_0)v} e^{+2i\alpha \log(-v/\mu)}$$

Single copy

$$\beta = \mathcal{N} \int \mathrm{d}v \, e^{i(E'+E_0)v} e^{-4iGM \, E' \log(-v/\mu)}$$

Double copy

Different energy dependence: additional power of momentum in GR

Pair Production

Number distribution of outgoing states: not thermal

$$|\beta|^2 = (\text{factor}) \frac{1}{e^{-4\pi\alpha} - 1}$$

$$\frac{1}{e^{8\pi GME'} - 1} \quad \text{in GR!}$$

Interpretation? Bose-Einstein distribution $\dfrac{1}{e^{\beta E - \beta\mu} - 1}$

Perhaps just take $\beta E \to 0$ with non-zero fixed $\beta\mu = 4\pi\alpha$

Thank you!

Conclusions

Bogoliubov coefficients: closely related to amplitudes

- Generalised amplitudes, different boundary conditions

Single copy of Hawking radiation — not thermal

Outlook

Corrections to background QFT picture?