



Scattering and the Asymptotic Structure of Spacetime

Testing the asymptotic properties of spacetime with QFT techniques

Based on [[arXiv:2511.10637](https://arxiv.org/abs/2511.10637)] with A. Herdershee, R. Roiban and F. Teng

Stefano De Angelis - *New Frontiers of Quantum Field and Gravity* @ PKU

Outline of the talk

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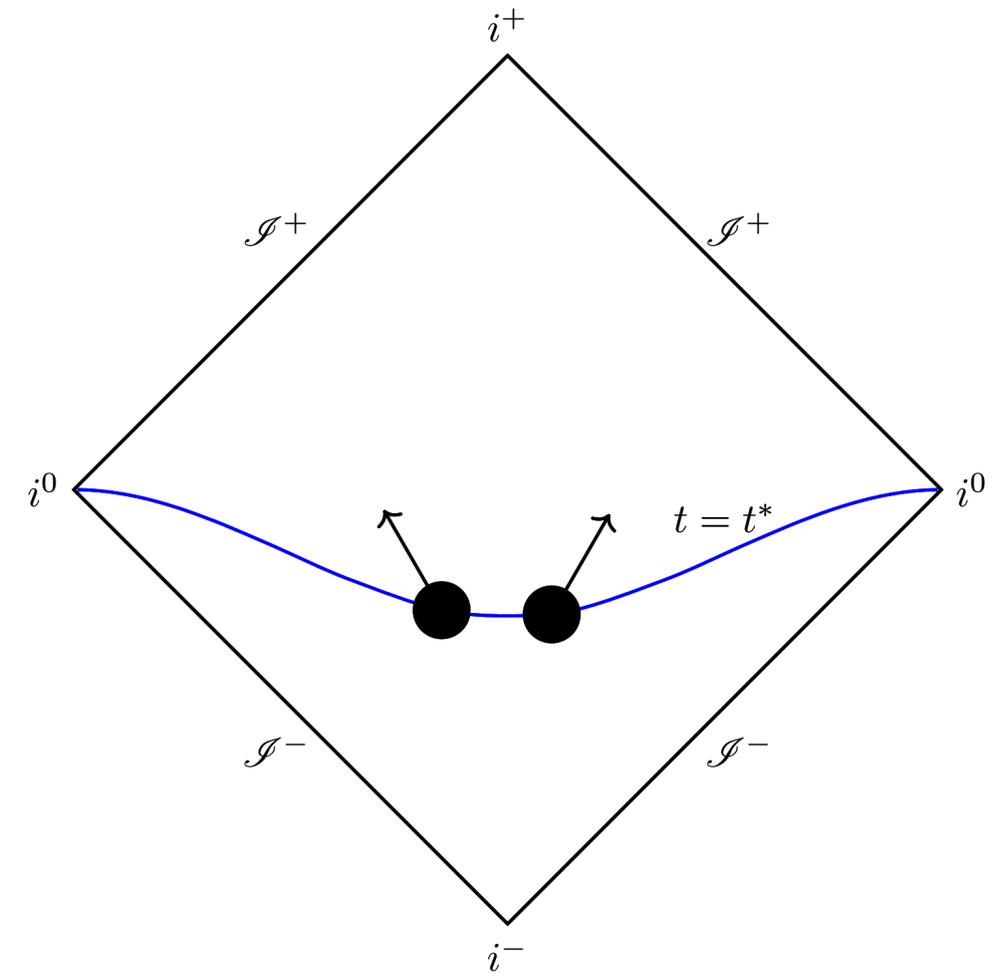
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- Outlook and conclusions

Isolated Systems in General Relativity

Conformal Compactifications and Asymptotic Simplicity

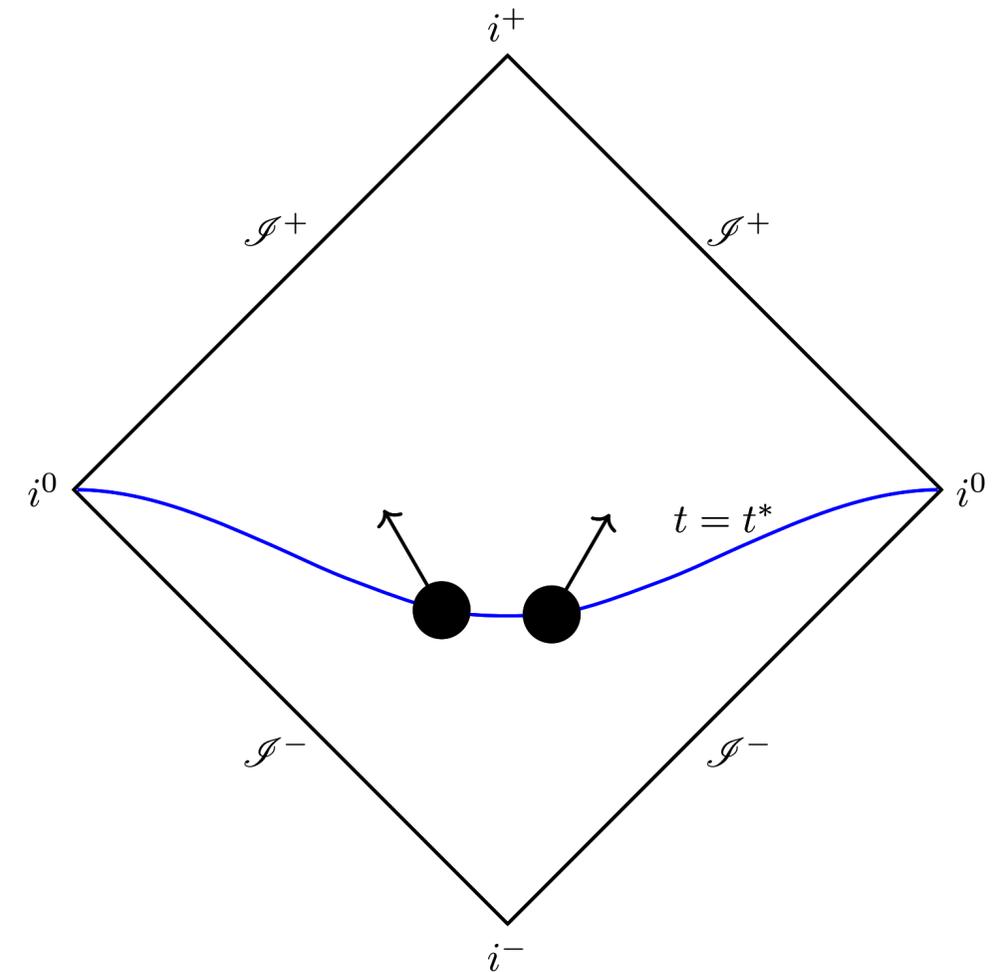


Isolated Systems in General Relativity

Conformal Compactifications and Asymptotic Simplicity

Conformal compactifications \Rightarrow metric smooth enough to \mathcal{I}^+ \Rightarrow glue a boundary at null infinity.

[Penrose, '65]



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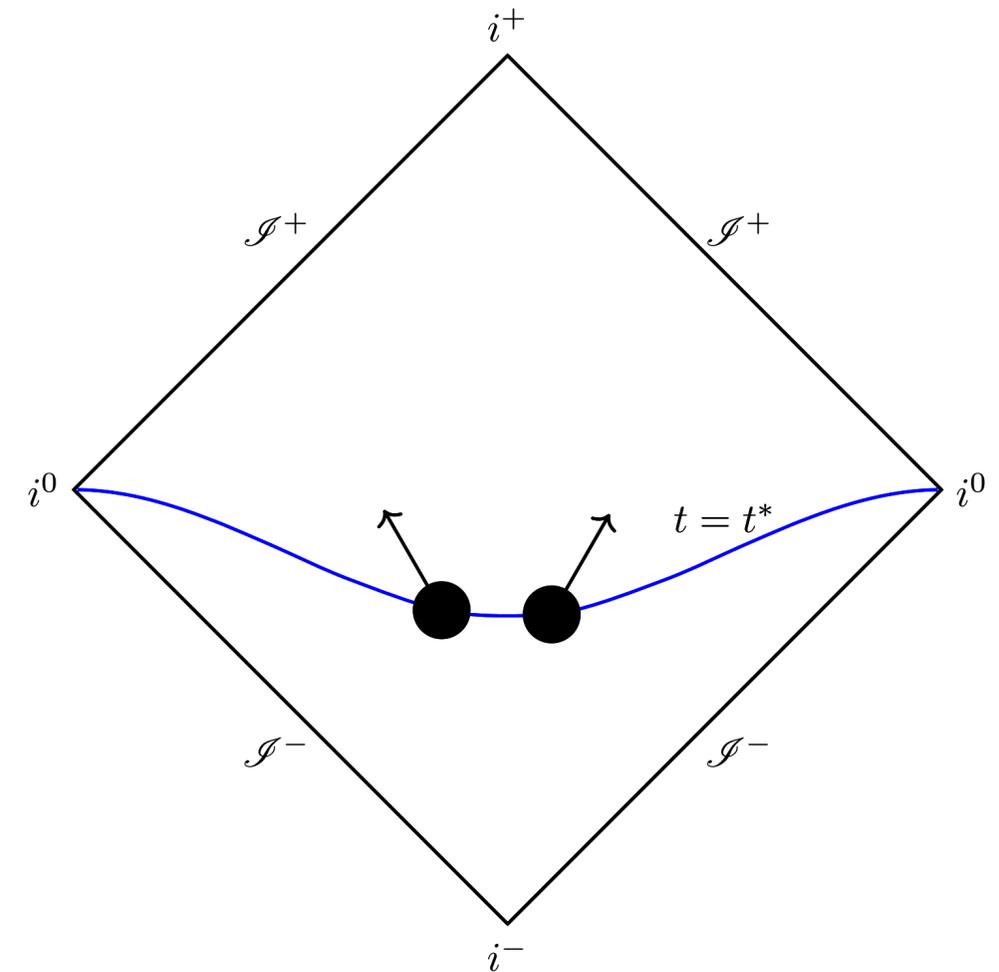
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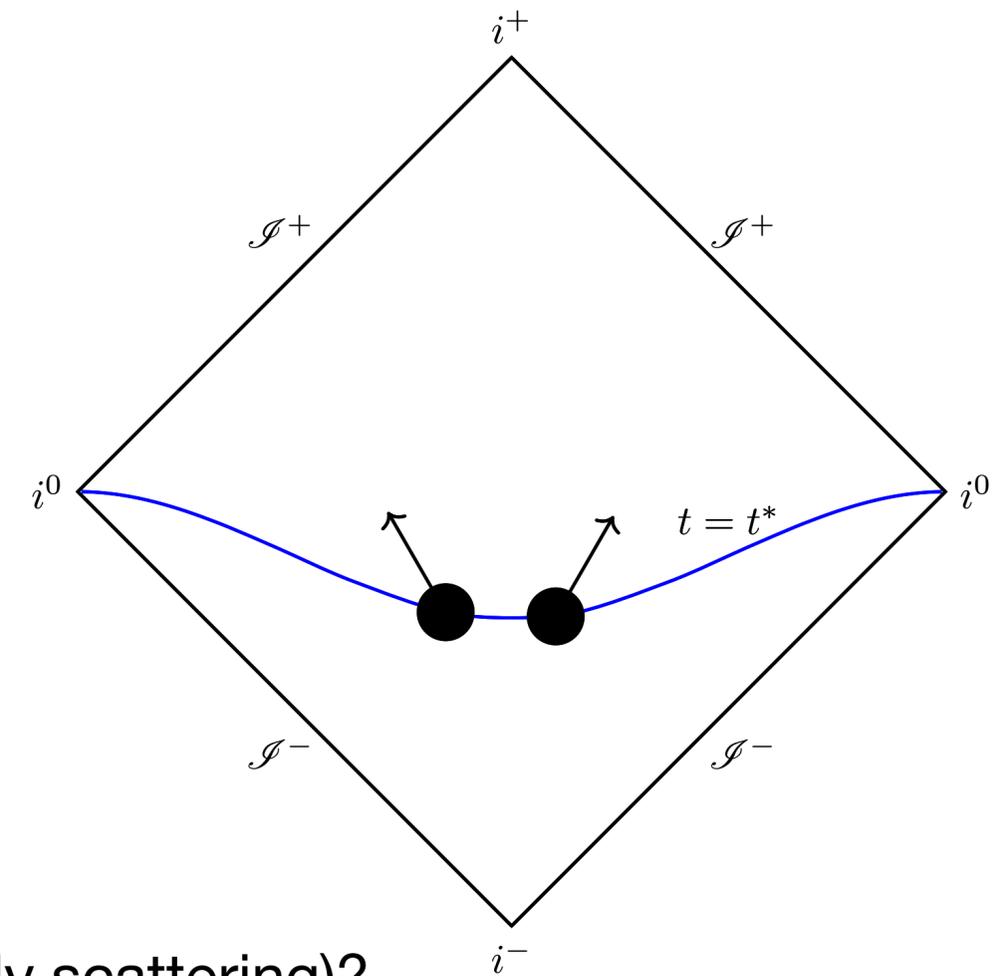
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What about **physically interesting spacetimes** (two-body and n-body scattering)?

[Damour, '86], [Christodoulou, 2002], [Kehrberger, 2021-2025]x6

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Defining rigorously a physically-interesting isolated system in GR?

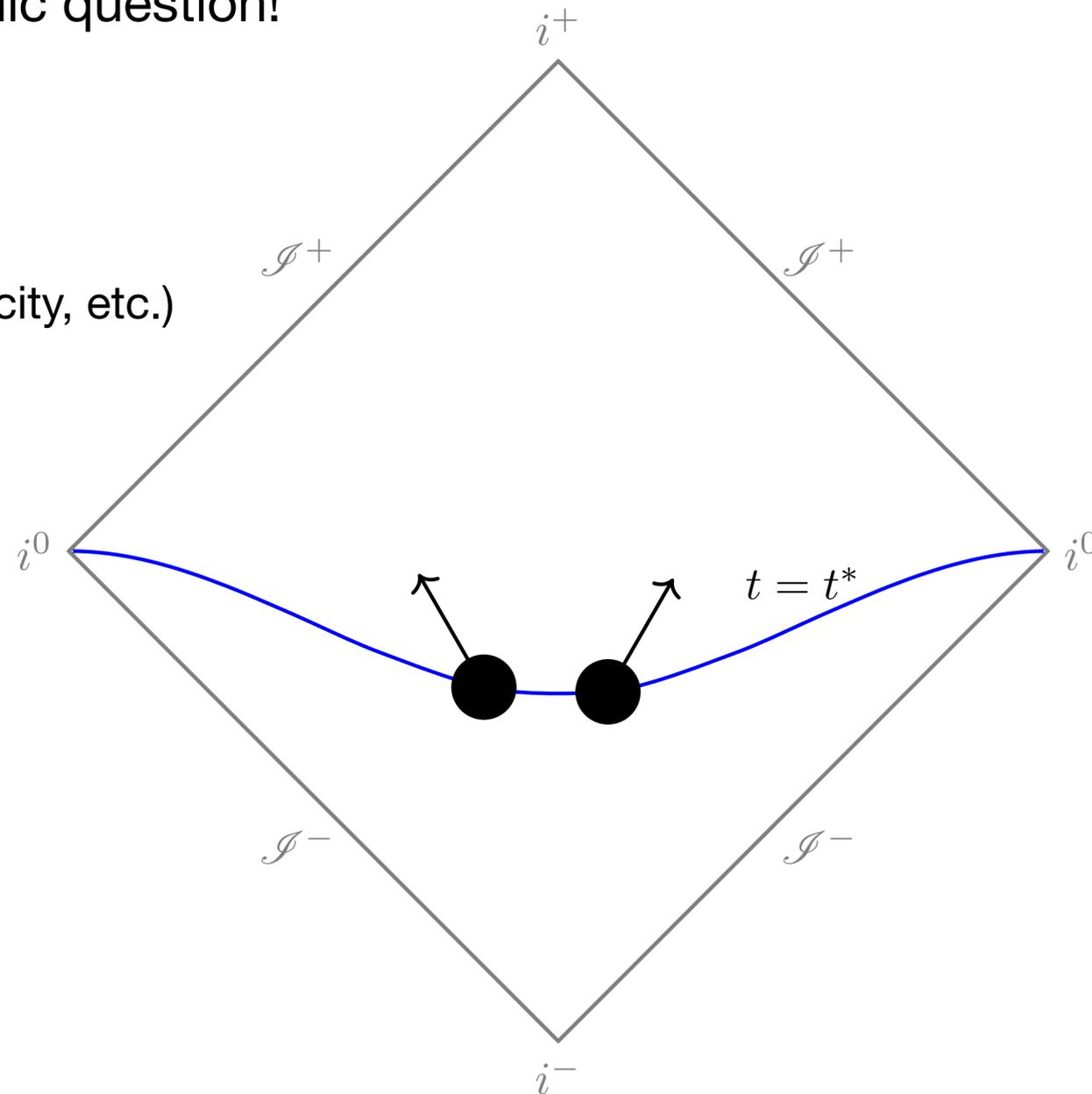
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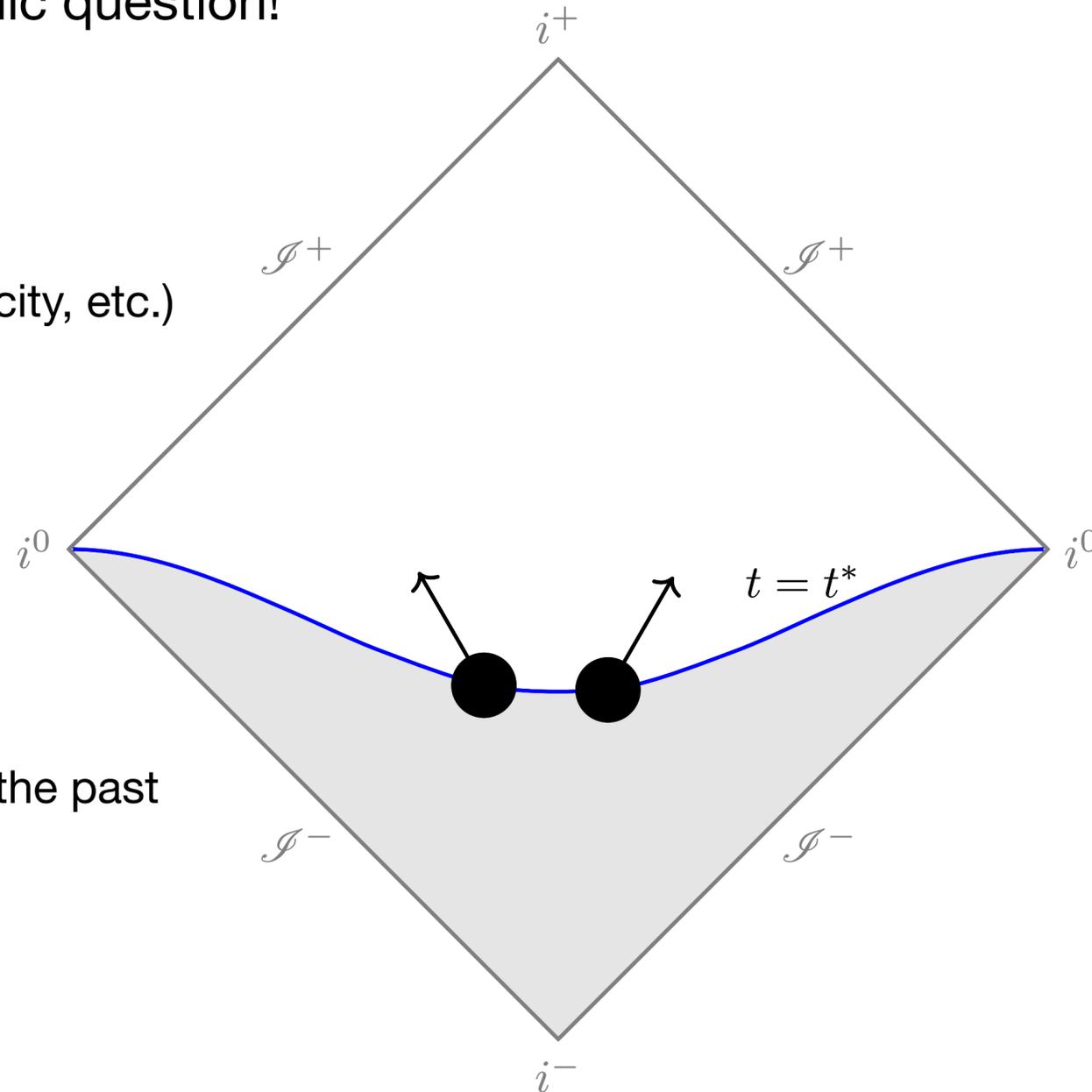


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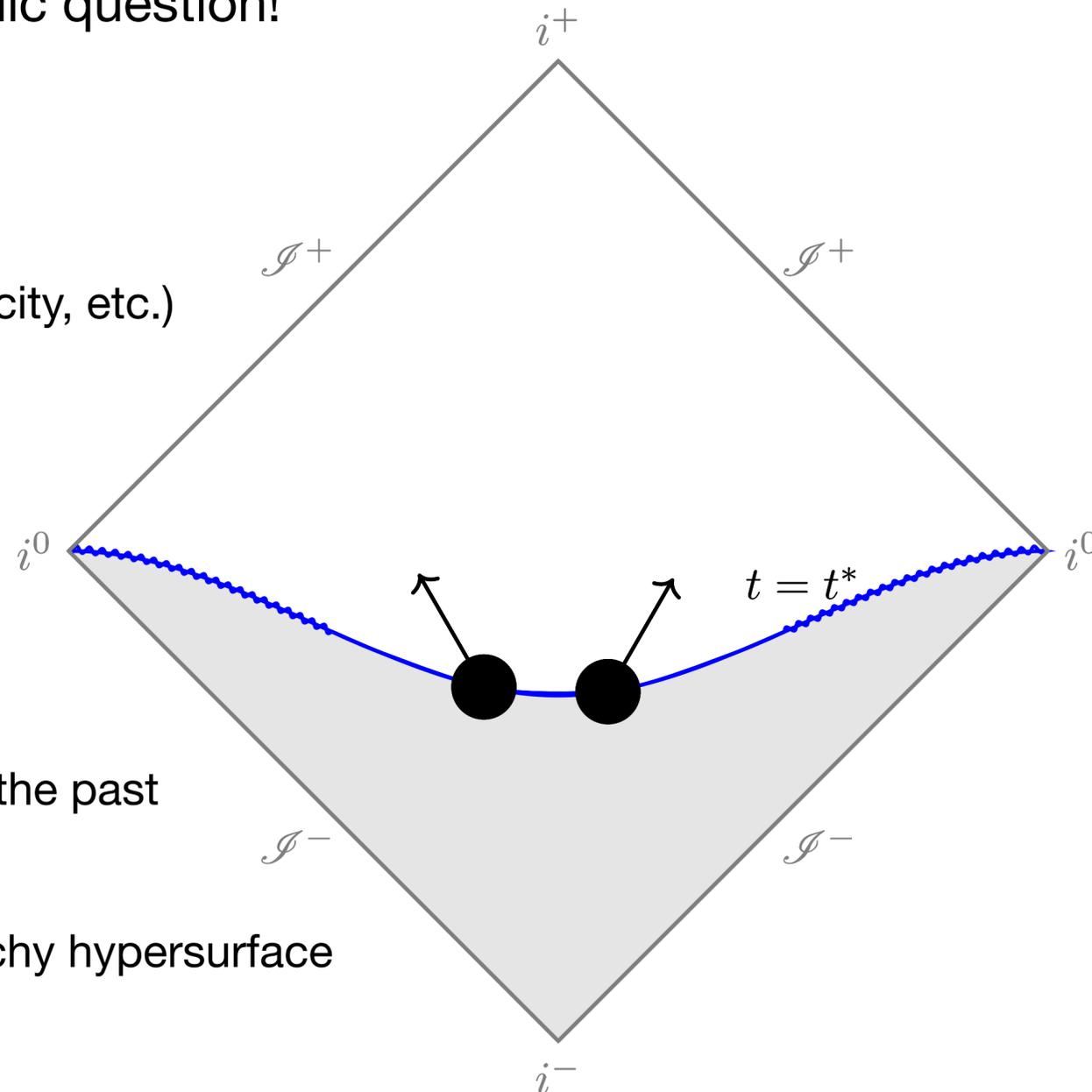
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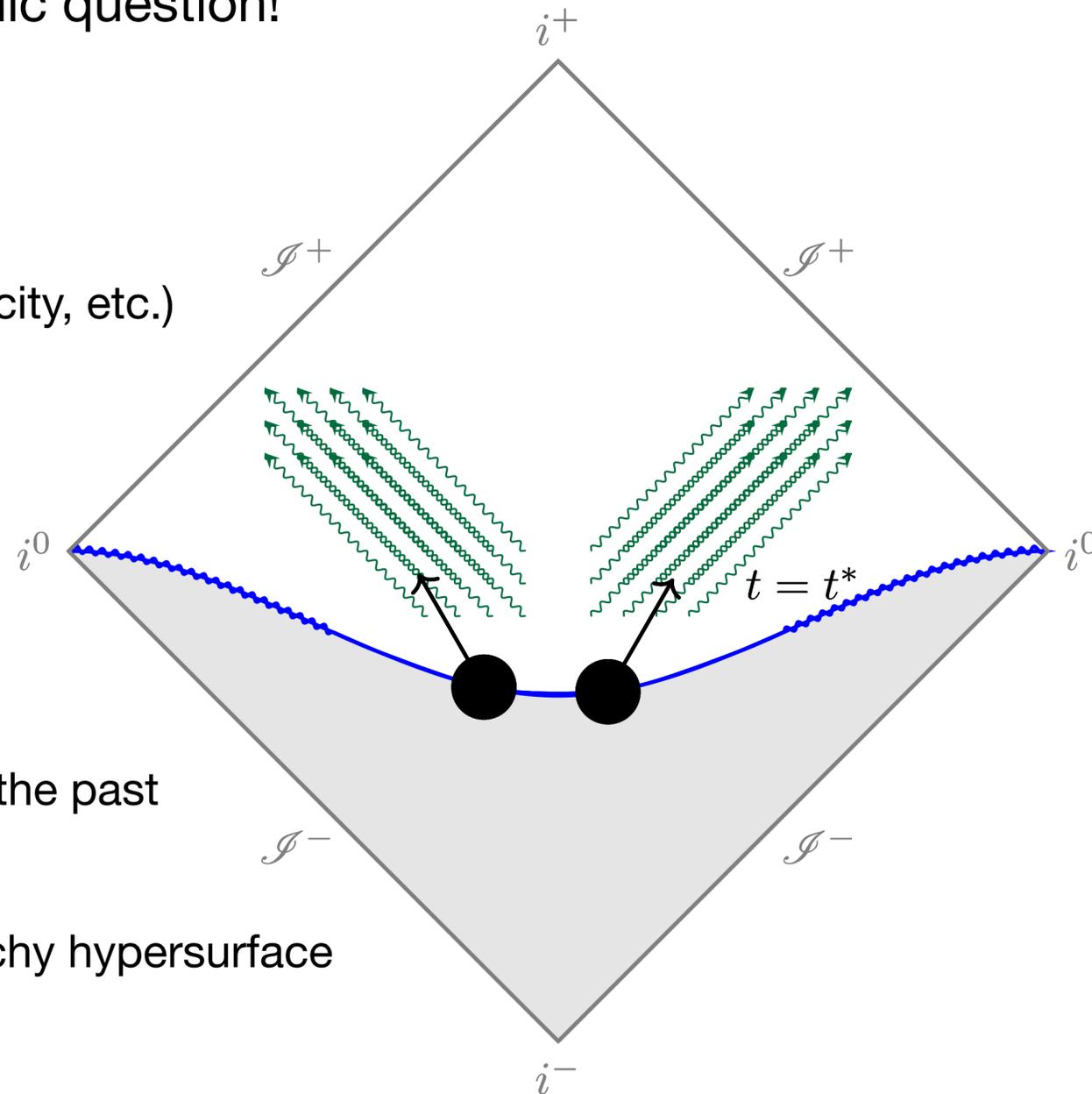
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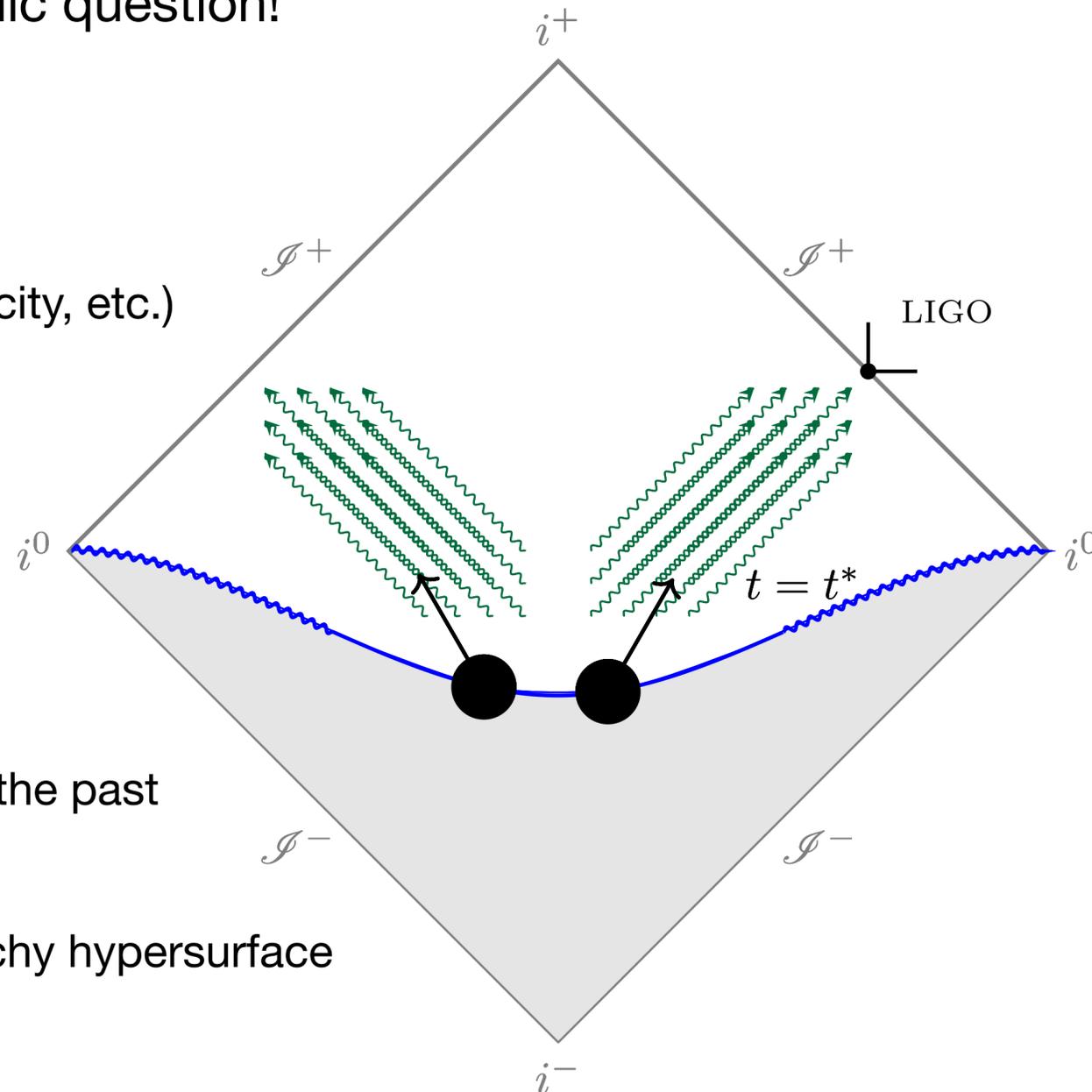
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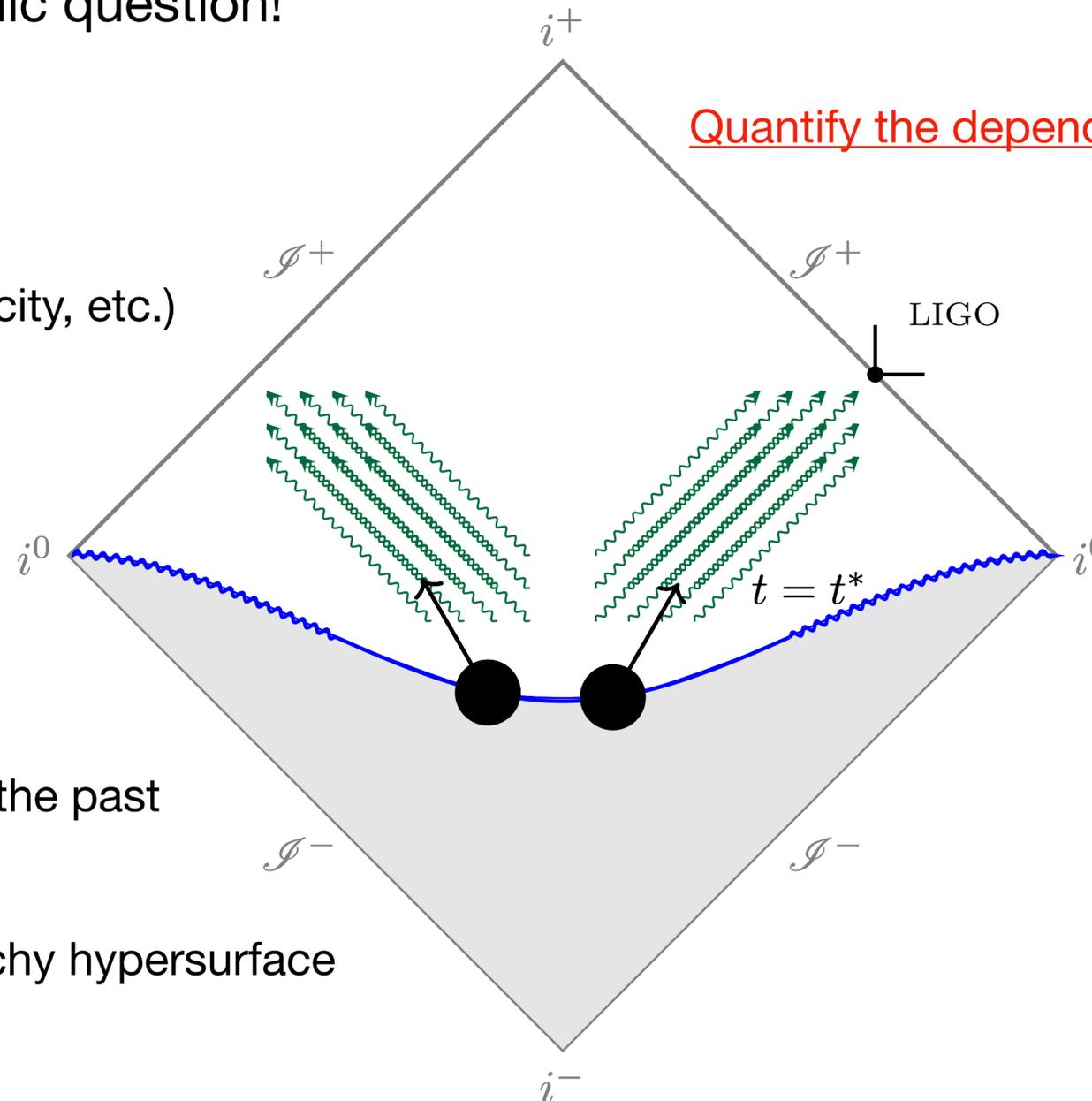
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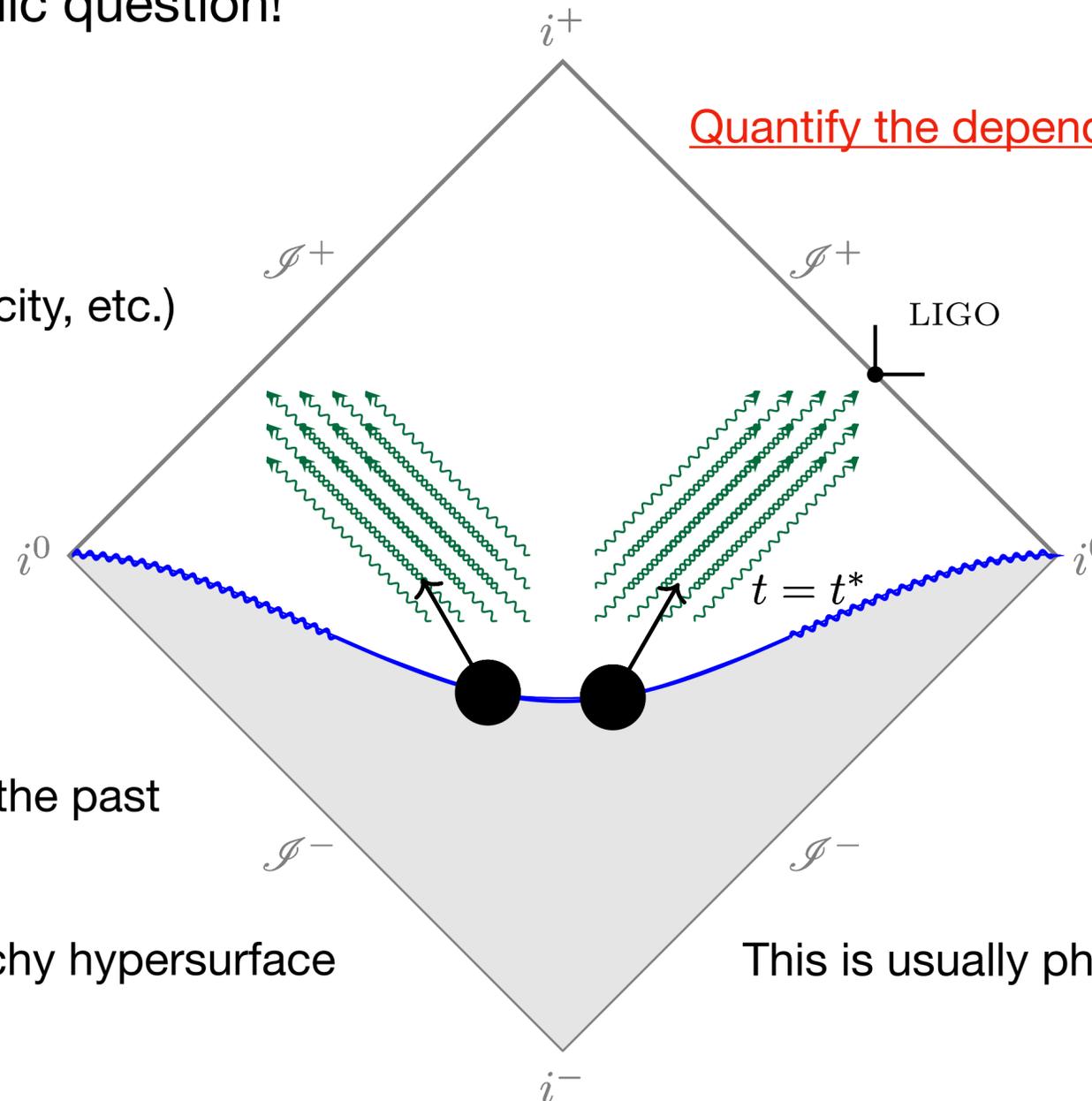
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This is usually phrased in terms of **BMS transformations**.

Isolated Systems in General Relativity

Waveform templates

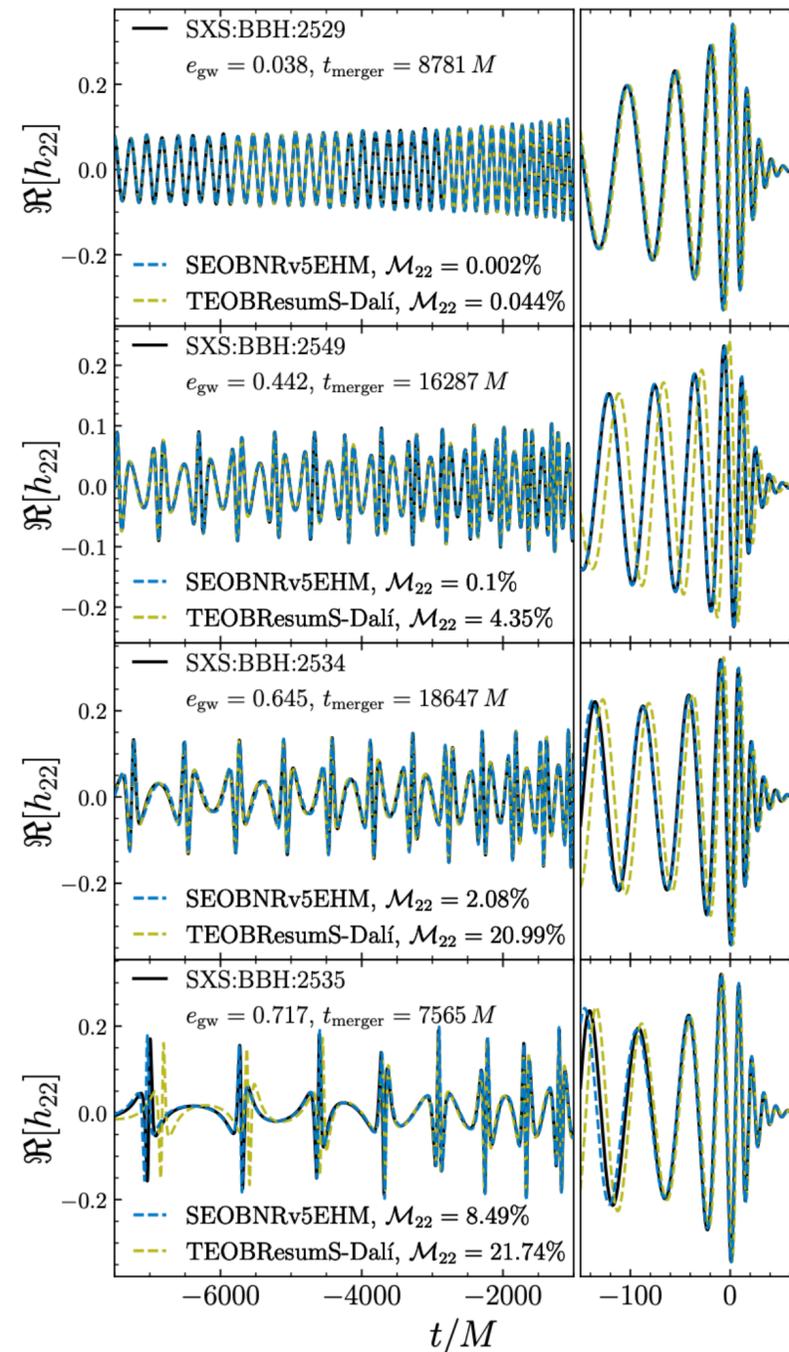


Figure from [arXiv:2412.12823]

Isolated Systems in General Relativity

Waveform templates

We **glue** different results computed in different regimes build accurate templates!

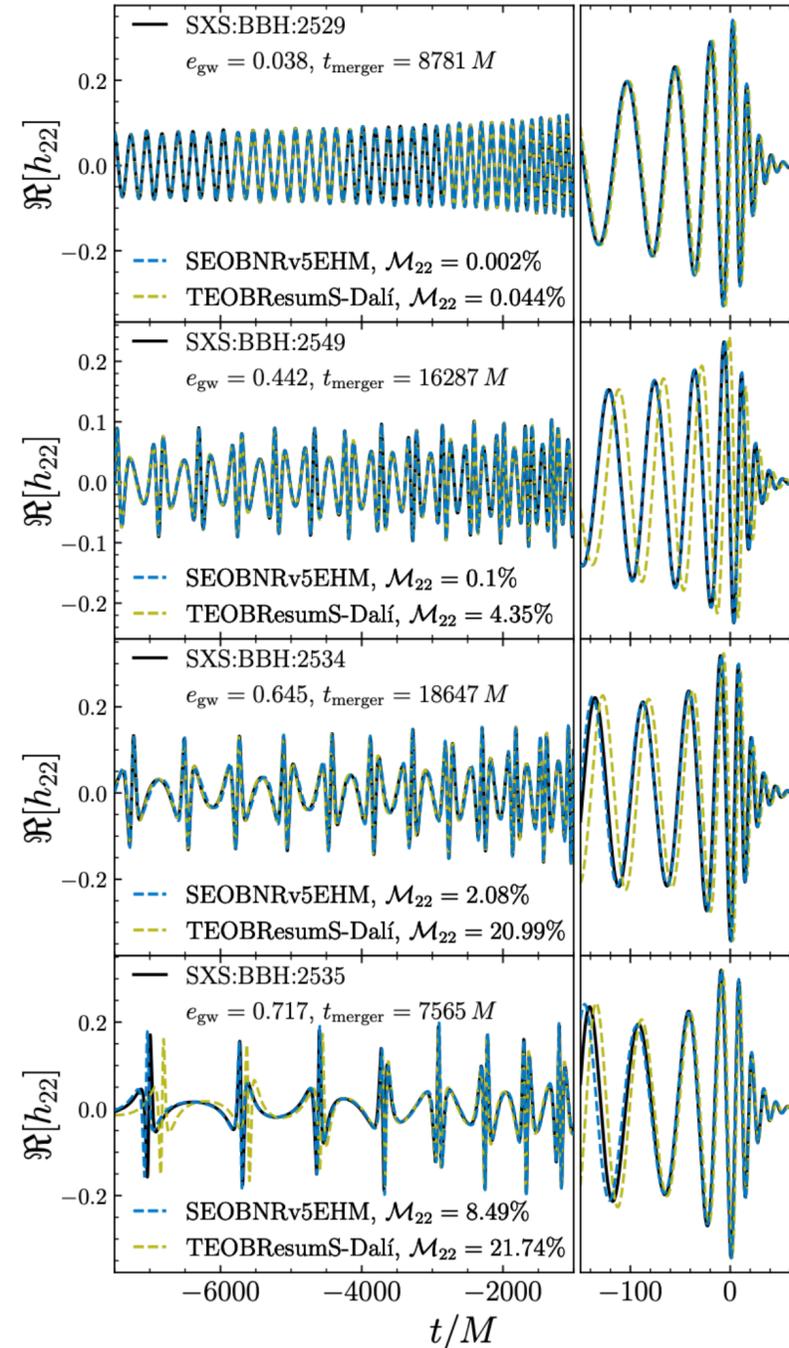
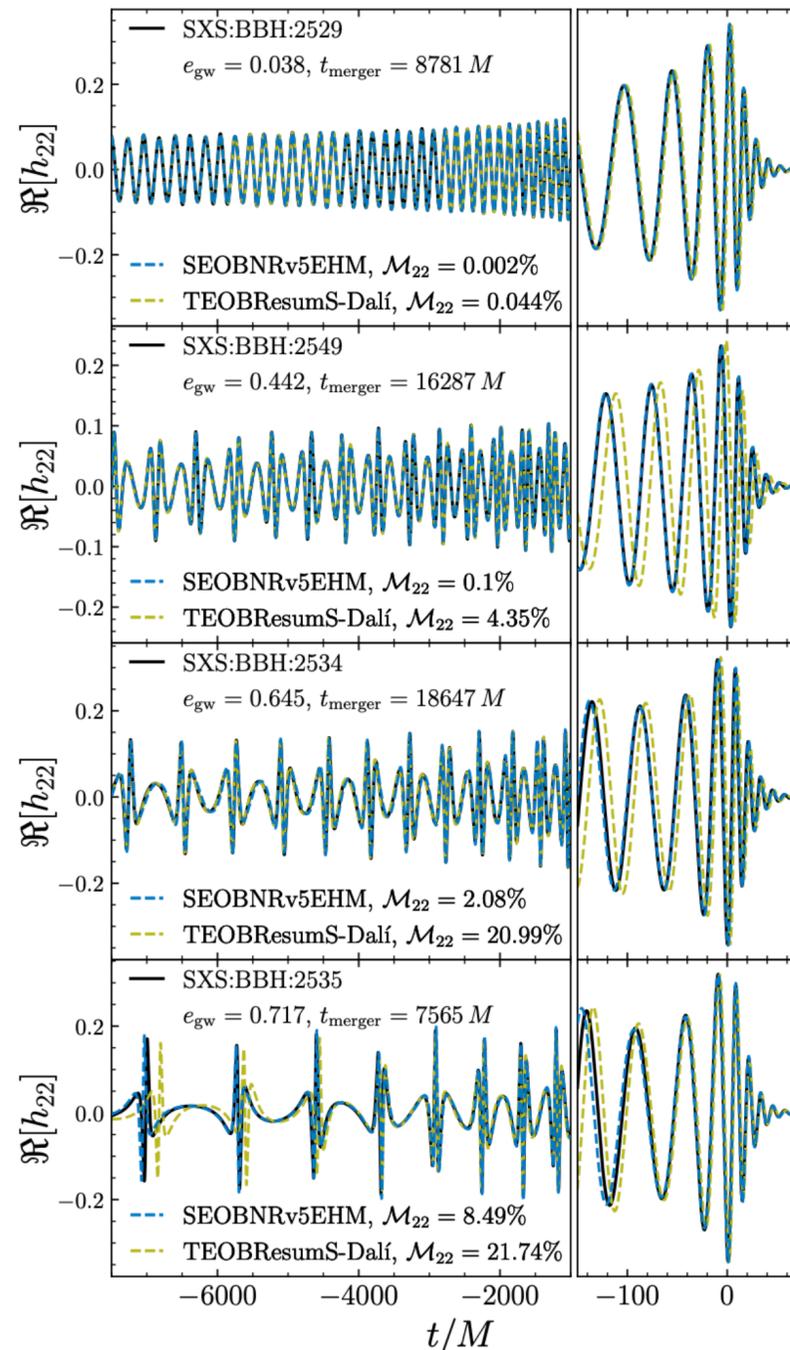


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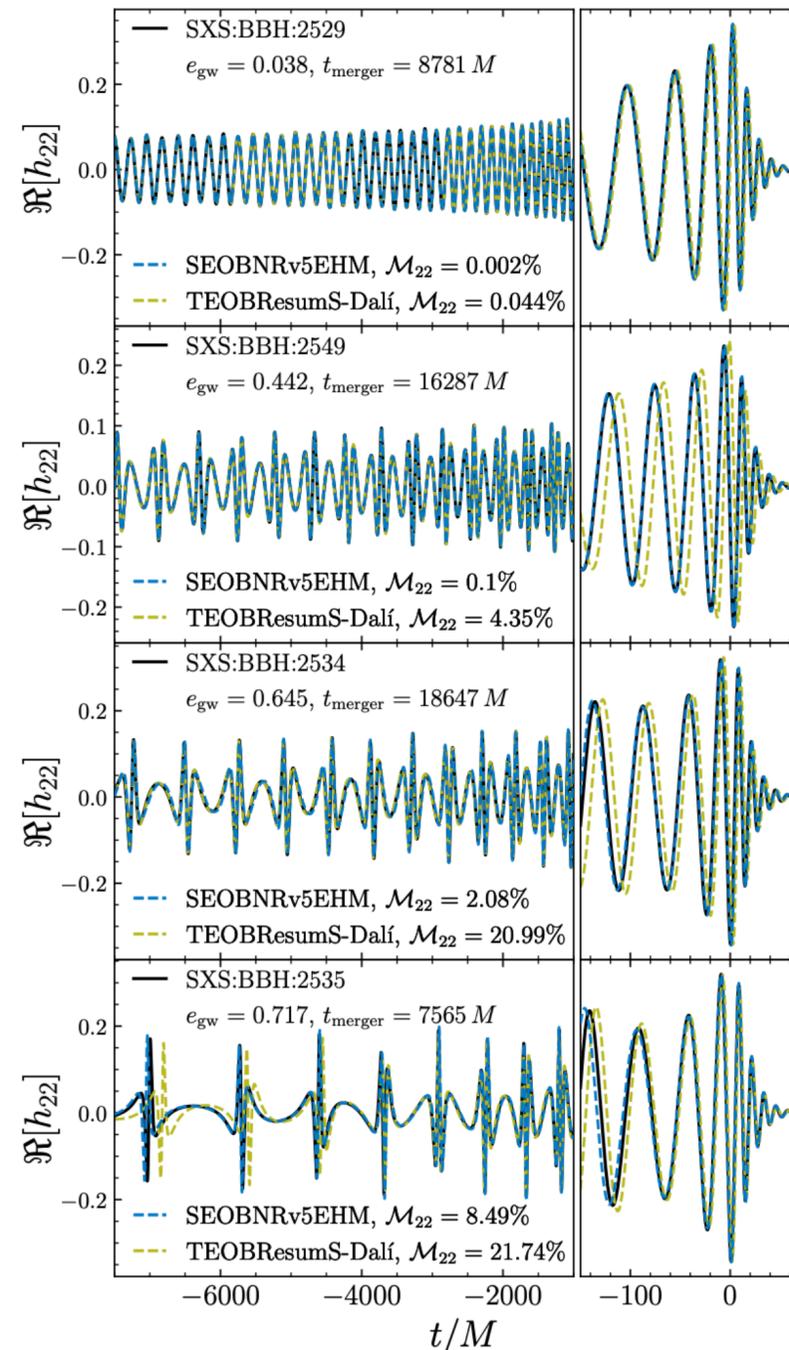
Numerical Relativity has intrinsic **incoming radiation!**

Figure from [arXiv:2412.12823]

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Numerical Relativity has intrinsic **incoming radiation!**

For example, “A Review of Gravitational Memory and **BMS** Frame Fixing in Numerical Relativity” [[arXiv:2405.08868](https://arxiv.org/abs/2405.08868)]

Figure from [[arXiv:2412.12823](https://arxiv.org/abs/2412.12823)]

Waveforms and Supertranslations

Field Theory Waveform vs Multipolar post-Minkowskian

See **David's** talk

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$$W(u) = \frac{1}{4\pi r} \int_0^\infty \frac{d\omega}{2\pi} \left[e^{i\omega u} W_{\text{MPM}}(\omega) + \text{c.c.} \right] = \frac{1}{4\pi r} \int \frac{d\omega}{2\pi} \left[e^{i\omega \left[u + m_1 u_1 \cdot n \log(u_1 \cdot n) + m_2 u_2 \cdot n \log(u_2 \cdot n) \right]} W_{\text{KMOC}}(\omega) + \text{c.c.} \right]$$

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Same for angular momentum: [Veneziano, Vilkovisky]
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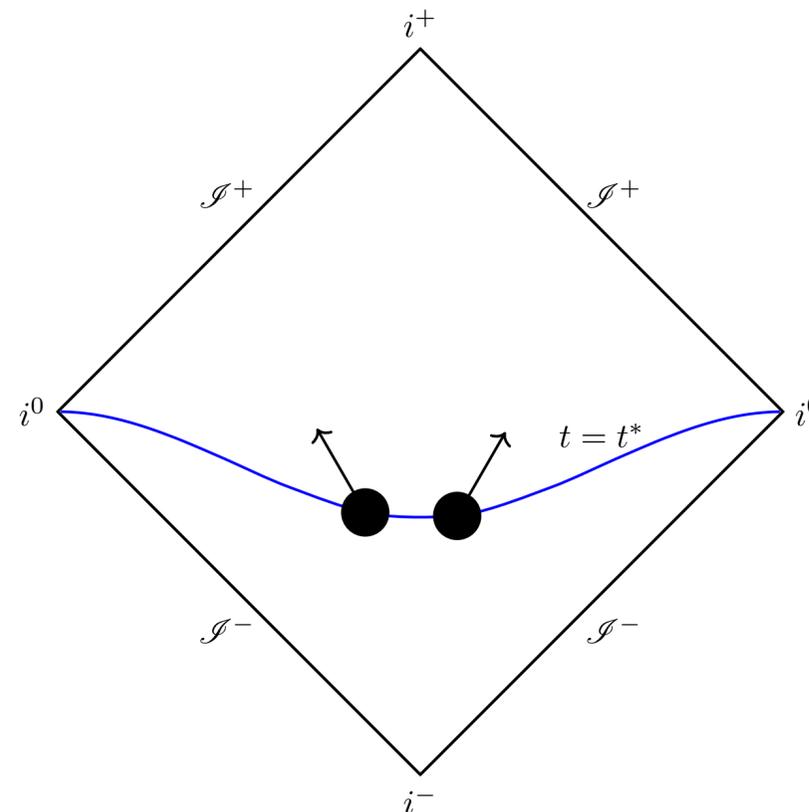
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[Bini, Damour, SDA, Geralico, Herderschee, Roiban, Teng]



Boundary conditions:
space-like hypersurface vs i^0 and \mathcal{I}^-

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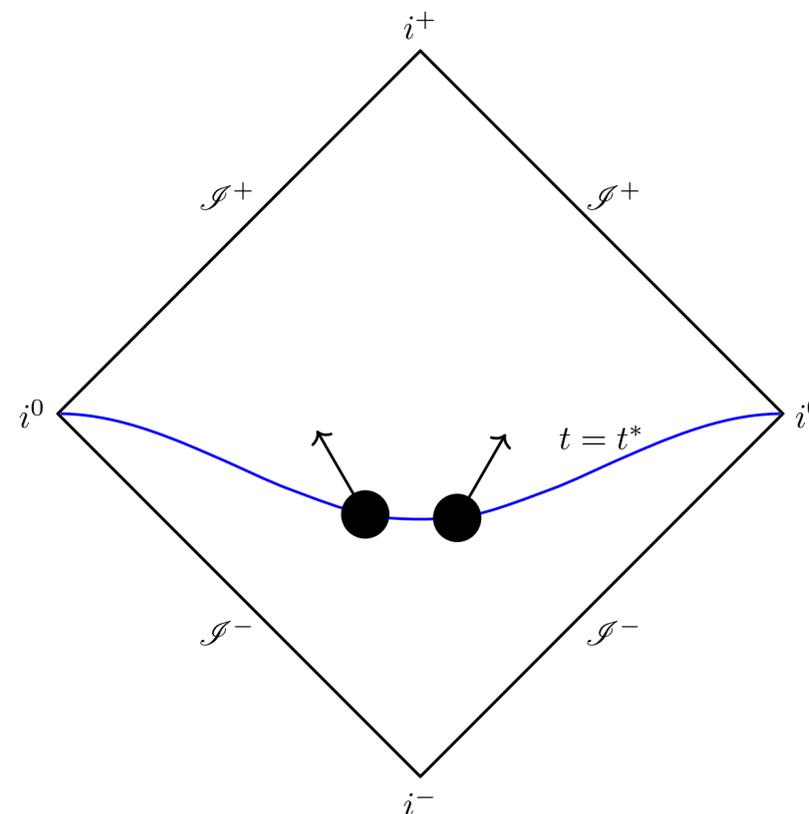
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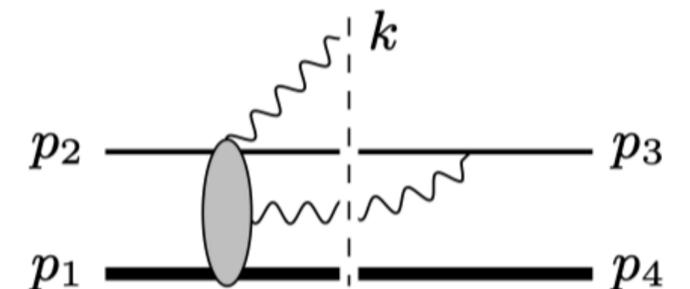
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Regulator:
dim.reg & disconnected contributions



A cruch course of Asymptotic Simplicity

Asymptotic smoothness and the Newman-Penrose scalars

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Asymptotic smoothness and the Newman-Penrose scalars

Newman-Penrose scalars are components of the Weyl/Rieman tensor

$$\Psi_4 = C_{\mu\nu\rho\sigma} N^\mu \varepsilon_-^\nu N^\rho \varepsilon_-^\sigma$$

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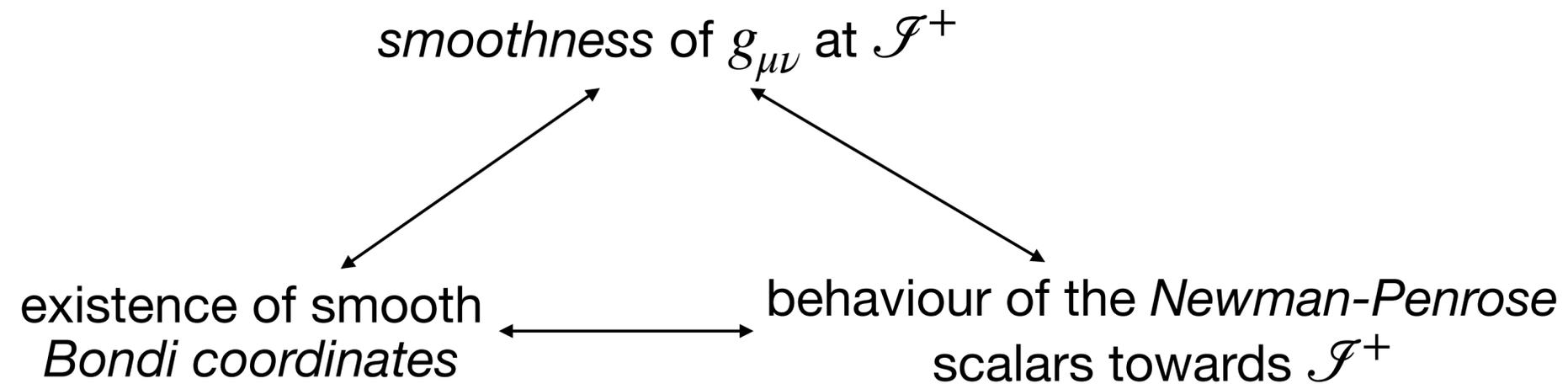
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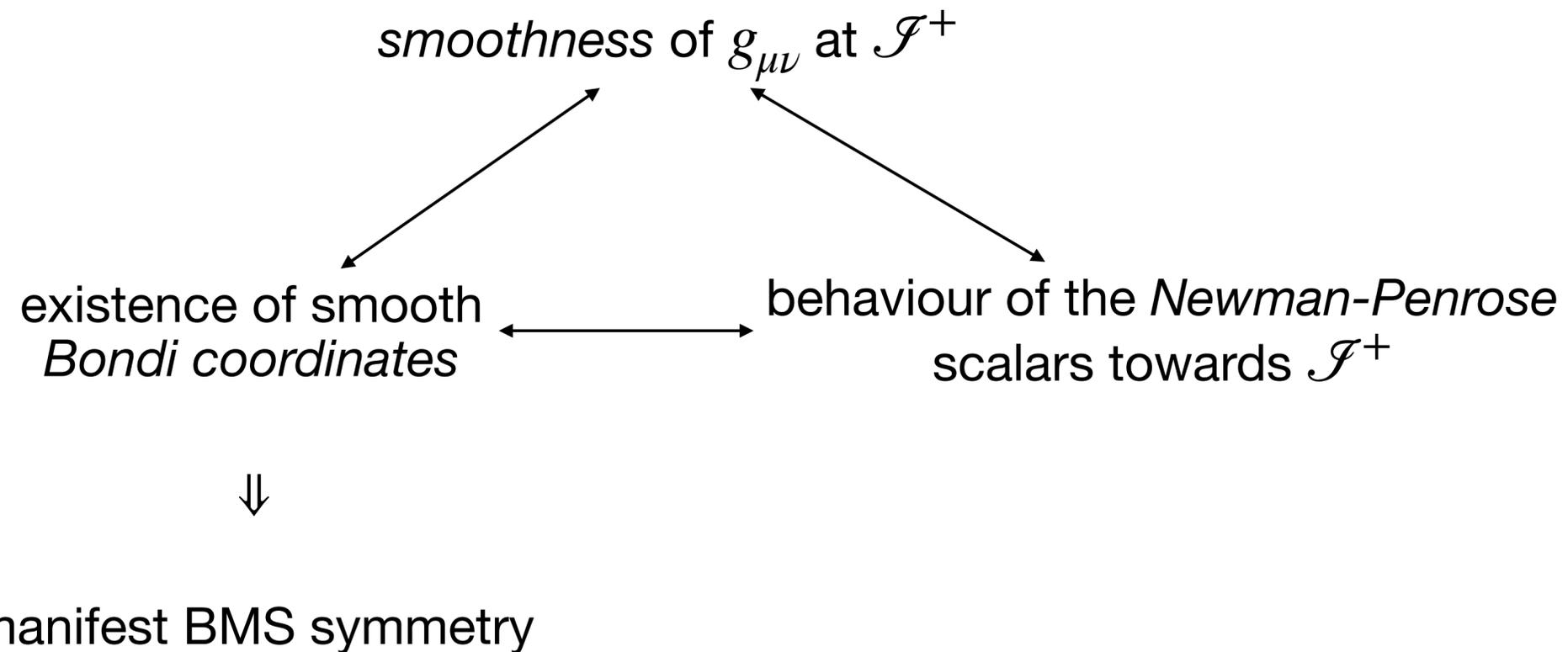
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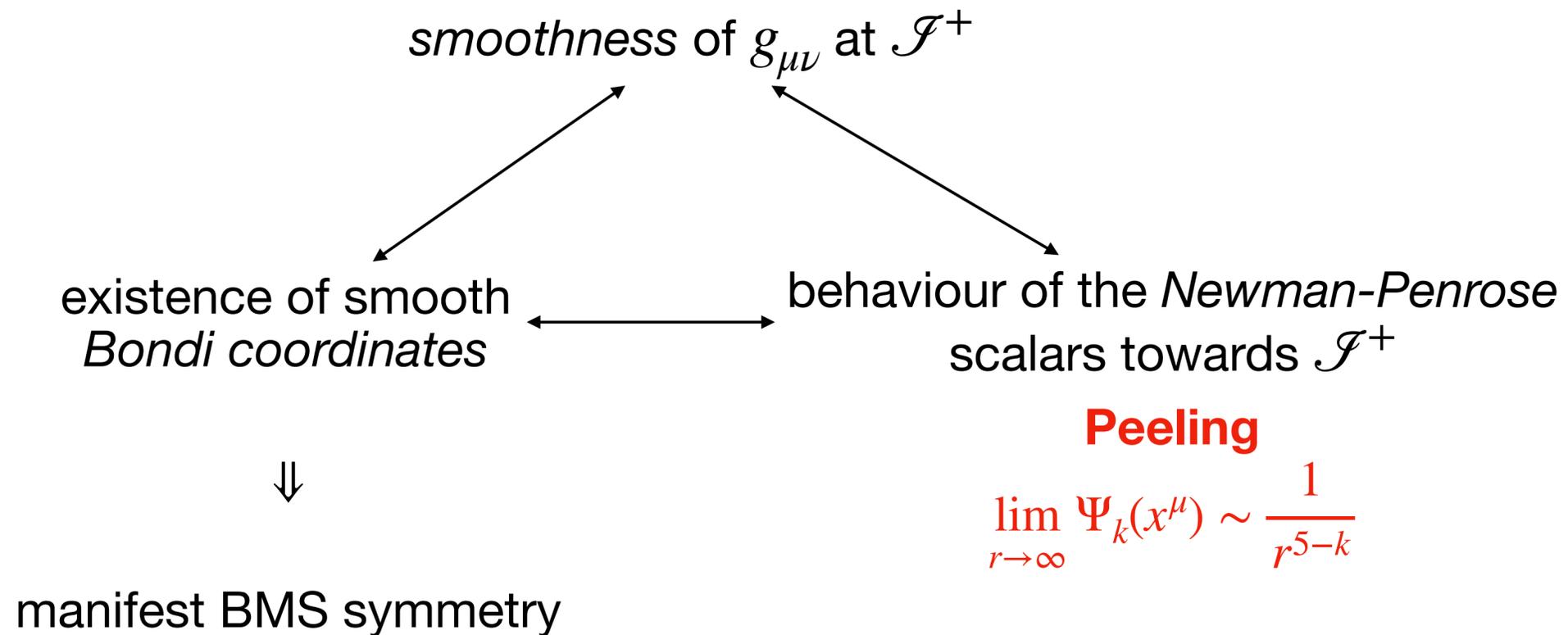
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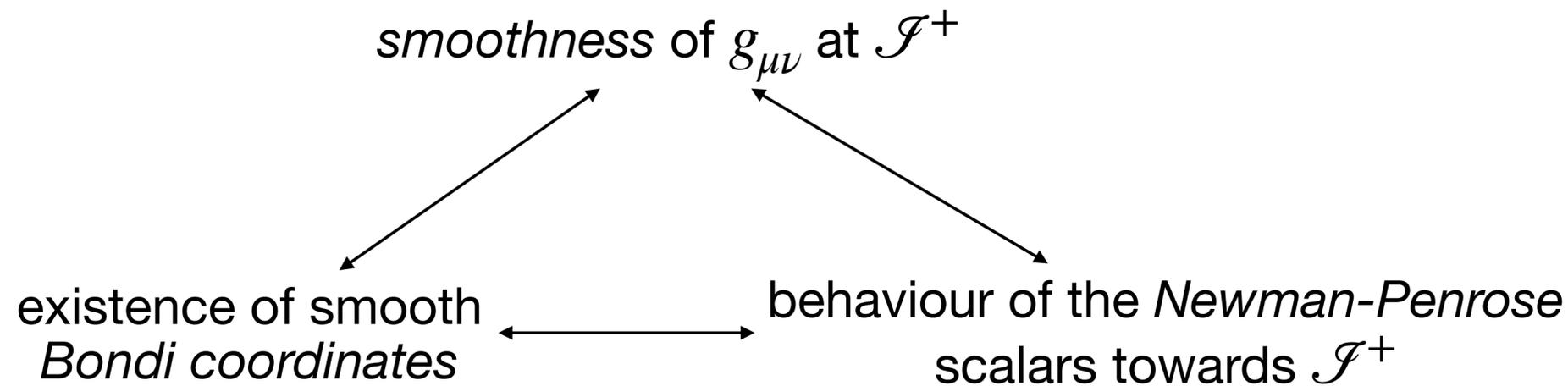
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Peeling

$$\lim_{r \rightarrow \infty} \Psi_k(x^\mu) \sim \frac{1}{r^{5-k}}$$

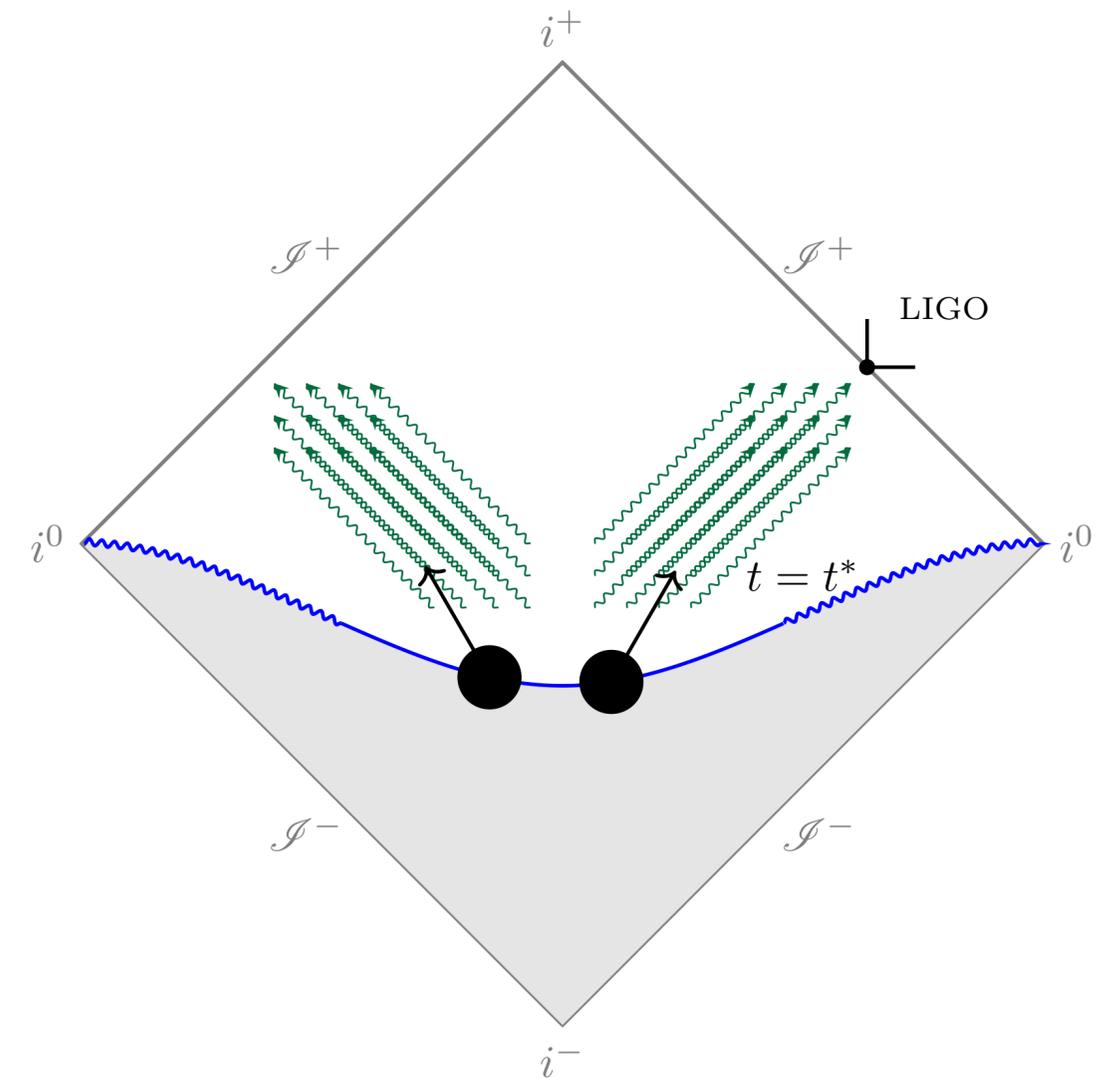
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manifest BMS symmetry

We will probe smoothness studying the decay of the NP scalars!

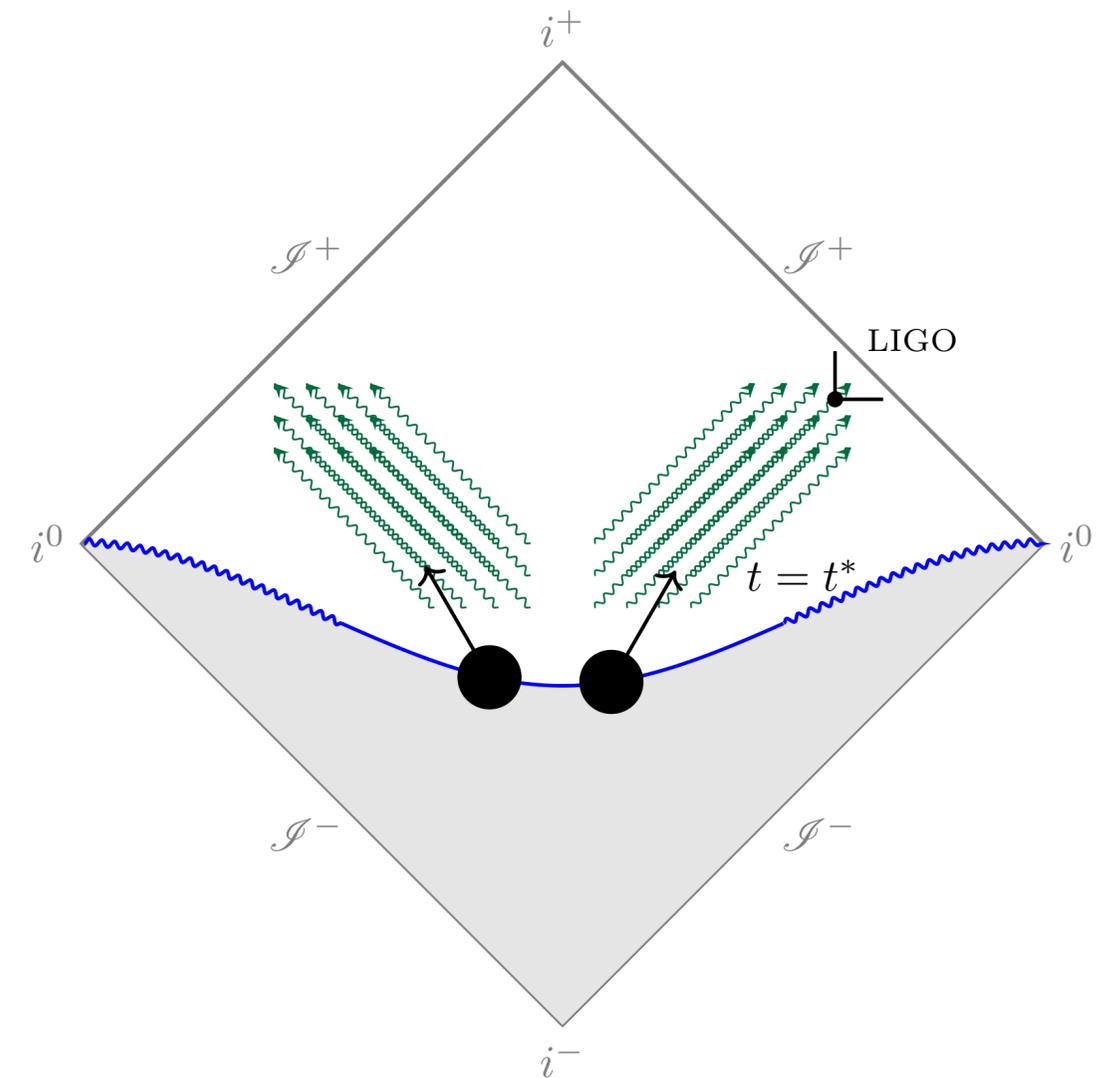
Two-body Scattering

The computational set-up for two-body scattering



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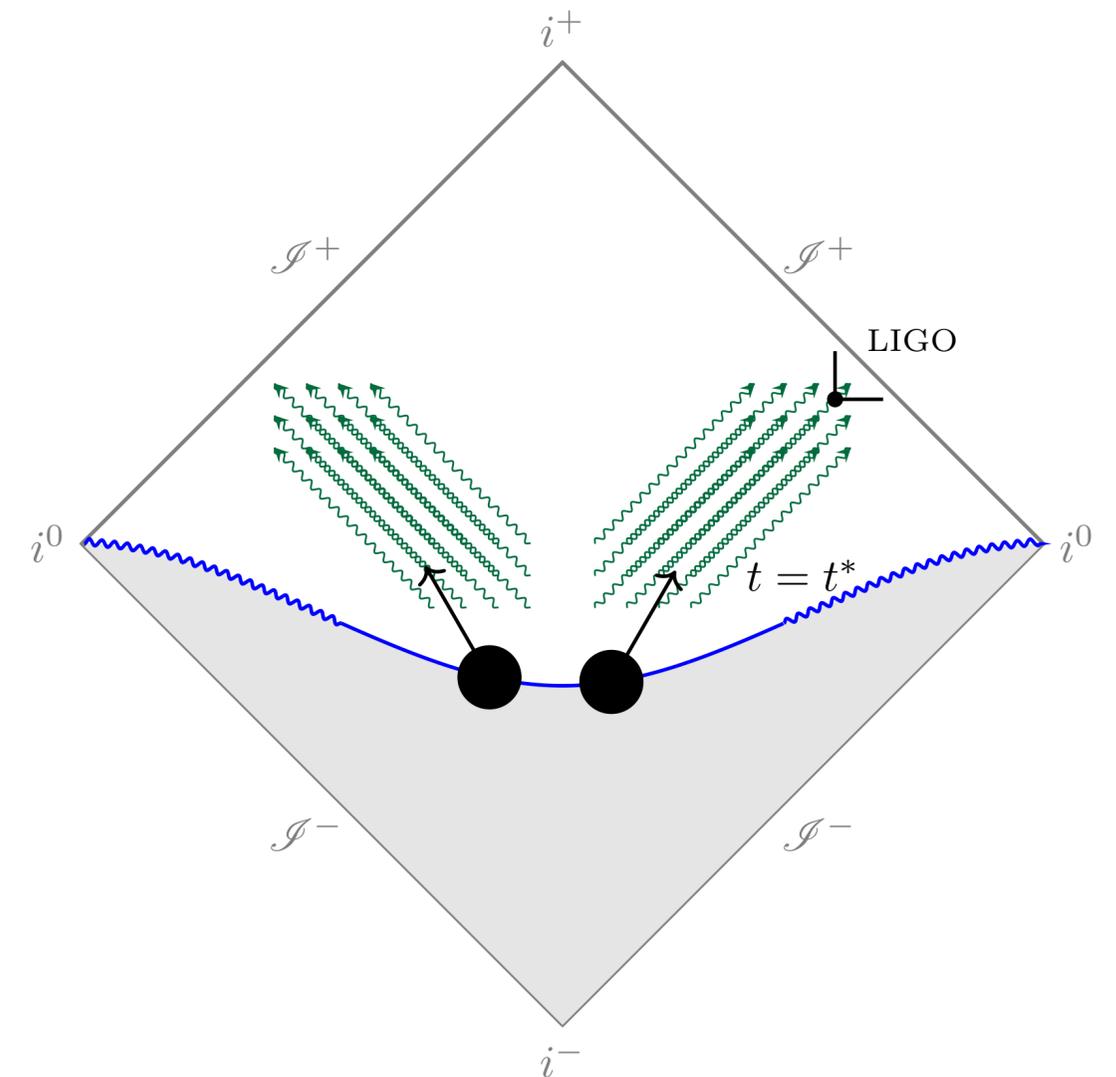
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The computational set-up for two-body scattering

We want to solve **Einstein's equations**

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

two-body stress-energy tensor



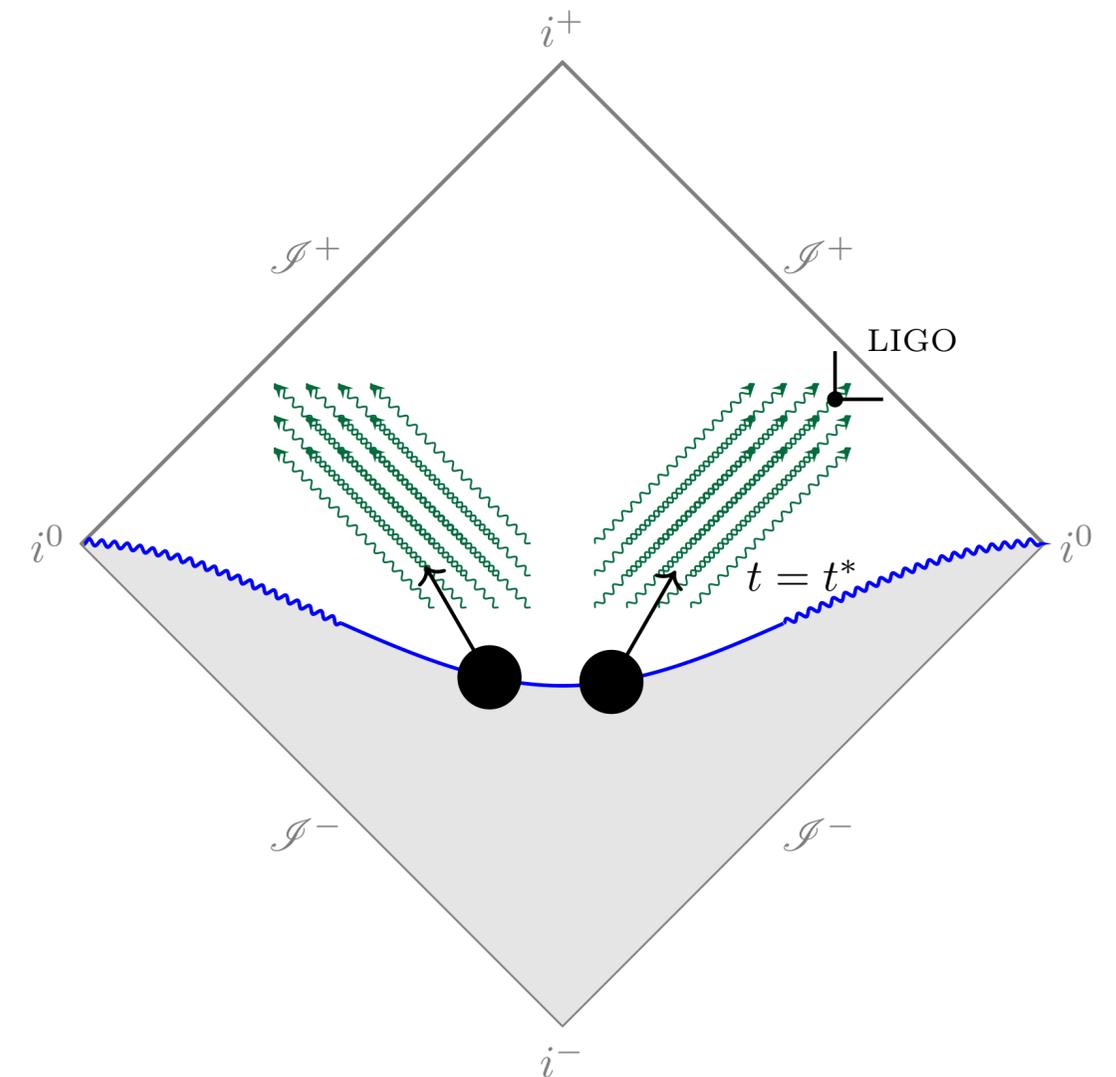
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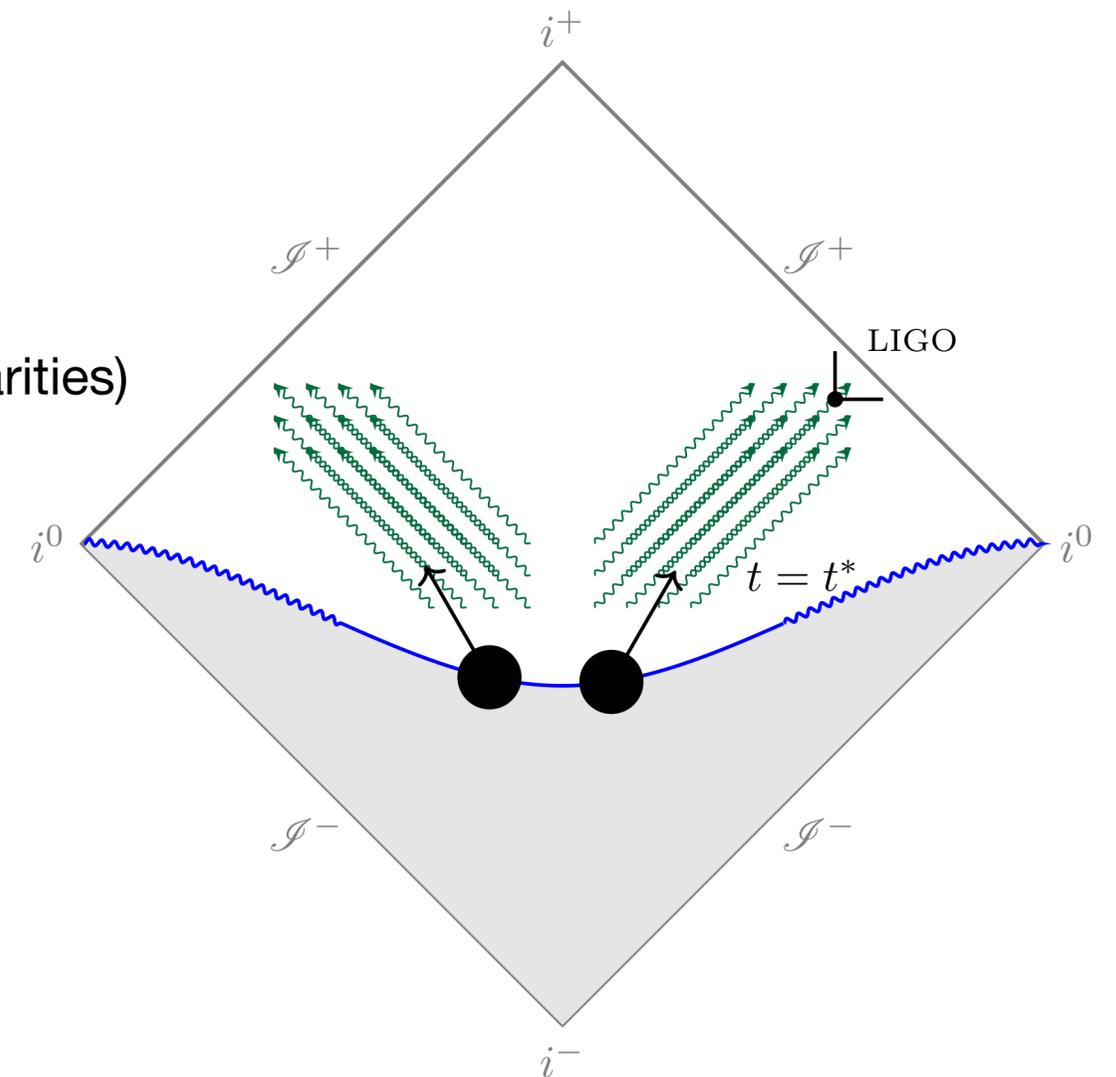
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$$\square h_{\mu\nu} = J_{\mu\nu} \quad \text{pseudo-stress-energy tensor (with non-linearities)}$$



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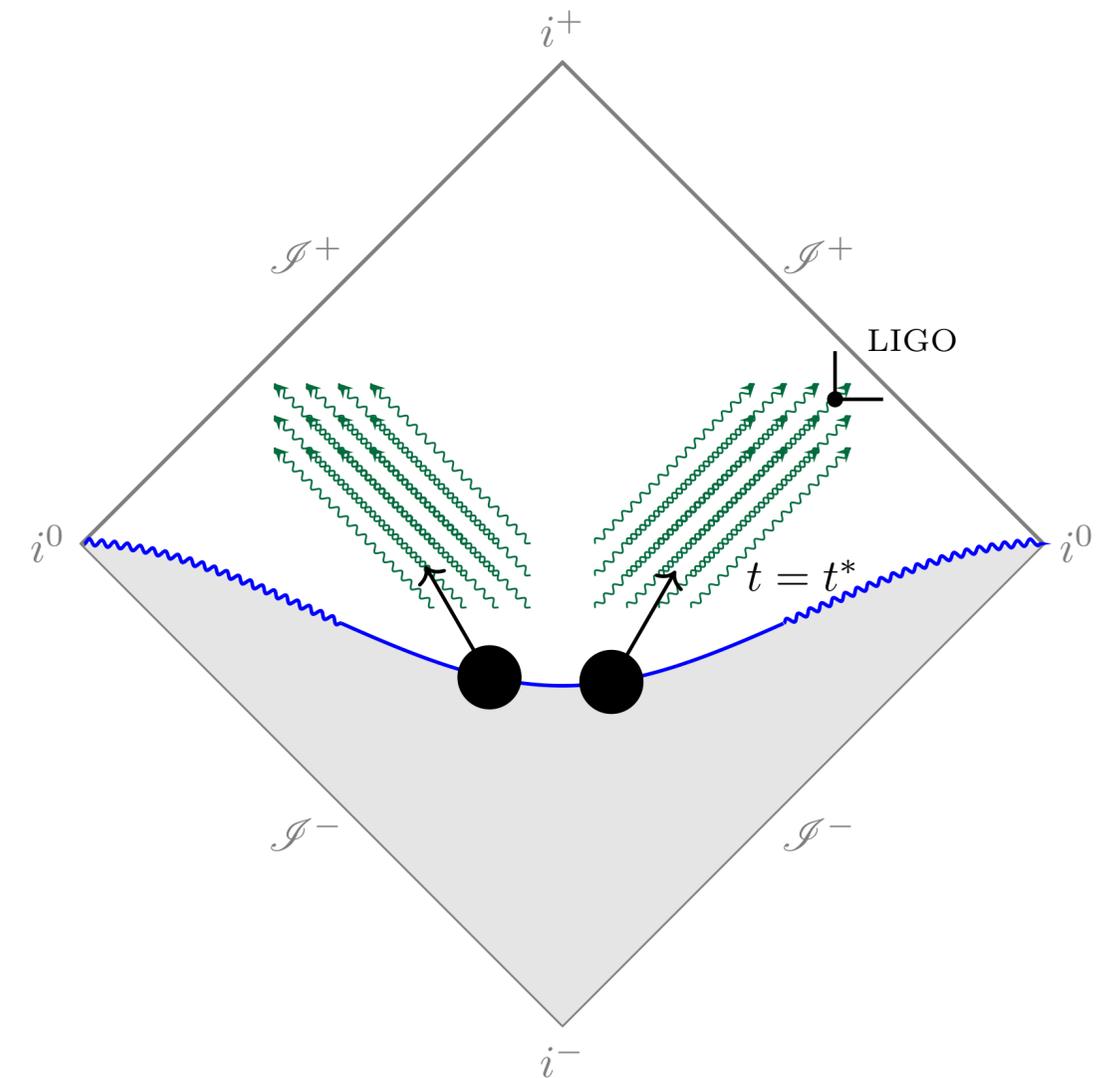
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$$h_{\mu\nu}(x) = - \int \frac{d^d k}{(2\pi)^d} \frac{e^{-ikx}}{(k_0 + i\epsilon)^2 - \vec{k}^2} \mathcal{P}_{\mu\nu}^{\mu'\nu'} J_{\mu'\nu'}(k)$$



Two-body Scattering

The computational set-up for two-body scattering

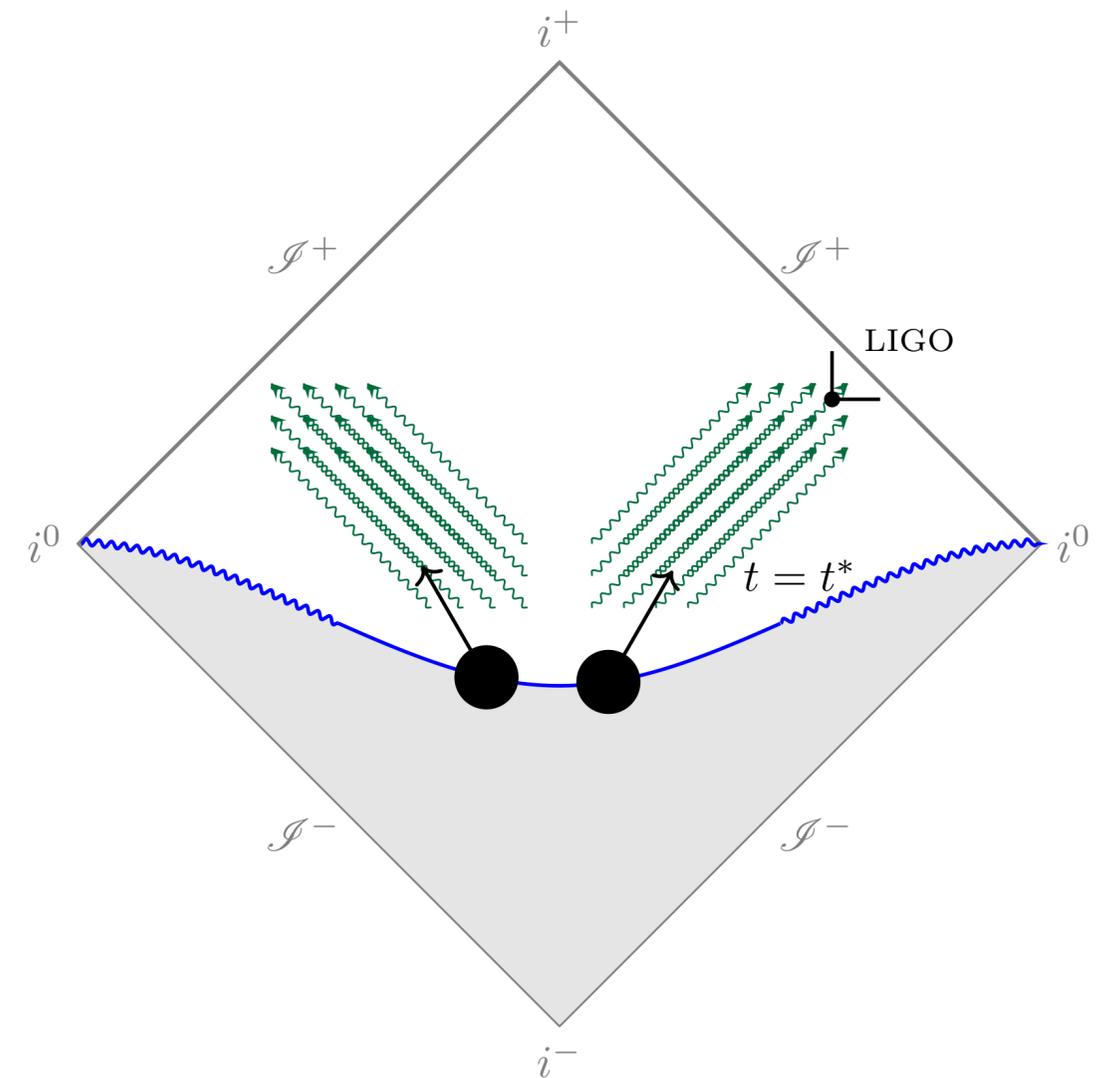
We want to solve **Einstein's equations**

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Expand around the flat metric: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$

$$h_{\mu\nu}(x) = - \int \frac{d^d k}{(2\pi)^d} \frac{e^{-ikx}}{(k_0 + i\epsilon)^2 - \vec{k}^2} \mathcal{P}_{\mu\nu}^{\mu'\nu'} J_{\mu'\nu'}(k)$$

source of to the graviton field



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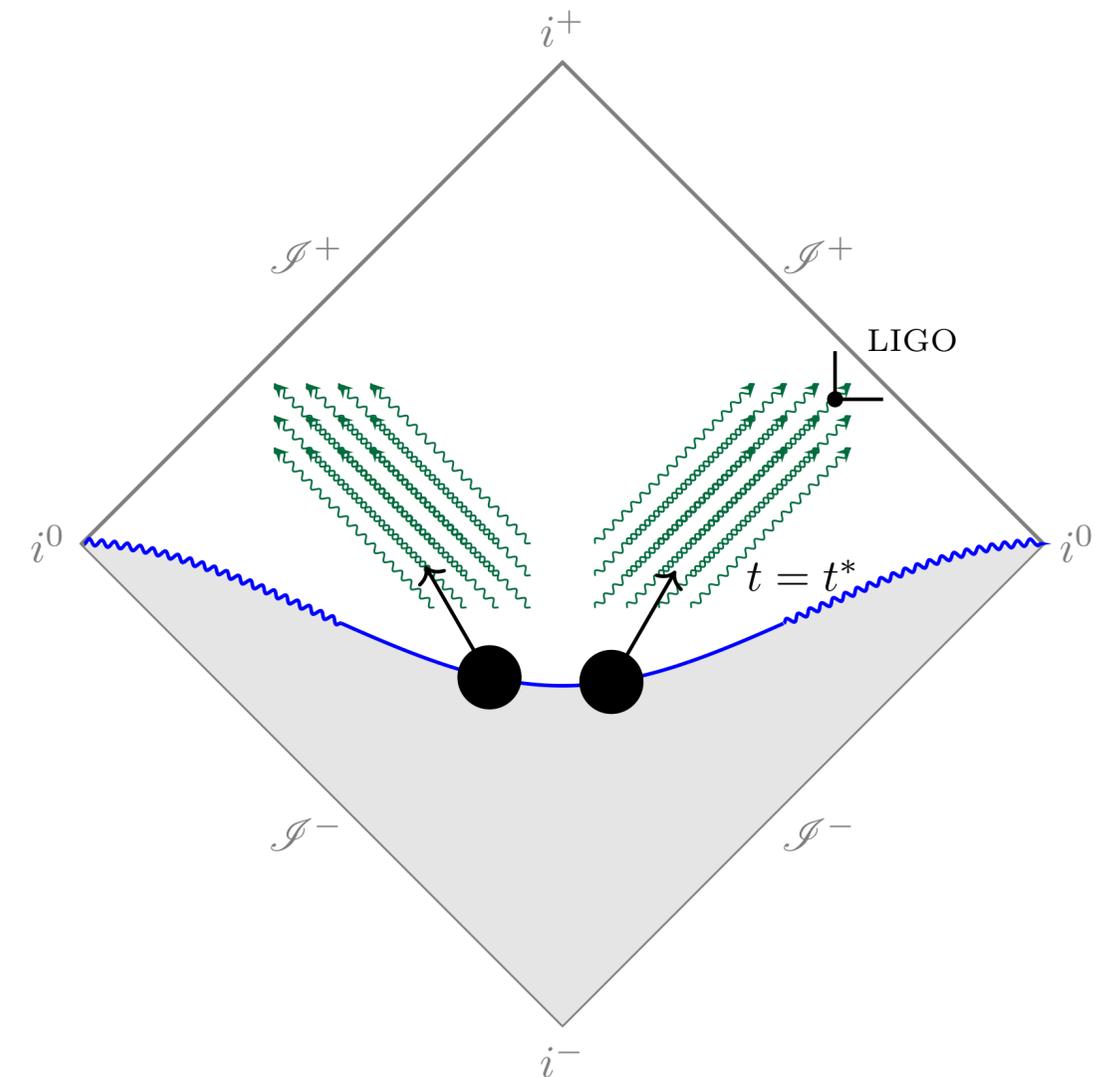
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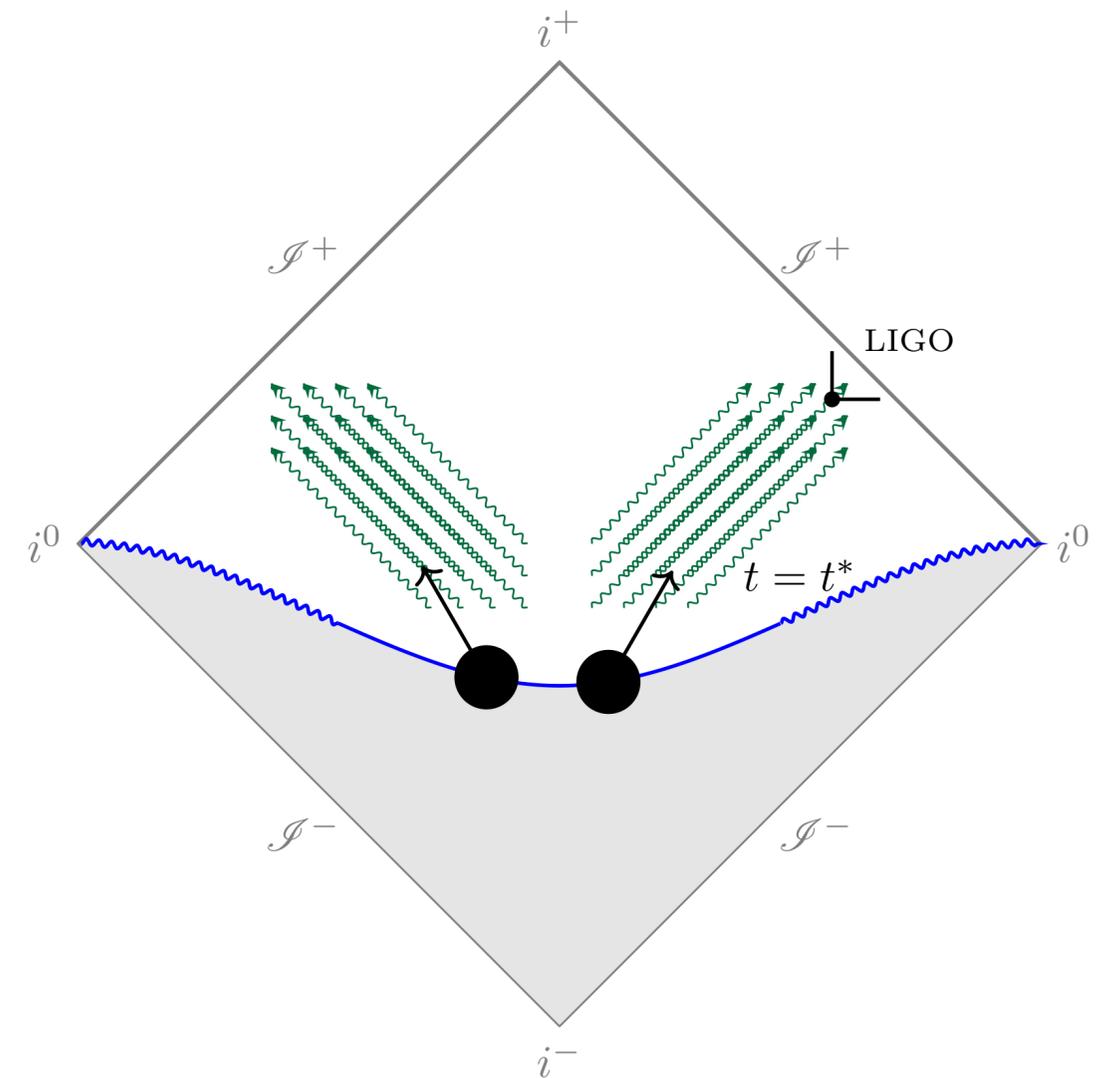
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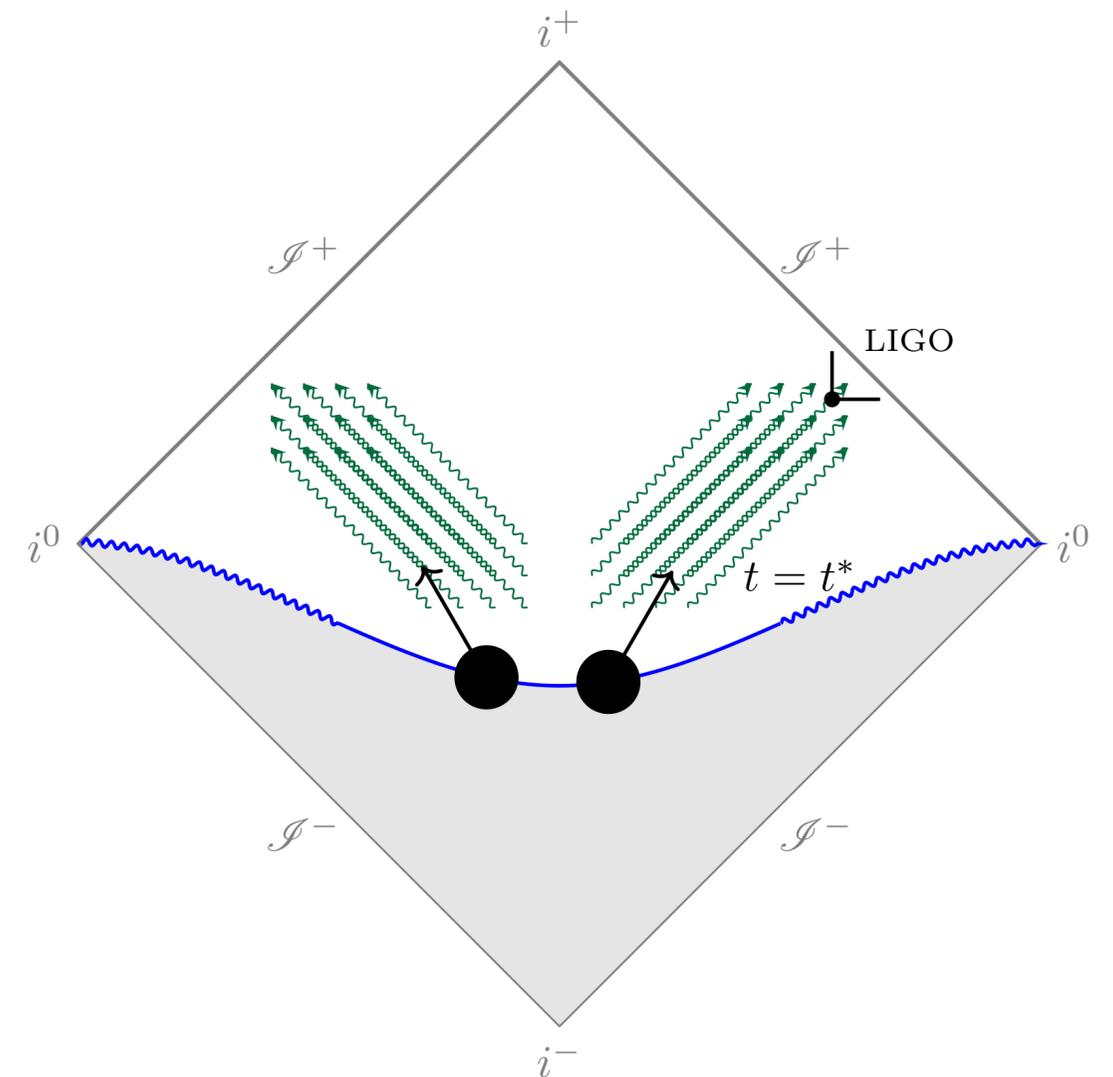
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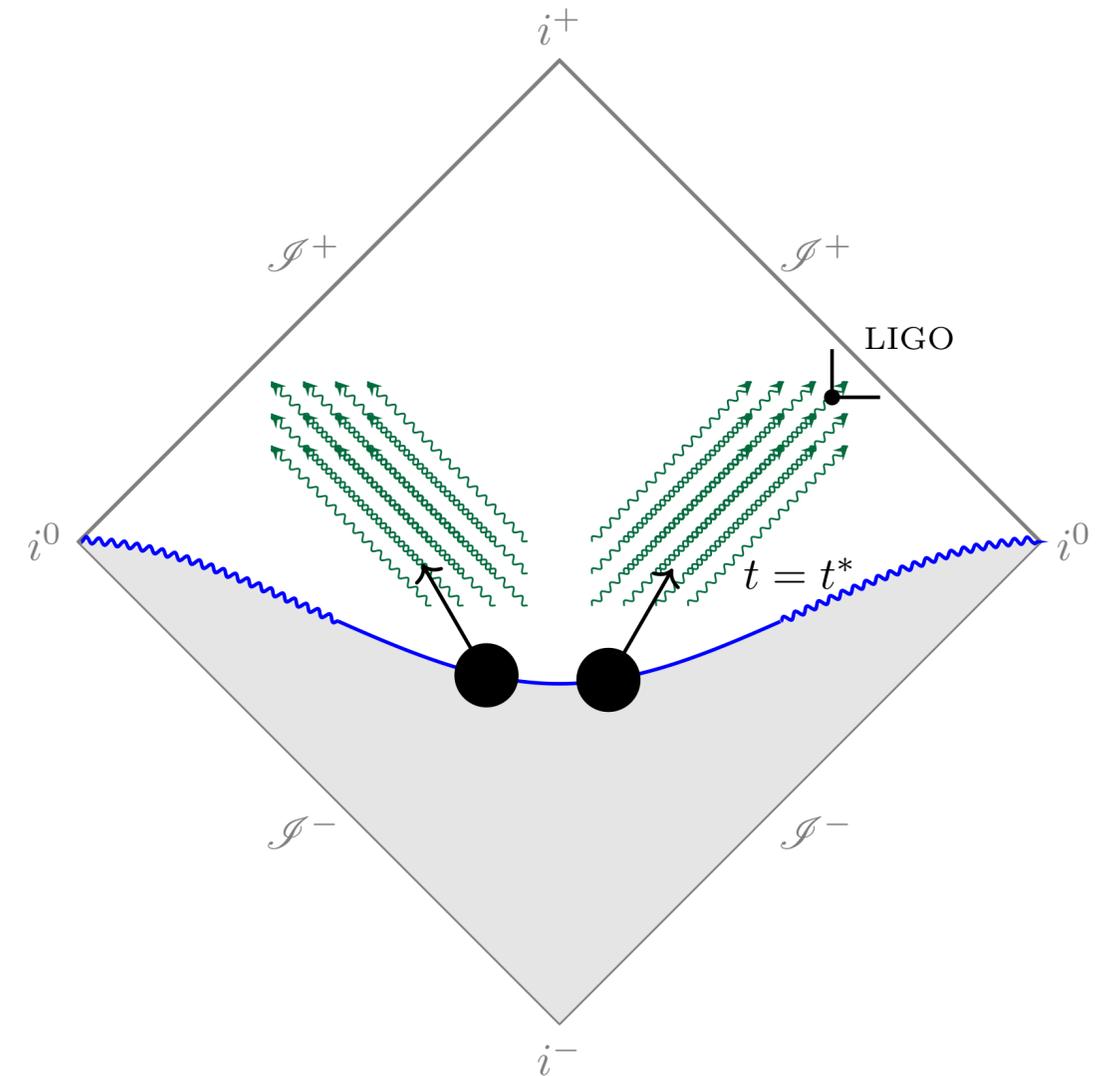
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- **Field-theory inspired computation**



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The hierarchies between the scales

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Expectation value of a current

see David, Giacomo, Donal, Radu

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Asymptotic expansion of the graviton field expansion value

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$J^{\mu\nu}(k)$ depends on $R_i < b$ (length scales) and a few more dimensionless parameters.

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$$R_i, b, u \ll t, r$$

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A reminder of old-fashioned methods to compute the asymptotic expansion of integrals $\int_C dz e^{irf(z)} g(z)$.

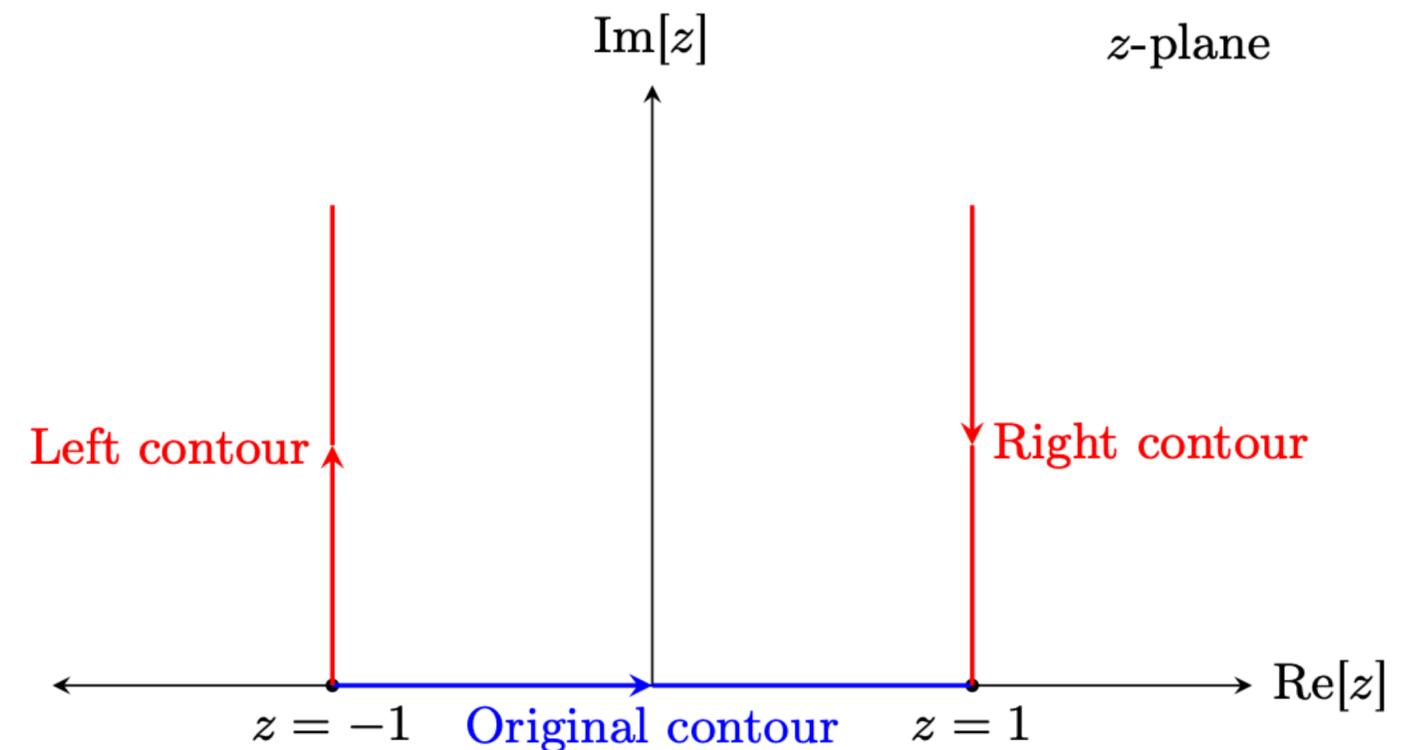
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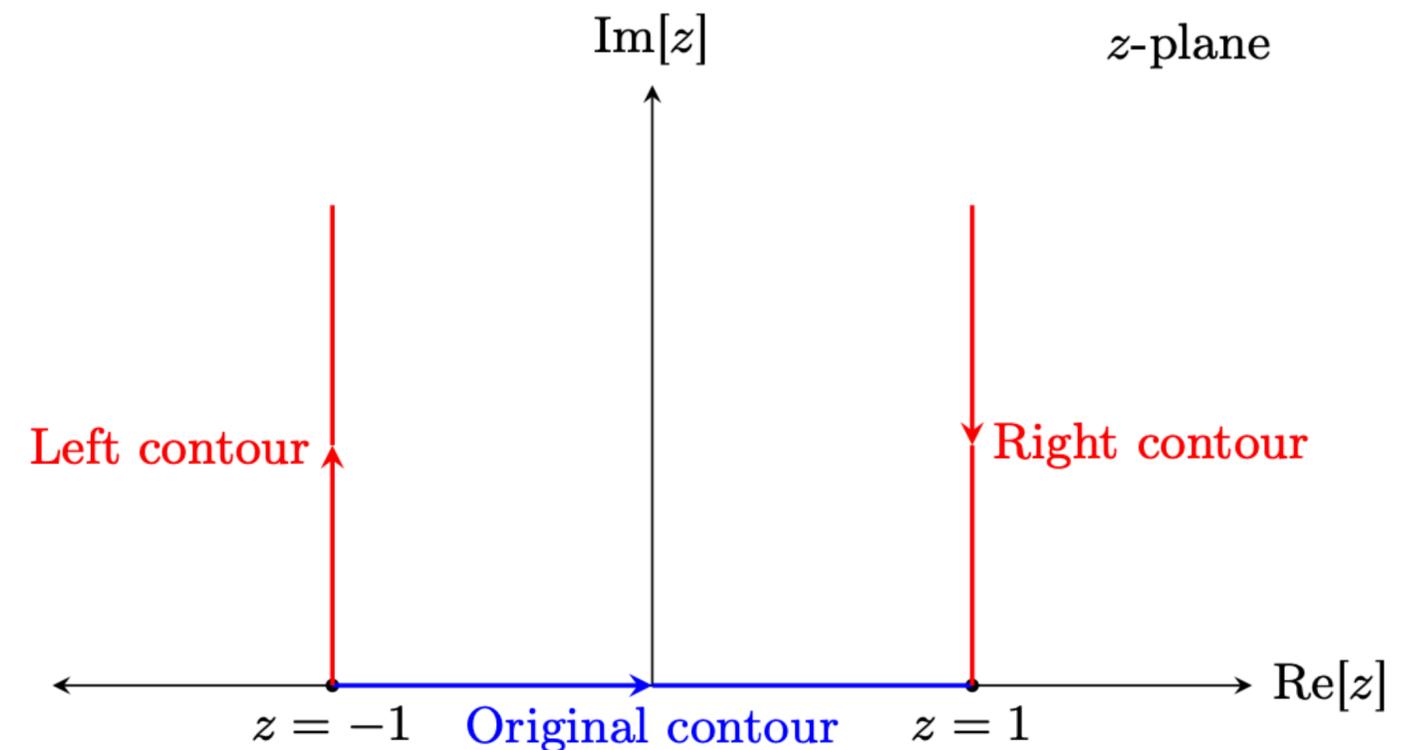
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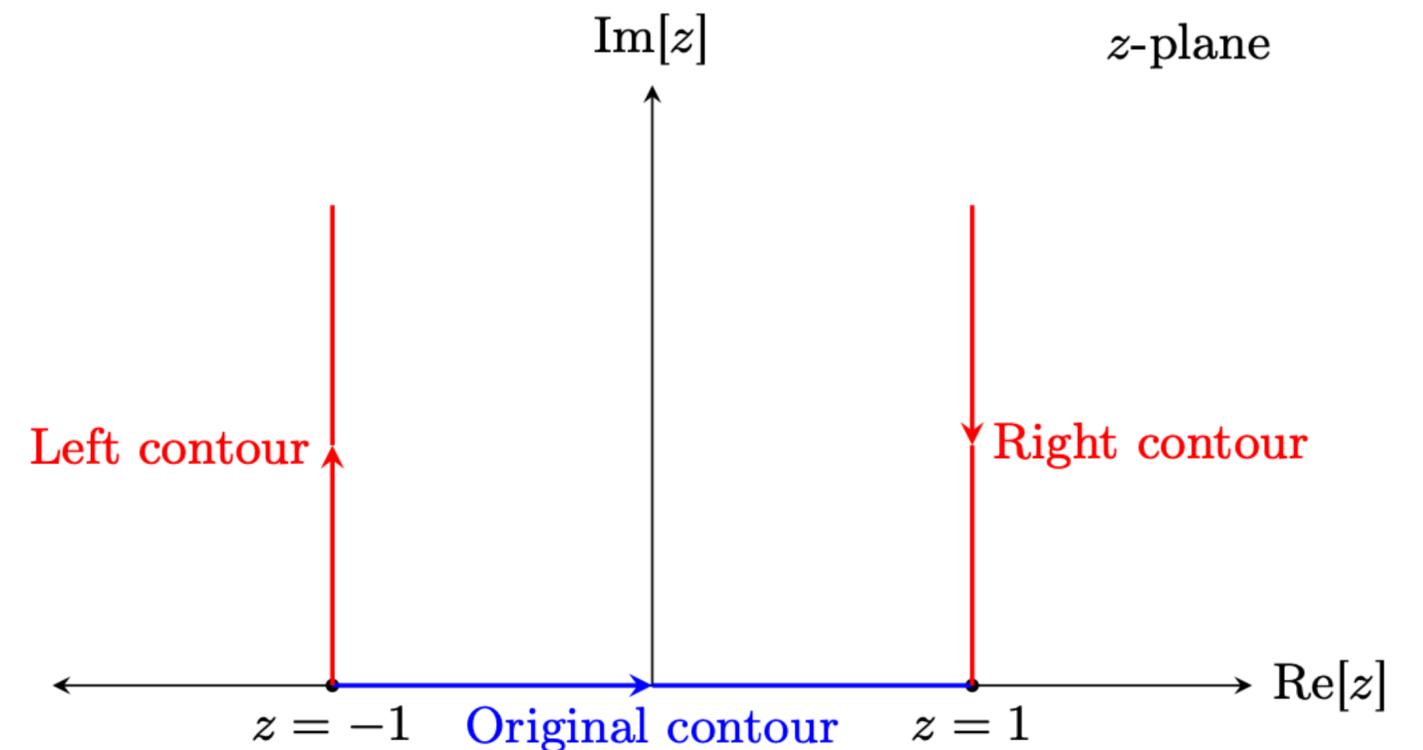
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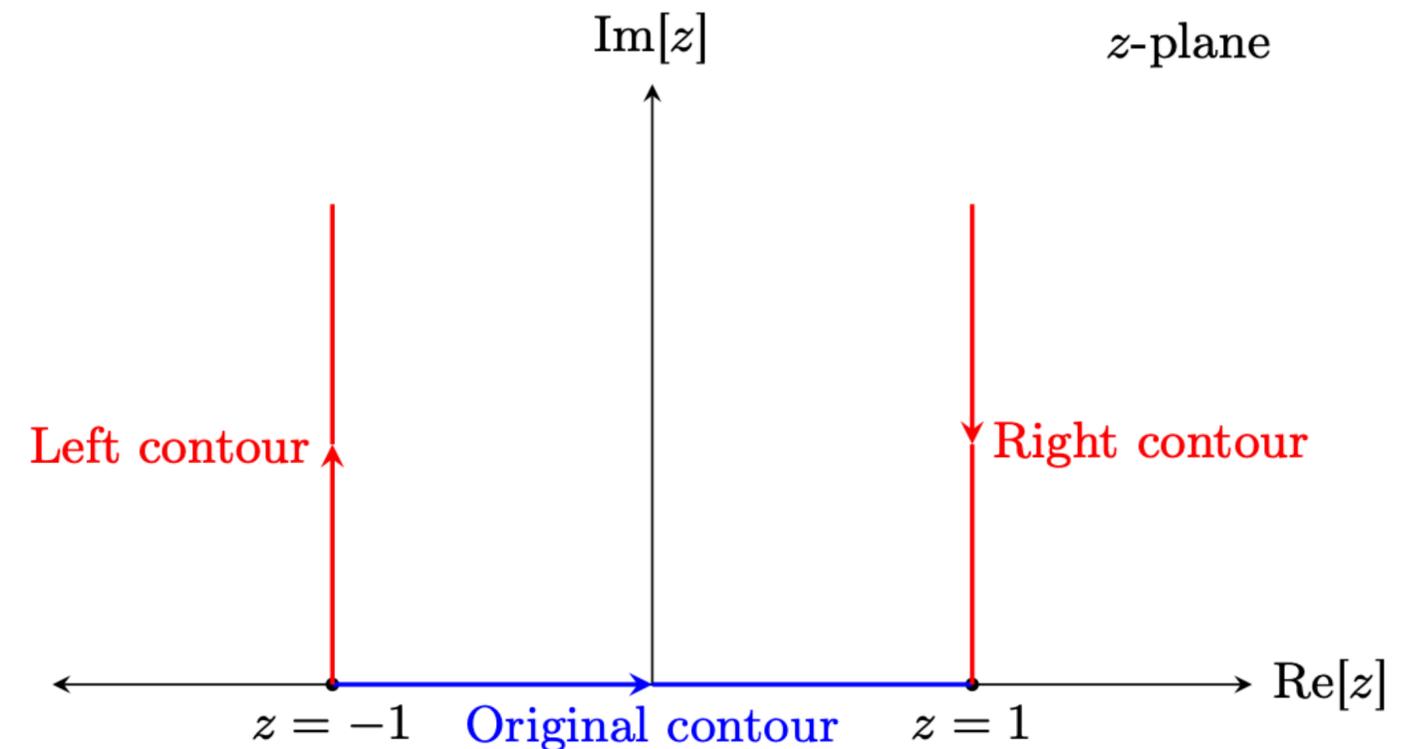
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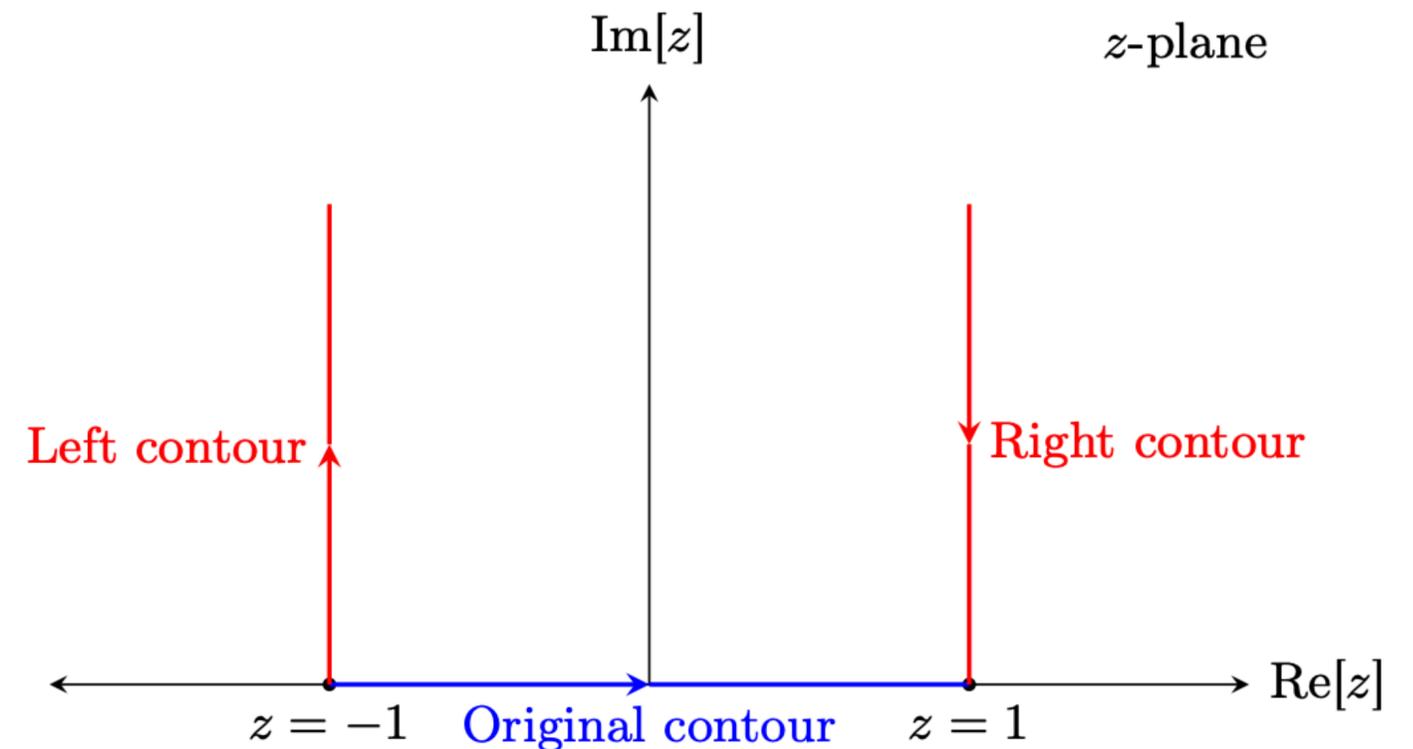
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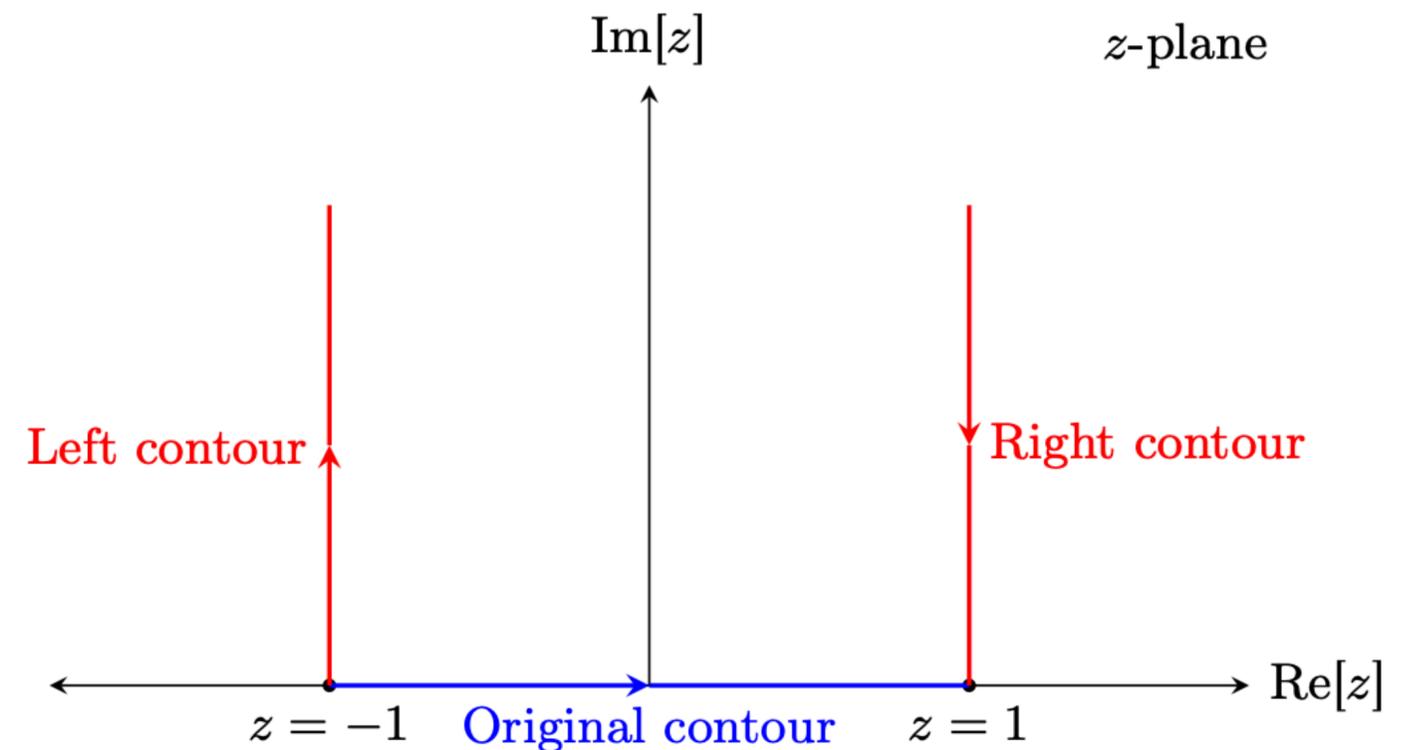
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We are missing a region!



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Coulomb region vs Radiation region

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We can do this because we have a **large hierarchy of scales** $R_i, b \ll \frac{1}{\lambda} \ll t, r .$

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$$\begin{aligned} J^{\mu\nu}(k) \Big|_{\omega^0 \ln \omega} &= -\frac{\kappa^4}{32\pi} \log(\omega - i\epsilon) F(Q_1, Q_2) \frac{Q_1 \cdot Q_2}{k \cdot Q_1} Q_1^\mu k \cdot Q_{[2} Q_{1]}^\nu + (1 \leftrightarrow 2) \\ &\quad -\frac{\kappa^4}{32\pi} \log(\omega + i\epsilon) \sum_{a=1}^3 \sum_{\substack{b=1 \\ b \neq a}}^3 F(Q'_a, Q'_a) \frac{Q'_a \cdot Q'_b}{k \cdot Q'_a} Q_a'^\mu (k \cdot Q'_{[b} Q_a]^\nu) \\ &\quad -\frac{\kappa^4}{64\pi} \log(k^2 + i\epsilon) \sum_{a=1}^3 Q'_a \cdot k \left[\sum_{b=1}^3 \frac{Q_a'^\mu Q_a'^\nu}{k \cdot Q'_a + i\epsilon} - \sum_{b=1}^2 \frac{Q_a^\mu Q_a^\nu}{k \cdot Q_a + i\epsilon} \right] \end{aligned}$$

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Analytic terms, e.g. the tree-level

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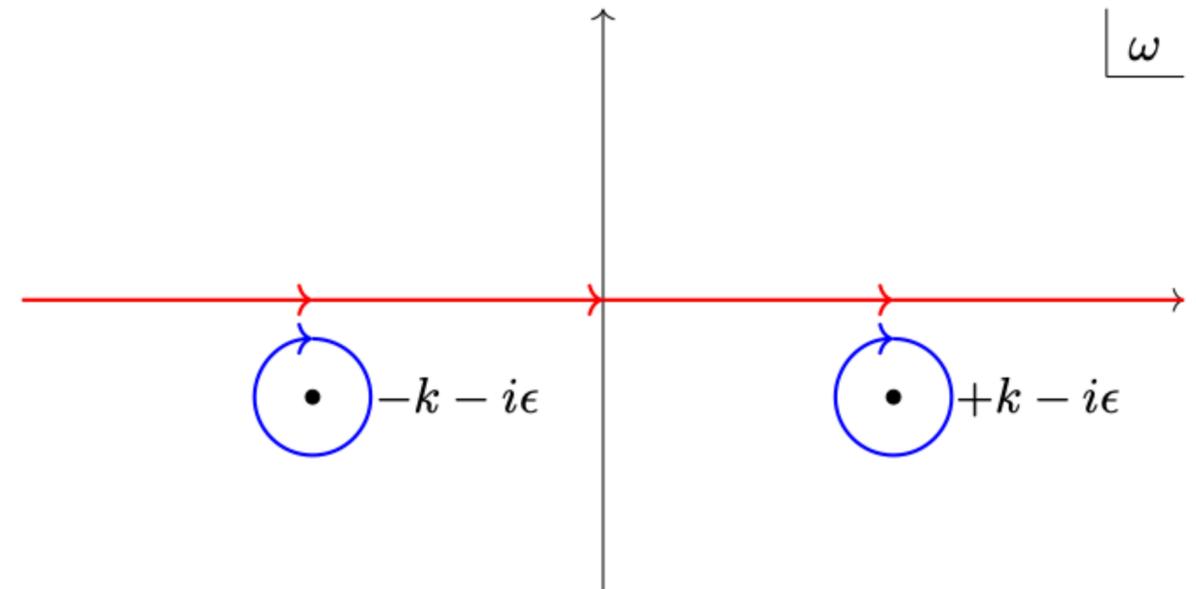
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At leading order in the perturbative expansion, in $\frac{Gm_i}{b}$, the expectation value is given by tree-level Feynman rules.

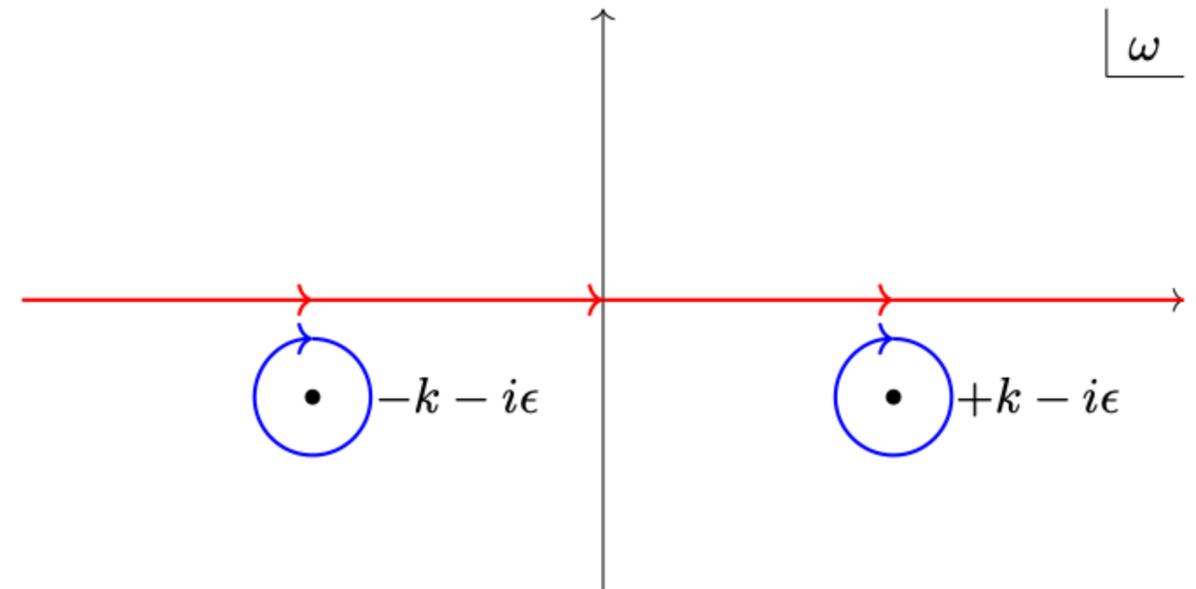


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The current $J^{\mu\nu}(k)$ is analytic at $k^2 = 0$ at leading order.

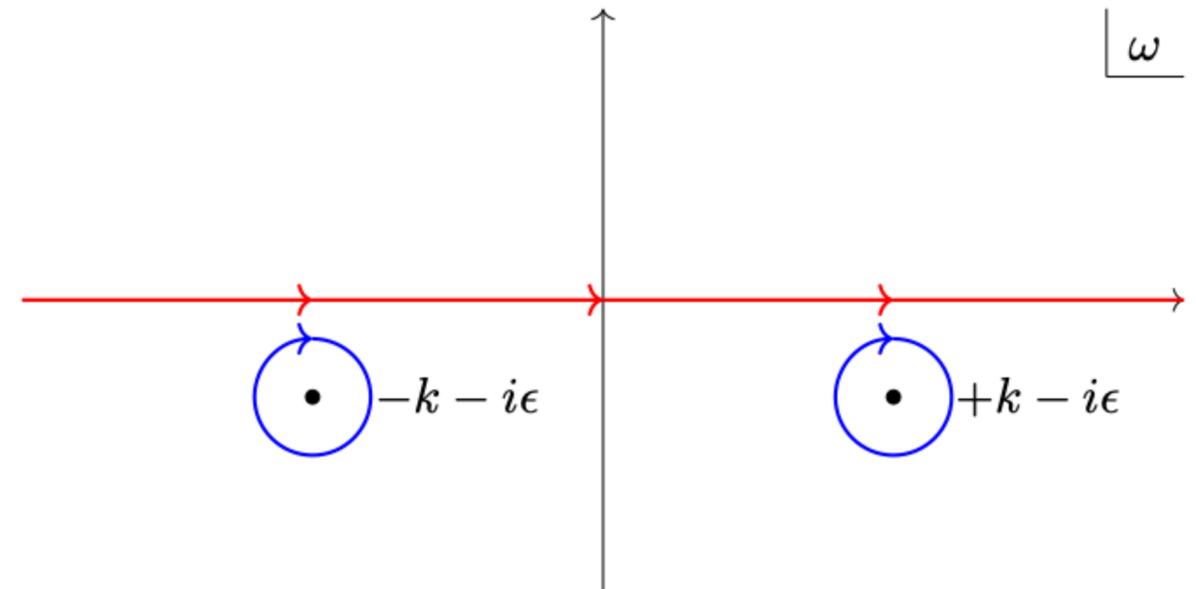
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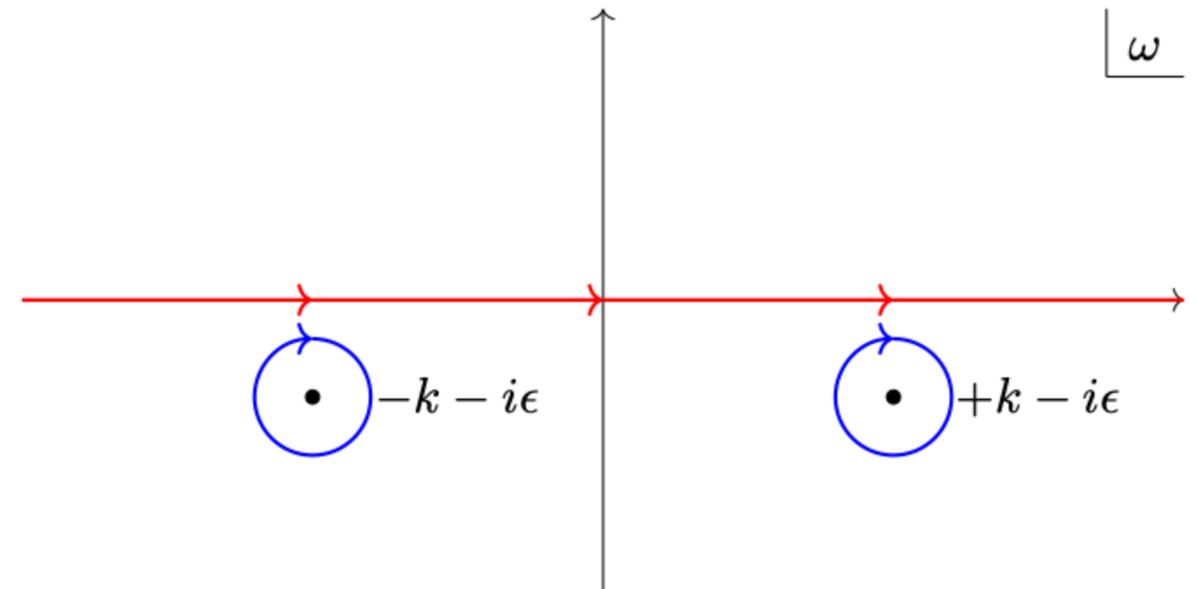


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Proof. Combine spinor variables, Coleman-Norton, and transversality of the Weyl tensor $k^\mu C_{\mu\nu\rho\sigma}(k)$. \square

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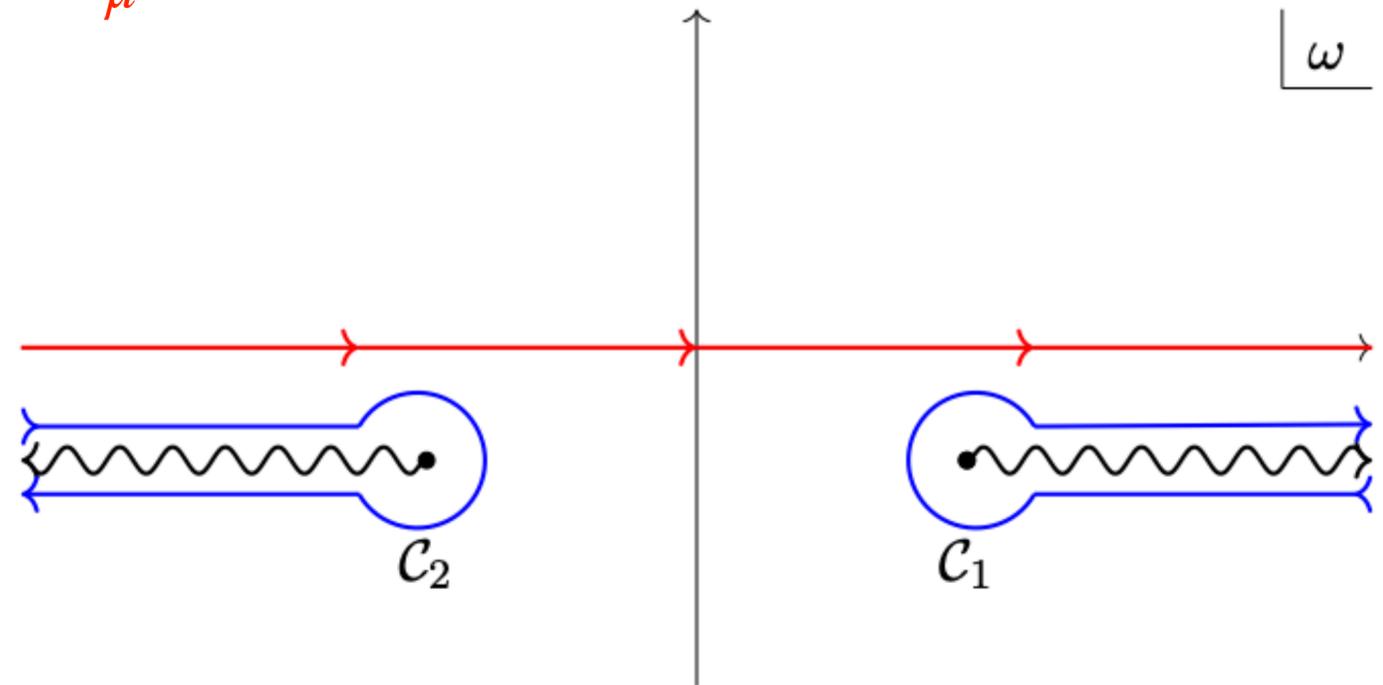
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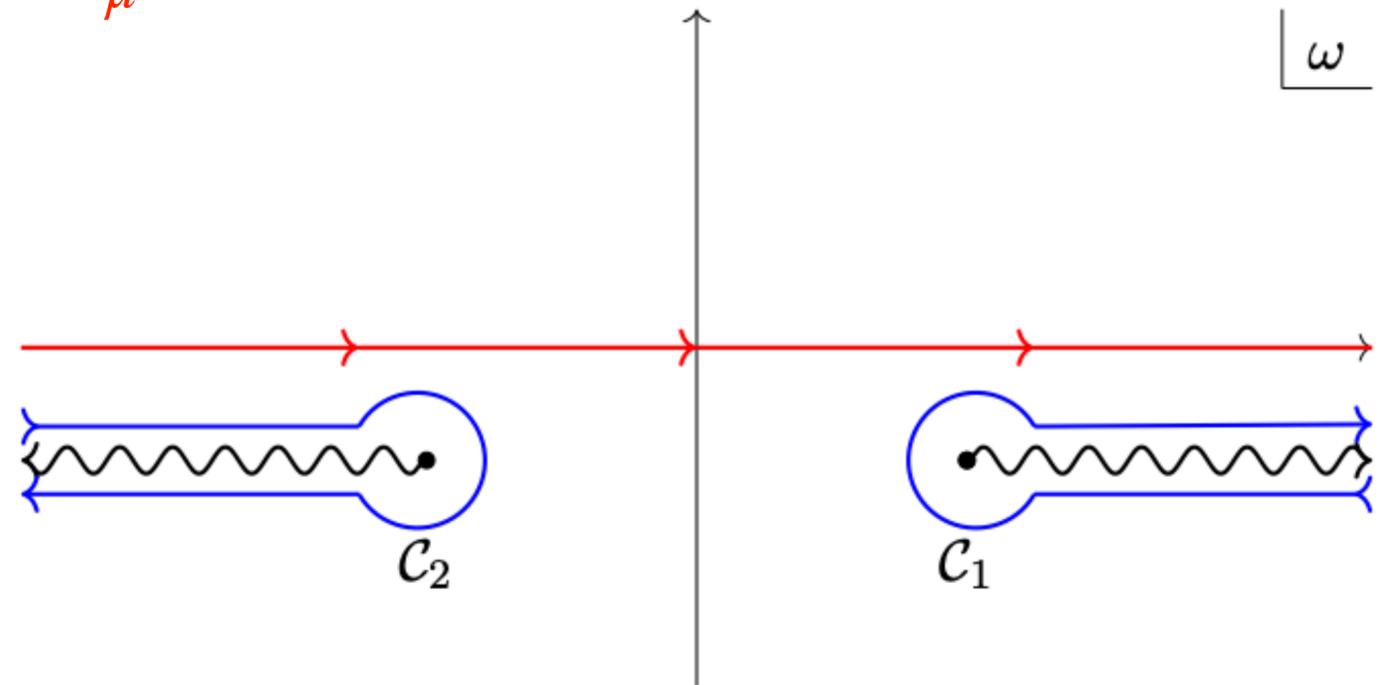
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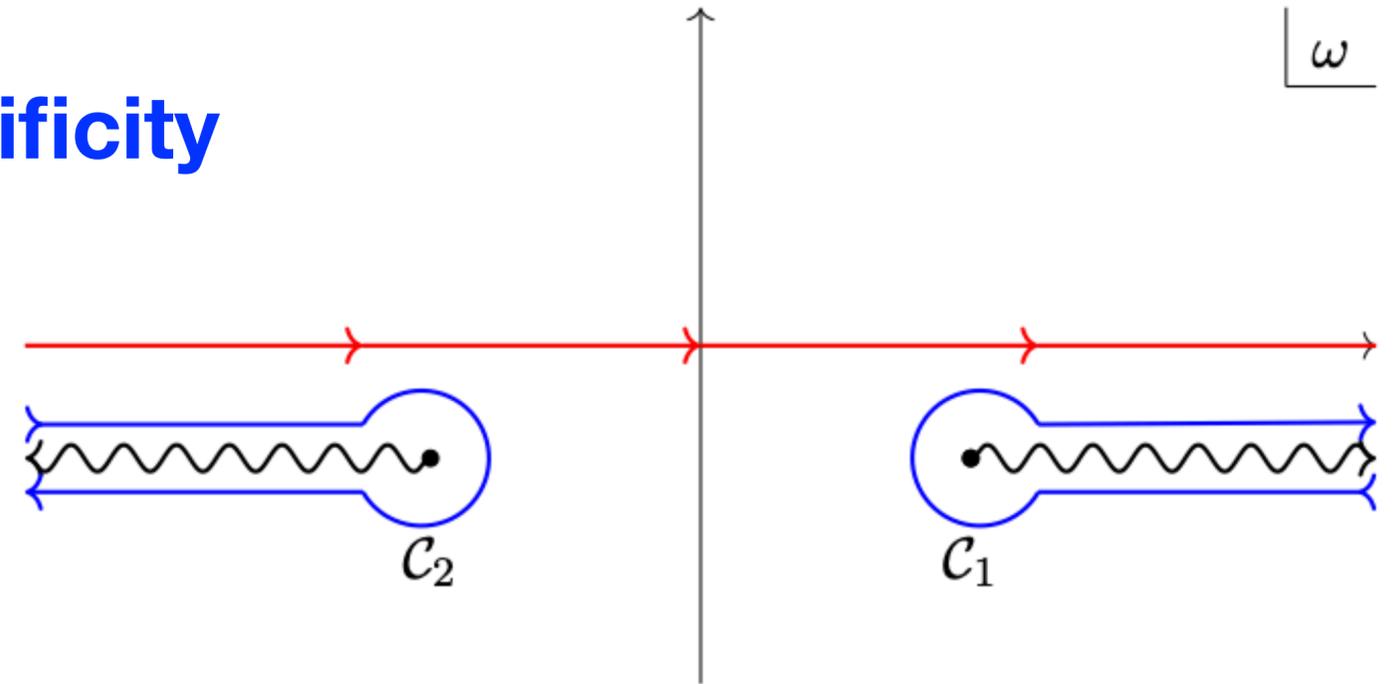
IR divergences exponentiate to a phase:

$$J^{\mu\nu}(k, \epsilon) = \left(\frac{-4\pi k^2}{\mu^2} \right)^{-2iG(\omega t + kn) \cdot (p_1 + p_2)} \tilde{J}_0^{\mu\nu}(k, \epsilon),$$



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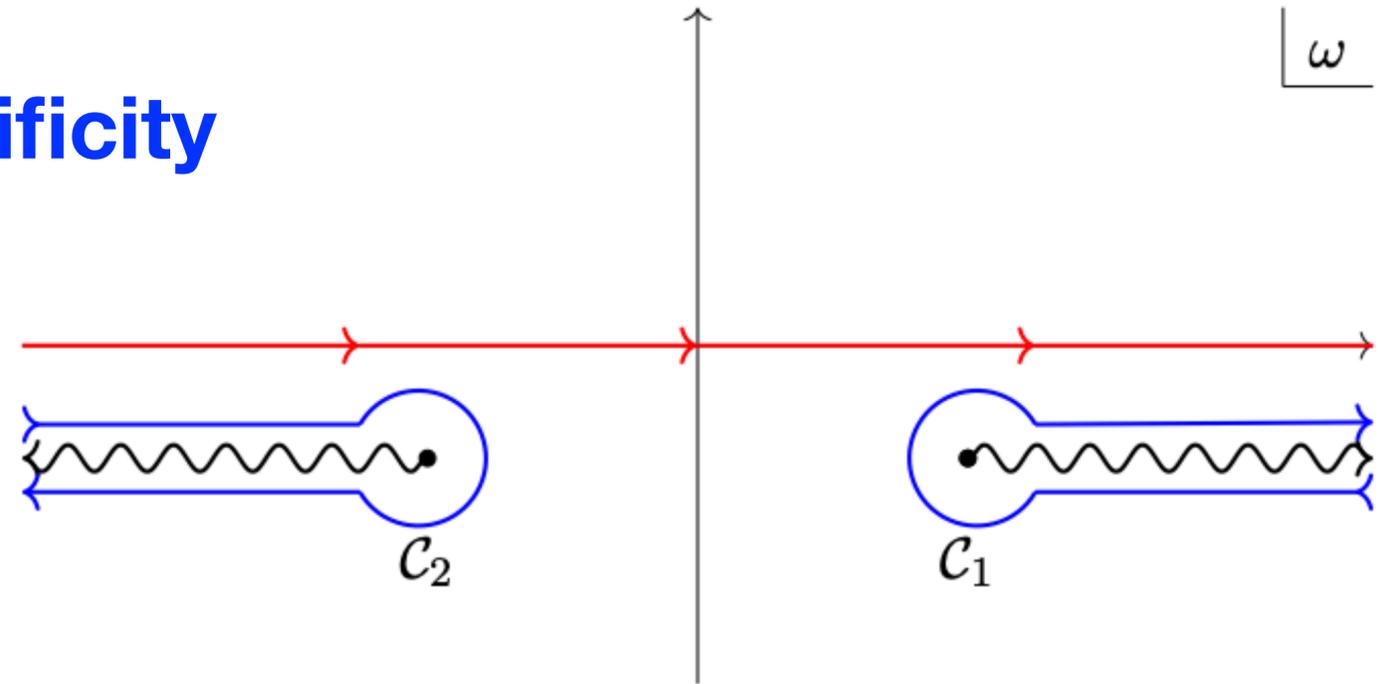
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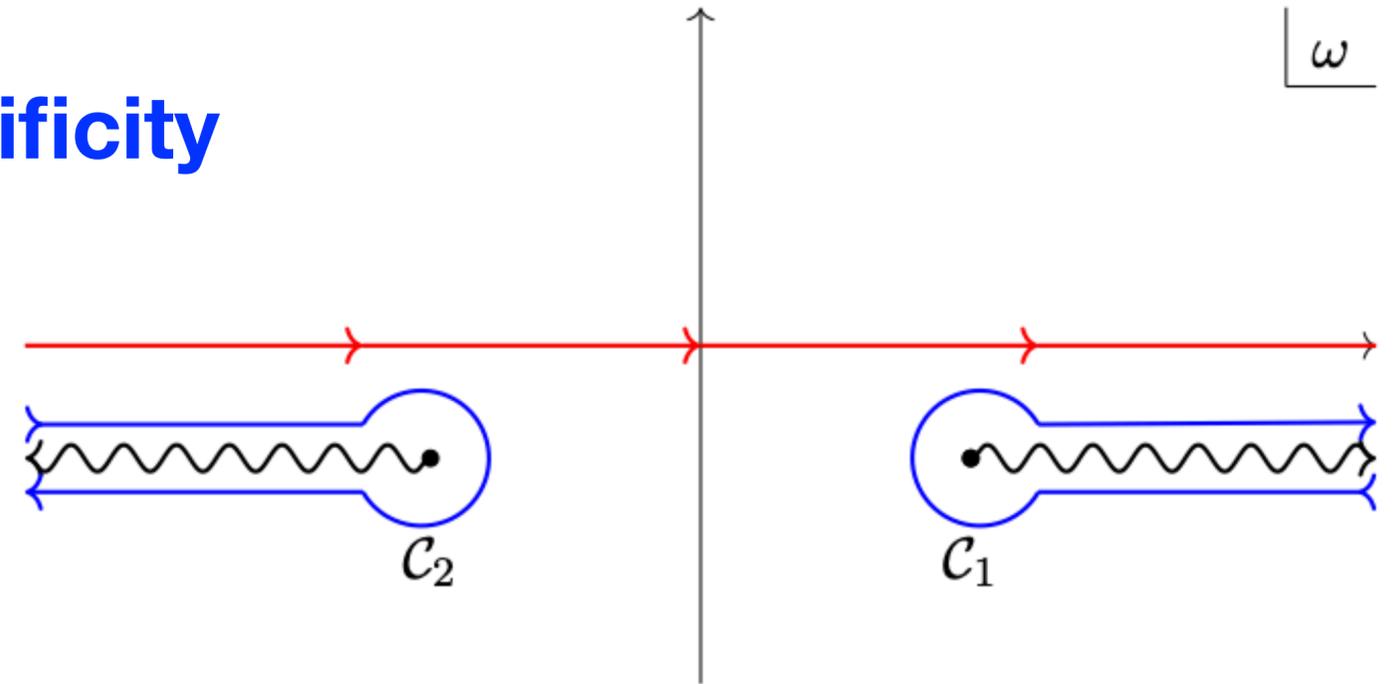


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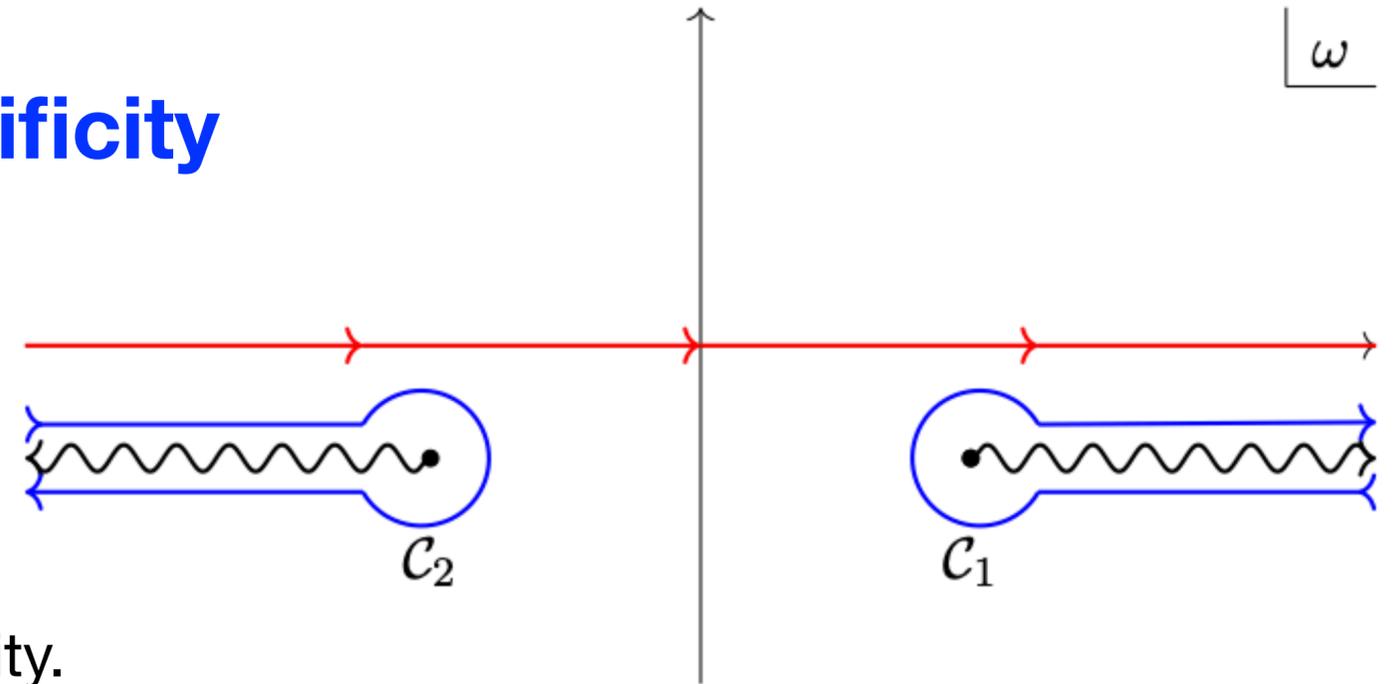


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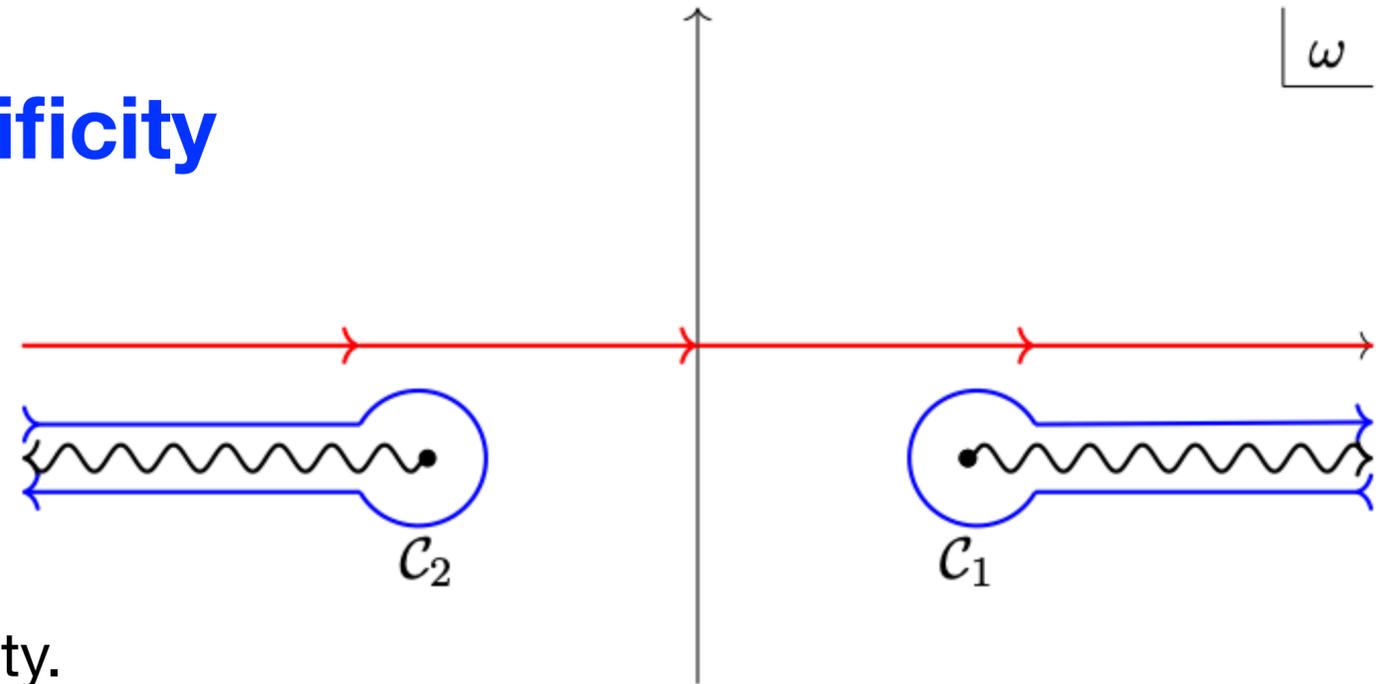


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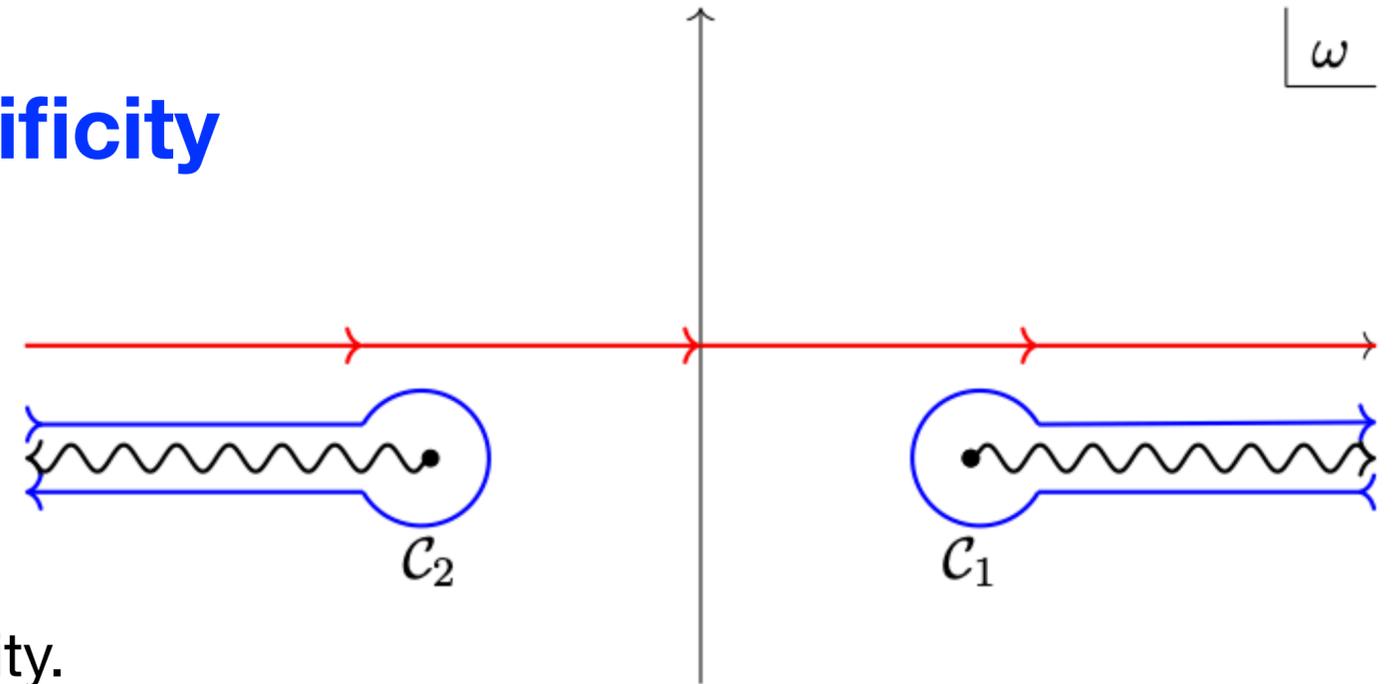
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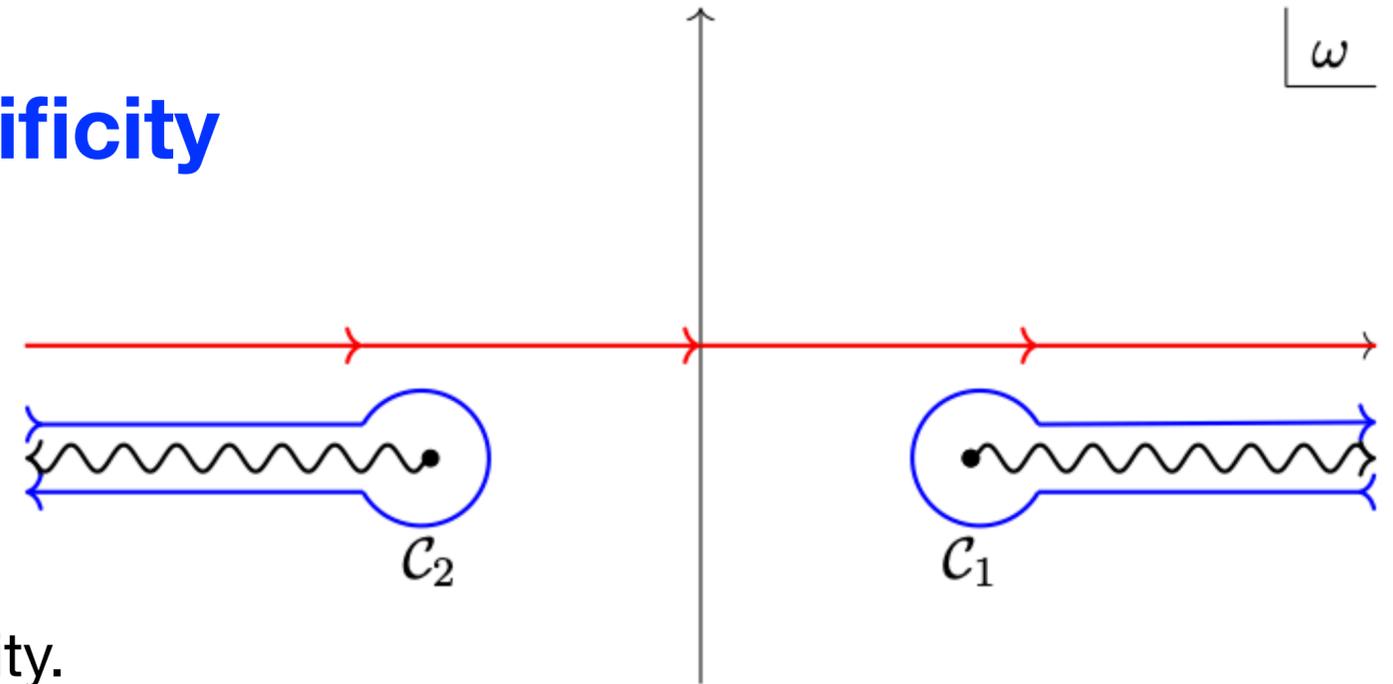
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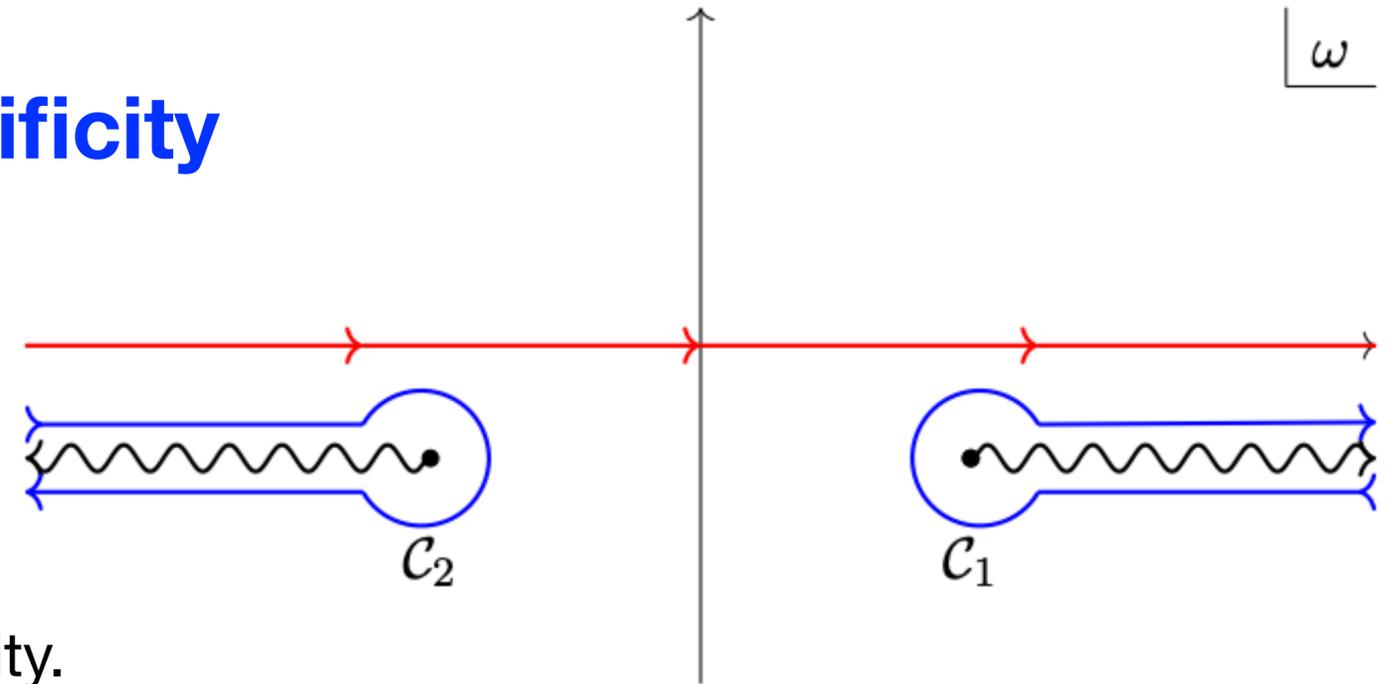
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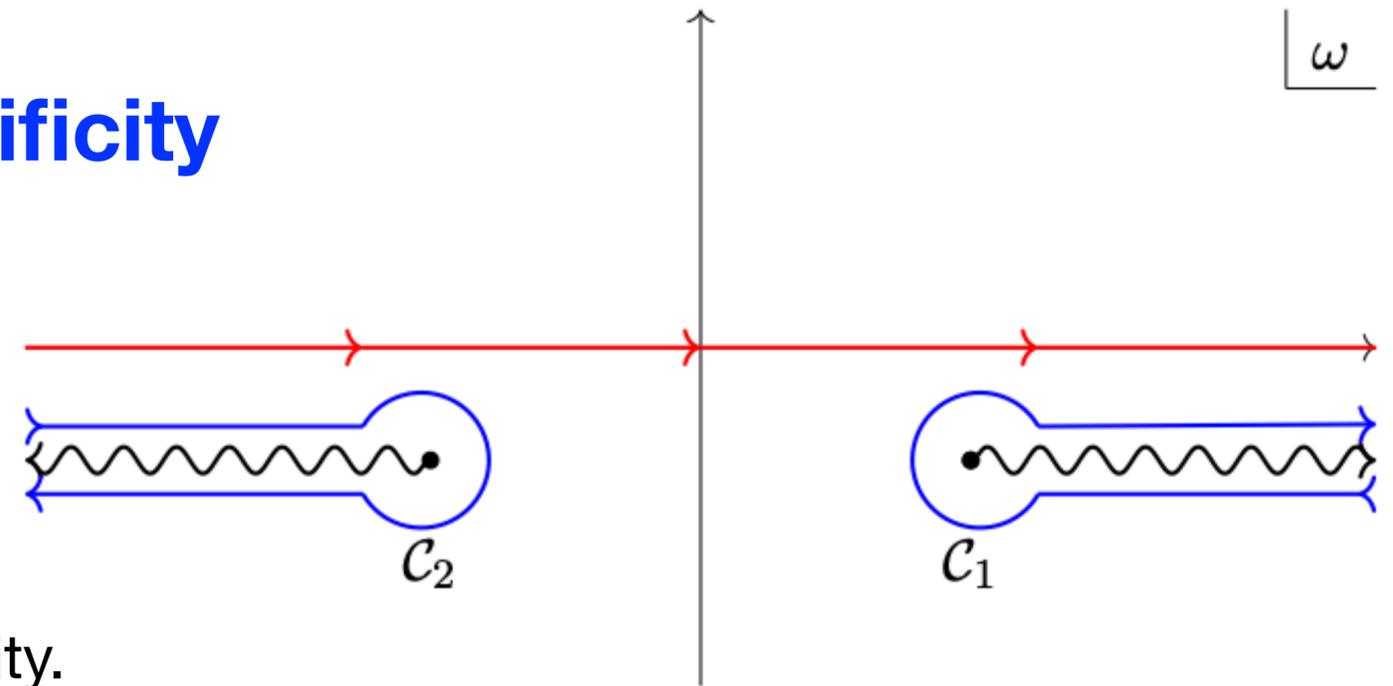
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A stronger violation of Asymptotic Simplicity

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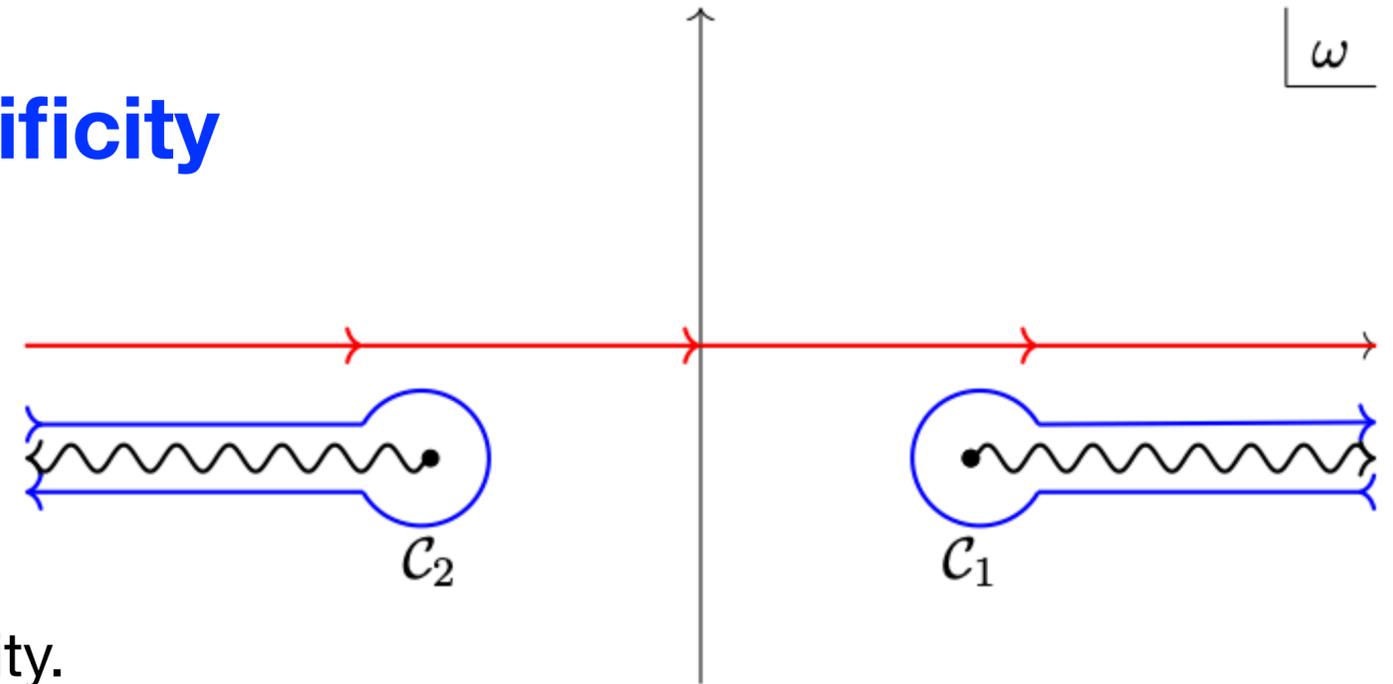
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PUNCHLINE:

The asymptotic structure of spacetimes in physically interesting setups is more complicated than expected!

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- Move on and leave the study of the *smoothness* to more mathematically oriented colleagues.

THANK YOU!