Non-perturbative bounds on scattering of neutral Goldstones

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New Frontiers of Quantum Field and Gravity

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Quantum Field Theories
scattering amplitudes

space of scattering amplitudes

S-matrix bootstrap: bounds on the space of scattering amplitudes

1. crossing symmetry
2. non-linear unitarity
3. maximal analyticity
4. additional requirements

This talk:

we bound scalar neutral m=0 particles

we assume that particles are Goldstones

focus on d=4

arXiv:2310.06027
[F. Acanfora, A. Guerrieri, K. Häring, DK; 2023]

S-matrix bootstrap

Beijing lectures
on the S-matrix bootstrap

https://gitlab.com/d.s.karateev/beijing-lectures-smatrix-bootstrap

S-matrix bootstrap

scattering amplitude:

$$T(s,t,u) = T(t,s,u) = T(u,t,s)$$

$(s+t+u=4m^2)$ ← **Mandelstam variables**

partial amplitudes:

$x \equiv \cos\theta$ ← **scattering angle**

$$t = -\frac{s-4m^2}{2}(1-x), \quad u = -\frac{s-4m^2}{2}(1+x)$$

$$T_\ell(s) \equiv \frac{\sqrt{1-4m^2/s}}{32\pi}\int_{-1}^{+1} dx\, P_\ell(x)\, T(s,t(x),u(x))$$

angular momentum: $\ell = 0,2,4,6,\ldots$

Legendre polynomial

non-linear unitarity:

$$ImT_\ell(s) \geq \frac{1}{16}|T_\ell(s)|^2$$

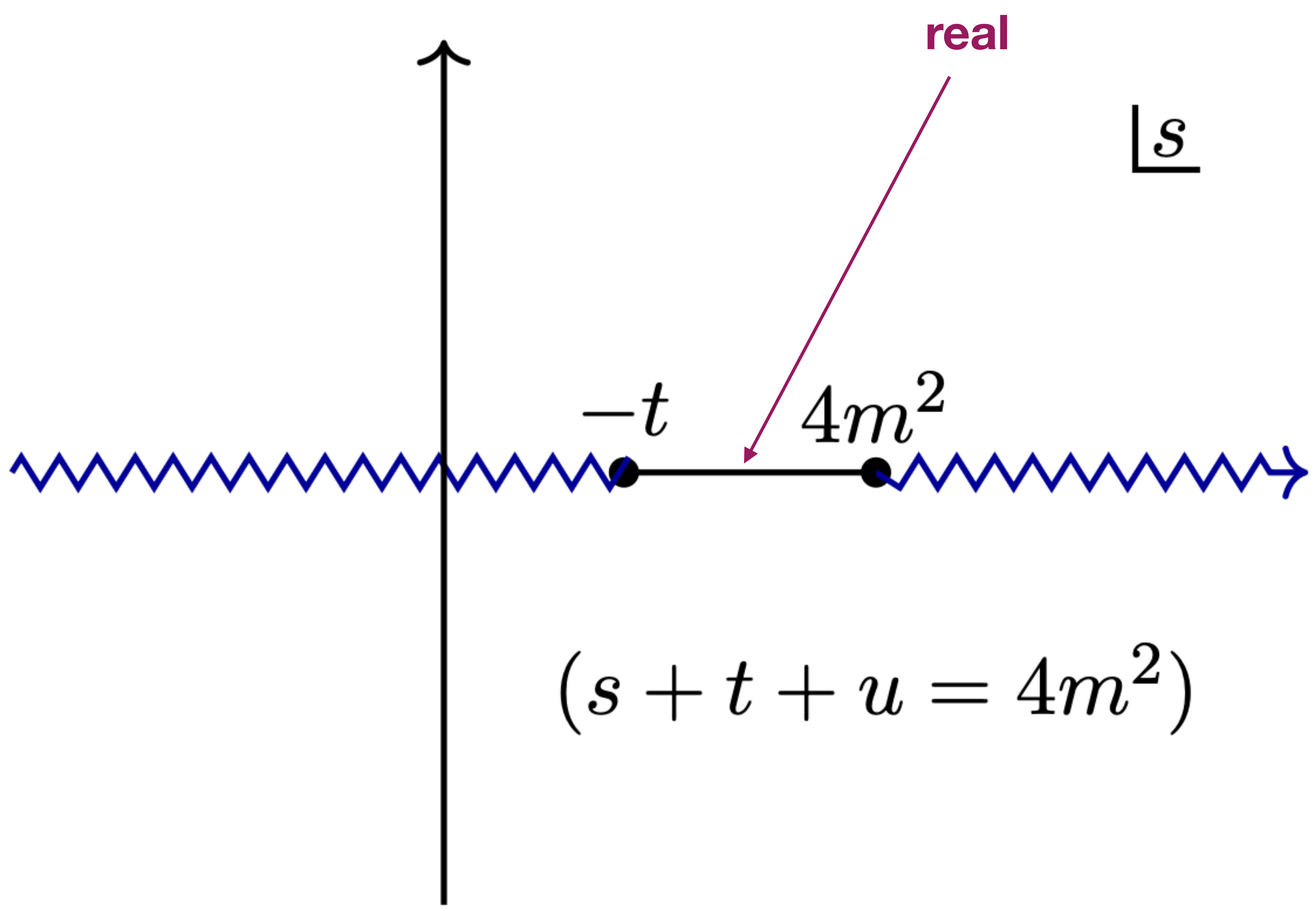
$s \geq 4m^2, \quad \ell = 0,2,4,6,\ldots$

S-matrix bootstrap

$T(s, t < 0)$

$t = fixed\ (real)$

Maximal analyticity:



S-matrix bootstrap

[M. Paulos, J. Penedones, J. Toledo, B. van Rees, P. Vieira; 2016]

Ansatz:

$$T(s,t,u) = \sum_{a,b,c=0}^{N_{max}} \alpha_{(abc)}\, \rho^a(s)\rho^b(t)\rho^c(u)$$

size of the ansatz

real dimensionless coefficients describe the space of amplitudes

$\rho-$ variables:

$$\rho(s) \equiv \frac{1-\sqrt{4m^2-s}}{1+\sqrt{4m^2-s}}$$

Coefficients to bound: $\alpha_{(abc)}$

Impose unitarity using SDPB

[D. Simmons-Duffin; 2015]

✓ crossing

✓ analyticity

✓ low energy expansion

EFT for the Goldstones

shift-symmetric: $\phi(x) \longrightarrow \phi(x) + c$

$$\mathscr{L}_{EFT} = -\frac{1}{2}(\partial\phi)^2 + c_4(\partial\phi)^4 + c_6(\partial_\mu\partial_\nu\partial_\rho\phi)(\partial_\mu\phi)(\partial_\nu\phi)(\partial_\rho\phi) + c_8(\partial_\mu\partial_\nu\phi)^4 + \dots$$

↓

non-perturbative expansion around s=0

$$T(s,t,u) = g_2\,(s^2+t^2+u^2) + g_3\,stu + g_4\left(s^2+t^2+u^2\right)^2 - \frac{g_2^2}{480\pi^2}\left(s^2(41s^2+t^2+u^2)\log\left(-s\sqrt{g_2}\right) + (s \leftrightarrow t) + (s \leftrightarrow u)\right) + O(s^5)$$

$g_2,\, g_3,\, g_4$ ← **physical, can be measured in experiments**

Dimensionless observables:

$$\bar{g}_3 \equiv \frac{g_3}{g_2^{3/2}}, \quad \bar{g}_4 \equiv \frac{g_4}{g_2^2}$$

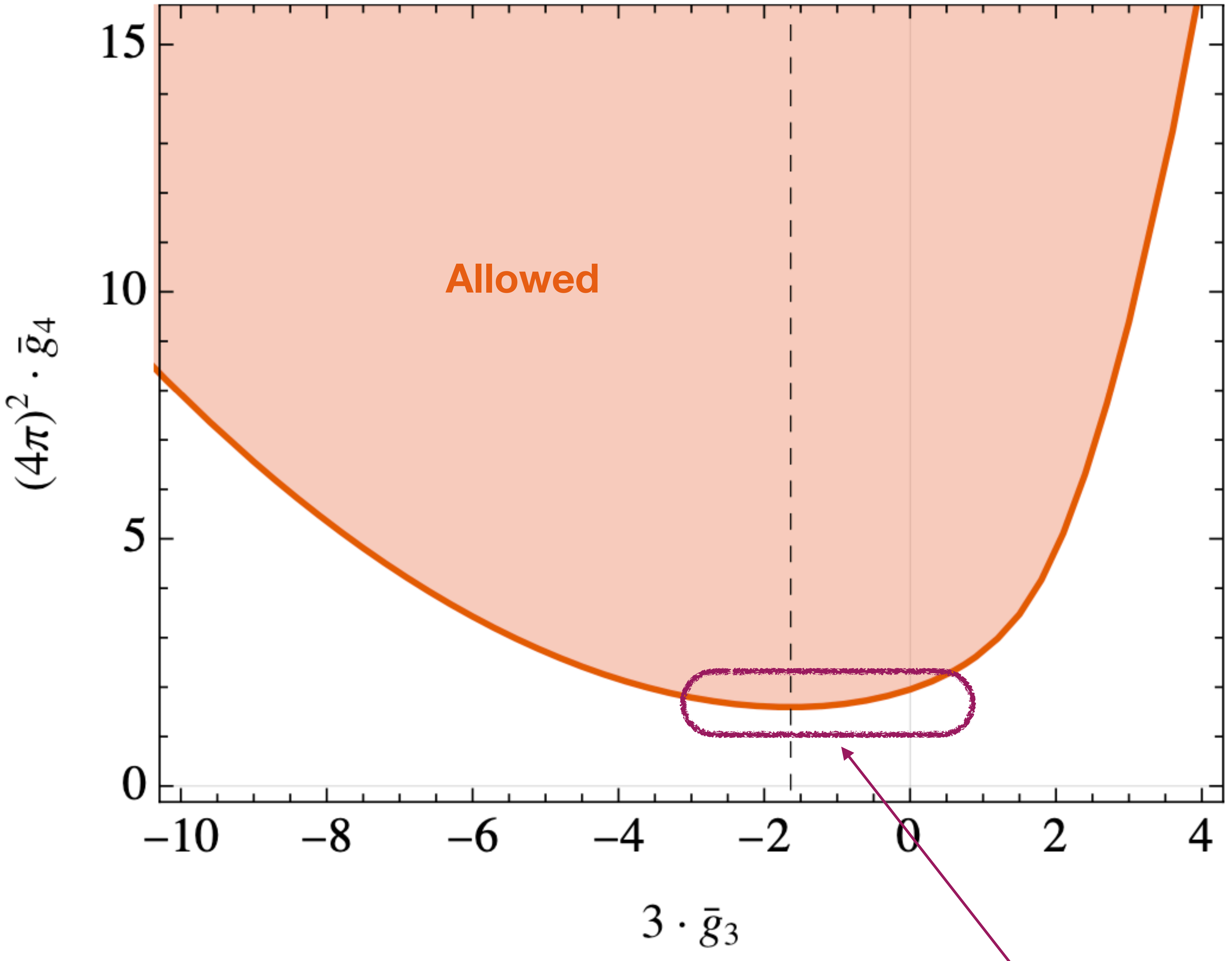
are there any bounds on these parameters from the S-matrix bootstrap?

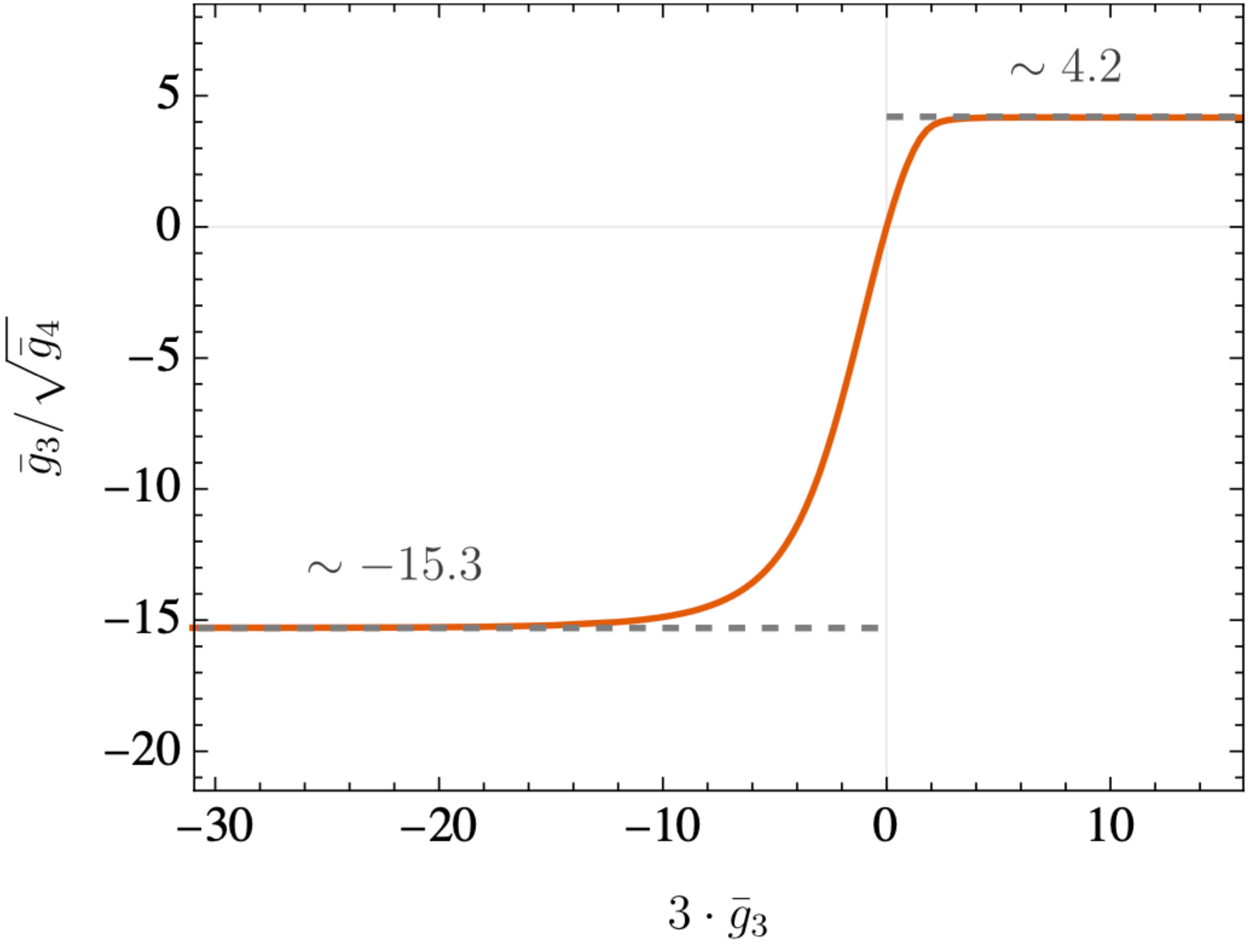
non-perturbative expansion around s=0 (Goldstones)

[A. Guerrieri, J. Penedones, P. Vieira; 2020]

Write an ansatz → $T(s, t, u)$

Apply the S-matrix bootstrap

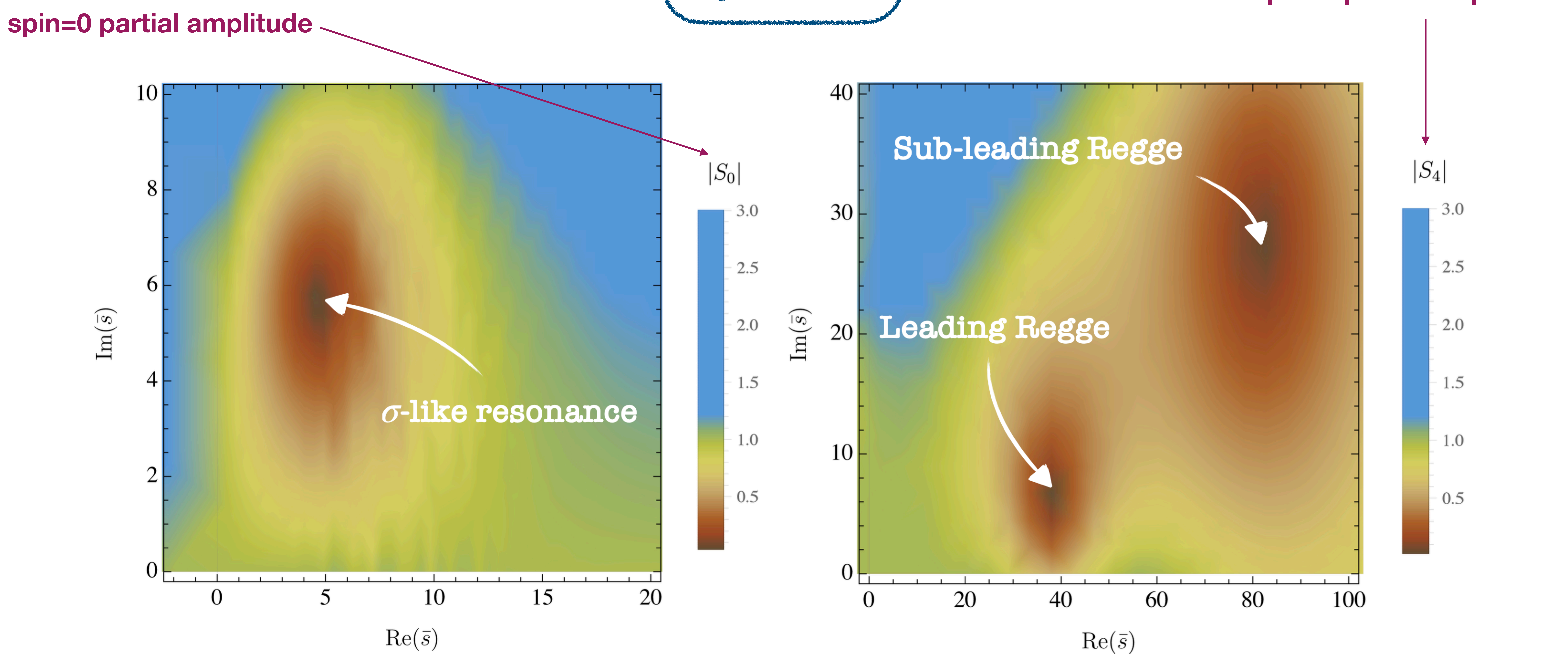




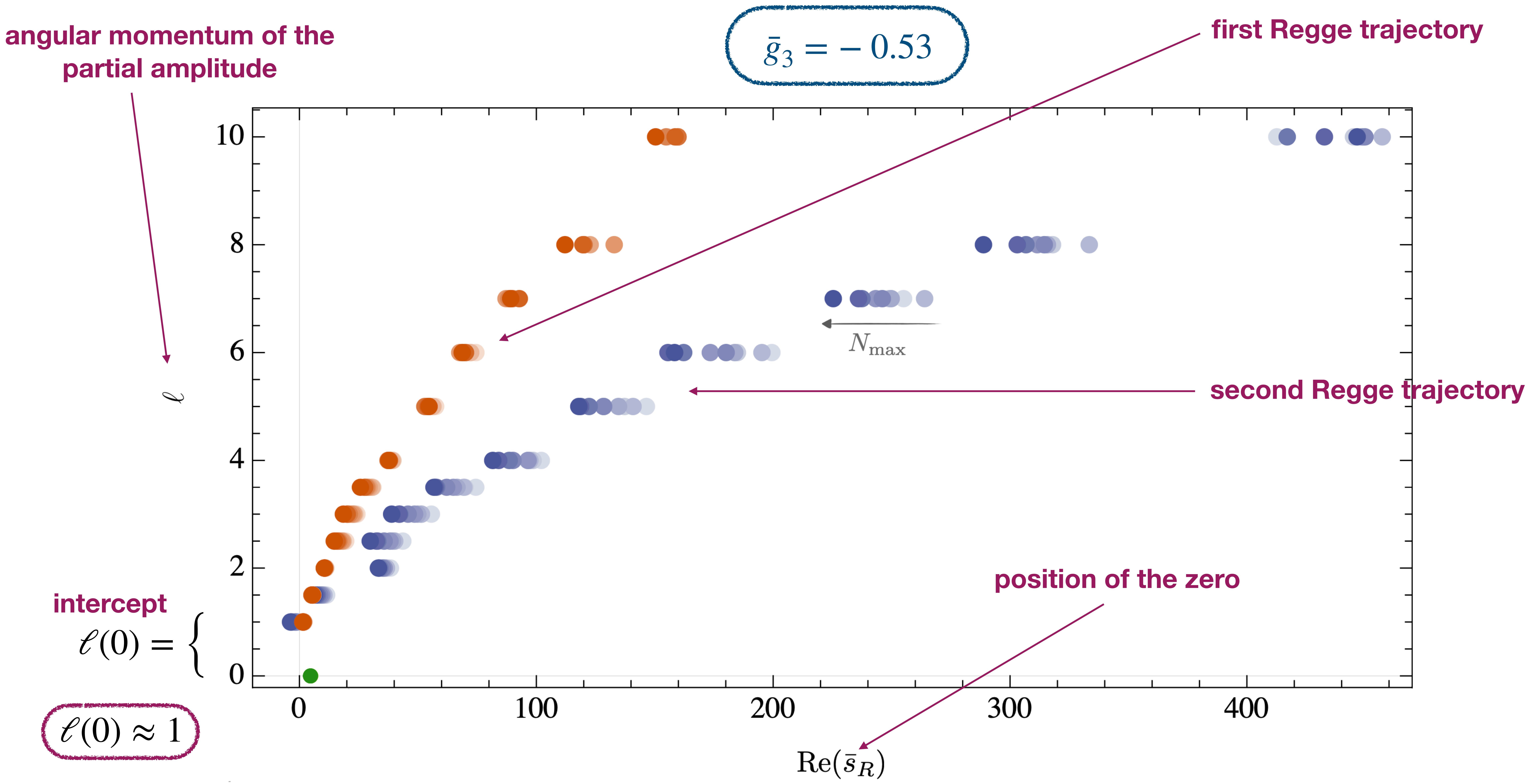
non-linear unitarity is crucial

$$-15.3 \leq \frac{\bar{g}_3}{\sqrt{\bar{g}_4}} \leq 4.2$$

$\bar{g}_3 = -0.53$



Zero on the first sheet = pole on the second sheet (for elastic amplitudes)



angular momentum of the partial amplitude
$\bar{g}_3 = -0.53$
first Regge trajectory
second Regge trajectory
$N_{\max}$
position of the zero
intercept
$\ell(0) =$
$\ell(0) \approx 1$
ℓ
$\mathrm{Re}(\bar{s}_R)$
0
2
4
6
8
10
0
100
200
300
400

Weakly coupled QFTs

model independent

non-perturbative expansion around s=0

$$T(s,t,u) = g_2\,(s^2+t^2+u^2) + g_3\,stu + g_4\left(s^2+t^2+u^2\right)^2 - \frac{g_2^2}{480\pi^2}\left(s^2(41s^2+t^2+u^2)\log\left(-s\sqrt{g_2}\right) + (s\leftrightarrow t) + (s\leftrightarrow u)\right) + O(s^5)$$

extended range of validity

extra assumptions: $|s| \leq M^2$

finite coefficients

cut-off

weak coupling

$$g_n = k_n M^{-2n}\alpha$$

Alternative dimensionless observables:

$$\tilde{g}_3 \equiv g_3\frac{M^2}{g_2}, \quad \tilde{g}_4 \equiv g_4\frac{M^4}{g_2}$$

Weakly coupled QFTs

No logs

Bellazzini, Elias Miró, Rattazzi, Riembau and Riva; 2020

Caron-Huot, Van Duong; 2021

Tolley, Wang, Zhou; 2021

With logs

Bellazzini, Riembau and Riva; 2021

Beadle, Isabella, Perrone, Ricossa and Riva; 2024

Comparison

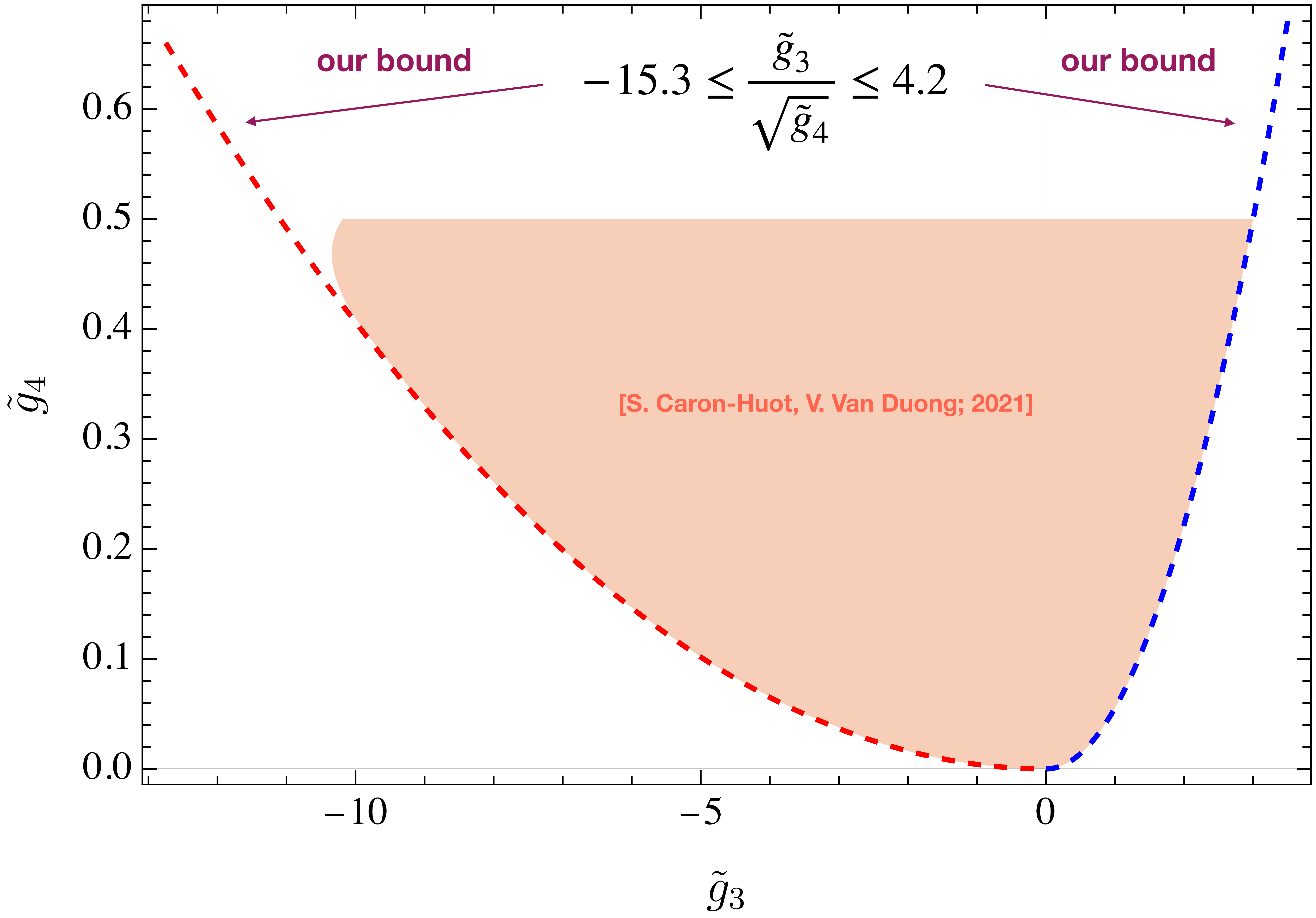
Dimensionless observables: $\bar{g}_3 \equiv \frac{g_3}{g_2^{3/2}}, \quad \bar{g}_4 \equiv \frac{g_4}{g_2^2}$ $\qquad -15.3 \leq \frac{\bar{g}_3}{\sqrt{\bar{g}_4}} \leq 4.2$

Alternative dimensionless observables: $\tilde{g}_3 \equiv g_3 \frac{M^2}{g_2}, \quad \tilde{g}_4 \equiv g_4 \frac{M^4}{g_2}$

relation: $\frac{\tilde{g}_3}{\sqrt{\tilde{g}_4}} = \frac{\bar{g}_3}{\sqrt{\bar{g}_4}}$

$$-15.3 \leq \frac{\tilde{g}_3}{\sqrt{\tilde{g}_4}} \leq 4.2$$

Comparison


our bound
$-15.3 \leq \frac{\tilde{g}_3}{\sqrt{\tilde{g}_4}} \leq 4.2$
our bound
[S. Caron-Huot, V. Van Duong; 2021]
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$\tilde{g}_4$
−10
−5
0
$\tilde{g}_3$

Summary

1. We studied non-perturbatively scattering of neutral Goldstones
2. Obtained bounds on Wilson coefficients (using non-linear unitarity)
3. Amplitudes on the boundary are extracted and investigated
4. Regge trajectories are discovered
5. Our bounds are compared with EFT positivity

Thank you!