Origin limits of large-charge correlators in planar N=4 SYM

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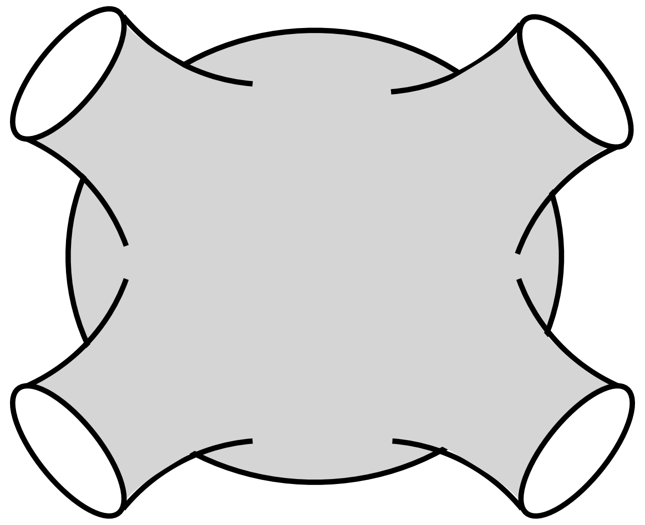
Based on ongoing work with Thiago Fleury, Erkan Kaluç and Didina Serban

Motivation

- **N = 4 SYM** : remarkable laboratory for exploring dynamics of 4d massless gauge theories
- Symmetries open the way to new methods for computing correlation functions and amplitudes at both weak and strong coupling
- Key example: Duality between N=4 SYM and type IIB string theory on AdS5*S5
- This duality triggered the development of **Integrability**, enabling exact studies of observables in the large-N limit
- Key success: Spectrum of single-trace operators at finite ’t Hooft coupling
- Ultimate goal: Correlation functions of arbitrary single-trace operators

Motivation

Compute correlation functions of single-trace operators in the large N limit



Simplest insertions are half-BPS operators $\mathcal{O}_i = \mathrm{Tr}\left[\phi(x_i, y_i)^{J_i}\right]$ with

$$\phi(x_i, y_i) = \sum_{I=1}^{6} y_i^I \phi^I(x_i)$$
$$\sum_{I=1}^{6} y_i^I y_i^I = 0$$

Dimension given by R-charge $\Delta_i = J_i$

Correlators are (UV&IR finite) functions of cross ratios and coupling constant

$$u_{ijkl} = \frac{x_{ij}^2 x_{kl}^2}{x_{ik}^2 x_{jl}^2} \qquad v_{ijkl} = \frac{y_{ij}^2 y_{kl}^2}{y_{ik}^2 y_{jl}^2} \qquad g^2 = \frac{g_{\mathrm{YM}}^2 N}{16\pi^2}$$

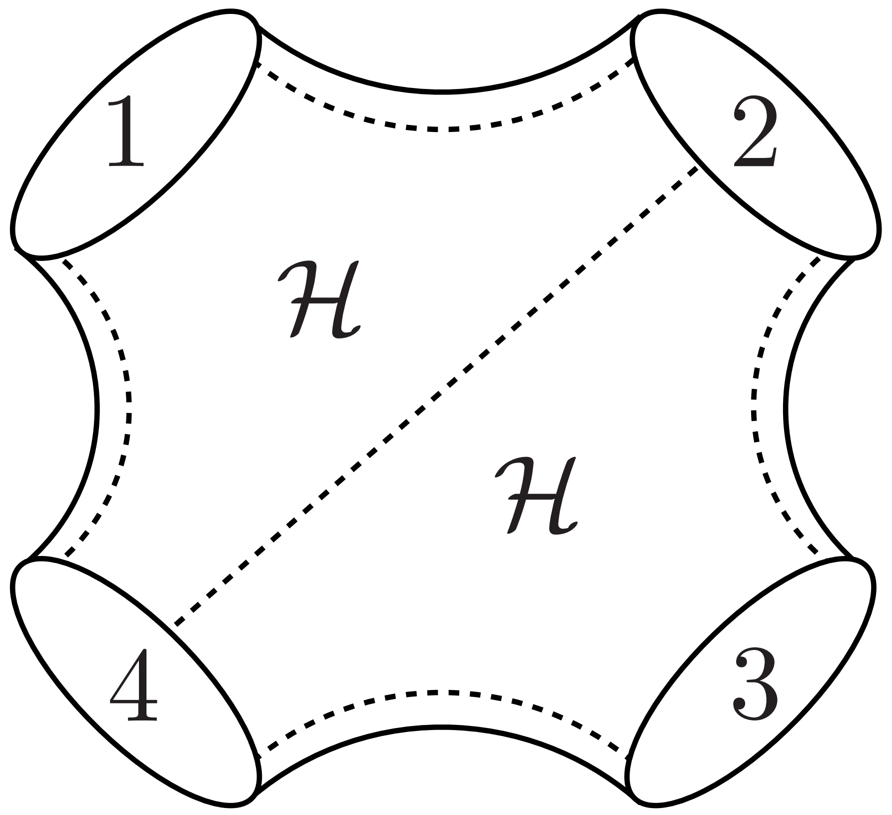
Motivation

[Fleury,Komatsu]
[Eden,Sfondrini]
[BB,Komatsu,Vieira]

Hexagonalization is an integrability-based method for studying correlators at finite coupling

Core idea: Planar single-trace correlators can be tessellated into hexagons

Ex. 4-point function



4 hexagons are needed here
(2 in the front/back)

Generally, break n-point functions into 2(n-2) hexagons

$$F_n \;\to\; \mathcal{H}_1 \otimes \mathcal{H}_2 \ldots \otimes \mathcal{H}_{2(n-2)}$$

Disk correlators / Polygons

[Fleury,Komatsu]
[Bargheer,Caetano,Fleury,Komatsu,Vieira]
[Coronado]
[Caron-Huot,Coronado]
[Caron-Huot,Coronado,Muhlmann]
[Fleury,Gonçalves]
[Bercini,Fernandes,Gonçalves]
[Crisanti,Eden,Gottwald,Mastrolia,Scherdin]
[Bargheer,Bekov,Bercini,Coronado]

Gluing hexagons together leads to sums and integrals over magnons

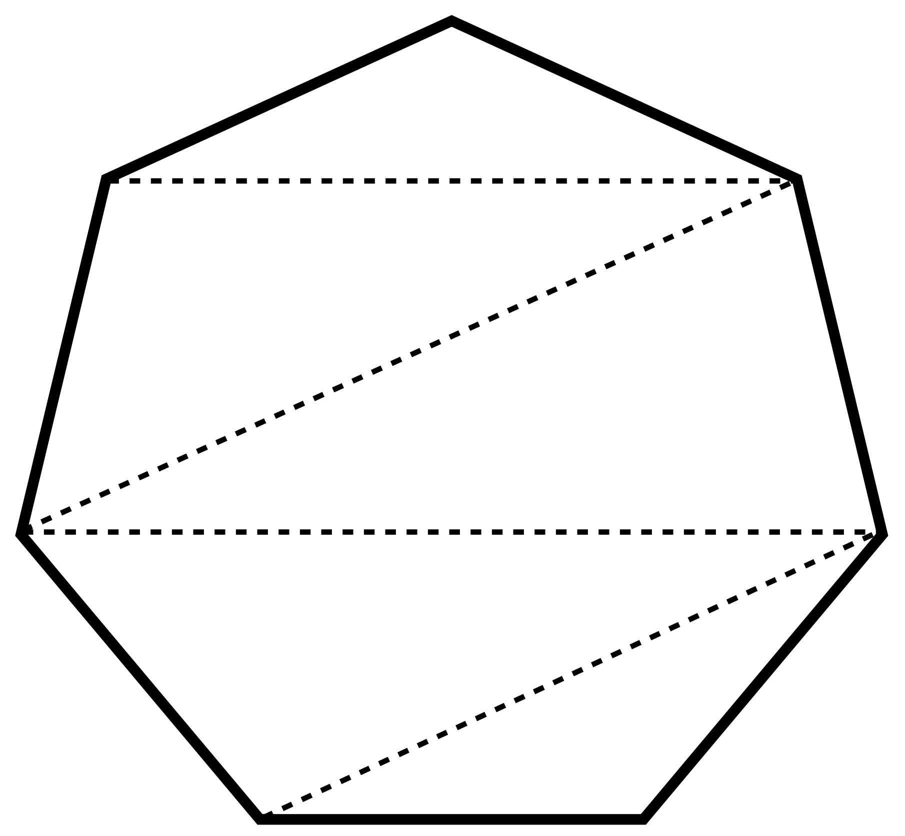
The more we glue the more intricate the sums become

4-pt function: 6 cuts carrying magnons all interacting with each other

Simplification: large-charge correlators = disk correlators

Hexagons glued together into bigger polygons

Polygons are easier to study than punctured spheres (less hexagons)



Ex. 7-pt function
covered by 5 hexagons

Toy model for re-summation of hexagon expansion in various kinematics

Disk correlators / Polygons

[Fleury,Komatsu]
[Bargheer,Caetano,Fleury,Komatsu,Vieira]
[Coronado]
[Caron-Huot,Coronado]
[Caron-Huot,Coronado,Muhlmann]
[Fleury,Gonçalves]
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[Bargheer,Bekov,Bercini,Coronado]

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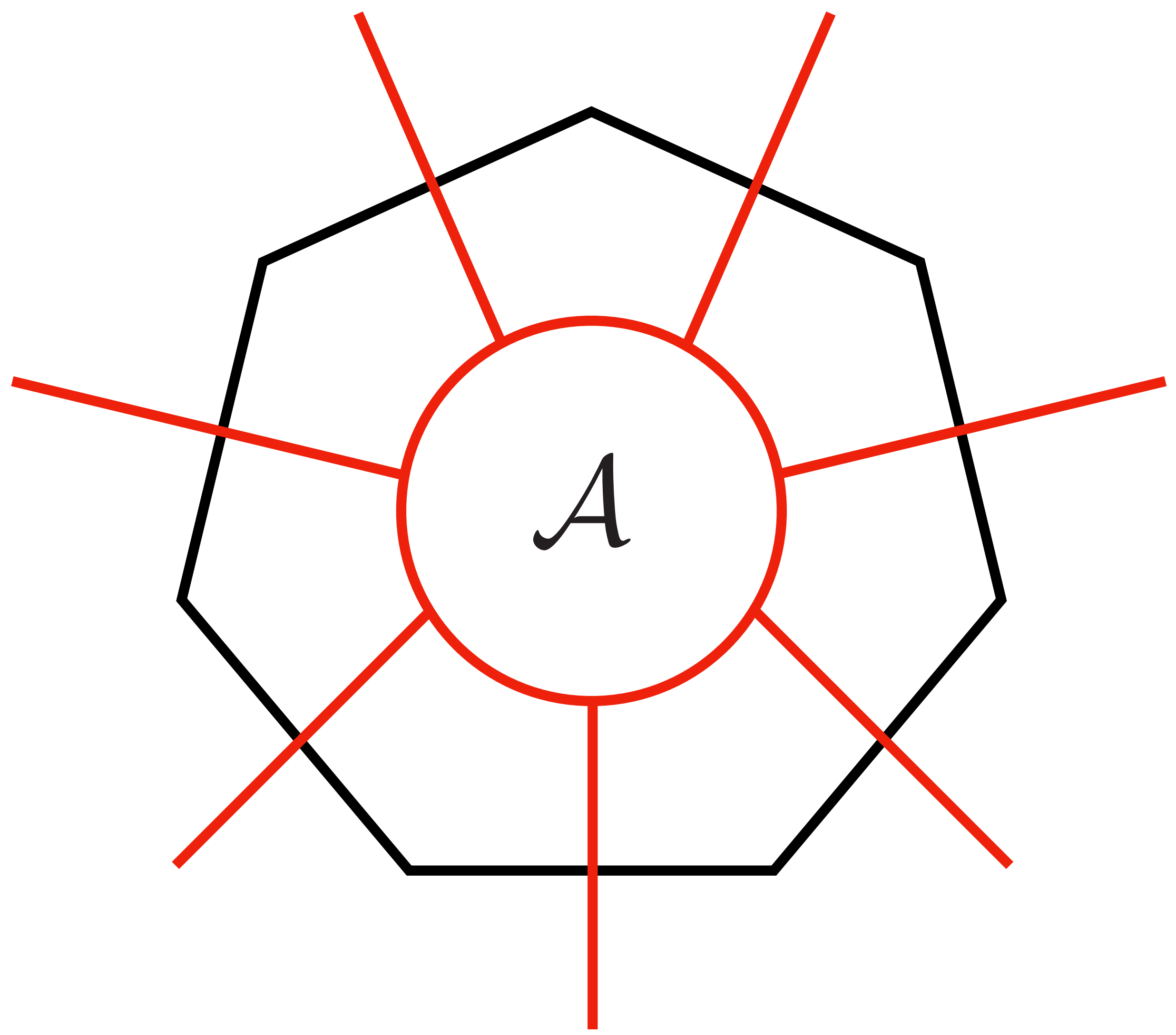


Toy model for re-summation of hexagon expansion in various kinematics

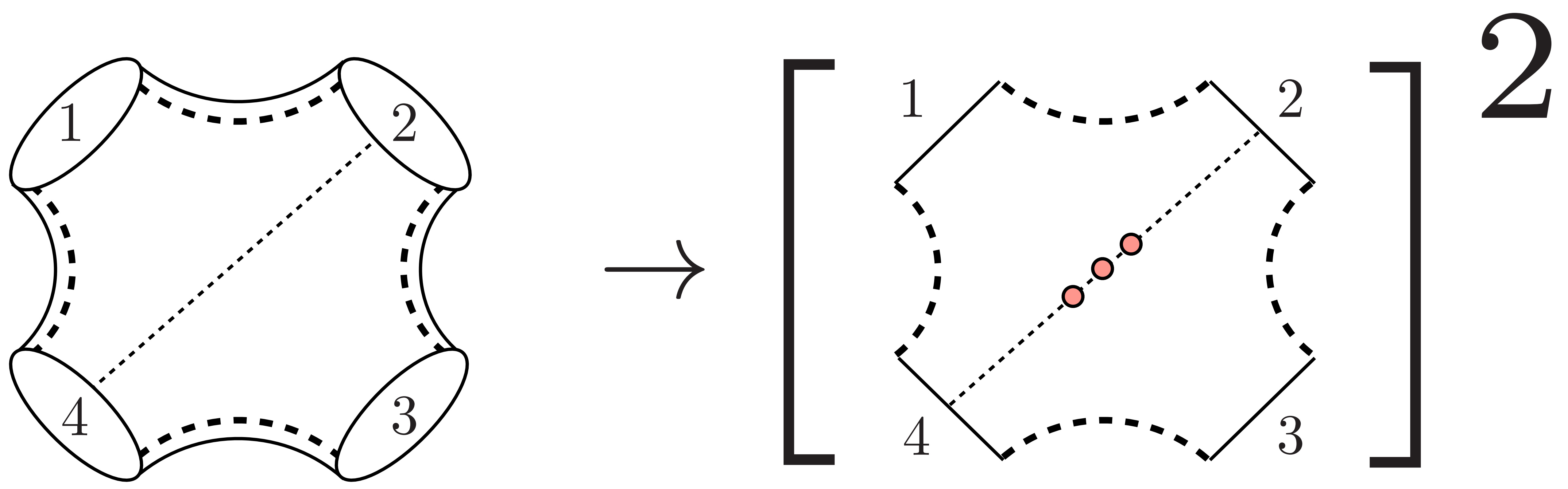
Interpretation as off-shell Scattering Amplitudes or Amplitudes on Coulomb Branch (IR reg.)

[Caron-Huot,Coronado]

Octagon

Prototype: Large-charge 4pt function or "Octagon"

[Coronado]
[Kostov,Petkova,Serban]
[Belitsky,Korchemsky]



Assume large R-charges are exchanged between consecutive operators

4-point function factorizes into two octagons - each containing a single sum over magnons

Octagon can be computed exactly and written as the Fredholm determinant of a Bessel kernel

$$\mathbb{O} = \det\left(1 - \boldsymbol{K}\right)$$

Hard to generalize to higher polygons (higher-point functions)

Null limit and Origins

Nice kinematic limits

Octagon develops large logarithms in cross ratios in null limit $U, V \to 0$

[Coronado]
[Kostov,Petkova,Serban]
[Belitsky,Korchemsky]

Exact re-summation is possible with a simple **Sudakov** behavior

$$\log \mathbb{O} \approx -\frac{\Gamma_{\text{oct}}}{16} \log^2 (UV) + \ldots$$

Coefficient exactly known $\Gamma_{\text{oct}} = \frac{2}{\pi^2} \log \cosh (2\pi g)$

Different from usual IR divergences controlled by cusp anomalous dimension!

Not isolated phenomenon: extend to various amplitudes with new anomalous dimensions

[Caron-Huot,Coronado]
[BB,Dixon,Liu,Papathanasiou]
[Bercini,Fernandes,Gonçalves]
[Bargheer,Bekov,Bercini,Coronado]
[Belitsky,Bork,Lee,Onishchenko,Smirnov]

It was observed for scattering amplitudes in so-called **Origin** limits

Origin limits = generalized light-like limits in which a maximum number of cross ratios vanish

Q: Origin limits for higher polygons? Classification? Exact description?

2d kinematics and positive region

Hard to explore Origins in full kinematics : lack “good” variables / cluster algebras

Focus on two-dimensional kinematics, with all operators lying in a plane $\mathbb{R}^{1,1}$

Configuration space = two copies of Grassmannian $\mathrm{Gr}(2,n)$

= two sets of n real spinors $\{(\lambda_i, \bar{\lambda}_i), i = 1, \ldots, n\}$

Distances: $x_{ij}^2 = (x_i - x_j)^2 \propto \langle ij \rangle \times [ij]$

where $\langle ij \rangle = \epsilon_{\alpha\beta} \lambda_i^\alpha \lambda_j^\beta$ and similarly for $[ij]$

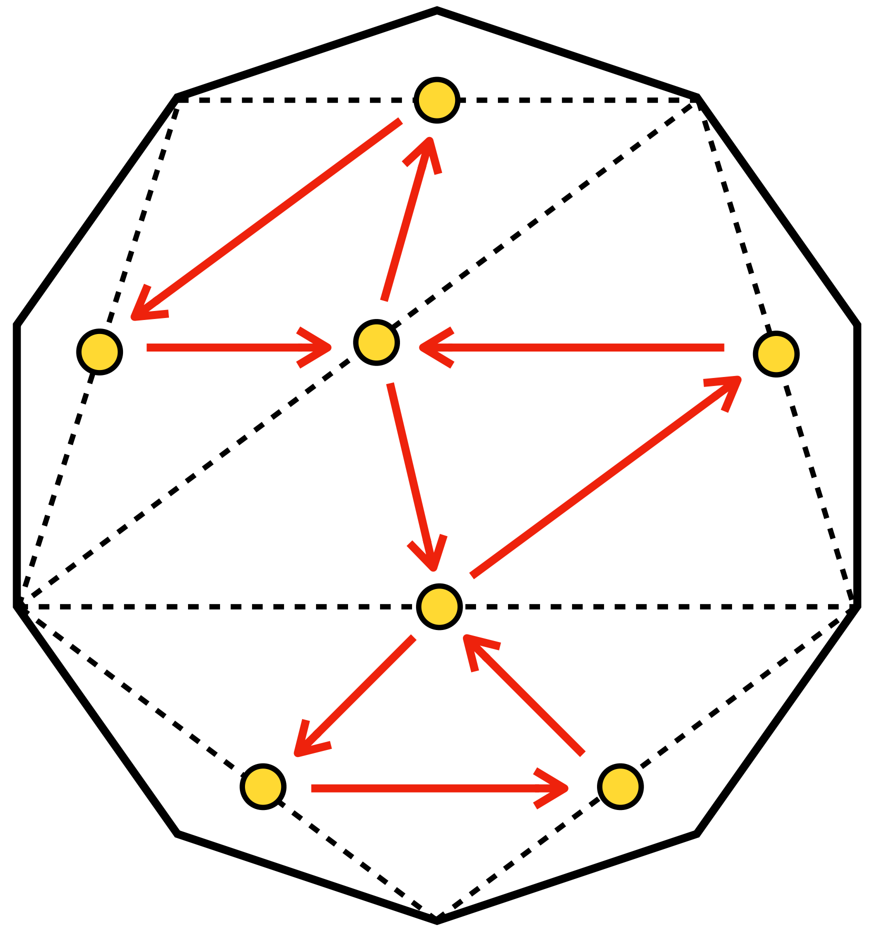
Positive region = domain defined by inequalities $\langle ij \rangle \geqslant 0 \quad 1 \leqslant i < j \leqslant n$

Boundary = where distances are going to zero = where correlators develop singularities

Facet = codim1 face where $\langle ij \rangle = 0$ for some i, j

Vertex = intersection of (n-3) facets = where singularities are in a sense “maximal”

Cluster algebra

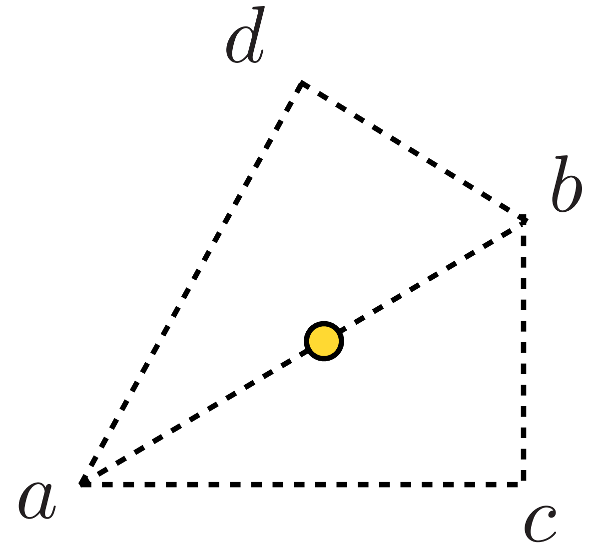


- Conformally invariant description using **cluster coordinates**

Each **vertex** is associated with set of cluster variables / triangulation

Cluster coordinate z is defined locally by a simple geometric rule

$$z_{ab} = \frac{\langle ad\rangle\langle cb\rangle}{\langle ac\rangle\langle bd\rangle}$$

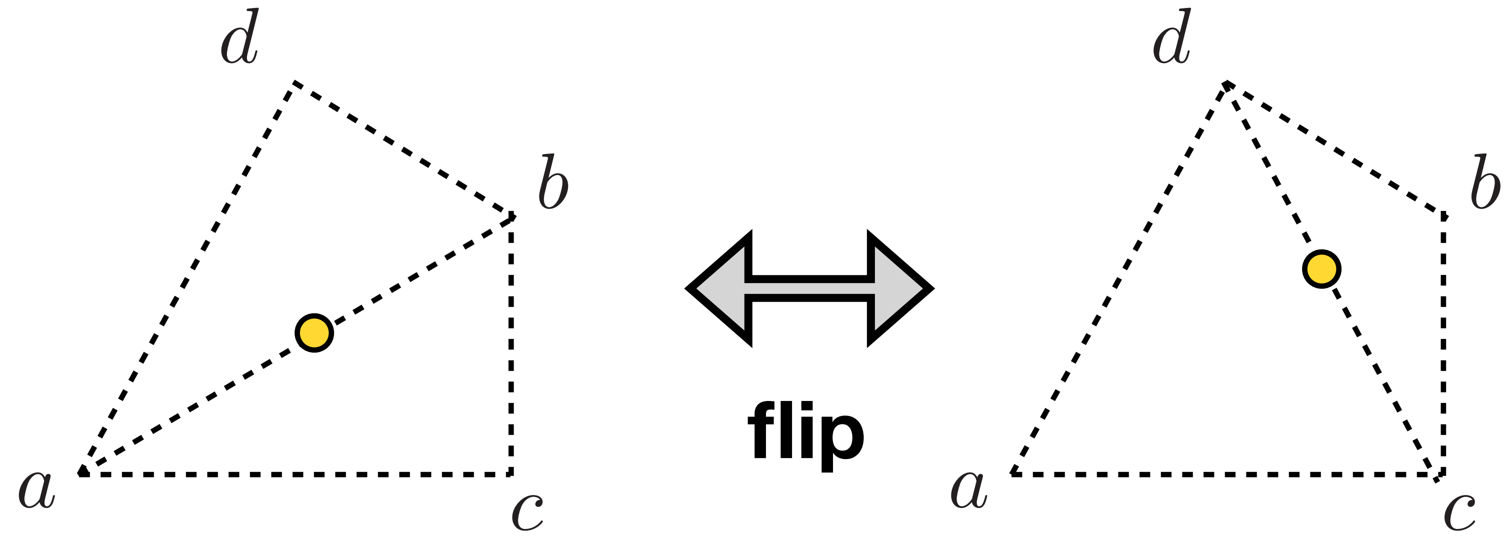


[Huge literature]

Vertex = point where all z variables are going to zero

- Move from one **vertex** to another using cluster **mutations**

They encode transformations of cluster coordinates associated with flips of edges in triangulation



Rules use information encoded in quiver diagram of the triangulation

$$z_i \to z_i' = 1/z_i$$
$$z_j \to z_j' = z_j\left(1 + z_i^{B_{ij}}\right)^{B_{ij}}$$

Summary : exchange graph

Exchange graph of the n-gon cluster algebra A_{n-3}

Vertices : triangulations = set of cluster coordinates

Edges : flips = mutations (bi-rational chart transformations)

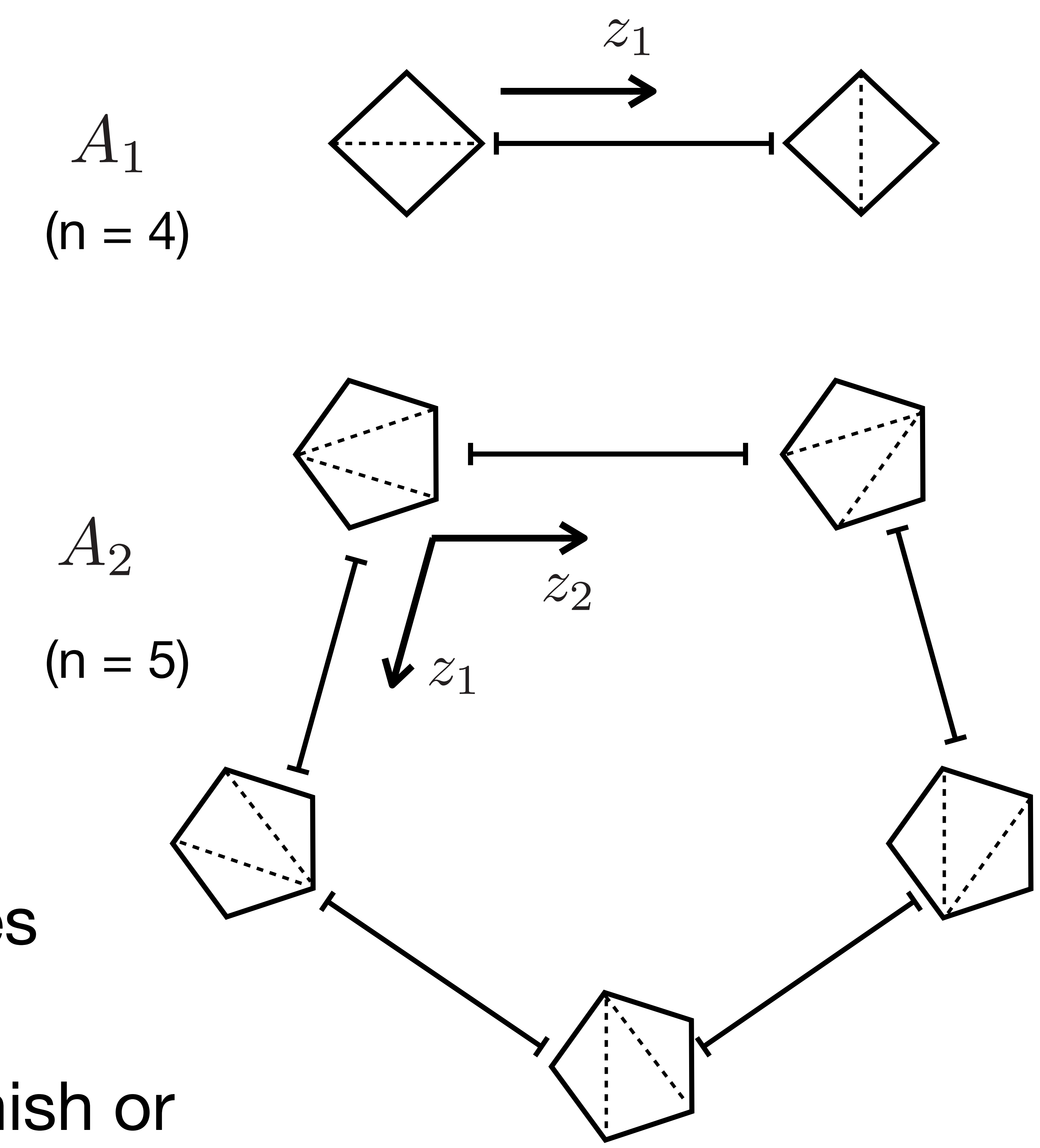
Positive region

Interior of the exchange graph corresponds to positive region, defined as the set of points reached by assigning positive values to the cluster coordinates

Boundary of this region are limits where cluster coordinates vanish or become large

Origin (cluster definition) "where singularities are maximal"

= Origin of a system of cluster coordinate (vertex <=> triangulation)

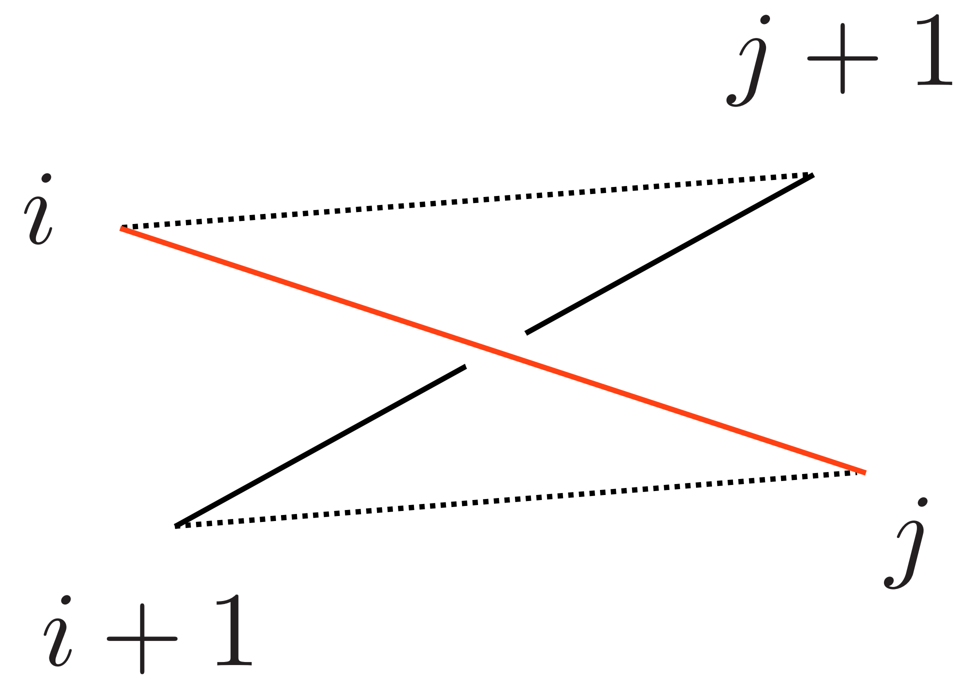


Similar to Origins in [BB,Dixon,Liu,Papathanasiou]

Alternative : binary geometry

[Arkani-Hamed,He,Lam]
[Arkani-Hamed,He,Lam,Thomas]
[Brown][DelDuca,Druc,Drummond,Duhr,
Dulat,Marzucca,Papathanasiou,Verbeek]

Attach a "u variable" (cross ratio) to each diagonal $u_{ij} = \frac{\langle i, j+1\rangle\langle i+1, j\rangle}{\langle i, j\rangle\langle i+1, j+1\rangle}$



Basis of $\frac{n(n-3)}{2}$ cross ratios in 2d (only n-3 are independent)

Relations take the elegant form

$$u_{ij} + \prod_{(kl)\,\text{crossing}\,(ij)} u_{kl} = 1$$

with product running over all chords crossing ij

Nice properties:

Variables are bounded when evaluated on positive region $0 < u_{ij} < 1$

Bounds are saturated at boundary of this region

Facet corresponds to limit where a "u" approaches 0, while crossing-related "u's" go to 1

$$u_{ij} \to 0 \quad \Rightarrow \quad u_{kl} \to 1$$

Vertex labeled by **binary sequence** indicating which (n – 3) u's are sent to 0 and which are set to 1

Origin limits (full definition)

We have both a left and a right polygon to parametrize

$$u_{ij}^{\text{Left}} = \frac{\langle i, j+1\rangle\langle j, i+1\rangle}{\langle i, j\rangle\langle i+1, j+1\rangle} \qquad u_{ij}^{\text{Right}} = \frac{[i, j+1][j, i+1]}{[i, j][i+1, j+1]}$$

Physical distances or cross ratios are products of left and right variables

$$\boxed{U_{ij} = u_{ij}^{\text{Left}} \times u_{ij}^{\text{Reft}}}$$

Number of vanishing cross ratios **maximized** when the sets of internal diagonals in the left and right triangulations are mutually disjoint

In that case, 2(n – 3) cross ratios approach zero, while the remaining ones approach one

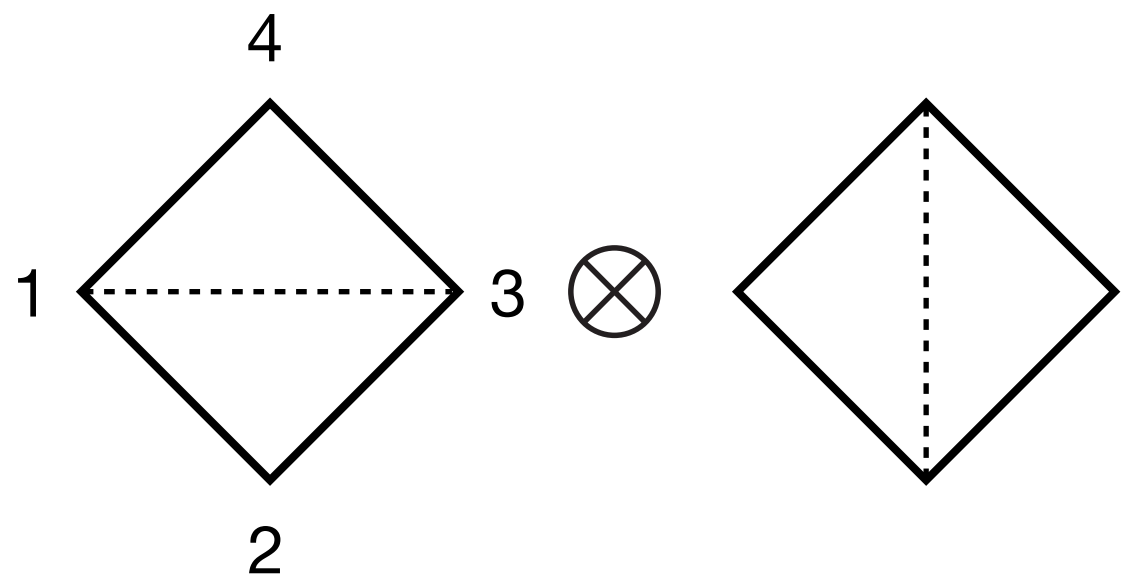
We call such configurations **Origins** or Origin limits, in analogy with scattering amplitudes

[BB,Dixon,Liu,Papathanasiou]

Examples I

Origin limits

4-point function

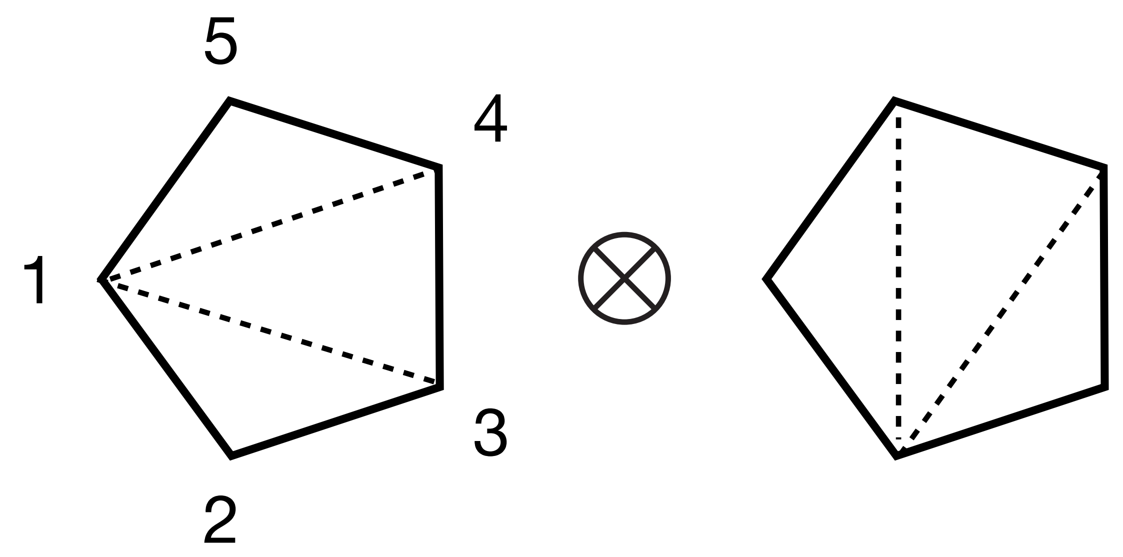


Left Right

$$U_{13}, U_{24} \to 0$$

same as null square limit

5-point function



$$U_{13}, U_{14}, U_{24}, U_{25} \to 0$$
$$U_{35} \to 1$$

Comments

1) Other choices for n=5 give cyclic images of the limit above

2) **No more than 4 cross ratios can approach zero simultaneously**. This is the maximal number allowed by the kinematics in 2d. (In contrast, in full kinematics, there is more freedom: the five cross ratios are independent and, in principle, can all be taken to zero simultaneously.)

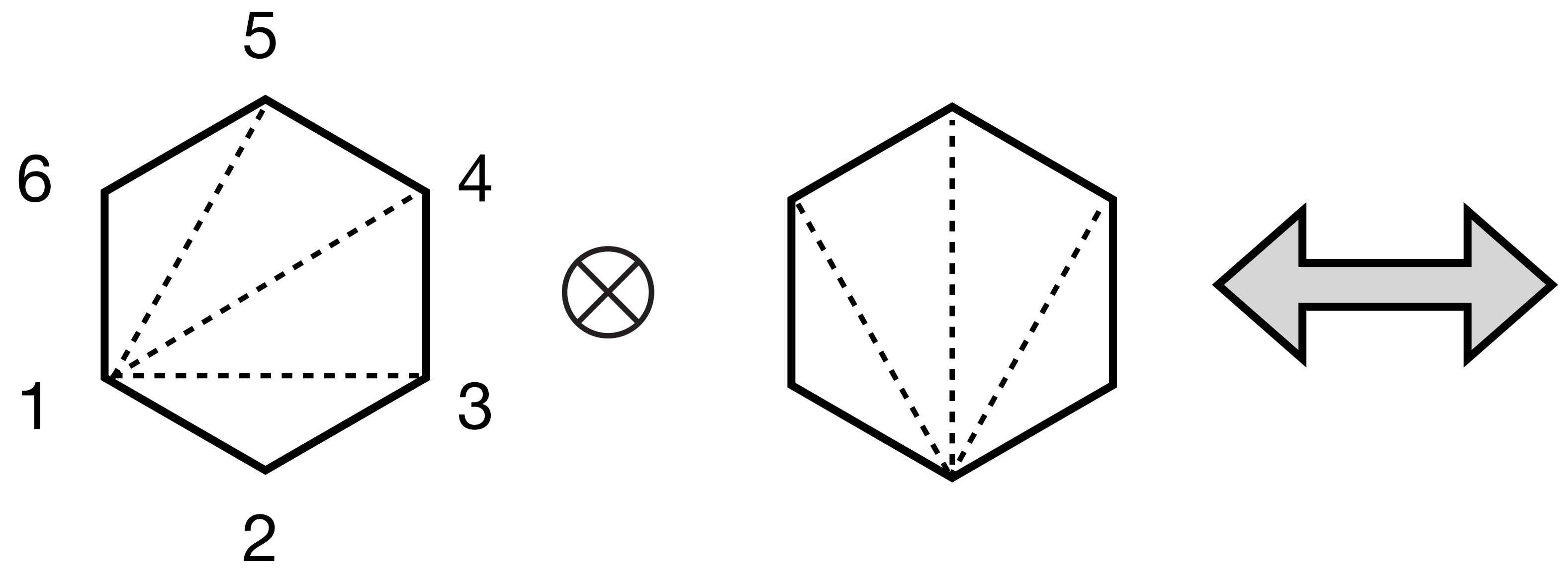
See e.g. [Bercini,Fernandes,Gonçalves]

Examples II

Origin limits

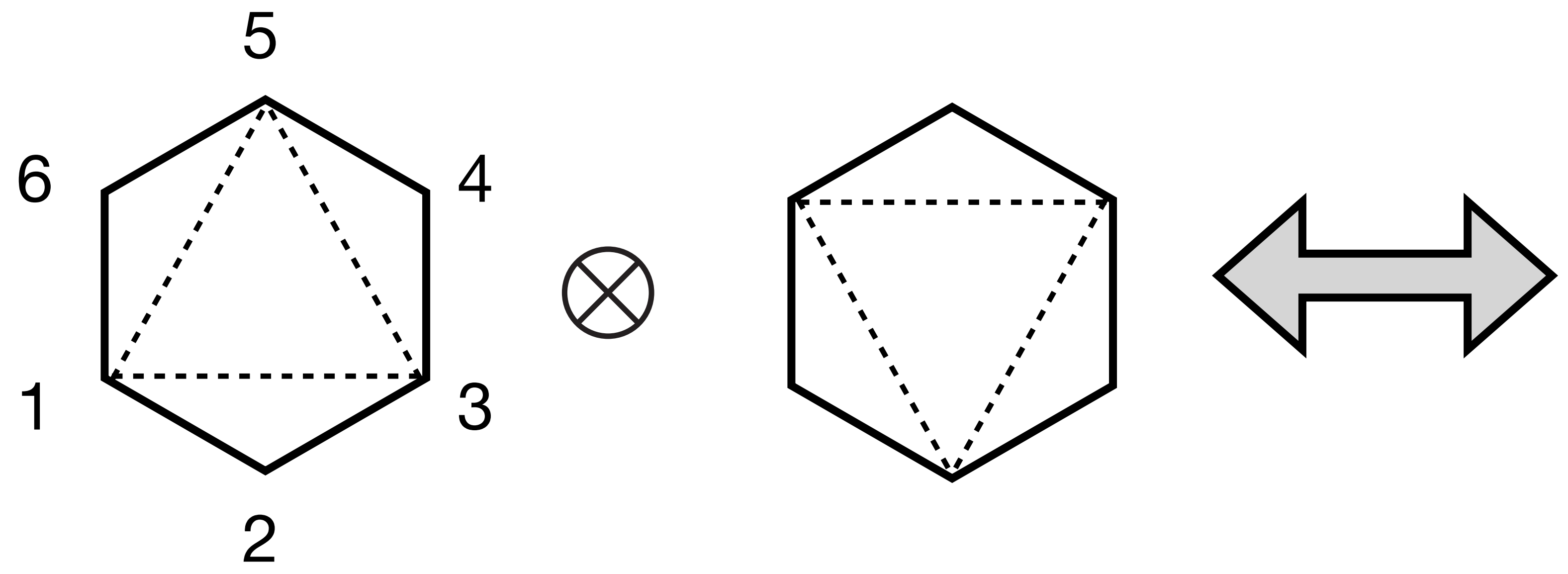
6-point function

Ex.1



$$U_{13}, U_{24}, U_{15}, U_{26}, U_{14}, U_{25} \to 0$$
$$U_{35}, U_{46}, U_{36} \to 1$$

Ex.2



$$U_{13}, U_{24}, U_{35}, U_{46}, U_{15}, U_{26} \to 0$$
$$U_{14}, U_{25}, U_{36} \to 1$$

same as null hexagon limit

...

Many Origins are dihedral images of one another. Classification of orbits?

Origin classes for hexagon

Define $(u_1, u_2, u_3, u_4, u_5, u_6; v_1, v_2, v_3) = (U_{13}, U_{24}, U_{35}, U_{46}, U_{15}, U_{26}; U_{14}, U_{25}, U_{36})$

Inequivalent classes of Origins are given by

Origin Class	u_1	u_2	u_3	u_4	u_5	u_6	v_1	v_2	v_3
O_1	0	0	1	0	0	1	0	0	1
O_2	0	1	0	0	1	0	0	0	1
O_3	0	0	1	0	1	0	0	0	1
O_4	1	0	0	0	0	1	0	0	1
O_5	0	0	1	0	0	0	0	1	1
O_6	0	0	0	0	0	0	1	1	1

Images are obtained by acting on above representatives with dihedral group

Orbits with 6,3,12,6,6,1 elements for O1 to O6 respectively

One-loop analysis

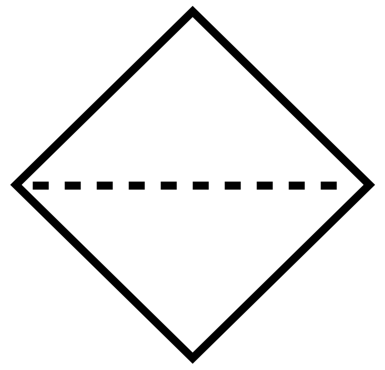
One-loop hexagonalization can be performed in closed form

[Fleury,Komatsu]
[Bargheer,Caetano,Fleury,Komatsu,Vieira]

Building block given by the Bloch-Wigner function

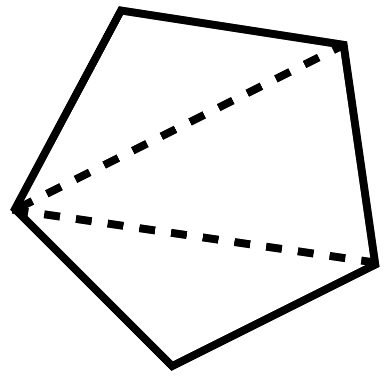
$$\mathcal{M}(z) = \frac{z+\bar{z}-\text{R-charge}}{2(z-\bar{z})}\left(2\text{Li}_2(-z) - 2\text{Li}_2(-\bar{z}) + \log\left(\frac{1+z}{1+\bar{z}}\right)\log\left(z\bar{z}\right)\right)$$

Ex. 4-pt



$$\mathcal{M}(z_1) + \mathcal{M}\left(\frac{1}{z_1}\right)$$

Ex. 5-pt



$$\mathcal{M}(z_1) + \mathcal{M}(z_2(1+z_1)) + \mathcal{M}\left(\frac{1+z_2}{z_1 z_2}\right) + \mathcal{M}\left(\frac{1+z_2(1+z_1)}{z_1}\right) + \mathcal{M}\left(\frac{1}{z_2}\right)$$

Find that the result is quadratic in logarithms in the Origin limits

R-charge drops out: Answer just a function of spacetime cross ratios

Higher-loop analysis

Identify states that dominate and **truncate** the hexagon sums

Use **Sommerfeld-Watson** (SW) transform to carry out sums and extract leading contribution

- ✓ 4pt function through all loops $U_1, U_2 \to 0$

$$\log \mathcal{C}_4 \approx -g^2 \log U_1 \log U_2 + \mathcal{O}(g^4)$$

- ✓ 5pt function through 5 loops at least $U_1, U_2, U_4, U_5 \to 0 \ \ (U_3 \to 1)$

$$\log \mathcal{C}_5 \approx -g^2 (\log U_1 \log U_2 + \log U_4 \log U_5 + \log U_1 \log U_5) + \mathcal{O}(g^4)$$

Higher polygons? In progress for all 6-point Origins…

Not always clear which states dominate in a generic Origin limit (need criterion)

Details depend on Origin & triangulation used for computation… but **Sudakov** behavior observed!

Can we do better than that? **Master formula** (all-loop, all-point)

All-loop master formula

Drawing inspiration from scattering amplitudes and generalizing observations made for 4-point function, we are led to conjecture that disk correlation functions exponentiate in any Origin limit and that the exponent is a quadratic polynomial in the logarithms of the U cross ratios

[BB,Dixon,Liu,Papathanasiou]
[Coronado]
[Belitsky,Korchemsky]
[Kostov,Petkova,Serban]

Conjecture: we expect the disk n-point function to take the form

[BB,Fleury,Kaluç,Serban - in progress]

$$\log \mathcal{C}_n \approx -\frac{1}{2}\oint_{C_n} \frac{(z-1/z)dz}{2\pi i z}\mathcal{G}(z,g)\times \mathcal{S}_n(z,\{\log U_{ij}\})$$

Three main ingredients

Variable z = spectral parameter

1) $\mathcal{G}$ is a function of the coupling constant known as tilted cusp anomalous dimension

2) $\mathcal{S}_n$ is the string integrand coding the kinematics: it is rational in z and quadratic in log U's

3) C_n (contour) is such as to enclose the poles of $\mathcal{S}_n$ in the plane of z

1) Tilted cusp anomalous dimension

Following the structure observed in scattering amplitude, we set [BB,Dixon,Liu,Papathanasiou]

$$\mathcal{G}(z,g) = \Gamma_\alpha(g) \quad \text{with} \quad z = -e^{2i\alpha}$$

$\Gamma_\alpha(g)$ is the tilted cusp anomalous dimension (1-parameter deformation of cusp)

$$\Gamma_\alpha = 4g^2 - 16\zeta_2 \cos^2\alpha \, g^4 + 32\zeta_4 \cos^2\alpha(3 + 5\cos^2\alpha)\, g^6 + \dots$$

with same zeta-structure as for cusp but with loop coefficients dressed with trigonometric numbers

All-order expression follows from deformation of the BES equation for cusp [Beisert,Eden,Staudacher]

$$\Gamma_\alpha(g) = 4g^2(1 + \mathbb{K}(\alpha))^{-1}_{11}$$

$$\mathbb{K}_{ij} = 2j(-1)^{ij+j} \int_0^\infty \frac{dt}{t} \frac{J_i(2gt)J_j(2gt)}{e^t - 1}$$

$$\mathbb{K}(\alpha) = 2\cos\alpha \begin{bmatrix} \cos\alpha\, \mathbb{K}_{\circ\circ} & \sin\alpha\, \mathbb{K}_{\circ\bullet} \\ \sin\alpha\, \mathbb{K}_{\bullet\circ} & \cos\alpha\, \mathbb{K}_{\bullet\bullet} \end{bmatrix}$$

In particular $\quad \Gamma_{\text{cusp}} = \Gamma_{\alpha=\pi/4} \qquad \Gamma_{\text{oct}} = \Gamma_{\alpha=0}$

2) String integrand

[Caetano,Toledo]
[Bargheer,Coronado,Vieira]
[BB,Kaluç,Serban - in progress]

At strong coupling correlation functions are described by minimal surfaces in AdS

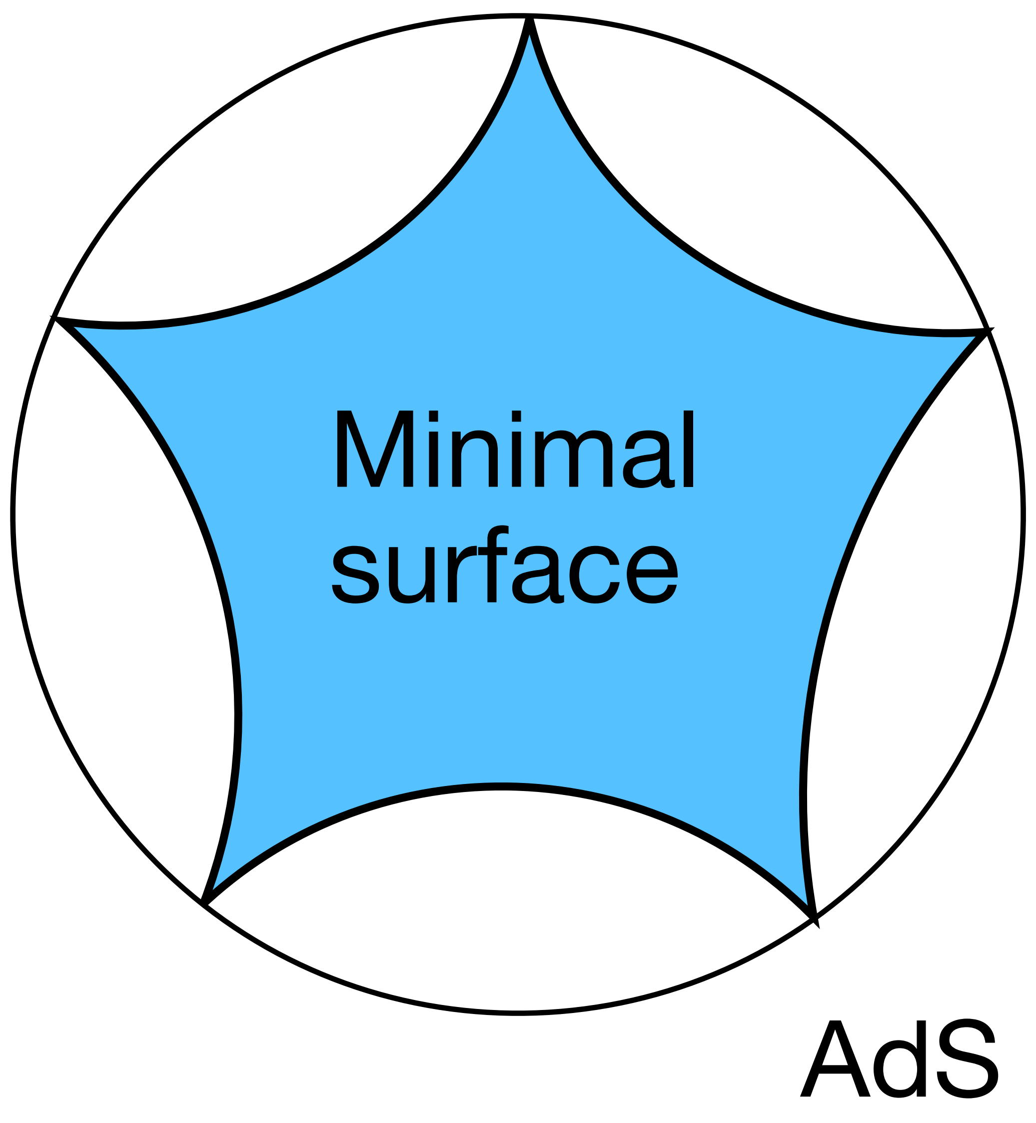
Precise stringy definition of min surface dual to a polygon is still lacking

However progress in computing the area can be made using **integrability**

Express area as the **free energy** of a system of **TBA** equations

Equations for a set of Y-functions (collective fields for string DoFs)

Equations simplify drastically in Origin limits where they become linear



$$\log Y_i(z) = \frac{1}{1 - K_{ij}(z)} \cdot I_j(z)$$

Free energy is obtained by integrating the string integrand $\mathcal{S}_n(z) = \sum_i I_i(1/z) \log Y_i(z)$

Generally, (n-3) Y functions are needed for n-point Origin, but complete dictionary still lacking

3) Contour evaluation I

In **all** cases the poles are on the unit circle and in **most** cases they are simple

$$\log \mathcal{C}_n \approx \sum_\alpha \Gamma_\alpha(g) \times P_\alpha(\{\log U_{ij}\})$$

where the set of alpha's and associated polynomials are determined by the string integrand

Ex.1 Simplest example is given by the 4-point function with pole at $z^2 = 1$

$$\log \mathcal{C}_4 \approx -\frac{1}{16}\Gamma_{\alpha=0}(g) \log^2 (U_1 U_2) + \frac{1}{16}\Gamma_{\alpha=\pi/2}(g) \log^2 (U_1/U_2)$$

with

$$\Gamma_{\alpha=0} = \Gamma_{\text{oct}} = \frac{2}{\pi^2} \log\cosh(2\pi g) \quad \text{and} \quad \Gamma_{\alpha=\pi/2} = 4g^2$$

3) Contour evaluation II

In **all** cases the poles are on the unit circle and in **most** cases they are simple

$$\log \mathcal{C}_n \approx \sum_{\alpha} \Gamma_\alpha(g) \times P_\alpha(\{\log U_{ij}\})$$

where the set of alpha's and associated polynomials are determined by string integrand

Ex.2 For 5-point function we have poles at $z^2(1-z^2)=1$ resulting in

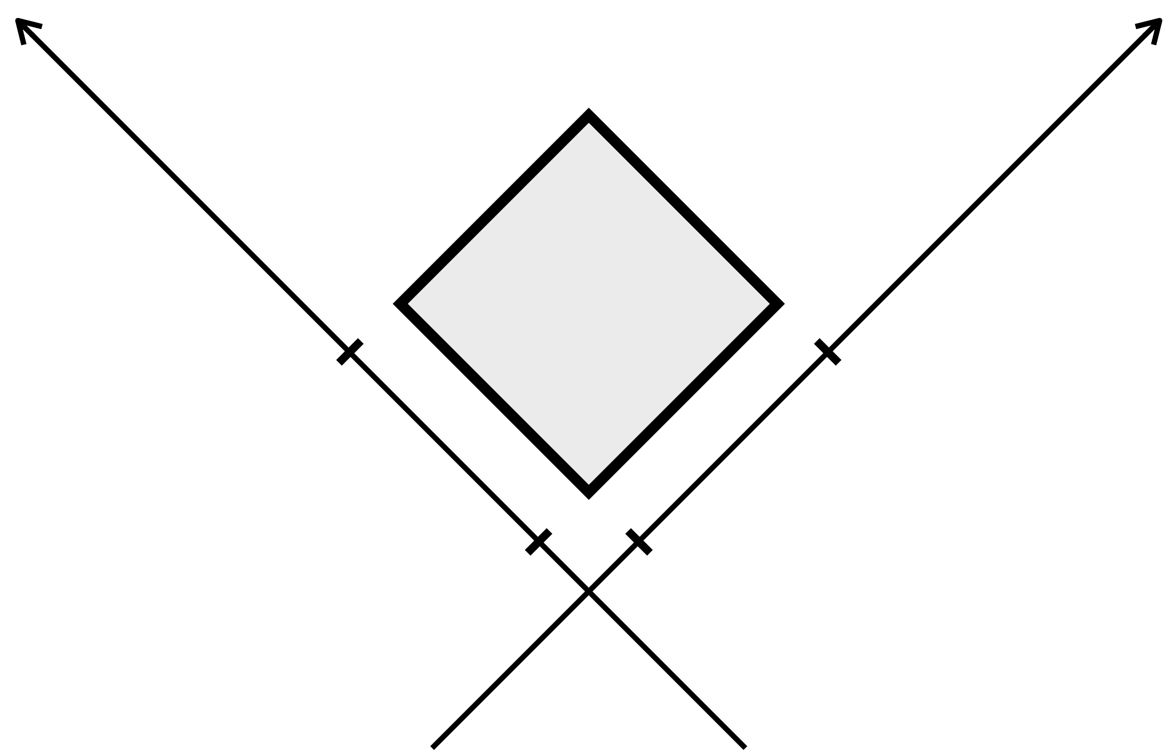
$$\log \mathcal{C}_5 \approx \Gamma_{\pi/12}(g)\, P_1(\{\log U_{ij}\}) + \Gamma_{5\pi/12}(g)\, P_2(\{\log U_{ij}\})$$

with $P_{1,2} = \frac{1}{2} P_{\text{one-loop}} \mp \frac{1}{2\sqrt{3}}(\log^2 U_1 + \log^2 U_2 + \log^2 U_4 + \log^2 U_5 + \log U_1 \log U_4 + \log U_2 \log U_5)$

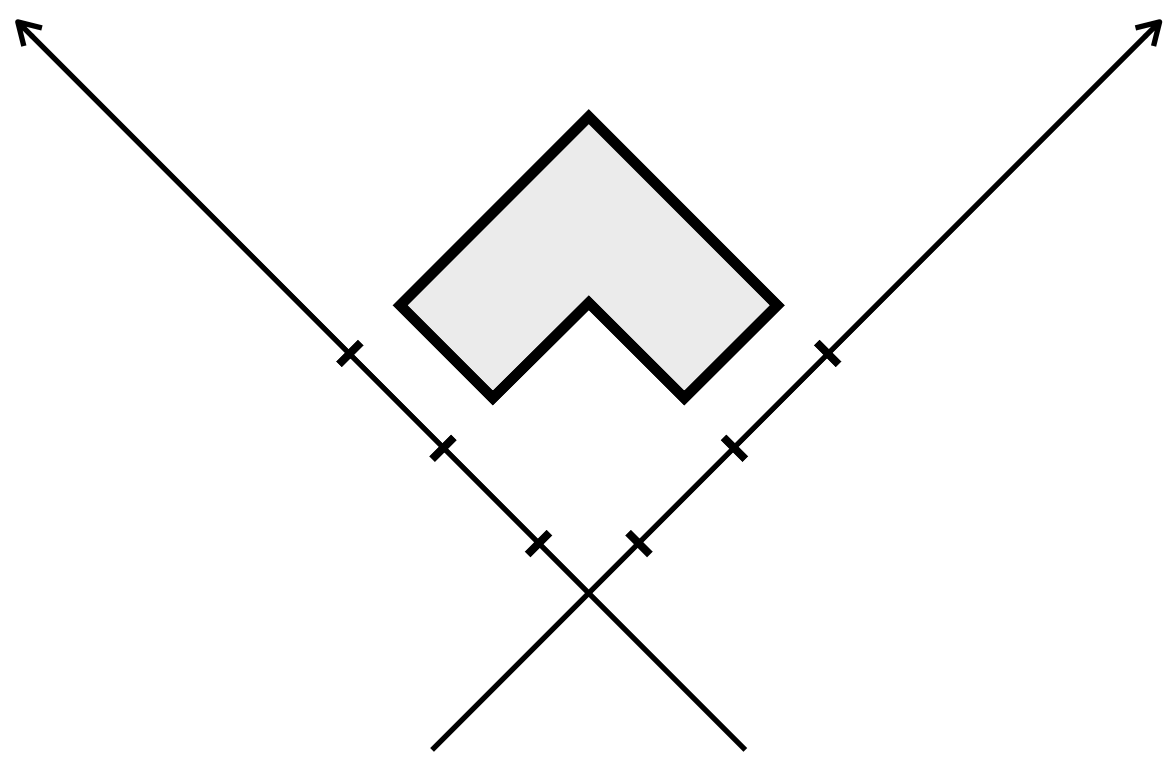
Evaluation at weak coupling in perfect agreement with previous data through 5 loops (at least)!

Null polygons in two dimensions

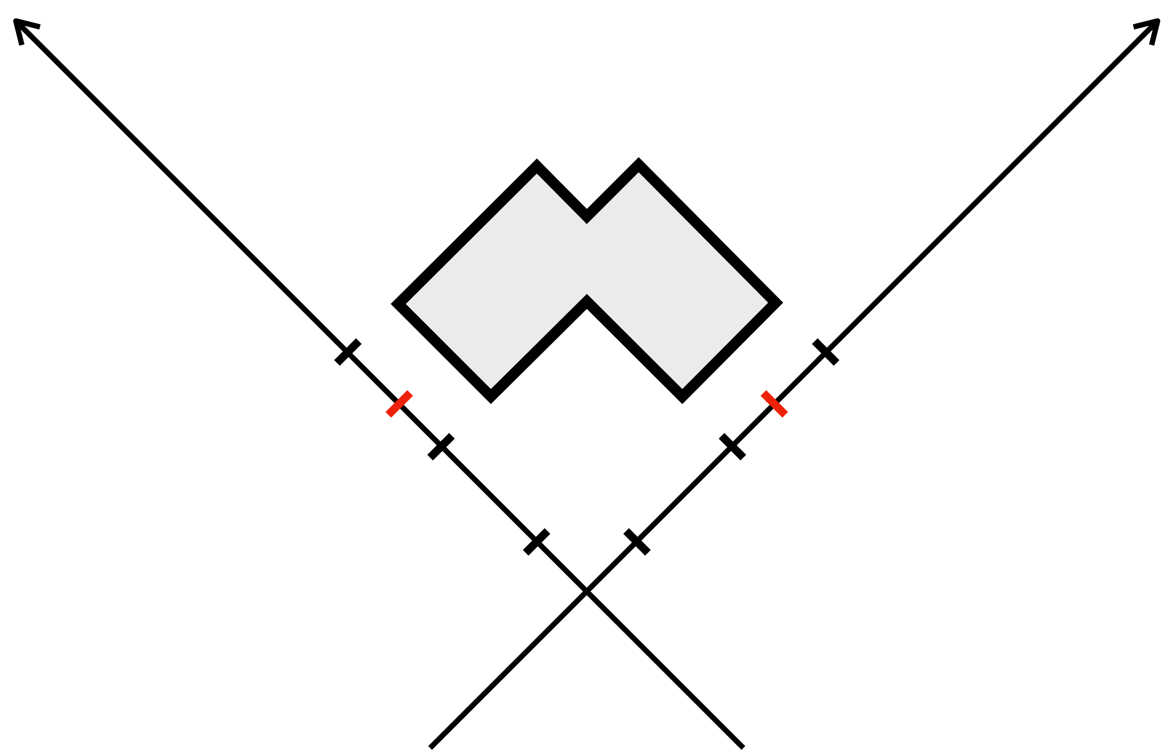
Consider limits where consecutive vertices are **null** separated (n even in 2d)



Ex.1 null square
no cross-ratios



Ex.2 null hexagon
no cross-ratios



Ex.3 null octagon
2 cross-ratios

Interesting due to their conjectured connection with Wilson loops and scattering amplitudes

[Caron-Huot,Coronado]

Null 2d kin. = **Origin** limits when n = 4 and 6 (null square and null hexagon are unique in 2d)

For n >= 8 we have (n-6) cross ratios - General structure?

Structuring null polygons

Null kinematics is not an Origin limit for higher n

Yet null kinematics is cornered by Origins = double-soft limits

Conjecture: Origins control the logarithmic divergent part of the null polygons

$$\log \mathcal{C}_n = \text{Quad-Log} + R_n$$

“Remaining object” should be finite and function of the cross ratios of the null 2d polygon

Caveat: examples from n = 4, 6 reveal that divergent part is NOT universal!

For n = 4 it is controlled by the octagon dimension but for n = 6 it also involves $\alpha = \pi/3, \pi/6$

At general n, we expect alpha’s associated with n-th roots of unity to play a role

Quid of the finite part? Hard to compute (need more data)!

[Caron-Huot,Coronado]

Does it have anything to do with the remainder functions of null polygonal WLs/Amplitudes?

[Caron-Huot,He]

Summary and outlook

Polygons like Amplitudes admit interesting Sudakov behaviors in Origin limits

A classification of Origins is possible in 2d using cluster algebra and U-coordinates

Behavior in all these limits is governed by the tilted cusp and the TBA string integrand

Generalization to full kinematics? Cluster algebra?

Full dictionary between Origins and TBA equations?

Lots of conjectures… How many of them can be proven?

Application to **massless Amplitudes** in null limit - New way of computing them?

Bridge gap between Hexagon method and Pentagon OPE?

Puzzles: Why is IR divergent part **NOT** universal? Are remainders the same as for amplitudes?