Giant graviton integrated correlator at large N made simple

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Work in progress

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Motivation

- Huge progress for correlators of operators with fixed dimensions in $\mathcal{N}=4$ super Yang-Mills (SYM), but much less results for heavy operators

- Heavy sectors are controlled by semi-classics, $\mathcal{N}=4$ SYM allows for exact computation and provides a testing ground

- AdS/CFT: heavy operators correspond to non-trivial background, or heavy objects (D-branes)

Outline

- Giant graviton operators in $\mathcal{N}=4$ SYM and their heavy-heavy-light-light correlators
- Brief introduction to integrated correlators
- Exact results of giant graviton integrated correlator at finite Yang-Mills coupling in the large-N expansion
- Final comments

Four-point correlators in $\mathcal{N}=4$ SYM

Superconformal primary operators

$$\mathcal{O}_p(x,Y) = \mathrm{Tr}\left(\Phi^{I_1}(x)\ldots\Phi^{I_p}(x)\right) Y_{I_1}\ldots Y_{I_p}$$

$$\mathcal{O}_{p_1,\cdots,p_n}(x,Y) = \frac{p_1\cdots p_n}{p}\mathcal{O}_{p_1}(x,Y)\cdots\mathcal{O}_{p_n}(x,Y)$$

$\Phi^I(x)$ are scalars, and Y_I are SO(6) R-symmetry null vectors.

They are 1/2 BPS operators.

Four-point correlators in $\mathcal{N}=4$ SYM

- We consider correlators in SU(N) $\mathcal{N}=4$ SYM of the following form

$$\langle \mathcal{H}(x_1,Y_1)\mathcal{H}(x_2,Y_2)\mathcal{O}_2(x_3,Y_3)\mathcal{O}_2(x_4,Y_4)\rangle$$

$\mathcal{H}$ is some linear combination of $\mathcal{O}_{\vec{p}}$

- Superconformal Ward identity

Eden, Petkou, Schubert, Sokatchev; Nirschl, Osborn

Fixed by symmetry

Our focus

$$\langle \mathcal{H}(x_1,Y_1)\mathcal{H}(x_2,Y_2)\mathcal{O}_2(x_3,Y_3)\mathcal{O}_2(x_4,Y_4)\rangle = \mathcal{G}_{\text{free}}(x_i,Y_i) + \mathcal{I}_4(x_i,Y_i)\,(d_{12})^{\Delta_{\mathcal{H}}-2}\,\mathcal{T}_{\mathcal{H}}(u,v)$$

Heavy-heavy-light-light correlators

We are interested in heavy operators $\Delta_{\mathcal{H}} \to \infty$; different regimes:

- The operators being heavy after large N: $\Delta_{\mathcal{H}} \ll N$

Coronado; Aprile, Vieira; …

Basso's talk

- The operators being heavy with fixed N: $\Delta_{\mathcal{H}} \gg N^2$

Caetano, Komatsu, Wang; Paul, Perlmutter, Raj; Brown, Galvagno, Grassi, Iossa, C.W.; …

- The operators of the talk are of order N: $\Delta_{\mathcal{H}} \sim N$

Holography: do no deform the background, but provide heavy objects.

Giant Gravitons

Giant Gravitons

Giant gravitons as determinant operators:

$$\det_m \varphi(x,Y) \quad \text{with} \quad \varphi(x,Y) \coloneqq Y \cdot \Phi(x)$$

- They may be expressed in terms of trace operators $\mathcal{O}_{\vec{p}}$
- We'll consider the dimension $m = \alpha N$ with $\alpha = 1$
- Holographically dual to D3-brane wrapping S^3 inside S^5

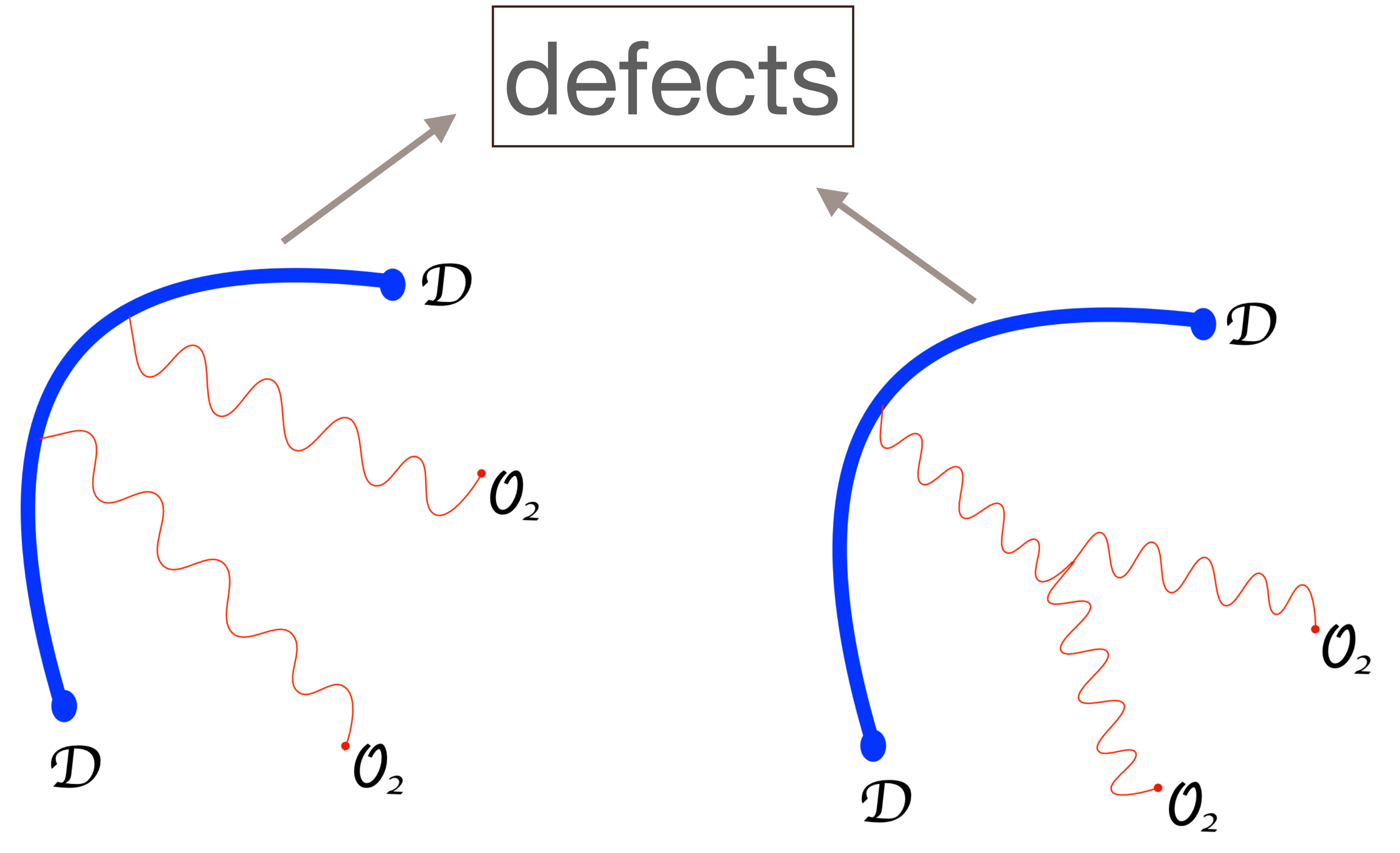
Heavy-heavy-light-light correlators

HHLL correlators of giant gravitons

in stress-tensor multiplet

$$\langle \mathcal{H}(x_1, Y_1)\mathcal{H}(x_2, Y_2)\mathcal{O}_2(x_3, Y_3)\mathcal{O}_2(x_4, Y_4)\rangle$$

Holographically interpreted as two gravitons scattering off D3-brane moving along geodesic



What is known

- For giant graviton with $\alpha = 1$
  - Weak coupling: 2 loops in planar limit + progress @ 3 loops
    Jiang, Komatsu, Vescovi; Jiang, Wu, Zhang; Wu, Jiang, Liu, Zhang
  - Strong coupling: tree-level supergravity Chen, Jiang, Zhou

We will consider a simpler quantity: the Integrated Correlators, for which we can say much more.

Integrated Correlators

Introduction to Integrated correlators

Schematic idea: consider 4-point function in $\mathcal{N}=2$ SCFT

$$\int d\mu(x_i)\, \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \partial_{h_1}\partial_{h_2}\partial_{h_3}\partial_{h_4}\, \log \mathcal{Z}(h_i)\big|_{h_i=0}$$

here $\mu(x_i)$ SUSY-preserving measure & $\mathcal{Z}(h_i)$ is deformed partition function, with h_i being deformation parameters.

Applying to $\mathcal{N}=4$ SYM: deform it to be $\mathcal{N}=2^*$ SYM (on a 4-sphere) + higher-dim operators (deform parameters: mass & higher-dim couplings)

Introduction to Integrated correlators

We will consider

$$\mathcal{I}_{\mathcal{H}}(\tau;N) = \frac{\partial_{\mathcal{H}}\partial_{\mathcal{H}}\partial_m^2 \log \mathcal{Z}(\tau,\tau';m)\,|_{\tau',m=0}}{\partial_{\mathcal{H}}\partial_{\mathcal{H}} \log \mathcal{Z}(\tau,\tau';m)\,|_{\tau',m=0}}$$

$\mathcal{Z}(\tau,\tau';m)$ is partition function of $\mathcal{N}=2^*$ SYM on 4-sphere & $\tau = \frac{\theta}{2\pi} + \mathrm{i}\frac{4\pi}{g_{\rm YM}^2}$

- It is the integrated correlator of

$$\langle \mathcal{H}(x_1,Y_1)\mathcal{H}(x_2,Y_2)\mathcal{O}_2(x_3,Y_3)\mathcal{O}_2(x_4,Y_4)\rangle$$

higher-dim operators are placed at northern & southern poles, introduced through coupling τ'

Introduction to Integrated correlators

- The integration measure: using $U = 1 + r^2 - 2r\cos(\theta),\ V = r^2$

Binder, Chester, Pufu, Wang

$$\mathcal{I}_{\mathcal{H}}(\tau, \bar{\tau}; N) = I_2\left[\mathcal{T}(u,v)\right] = -\frac{2}{\pi}\int_0^\infty dr \int_0^\pi d\theta \frac{r^3 \sin^2\theta}{u\, v}\mathcal{T}(u,v)$$

- Alternative version: $\mathcal{T}(u,v) = \text{factor} \times T(x_i)$

C.W., Zhang; Brown, Heslop, C.W., Xie

conformally covariant

$$\mathcal{I}_{\mathcal{H}}(\tau, \bar{\tau}; N) = I_2(\mathcal{T}(u,v)) = \int \frac{d^4x_1 \dots d^4x_4}{\text{vol}\, SO(1,5)} T(x_i)$$

They are periods of conformal Feynman integrals

Introduction to Integrated correlators

This is useful because the sphere partition function of $\mathcal{N}=2^*$ SYM can be computed by supersymmetric localisation

Pestun; Nekrasov; …

$$\mathcal{Z}(\tau,\tau';m) = \int d^{N-1}a \left| \exp\left(\mathrm{i}\pi\tau \sum_i a_i^2 + \mathrm{i} \sum_{p>2} \pi^{p/2} \tau'_p \sum_i a_i^p \right) \right|^2 Z_{\text{1-loop}}(a;m) \left| Z_{\text{inst}}(\tau,\tau',a;m) \right|^2$$

finite-dimensional

perturbation

Instantons

where

$$Z_{\text{1-loop}}(a;m) = \frac{1}{H(m)^N} \prod_{i<j} \frac{H^2(a_{ij})}{H(a_{ij}+m)H(a_{ij}-m)}, \qquad H(x) := e^{-(1+\gamma)x^2} G(1+\mathrm{i}x) G(1-\mathrm{i}x)$$

Barnes G-function

More on instantons later!

Integrated correlators at finite coupling

The goal is to determine the giant graviton integrated correlator at finite coupling

$$\tau = \tau_1 + \mathrm{i}\tau_2 := \frac{\theta}{2\pi} + \mathrm{i}\frac{4\pi}{g_{\mathrm{YM}}^2}$$

This has been done for $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2 \rangle$ and some $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_p\mathcal{O}_p \rangle$.

- These results provide non-perturbative constraints on the correlators.
- They have been used in numerical and analytical bootstraps (especially beyond planar limit, and finite N and finite coupling).

Integrated correlators at finite coupling

A direct evaluation of localisation formula is not easy (especially for finite τ)

- Localisation computation is on S^4, so we need map it to R^4. This leads to a Gram-Schmidt procedure, which can be very tedious for operators with large dimensions (giant gravitons have dimension N)

- $Z_{\text{inst}}(\tau, \tau', a; m)$ is in fact not known in general, due to higher-dimensional operators (1-instanton is known); so we do not know how to calculate it, even just in principle

Gerchkovitz, Gomis, Ishtiaque, Karasik, Komargodski, Pufu

S-duality to the Rescue

$\mathcal{N}$=4 SYM and S-duality

$\mathcal{N}$=4 SYM with gauge group G & Yang-Mills coupling

$$\tau = \tau_1 + i\tau_2 := \frac{\theta}{2\pi} + \frac{4\pi i}{g^2_{\rm YM}}$$

S-duality of $\mathcal{N}$=4 SYM:

Montonen, Olive; Goddard, Nuyts, Olive

$$T : (G, \tau) \to (G, \tau + 1)$$

$$\hat{S} : (G, \tau) \to ({}^L G, -\frac{1}{r\,\tau})$$

where r is the square of the ratio of the long and short roots of the Lie algebra of G, and ${}^L G$ is the Langlands dual.

$\mathcal{N}=4$ SYM and S-duality

- For SU(N) theory, giant graviton correlators (as all the superconformal primaries) are S-duality invariant. (D3-branes are self-dual)
- The correlators should be described by non-holomorphic modular functions.
- In general, we know very little about non-holomorphic modular functions, but integrated correlators appear to be described by very special modular functions.

Exact results of integrated correlators

We found the integrated correlator can be written as a remarkably simple lattice sum formula for any N and any τ

Dorigoni, Green, CW; …

$$\mathcal{C}_N(\tau,\bar{\tau}) = \sum_{(m,n)\in\mathbb{Z}^2} \int_0^\infty e^{-tY_{m,n}(\tau,\bar{\tau})} B_N(t)\, dt$$

with $Y_{m,n}(\tau,\bar{\tau}) := \pi\frac{|m+n\tau|^2}{\tau_2}$; the formula is manifestly SL(2, Z) invariant

$$T:\tau\to\tau+1 \quad \Leftrightarrow \quad m\to m+n,\, n\to n\,,$$
$$S:\tau\to -1/\tau \quad \Leftrightarrow \quad m\to -n,\, n\to m\,.$$

Exact results of integrated correlators

Writing $B_N(t)$ as power series leads to SL(2, Z) spectral decomposition

Collier, Permulter; …

$$\mathcal{C}_N(\tau,\bar{\tau}) = \frac{1}{2}\mathcal{C} + \sum_{s=2}^{\infty} c_N(s)\, E(s;\tau)$$

$$= \mathcal{C} + \int_{\text{Re } s=1/2} \frac{ds}{2\pi \text{i}} \frac{\pi(-1)^s}{\sin(\pi s)} c_N(s)\, E(s;\tau) + \sum_n c_n \Phi_n(\tau)$$

no Maass cusp forms

Non-holomorphic Eisenstein series (with perturbative & instantons)

$$E(s;\tau) = \frac{1}{\pi^s} \sum_{(m,n)\neq(0,0)} \frac{\tau_2^s}{|m+n\tau|^{2s}} = \frac{1}{\Gamma(s)} \int_0^\infty e^{-tY_{m,n}}\, t^{s-1} dt$$

Giant graviton integrated correlator

We propose the giant graviton integrated correlator for any N and any τ

$$\mathcal{C}_{\mathcal{D}}(\tau;N) = \mathcal{C} + \int \frac{ds}{2\pi \mathrm{i}}\, g_N(s)\,(2s-1)^2\, E^*(s;\tau)$$

where $\mathcal{C} = \lim_{s\to 1} g_N(s)$

Crucially, $g_N(s)$ can be completely determined by perturbative contributions

$$\mathcal{C}_{\mathcal{D}}(\tau;N)|_{\text{pert}} = \frac{3N\,\zeta(3)}{2\pi^2}\lambda - \left[\frac{3N(N+1)^2+2}{N(N+1)} + \frac{(N^2-1)(N+2)}{N(1-(-N)^{N+1})}\right]\frac{15\zeta(5)}{32\pi^4}\lambda^2 + \left[\frac{17N^5+64N^4+90N^3+68N^2+43N+6}{N^2(N+1)^2} + \frac{(N-1)(N+2)\left(2N^3+10N^2+17N+3\right)}{N^2(1-(-N)^{N+1})}\right]\frac{35\zeta(7)}{512\pi^6}\lambda^3 + \ldots$$

Exact solution at large N

The large-N structure for the SL(2, Z) spectral overlap

$$\frac{1}{1-(-N)^{N+1}}$$

$$g_N(s) = g_N^{(1)}(s) + g_N^{(2)}(s)$$

The exact solution in the large-N expansion (fixed τ)

$$g_N^{(1)}(s) = f_N(s) + h_N(s)$$

where

$$f_N(s) = \frac{N\pi}{\sin(\pi s)}\, {}_3F_2(1-N, 1-s, s; 2, 2; 1)\,, \quad h_N(s) = \frac{\pi\, {}_2F_1\left(-N-1, -s; -N-s; -\frac{1}{N}\right)(s+1)_N}{\sin(\pi s)\,(1-2s)\,N!\left(1+\frac{1}{N}\right)^s}$$

Exact solution at large N

After performing the spectral integral:

$$\frac{1}{1-(-N)^{N+1}}$$

$$\mathcal{C}_{\mathcal{D}}(\tau;N) = \mathcal{C} - E(1;\tau) + \sum_{k=0}^{\infty} \frac{1}{N^{k+1/2}} \sum_{\ell=0}^{k} c_{k,\ell}\, E(\ell+3/2;\tau) + O\left(\exp\left(-4\sqrt{\pi N}\frac{|p+q\tau|}{\tau_2}\right)\right) + \ldots$$

Supergravity

Stringy α' -expansion

world-sheet instanton of (p, q)-string

Recall $1/\sqrt{N} \sim \alpha'$ (with fixed YM coupling)

e.g.

$$c_{0,0} = -\frac{1}{2},\ c_{1,0} = \frac{1}{2^3},\ c_{1,1} = -\frac{3}{2^5} \qquad c_{2,0} = -\frac{25}{2^{10}},\ c_{2,1} = \frac{9}{2^7},\ c_{2,2} = -\frac{405}{2^{12}}$$

Exact solution at large N

Read off large-N ’t Hooft expansion to all orders (first two orders agree with known results Brown, Galvagno, C.W.)

$$\mathcal{C}(\lambda;N)|_N \sim 2-\frac{4\pi^2}{3\lambda}-\sum_{n=1}^{\infty}\frac{16n^2\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)^2\Gamma\left(n+\frac{1}{2}\right)}{\lambda^{n+\frac{1}{2}}\,\pi^{3/2}\,\Gamma(n+1)}\,,$$

$$\mathcal{C}(\lambda;N)|_{N^0} \sim -2\gamma-2-\log\left(\frac{\lambda}{16\pi^2}\right)+\sum_{n=1}^{\infty}\frac{8n^2\zeta(2n+1)\Gamma\left(n-\frac{1}{2}\right)\Gamma\left(n+\frac{1}{2}\right)^2}{\lambda^{n+\frac{1}{2}}\,\pi^{3/2}\,\Gamma(n+1)}\,.$$

$$\mathcal{C}(\lambda;N)|_{N^{-1}} \sim -\frac{\sqrt{\lambda}}{6}+\sum_{n=1}^{\infty}\frac{n^2(2n-7)\Gamma\left(n-\frac{1}{2}\right)\Gamma\left(n+\frac{1}{2}\right)\Gamma\left(n+\frac{5}{2}\right)\zeta(2n+1)}{6\pi^{3/2}\Gamma(n+2)\lambda^{n+\frac{1}{2}}}\,,$$

$$\mathcal{C}(\lambda;N)|_{N^{-2}} \sim \frac{\sqrt{\lambda}}{24}-\sum_{n=1}^{\infty}\frac{n^2\Gamma\left(n+\frac{1}{2}\right)^2\Gamma\left(n+\frac{7}{2}\right)\zeta(2n+1)}{6\pi^{3/2}\Gamma(n+2)\lambda^{n+\frac{1}{2}}}\,.$$

We’ve omitted exponentially decayed terms $O(\exp(-\sqrt{\lambda}))$, related to the world-sheet instantons

Comments

We do not have a rigorous proof for the proposed exact result.

- Is consistent with all known results.
- Has been verified to very high loops for generic N, and to one-instanton for some fixed N’s.
- A possible proof is through some recursion relations, as was done for $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2 \rangle$ Dorigoni, Green, CW, Xie

More comments

- For the U(N) theory: no $O(N^{-N})$-like term, and we can obtain the exact result for generic N.
- The results provide constraints on the correlator; e.g. one & two loops for generic N are determined by combining with OPE.
- Generic sub-determinant operators ($\alpha < 1$), and dual giant gravitons?

AMPLITUDES 2026

Local Organisers: *Andreas Brandhuber, Ricardo Monteiro, Mary Thomas, Gabriele Travaglini, Congkao Wen, Chris White*

Amplitudes 2026 will be held at **Queen Mary University of London** (QMUL) from **June 29 to July 3, 2026**. This marks the 18th installment of the annual conference series that gathers researchers interested in both the formal and practical aspects of scattering amplitudes. The topics discussed span a broad spectrum, ranging from pure mathematics to collider physics and gravitational waves.

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