Review of MC generators

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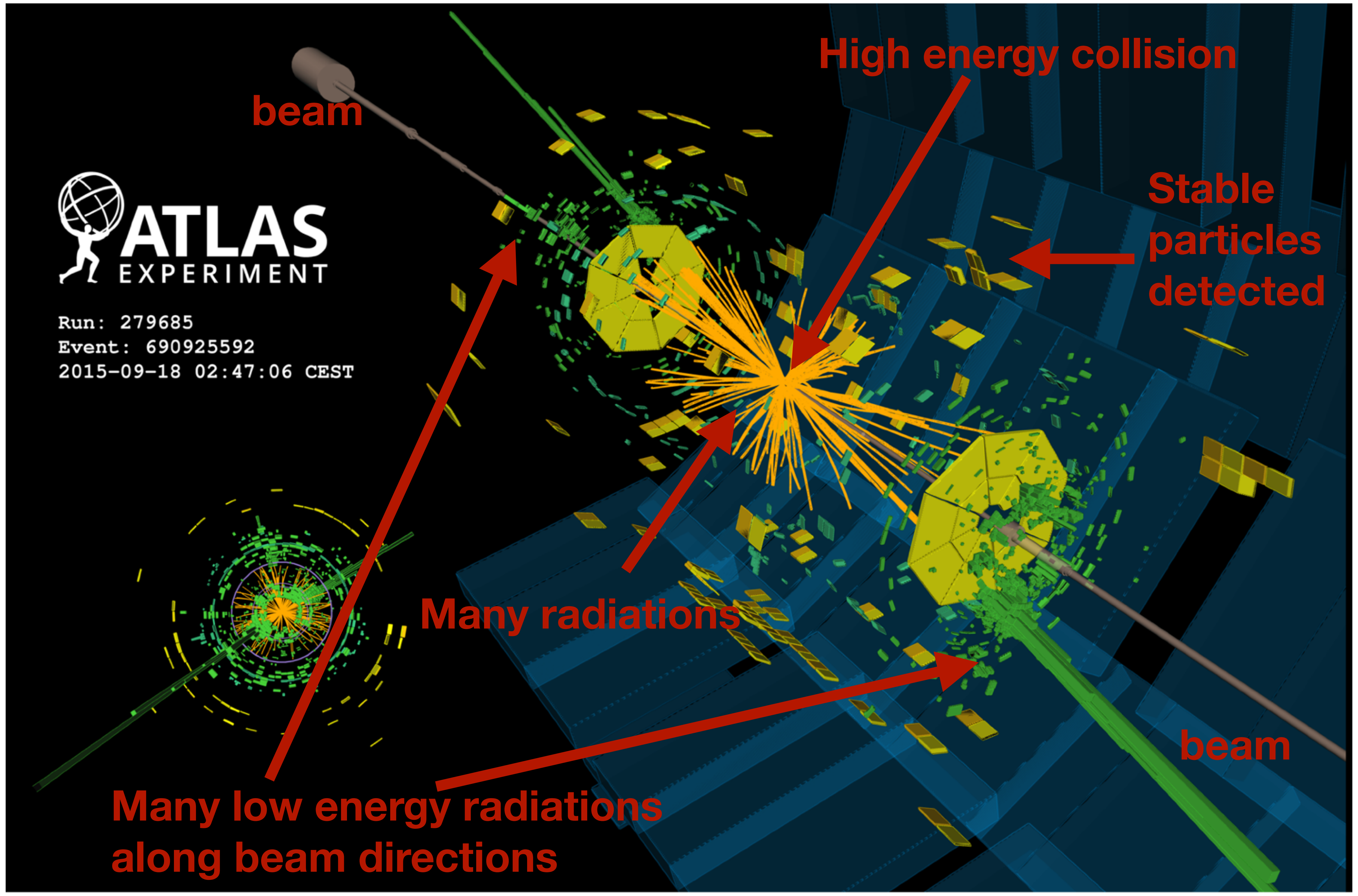
TeV物理前沿专题研讨会暨第31届LHC Mini-Workshop

2025-10-11

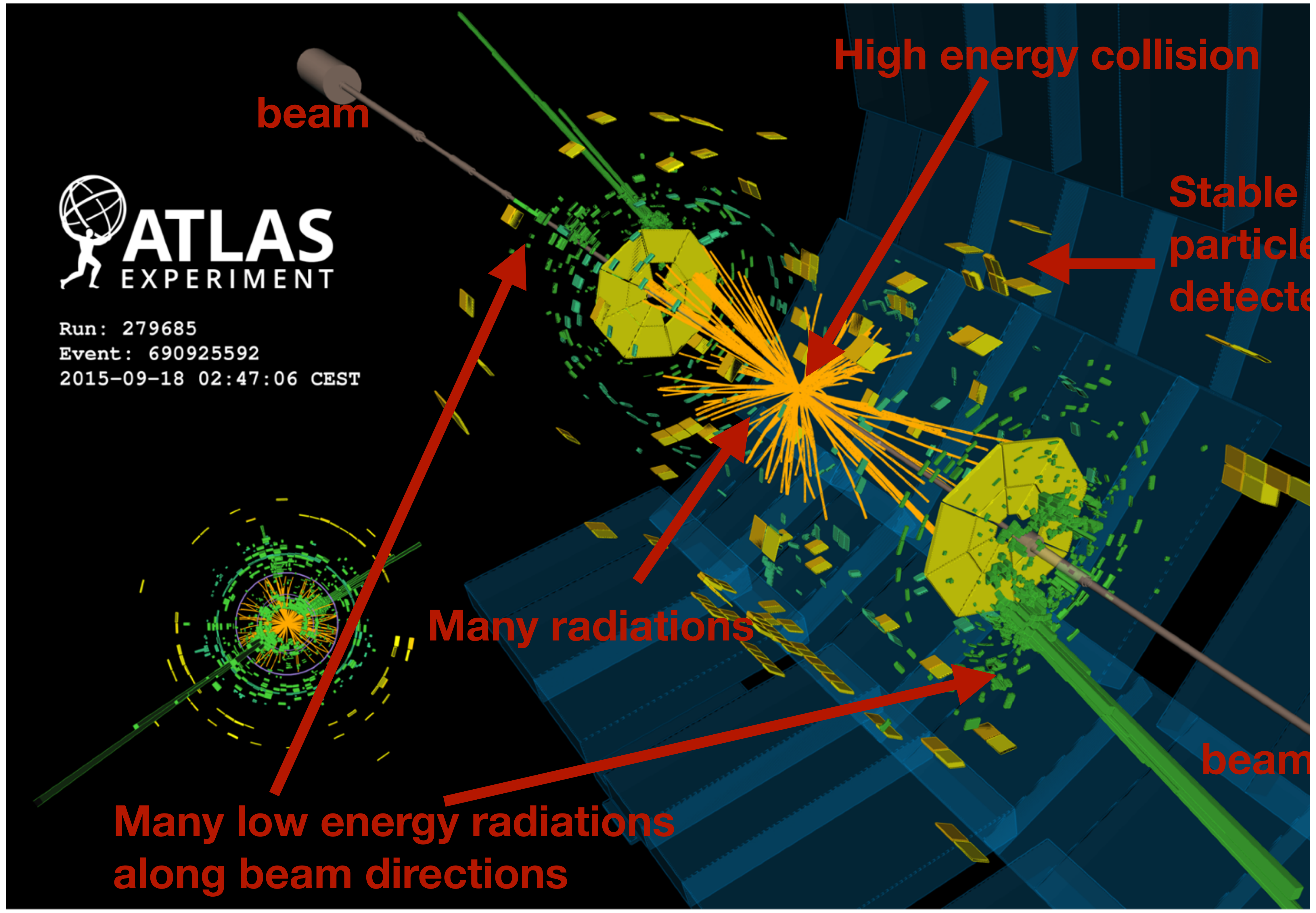
Outline

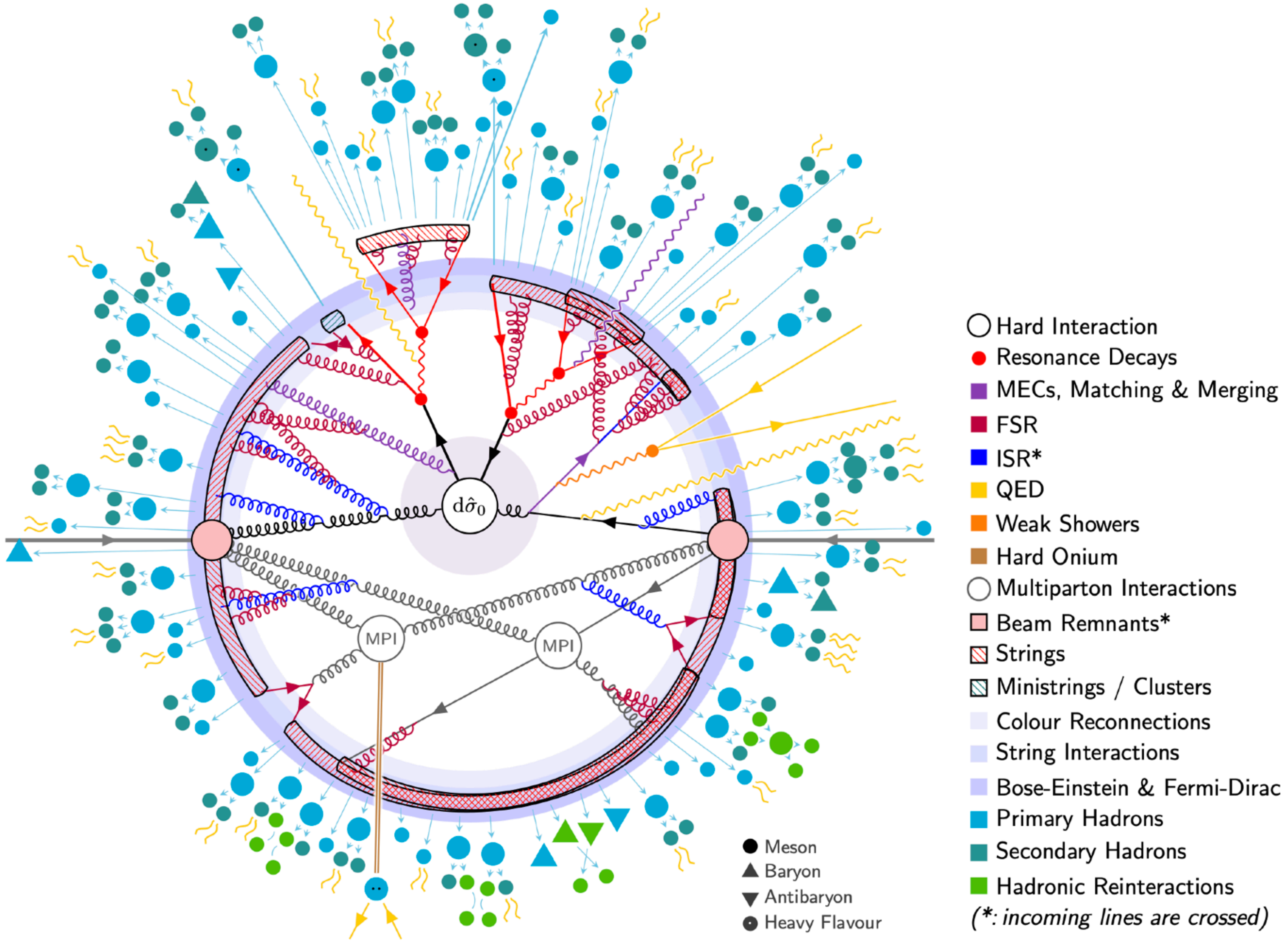
1. Introduction to MCEGs
2. Parton Showers
3. Hadronization models
4. Summary

1. Introduction to MCEGs



1. Introduction to MCEGs





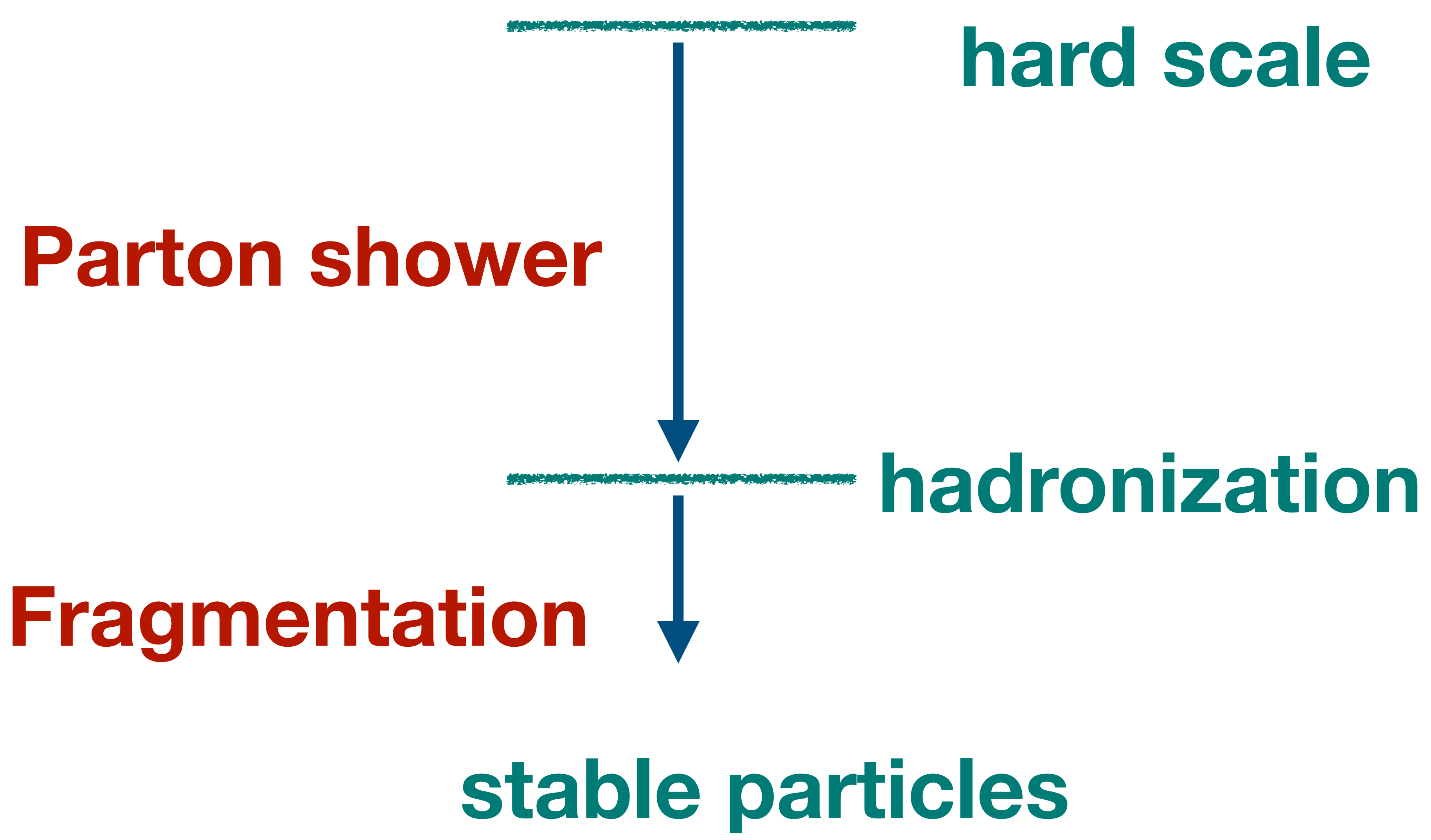
From PYTHIA 8.3

1. Introduction to MCEGs

The purpose of Monte Carlo event generators is to generate events in as much details as nature (generate average and fluctuation right)

$$\mathscr{P}_{\text{event}} = \mathscr{P}_{\text{Hard}} \otimes \mathscr{P}_{\text{Decay}} \otimes \mathscr{P}_{\text{ISR}} \otimes \mathscr{P}_{\text{FSR}} \otimes \mathscr{P}_{\text{MPI}} \otimes \mathscr{P}_{\text{Had}} \cdots$$

- ❑ **Hard process in high energy**
- ❑ **Transition from high energy to low energy**
  —parton shower
- ❑ **Low energy soft regime**
  —fragmentation

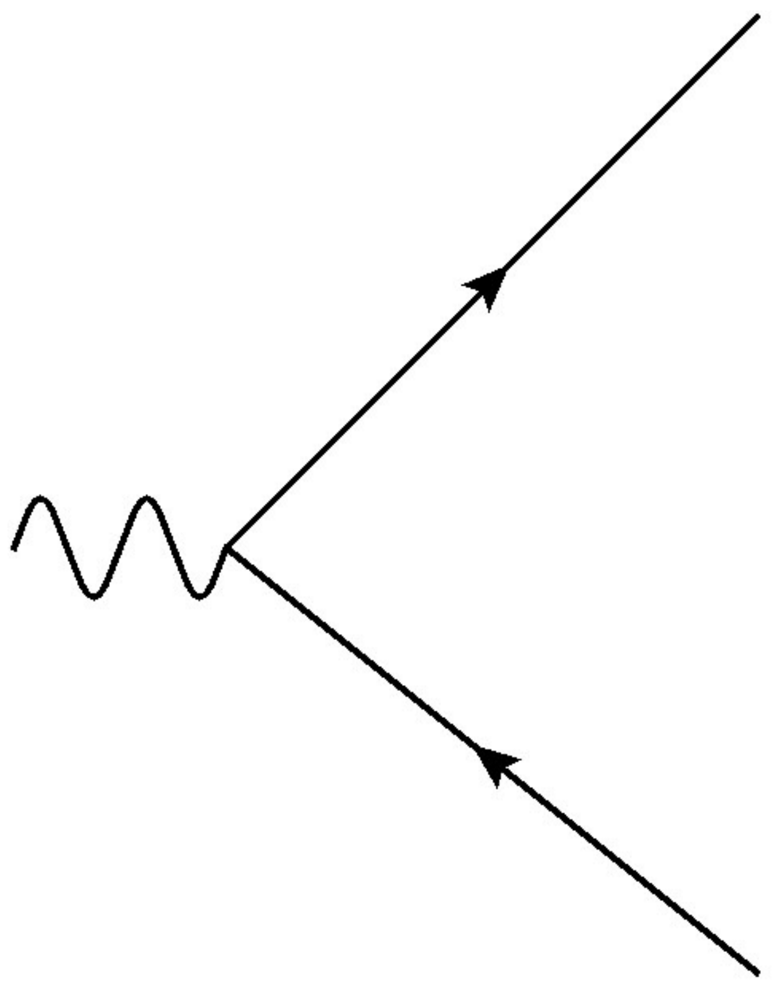


Parton shower: a model for the evolution from high scale to hadronization scale

2. Parton Showers

Parton showers approximate higher-order real-emission corrections to the hard scattering process

- Generate cascades of radiation automatically
- Locally conserved four momentum
- Locally conserved flavor
- Unitarity by construction



Parton showers

- sample infrared configurations
- simulate the evolution of parton (resummation)

Parton shower indispensable tools for particle physics phenomenology

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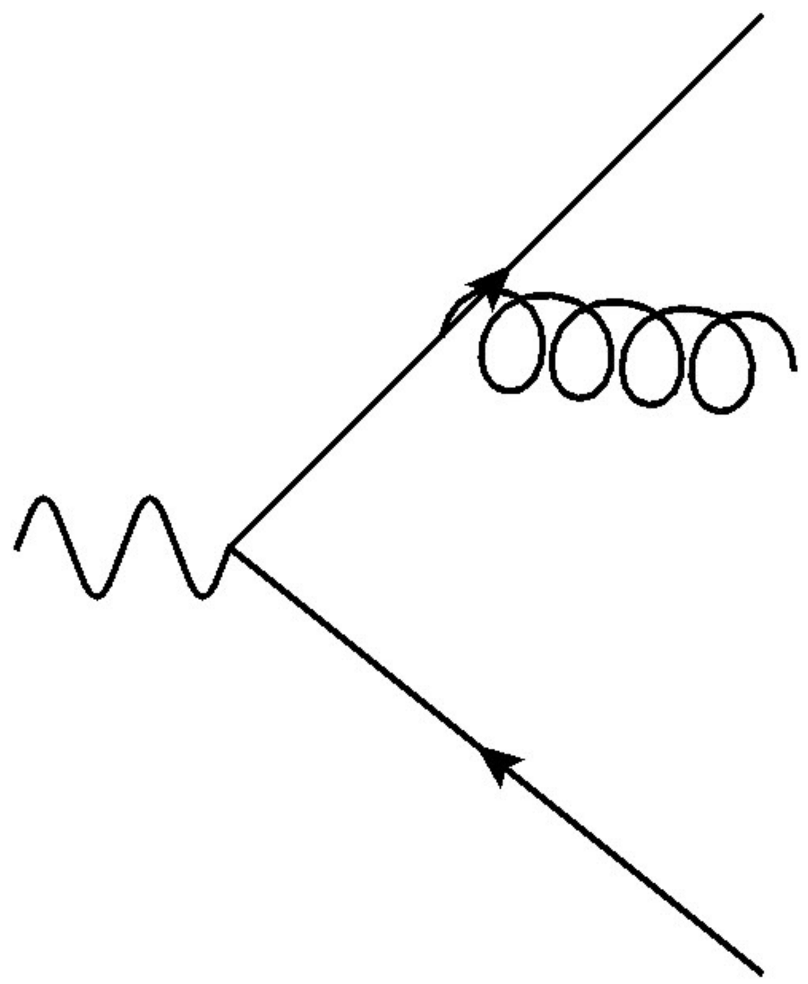

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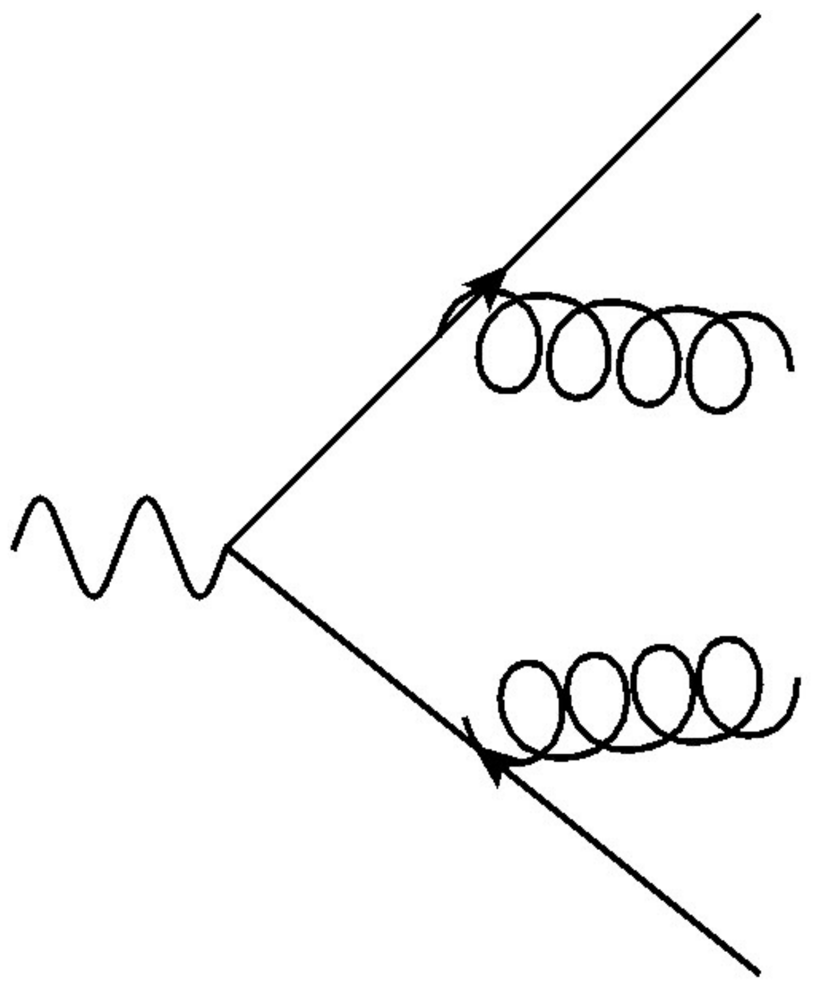

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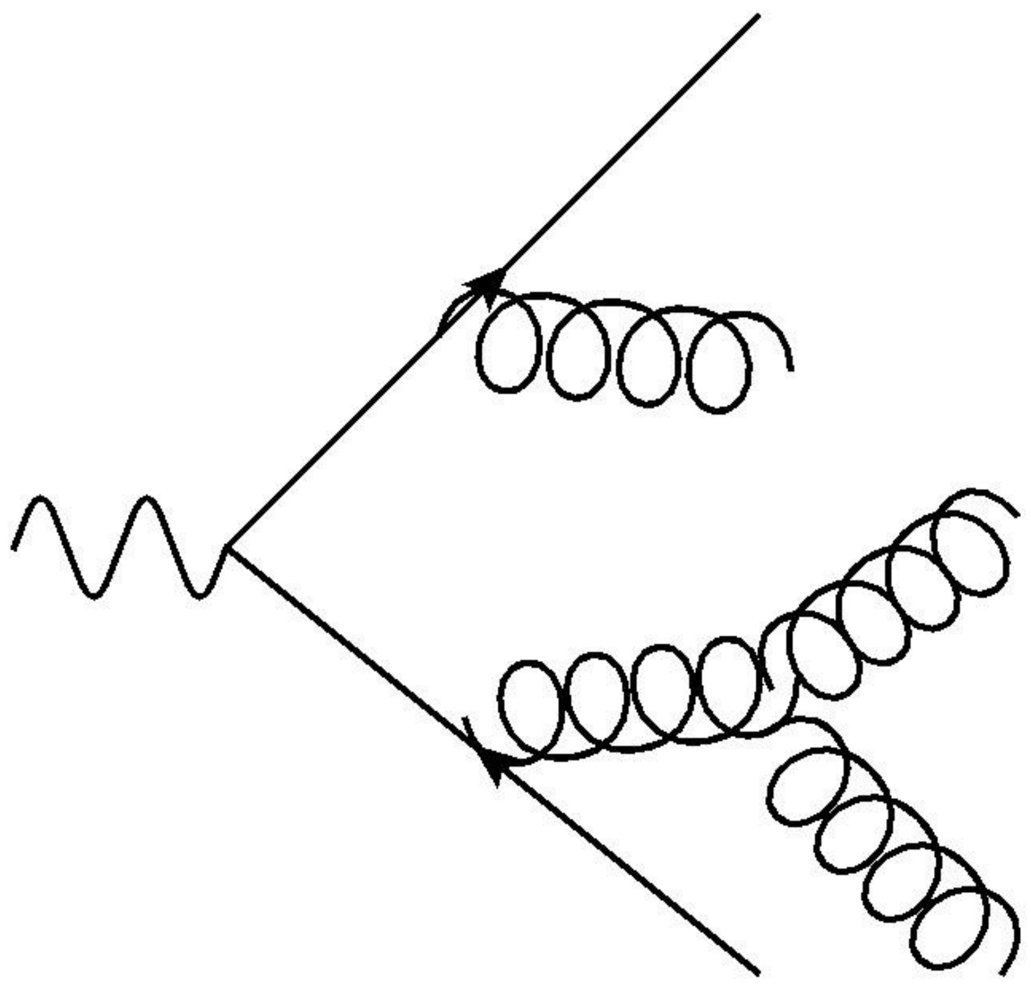

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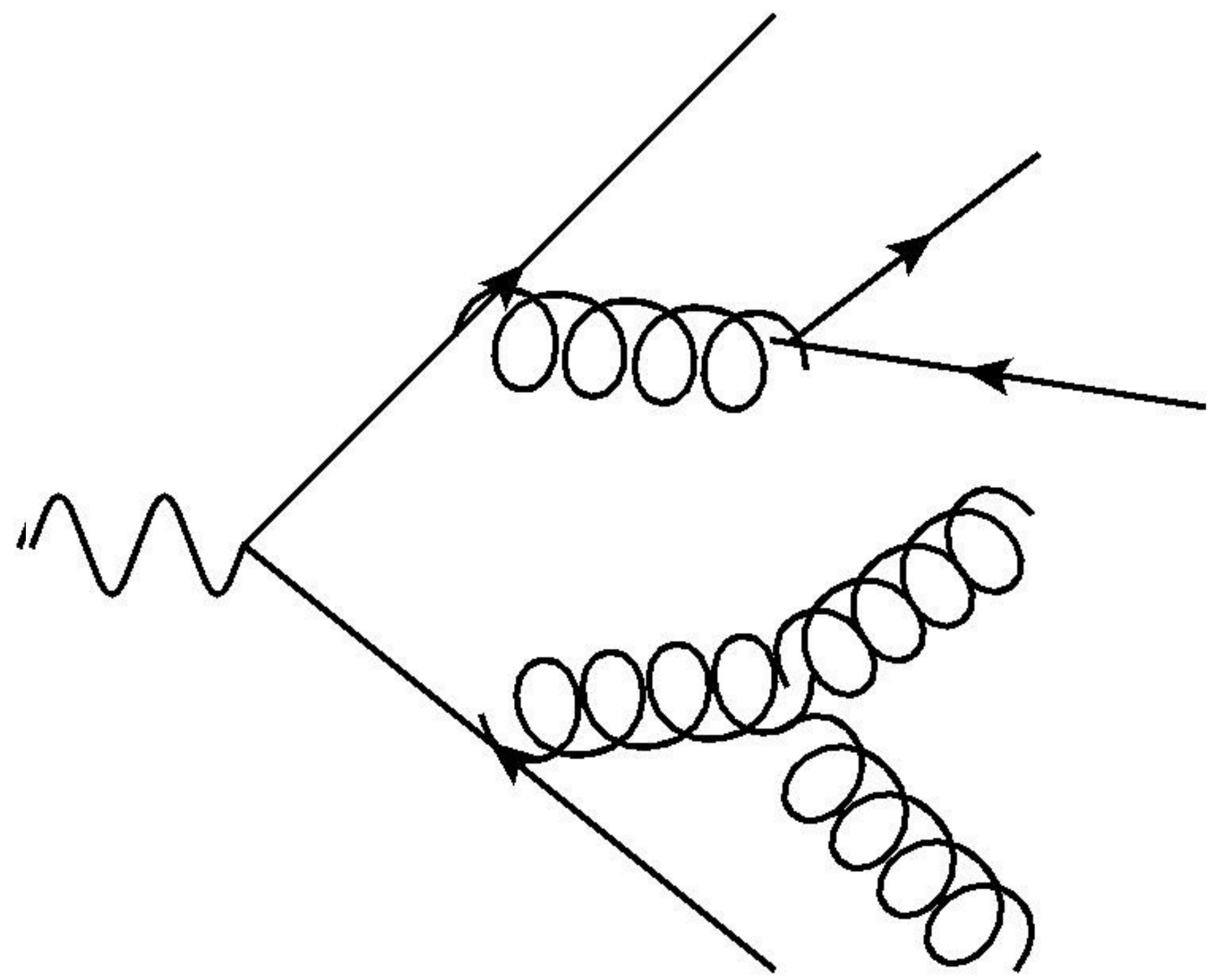

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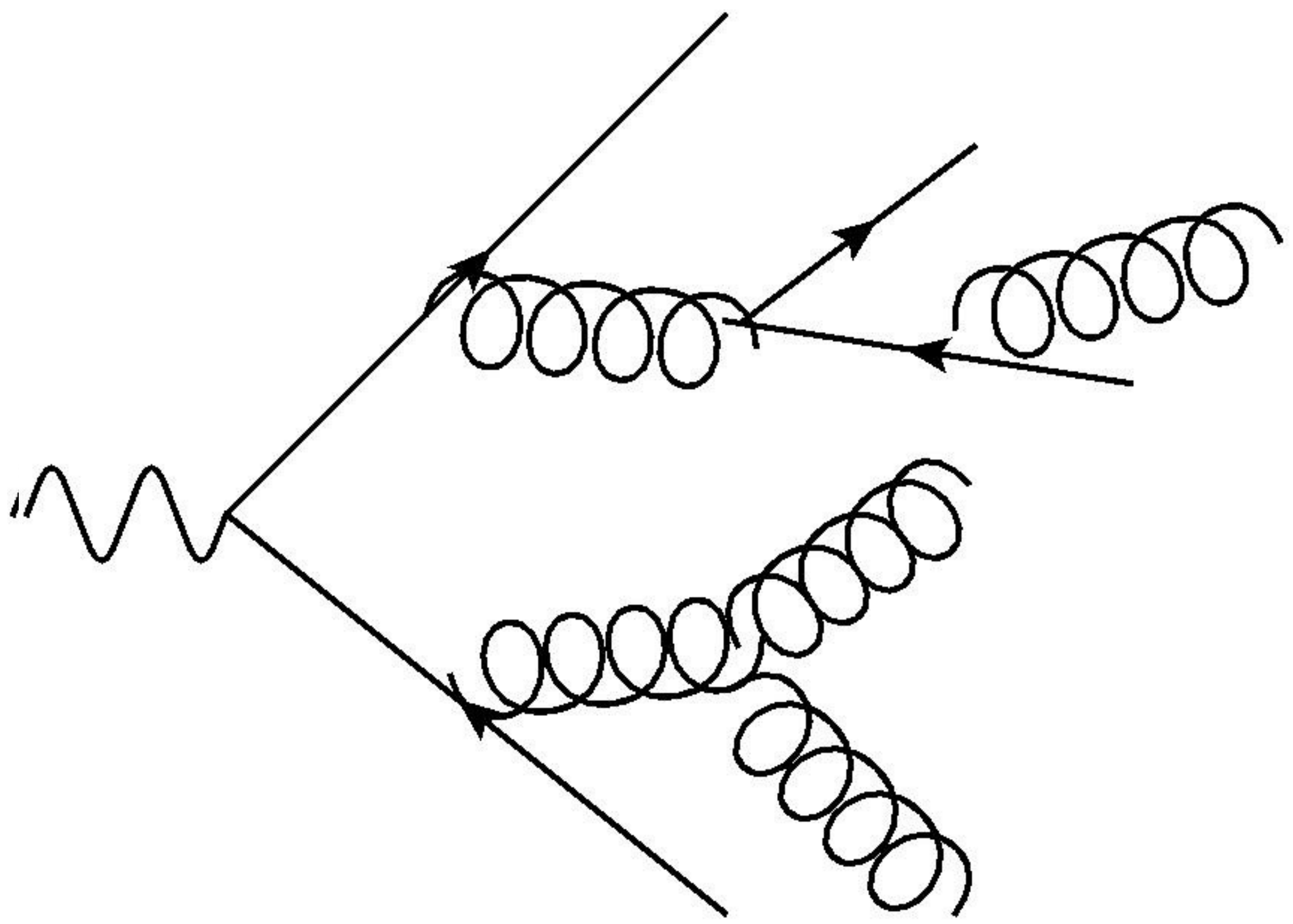

Parton showers

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2. Parton Showers

Sudakov form factor: Non-branching probability $\exp\left[\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right]$

Probability that there is no branching from Q to q is $\Delta_i\left(Q^2,q^2\right)$

choose kinematic variable as the evolution scale

$$\Delta\left(Q^2,q^2\right)=\exp\left\{\int_{Q^2}^{q^2} d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right\}$$

Probability for one observed branching $1-\Delta\left(Q^2,q^2\right)$

Probability one branching between the scale q^2 to q^2+dq^2

$$\frac{d}{dq^2}\Delta\left(Q^2,q^2\right)=\Delta\left(Q^2,q^2\right)\times d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}$$

Additional radiations can be added according to the function $\Delta\left(Q^2,q^2\right)$

2. Parton Showers

Phase space mapping $\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2} = \int_{q^2}^{Q^2}\frac{\mathrm{d}k^2}{k^2}\frac{\alpha_s}{2\pi}\int_{Q_0^2/k^2}^{1-Q_0^2/k^2}\mathrm{d}zP_{ji}(z)$

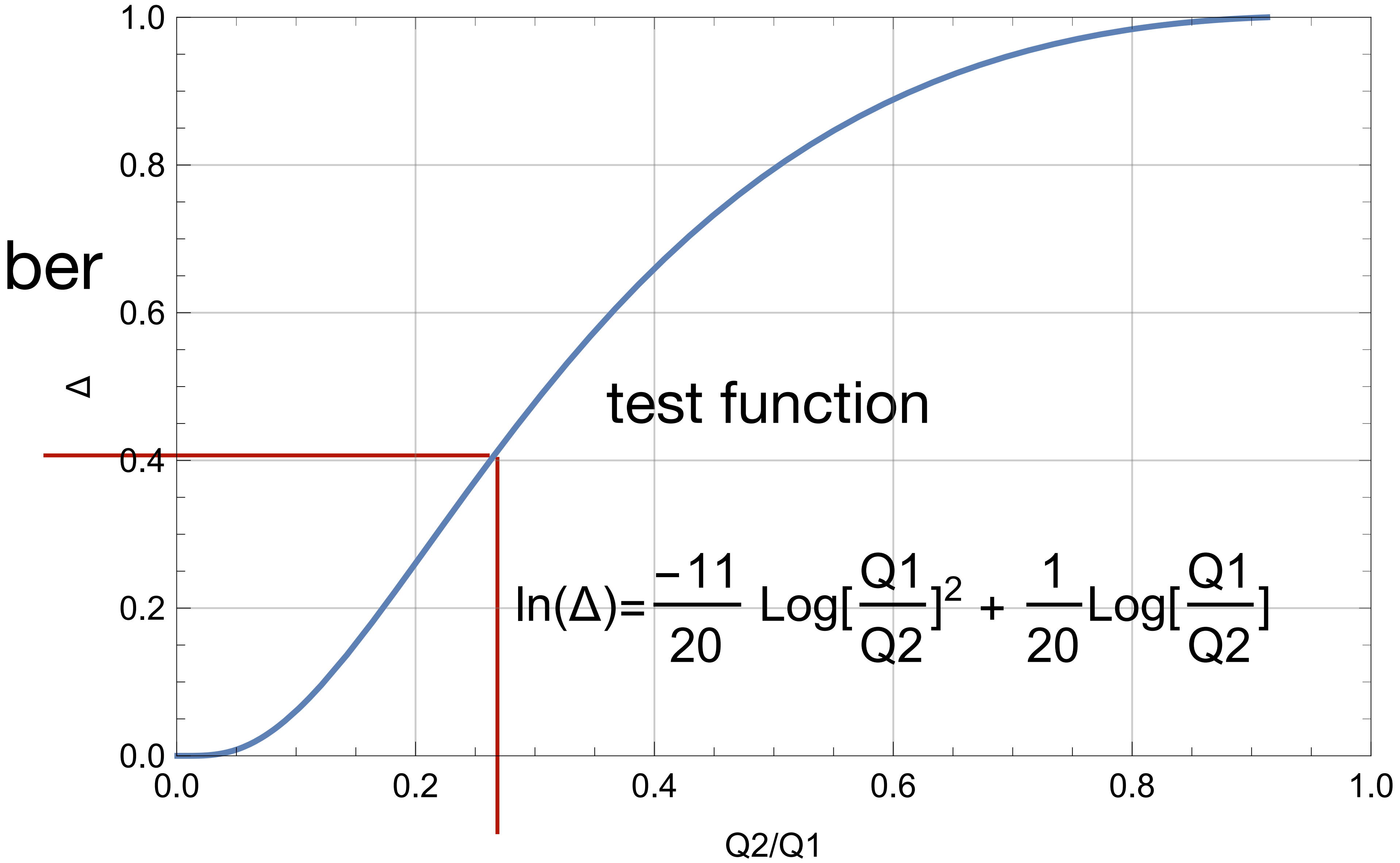
$$\frac{d\theta^2}{\theta^2}=\frac{dq^2}{q^2}=\frac{dk_\perp^2}{k_\perp^2}$$

many choices for the evolution variables

Monte-Carlo Technique and resummation

Sudakov factor $\Delta\left(Q_1^2,Q_2^2\right)$

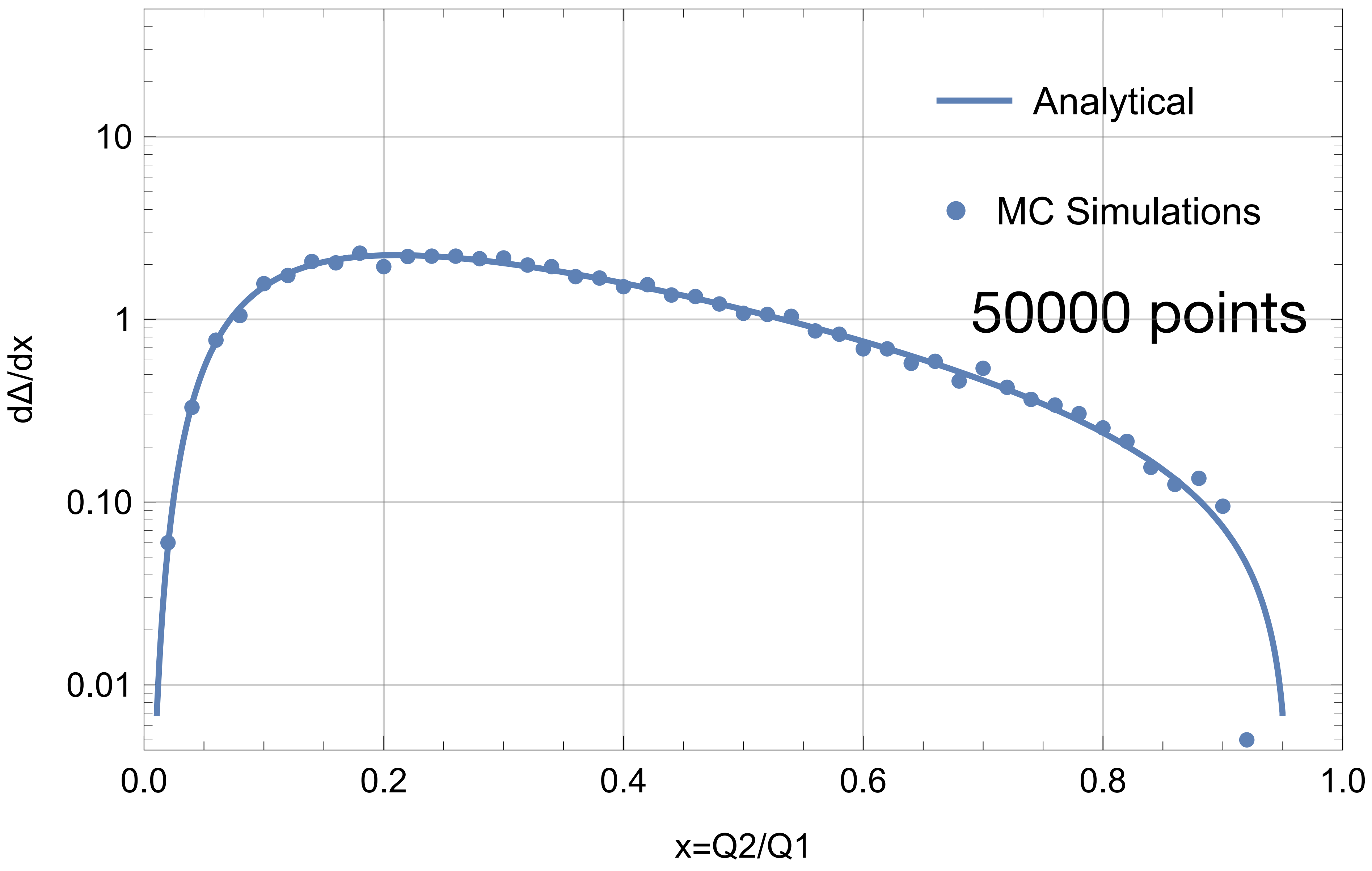
Generate random number $R\in(0,1)$



Solve $R=\Delta$ for Q1/Q2

Q_2/Q_1 distribution generated by

$$\frac{d}{dq^2}\Delta\left(Q^2,q^2\right)=\Delta\left(Q^2,q^2\right)\times d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}$$



new phase space point generated according to the new scales

2. Parton Showers

For multi-scale problem

固定阶计算

$$
\begin{array}{lllllll}
 & & \textbf{NLO} & \textbf{NNLO} & \textbf{NNNLO} & & \\
z\frac{1}{\sigma_0}\frac{d\sigma}{dz} \propto & z + & \frac{\alpha_s}{4\pi}a_{1,2}\ln z + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,4}\ln^3 z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,6}\ln^5 z & +\cdots & \textbf{LL} \\
 & + & \frac{\alpha_s}{4\pi}a_{1,1} + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,3}\ln^2 z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,5}\ln^4 z & +\cdots & \textbf{NLL} \\
 & + & \frac{\alpha_s}{4\pi}a_{1,0}z + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,2}\ln z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,4}\ln^3 z & +\cdots & \textbf{NNLL} \\
 & & + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,1} + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,3}\ln^2 z & +\cdots & \textbf{NNNLL} \\
 & & + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,0}z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}\ln z & +\cdots & \textbf{NNNNLL} \\
 & & & + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2} & +\cdots & \\
 & & & + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}z & +\cdots &
\end{array}
$$

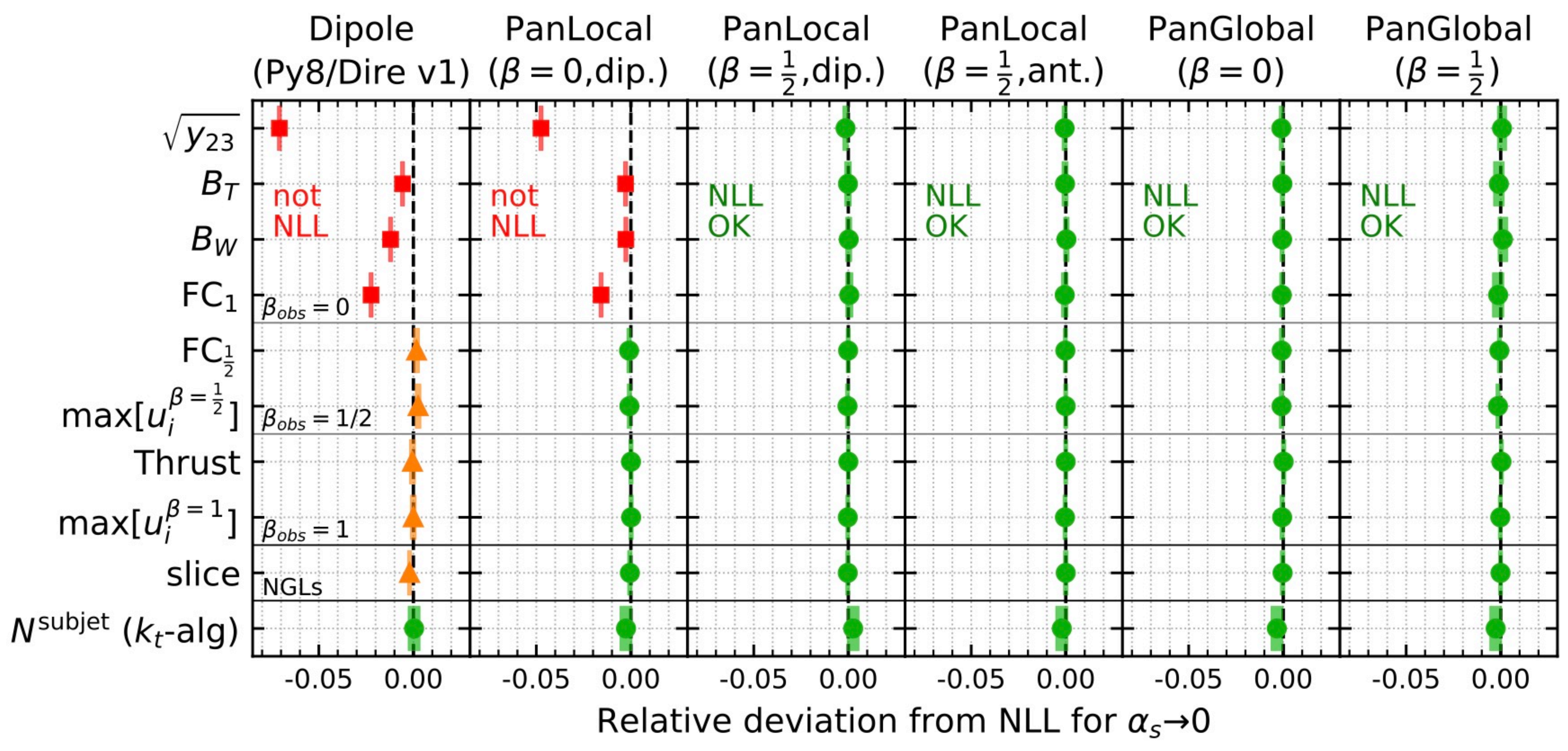
重求和

For observables that involve scale hierarchies resummation is required

NLL: PanScales, Alaric, Herwig et al

with higher order effects: Vincia, DIRE et al

PanScales:



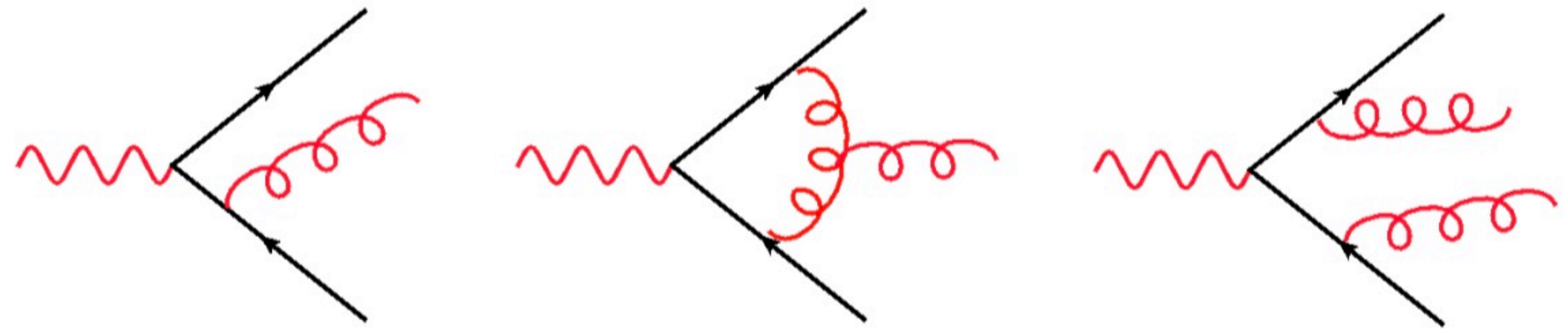
arXiv:2002.11114

2. Parton Showers

To which order can Parton Showers do?

NLO corrections to resummation kernel

What we expect for NLO showers

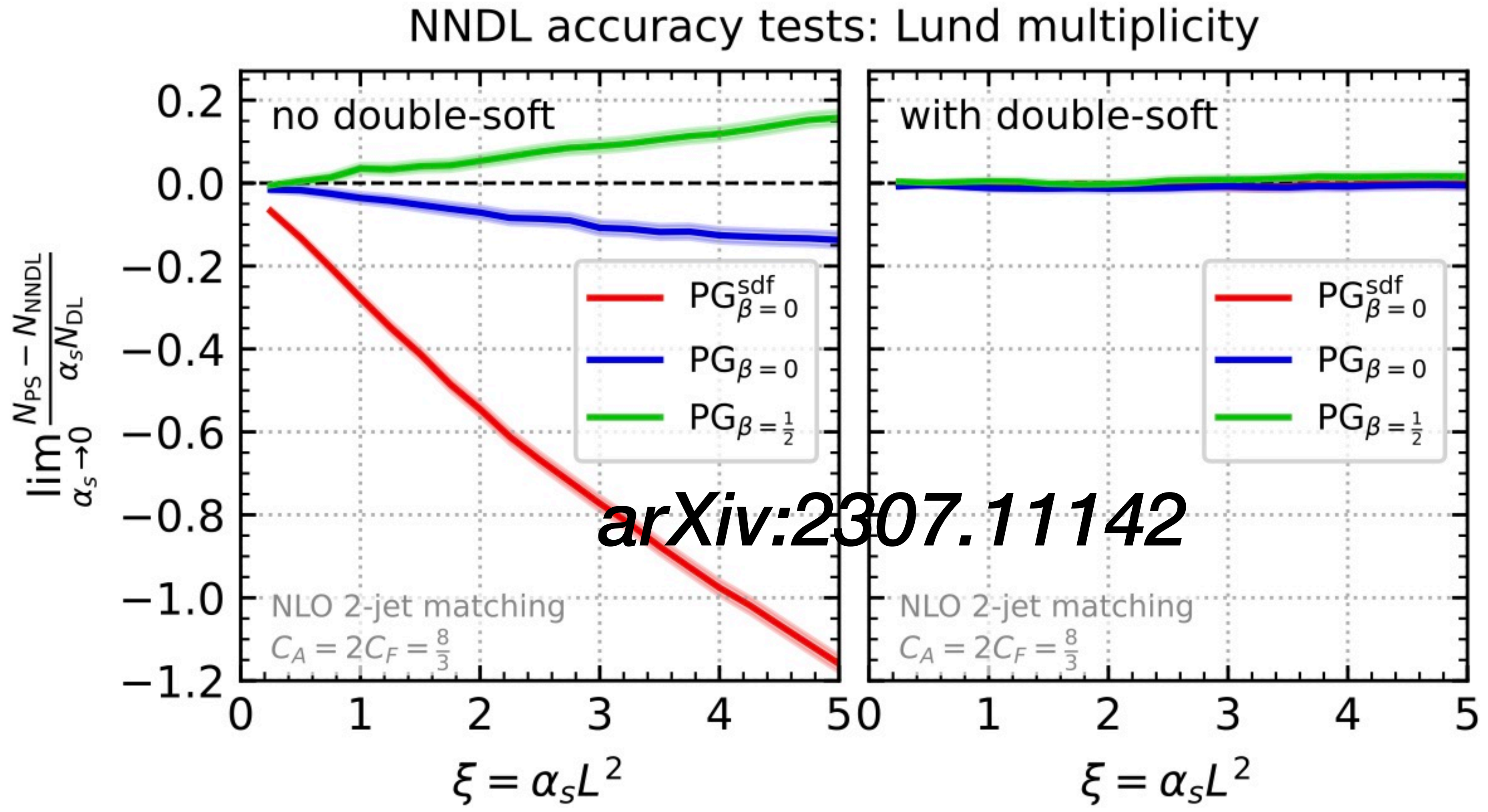


NLO parton shower

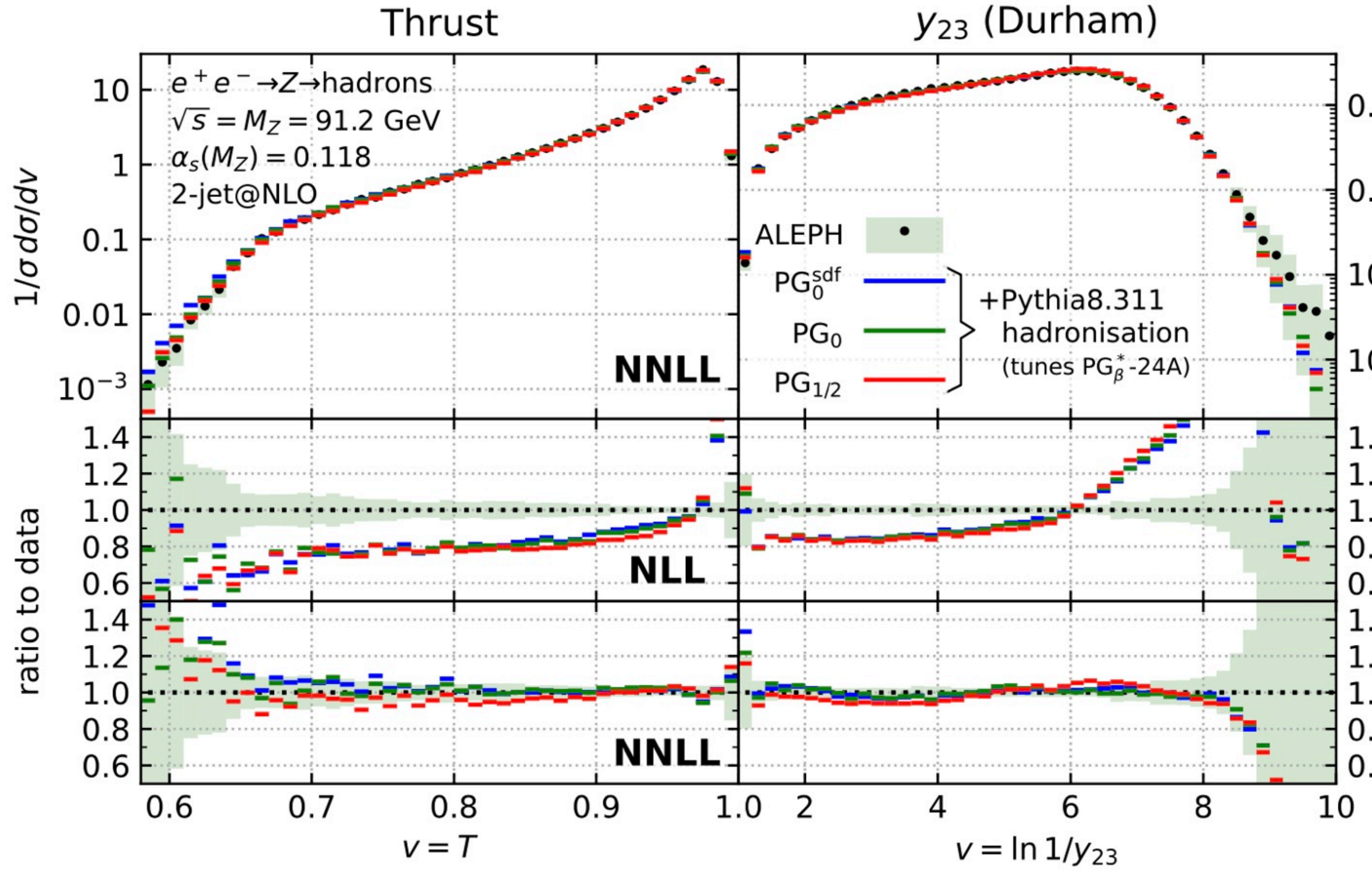
$$\frac{d}{dQ^2}\underbrace{\left(1-\Delta\left(Q_0^2,Q^2\right)\right)}_{\text{branching probability}} = -\underbrace{\int \frac{d\Phi_3}{d\Phi_2}\delta\left(Q^2-Q^2\left(\Phi_3\right)\right)\left(a_3^0+a_3^1\right)\Delta\left(Q_0^2,Q^2\right)}_{\text{born and virtual correction}}$$

$$-\underbrace{\int \frac{d\Phi_4}{d\Phi_2}\delta\left(Q^2-Q^2\left(\Phi_4\right)\right)a_4^0\,\Delta\left(Q_0^2,Q^2\right)}_{\text{real correction}}$$

HTL, Skands, arXiv:1611.00013



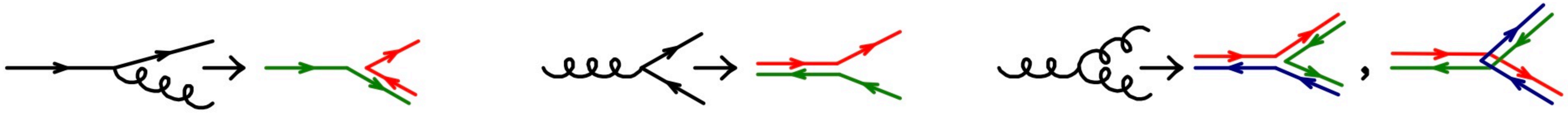
arXiv:2307.11142



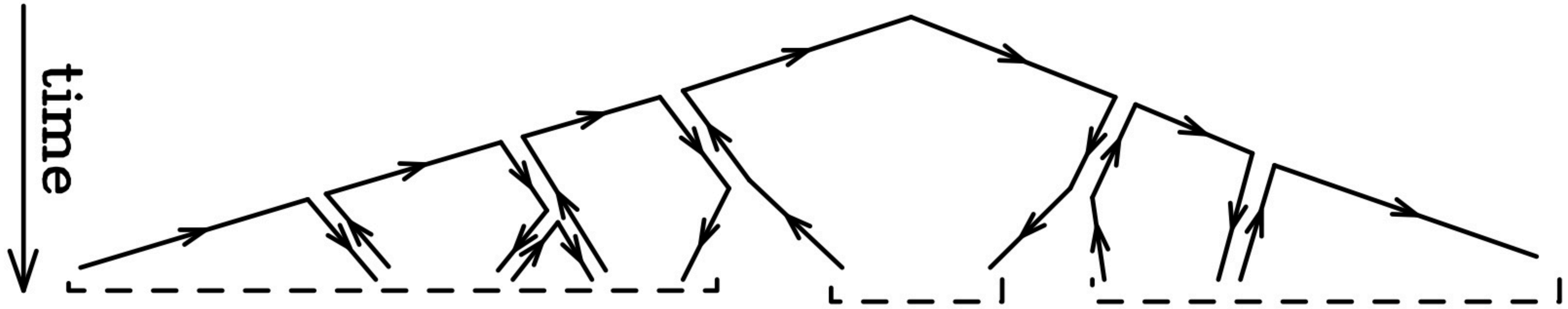
arXiv:2406.02661

2. Parton Showers

Leading Color Approximation: Dipole Shower

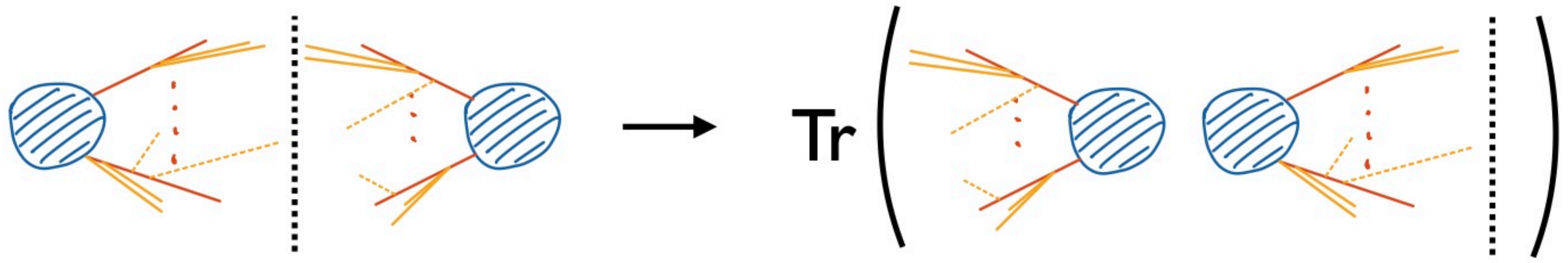


QCD radiation in this approximation is always simulated as the radiation from a single color dipole, rather than a coherent sum from a color multipole.



a color density operator   *Deductor, arXiv:1902.02105*

simulates parton showers at the amplitude level with full color information   *CVolver, arXiv:2502.12133*

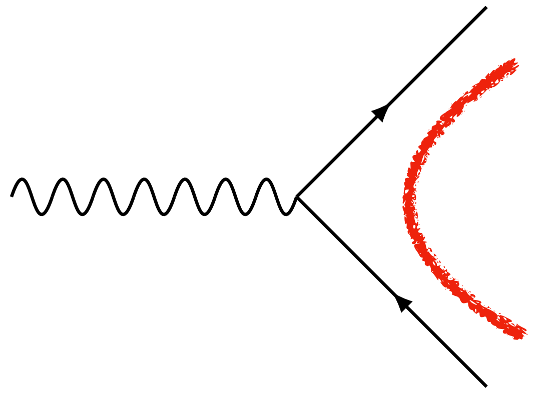
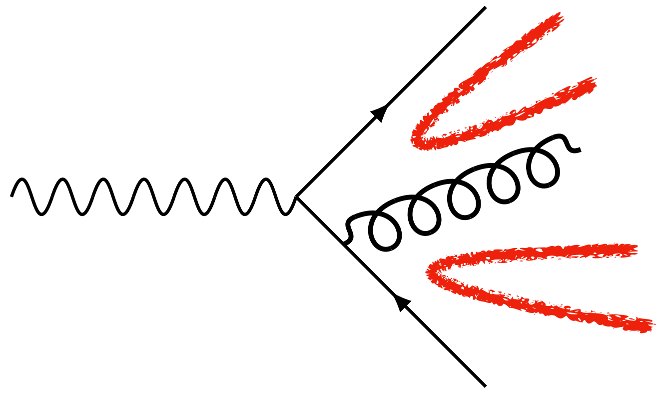


$$\mathbf{A}_n(E) = \mathbf{V}_{E,E_n}\mathbf{D}_n^{\mu}\mathbf{A}_{n-1}(E_n)\mathbf{D}_{n\mu}^{\dagger}\mathbf{V}_{E,E_n}^{\dagger}\Theta(E \le E_n),$$

2. Parton Showers

LO parton shower

From parton shower

$1 - \Delta_i(Q^2, Q_0^2)$

$$\sigma^{\text{PS}}_{\text{NLO}} = \sigma_0 \Pi_i \left(\boxed{\Delta_i(Q^2, Q_0^2)} + \boxed{\int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \Delta_i(Q^2, q^2) \int dz P_{ji}(z)} \right)$$

0-radiation **1-radiation (Sudakov suppressed)**

From the definition of Sudakov factor, we have

$$\mathscr{P}(\text{unresolved}) + \mathscr{P}(\text{resolved}) = 1$$

probability conservation from the definition of Δ

From NLO calculations

$$\sigma_{\text{NLO}} = \sigma_0 + \left(\int d\Phi_n V + \int d\Phi_{n+1} S \right) \mathscr{O}_n + \int d\Phi_{n+1} (R \mathscr{O}_{n+1} - S \mathscr{O}_n)$$

virtual **integrated subtraction** **subtracted real**

$$\sigma_{\text{NLO}} = \sigma_0^n + \int_0^{t_n} d\sigma^n_{(1)} + \int_{t_n} d\sigma^{n+1}_{(1)}$$

t_n as the resolution scale for 1-radiation

LO parton showers reproduce the NLO singular behavior of the underlying hard process with unitarity assumption

$V + \int R = 0.$

Resummation from Showers + Hard emissions From fixed orders

2. Parton Showers

Sunshine by @vector_corp on freepik.es

Sunshine

Sudakov Nesting of Hard Integrals

Using generalized parton shower to generate fixed order corrections

Fixed order should look like

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\Phi_n} = |M_0|^2 \boxed{\prod_{i=0}^{n-1} \mathrm{ant}_{i\mapsto i+1}}$$

matrix element ratio $(0 \to 1) \times (1 \to 2) \times \cdots \times (n-1 \to n)$

Usually showers will give $(0 \to n)$

$$\frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n} = |M_0|^2 \prod_{i=0}^{n-1} \mathrm{ant}_{i\mapsto i+1} \boxed{\Delta_i(t_i, t_{i+1})}$$

Sudakov factor from showers $\Delta_0 \times \Delta_1 \times \cdots \times \Delta_{(n-1)}$

Altmann, HTL, Scyboz, Skands, arXiv:2507.00111

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keep the parent events after branching, and ask the event branches m times at stage $0 \to 1$, then shower them afterwards

$$\frac{\mathrm{d}\mathcal{P}_{m0\ldots0}}{\mathrm{d}\Phi_n} = \frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n}\prod_{j=1}^{m}\int_{t_1}^{\tilde{t}_{j-1}} \mathrm{ant}_{0\mapsto1}(\tilde{t}_j)\,\mathrm{d}\tilde{t}_j$$

keep all the intermediate states and shower them m_k times from $k-1$ partons to k partons

$$\frac{\mathrm{d}\mathcal{P}_{m_1m_2\ldots m_n}}{\mathrm{d}\Phi_n} = \frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n}\prod_{k=1}^{n}\prod_{j=1}^{m_k}\int_{t_k}^{\tilde{t}_{k_j-1}} \mathrm{ant}_{k-1\mapsto k}(\tilde{t}_{k_j})\,\mathrm{d}\tilde{t}_{k_j}$$

sum m_k to infinity

$$\sum_{m_k\geq0}\frac{\mathrm{d}\mathcal{P}_{m_1m_2\ldots m_n}}{\mathrm{d}\Phi_n} = \frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n}\prod_{k=1}^{n}\frac{1}{\Delta_k(t_{k-1},t_k)}$$

$$\textsc{Sunshine}: \quad \sum_{m_k\geq0}\frac{\mathrm{d}\mathcal{P}_{m_1m_2\ldots m_n}}{\mathrm{d}\Phi_n} = |M_0|^2\prod_{i=0}^{n-1}\mathrm{ant}_{i\mapsto i+1}\,.$$

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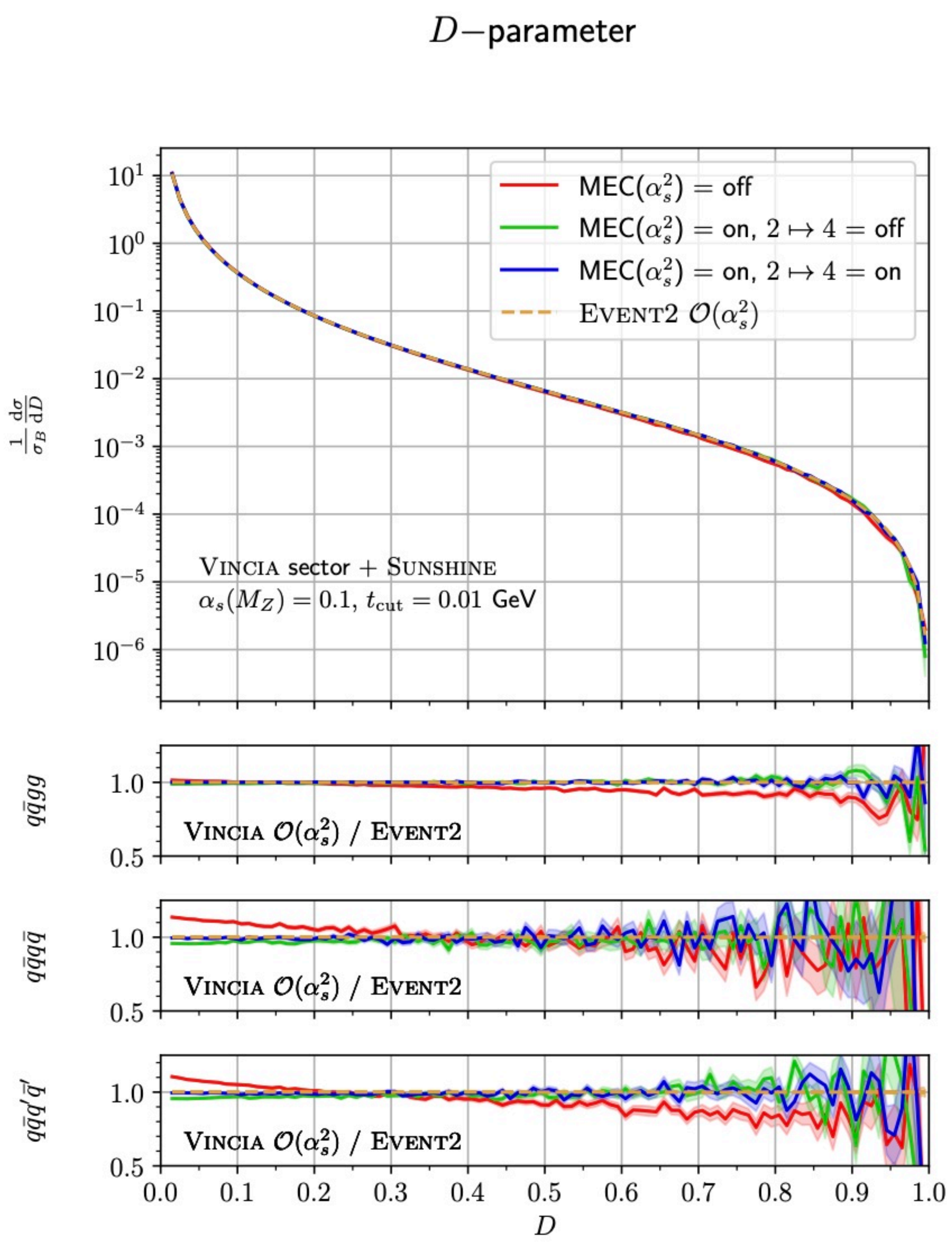
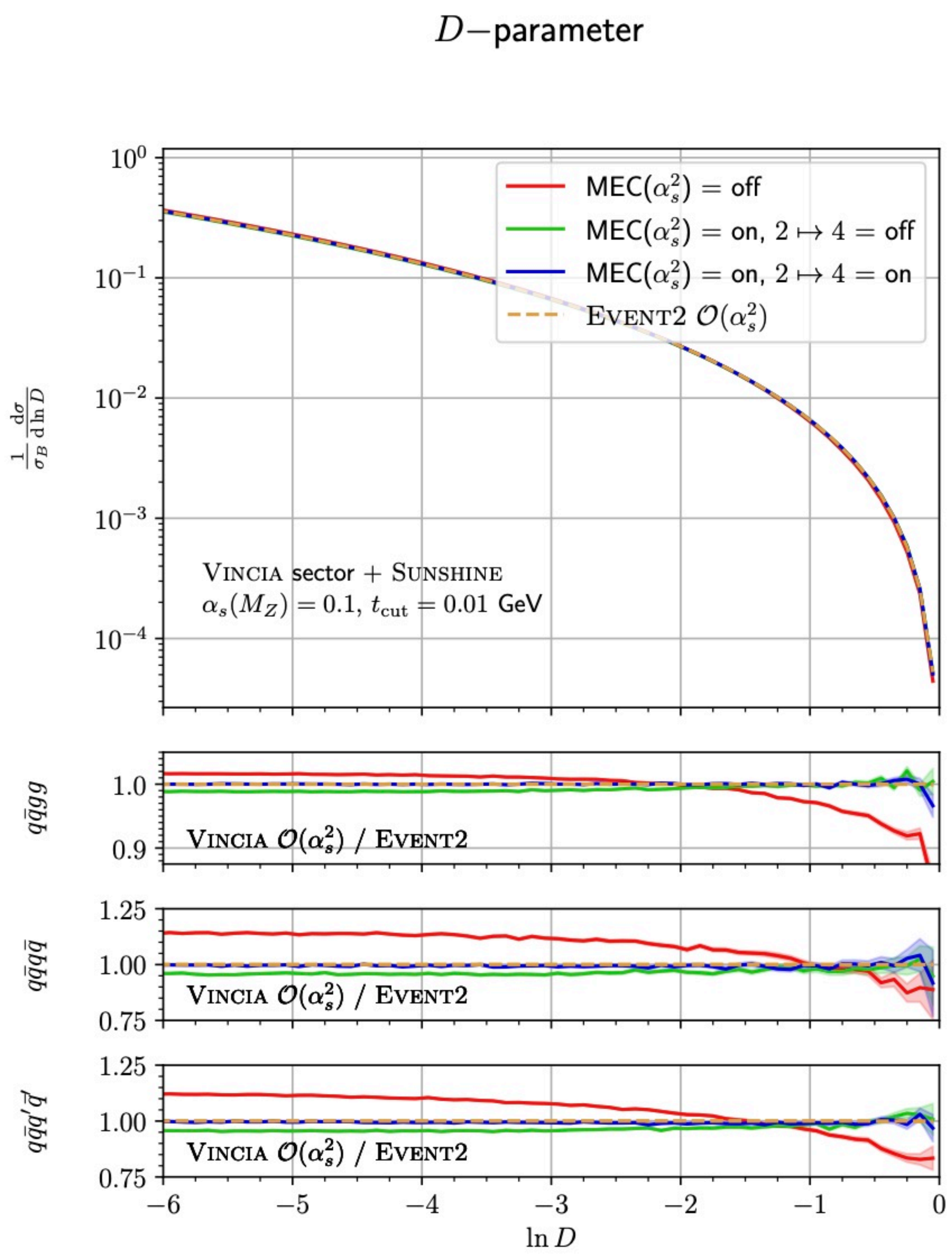
Sudakov Nesting of Hard Integrals

Using generalized parton shower to generate fixed order corrections





Altmann, HTL, Scyboz, Skands, 2025

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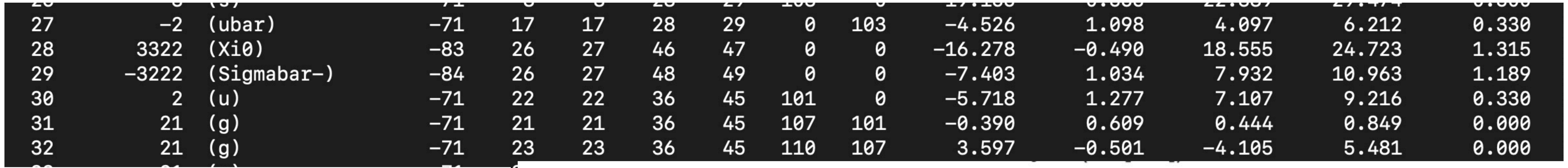
3. Hadronization models

hadronization effects

27	-2	(ubar)	-71	17	17	28	29	0	103	-4.526	1.098	4.097	6.212	0.330
28	3322	(Xi0)	-83	26	27	46	47	0	0	-16.278	-0.490	18.555	24.723	1.315
29	-3222	(Sigmabar-)	-84	26	27	48	49	0	0	-7.403	1.034	7.932	10.963	1.189
30	2	(u)	-71	22	22	36	45	101	0	-5.718	1.277	7.107	9.216	0.330
31	21	(g)	-71	21	21	36	45	107	101	-0.390	0.609	0.444	0.849	0.000
32	21	(g)	-71	23	23	36	45	110	107	3.597	-0.501	-4.105	5.481	0.000
33	21	(g)	-71	24	24	36	45	106	110	1.334	0.455	-1.320	1.932	0.000
34	21	(g)	-71	25	25	36	45	105	106	7.964	-0.514	-7.933	11.253	0.000
35	-3	(sbar)	-71	11	11	36	45	0	105	16.893	-1.870	-20.679	26.772	0.500
36	111	(pi0)	-83	30	35	50	51	0	0	-3.511	0.738	4.002	5.377	0.135
37	211	pi+	83	30	35	0	0	0	0	0.002	0.218	0.085	0.273	0.140
38	-211	pi-	83	30	35	0	0	0	0	-1.767	-0.071	2.475	3.045	0.140
39	211	pi+	83	30	35	0	0	0	0	-0.182	0.285	0.651	0.747	0.140
40	-211	pi-	83	30	35	0	0	0	0	0.016	0.232	0.209	0.342	0.140
41	211	pi+	83	30	35	0	0	0	0	-0.413	0.450	-0.145	0.643	0.140
42	-211	pi-	84	30	35	0	0	0	0	2.478	-0.473	-2.622	3.642	0.140
43	2212	p+	84	30	35	0	0	0	0	6.374	-0.009	-6.640	9.252	0.938
44	111	(pi0)	-84	30	35	52	53	0	0	0.270	0.111	-0.364	0.486	0.135
45	-3122	(Lambdabar0)	-84	30	35	54	55	0	0	20.414	-2.024	-24.136	31.696	1.116
46	3122	(Lambda0)	-91	28	0	56	57	0	0	-14.222	-0.534	16.090	21.510	1.116
47	111	(pi0)	-91	28	0	58	59	0	0	-2.056	0.043	2.465	3.213	0.135
48	-2112	nbar0	91	29	0	0	0	0	0	-5.613	0.671	6.203	8.445	0.940
49	-211	pi-	91	29	0	0	0	0	0	-1.790	0.363	1.728	2.518	0.140
50	22	gamma	91	36	0	0	0	0	0	-3.222	0.667	3.613	4.887	0.000
51	22	gamma	91	36	0	0	0	0	0	-0.289	0.071	0.388	0.490	0.000
52	22	gamma	91	44	0	0	0	0	0	0.028	-0.020	-0.008	0.036	0.000
53	22	gamma	91	44	0	0	0	0	0	0.242	0.131	-0.356	0.450	0.000
54	-2212	pbar-	91	45	0	0	0	0	0	18.123	-1.732	-21.512	28.198	0.938
55	211	pi+	91	45	0	0	0	0	0	2.291	-0.292	-2.624	3.498	0.140
56	2212	p+	91	46	0	0	0	0	0	-10.893	-0.393	12.398	16.535	0.938
57	-211	pi-	91	46	0	0	0	0	0	-3.329	-0.140	3.692	4.975	0.140
58	22	gamma	91	47	0	0	0	0	0	-0.678	0.003	0.911	1.136	0.000
59	22	gamma	91	47	0	0	0	0	0	-1.378	0.040	1.554	2.077	0.000

3. Hadronization models

hadronization effects



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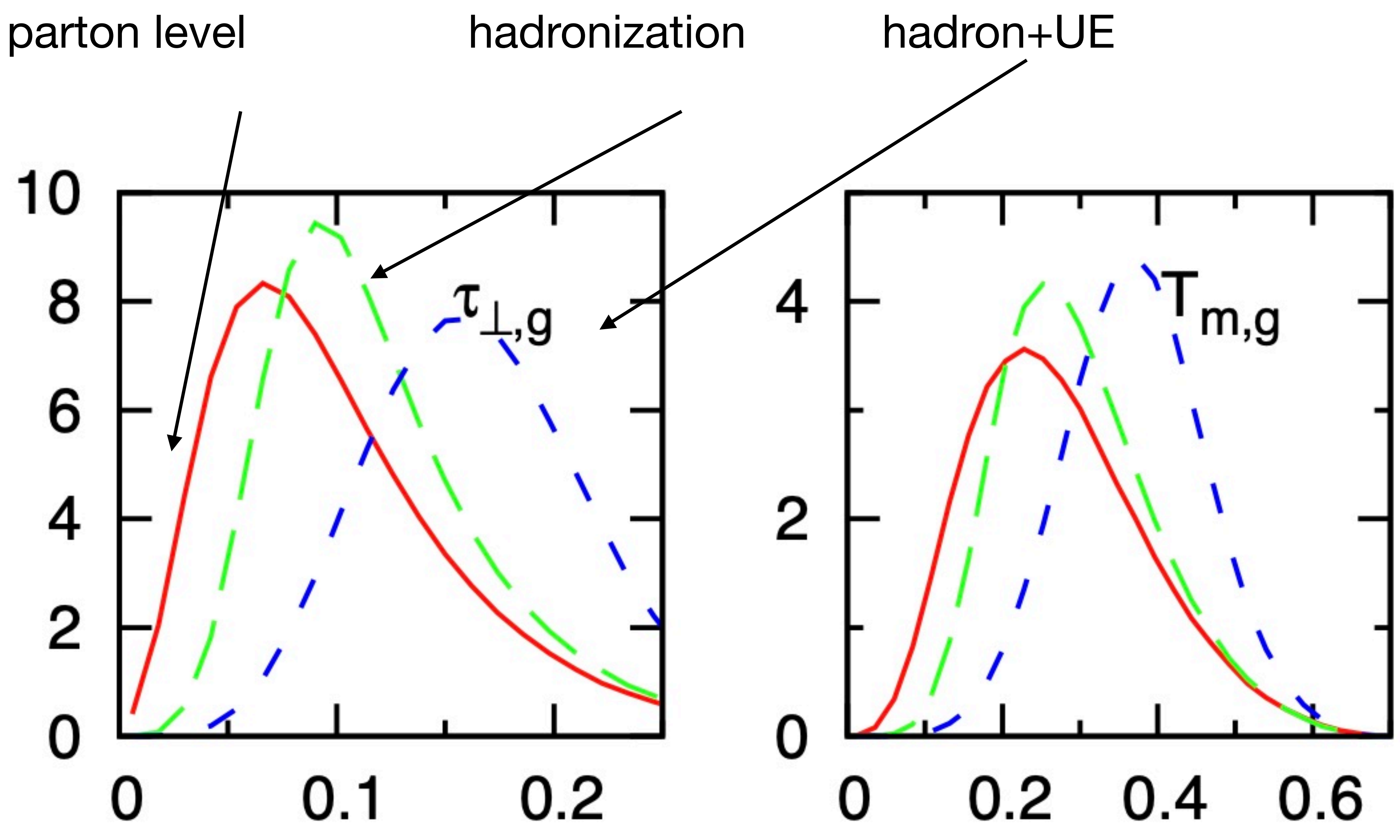
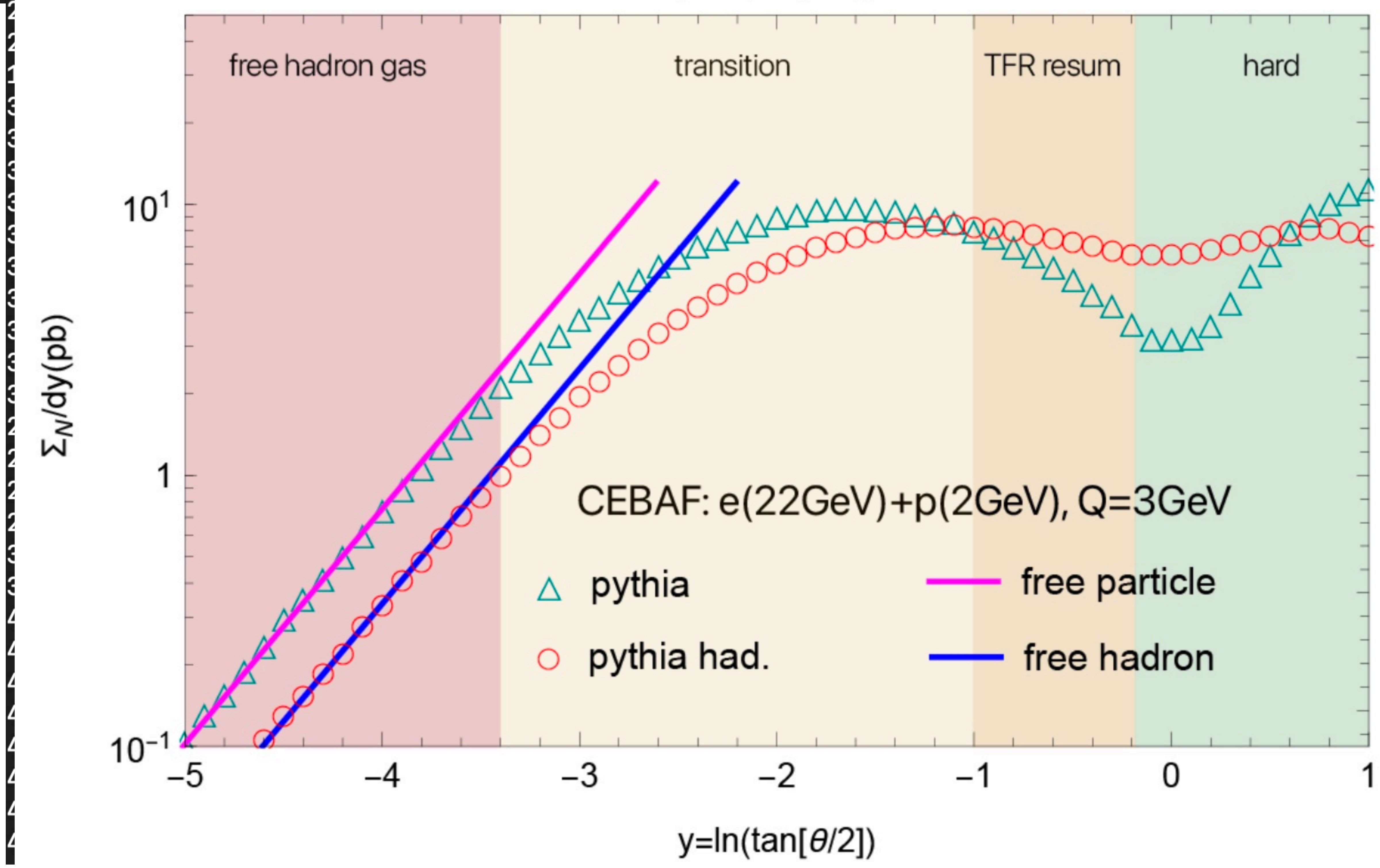
31	21	(g)	-71	21	21	36	45	107	101	-0.390	0.609	0.444	0.849	0.000
32	21	(g)	-71	23	23	36	45	110	107	3.597	-0.501	-4.105	5.481	0.000

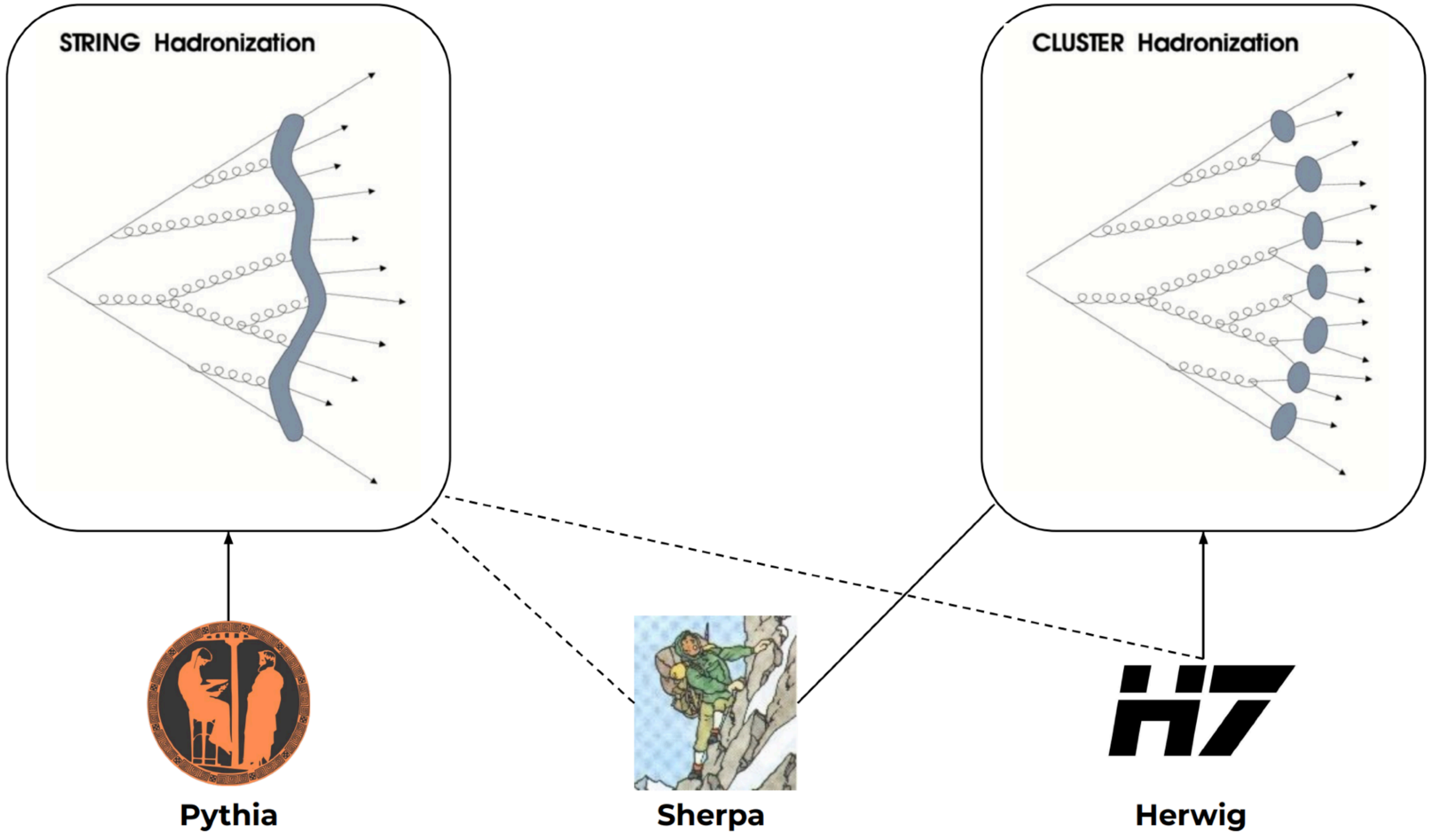


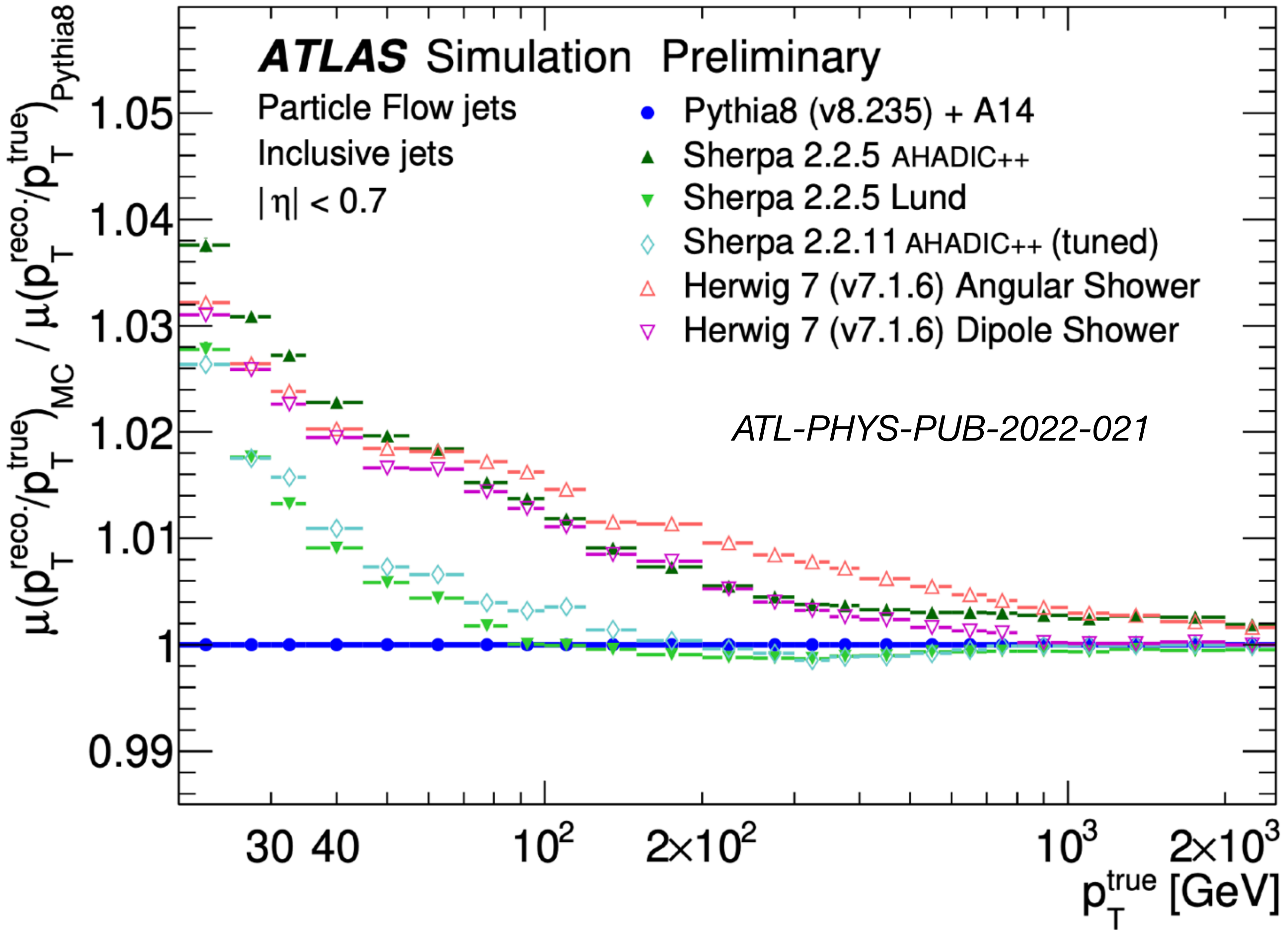
Banfi, Salam and Zanderighi arXiv:1001.4082



Cao, HTL, Mi, arXiv:2312.07655

3. Hadronization models

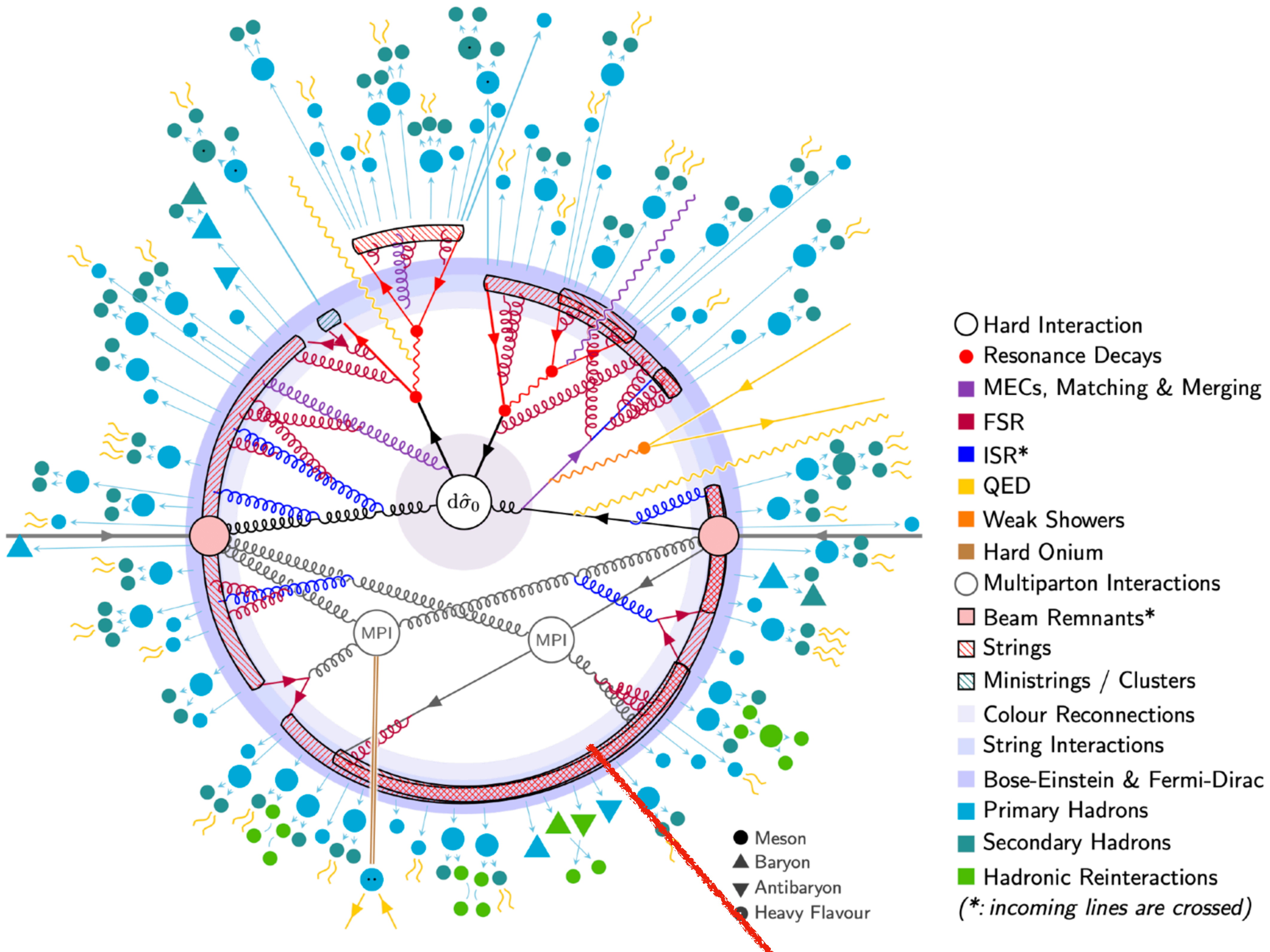




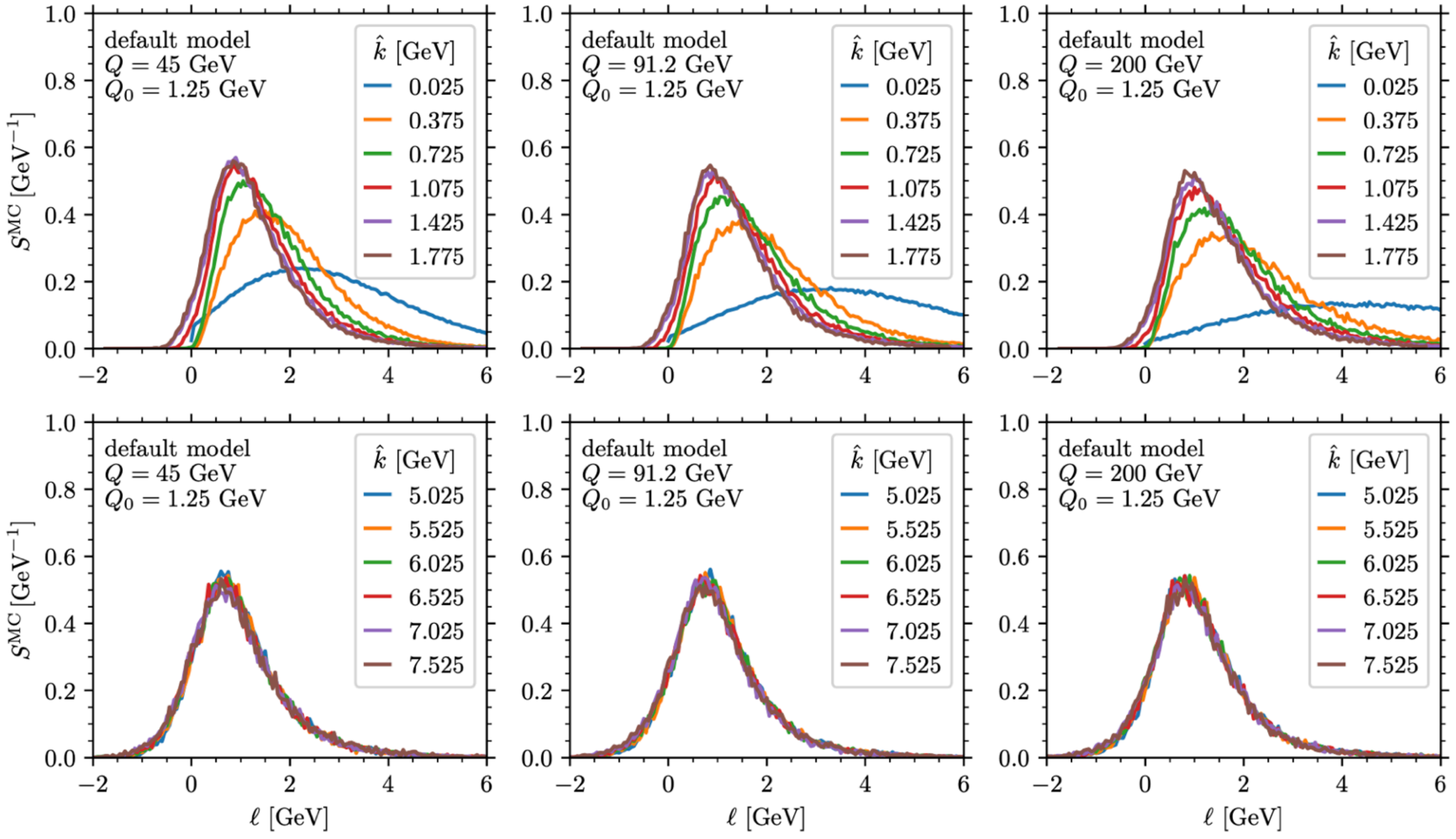
differences between

different models and tunes

3. Hadronization models



From PYTHIA



Physics should be independent on the transition scales

Matching the evolution of the perturbative evolution with hadronization arXiv:2404.09856

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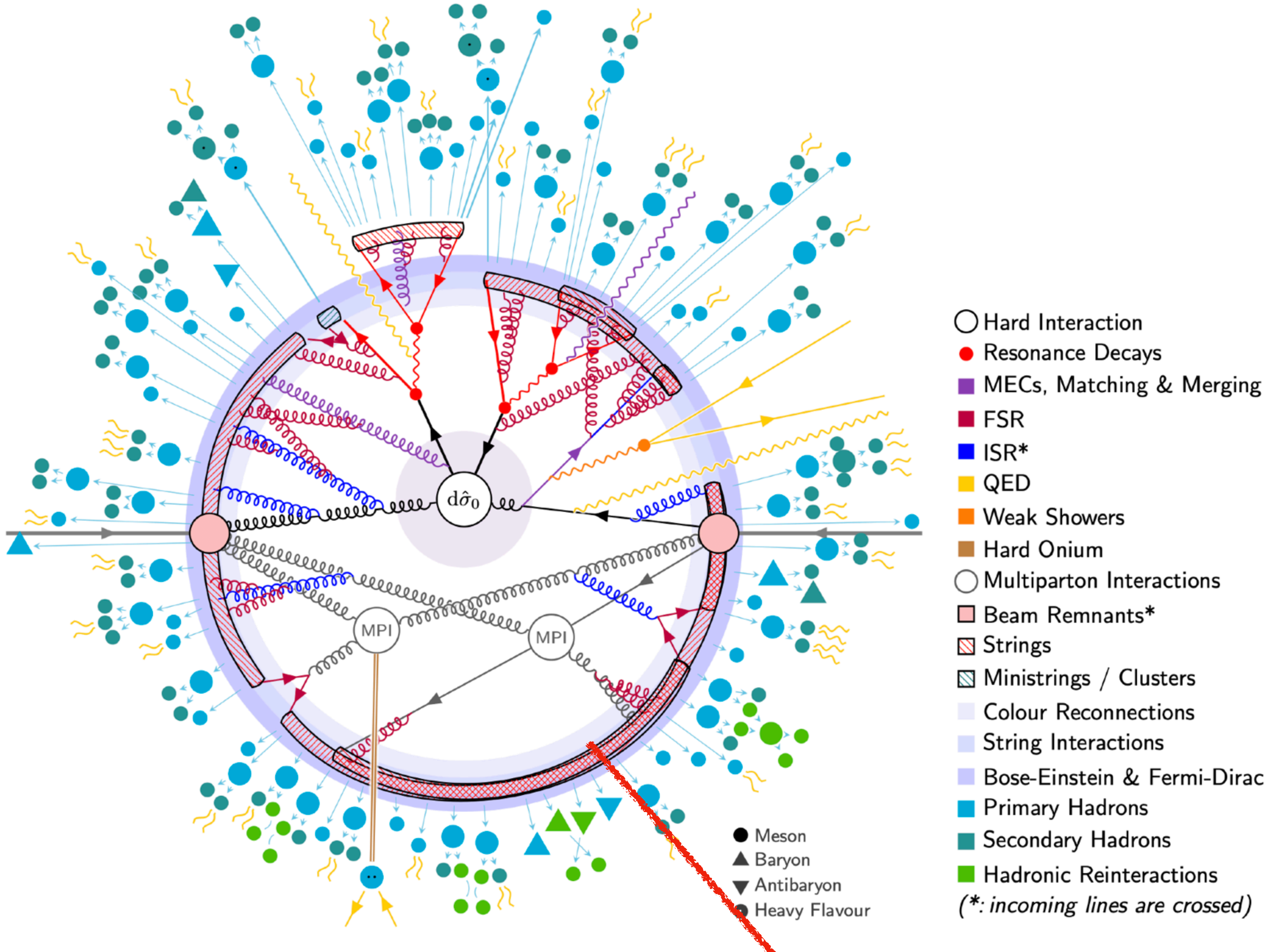

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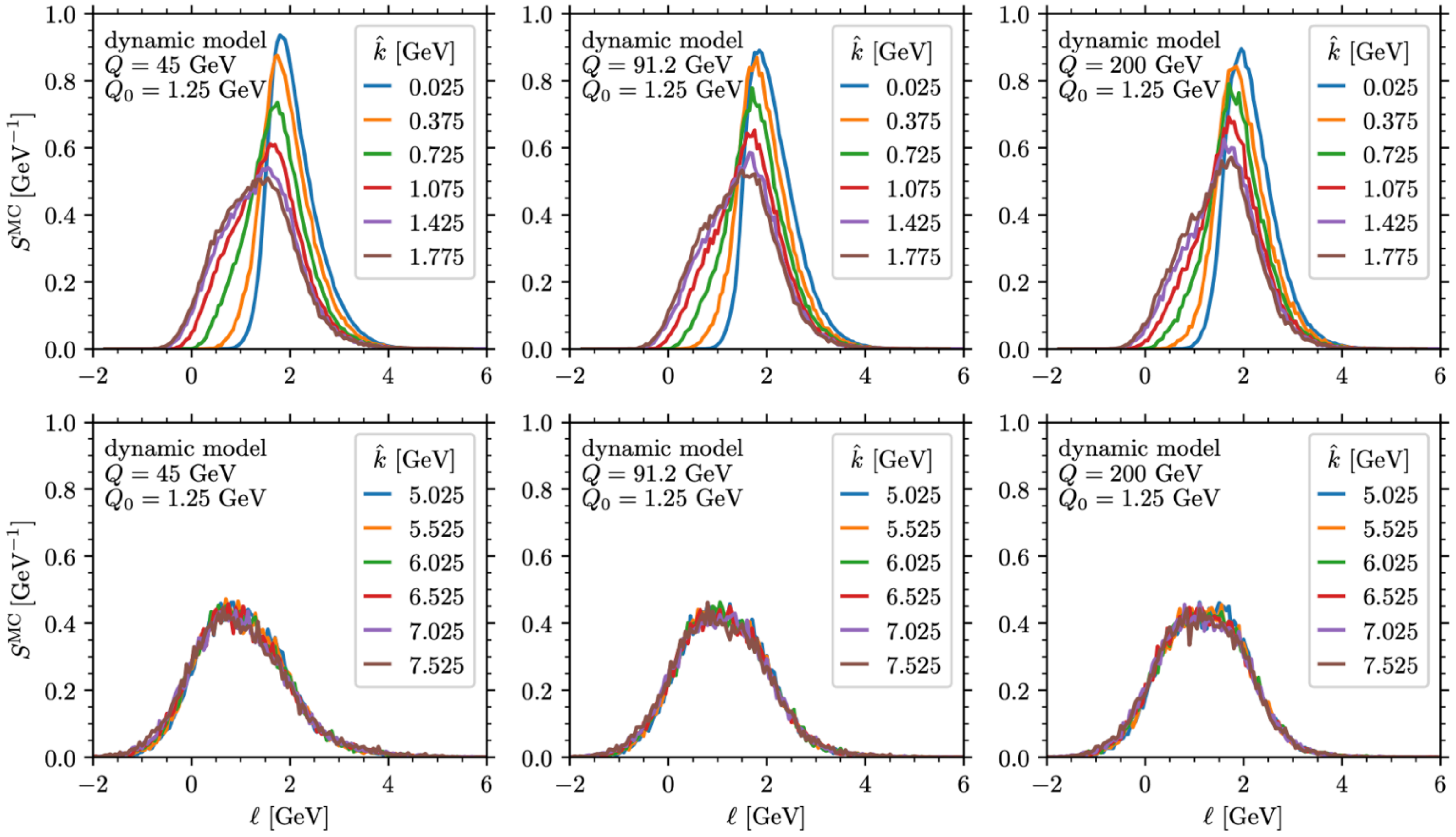

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4. Summary

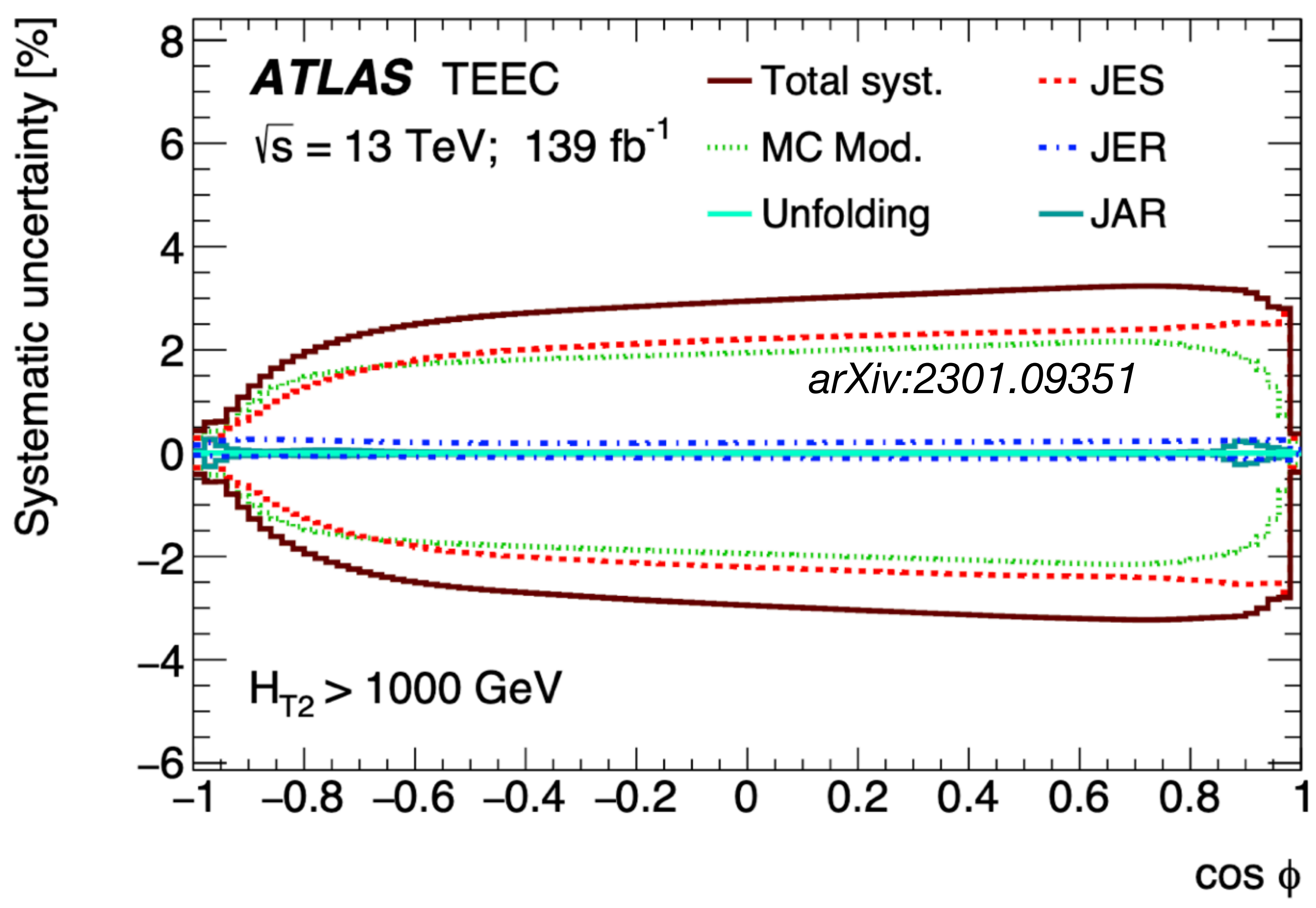
MCEGs are essential computational tools for experimentalists and theorists

Starting from hard processes to generate the perturbative and nonperturbative QCD radiations

Recently, a lot progresses on improving the logarithmic resummation order of Parton Showers

Also, subleading color effects are discussed

Hadronization model, multiple parton interactions (MPI), and underlying event descriptions introduce uncertainties



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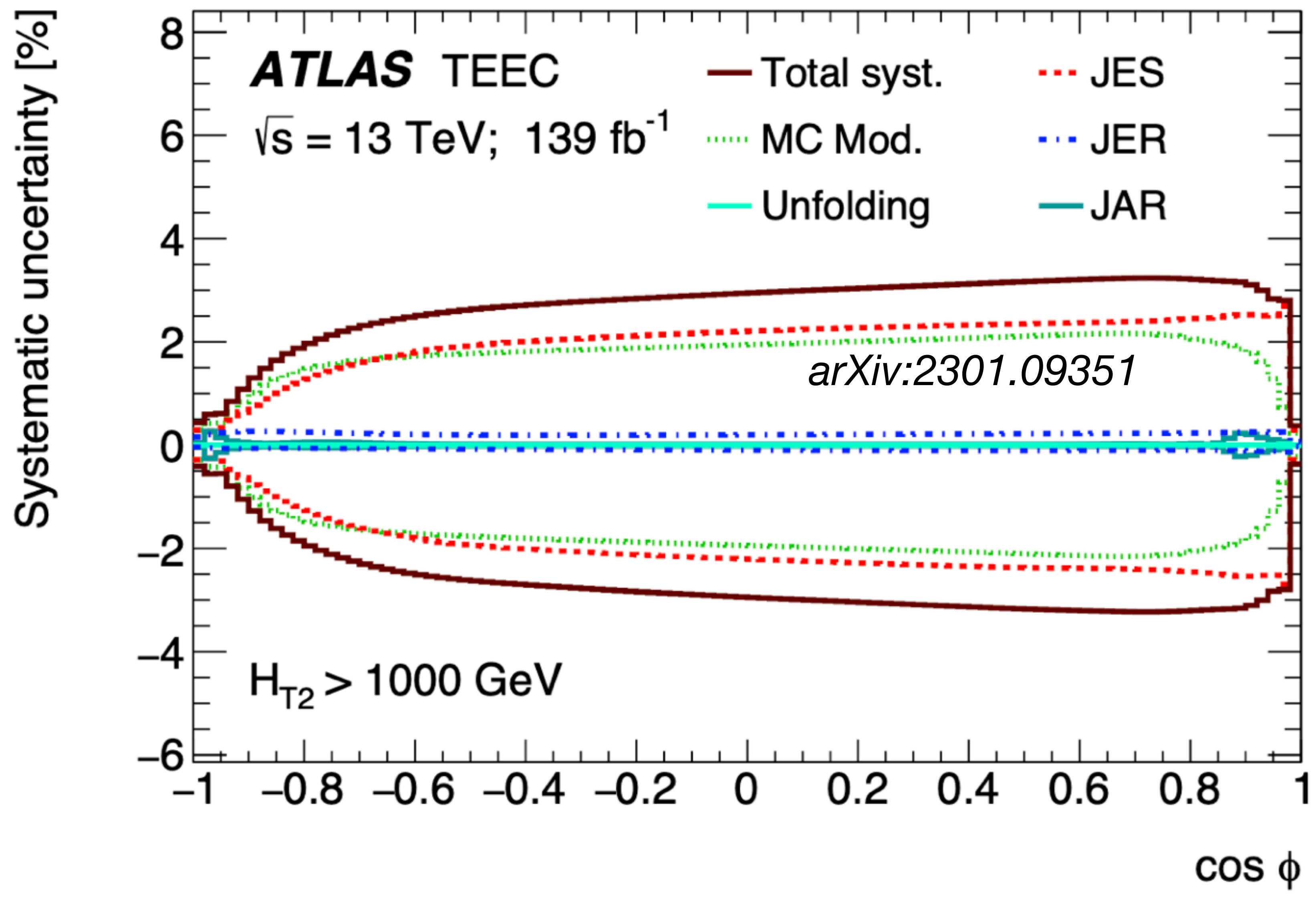

Thank you!