Review of QCD precision calculations

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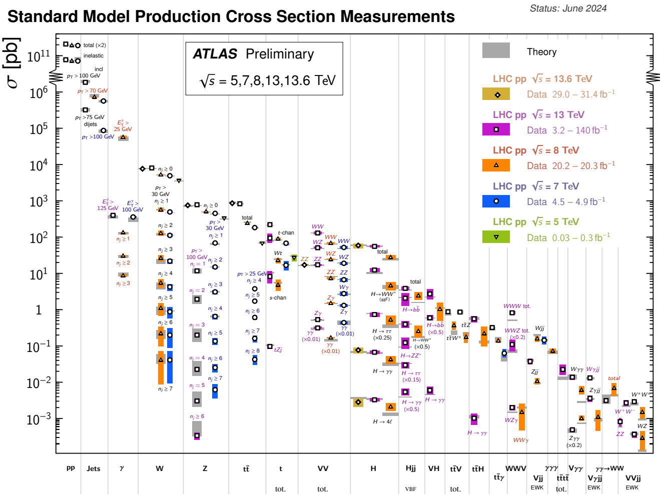
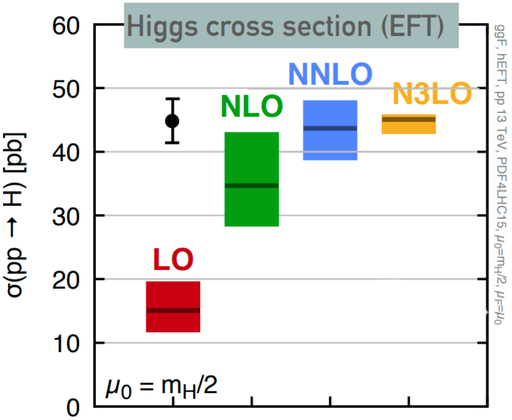
SM precision measurements see the talk by Yusheng Wu



SM has been quite successful in describing experimental data

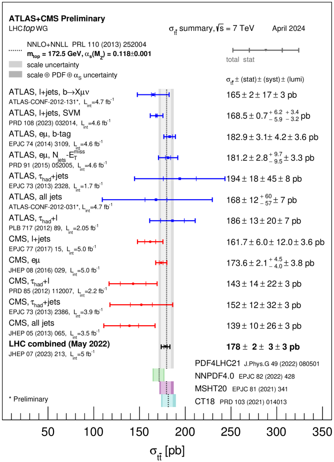
Why precision? Higher-order corrections can be large

- Cross section for $gg \to H$ increases by a factor of 3 from LO $\to$ N3LO

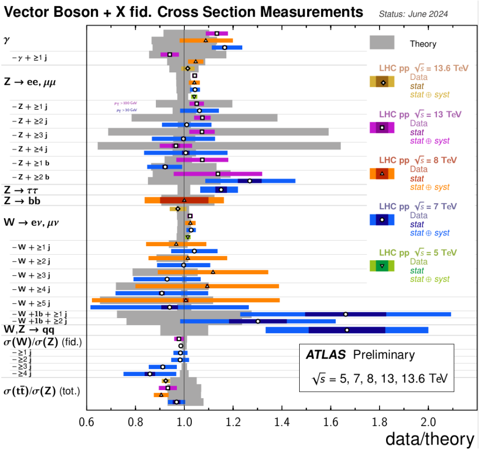


plot from M. Grazzini

Why precision? Bucket Effect: short panel on theory side

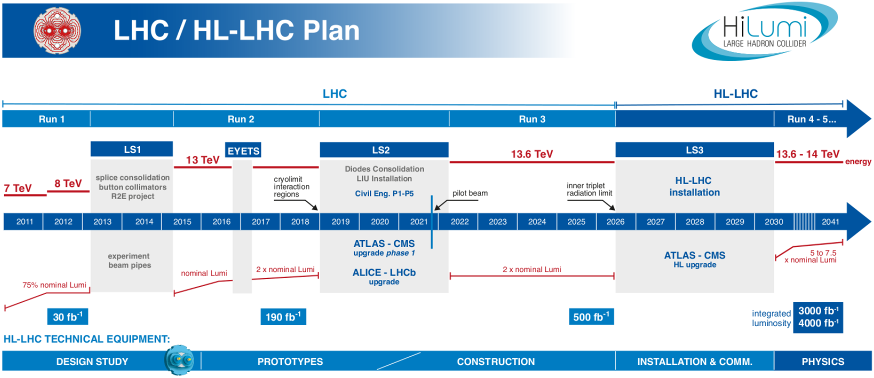


Top-pair production



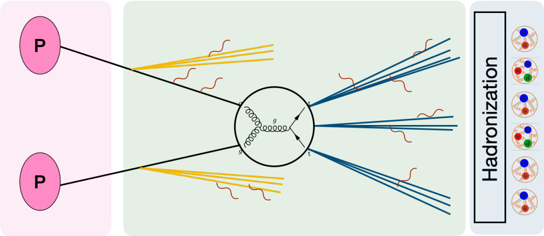
V+X production

LHC/HL-LHC Plan



HL-LHC will collect 95% more data, prime time for precision calculations

How? QCD factorization

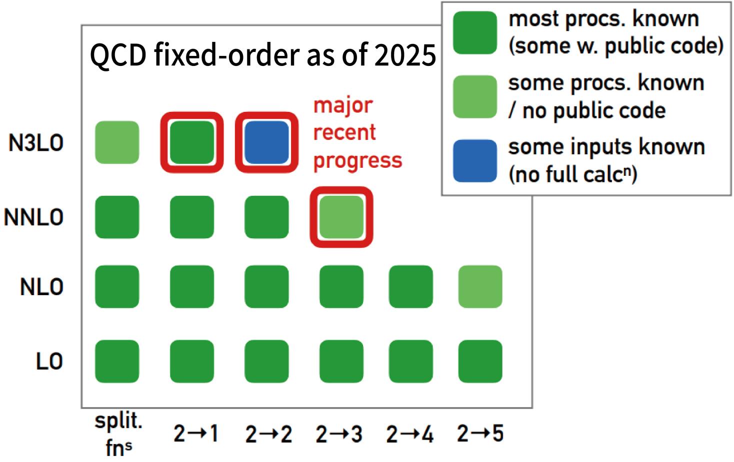


$$d\sigma = \underbrace{f_{P\to a}(x_1)\times f_{P\to b}(x_2)}_{\text{Parton distribution function}} \otimes \underbrace{d\hat{\sigma}}_{\text{Perturbative}} \otimes \underbrace{F}_{\text{Fragmentation}}$$

- Non-perturbative PDFs: global fitting or LQCD
- Non-perturbative FFs: global fitting
- $\hat{\sigma}$ is perturbatively calculable

$$\hat{\sigma} = \underbrace{\hat{\sigma}^{(0)}}_{\text{LO}} + \underbrace{\alpha_s\hat{\sigma}^{(1)}}_{\text{NLO}} + \underbrace{\alpha_s^2\hat{\sigma}^{(2)}}_{\text{NNLO}} + \underbrace{\alpha_s^3\hat{\sigma}^{(3)}}_{\text{N3LO}} + \cdots$$

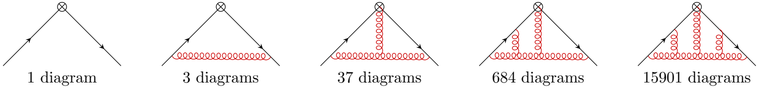
Status of fixed-order calculation in QCD



Focus on fixed-order in this talk, see the talk by Haitao Li for Parton shower

Overview of fixed-order calculation

- Feynman diagram is used to organize the expansion in α_s



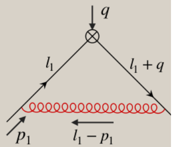
- The partonic cross section is given by the integral of squared amplitude over the final-state phase space.

$$d\hat{\sigma} \sim \int |\mathcal{M}_{i\to f}|^2 \,\mathrm{dPS}_n$$

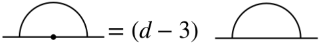
- Standard approach to evaluate $\hat{\sigma}$
  - Evaluate the amplitude by (semi-)analytic/numeric approach
  - Subtract the infrared divergences in phase-space integration
  - Numerically integrate the remaining finite integrals by Monte Carlo

Overview of procedure in amplitude calculation

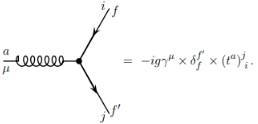
- Generate Feynman diagrams



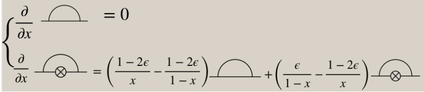
- Reduce amplitude via *master integrals* by IBP identities



- Substitute Feynman rules to generate (squared-)amplitude



- Solve the master integrals by differential equation



IBP and differential equation are often the bottleneck

IBP reduction

- Feynman integrals are related by IBP identities:

$$0 = \int \prod_{i=1}^{L} d^D l_i \, \frac{\partial}{\partial l_k^\mu} \left(\frac{v^\mu}{D_1^{n_1} \cdots D_m^{n_m}} \right)$$

- First proposed by [Chetyrkin, Tkachov,1981] to calculate 4-loop β function
- Systematic approach: Laporta Algorithm [Laporta, 2000]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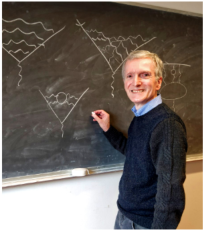
PARTICLE PHYSICS

The Algorithm That Lets Particle Physicists Count Higher Than Two



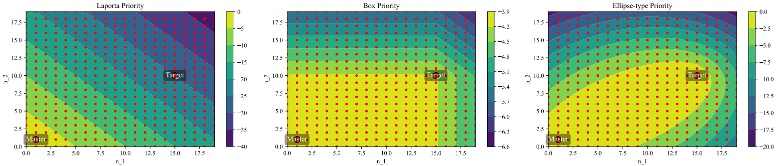
- Complexity increases factorially with $\#$ of loops and $\#$ of legs, require $\mathcal{O}(\text{TB})$ RAM and $\mathcal{O}(10\text{TB})$ disk space

New methods for IBP reduction

- Finite-fileds method [von Manteufell, Schabinger,Peraro, Abreu, Page, Ita, Klappert, Lange...,15,...]
  - Reconstruct the analytic results by numerical sampling; implemented in `FIRE`, `Kira`, `Finiteflow`, `Firefly`...
  - Avoid intermediate expression swell
- Algebraic geometry methods [Zhang, Bohem, Kosower,Peraro, Page, Abreu, Ita, von Manteuffel, Schabinger,...,11...]
  - Reduce the number of IBP equations generated by solving syzygy equations; implemented in `NeatIBP`
- Intersection theory [Mizera, Mastrolia, Frellesvig, Brunello, Crisanti, Mattiazzi,Gasparotto, Smith, Chen, Feng,Yang, Xu, Pokraka, Caron-Huot, Giroux, Weinzierl, Fontana, Peraro...,19,..]
  - Directly project out the coefficients of master integrals
- Block-triangular form [Guan, Liu, Ma,Wu,18,20,24]
  - Find linear relations among unreduced integrals; implemented in `Blade`
- Generating functions [Guan, Li, Ma, Hu, Feng, Shen,Zhang, Liu, Zhang,23,24,25]

New methods for IBP reduction

- Improved seeding [Driesse,Jakobsen,Mogull,Plefka,Sauer, Usovitsch,24; Guan, Liu, Ma, Wu,24; Bern, Herrmann, Roiban,Ruf, Smirnov, Smirnov,Zeng,24]
  - A direct improvement over Laporta algorithm
  - Generate fewer IBP equations, implemented in `Blade` and `Kira3`
- AI-priority function approach [Hippel,Wilhelm,25;Song,TZY,Cao,Luo,Zhu,25;Zeng,25]
  - Operate on top of Laporta algortihm or improved seeding
  - Further reduce the number of IBP equations generated



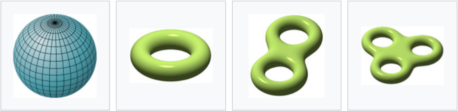
Laporta seeding $\rightarrow$ Box seeding $\rightarrow$ Ellipse seeding

[Song,TZY,Cao,Luo,Zhu,25]

Differential equation and special functions

- Proposed in [Gehrmann,E. Remiddi,99] to compute 2-loop 4-point integrals
- ϵ-factorized DE [Henn,13]: $d\vec{f}/dx = \epsilon\, A(x)\vec{f}(x)$
  - Relatively easy to achieve if the solution is polylogs (genus 0)
  - Can also be achieved for integrals beyond polylogs (genus 1,2,...)

    [Weinzierl,Claude,Tancredi,Pogel,Marzucca,McLeod,Page, Wang,Xu,... ;ϵ-collaboration,25]
  - Once achieved, solution can be easily written down order by order in ϵ



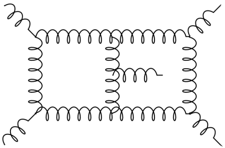
genus 0 genus 1 genus 2 genus 3

- Solve the DE by semi-analytic method (series expansion): `AMFlow`, `DiffExp`, `AmpRed`

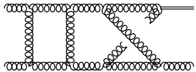
All these advancements enable to push the boundaries

Recent highlights in amplitude: Frontiers in multiplicity

- 2-loop 5-leg [Abreu, Agarwal, Badger, Buccioni, Chawdhry, Chicherin, Czakon, de Laurentis, Febres-Cordero, Gambuti, Gehrmann, Henn, Ita, Lo Presti, Manteuffel, Ma, Mitov, Page, Peraro, Pochelet, Schabinger, Sotnikov, Tancredi, Zhang,...]
  - Massless, done $(3j, 3\gamma, 2\gamma + j, ...)$
  - One/two off-shell legs, LC $(Vjj, Hjj, V\gamma\gamma, VVj...)$



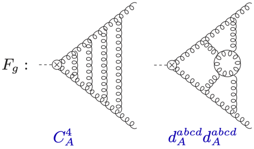
- 3-loop 4-leg [Bargiela, Bobadilla, Canko, Caola, Chen, Guan, Jakubcik, Gambuti, Gehrmann, Henn, Lim, Mella, Mistlberger, Wasser, Manteuffel, Syrrakos, Smirnov, Tancredi,...]
  - Massless (done)
  - One off-shell leg (LC)



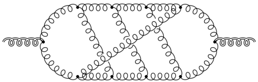
- First results for 2-loop 6-leg and 3-loop 5-leg integrals [Henn, Liu, Matijasic, Miczajka, Peraro, Xu, Xu, Zhang,24,24,25; Abreu,Monni,Usovitsch,25]

Recent highlights in amplitude: Frontiers in loops

- 4-loop 3-leg [Henn, Lee, Manteuffel, Schabinger, Smirnov, Smirnov, Stainhauser,...]
  - Form factors ($\gamma^* \to q\bar{q}$, $H \to gg, q\bar{q}$)

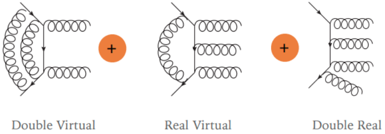


- 5-loop 2-leg [Baikov, Chetyrkin, Kühn, Herzog, Ruijl, Ueda, Vermaseren, Vogt, Luthe, Maier, Marquard, Schröder, Falcioni]
  - β function
  - Field renormalization constants
  - R-value



Infrared subtraction at NNLO

- At NNLO, KLN theorem ensures the cancellations of infrared divergences among VV, RV, RR contributions



- Each contritution is divergent, design a subtraction scheme such that each term after subtraction is finite

$$d\hat{\sigma}_{\mathrm{NNLO}} = \int_n \left(d\hat{\sigma}^{VV} - d\hat{\sigma}^{U} \right) + \int_{n+1} \left(d\hat{\sigma}^{RV} - d\hat{\sigma}^{T} \right) + \int_{n+2} \left(d\hat{\sigma}^{RR} - d\hat{\sigma}^{S} \right)$$

- Well under control up to NNLO (many schemes exist)
  - Antenna, Stripper, Nested, Torino, Colorful, Geometric, Qt, N-jettiness, Projection-To-Born

Towards N3LO: New frontier

- Our current ability of going to N3LO is rather limited
- General subtraction schemes don't exist
- Partial N3LO results based on
  - Direct (semi-)analytic calculation (IBP+DE) from reverse unitarity
  - Slicing techniques based on factorization theorems: Qt, N-jettines

$$\frac{d^2\sigma_F^{\mathrm{N3LO}}}{dO d\tau} = \left(\frac{d^2\sigma_{F+\mathrm{jet}}^{\mathrm{NNLO}}}{dO d\tau}\Big|_{\tau > \tau_{\mathrm{cut}}} \right) + \left(\frac{d^2\sigma_F^{\mathrm{N3LO}}}{dO d\tau}\Big|_{\tau < \tau_{\mathrm{cut}}} \right)$$

  - Projection-To-Born ($\mathcal{O} \to$ full, $\tilde{\mathcal{O}} \to$ born kinematics)

$$\frac{d\sigma_F^{\mathrm{N3LO}}}{d\mathcal{O}} = \left(\frac{d\sigma_{F+\mathrm{jet}}^{\mathrm{NNLO}}}{d\mathcal{O}} - \frac{d\sigma_{F+\mathrm{jet}}^{\mathrm{NNLO}}}{d\tilde{\mathcal{O}}} \right) + \frac{d\sigma_F^{\mathrm{N3LO}}}{d\tilde{\mathcal{O}}}$$

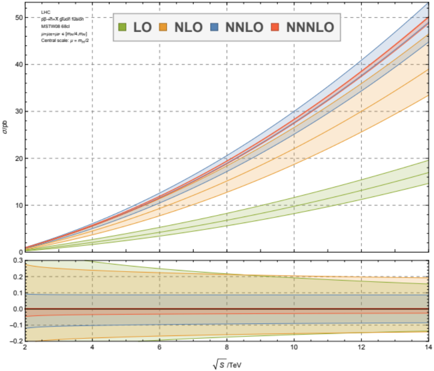
  - Apply Antenna to $e^+e^- \to jj$ at N3LO [Chen, Jakubcik, Marcoli, Stagnitto,25]

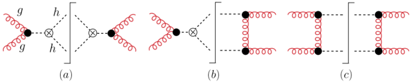
Selected Results

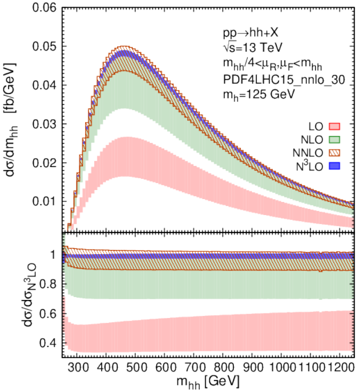
Apologies for the biased and incomplete selection

Higgs production@N3LO see more in talk by Jian Wang

- $gg \to H$ [C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger,15]
- $gg \to HH$ [Chen, Li,Shao, Wang,19]



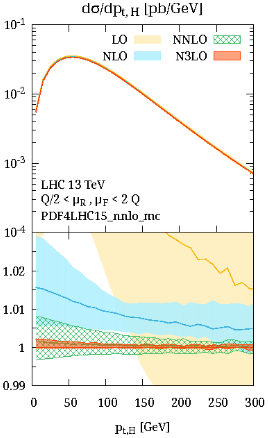




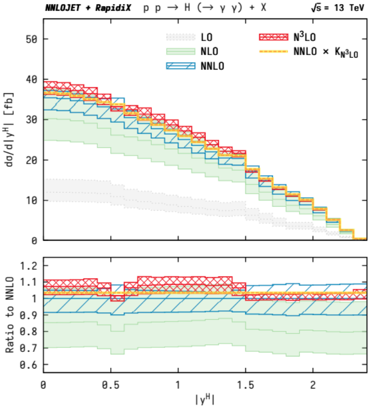
Scale uncertainty reduces substantially

Higgs production@N3LO

- VBF $\rightarrow H$ [Dreyer,Karlberg,16]



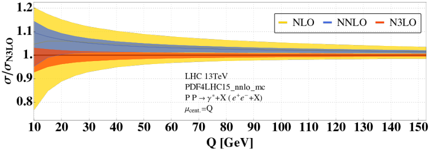
- Fully differential $gg \rightarrow H$ [Chen, Gehrmann, Glover, Huss, Mistlberger,Pelloni,21]



Vector boson production@N3LO

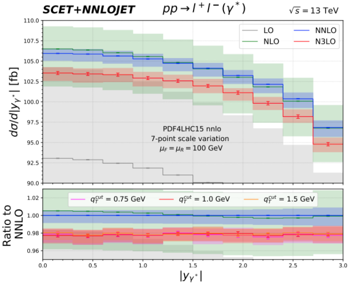
- Total cross section for γ^*, Z, W

  [Duhr, Dulat,Mistlberger,20,20,21]



- Rapidity distribution for γ^*

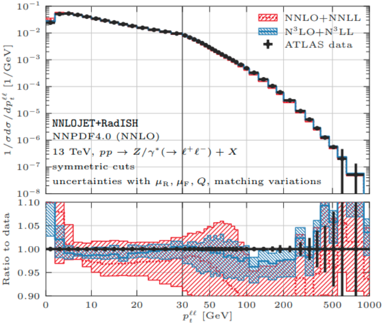
  [Chen, Gehrmann, Glover, Huss, TZY,Zhu,21]



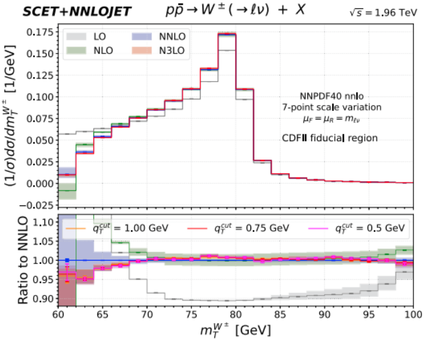
Scale band at N3LO is outside the band of NNLO

Vector boson production@N3LO

- Fiducial predic. for γ^*/Z [Chen, Gehrmann, Glover, Huss, Monni, Re,Rottoli, Torrielli,22]

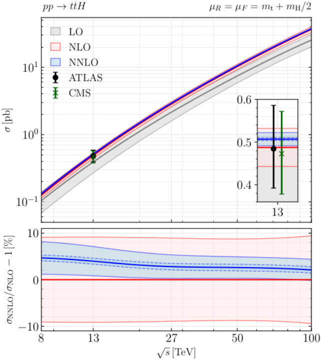


- Fiducial predic. for W [Chen, Gehrmann, Glover, Huss, TZY,Zhu,22]



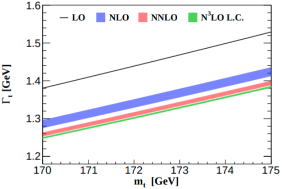
Top quark production and decay

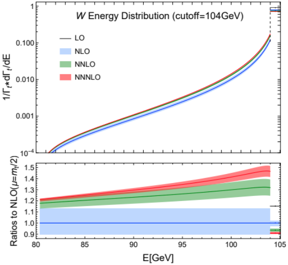
- $t\bar{t}$ remains at NNLO[Czakon,Fiedler,Mitov,13]
- $t\bar{t}H$, $t\bar{t}W$ [Catani, Devoto, Grazzini,Kallweit, Mazzitelli, Savoini,Buonocore,Rottoli,23,23]



- Top-Quark Decay@N3LO

[Chen,Li,Li,Wang,Wang,Wu,25;Chen,Chen,Guan,Ma,25]

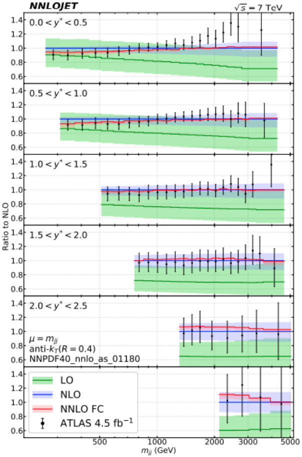




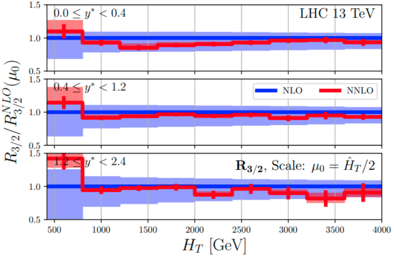
See more in talks by Lilin Yang and Haifeng Li

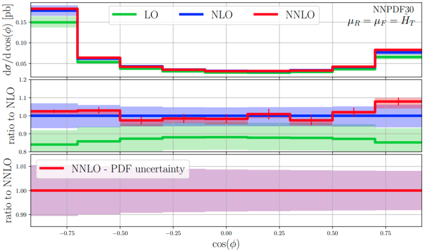
Jets/photon production at NNLO

- $pp \to 2j$ [Chen, Currie, Ridder, Gehrmann, Glover, Huss, Pires,Mo,17,19,22]



- $pp \to 3j$ [Czakon, Mitov, Poncelet,22,23]

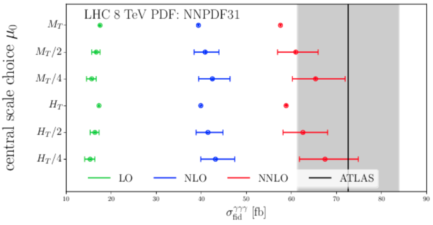




Jets/photon production at NNLO

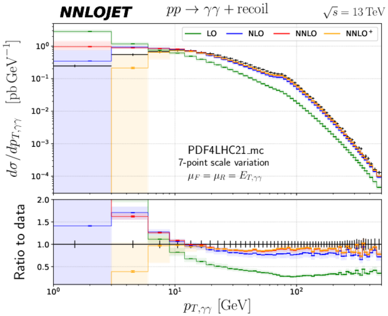
- $pp \to 3\gamma$

  [Chawdhry,Czakon,Mitov,Poncelet,19; Kallweit,Sotnikov, Wiesemann,20]



- $pp \to \gamma\gamma j$

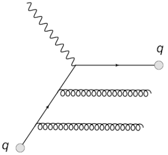
  [Chawdhry,Czakon,Mitov,Poncelet,21;Buccioni,Chen,Feng, Gehrmann,Huss,Marcoli,24]

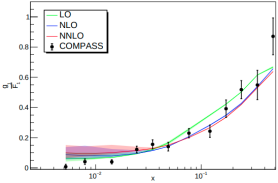


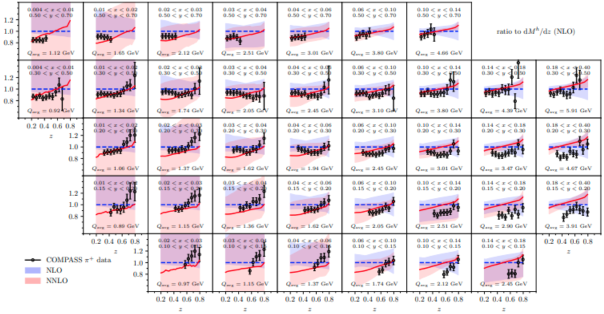
Large corrections

Process with identified Hadrons: New direction

- SIDIS at NNLO [Bonino, Gehrmann, Lochner, Schonwald, Stagnitto,24,24..; Goyal, Lee, Moch, Pathak, Rana, Ravindran,24,24..]



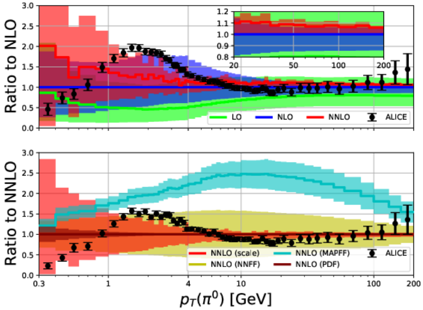




Process with identified Hadrons

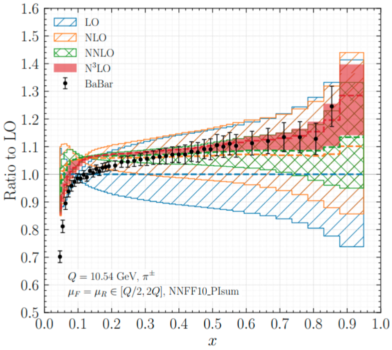
- $pp \to \text{Hadron} + X$ at NNLO

  [Czakon,Mitov,Poncelet,25]



- $e^+e^- \to \text{Hadron} + X$ at N3LO

  [He, Xing, TZY, Zhu,25]



NNLOJET: A 20-year effort

- Available processes (14 in total) in version 1.0 at NNLO [NNLOJET collaboration,25]

$$e^+e^- \to 2 \text{ jets or } 3 \text{ jets}\,,$$
$$ep \to \text{lepton} + 1 \text{ jet or } 2 \text{ jets}\,,$$
$$pp \to 1 \text{ jet} + X \text{ or } 2 \text{ jets}\,,$$
$$pp \to (\gamma^* Z) + 0 \text{ or } 1 \text{ jet}\,,$$
$$pp \to W^\pm + 0 \text{ or } 1 \text{ jet}\,,$$
$$pp \to \gamma + X \text{ or } \gamma + 1 \text{ jet or } \gamma\gamma\,,$$
$$pp \to H + X \text{ or } H + 1 \text{ jet}$$

- More processes will be published in future versions (including DY@N3LO, H@N3LO)

Antenna subtraction at NNLO

A. Gehrmann-De Ridder (Zurich, ETH), T. Gehrmann (Zurich U.), E.W.Nigel Glover (Durham U., IPPP and Durham U.)
May, 2005

NNLOJET: a parton-level event generator for jet cross sections at NNLO QCD accuracy

NNLOJET Collaboration • A. Huss (CERN) Show All(37)
Mar 28, 2025

Conclusions

- Higher-order corrections are crucial to precision physics studies
- Many advanced techniques have been developed to push the boundaries of the field (finite-filed,improved seeding, AI priority functions, $\epsilon-$factorized DE...)
- QCD NNLO calculations for $2 \to 3$ have become a reality!
- QCD@N3LO results start to appear!
- More exciting developments will continue. Stay tuned!

Thank you for your attention!