Quantum entanglement experiment

between quarks, vector bosons, leptons

Qiang Li (Peking University) 2025/10 Qingdao

"Can Quantum Mechanical Description of Physical Reality Be Considered Complete?"

A. Einstein B. Podolski N. Rosen

Physical reality must be local! - Podolsky

EPR Paradox

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

1.

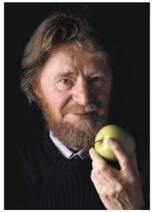
ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one

1964 QM with hidden variables differs from QM



1928~1990
John Stewart Bell

In the 1980s, he was always mentioned as a candidate for the Nobel Prize.

Bell's Inequality

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected

QM with hidden variables differs from standard QM

...ates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is chara...

He shows that von Neumann's proof was bogus.

...reproduces exactly the quantum mechanical predictions.

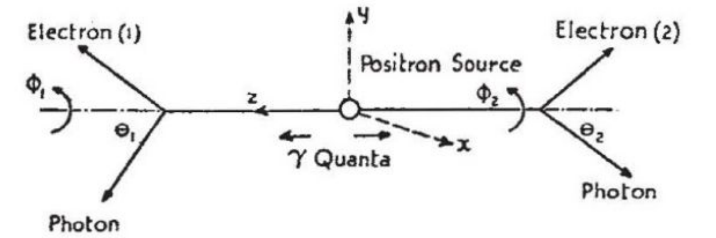
Phys.Rev.D 111 (2025) 3, 036008, *JHEP* 10 (2024) 211, arXiv:2506.16077 (CPC)
JPG52 (2025) 075002, MPLA (2025) 2530008, PRD111 (2025) 11, 116018

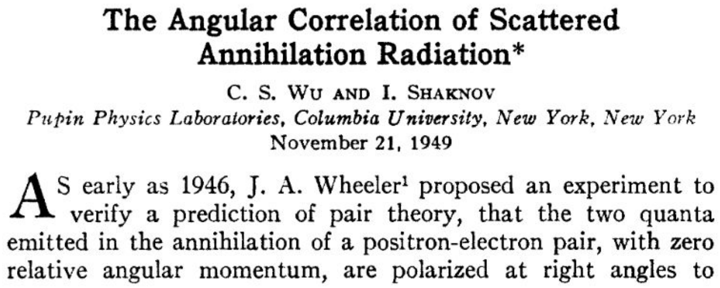
TeV物理前沿专题研讨会暨第31届LHC Mini-Workshop

Quantum entanglement measurements — history and today

- C. N. Yang (IJMPA 2015): the first experiment on quantum entanglement is the **Wu-Shaknov** Experiment (PR1950) which measures **the angular correlation of two Compton-scattered photons arising from *e*+*e*− annihilation**

- The violation of **Bell inequality (PPF1964)** was demonstrated in 1970s and afterwards using entangled photons, confirming the non-locality of our universe.
- Alain Aspect, John Clauser and Anton Zeilinger won the Nobel Prize in Physics in 2022 for demonstrating the potential to investigate and control particles (photons) that are in entangled states.





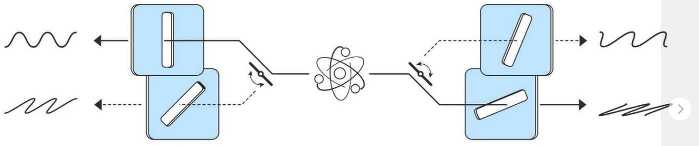
The Angular Correlation of Scattered Annihilation Radiation*

C. S. Wu and I. Shaknov

Pupin Physics Laboratories, Columbia University, New York, New York

November 21, 1949

As early as 1946, J. A. Wheeler[1] proposed an experiment to verify a prediction of pair theory, that the two quanta emitted in the annihilation of a positron-electron pair, with zero relative angular momentum, are polarized at right angles to



Alain Aspect developed this experiment, using a new way of exciting the atoms so they emitted entangled photons at a higher rate. He could also switch between different settings, so the system would not contain any advance information that could affect the results.

Early entanglement

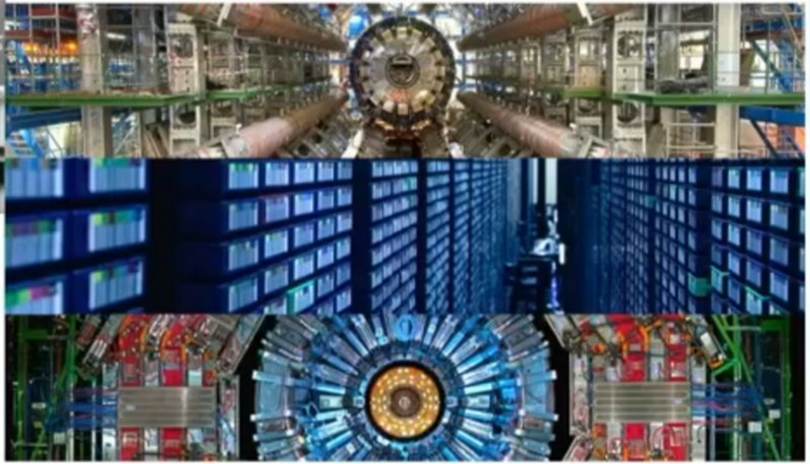
The concept of quantum entanglement was brought to light in May 1935 by Einstein, Boris Podolsky, and Nathan Rosen, who were all at the Institute for Advanced Study at the time. Their groundbreaking paper, "Can quantum-mechanical description of physical reality be considered complete?," delved into the novel idea.[10] In that influential work, later dubbed the EPR paper, the trio investigated a pair of particles deliberately prepared with a separation far exceeding the range of their mutual interaction and with a total momentum of zero. Their exploration revealed a dilemma: There is an inherent inconsistency among locality, separability, and completeness when describing physical systems with wavefunctions.

The EPR paper asserts that by measuring the position of the first particle without disturbing the second, it should be possible to predict with certainty the value of the second particle's position because of the perfectly correlated positions of the particle pair. On the other hand, by performing a measurement of the first particle's momentum rather than position, it should be possible to predict with certainty the value of the second particle's momentum because of the perfectly anticorrelated momenta of the particle pair. The uncertainty principle in quantum mechanics, however, prevents the precise attribution of values to both the position and momentum of a single particle.

Bell's inequality – full circle

John Stewart Bell

https://home.cern/news/press-release/physics/lhc-experiments-cern-observe-quantum-entanglement-highest-energy-yet



It is symbolic that the test of Bell inequality is coming back to CERN, where the original idea was developed

Quantum entanglement measurements — history and today

Many measurements with meson/baryon pairs performed

PLB 642 (2006) 315 (KLOE) *Phys.Rev.Lett.* 99 (2007) 131802 (Belle), Nature 606 (2022) 64/ Nat Commun 16, 4948 (2025) (BESIII)

Flavor/spin/time/…. entanglement , interplay with CP …

- **e+e- entanglement at Belle** measured using time-dependent flavor asymmetry

e+ e- $\rightarrow \Upsilon(4s) \rightarrow B^0 \bar{B}^0$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|B^0\rangle_1 \otimes |\overline{B}^0\rangle_2 - |\overline{B}^0\rangle_1 \otimes |B^0\rangle_2\right]$$

entangled state

→ data described by quantum mechanics

nature communications

$\eta_c \rightarrow \Lambda(p\pi^-)\bar{\Lambda}(\bar{p}\pi^+)$

Article
https://doi.org/10.1038/s41467-025-59498-4

Test of local realism via entangled $\Lambda\bar{\Lambda}$ system

Received: 18 September 2024
Accepted: 25 April 2025
Published online: 28 May 2025
Check for updates

BESIII Collaboration*

The non-locality of quantum correlations is a fundamental feature of quantum theory. The Bell inequality serves as a benchmark for distinguishing between predictions made by quantum theory and local hidden variable theory (LHVT). Recent advancements in photon-entanglement experiments have addressed potential loopholes and have observed significant violations of variants of Bell inequality. However, examples of Bell inequalities violation in high energy physics are scarce. In this study, we utilize $(10.087 \pm 0.044) \times 10^9$ J/ψ events collected with the BES-III detector at the BEPCII collider, performing non-local correlation tests using the entangled hyperon pairs. The massive-entangled $\Lambda\bar{\Lambda}$ systems are formed and decay through strong and weak interactions, respectively. Through measurements of the angular distribution of $p\bar{p}$ in $J/\psi \rightarrow \gamma\eta_c$ and subsequent $\eta_c \rightarrow \Lambda(p\pi^-)\bar{\Lambda}(\bar{p}\pi^+)$ cascade decays, a significant violation of LHVT predictions is observed. The exclusion of LHVT is found to be statistically significant at a level exceeding 5.2σ in the testing of three Bell-like inequalities.

https://www.koushare.com/live/details/44767

Event at Galileo Galilei Institute

https://indico.ihep.ac.cn/event/24387/timetable/?view=standard

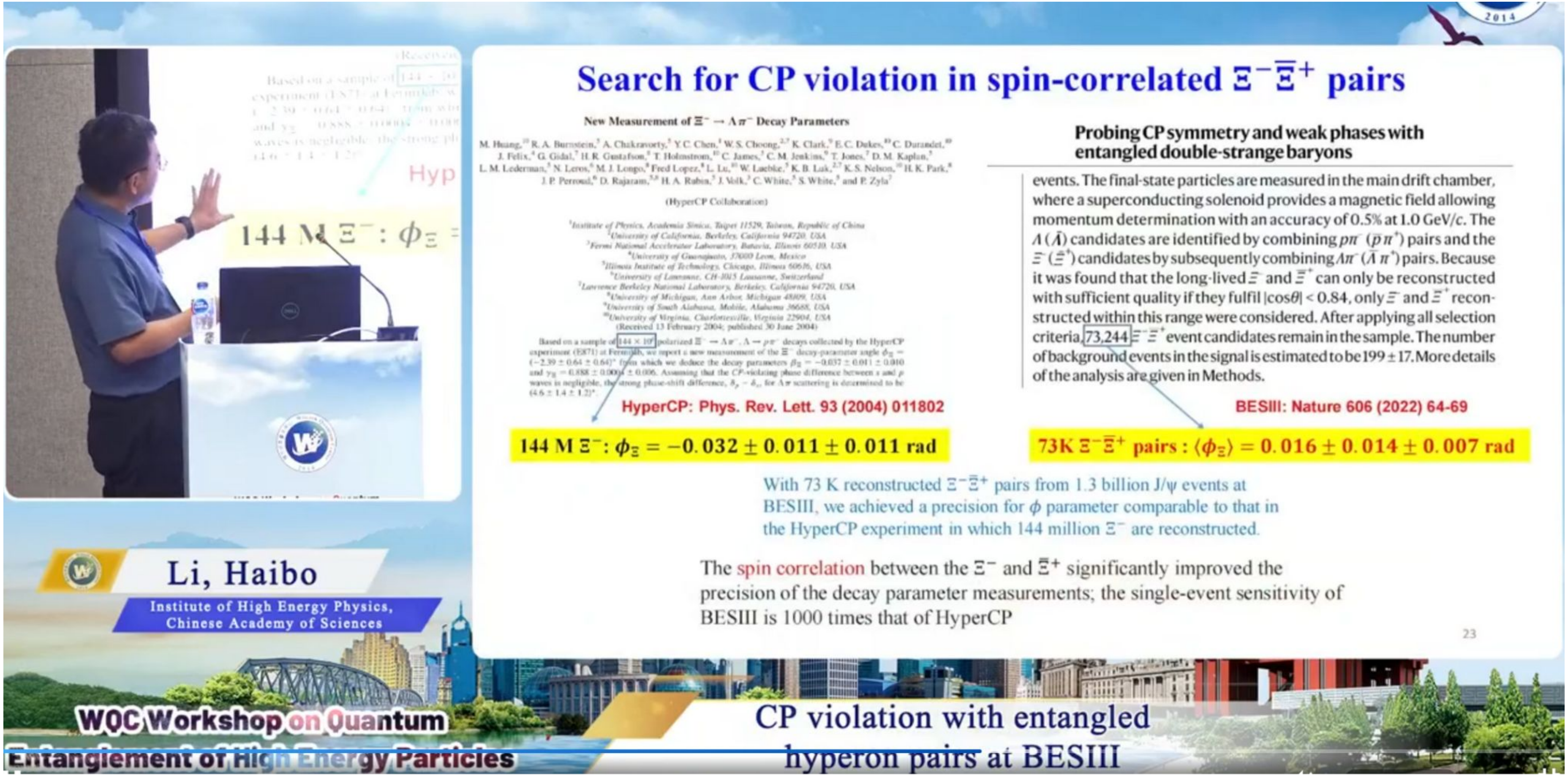
https://agenda.infn.it/event/44563/overview

Workshop on Quantum
Entanglement at the Energy Frontier

Apr 25-27, 2025 @W202, School of Physics, Peking University

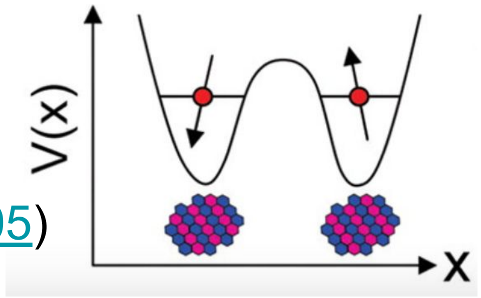
Quantum Observables for Collider Physics 2025

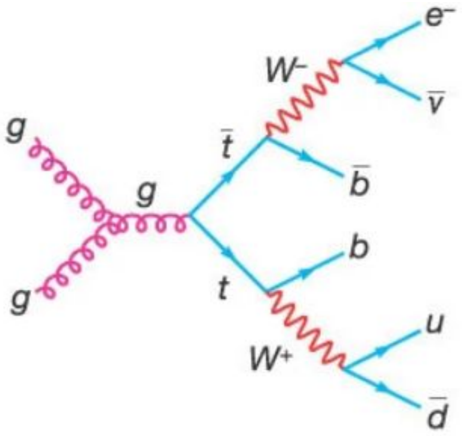
Apr 07, 2025 - Apr 10, 2025

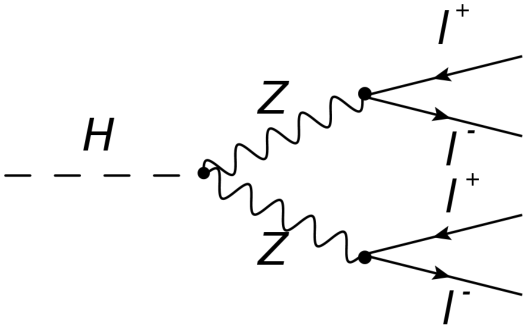

Search for CP violation in spin-correlated Ξ⁻Ξ̄⁺ pairs
New Measurement of Ξ⁻ → Λπ⁻ Decay Parameters
(HyperCP Collaboration)
Probing CP symmetry and weak phases with entangled double-strange baryons
events. The final-state particles are measured in the main drift chamber, where a superconducting solenoid provides a magnetic field allowing momentum determination with an accuracy of 0.5% at 1.0 GeV/c. The Λ (Λ̄) candidates are identified by combining pπ⁻ (p̄π⁺) pairs and the Ξ⁻ (Ξ̄⁺) candidates by subsequently combining Λπ⁻ (Λ̄π⁺) pairs. Because it was found that the long-lived Ξ⁻ and Ξ̄⁺ can only be reconstructed with sufficient quality if they fulfil |cosθ| < 0.84, only Ξ⁻ and Ξ̄⁺ reconstructed within this range were considered. After applying all selection criteria, 73,244 Ξ⁻Ξ̄⁺ event candidates remain in the sample. The number of background events in the signal is estimated to be 199 ± 17. More details of the analysis are given in Methods.
HyperCP: Phys. Rev. Lett. 93 (2004) 011802
BESIII: Nature 606 (2022) 64-69
144 M Ξ⁻: φ_Ξ = −0.032 ± 0.011 ± 0.011 rad
73K Ξ⁻Ξ̄⁺ pairs : ⟨φ_Ξ⟩ = 0.016 ± 0.014 ± 0.007 rad
With 73 K reconstructed Ξ⁻Ξ̄⁺ pairs from 1.3 billion J/ψ events at BESIII, we achieved a precision for φ parameter comparable to that in the HyperCP experiment in which 144 million Ξ⁻ are reconstructed.
The spin correlation between the Ξ⁻ and Ξ̄⁺ significantly improved the precision of the decay parameter measurements; the single-event sensitivity of BESIII is 1000 times that of HyperCP
23
Li, Haibo
Institute of High Energy Physics, Chinese Academy of Sciences
WQC Workshop on Quantum Entanglement of High Energy Particles
CP violation with entangled hyperon pairs at BESIII

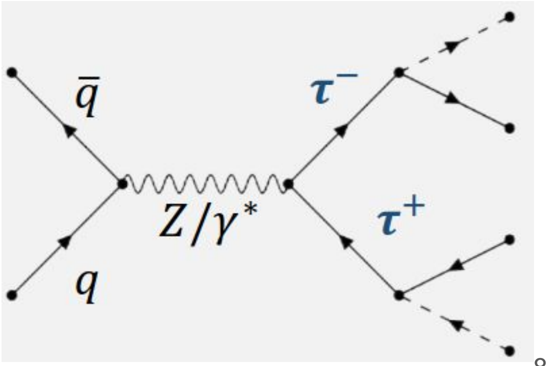
QE between top quarks, tau leptons and …

- The ATLAS and CMS Collaborations recently observed quantum entanglement involving top quarks at a center-of-mass energy of 13 TeV
    - marking the highest energy measurements of QE
- Recent proposals on vector bosons, and tau leptons
- Less attention has been given to **electrons and muons**
    - **Confined electron pairs** (Science309(5744)1116955,2005)
        - confined in semiconductor quantum dots
        - entangled states were prepared, coherently manipulated









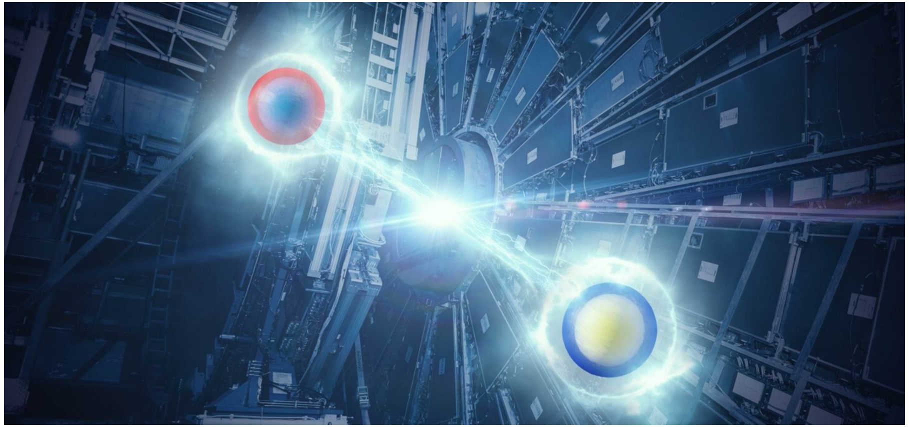
Quantum entanglement at high energy

Nature volume 633, pages 542–547 (2024)

LHC experiments at CERN observe quantum entanglement at the highest energy yet

The results open up a new perspective on the complex world of quantum physics

18 SEPTEMBER, 2024



Article

Observation of quantum entanglement with top quarks at the ATLAS detector

https://doi.org/10.1038/s41586-024-07824-z

Received: 14 November 2023

Accepted: 12 July 2024

Published online: 18 September 2024

Open access

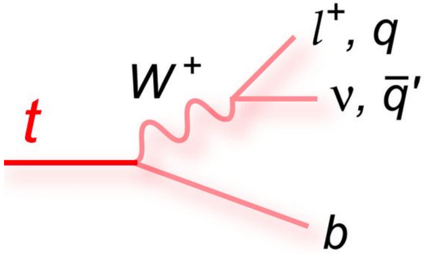
Check for updates

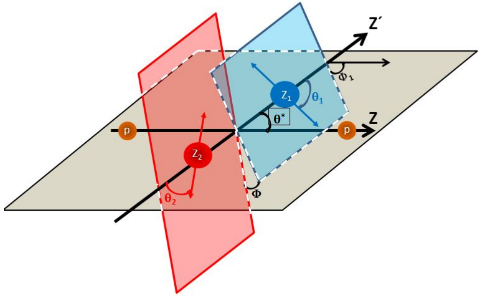
The ATLAS Collaboration*✉

Entanglement is a key feature of quantum mechanics[1–3], with applications in fields such as metrology, cryptography, quantum information and quantum computation[4–8]. It has been observed in a wide variety of systems and length scales, ranging from the microscopic[9–13] to the macroscopic[14–16]. However, entanglement remains largely unexplored at the highest accessible energy scales. Here we report the highest-energy observation of entanglement, in top–antitop quark events produced at the Large Hadron Collider, using a proton–proton collision dataset with a centre-of-mass energy of $\sqrt{s} = 13$ TeV and an integrated luminosity of 140 inverse femtobarns $(\mathrm{fb})^{-1}$ recorded with the ATLAS experiment. Spin entanglement is detected from the measurement of a single observable D, inferred from the angle between the charged leptons in their parent top- and antitop-quark rest frames. The observable is measured in a narrow interval around the top–antitop quark production threshold, at which the entanglement detection is expected to be significant. It is reported in a fiducial phase space defined with stable particles to minimize the uncertainties that stem from the limitations of the Monte Carlo event generators and the parton shower model in modelling top-quark pair production. The entanglement marker is measured to be $D = -0.537 \pm 0.002\,(\mathrm{stat.}) \pm 0.019\,(\mathrm{syst.})$ for $340\ \mathrm{GeV} < m_{t\bar{t}} < 380\ \mathrm{GeV}$. The observed result is more than five standard deviations from a scenario without entanglement and hence constitutes the first observation of entanglement in a pair of quarks and the highest-energy observation of entanglement so far.

Why QE at high energy? (ref)

- **Understand quantum nature & seek for BSM effects.**
- Particle scattering/decay of unstable particles provide **a natural laboratory**
  - the momenta of observed particles are essentially commuting observables. Therefore, there is always some hidden variable theory that can explain the observed momentum data
  - However, one can focus on **spin correlation** emerges in different phase-space region
- It is plausible that **quantum mechanics undergoes modifications** (ref) **at some short distance scales** to achieve compatibility with gravity. Such modifications could, in principle, **be (only) detected by measuring Bell-type observables** or through quantum process tomography (ref)
- offers the potential to uncover **new insights into quantum field theory**.





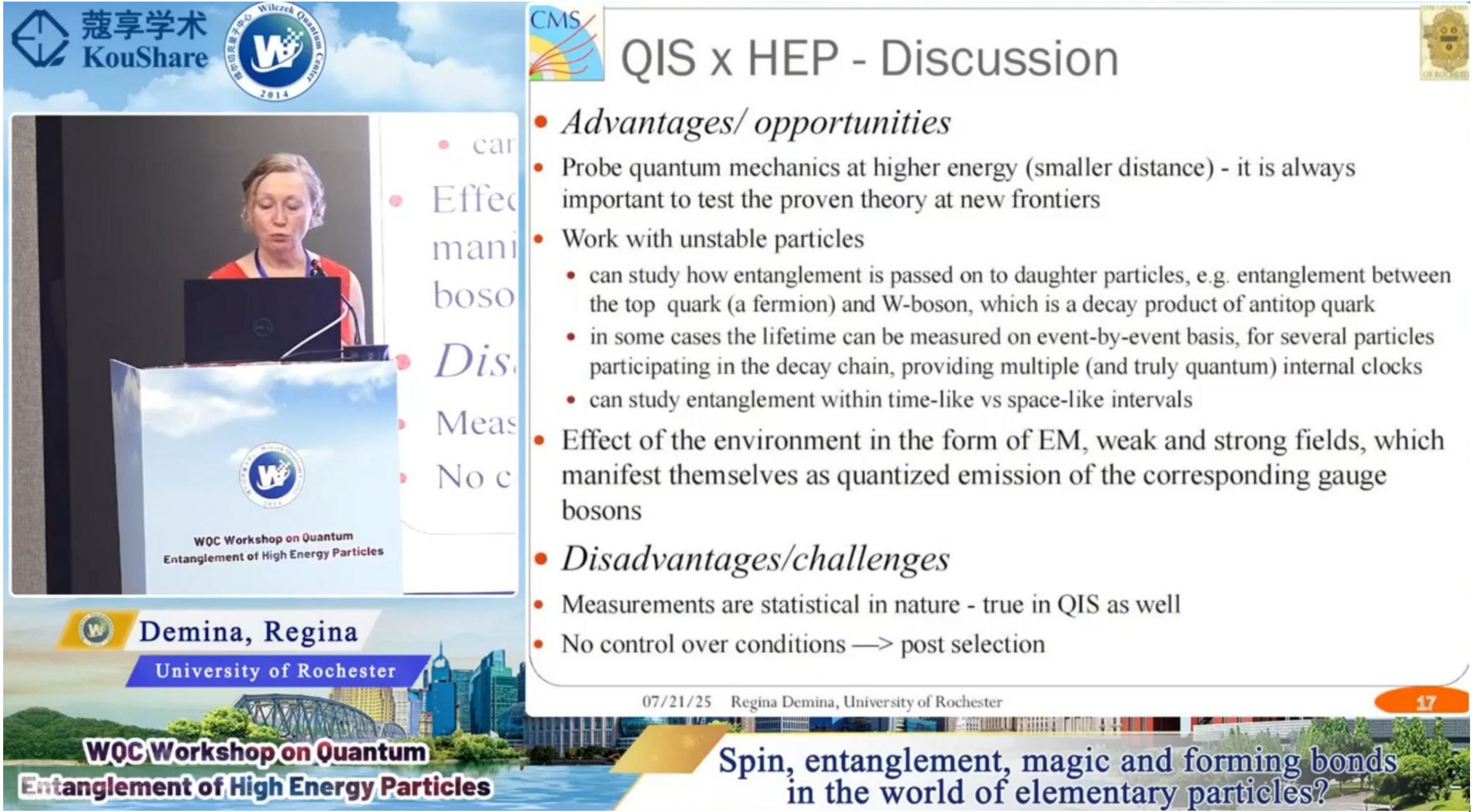
https://scipost.org/10.21468/SciPostPhys.3.5.036


SciPost

SciPost Phys. 3, 036 (2017)

Maximal entanglement in high energy physics

Alba Cervera-Lierta[1], José I. Latorre[1,2], Juan Rojo[3] and Luca Rottoli[4]


蔻享学术
KouShare
Wilczek Quantum Center
2014
QIS x HEP - Discussion
Advantages/ opportunities
Probe quantum mechanics at higher energy (smaller distance) - it is always important to test the proven theory at new frontiers
Work with unstable particles
can study how entanglement is passed on to daughter particles, e.g. entanglement between the top quark (a fermion) and W-boson, which is a decay product of antitop quark
in some cases the lifetime can be measured on event-by-event basis, for several particles participating in the decay chain, providing multiple (and truly quantum) internal clocks
can study entanglement within time-like vs space-like intervals
Effect of the environment in the form of EM, weak and strong fields, which manifest themselves as quantized emission of the corresponding gauge bosons
Disadvantages/challenges
Measurements are statistical in nature - true in QIS as well
No control over conditions —> post selection
07/21/25 Regina Demina, University of Rochester
17
WQC Workshop on Quantum Entanglement of High Energy Particles
Demina, Regina
University of Rochester
WQC Workshop on Quantum Entanglement of High Energy Particles
Spin, entanglement, magic and forming bonds in the world of elementary particles?

https://arxiv.org/abs/2504.00086

Quantum Information meets High-Energy Physics: Input to the update of the European Strategy for Particle Physics

Contact Persons: Yoav Afik [*1]**, Federica Fabbri** [†2,3]**, Matthew Low** [‡4]**, Luca Marzola** [§5,6]**,**

Abstract: Some of the most astonishing and prominent properties of Quantum Mechanics, such as entanglement and Bell nonlocality, have only been studied extensively in dedicated low-energy laboratory setups. The feasibility of these studies in the high-energy regime explored by particle colliders was only recently shown, and has gathered the attention of the scientific community. For the range of particles and fundamental interactions involved, particle colliders provide a novel environment where quantum information theory can be probed, with energies exceeding by about 12 orders of magnitude those employed in dedicated laboratory setups. Furthermore, collider detectors have inherent advantages in performing certain quantum information measurements, and allow for the reconstruction of the state of the system under consideration via quantum state tomography. Here, we elaborate on the potential, challenges, and goals of this innovative and rapidly evolving line of research, and discuss its expected impact on both quantum information theory and high-energy physics.

Debates

Home > The European Physical Journal C > Article

Can Bell inequalities be tested via scattering cross-section at colliders ?

Regular Article | Theoretical Physics | Open access | Published: 21 November 2024
Volume 84, article number 1195, (2024) Cite this article

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Song Li, Wei Shen & Jin Min Yang

Eur. Phys. J. C (2024) 84:1195

Existing methods for Bell tests using colliders rely on the spin correlation coefficients C_{ij}, which essentially require the spin correlations to have a bilinear form. This allows us to derive the spin correlation in any direction from a finite number of spin correlation values through the relation

$$P(\mathbf{n}^a, \mathbf{n}^b) = n_i^a C_{ij} n_j^b = n_i^a P(\mathbf{e}_i^a, \mathbf{e}_j^b) n_j^b. \quad (2.1)$$

- the reconstruction of spin correlations from scattering cross-sections relies on **the bilinear form** of the spin correlations, but **not all local hidden variable models (LHVMs) have such a property.**
- Despite of this, we find that reconstructing spin correlations through scattering cross-sections **can still exclude a broad class of LHVMs**

Debates (LHVT mimicking, and circular argument)

https://arxiv.org/abs/2507.15949

Colliders are Testing neither Locality via Bell's Inequality nor Entanglement versus Non-Entanglement

Steven A. Abel,[a] Herbi K. Dreiner,[b] Rhitaja Sengupta,[b] Lorenzo Ubaldi[c]

https://arxiv.org/abs/2507.15947

ππ expectation follows BI, while spinspin not. However, QE assumption is assumed between them.

$$P_{\mathrm{QM}}(\cos\theta_{\pi\pi}) = \frac{(d\sigma/d\cos\theta_{\pi\pi})(e^+e^-\to\pi^+\pi^-\nu_\tau\bar{\nu}_\tau)}{\sigma(e^+e^-\to\pi^+\pi^-\nu_\tau\bar{\nu}_\tau)} = \tfrac{1}{2}(1-\tfrac{1}{3}\cos\theta_{\pi\pi}) .$$

$$P_{\sigma_\tau\sigma_\tau}(\theta) = \frac{P_{\pi\pi}(\theta)-c_1}{c_2} = -\cos(\theta) .$$

A critical appraisal of tests of locality and of entanglement versus non-entanglement at colliders

Philip Bechtle[a], Cedric Breuning[a], Herbi K. Dreiner[a,b], Claude Duhr[a,b]

Debates

https://indico.cern.ch/event/1460916

14:00 → 14:30 **Testing locality at colliders via Bell's inequality ?**

Speaker: Herbi Dreiner

Bell-CMS-Top-Group...

14:30 → 14:40 **Questions**

14:40 → 15:10 **Quantum Entanglement and Bell Inequality Violation in Semi-Leptonic Top Decays**

Speaker: Tao Han

2024.CERN.pdf | 2024.CERN.pdf | 2024.CERN.pptx

15:10 → 15:20 **Questions**

15:20 → 15:50 **General discussion**

Physics Letters B 280 (1992) 304–312
North-Holland

PHYSICS LETTERS B

Testing locality at colliders via Bell's inequality?

S.A. Abel [a], M. Dittmar [b] and H. Dreiner [a]

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[b] *Department of Physics, University of California, Riverside, CA 92521, USA*

(1992)

Received 5 December 1991; revised manuscript received 6 January 1992

We consider a measurement of correlated spins at LEP and show that it does *not* constitute a *general* test of local-realistic theories via Bell's inequality. The central point of the argument is that such tests, where the spins of two particles are inferred from a scattering distribution, can be described by a local hidden variable theory. We conclude that with present experimental techniques it is not possible to test locality via Bell's inequality at a collider experiment. Finally we suggest an improved fixed-target experiment as a viable test of Bell's inequality.

New observables?!
Good for anything?
More BSM sensitivity?
Probably no!
New future potential?
So interesting!

Main Statement — Theorem

For all experiments where the correlated observables **commute** we can construct an LHVT using the QM function, which exactly reproduces the data.

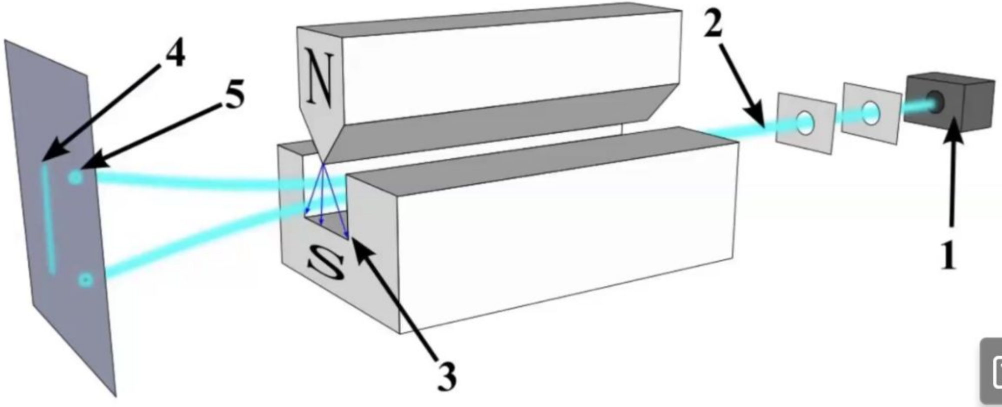
In collider experiments we measure 4-momenta. These all commute. Ergo: all results can be reproduced by an LHVT.

***Bohr/Pauli* have already shown that the spin of a free electron is not measurable.**

Bell's Inequality

- Bell showed for the first time you can distinguish **experimentally** between entangled state and local realistic state

- Problems with this



- Impossible to measure spin-½ with Stern-Gerlach for charged particles, i.e. electrons (or taus)

2022 Nobel Prize for physics:
"pioneering quantum information science"

EPR :

Go! QM Go!

QFT: most precise theory in science!

Our goals:

In the framework of QFT, in the HE regime at colliders,

- We lay out the QM predictions / information.
- We calculate the QM correlations / entanglement
- Hope to establish the quantum tomography.
- Understand quantum nature & seek for BSM effects.

Tao Han

"...EPR paradox is long gone. What we care is how to do quantum information… at high energy…"

Debates

https://arxiv.org/abs/2508.10979

Addressing Local Realism through Bell Tests at Colliders

Matthew Low*

This mirrors **the situation at high-energy colliders, where particle spins are not measured directly but inferred from the angular distributions of their decay products**. We show that, in this setup, **a test of local realism is not possible**.

Quantum correlations, however, are still present, measurable, and informative in high-energy colliders.

Private communications with ML

$$\Gamma^{F_+} \propto \mathbb{1} + \alpha_1\, \mathbf{q}_+ \cdot \vec{\sigma}\,.$$

what is measured is the spin correlations. In relating spin correlations to the density matrix, one needs to use the spin analyzing power. This is why you can't model-independently measure non-zero vs. zero entanglement.

One of the goals of Bell's inequality is to exclude local realism, without making any assumptions. This cannot be done at a collider. **However, once you make an assumption, for example that a collider produces a quantum state, then you can classify your state as Bell local or Bell nonlocal.**
Or **You are measuring Bell nonlocality but not commenting on local realism**

All of these quantum studies at colliders are very interesting and very useful, they simply require a few assumptions, just like all measurements.

Physics Letters B 539 (2002) 235–241

www.elsevier.com/locate/npe

https://linkinghub.elsevier.com/retrieve/pii/S0370269302020981

QCD-corrected spin analysing power of jets in decays of polarized top quarks

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Received 15 May 2002; accepted 10 June 2002

Editor: P.V. Landshoff

	b	l	$\bar{d}\,/\,\bar{s}$
LO analysing power	−0.41	1	1

VOLUME 87, NUMBER 24 PHYSICAL REVIEW LETTERS 10 DECEMBER 2001

Top-Quark Spin Correlations at Hadron Colliders: Predictions at Next-to-Leading Order QCD

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The collider experiments at the Tevatron and the LHC will allow for detailed investigations of the properties of the top quark. This requires precise predictions of the hadronic production of $t\bar{t}$ pairs and of their subsequent decays. In this Letter we present for the reactions $p\bar{p}, pp \to t\bar{t} + X \to \ell^+\ell'^- + X$ the first calculation of the dilepton angular distribution at next-to-leading order in the QCD coupling, keeping the full dependence on the spins of the intermediate $t\bar{t}$ state. The angular distribution reflects the degree of correlation of the t and $\bar{t}$ spins which we determine for different choices of t and $\bar{t}$ spin bases. In the case of the Tevatron, the QCD corrections are sizable, and the distribution is quite sensitive to the parton content of the proton.

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https://arxiv.org/abs/2510.08031

Quantum Entanglement without Spin-Analyzing Power Dependence at the Colliders

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ABSTRACT: We study the quantum entanglement at the colliders which is independent of the spin-analyzing powers. Taking $\Lambda(\to p\pi^-)\bar{\Lambda}(\to \bar{p}\pi^+)$ as an example, we investigate whether quantum entanglement in fermion pairs produced at colliders can be certified by using only angular information from final-state decays, while remaining independent of the parity-violating decay parameters α_Λ and $\alpha_{\bar{\Lambda}}$. Building on a general decomposition of any

Let’s start

Quantum State

For a state vector $|\phi_i\rangle$

Density matrix

a state

$$\rho = \sum_i n_i \left|\phi_i\right\rangle \left\langle\phi_i\right|$$

an observable

$$\langle\mathcal{O}\rangle = \mathrm{Tr}(\mathcal{O}\rho)$$

For a pure state: $n_i = 1$; for a mixed state: $\Sigma_i n_i = 1$.

For a **single qubit** (*i.e.*, a doublet of spin, iso-spin etc.):

$$\rho = \frac{1}{2}\left(\mathbb{I}_2 + \sum_i B_i \sigma_i\right)$$

For a bipartite system (*i.e.*, ½⊗½)

$$\rho = \frac{1}{4}\left(\mathbb{I}_4 + \sum_i \left(B_i^{\mathcal{A}}\left(\sigma_i \otimes \mathbb{I}_2\right) + B_i^{\mathcal{B}}\left(\mathbb{I}_2 \otimes \sigma_i\right)\right) + \sum_{i,j} C_{ij}\left(\sigma_i \otimes \sigma_j\right)\right)$$

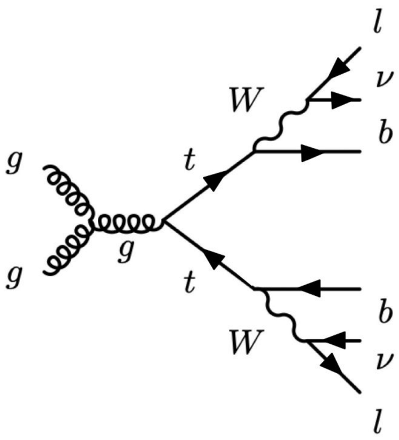
$B_i^{A,B}$ the polarizations, C_{ij} the spin-correlation matrix

The 15 coefficients → **Quantum Tomography** for the bipartite.

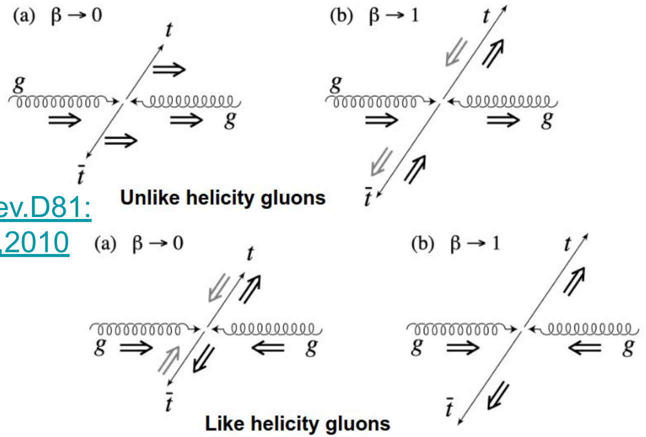
Top quark

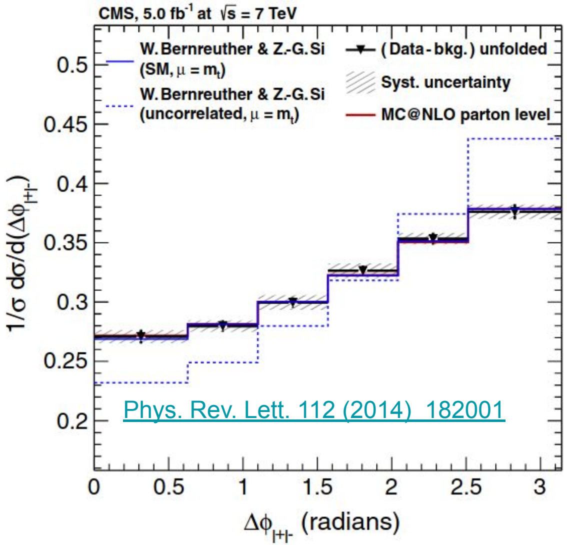
- The most massive fundamental particle : $m_t \approx 173$ GeV
- Large width : $\Gamma_t \sim 1$ GeV
  - Short lifetime : $\tau = 1/\Gamma_t \sim 10^{-25}$ s
    - ✓ decay before hadronisation : $\sim 10^{-23}$ s
      - In case of top pair production, $t\bar{t}$ spins can be measured from decay products
      - The effect due to spin correlation has already been measured in several experiments.

BR(t→Wb)~100% + weak interaction is maximally parity-violating → correlations are observable!



Phys.Rev.D81: 074024,2010



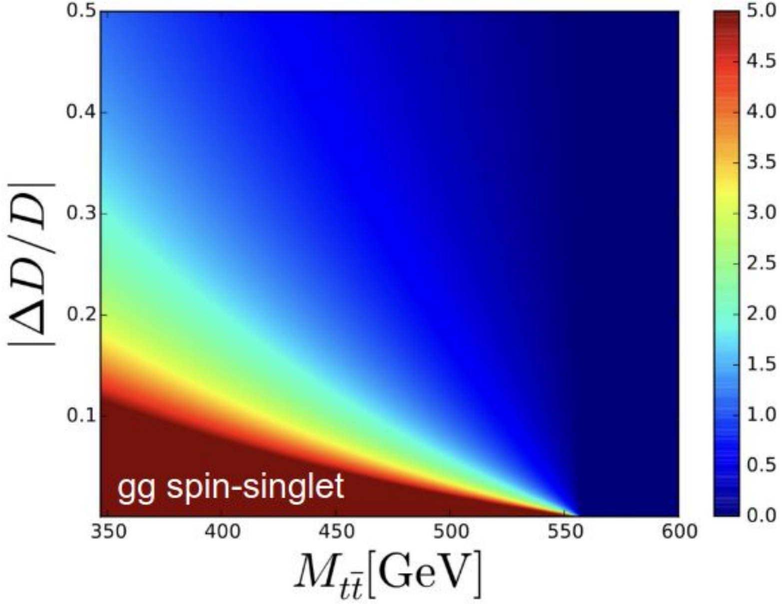


Phys. Rev. Lett. 112 (2014) 182001

Top quark QE in a nutshell

Eur. Phys. J. Plus (2021) 136 (March 2020) → first analysis of top quark pair production from the quantum information point of view: “bipartite qubit system”

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1\Omega_2} = \frac{1}{4\pi^2}\left(1 + \alpha_1\, \cancelto{0}{\mathbf{B}_1} \cdot \hat{\ell}_1 + \alpha_2\, \cancelto{0}{\mathbf{B}_2} \cdot \hat{\ell}_2 + \alpha_1\alpha_2\, \hat{\ell}_1\, \mathbb{C}\, \hat{\ell}_2\right)$$



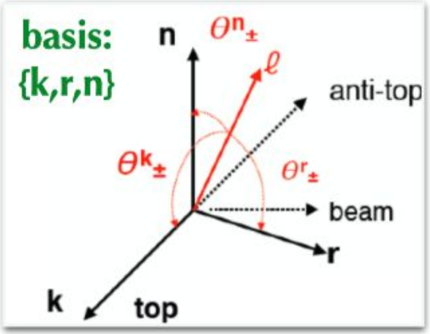
$$\mathrm{Tr}\,[\mathbb{C}] < -1$$ *Peres-Horodecki criterion*

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$ *a simple observable*

Peres-Horodecki criterion

$$D = \frac{\mathrm{Tr}\,[\mathbb{C}]}{3} \Rightarrow D < -\frac{1}{3}$$ *a quantum entanglement marker!*

Spin correlations



Fictitious state, event based (helicity) frame, beam frame

- Probed by angular distribution of decay products in helicity basis:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2}\left(1 + \underbrace{\mathbf{B}^+\cdot\hat{\ell}^+ + \mathbf{B}^-\cdot\hat{\ell}^-}_{\text{polarization}} - \underbrace{\hat{\ell}^+\cdot\mathbb{C}\cdot\hat{\ell}^-}_{\text{spin correlations}}\right)$$

$$B^{+/-} = \begin{pmatrix} \times \\ \times \\ \times \end{pmatrix} \quad C = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$

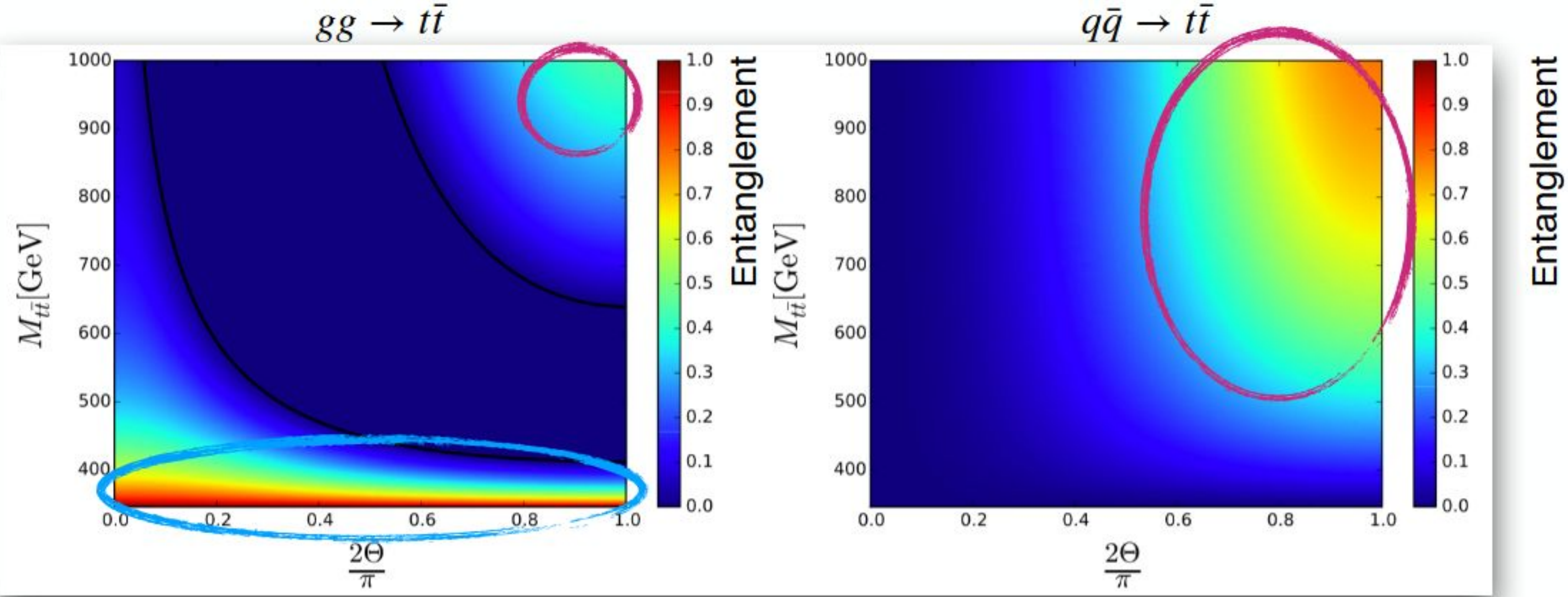
- Spin dependence of $t\bar{t}$ production **completely characterized by 15 coefficients**
  - can be probed individually by measuring 1D angular distributions

$$\boxed{\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{1}{2}\left(1 + [\text{Coef.}]\,x\right) f(x)}$$

Observable	Coefficient	Coefficient function
$\cos\theta_1^k$	B_1^k	b_k^+
$\cos\theta_2^k$	B_2^k	b_k^-
$\cos\theta_1^r$	B_1^r	b_r^+
$\cos\theta_2^r$	B_2^r	b_r^-
$\cos\theta_1^n$	B_1^n	b_n^+
$\cos\theta_2^n$	B_2^n	b_n^-
$\cos\theta_1^k\cos\theta_2^k$	C_{kk}	c_{kk}
$\cos\theta_1^r\cos\theta_2^r$	C_{rr}	c_{rr}
$\cos\theta_1^n\cos\theta_2^n$	C_{nn}	c_{nn}
$\cos\theta_1^r\cos\theta_2^k + \cos\theta_1^k\cos\theta_2^r$	$C_{rk} + C_{kr}$	c_{rk}
$\cos\theta_1^r\cos\theta_2^k - \cos\theta_1^k\cos\theta_2^r$	$C_{rk} - C_{kr}$	c_n
$\cos\theta_1^n\cos\theta_2^r + \cos\theta_1^r\cos\theta_2^n$	$C_{nr} + C_{rn}$	c_{nr}
$\cos\theta_1^n\cos\theta_2^r - \cos\theta_1^r\cos\theta_2^n$	$C_{nr} - C_{rn}$	c_k
$\cos\theta_1^n\cos\theta_2^k + \cos\theta_1^k\cos\theta_2^n$	$C_{nk} + C_{kn}$	c_{kn}
$\cos\theta_1^n\cos\theta_2^k - \cos\theta_1^k\cos\theta_2^n$	$C_{nk} - C_{kn}$	$-c_r$
$\cos\varphi$	D	$-(c_{kk} + c_{rr} + c_{nn})/3$
$\cos\varphi_{\mathrm{lab}}$	$A^{\mathrm{lab}}_{\cos\varphi}$	—
$\lvert\Delta\phi_{\ell\ell}\rvert$	$A_{\lvert\Delta\phi_{\ell\ell}\rvert}$	—
$\lvert\Delta\eta_{\ell\ell}\rvert$	$\lvert\Delta\eta_{\ell\ell}\rvert$	—

- SM predicts entangled states:
  - at the production threshold region in gg fusion production
  - at the boosted region for central production of the $t\bar{t}$ system

high relative velocity of top quarks
→ space-like separated events



low relative velocity of top quarks
→ time-like separated events

Afik, De Nova
Eur. Phys. J. Plus **136**, 907

- **Peres-Horodecki criterion:**
  using simpler observables, a **sufficient condition to observe entanglement in top quarks** is:

$$\boxed{\Delta_E = C_{nn} + |C_{rr} + C_{kk}| > 1}$$

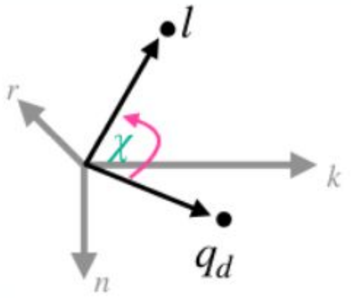
Afik, De Nova
Eur. Phys. J. Plus **136**, 907

- **Two approaches:**
  - Use **full angular information of two decay products** to measure full matrix C and then construct Δ_E
  - Use χ **and** $\tilde{\chi}$ **distributions** to measure D and $\tilde{D}$
    - potentially improved sensitivity since we can use simpler 1D angular distributions

$$\frac{d\sigma}{d\cos\chi} = A(1 + D\cos\chi) \quad \to \frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$ in dilepton events

$$\frac{d\sigma}{d\cos\tilde{\chi}} = A(1 + \tilde{D}\cos\tilde{\chi})$$

χ = opening angle between the 2 decay products



$\tilde{\chi} = \chi$ with inverted sign of n-component in one of the decay products

- Four maximally entangled states: $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)$

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)$$

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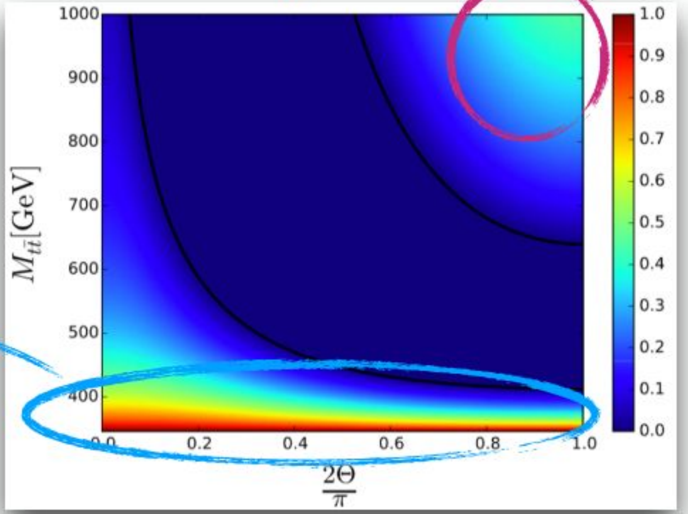
$gg \to t\bar{t}$



- Spin-triplet vector state ($\Phi^+ - \Phi^-$, Ψ^+, $\Phi^+ + \Phi^-$)
- At high $m_{t\bar{t}}$ and low $|\cos\Theta|$: $C_{kk} < 0$ and $C_{rr} < 0$

$$\Delta_E = C_{nn} - C_{rr} - C_{kk} = 3\tilde{D} > 1$$

$$\to \tilde{D} > 1/3$$

- Spin-singlet pseudoscalar state Ψ^-
- At low $m_{t\bar{t}}$: $C_{rr} > 0$ and $C_{kk} > 0$

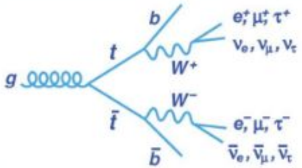
$$\Delta_E = C_{nn} + C_{rr} + C_{kk} = Tr[C] = -3D > 1$$

$$D = -\frac{\text{tr}[C]}{3} \quad \to D < -1/3$$

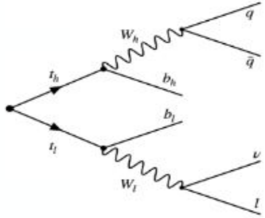
$$\Delta_E = C_{nn} + |C_{rr} + C_{kk}| > 1$$

Sufficient condition for entanglement

→ **measure D, $\tilde{D}$ to access entanglement information in top quark events!**



Dilepton vs lepton+jets



Dilepton

arXiv:2406.03976
submitted to ROPP

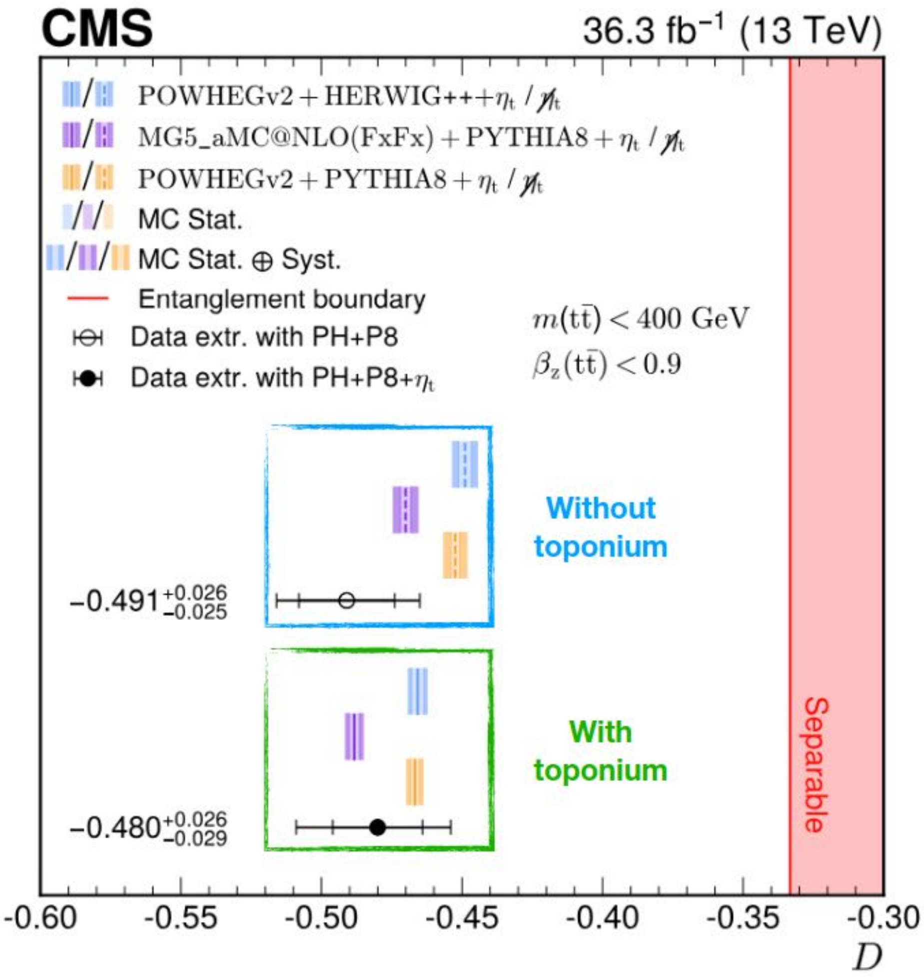
- **36.3 fb^{-1} of 2016 data @13 TeV**
  - based on PRD 100 (2019) 072002
- Lower branching ratio
- top spin info 100 % transmitted to charged leptons → easy to identify
- Lower p_T cuts for leading/subleading lepton (25/20 GeV) → higher efficiency at the threshold
- Worse $m_{t\bar{t}}$ resolution → not ideal for differential measurement
- Best for threshold region
  - high entanglement
  - mostly **time-like separated events**

Lepton + jets

CMS-PAS-TOP-23-007

- **138 fb^{-1} of data @13 TeV** collected in **full Run 2**
- Higher branching ratio
- top spin info ~100 % transmitted to down-type quarks → hard to identify
- Higher p_T cut for single lepton (30 GeV) and for 4 jets (30 GeV) → lower efficiency at the threshold but OK for high $m_{t\bar{t}}$
- Better $m_{t\bar{t}}$ resolution → good for differential measurement
- Advantage for high $m_{t\bar{t}}$
  - high entanglement
  - mostly **space-like separated events**

Dilepton results



- PowhegBox+Pythia8 (NLO) as **nominal $t\bar{t}$ sample**
- **toponium model** generated with MG5 aMC@NLO(LO)+Pythia8

B. Fuks et al.
Phys Rev D 104 034023

  - $337 < m_{\eta_t} < 349$ GeV
  - only pseudoscalar colour singlet and spin-0 η_t state accounted for

- **Same uncertainties considered in 2016 spin corr analysis + additional ones for toponium:**
  - *toponium cross section varied by 50%* due to missing octet contributions
  - *binding energy uncertainty varied by ±0.5 GeV*
- **Leading theory-based uncertainties:**
  - Toponium normalization
  - NNLO QCD reweighing
  - Parton Shower
- **Leading experimental uncertainties:**
  - Jet energy scale

Lepton + jets: fit strategy

- The total cross section is a **linear combination of Σ_m templates** with P and C coefficients Q_m:

$$\boxed{\Sigma_{tot} = \Sigma_0 + \Sigma_{m=1}^{15} Q_m \Sigma_m}$$

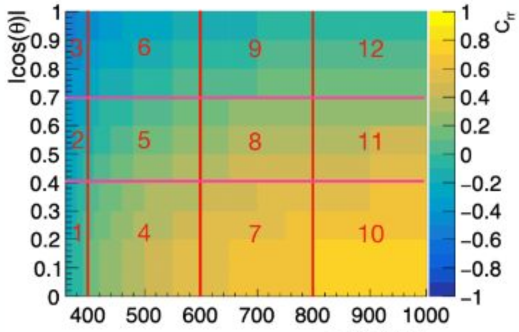
$$\Sigma_m = \sigma_{norm}\{\sin\theta_p \cos\phi_p, \sin\theta_p \sin\phi_p, \ldots, \cos\theta_p \cos\theta_{\bar{p}}\}$$

Q_m can be extracted by fitting Σ_{tot}

- **Events are reweighted at the generator level**

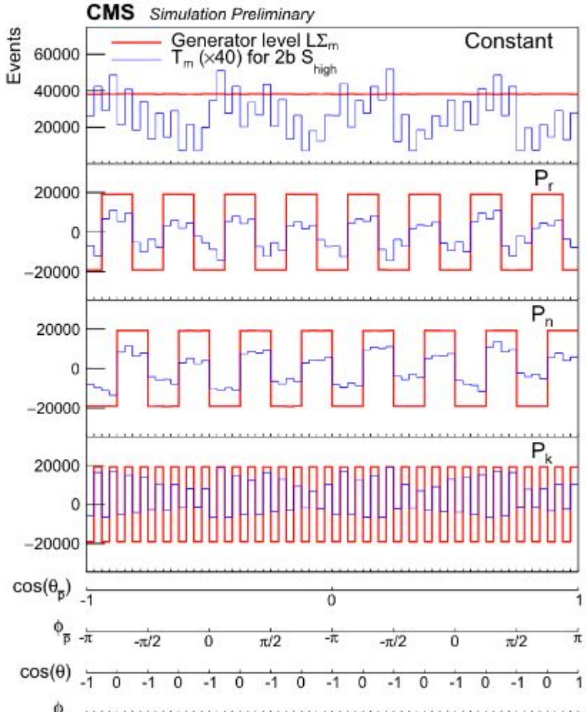
$$w_m = \frac{\Sigma_m}{\Sigma_{tot}}$$

- To minimize bias due to variations of T_m within a bin, measurements are performed in **sufficiently small bins**



$L\Sigma_m$ = templates used at gen level

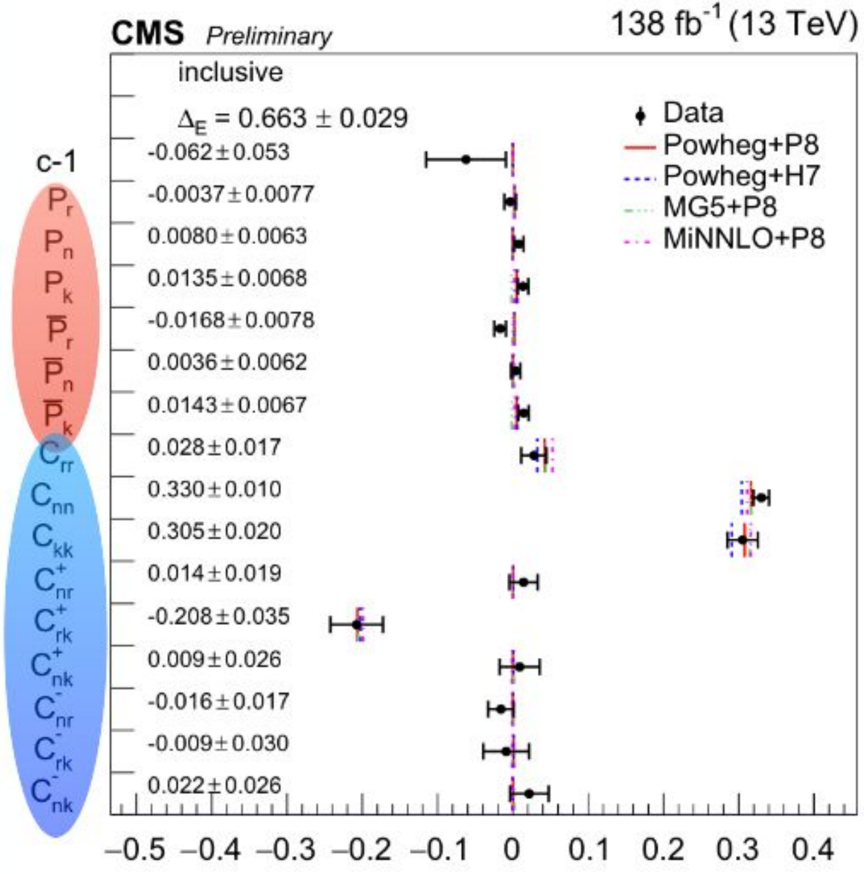
T_m = templates defined at reco level



- **Full measurement of P and C performed inclusively and differentially**
- **Good agreement with SM prediction**

$$B^{+/-} = \begin{pmatrix} \times \\ \times \\ \times \end{pmatrix}$$

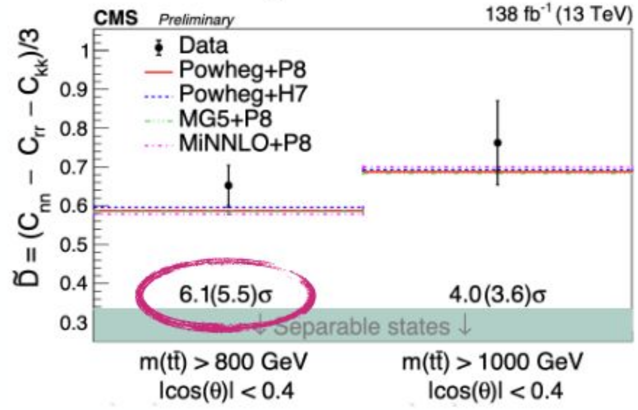
$$C = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$



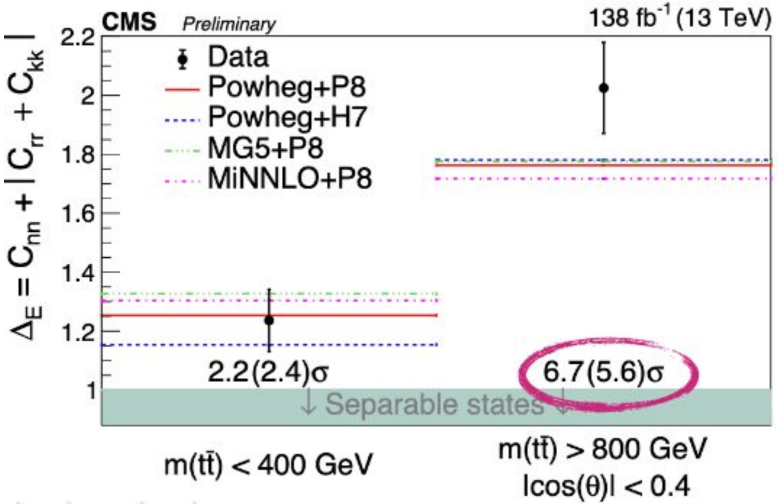
Lepton+jets: entanglement

- **Entanglement observed for first time in space-like separated events!**
  - highest sensitivity in full matrix measurement
- No sensitivity in low $m_{t\bar{t}}$ region

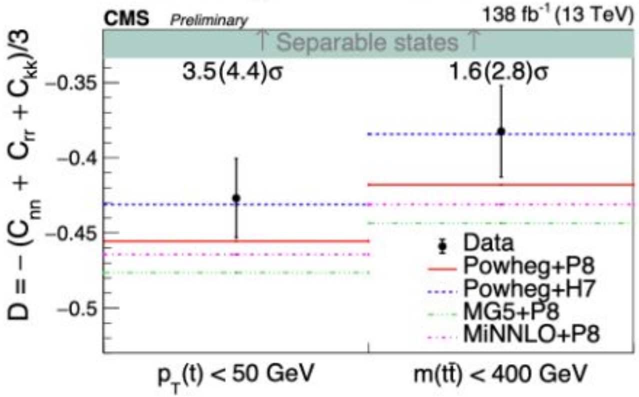
High $m_{t\bar{t}}$ $\tilde{D} > 1/3$



Full matrix $\Delta_E = C_{nn} + |C_{rr} + C_{kk}| > 1$



Low $m_{t\bar{t}}$ $D < -1/3$



QE between qutrits: H$\longrightarrow$ VV

For two-triplet system, we can expand the density matrix as

$$\rho = \frac{1}{9}\left[\mathbb{1}\otimes\mathbb{1}\right] + \sum_{a=1}^{8} f_a\left[T^a\otimes\mathbb{1}\right] + \sum_{a=1}^{8} g_a\left[\mathbb{1}\otimes T^a\right] + \sum_{a,b=1}^{8} h_{ab}\left[T^a\otimes T^b\right]$$

1+8+8+8*8=81

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+\, d\Omega_-} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left[\underbrace{\rho_{V_1V_2}}_{\text{Production}}\underbrace{(\Gamma_1\otimes\Gamma_2)}_{\text{Decay}}\right]$$

All coefficients → Quantum Tomography

- No direct spin measurements: inferred by angular distributions.
- Both the state before decay & the final state decay products inherit the SAME quantum information.

The polarization density matrix of the ZZ system is:

$$\rho = \sum p_\ell |\ell\rangle\langle\ell|,\ p_\ell \geq 0,\ \sum p_\ell = 1,$$

where

$$|\psi_{ZZ}\rangle = \frac{1}{\sqrt{2+\beta^2}}\left(|+-\rangle - \beta|00\rangle + |-+\rangle\right),\ \beta = 1 + \frac{m_H^2-(m_1+m_2)^2}{2m_1m_2},$$

assuming spin-0 and even parity.

$$\rho \propto \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_+a_+^* & 0 & a_+a_0^* & 0 & a_+a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_0a_+^* & 0 & a_0a_0^* & 0 & a_0a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_-a_+^* & 0 & a_-a_0^* & 0 & a_-a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

generally applying to a pure state.

Method 1: Quantum Tomography

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+\, d\Omega_-} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left[\rho_{V_1 V_2}\left(\Gamma_1 \otimes \Gamma_2\right)\right],$$

$$\Gamma(\theta,\phi) = \frac{1}{4}\begin{pmatrix} 1+\cos^2\theta - 2\eta_\ell\cos\theta & \frac{1}{\sqrt{2}}(\sin 2\theta - 2\eta_\ell \sin\theta)e^{i\phi} & (1-\cos^2\theta)e^{i2\phi} \\ \frac{1}{\sqrt{2}}(\sin 2\theta - 2\eta_\ell\sin\theta)e^{-i\phi} & 2\sin^2\theta & -\frac{1}{\sqrt{2}}(\sin 2\theta + 2\eta_\ell\sin\theta)e^{i\phi} \\ (1-\cos^2\theta)e^{-i2\phi} & -\frac{1}{\sqrt{2}}(\sin 2\theta + 2\eta_\ell\sin\theta)e^{-i\phi} & 1+\cos^2\theta - 2\eta_\ell\cos\theta \end{pmatrix};$$

$$\longrightarrow \quad \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + A^1_{LM} Y^M_L(\theta_1,\phi_1) + A^2_{LM} B_L Y^M_L(\theta_2,\phi_2) + C_{L_1 M_1 L_2 M_2} B_{L_1} B_{L_2} Y^{M_1}_{L_1}(\theta_1,\phi_1) Y^{M_2}_{L_2}(\theta_2,\phi_2)\Big],$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^M_L(\Omega_j) d\Omega_j = \frac{B_L}{4\pi} A^j_{LM}, \qquad j = 1,2;$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_1) d\Omega_1 d\Omega_2 = \frac{B_{L_1} B_{L_2}}{4\pi} C_{L_1 M_1 L_2 M_2}.$$

Integral $\longrightarrow$ events summed

Notice that the theoretical form of the density matrix imposes strong constraints on the various coefficients: this assumption can be relaxed though

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}\left(1-\sqrt{2}A^1_{2,0}\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

More details in

[PRD 107 (2023) 1, 016012]
[JHEP 10 (2024) 211]
[PRD 111 (2025) 3, 036008]

Method 1: Quantum Tomography

- The normalized joint angular distribution in terms of spherical harmonics and A, C and B coefficients

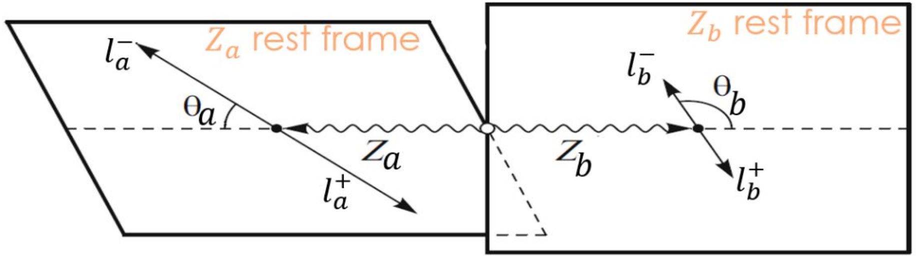
$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{1}{(4\pi)^2}[1 + A^a_{L,M} B^a_L Y_{L,M}(\theta_a,\phi_a) + A^b_{L,M} B^b_L Y_{L,M}(\theta_b,\phi_b) + C_{L_a,M_a,L_b,M_b} B^a_{L_a} B^b_{L_b} Y_{L_a,M_a}(\theta_a,\phi_a) Y_{L_b,M_b}(\theta_b,\phi_b)]$$

- We can compute full spin density matrix using experimental data by using the following tomographic reconstruction:

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y^*_{M,L}(\Omega_j)\, d\Omega_a d\Omega_b = \frac{B^j_L}{4\pi} A^j_{L,M} \quad j = a, b\,,$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y^*_{M_a,L_a}(\Omega_a) Y^*_{M_b,L_b}(\Omega_b)\, d\Omega_a d\Omega_b = \frac{B^a_{L_a} B^b_{L_b}}{(4\pi)^2} C_{L_a,M_a,L_b,M_b}$$

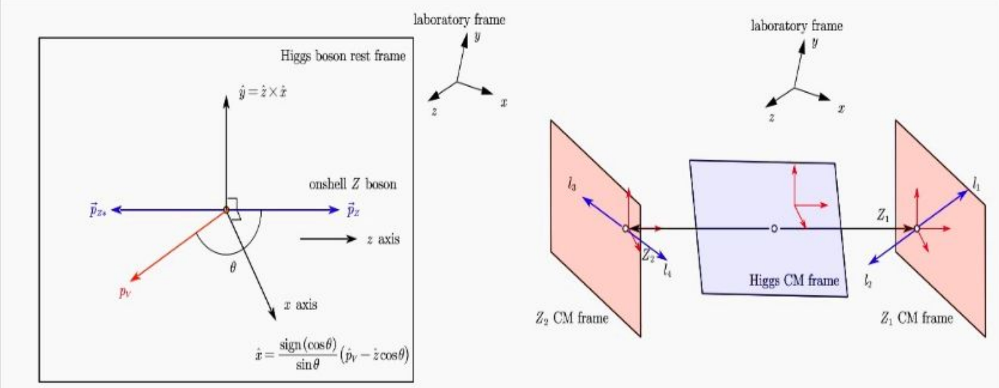
$$d\Omega_a = \sin\theta_a d\theta_a d\phi_a$$



Method 1: Quantum Tomography

In two spin-1 massive bosons' system:

- The z-axis is the direction of the on-shell Z boson's 3-momentum.
- The $\hat{x}$ axis is in the production plane: $\hat{x} = \frac{sign(cos\theta)(\hat{p}_p - cos\theta\hat{z})}{sin\theta}, \hat{p}_p = (0,0,1)$
- The $\hat{y} = \hat{z}\times\hat{x}$
- J_Z is the polarization operator.
- The eigenstates of J_Z is the basis of the spin space.



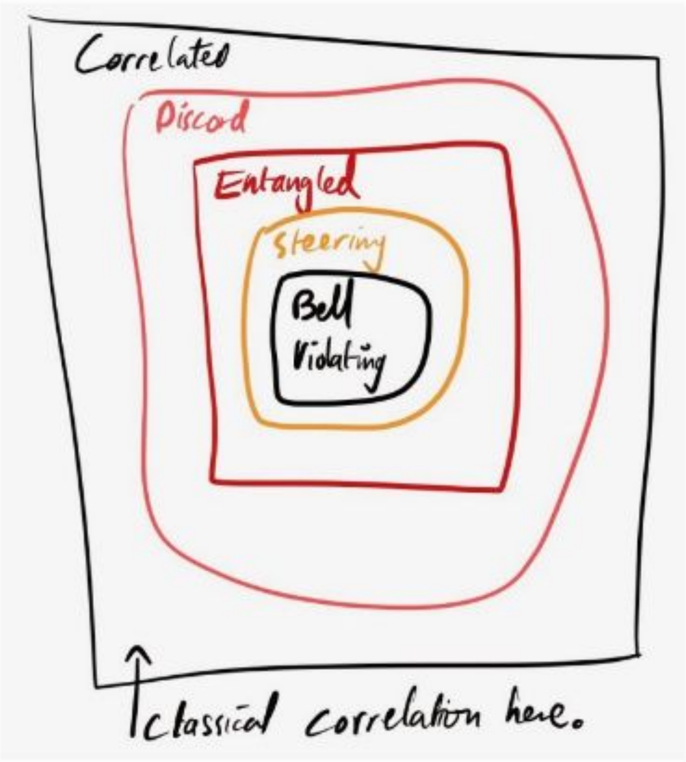
Two Lorentz Transformation:

- Higgs rest frame → determine Z axis
- Z boson rest frame(boost along Z vector) → lepton's polar angles

Obtain :(θ_1, φ_1) in Z_1 rest frame, (θ_2, φ_2) in Z_2 rest frame. The coefficients can A^I_{LM} *and* $C_{L_1M_1L_2M_2}$ can be calculated

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^1_{LM}T^L_M \otimes \mathbb{1}_3 + A^2_{LM}\mathbb{1}_3 \otimes T^L_M + C_{L_1M_1L_2M_2}T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right]$$

Method 1: Quantum Tomography



SUSY2024

The theoretical form of the density matrix imposes strong constraints and leads to a entanglement criteria

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c \\ 0 & 0 & 0 & 0 & 0 & b & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & f & 0 \\ 0 & 0 & 0 & 0 & d & 0 & 0 & 0 & 0 \\ 0 & b^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & f^* & 0 & 0 & 0 & 0 & 0 \\ c^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{29}$$

which has eigenvalues $a, d, g, \pm|b|, \pm|c|, \pm|f|$. Therefore if $b \neq 0$, $c \neq 0$ or $f \neq 0$ the density matrix is entangled. Note that the reverse is also true: if $b = c = f = 0$ the state is obviously separable, as ρ is diagonal in the separable basis. This represents a noteworthy example beyond a two-qubit system, where, thanks to an underlying symmetry, the Peres-Horodecki condition for entanglement is not just sufficient, but also necessary.

When applied this condition to our density matrix (26), it turns out that the ZZ system is entangled if and only if

$$\boxed{C_{2,1,2,-1} \neq 0 \qquad \text{or} \qquad C_{2,2,2,-2} \neq 0\,.}$$

Method 1: Quantum Tomography

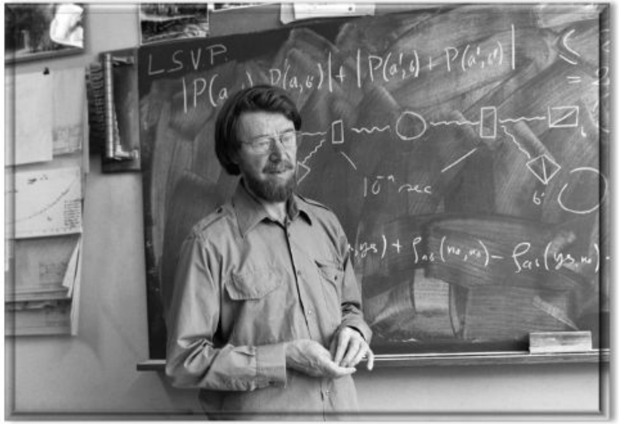
- The most original form of Bell inequalities (Clauser-Horne-Shimony-Holt Inequality):

$$\mathrm{P}(\mathrm{A}_1\mathrm{B}_1|\mathrm{AB},\lambda) = \mathrm{P}(\mathrm{A}_1|\mathrm{A},\lambda)\mathrm{P}(\mathrm{B}_1|\mathrm{B},\lambda)$$

Classical local hidden variable theory:

$$\boldsymbol{I_3 = \langle O_{Bell}\rangle = Tr\{\rho O_{Bell}\} \leq 2}$$

ρ **: Polarization density matrix (PDM)**



- More general form (Collins-Gisin-Linden-Massar-Popescu Inequality)

$$\begin{aligned}\mathcal{I}_d = \sum_{k-0}^{[d/2]-1}(1-\frac{2k}{d-1})\{+[&P(A_1 = B_1 + k) + P(B_1 = A_2 + k + 1) + P(A_2 = B_2 + k)\\ &+ P(B_2 = A_1 + k) - [P(A_1 = B_1 - k - 1) + P(B_1 = A_2 - k)\\ &+ P(A_2 = B_2 - k - 1) + P(B_2 = A_1 - k - 1)]\}\end{aligned}$$

3-dimensional form:

$$\begin{aligned}\mathcal{I}_3 = &P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1)\\ &- [P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)].\end{aligned}$$

- The expectation value of the Bell operator can be written as:

$$\begin{aligned}\mathcal{B} = \Big[&\frac{2}{3\sqrt{3}}(T_1^1 \otimes T_1^1 - T_0^1 \otimes T_0^1 + T_1^1 \otimes T_{-1}^1) + \frac{1}{12}(T_2^2 \otimes T_2^2 + T_2^2 \otimes T_{-2}^2)\\ &+ \frac{1}{2\sqrt{6}}(T_2^2 \otimes T_0^2 + T_0^2 \otimes T_2^2) - \frac{1}{3}(T_1^2 \otimes T_1^2 + T_1^2 \otimes T_{-1}^2) + \frac{1}{4}T_0^2 \otimes T_0^2\Big] + \text{h.c.}.\end{aligned}$$

- Bell inequality expectation value can be calculated:

$$\mathcal{I}_3 = \frac{1}{36}\left(18 + 16\sqrt{3} - \sqrt{2}\left(9 - 8\sqrt{3}\right)A_{2,0}^1 - 8\left(3 + 2\sqrt{3}\right)C_{2,1,2,-1} + 6C_{2,2,2,-2}\right)$$

Method 2: Polarization Fit

- Assuming the SM H–ZZ coupling with CP conservation, we have (LO)

$$C_{222-2} = 3\frac{1}{\mathcal{N}} a_{11} a^*_{-1-1};$$

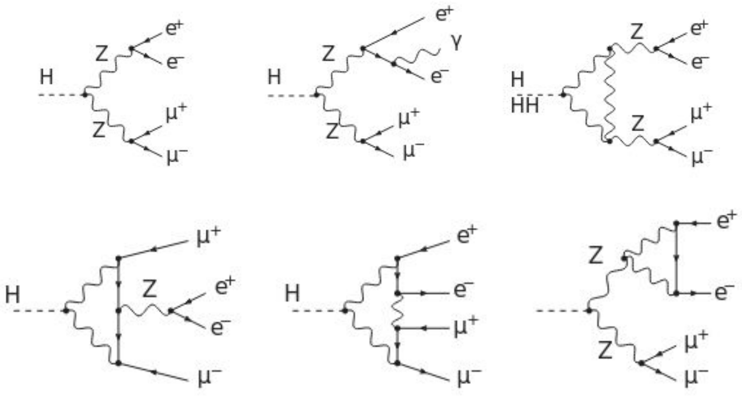
  a necessary and sufficient condition for entanglement is $C_{222-2} \neq 0$ (the so-called tomography method).

- Since $a_{11} = a_{-1-1}$, the ZZ pair is entangled whenever these amplitudes are nonzero (the so-called polarization method).
- Thus, the test of entanglement consists of comparing the SM prediction with the case where only longitudinal (LL) ZZ polarization is present, for which $a_{11} = a_{-1-1} = 0$ and $a_{00} \neq 0$.

Comparison between the two methods:

Method	Handle	Advantages
Tomography	$C_{2,1,2,-1} \neq 0$ or $C_{2,2,2,-2} \neq 0$	less model-dependence
Polarization	$C_{2,2,2,-2} \neq 0 \;\Leftarrow\; a_{11} \neq 0$	higher sensitivity

Giovanni Pellicciioli



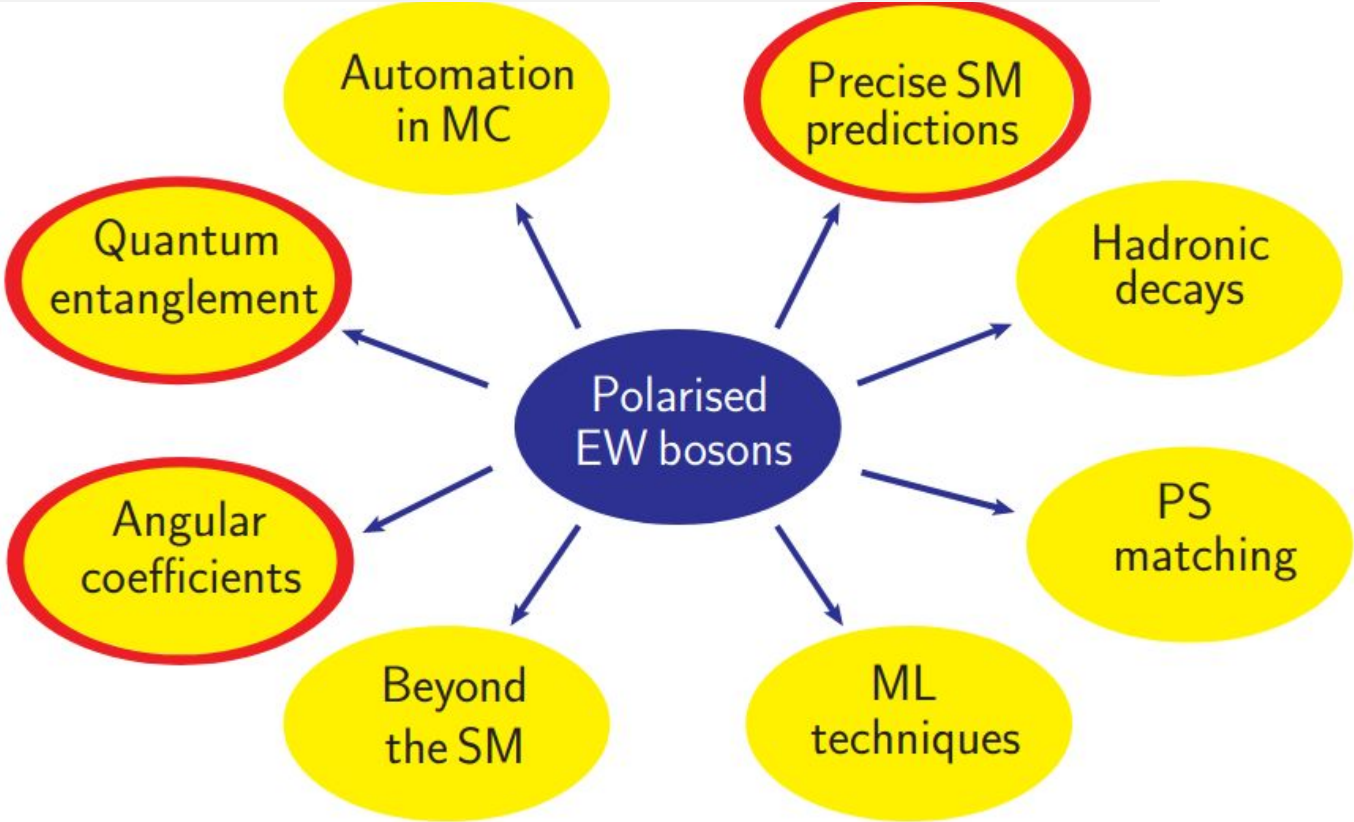
Fiducial cuts on decay products introduce angular modulation $f_{\rm cut}$ which is not a combination of $\ell \leq 2$ spherical harmonics,

$$\int_{\rm cut} \frac{d\sigma}{d\cos\theta^*\, d\phi^*\, dX} dX = \frac{d\sigma}{d\cos\theta^*\, d\phi^*} f_{\rm cut}(\theta^*, \phi^*)\,.$$

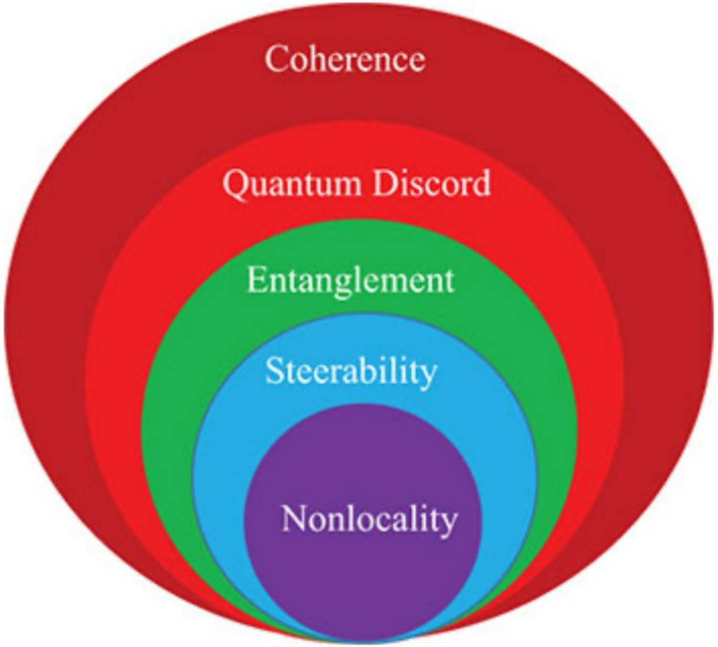
"Maybe better to not make strong statement"...

- ★ off-shell and radiative-decay effects: small for EW prod., larger for $h \to 4\ell$
- ★ QCD and EW corr. sizeably change angular coeff.
- ★ choose suitable Lorentz frame, especially with radiative corr.
- ★ LO picture may be enough for some quantum obs.
- ★ scrutinise NLO effects: tools available
- ★ fiducial cuts/ν-reco. disrupt $\ell \leq 2$ expansion: extrapolation needed

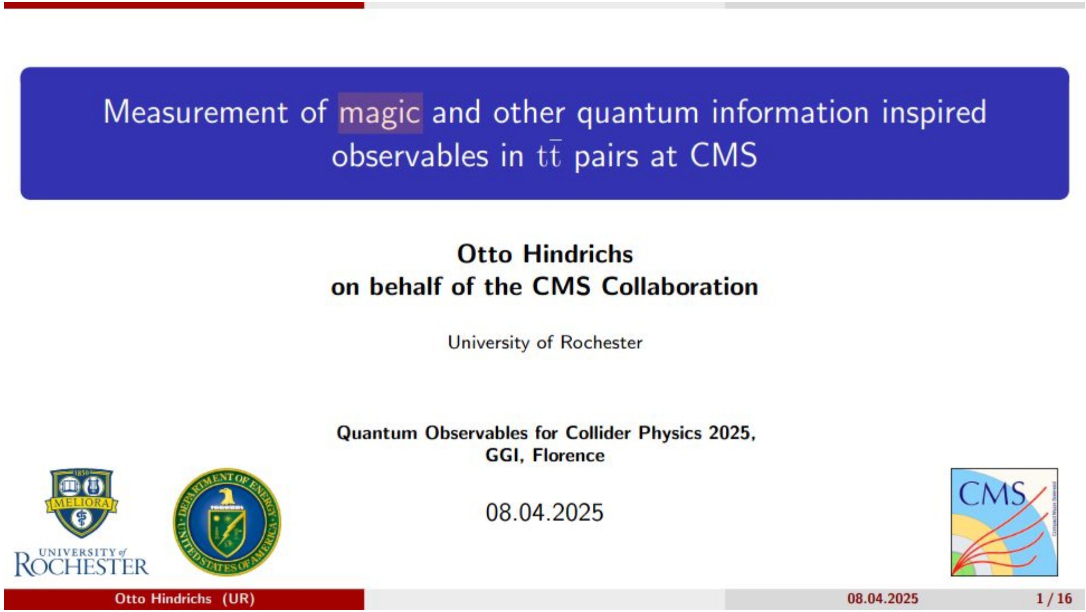
What is needed from the theory side?



Quantum Collisions: a rich hierarchy of information to explore



“ It is important to note that there is **a whole hierarchy of quantum correlations that can be studied**. For instance, discord is a measure of non-classical correlations that can interconnect the components of a system even if they are not entangled”


Measurement of magic and other quantum information inspired observables in $t\bar{t}$ pairs at CMS

Otto Hindrichs
on behalf of the CMS Collaboration

University of Rochester

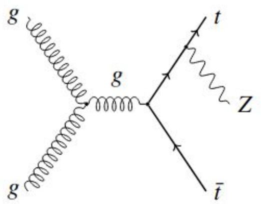
Quantum Observables for Collider Physics 2025,
GGI, Florence

08.04.2025

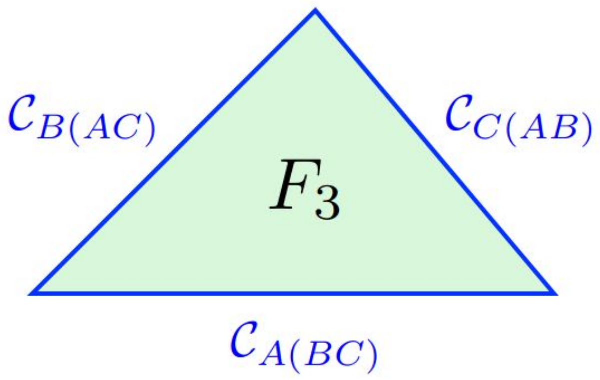
Otto Hindrichs (UR) 08.04.2025 1 / 16

CMS-PAS-TOP-25-001

Quantum Collisions: more funs



- **Three-partite entanglement**
  - 3-body Decay: *Phys.Rev.Lett.* 132 (2024) 15, 151602; arXiv:2502.19470
  - 2 to 3 process (ttZ): arXiv:2404.03292
- **Quantum Process Tomography** (operating initial particles' flavor and spin)
  - arXiv:2412.01892



concurrence triangle

PHYSICAL REVIEW A, VOLUME 62, 062314

Three qubits can be entangled in two inequivalent ways

W. Dür, G. Vidal, and J. I. Cirac
Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria
(Received 26 May 2000; published 14 November 2000)

PHYSICAL REVIEW A, VOLUME 65, 052112

Four qubits can be entangled in nine different ways

F. Verstraete,[1,2] J. Dehaene,[2] B. De Moor,[2] and H. Verschelde[1]
[1]*Department of Mathematical Physics and Astronomy, Ghent University, Krijgslaan 281 (S9), B-9000 Gent, Belgium*
[2]*Department of Electrical Engineering, Katholieke Universiteit Leuven, Research Group SISTA Kasteelpark Arenberg 10, B-3001 Leuven, Belgium*
(Received 29 November 2001; published 25 April 2002)

Quantum Process Tomography: one further step

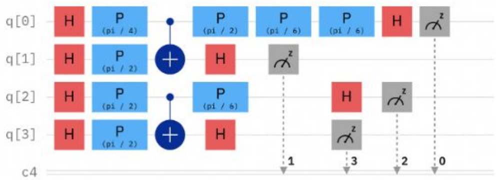
- **Spin and flavour measurements** in collider experiments as a **quantum instrument**
- **Choi matrix**, which completely determines input-output transitions, can be both **theoretically computed** and **experimentally reconstructed**
- **Polarized Beam collisions, or,**

ref **lepton scattering on polarized target experiments (see next)**

Particle Collider = Quantum Computer

=

C. Altomonte, A.Barr [2312.02242]



$|\psi\rangle_{\rm in}$ ⟶ Unitary Evolution ⟶ Measurements

Reconstruction of Choi matrix $\boxed{e^-e^+ \to t\bar{t}}$

- Reconstruction of the diagonal part:

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

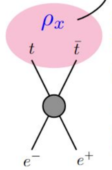
- Consider 4 purely polarised beam settings:

$$\{|i\rangle\} = \{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\} \qquad \rho_0^i = |i\rangle\langle i|$$

$$\mathcal{I}_x(|i\rangle\langle i|) = \varrho_x^i = \mathrm{Tr}\varrho_x^i \cdot \frac{\varrho_x^i}{\mathrm{Tr}\varrho_x^i}$$

measurable

$$\frac{\sigma_x[e^-e^+(\rho_0^i) \to t\bar{t}]}{\sigma[e^-e^+(\rho_0^{\rm mix}) \to t\bar{t}]}$$



Reconstruction of spin-density matrix is possible

Quantum State Tomography

Y.Afik, J.Nova [2003.02280], Ashby-Pickering, Barr, Wierzchucka [2209.13990]

QE between free electrons and muons

- None similar experiment has been done with free-traveling electrons as **measuring the spin of a single traveling electron poses a significant challenge** due to interference from its orbital motion

Original Articles

Does a flying electron spin?

B. M. Garraway & S. Stenholm

Pages 147-160 | Published online: 08 Nov 2010

Cite this article https://doi.org/10.1080/00107510110102119

bobbi_john jfcbat.com <bobbi_john@jfcbat.com>
To: Qiang Li <qliphy@gmail.com>

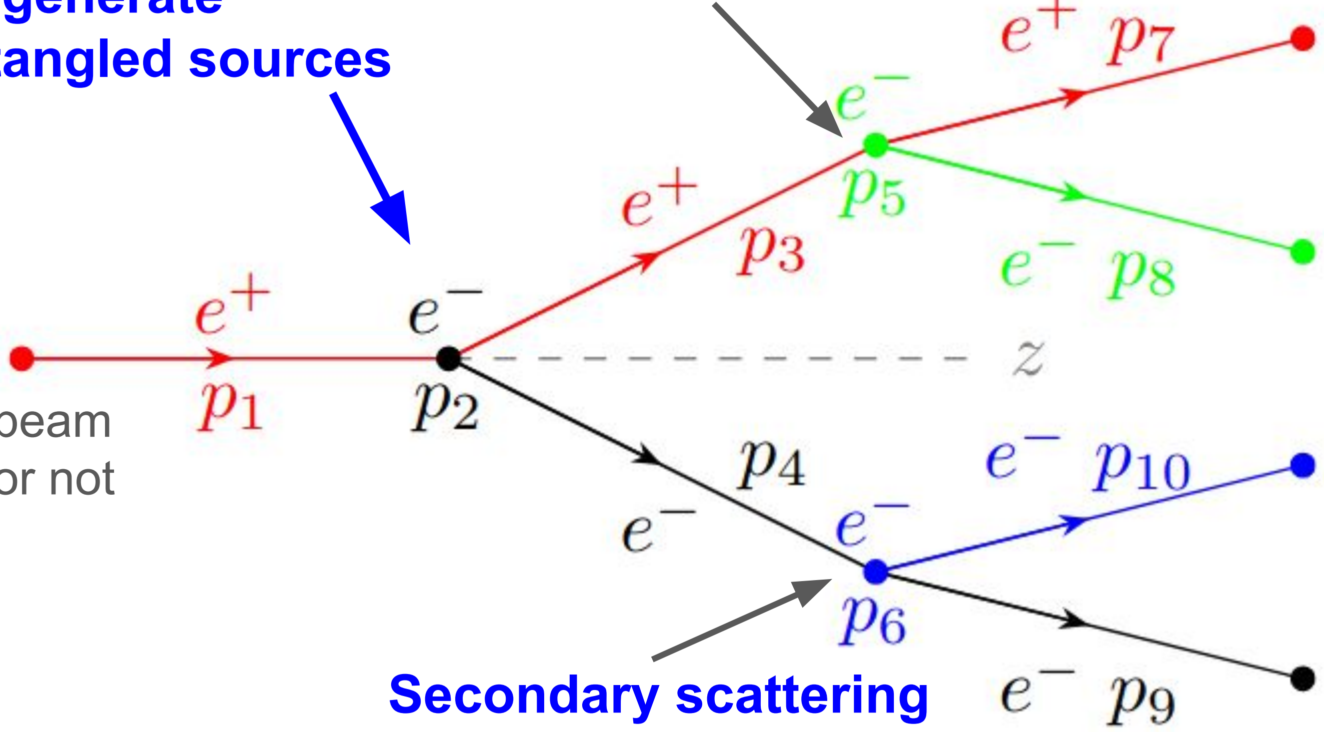
Dear Qiang Li
Please be aware that Stern Gerlach magnets, because they also have a finite magnetic field, cannot separate a charged g=2 electron beam into 2 spin states. This fact was first noted (and proven for g=2) by Nils Bohr. It is obscurely reported (only) in Mott and Massey's book, Theory of Atomic scattering.
John Clauser

- **Our proposal:**
  - a first measurement on **polarization correlations**
    - **between charged lepton beams**
  - through **joint measurements** of their individual polarization-sensitive scatterings off two separate targets.

Lepton scattering experiment proposal

Primary scattering to generate entangled sources

Secondary scattering as a polarimetry

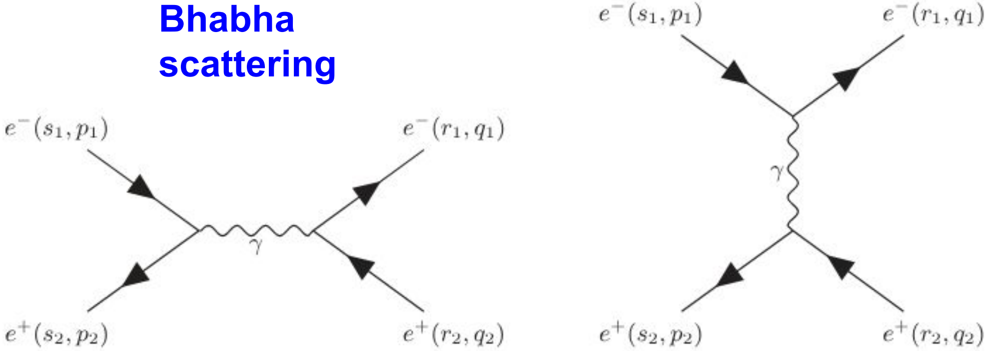


Incoming beam polarized or not

Secondary scattering as a polarimetry

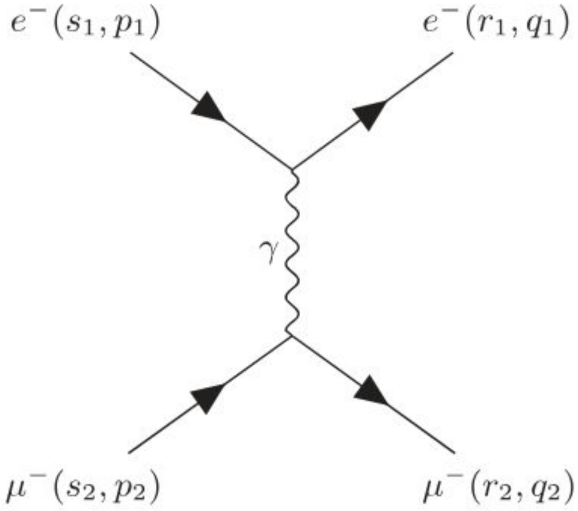
Tree-level entanglement in quantum electrodynamics

Bhabha scattering

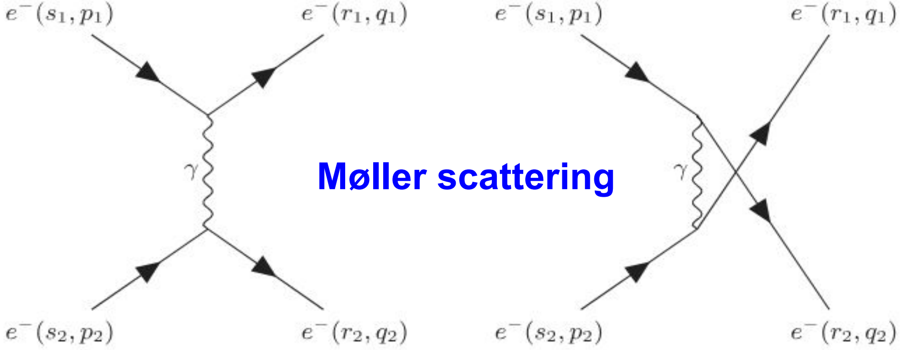


(a) s-channel (b) t-channel

electron-muon scattering
(Mott scattering)

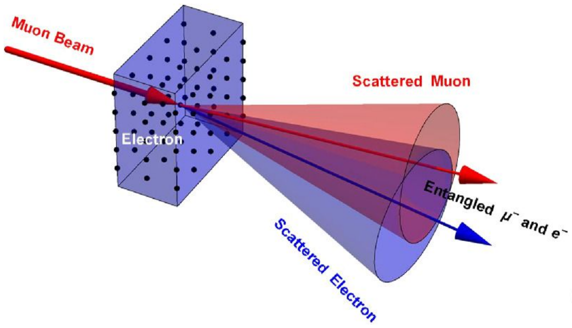


Møller scattering



(a) t-channel (b) u-channel

Controllable Source: muon on-target experiments

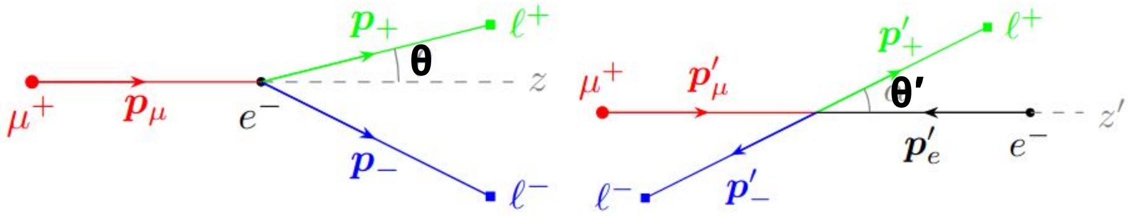


$$\rho_{s_3' s_4' s_3 s_4} = \langle s_3' s_4' | \rho_f | s_3 s_4 \rangle = \frac{1}{4} \sum_{s_1 s_2} \left(\frac{\mathcal{M}_{s_3' s_4' s_1 s_2} \mathcal{M}^*_{s_3 s_4 s_1 s_2}}{\sum_{s_3'' s_4''} \left| \mathcal{M}_{s_3'' s_4'' s_1 s_2} \right|^2} \right). \tag{1}$$

where s_1 and s_2 are the helicities of the muon and electron before scattering, and $s_3^{(')}$ and $s_4^{(')}$ are those after scattering. The polarized scattering amplitude $\mathcal{M}$ is calculated using the helicity spinors of each particle with momentum p_i as

Let $\theta_e(\theta'_e)$ and $\theta_\mu(\theta'_\mu)$ denote the polar angles of the final state electron and muon momenta in the lab (center of mass) frame

$$i\mathcal{M}_{s_3 s_4 s_1 s_2} = \bar{u}(p_3, s_3)(-ie\gamma^\mu)u(p_1, s_1)\frac{(-ig_{\mu\nu})}{(p_1 - p_3)^2}\bar{u}(p_4, s_4)(-ie\gamma^\nu)u(p_2, s_2), \tag{2}$$



(a) lab frame (b) COM frame

Controllable Source: Concurrence, CHSH inequality

PRL 80 (1998) 2245
PRL 23 (1969) 880

- Entanglement can be quantified by *concurrence*

$$\mathcal{C}(\rho_f) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \in [0, 1]$$

for a two-qubit system, where λ_i ($\lambda_i \geq \lambda_j, \ \forall i < j$) are the square roots of the eigenvalues of the matrix $\rho_f(\sigma_2 \otimes \sigma_2)\rho_f^*(\sigma_2 \otimes \sigma_2)$. If $\mathcal{C} > 0$, the two-qubit system is entangled.

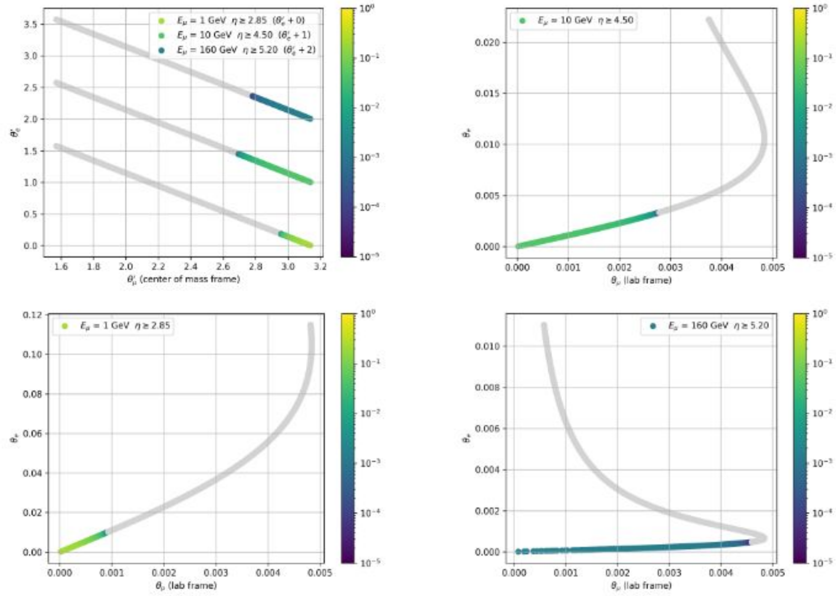
- The *CHSH inequality*, $I_2 \leq 2$, is the Bell inequality for a two-qubit system. The optimal (maximal) I_2

$$I_2 = 2\sqrt{\lambda_1 + \lambda_2},$$

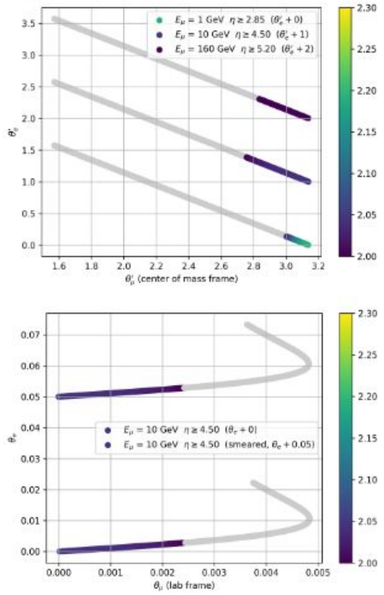
where λ_1 and λ_2 are the two largest eigenvalues of the matrix $C^{\mathrm{T}}C$, and C is the correlation matrix calculated by $C_{ij} = \mathrm{Tr}\left(\rho_f\left(\sigma_i \otimes \sigma_j\right)\right)$. $I_2 = 2\sqrt{2}$ is the upper limit of the quantum mechanics.

Controllable Source: muon on-target experiments

Concurrence $\mathcal{C}(\rho_f)$:



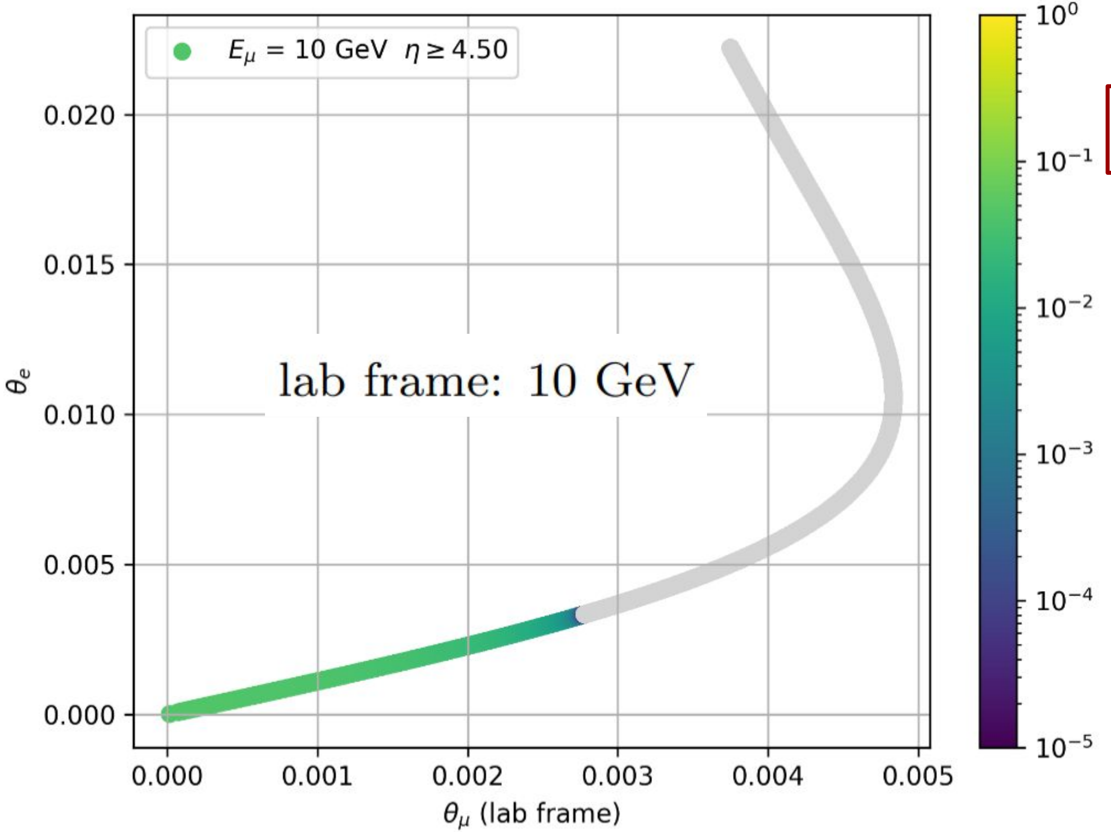
Optimal I_2:



- Simulated by MG5_aMC@NLO 3.5.5 [32] in tree-level QED with non-zero lepton masses
- The light gray regions depict $\mathcal{C}(\rho_f) = 0$ or $I_2 \leq 2$
- Assuming a **1-day** run with a **10 GeV** muon beam of flux $\mathbf{10^5}$/s on 10 cm Al targets, the expected number of events with $\mathcal{C}(\rho_f) > 0$ is $\mathbf{2.6 \times 10^4}$

$E_{\rm beam}$/GeV	$E_{\rm COM}$/GeV	$\mathcal{C}(\rho_f)_{\max}$	$\theta_{\mu,\max}$/mrad	$\theta_{e,\max}$/mrad	$E_{\mu,\min}$/GeV	$E_{e,\min}$/GeV	$\sigma_{\rm E}$/μb	$\sigma_{{\rm E},\,\theta\geq 0.5\,{\rm mrad}}$/μb
1	0.111	**0.22**	0.9	10.2	0.92	0.08	0.56	0.56
10	0.146	**0.044**	**2.8**	**3.3**	**5.2**	**4.5**	**0.39**	**0.39**
160	0.418	0.0014	4.6	0.5	10	145	0.027	0.022

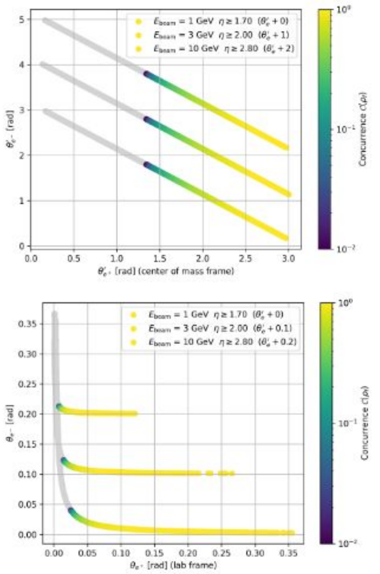
Controllable Source: muon on-target experiments



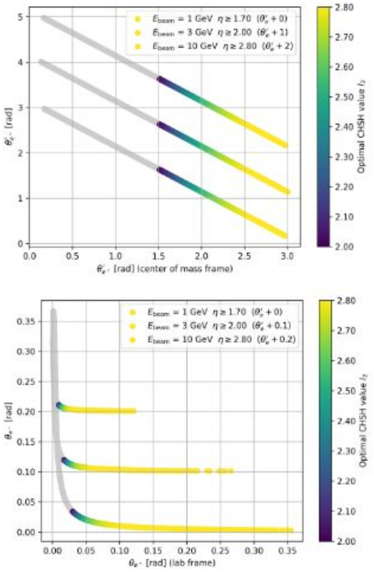
concurrence $\mathcal{C}(\rho_f)$

Controllable Source: positron on-target experiments

Concurrence $\mathcal{C}(\rho_f)$:



Optimal I_2:



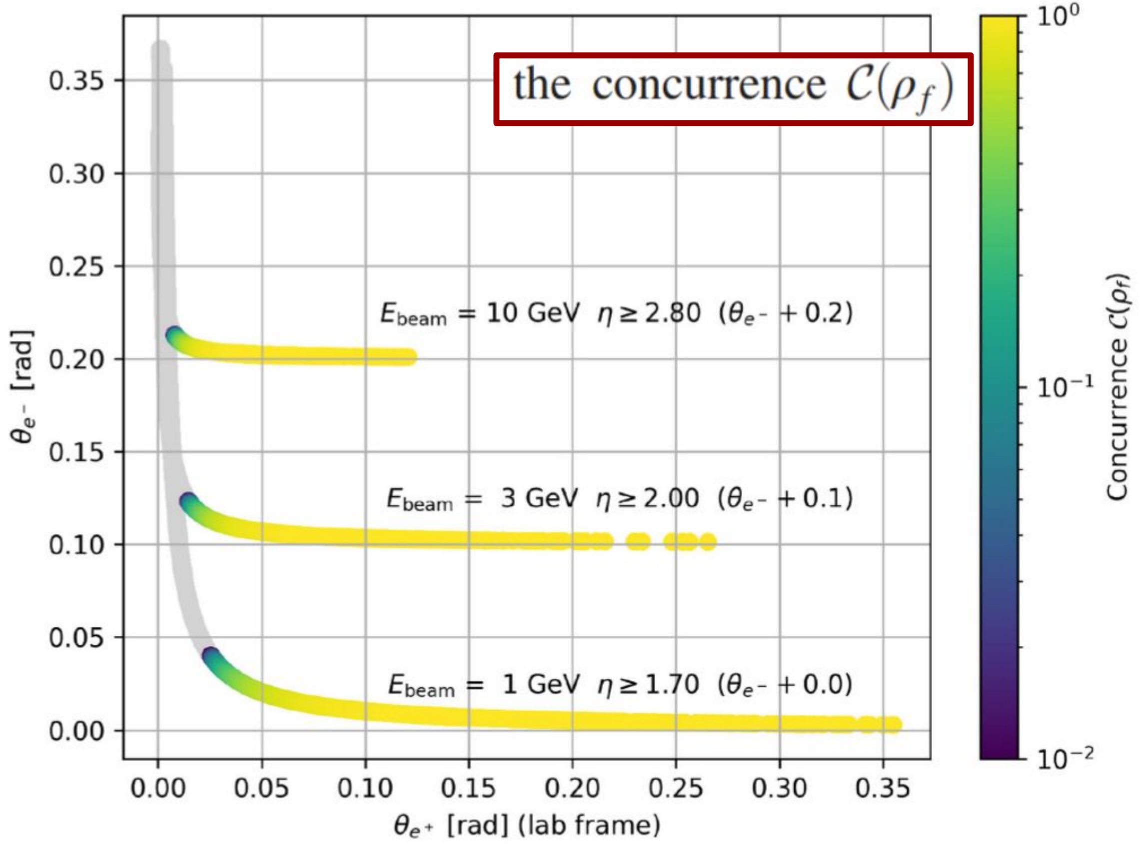
- The angular ranges exhibiting $\mathcal{C}(\rho_f) > 0$ in the center-of-mass frame are significantly broader
- The theoretical upper limits for both $\mathcal{C}(\rho_f)$ and I_2 in quantum mechanics are nearly reached as θ'_{e^+} approaches 3

STCF

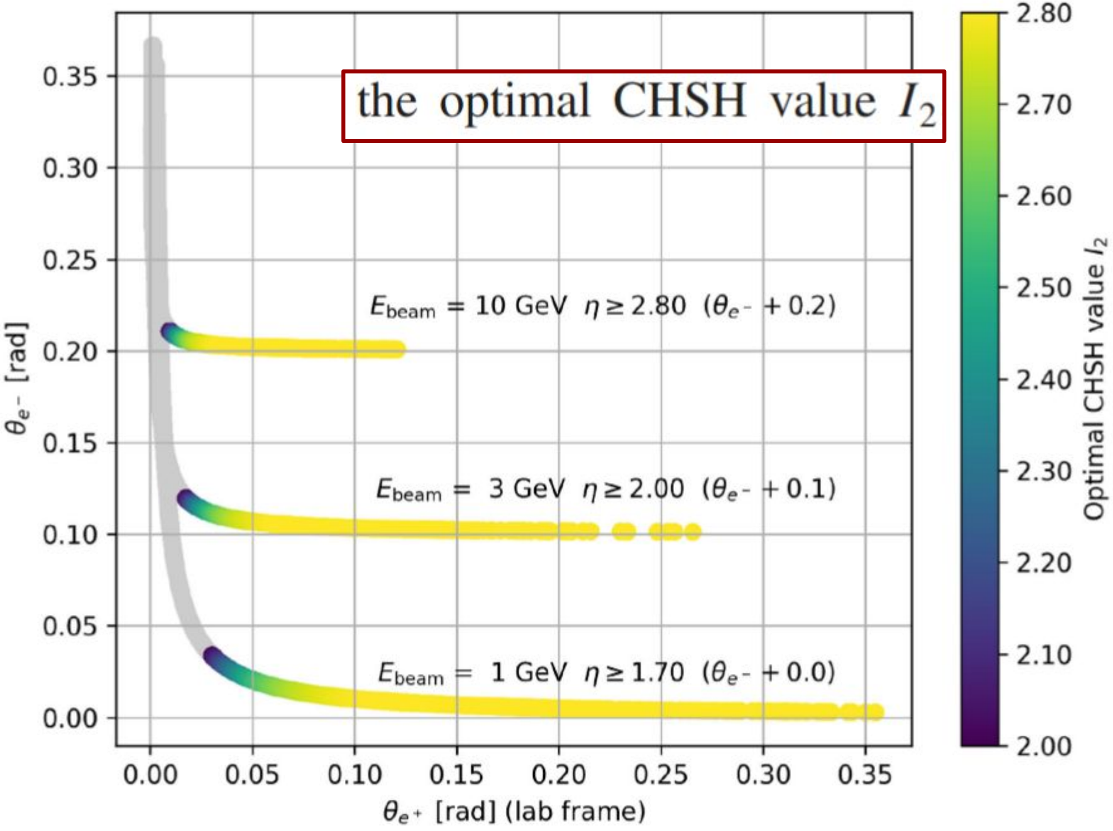
- Assuming a **1 GeV** positron beam with a flux of $\mathbf{10^{12}/s}$ directed at a 10 cm thick Al target, the expected entangled event rate is $\mathbf{1.9} \times \mathbf{10^9/s}$
- A golden region for measurements:
  - $E_{\text{beam}} = \mathbf{1\ GeV}$, $0.05\ \text{rad} \leq \theta_{e^+} \leq 0.1\ \text{rad}$
  - 23.4% of all events with $\mathcal{C}(\rho_f) > 0$
  - $E \geq \mathbf{0.094\ GeV}$, $\theta \geq \mathbf{0.0103\ rad}$
  - $\mathcal{C}(\rho_f)$ reaching up to $\mathbf{0.953}$ and I_2 up to $\mathbf{2.8281}$

E_{beam}/GeV	E_{COM}/GeV	$\mathcal{C}^{\max}(\rho_f)$	$I_2^{\max}$	$E_{e^+}^{\min}$/GeV	$E_{e^-}^{\min}$/GeV	$\theta_{e^+}^{\min}$/rad	$\theta_{e^-}^{\min}$/rad	σ_{E}/μb
1	0.032	0.9996	2.8281	0.008	0.389	0.0255	0.0028	243.6
3	0.055	0.9997	2.8282	0.023	1.166	0.0147	0.0016	82.1
10	0.101	0.9997	2.8282	0.074	3.890	0.0081	0.0009	26.5

Controllable Source: positron on-target experiments



Controllable Source: positron on-target experiments



A new kind of Wu experiment

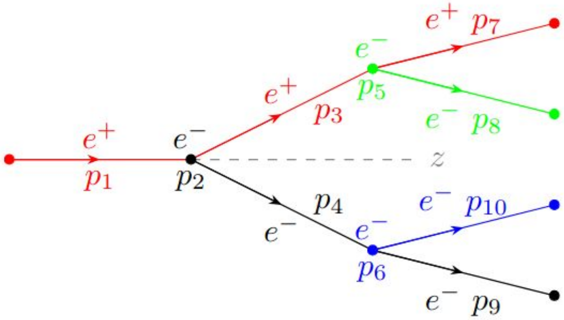


FIG. Proposed cascade experiment for measuring polarization correlations of the primary products

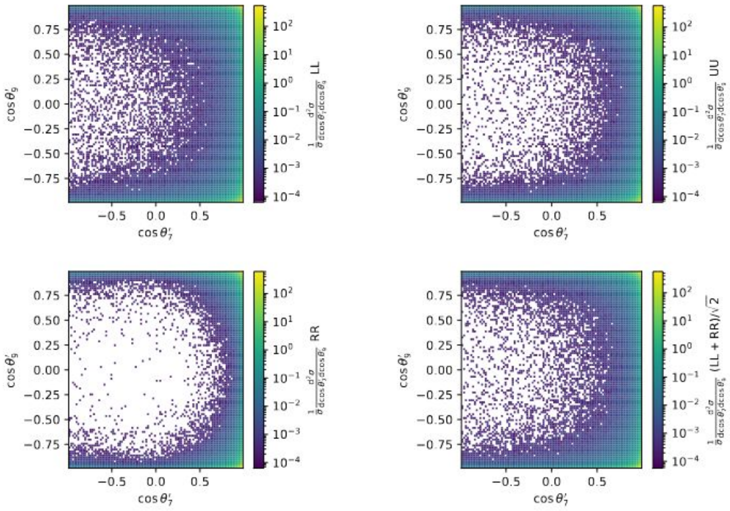


FIG. Joint angular distribution densities of the two secondary scattering processes

Simulation setup:

- $0.05 \text{ rad} \leq \theta_3 \leq 0.1 \text{ rad}$ in a 1 GeV positron on-target experiment
- The spins of target electrons 5 and 6 are aligned with the beam direction
- Consider the main component of the primary state, $(LL + RR)/\sqrt{2}$

Assuming the two secondary targets are 10 cm thick iron, the event rate in $\cos\theta_7' \leq 0.5 \wedge -0.75 \leq \theta_9' \leq 0.75$ is $\mathbf{1.4 \times 10^2}$/s for the state $(LL + RR)/\sqrt{2}$.

A new kind of Wu experiment

Take $0.05\ \mathrm{rad} \leq \theta_3 \leq 0.1\ \mathrm{rad}$ in a 1 GeV positron on-target experiment as an example:

- The state of the primary products is approximately 1% $(RL+LR)/\sqrt{2}$, 1% $(RL-LR)/\sqrt{2}$, 7% $(RR-LL)/\sqrt{2}$, and 90% $(RR+LL)/\sqrt{2}$ in the lab frame
- The optimized ratio of the yields of $(LL+RR)/\sqrt{2}$ to UU is $1.29 \pm 0.03(\mathrm{MC})$, corresponding to 4.4×10^3 post-optimization efficient signal event counts and an expected signal yield over a **27-second** run; the result for $(LR+RL)/\sqrt{2}$ is $0.78 \pm 0.02(\mathrm{MC})$ in comparison
- Other uncertainties, such as those from process modeling and background suppression, may dominate the real experimental analysis
- For the 20% polarized targets, the ratios are 1.010 ± 0.009 and 0.986 ± 0.009 generated from 25 times the number of Monte Carlo events, corresponding to 2.5×10^4 efficient event counts accumulated in **680 seconds**

A new kind of Wu experiment

- The **high event rate** can help mitigate the decline in resolving power associated with low target polarization purities in real-world applications

- **A simplified state tomography** can be performed assuming **prior knowledge** from the primary scattering

- Further studies with **polarized muon beam** in underway
  - 70% polarized beam at HIAF is possible with 10^5/s
  - muon decay can act as a polarimetry

Proposal Summary

- GeV-scale muon and positron on-target experiments are examined as **controllable entangled lepton pair sources** through the kinematic approach
- Quantum entanglement and the CHSH inequality violation are present in the primary scattering products
- A **first measurement of the correlation between entangled free-traveling lepton pairs** is proposed to verify the entanglement
- The electron-positron beam polarization correlation measurement can be conducted with **a high event rate at many domestic positron beam facilities**

Process	Incident flux	Primary event rate	Secondary coincidence rate
$\mu^- e^- \to \mu^- e^-$	$10^5/\mathrm{s}$	$2.6 \times 10^4/\mathrm{d}$	(not estimated)
$e^+ e^- \to e^+ e^-$	$10^5/\mathrm{s}$	$1.9 \times 10^2/\mathrm{s}$	$4.4 \times 10^2/\mathrm{y}$
$e^+ e^- \to e^+ e^-$	$10^{12}/\mathrm{s}^*$	$\mathbf{1.9 \times 10^9/s}$	$\mathbf{1.4 \times 10^2/s}$

*Possibly from the beam dump of the STCF.

PKMu (R&D) & MUonE (ongoing)

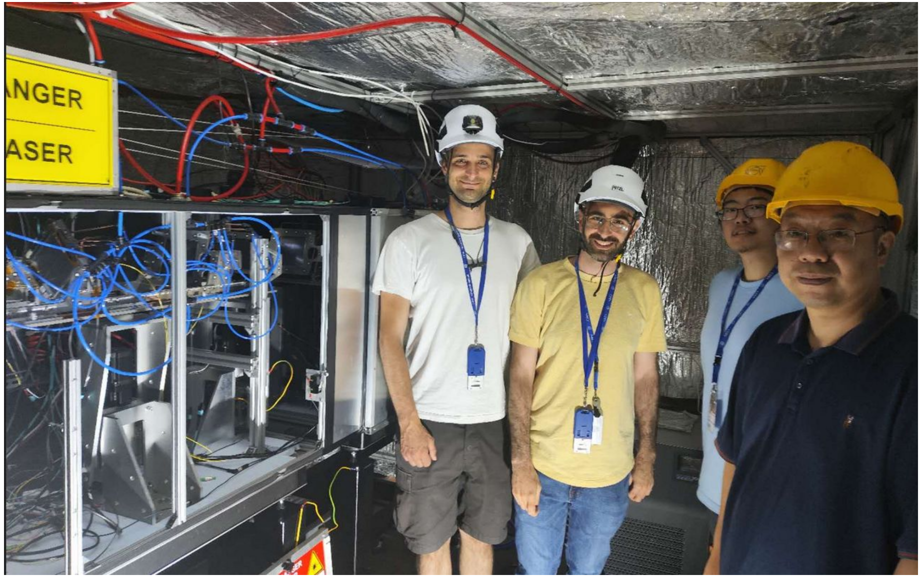
PKMu: Muon on target experiment proposed by PKU for multi-purpose including cosmic ray, dark boson, and **quantum entanglement**.

HIRIBL: 1-10GeV 10^6-10^7/s muon beam line from the HIAF facility from the imp, cas, China.

MUonE: a Muon Electron scattering experiment at CERN exploiting 150-160 GeV Muon beam, aims at an independent and precise determination of the leading hadronic contribution to the muon g-2.

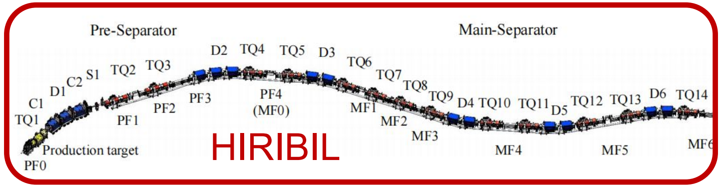
HIAF, Hui Zhou

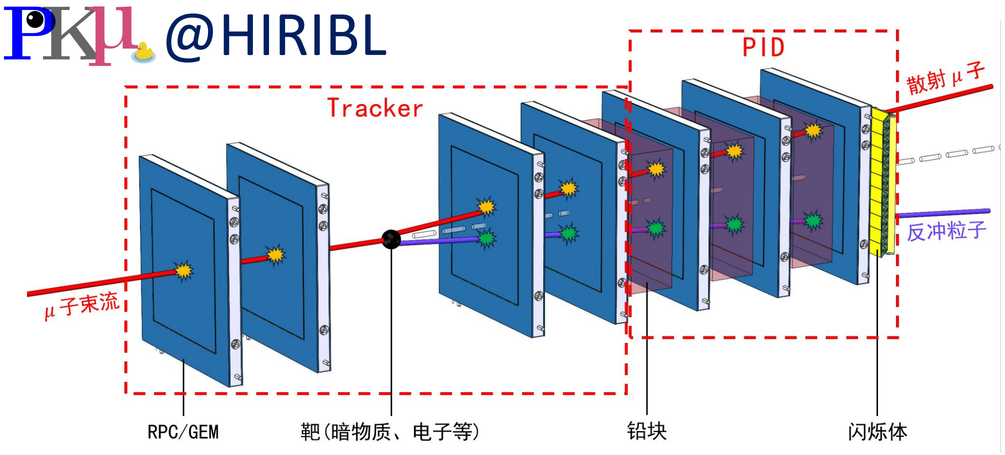
MUonE CERN



Muon Scattering Experiment at HIAF-HIRIBL

PKMu(Probing and Knocking with Muons) Proposed by Peking University together with HIAF-HFRS from Institute of Modern Physics, Chinese Academy of Sciences, China: **using 1-10 GeV Muon to probe new physics beyond the Standard Model**





References:[1] Phys. Rev. D 110, 016017 [2] arxiv:2410.20323 [3] arXiv:2411.12518 [4] Nucl. Instrum. Methods. Phys. Res. A 663 (2012) 22-25

Thanks for your attention!

The world of elementary particles

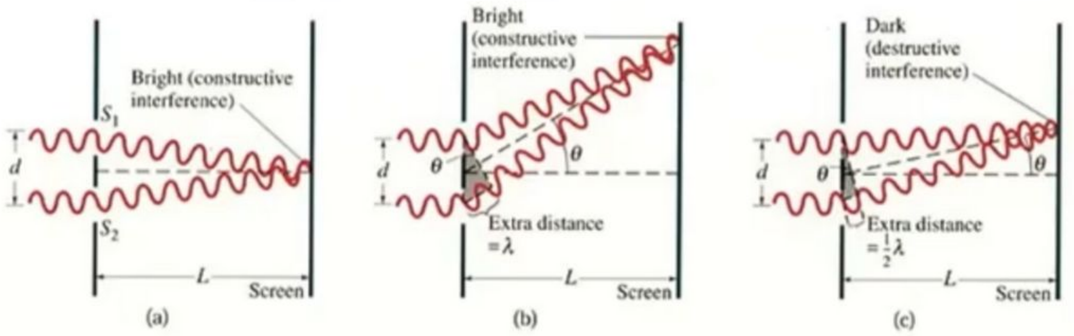
- The behavior of the elementary particles is described by quantum field theories: QED and QCD
- Quantumness should be engrained in their behavior
- Yet, it is not so easy to observe
- For a typical momentum of 100 GeV/c the precision of measurement is ~1GeV/c, typical position precision is 100 um

$$\Delta x \cdot \Delta p > h / 2\pi$$

$$\Delta t \cdot \Delta E > h / 2\pi$$

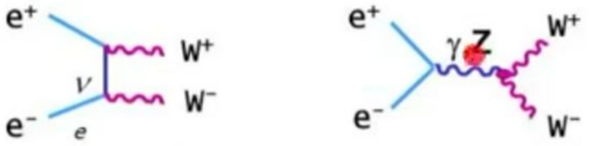
- We are ~10 orders of magnitude away from the uncertainty principle

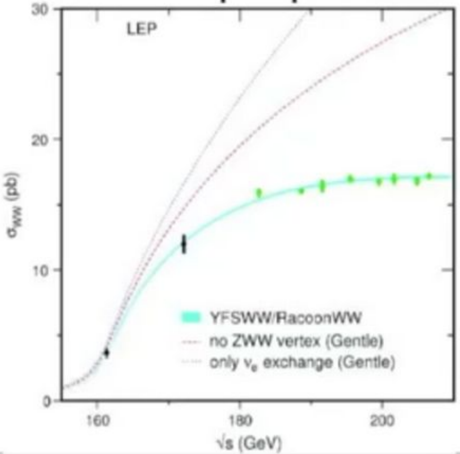
“Double slit experiment”



- Effects of amplitude interference are observed in multiple processes

W pair production at LEPII:





- **And then there is spin**

https://arxiv.org/abs/2403.14757

New physics in spin entanglement

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Abstract We propose a theory that preserves spin-summed scattering and decay rates at tree level while affecting particle spins. This is achieved by breaking the Lorentz group in a non-local way that tries avoiding stringent constraints, for example leaving unbroken the maximal sub-group SIM(2). As a phenomenological application, this new physics can alter the spins of top-antitop pairs (and consequently their entanglement) produced in *pp* collisions without impacting their rates. Some observables affected by loops involving top quarks with modified entanglement receive corrections.

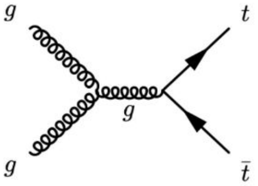
Quantum State

- **Pure state** $\rightarrow$ Wave function $|\Psi\rangle$
  1. $|\Psi\rangle = \sum_n \alpha_n \cdot |\phi_n\rangle \,,\ \langle\Psi|\Psi\rangle = \sum_n |\alpha_n|^2 = 1$
  2. Coherent mixture of quantum states $\rightarrow$ α_n are complex amplitudes
  3. Expectation values: $\langle A\rangle = \langle\Psi|A|\Psi\rangle = \sum_{n,m} \alpha_m^* \alpha_n \langle\phi_m|A|\phi_n\rangle$
- **Mixed state** $\rightarrow$ Generalization to density matrix ρ
  1. $\rho = \sum_n p_n \cdot |\phi_n\rangle\langle\phi_n| \,,\ \mathrm{tr}\rho = \sum_n p_n = 1,\ p_n \geq 0$
  2. Incoherent mixture of quantum states $\rightarrow$ p_n are probabilities
  3. Expectation values: $\langle A\rangle = \mathrm{tr}(\rho A) = \sum_n p_n \langle\phi_n|A|\phi_n\rangle$

Density matrix, Peres-Horodecki criterion ref

	pure state	mixed states
wavefunction	$\vert\psi\rangle$	$f(\vert\psi_i\rangle)$
density matrix	$\rho = \vert\psi\rangle\langle\psi\vert$	$\rho = \sum_i p_i \vert\psi_i\rangle\langle\psi_i\vert$
trace of ρ	$Tr(\rho) = 1$	$Tr(\rho) = 1$
trace of ρ^2	$Tr(\rho^2) = 1$	$Tr(\rho^2) < 1$
entropy	$S(\rho) = 0$	$S(\rho) = -Tr(\rho \ \ln \rho) > 0$

- **density matrix for 1 spin**
  - $\rho = \frac{1}{2}\left(I_2 + \sum_i B_i \sigma^i \otimes I_2\right)$
    - ✓ $B_i = \langle \sigma^i \rangle$
      - B_i **3 parameters → Polarization**
- **density matrix for 2 spins**
  - $\rho = \frac{1}{4}\left(I_4 + \sum_i (B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i)\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j)$
    - ✓ $B_i^{\pm} = \langle \boldsymbol{\sigma}_{\pm}^{i} \rangle$
      - **6 paramters → Polarizations**
    - ✓ $C_{ij} = \langle \boldsymbol{\sigma}_{+}^{i} \boldsymbol{\sigma}_{-}^{j} \rangle$
      - **9 parameters → Correlations**



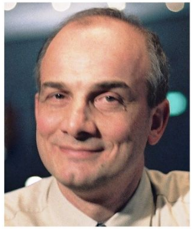
- From pure state to mixed state.

"A quantitatively characterization of the degree of the entanglement between the subsystems of a system in a mixed state, is not unique! "

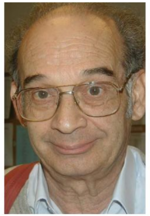
$$\rho_{AB} \stackrel{?}{=} \sum_{i=1}^{N} p_i \rho_A^{(i)} \otimes \rho_B^{(i)}, \quad \left(\sum_i^N p_i = 1,\ p_i > 0 \right)$$

"Finally, we prove that the weak membership problem for the convex set of separable normalized bipartite density matrices is ***NP-HARD***. "

——Leonid Gurvits



- For 2×2 and 2×3 system, it is solved by Peres, and Horodeckis 1996 (Peres-Horodecki criterion, concurrence).



Asher Peres
(1934/01/30-2005/01/01)

Ryszard Horodecki
(1943/09/30-)

Paweł Horodecki
(1971-)

Michał Horodecki
(1973-)

https://indico.cern.ch/event/1462029/contributions/6637480/attachments/3132597/5557580/QCD@LHC_PLamba.pdf

Quantum observables

1. For Entanglement:

   ✓ *Peres-Horodecki criterion* (Peres 9604005, Horodecki 9703004): For the following LO spin density matrix, it give us sufficient and necessary condition for entanglement $C_{2,2,2,-2} \neq 0$ or $C_{2,1,2,-1} \neq 0$.

   ✓ *Concurrence upper and lower bound*: Greater than zero is signal of entanglement. Concurrence 1 corresponds to maximal entanglement state.

$$(\mathcal{C}(\rho))^2 \geq \frac{2}{9} \max \Bigg[\Big(-2 - 2\sum_{L,M} |A^a_{L,M}|^2 + \sum_{L,M} |A^b_{L,M}|^2 + \sum_{L_a,L_b,M_a,M_b} |C_{L_a,M_a,L_b,M_b}|^2 \Big), \Big(-2 + \sum_{L,M} |A^a_{L,M}|^2 - 2\sum_{L,M} |A^b_{L,M}|^2 + \sum_{L_a,L_b,M_a,M_b} |C_{L_a,M_a,L_b,M_b}|^2 \Big) \Bigg],$$

$$(\mathcal{C}(\rho))^2 \leq \frac{2}{3} \min \Bigg[\Big(2 - \sum_{L,M} |A^a_{L,M}|^2 \Big), \Big(2 - \sum_{L,M} |A^b_{L,M}|^2 \Big) \Bigg].$$

$$\boxed{\text{Upper bound is independent of all C coefficients!!!}}$$

- **At the LHC, top quarks are produced in a mixed state**
  → can be represented as a density operator:

$$\rho = \frac{I_4 + \Sigma_i \left(B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i\right) - \Sigma_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

$$B^{+/-} = \begin{pmatrix} \times \\ \times \\ \times \end{pmatrix} \qquad C = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$

- **Peres-Horodecki criterion:**
  if a system is separable, $\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$

  Peres, Phys. Rev. Lett. **77**, 1413
  Horodecki, Phys. Lett. A **232**, 5

  the transpose with respect to a subspace of ρ is a non-negative operator

$$\rho^{T2} = \sum_i p_i \rho_A^i \otimes (\rho_B^i)^T$$

  → system is entangled if at least 1 eigenvalue is negative

- **For $t\bar{t}$ system, spin density matrix is separable if all eigenvalues are positive**

  → top quarks are entangled in a certain phase space if at least one eigenvalue is < 0

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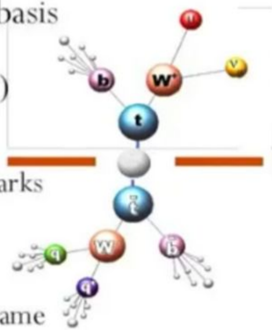
CMS

Spin correlation and entanglement

Polarization, P and spin correlation matrix, C determines the angular distribution of the decay products in the helicity basis as in [1212.4888]

$$\frac{d\sigma}{d\Omega d\bar{\Omega}} = \sigma_{norm}(1+\kappa\vec{P}\cdot\Omega+\bar{\kappa}\vec{\bar{P}}\cdot\bar{\Omega}-\kappa\bar{\kappa}\Omega\cdot C\cdot\bar{\Omega})$$



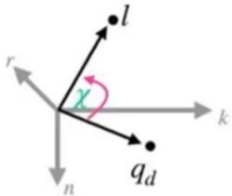
κ - spin analyzing power of top/antitop decay products – leptons or quarks

Ω – unit vector in the direction of the decay product

2·3(P)+3·3(C) =**15 coefficients Q_n**

Or use χ –opening angle between the two decay products in rotated tt frame (a.k.a helicity angle)

$$\frac{d\sigma}{d\cos\chi} = A(1+D\kappa\bar{\kappa}\cos\chi)$$

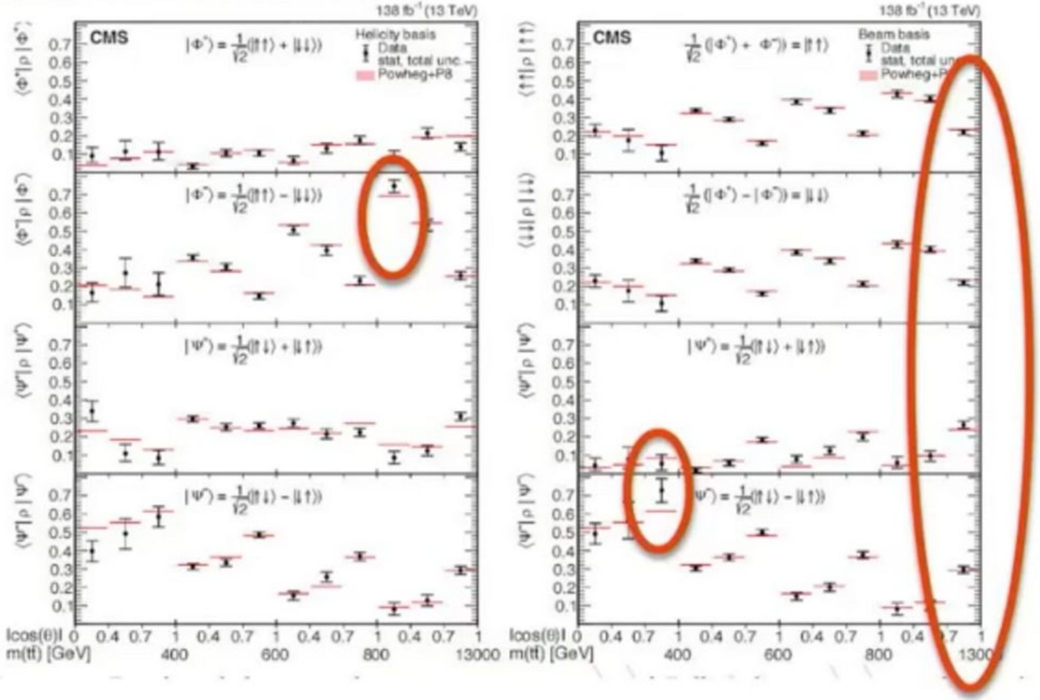


and χ, where the sign of n-component in one of the decay products is inverted:

$$\frac{d\sigma}{d\cos\tilde{\chi}} = A(1+\tilde{D}\kappa\bar{\kappa}\cos\tilde{\chi})$$

Decomposition into eigenstates

- Decomposition into the eigenstates in the helicity (Bell states) and beam (pseudoscalar and vector) bases.
- At the threshold (M_{tt}<400 GeV) 70% in the pseudo scalar state and 30% in vector state
- At high M_{tt}, low $cos\theta$ - >70% F- (pure Bell state)
- At high M_{tt}, high $cos\theta$ in beam basis maximally mixed state



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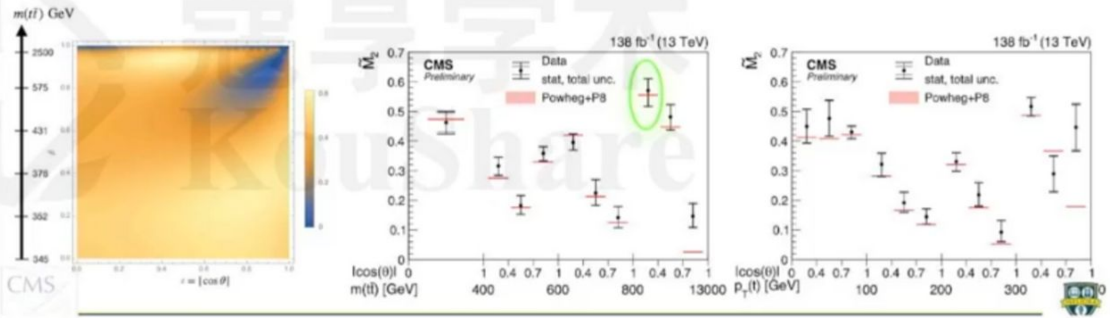
Magic in top events – results on data

- Magic is quite easy to evaluate based on the spin correlation matrix.
- Preliminary result CMSTop-25-001. Paper in preparation.
- The plan of cause is not to use LHC as a quantum computer, but to establish a common language between QIS and HEP, or at least have a "dictionary"



WQC Workshop on Quantum Entanglement of High Energy Particles

Spin, entanglement, magic and forming bonds in the world of elementary particles?