Theory of “toponium”

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暨第31届LHC Mini-Workshop
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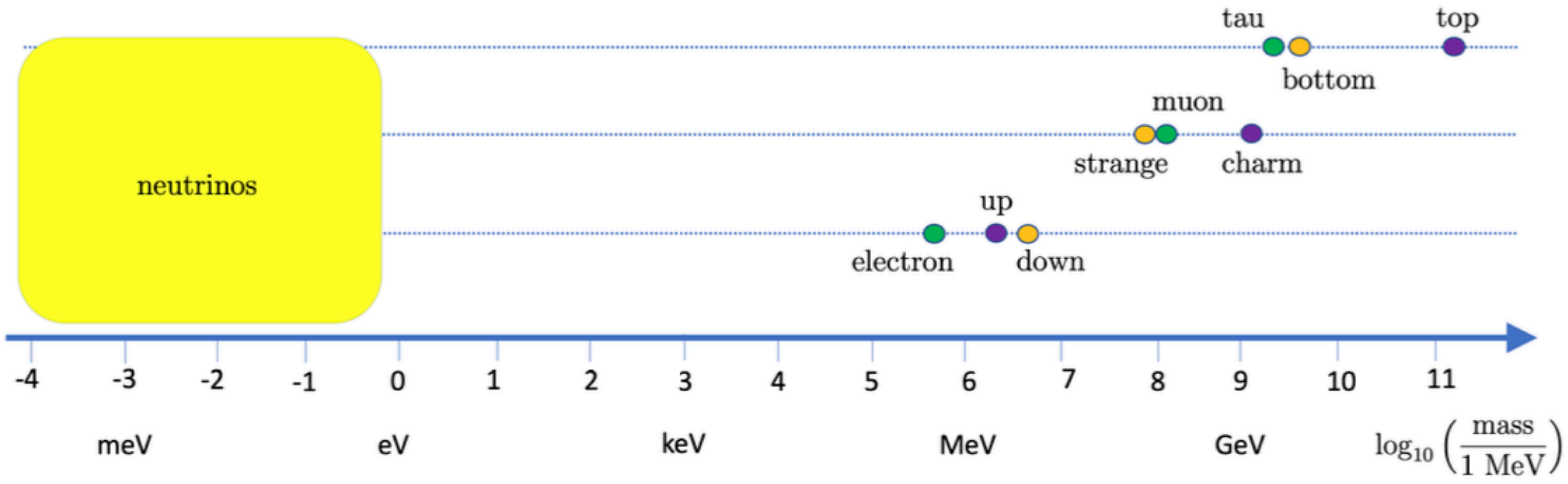
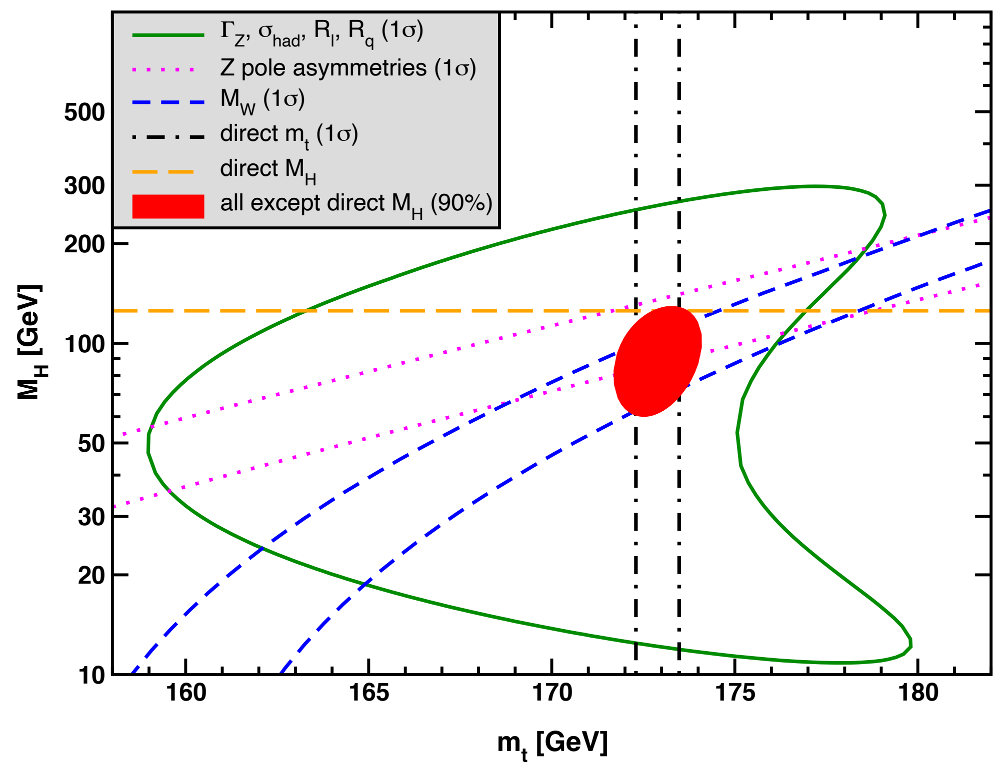
The top quark



- ➤ The heaviest fundamental particle
- ➤ Large Yukawa coupling with the Higgs field
- ➤ Top quark mass: a crucial parameter of the SM
- ➤ Decay before hadronization: “bare quark”



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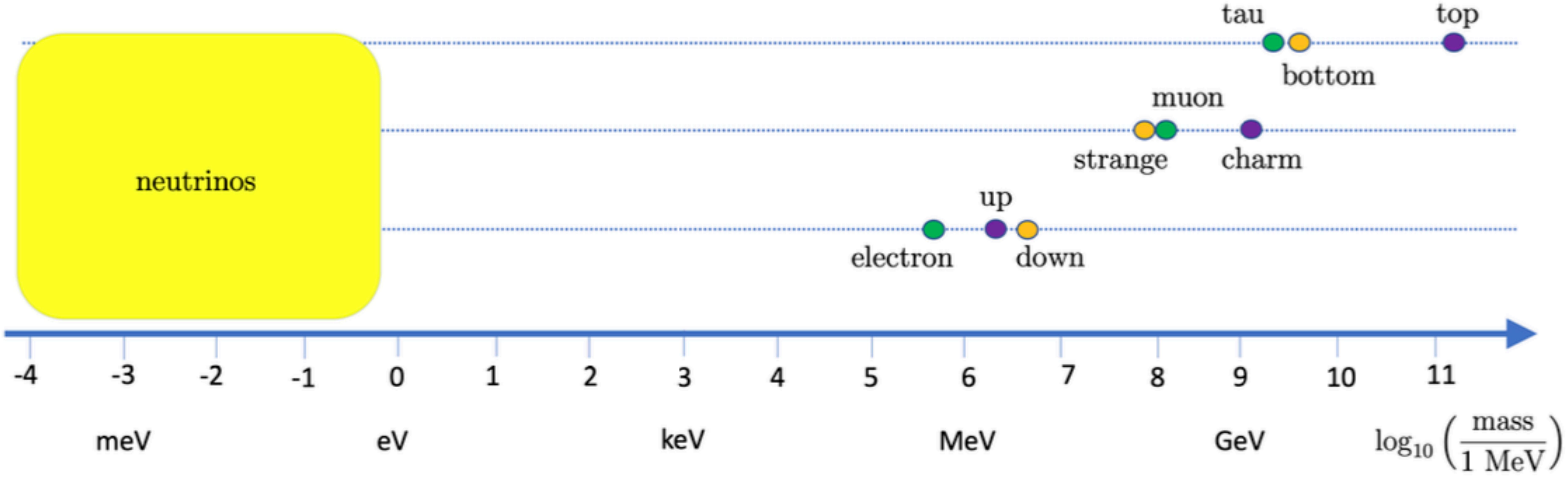

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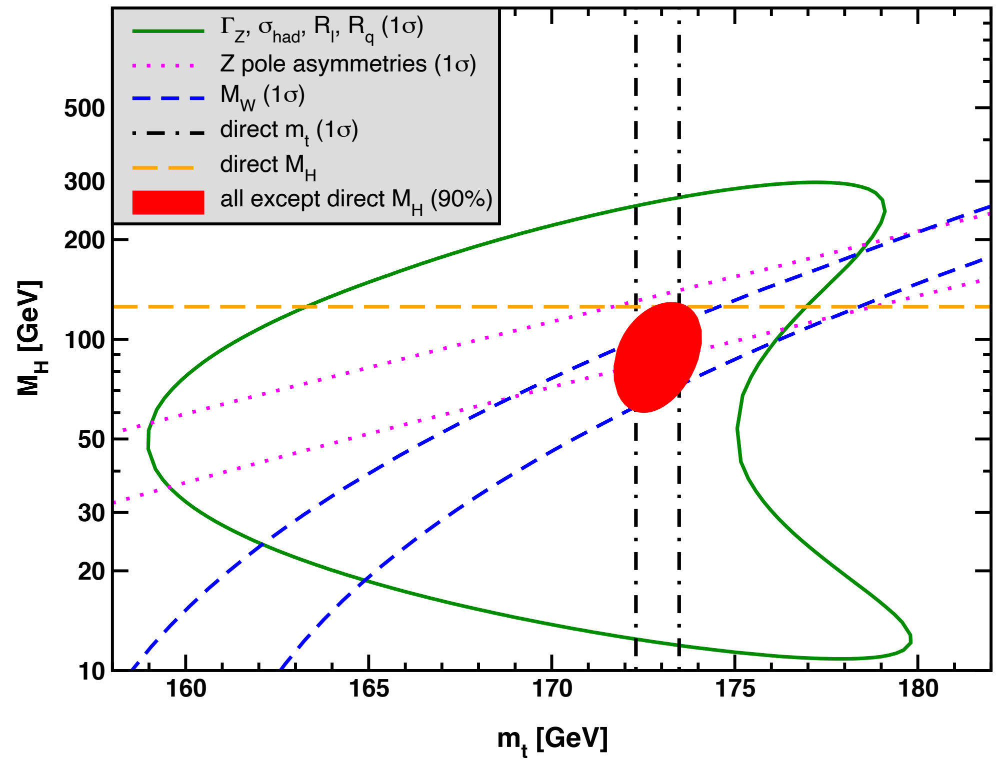
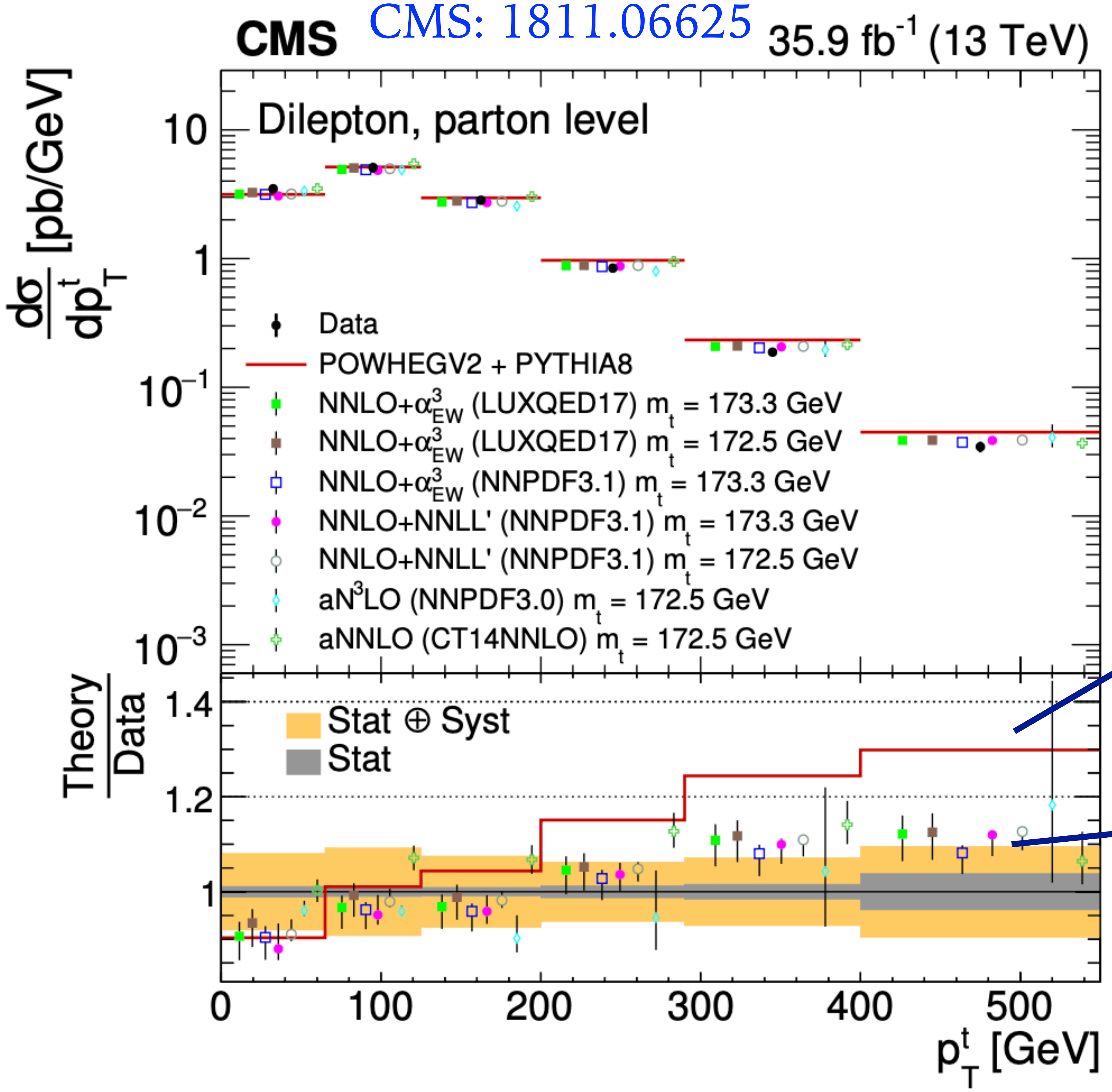


What is the “toponium” then?

Top quark pair production

CMS: 1811.06625



State-of-the-art theoretical prediction

NNLO+NNLL' in QCD + NLO in EW

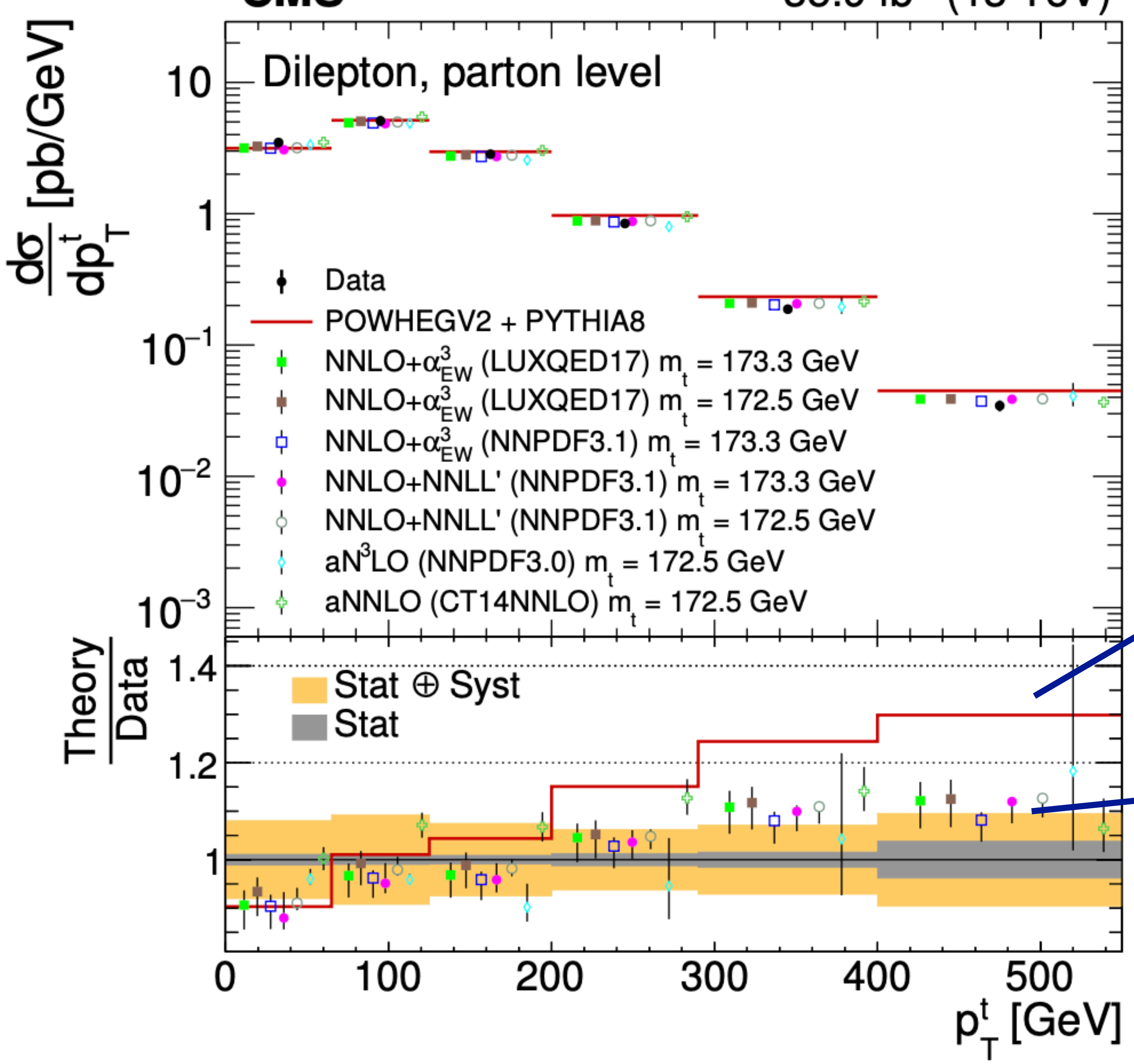
Pecjak, Scott, Wang, LLY: 1601.07020

Czakon et al.: 1803.07623, 1901.08281

NLO alone cannot describe data

Including higher-order corrections

Top quark pair production



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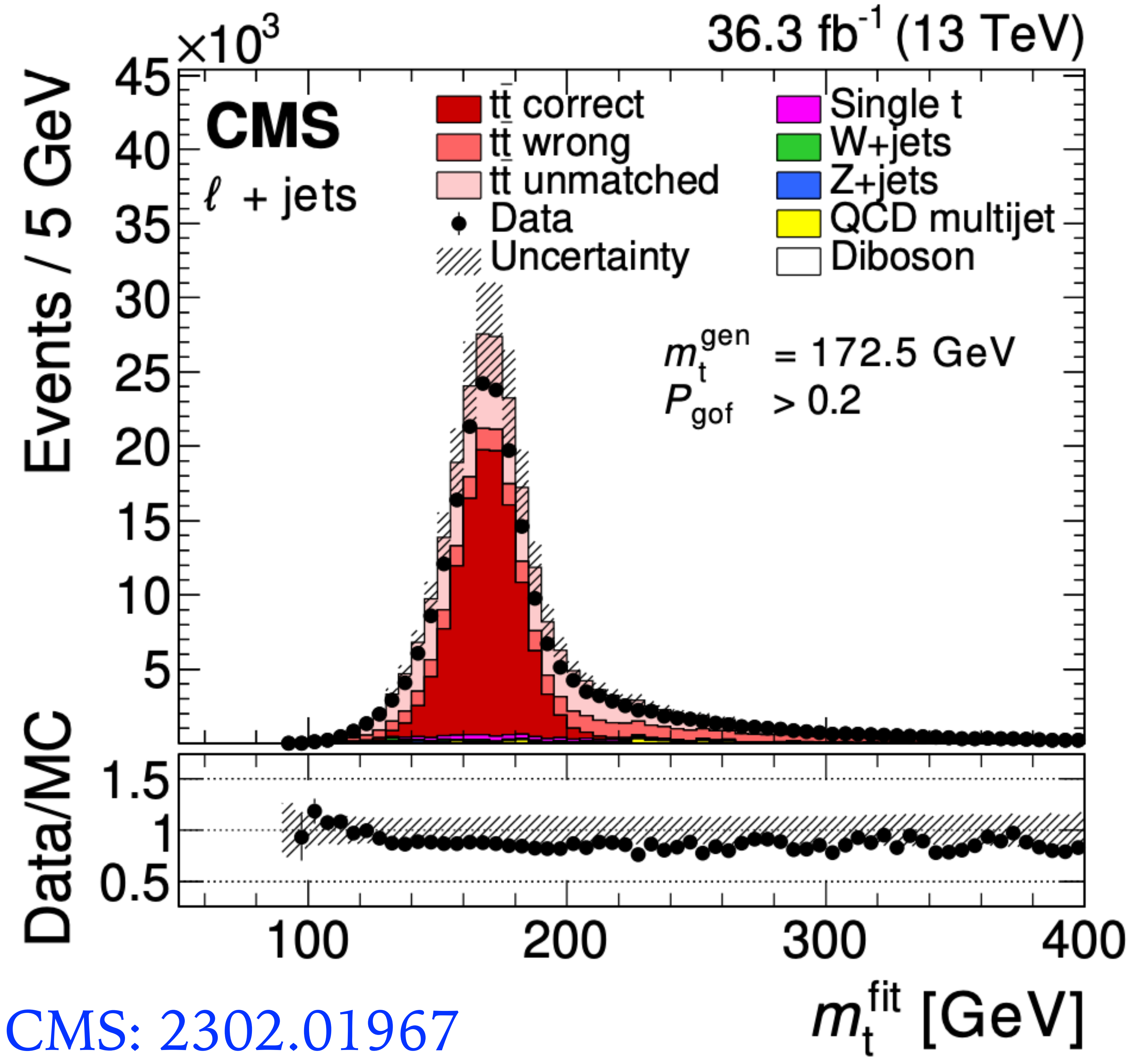
With such precision data and predictions, one would like to extract m_t from them…

Top quark mass measurement

Direct measurement

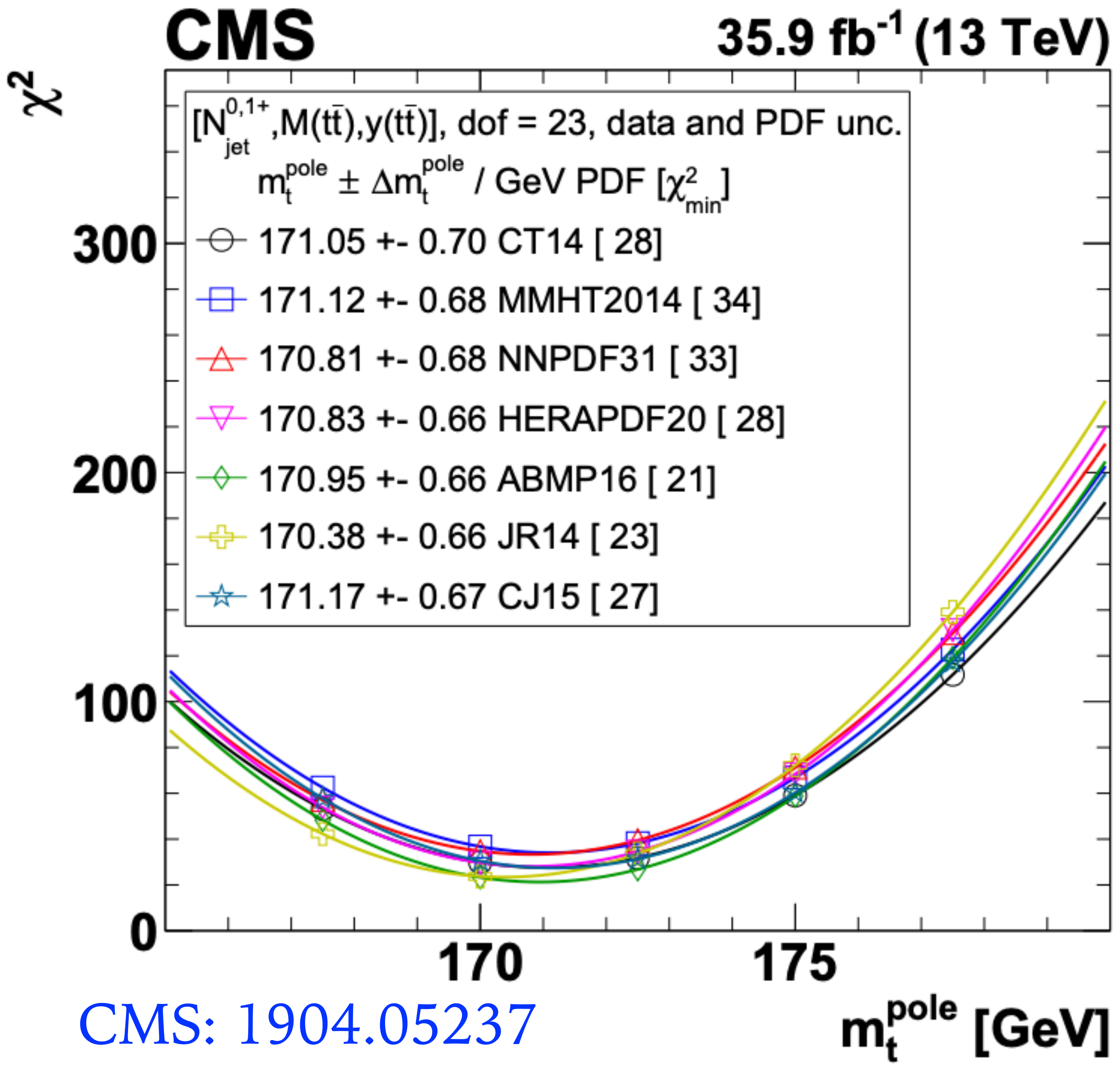


CMS: 2302.01967

Reconstruct the invariant mass of decay products

vs

Indirect measurement



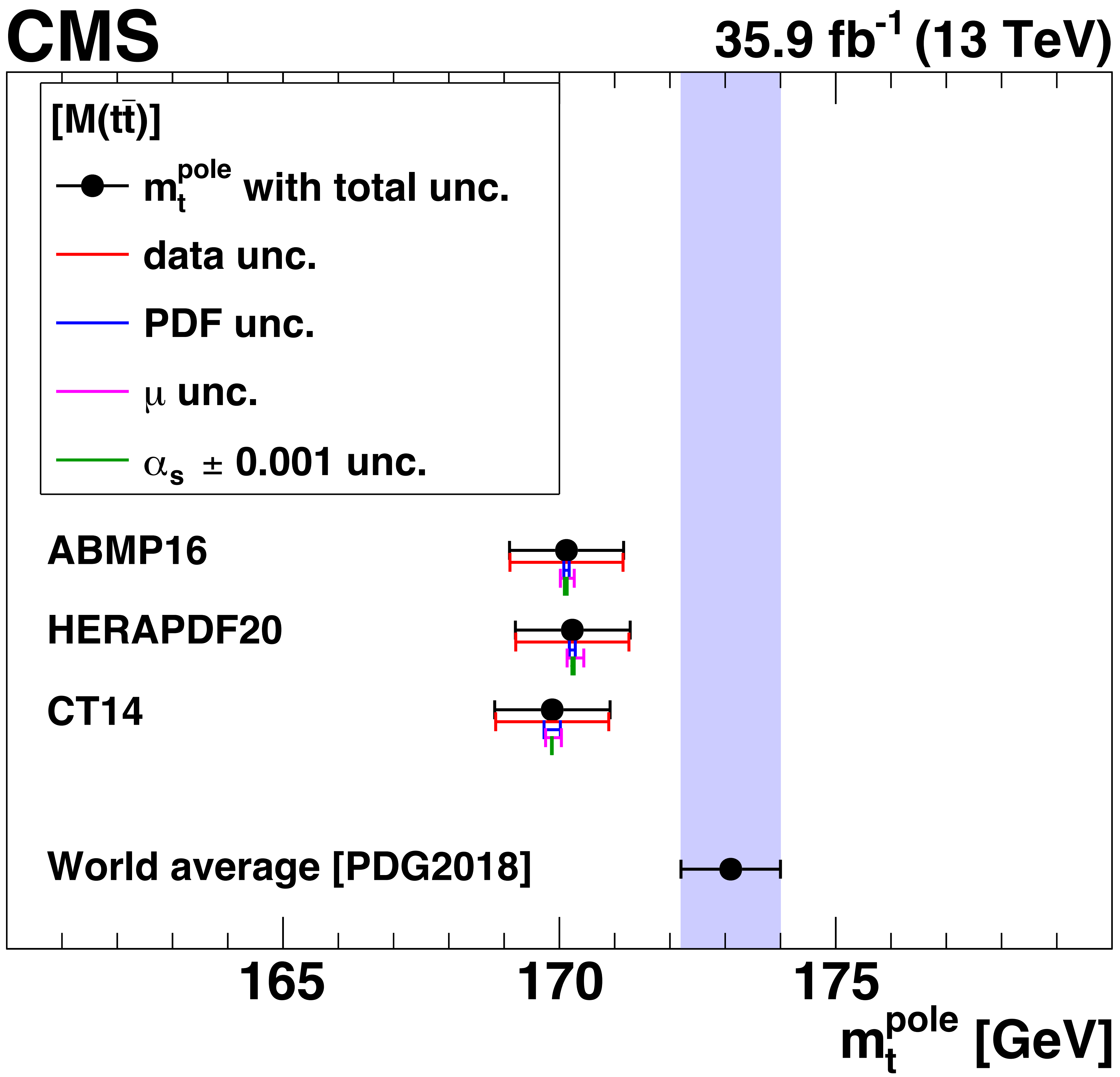
CMS: 1904.05237

$$\frac{d\sigma}{d\mathcal{O}}^{\mathrm{TH}}(m_t) = \frac{d\sigma}{d\mathcal{O}}^{\mathrm{Exp}}$$

Top quark mass measurement

ATLAS: 1905.02302

CMS: 1904.05237



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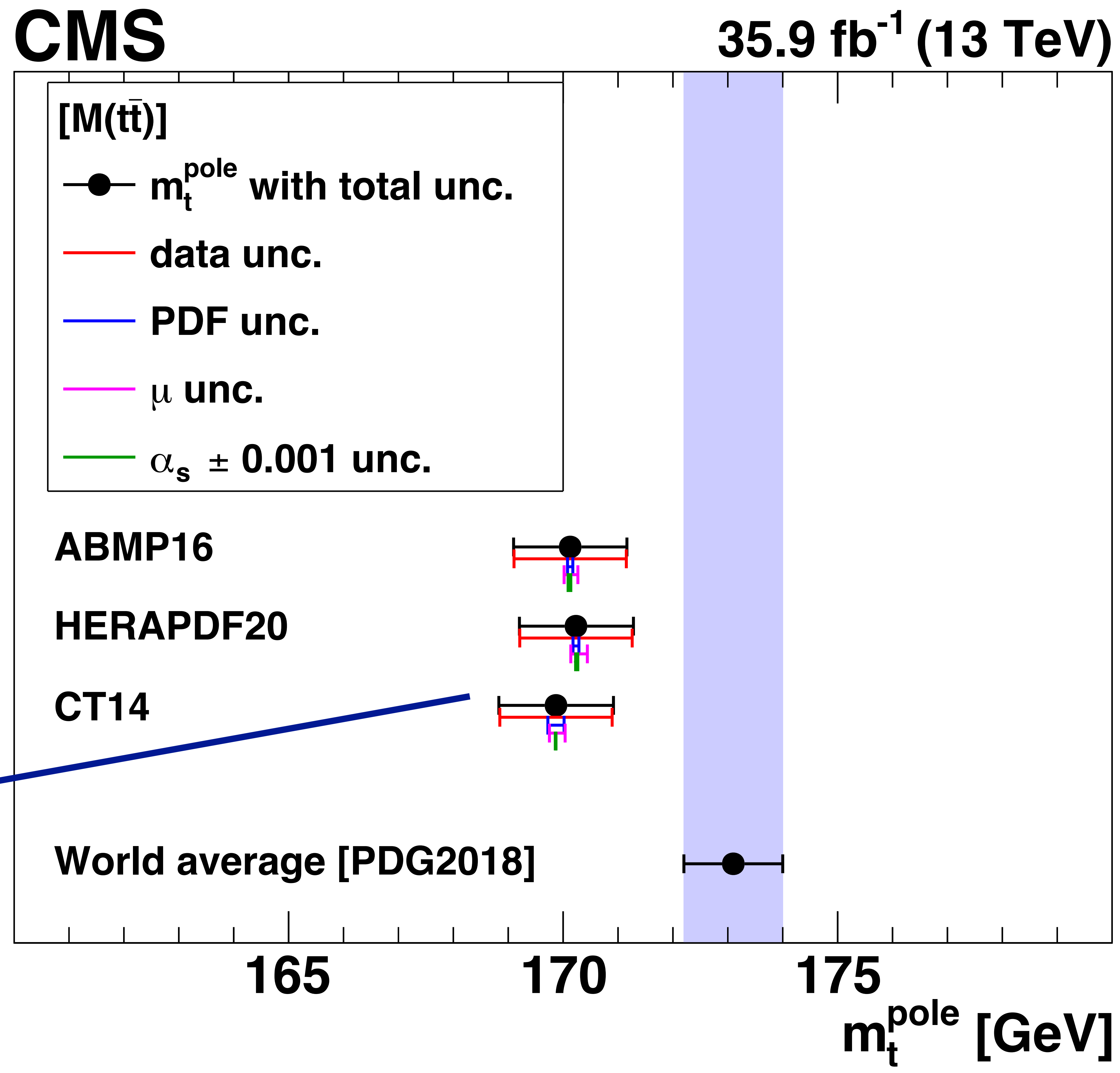

Indirect measurements

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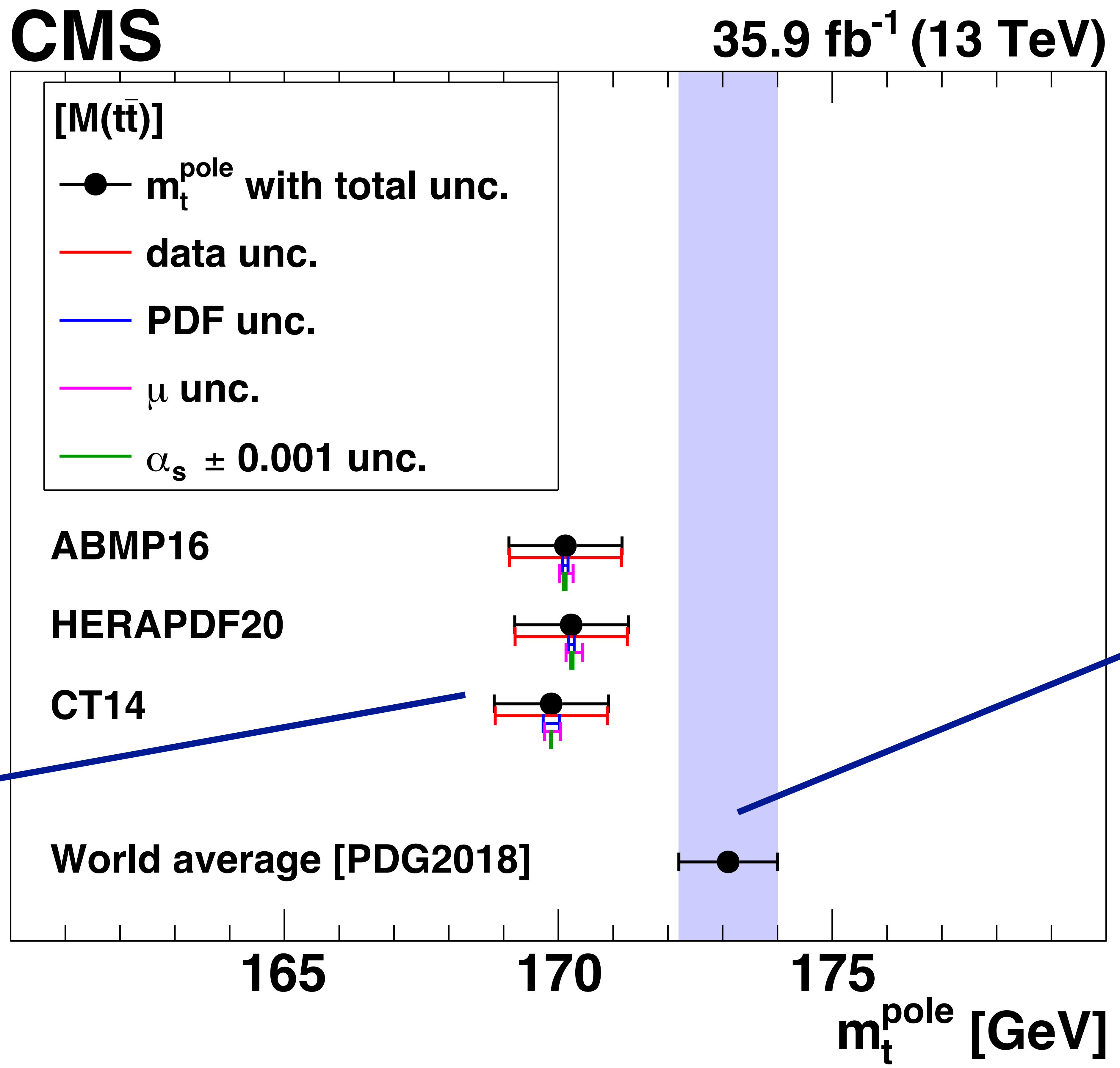

Dominated by direct measurements

Indirect measurements

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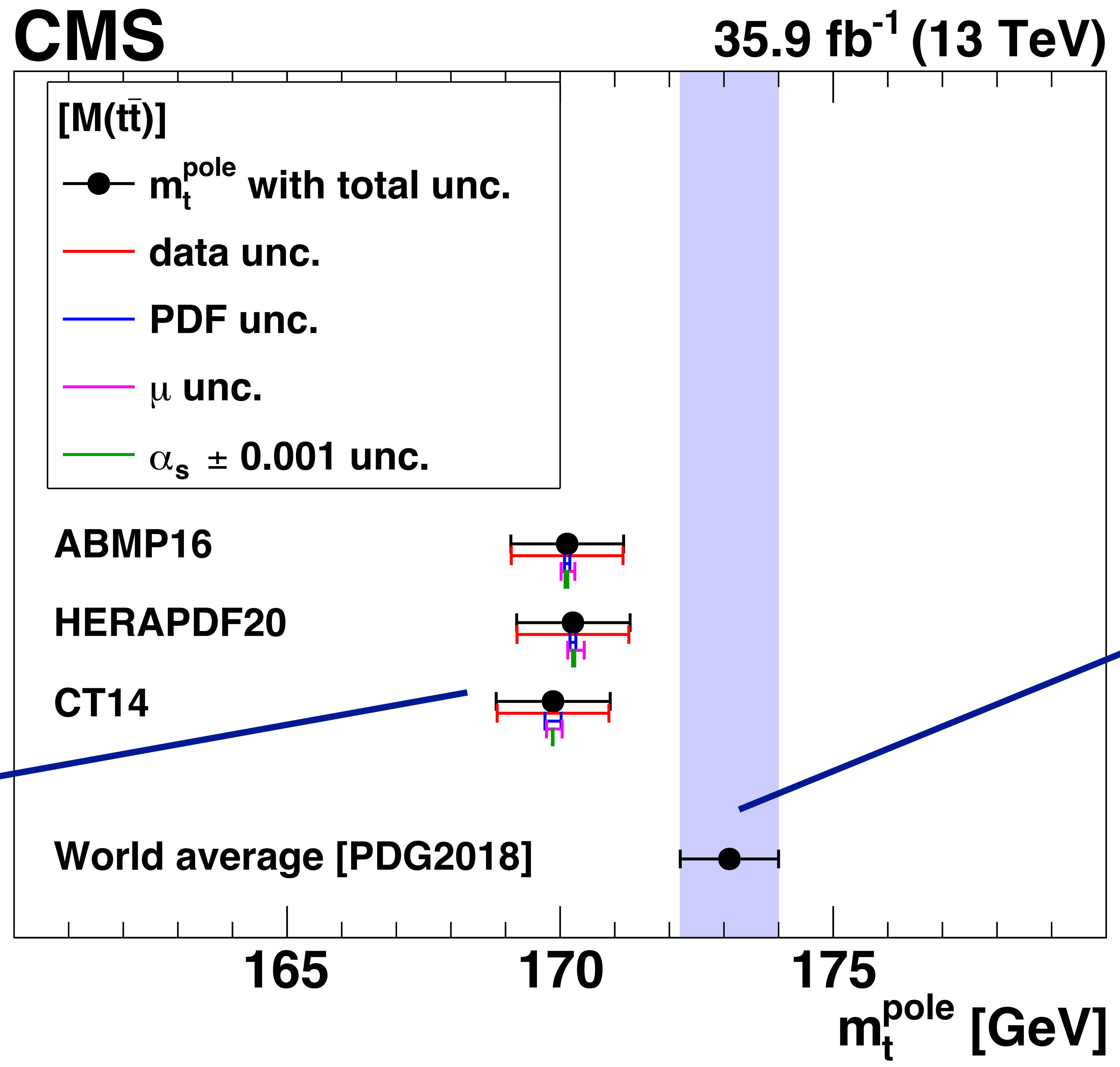


Indirect measurements

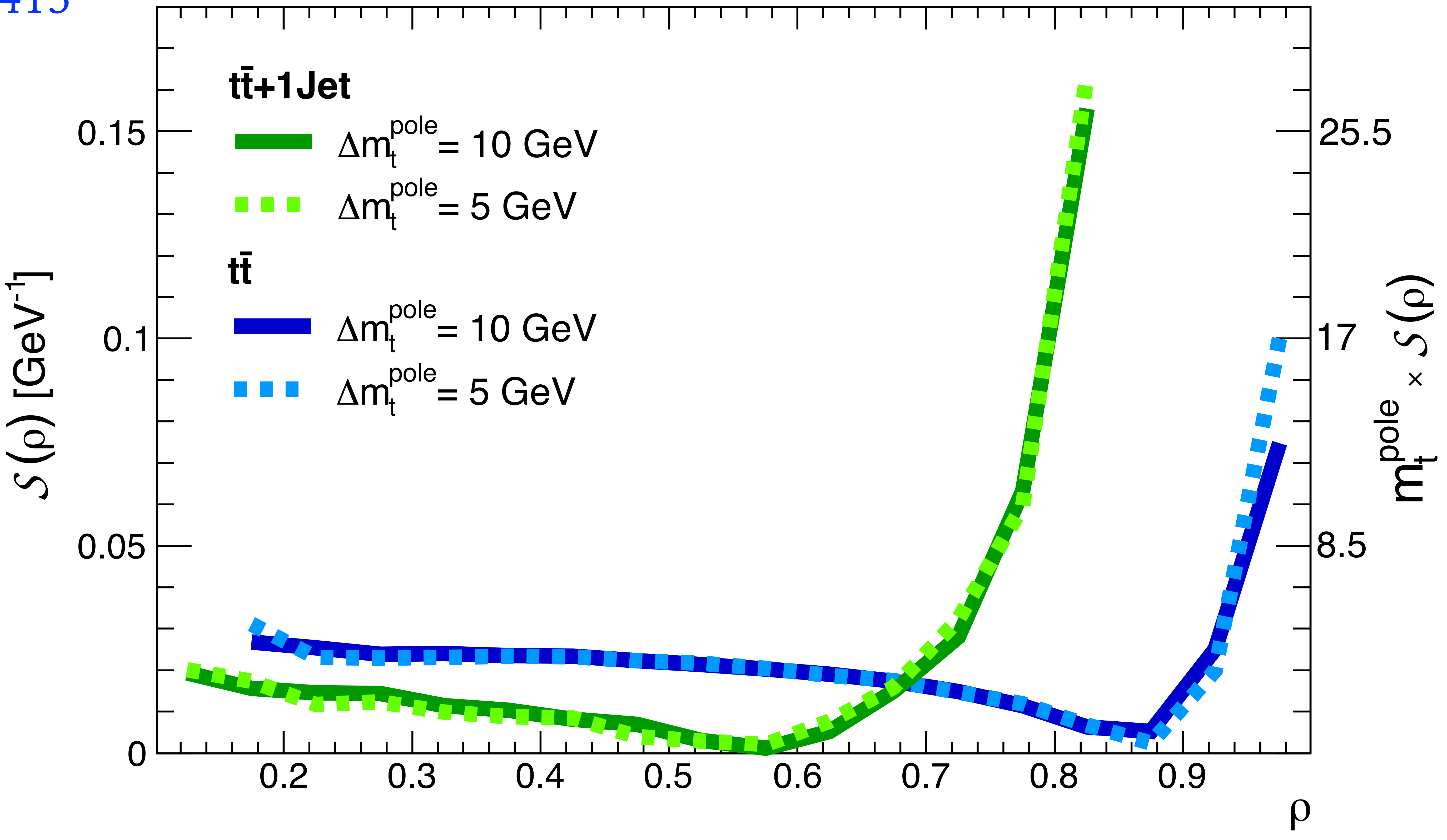
Dominated by direct measurements

Tension! Why?

Threshold region and mass measurement

Alioli et al.: 1303.6415

More sensitive to m_t



Threshold region and mass measurement

$M_{t\bar{t}} \to 2m_t$ in $t\bar{t}$+jets production

Alioli et al.: 1303.6415

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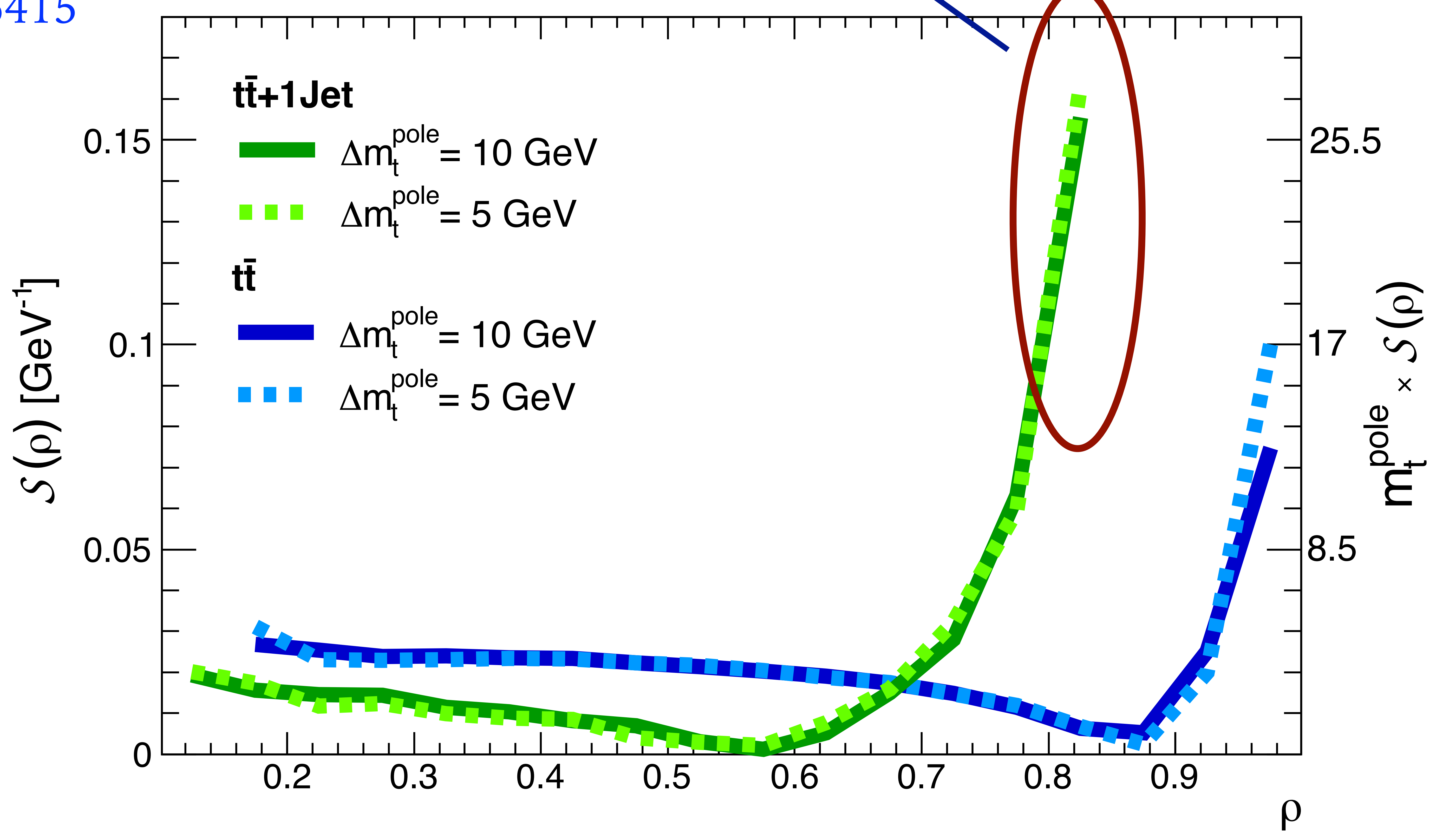

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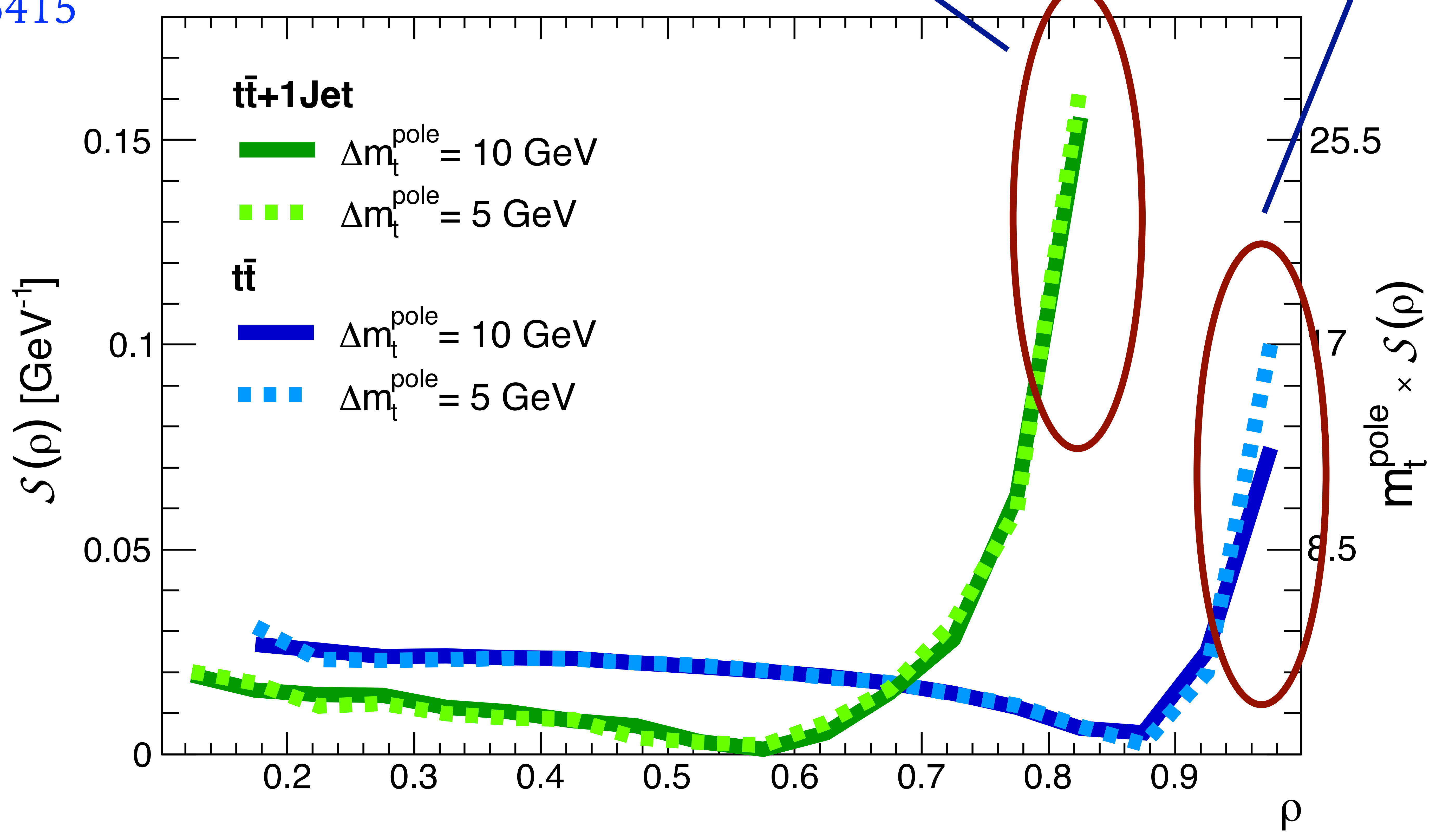

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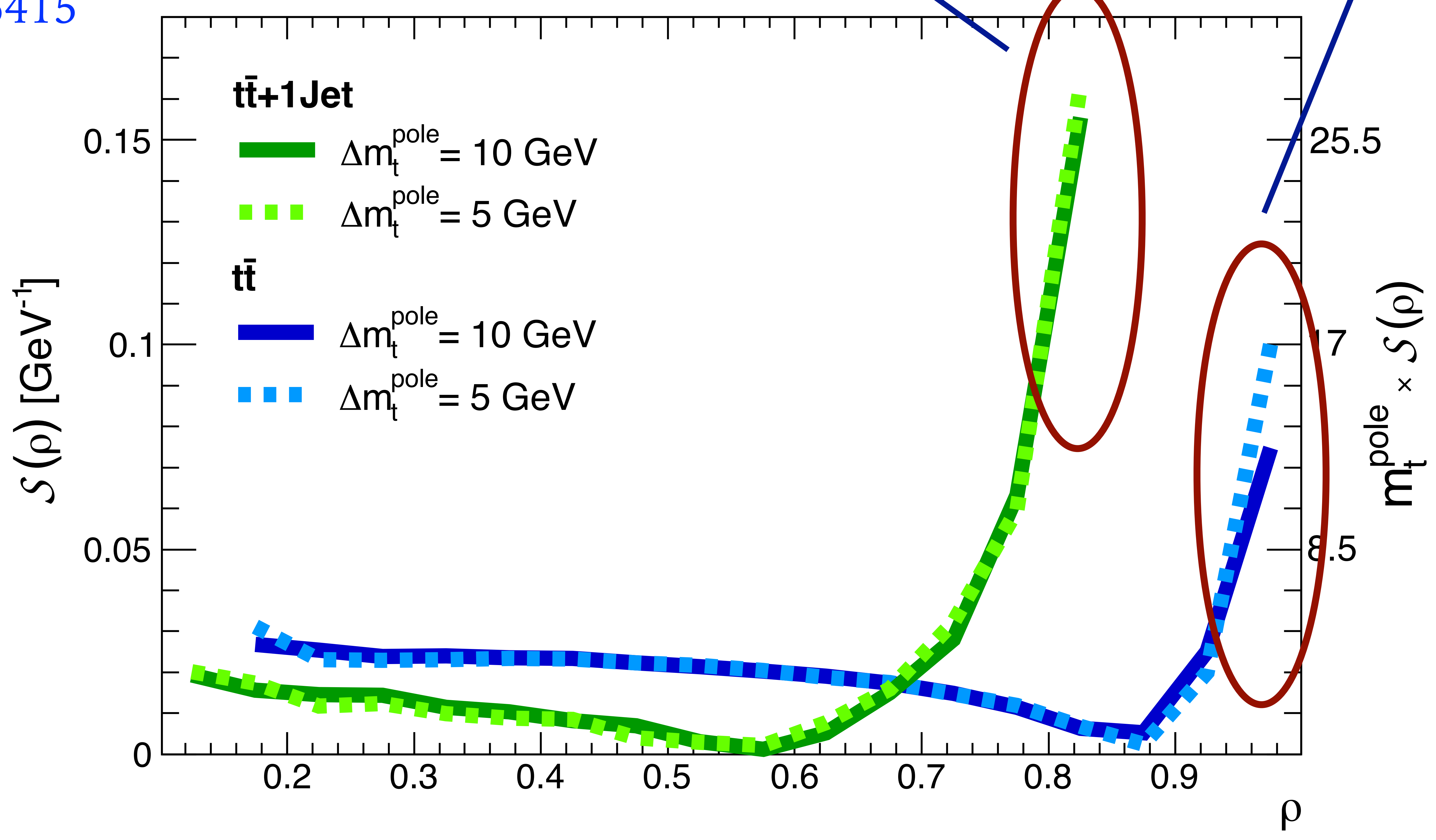
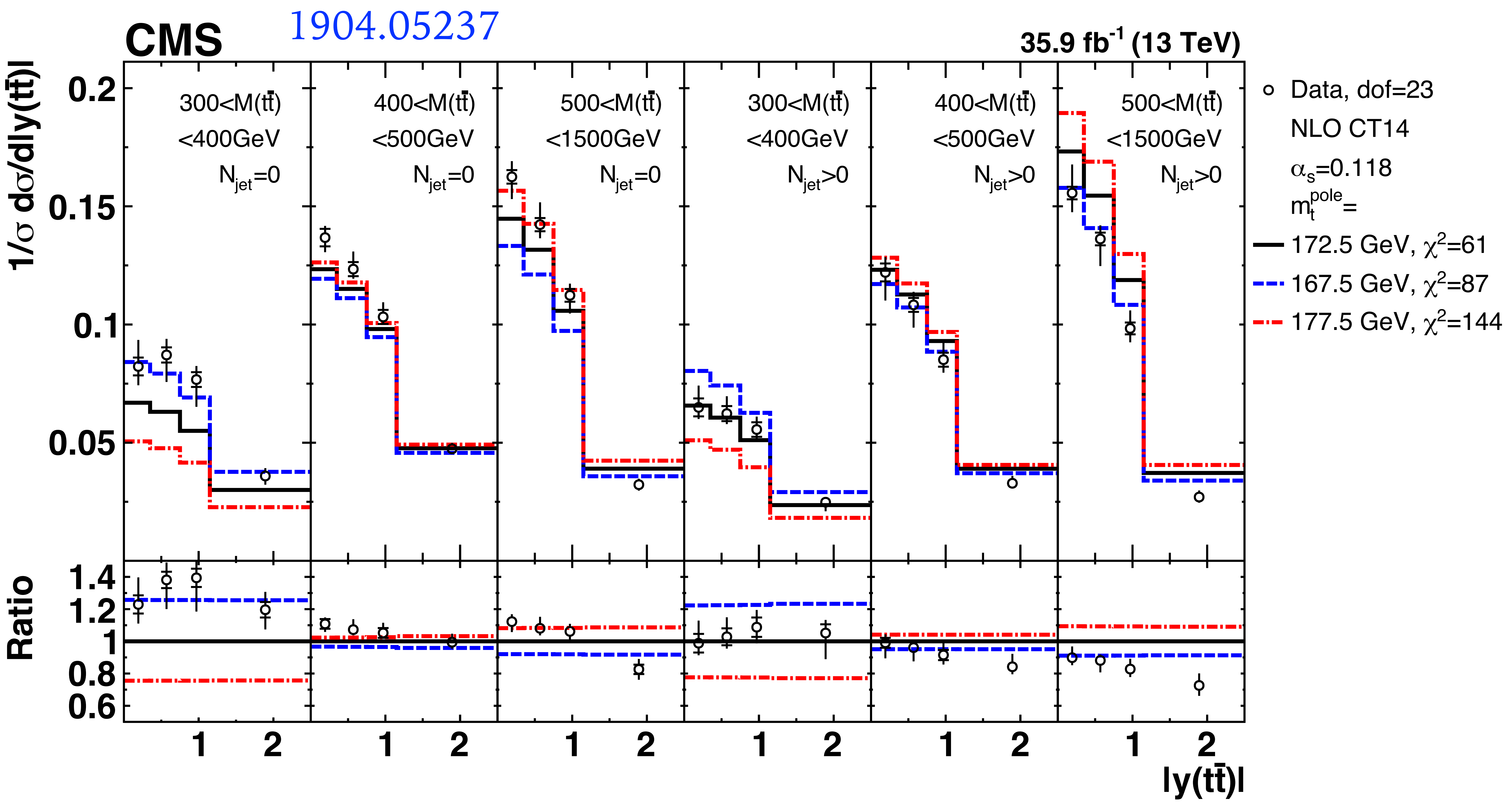


It is well-known that threshold region is most sensitive to m_t

Threshold region and mass measurement



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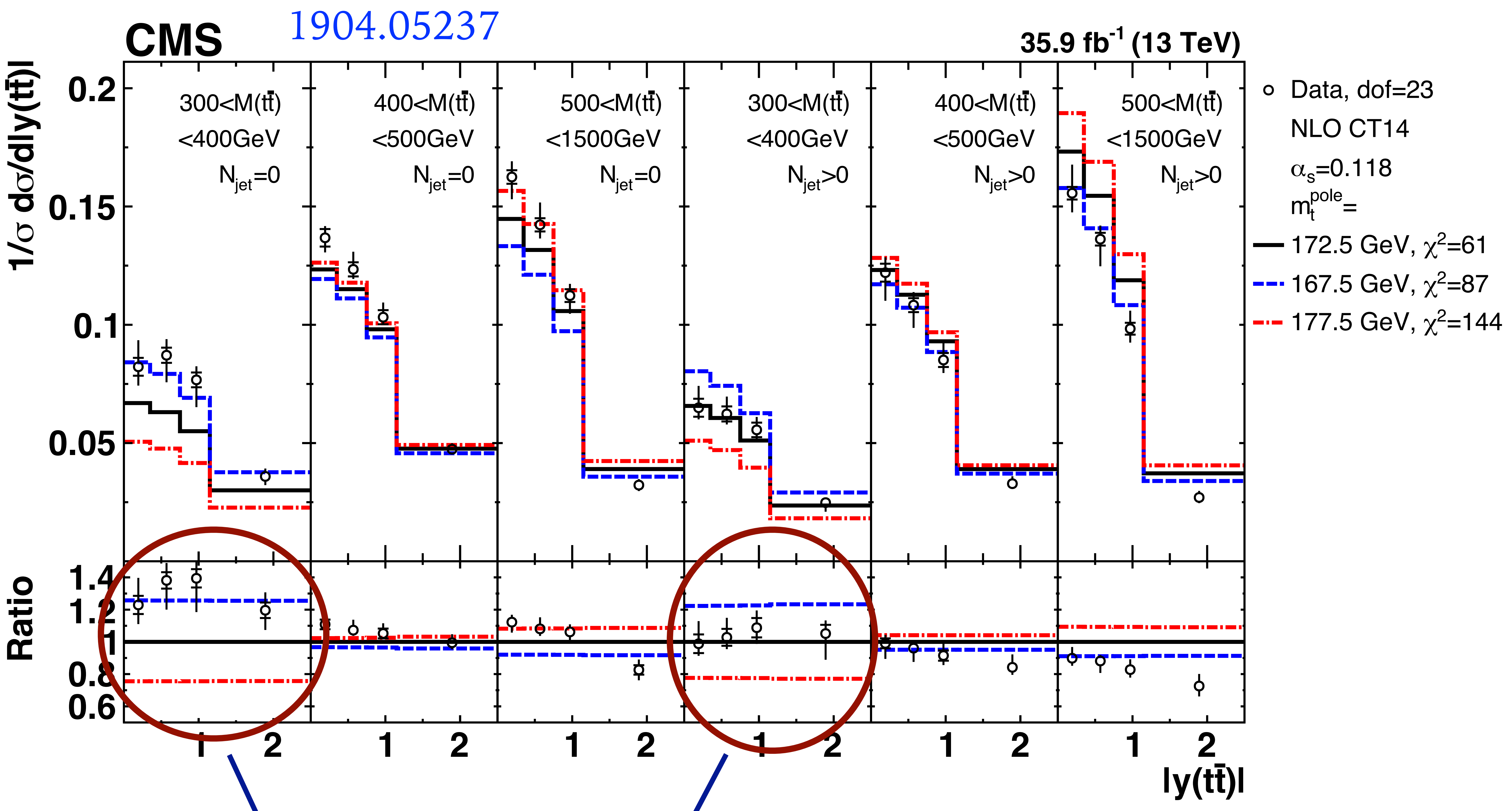

Data also shows that most of the mass-sensitivity comes from the low $M_{t\bar{t}}$ region

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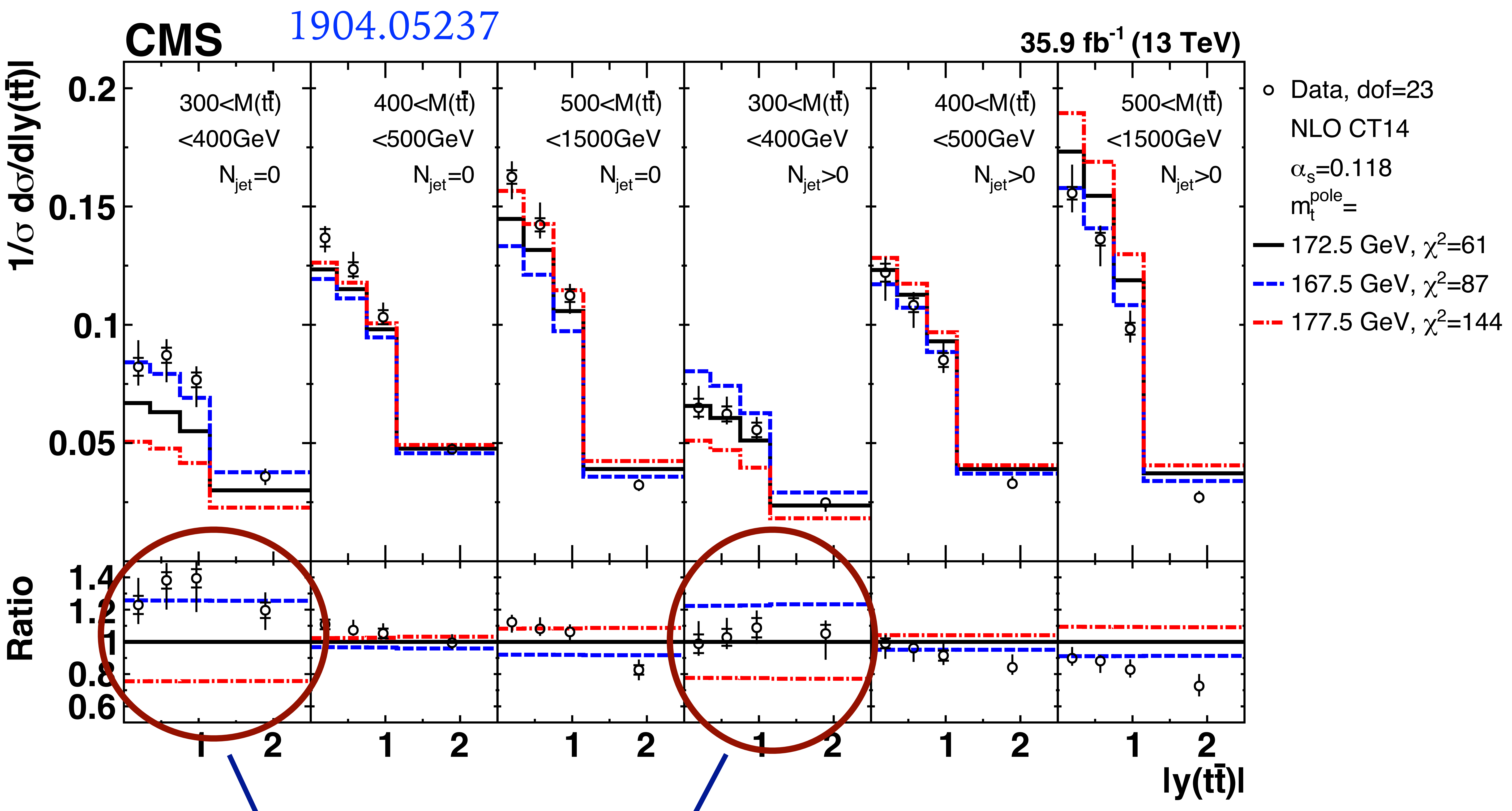

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What’s going on here?

Theoretical predictions vs experimental data

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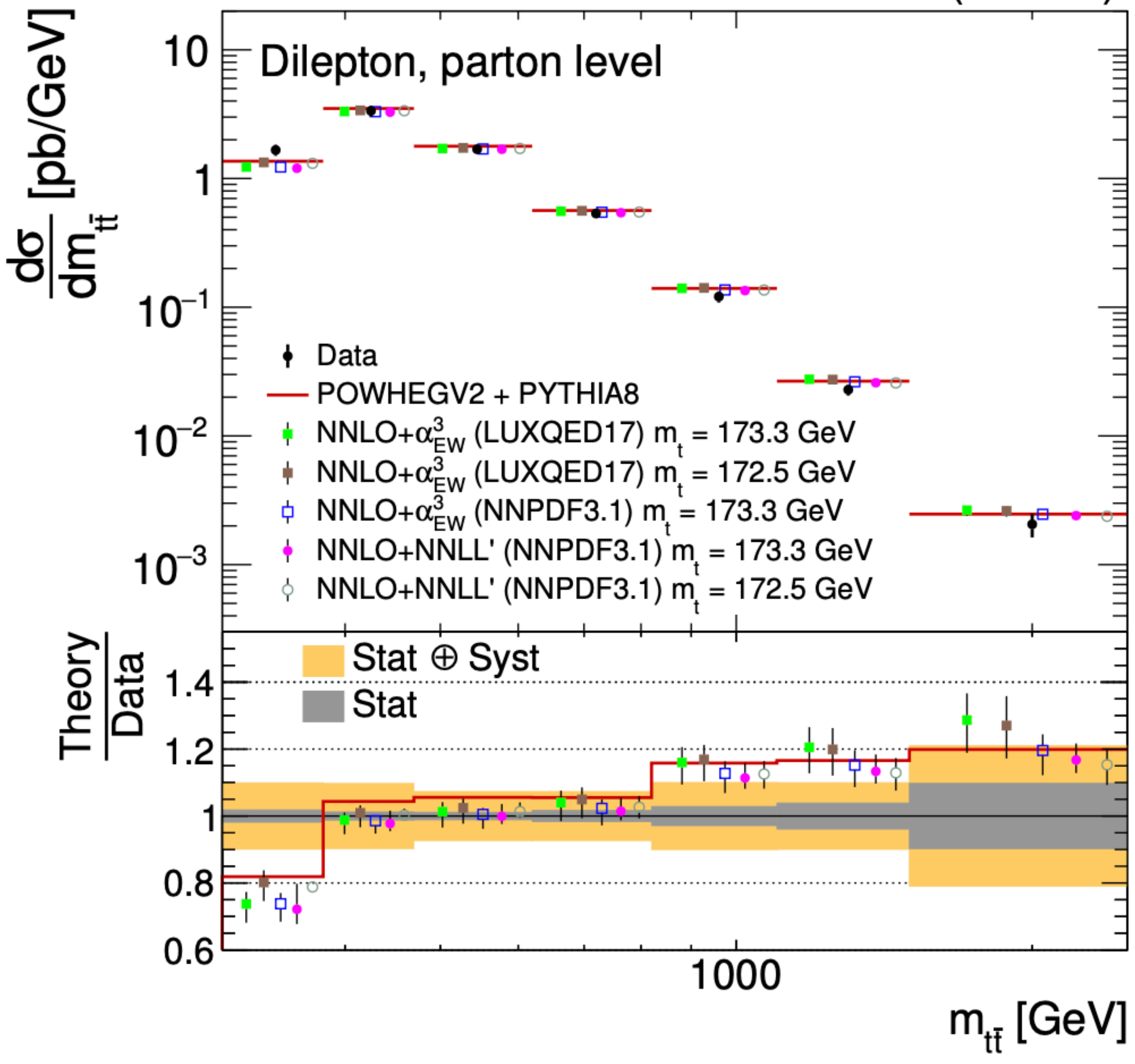

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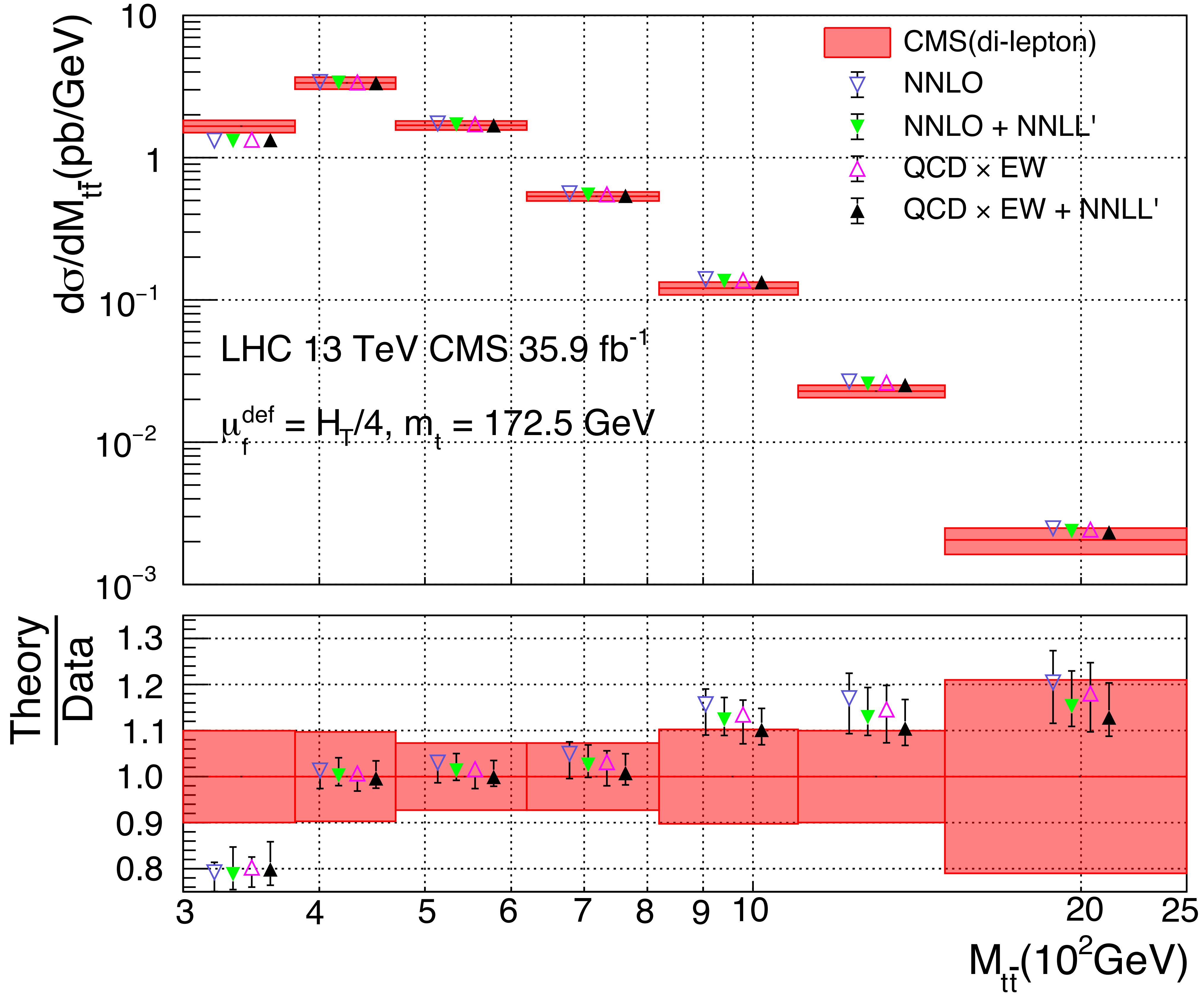

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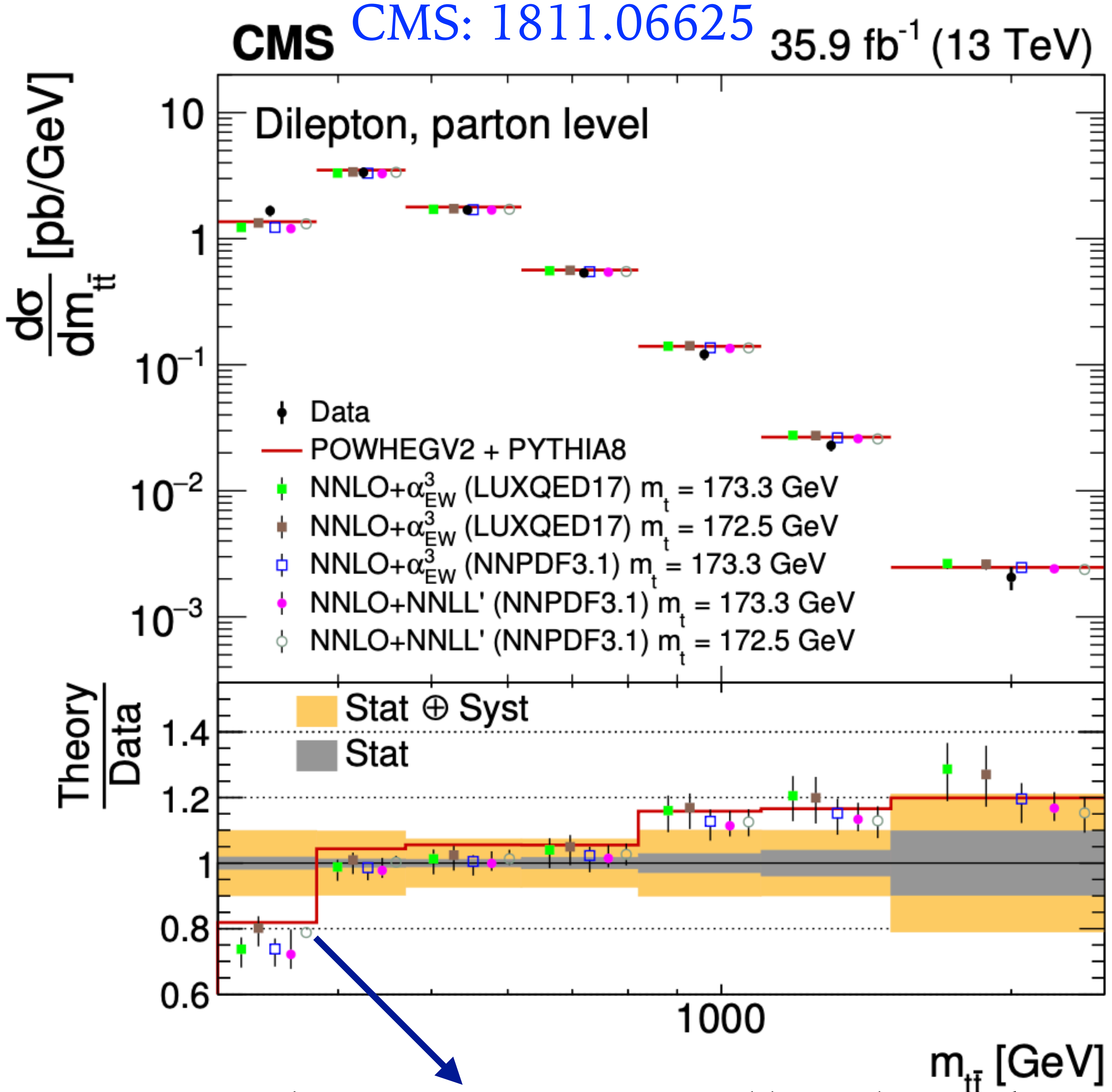

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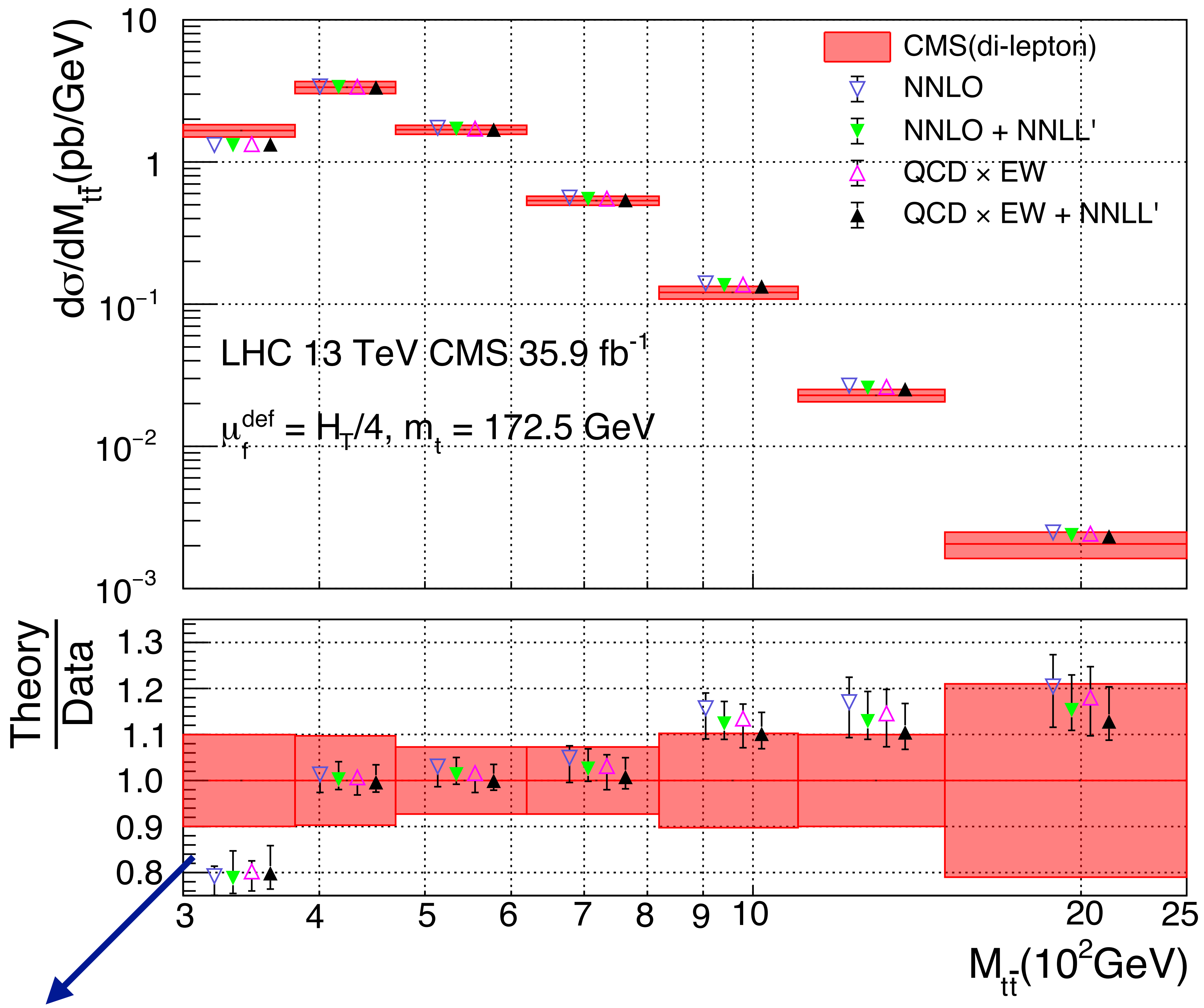

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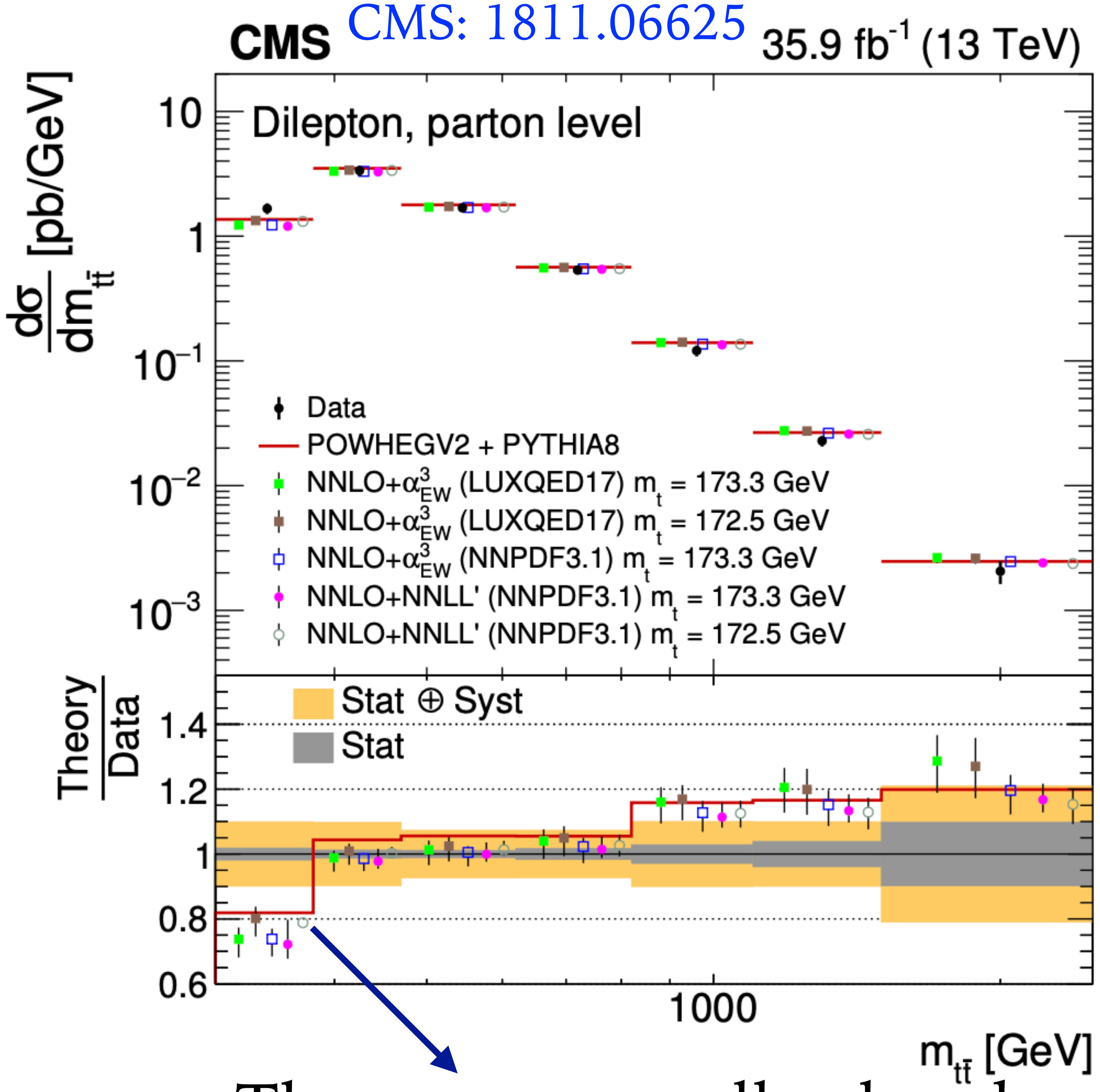

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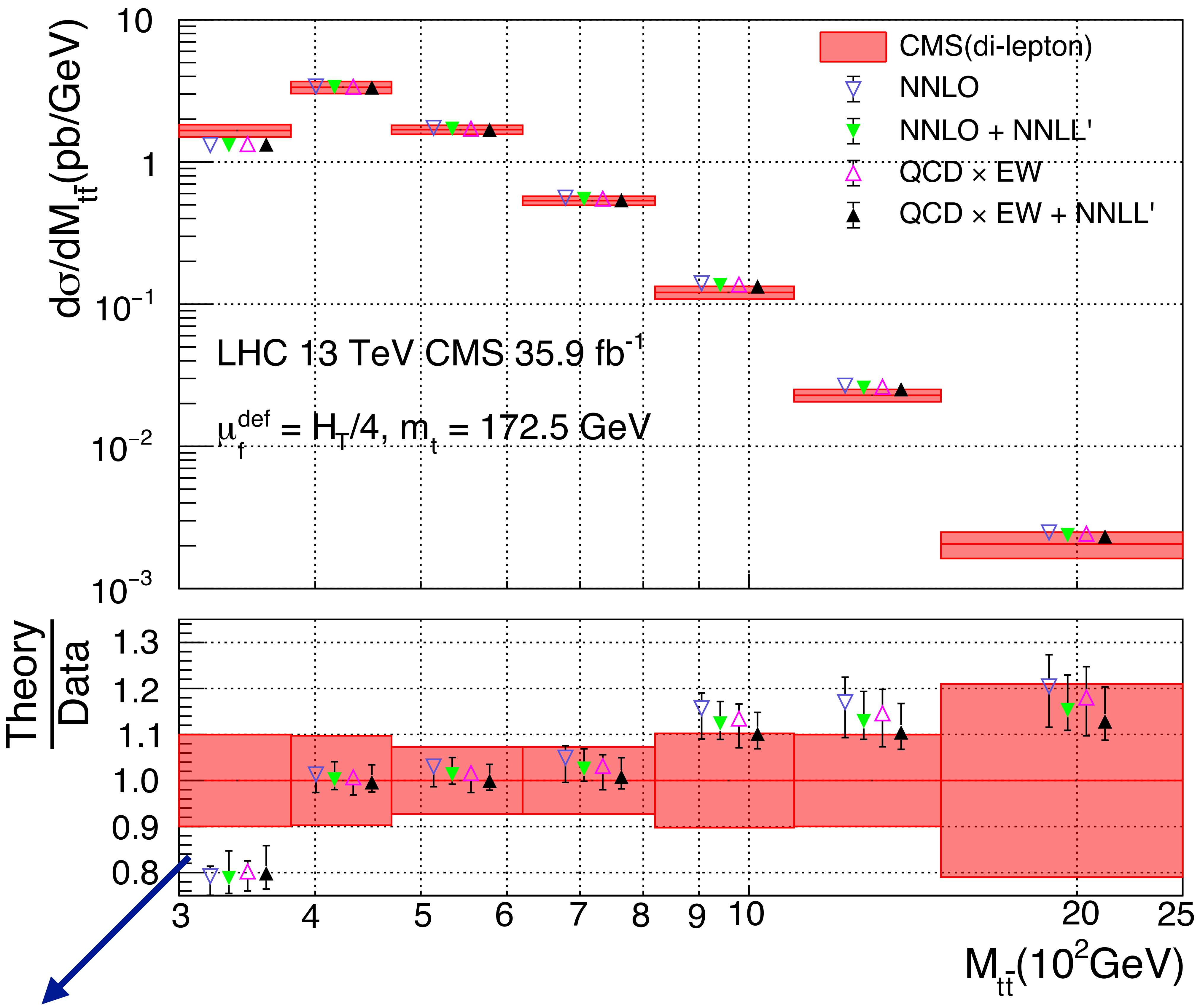

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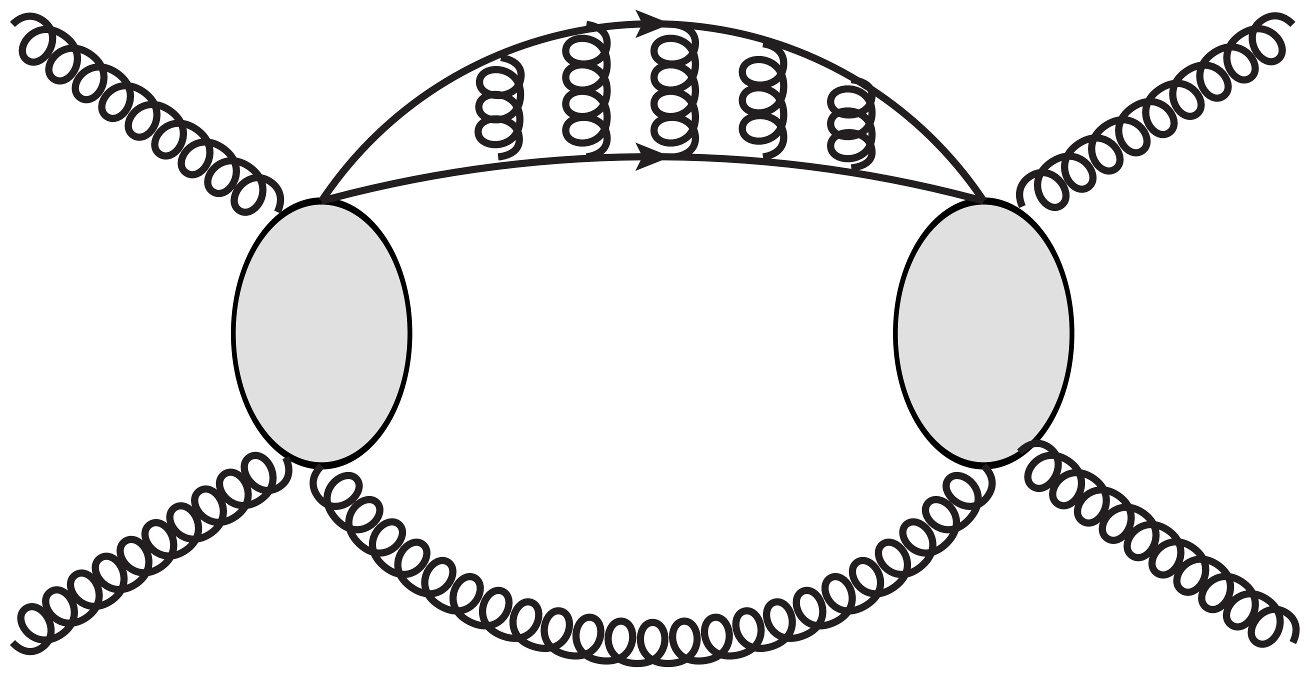
More corrections? New physics?

Non-relativistic Sommerfeld enhancement



$$\frac{\mathrm{d}\sigma}{\mathrm{d}M_{t\bar{t}}\mathrm{d}\Theta} \sim \beta\left(1 + \frac{\alpha_s}{\beta} + \frac{\alpha_s^2}{\beta^2} + \frac{\alpha_s^3}{\beta^3} + \cdots\right)$$

$$\beta = \sqrt{1 - \frac{4m_t^2}{M_{t\bar{t}}^2}} \qquad \beta \sim \alpha_s \ll 1$$

3-velocity of the top quark in the $t\bar{t}$ center-of-mass frame

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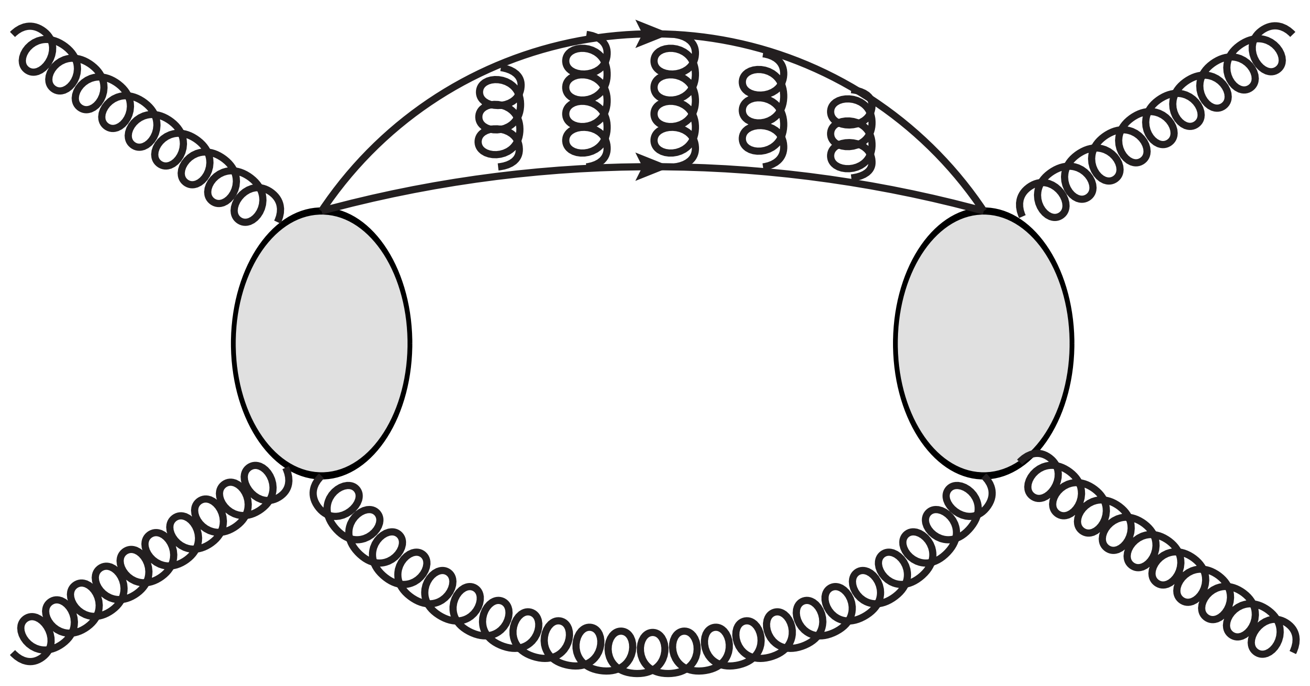

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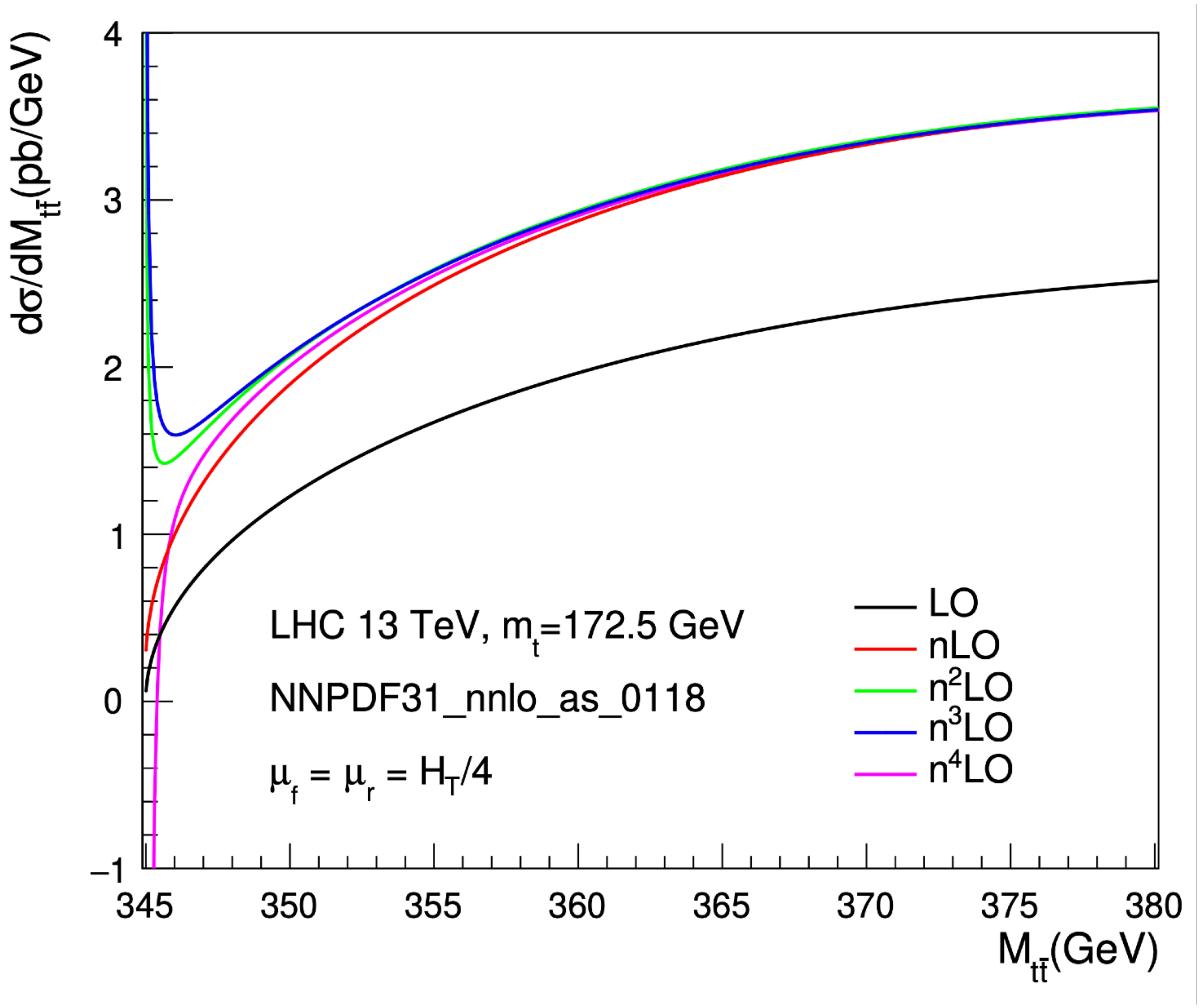

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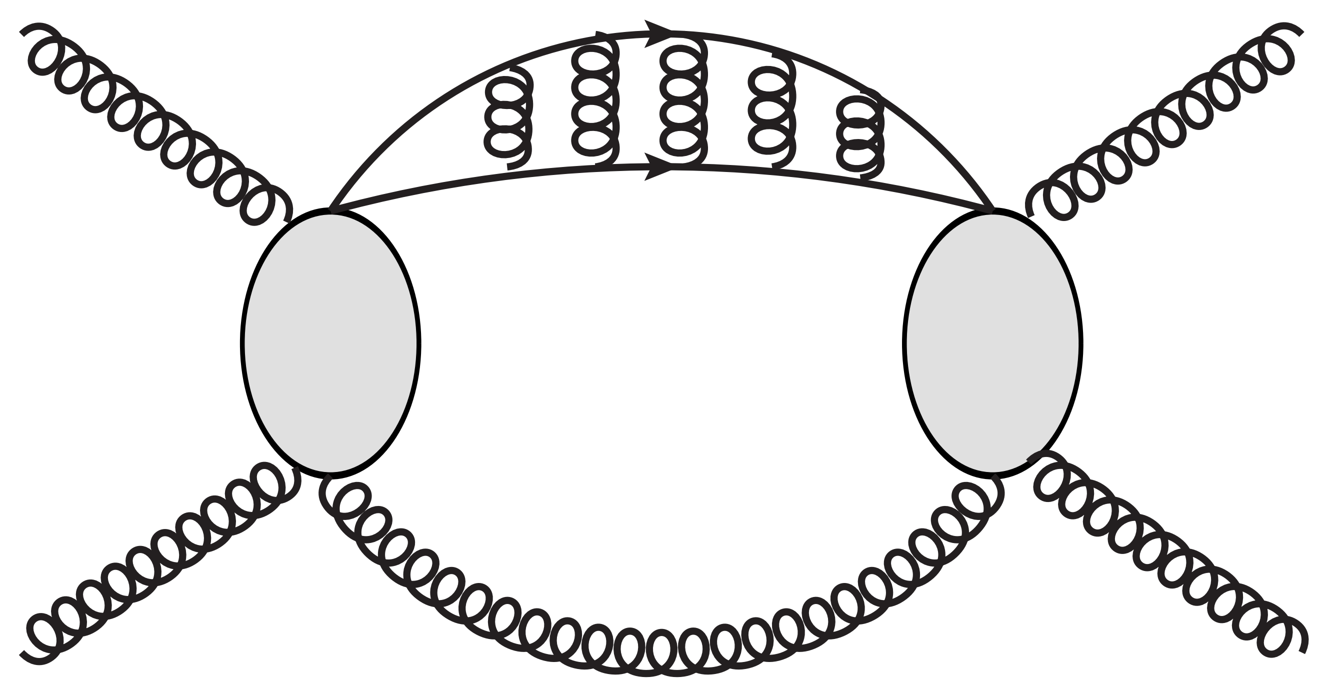

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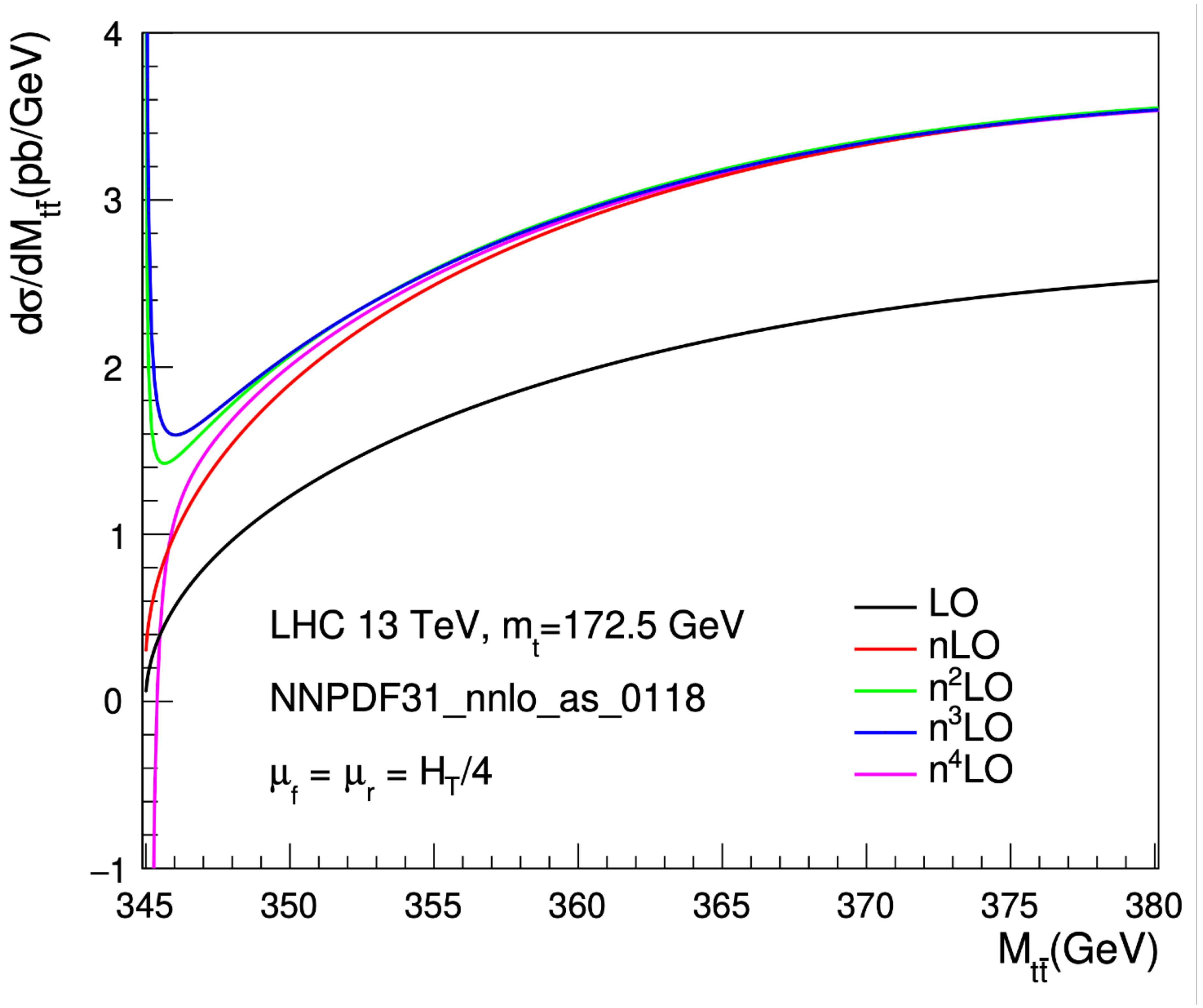
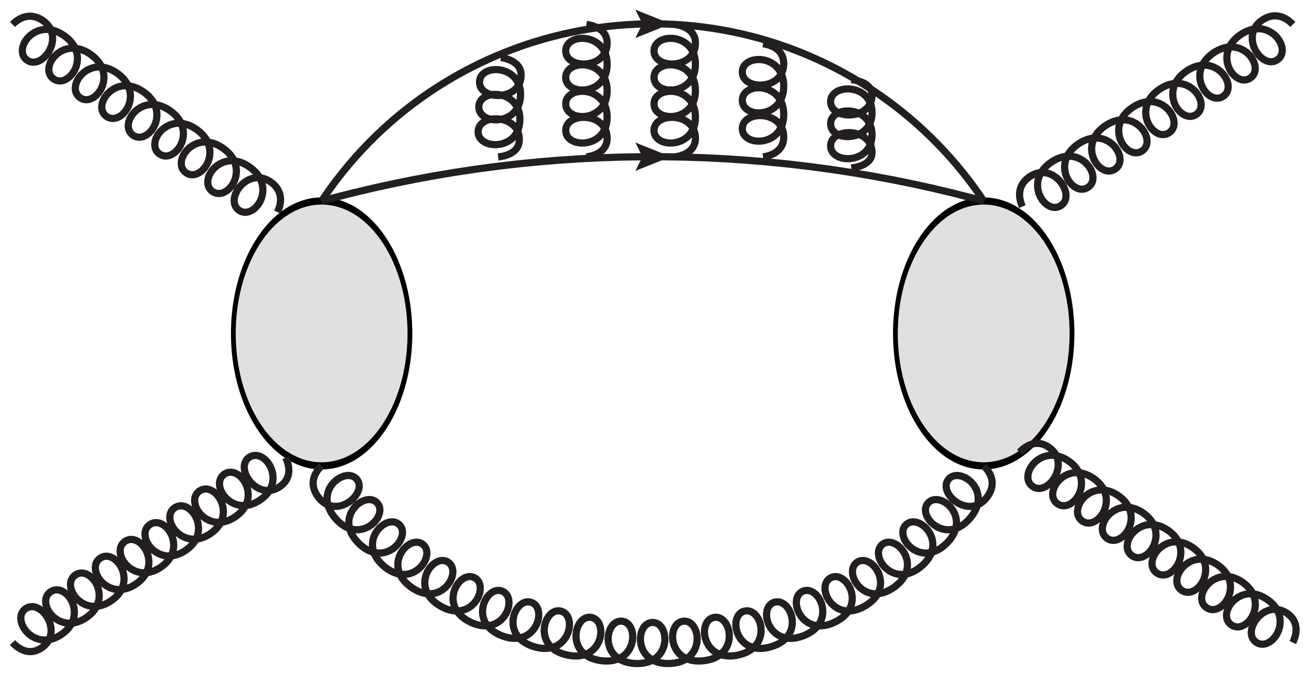


- No big deal at NLO
- Integrable at NNLO $\qquad \mathrm{d}M_{t\bar{t}} \sim \beta \mathrm{d}\beta$
- Subtleties beyond NNLO

Non-relativistic Sommerfeld enhancement



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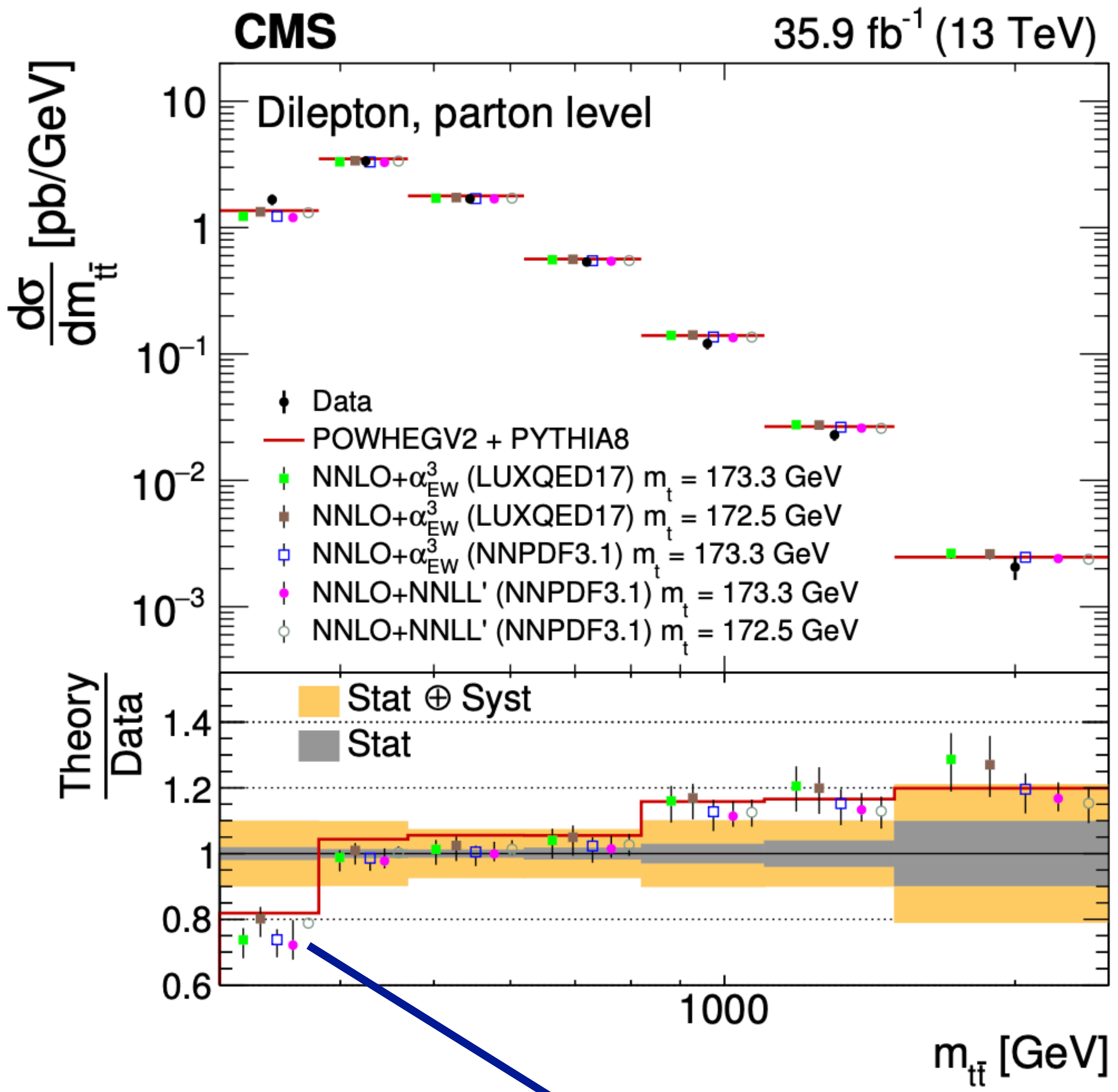

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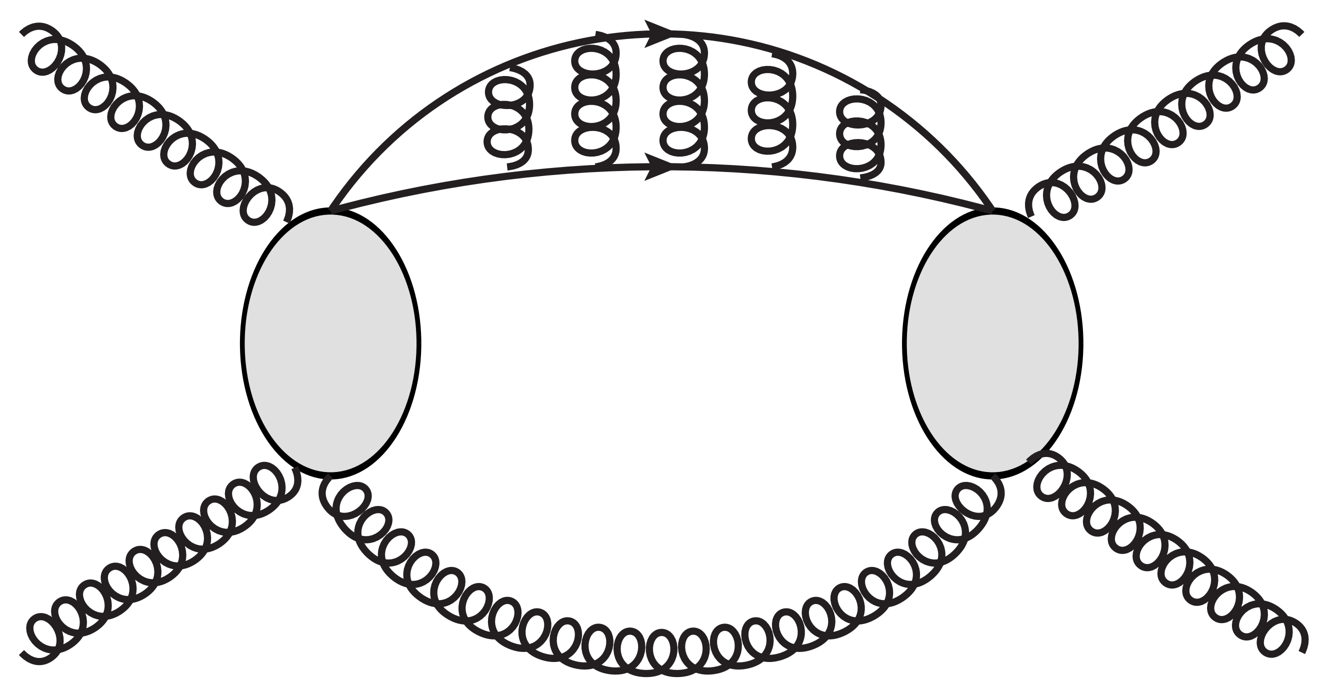

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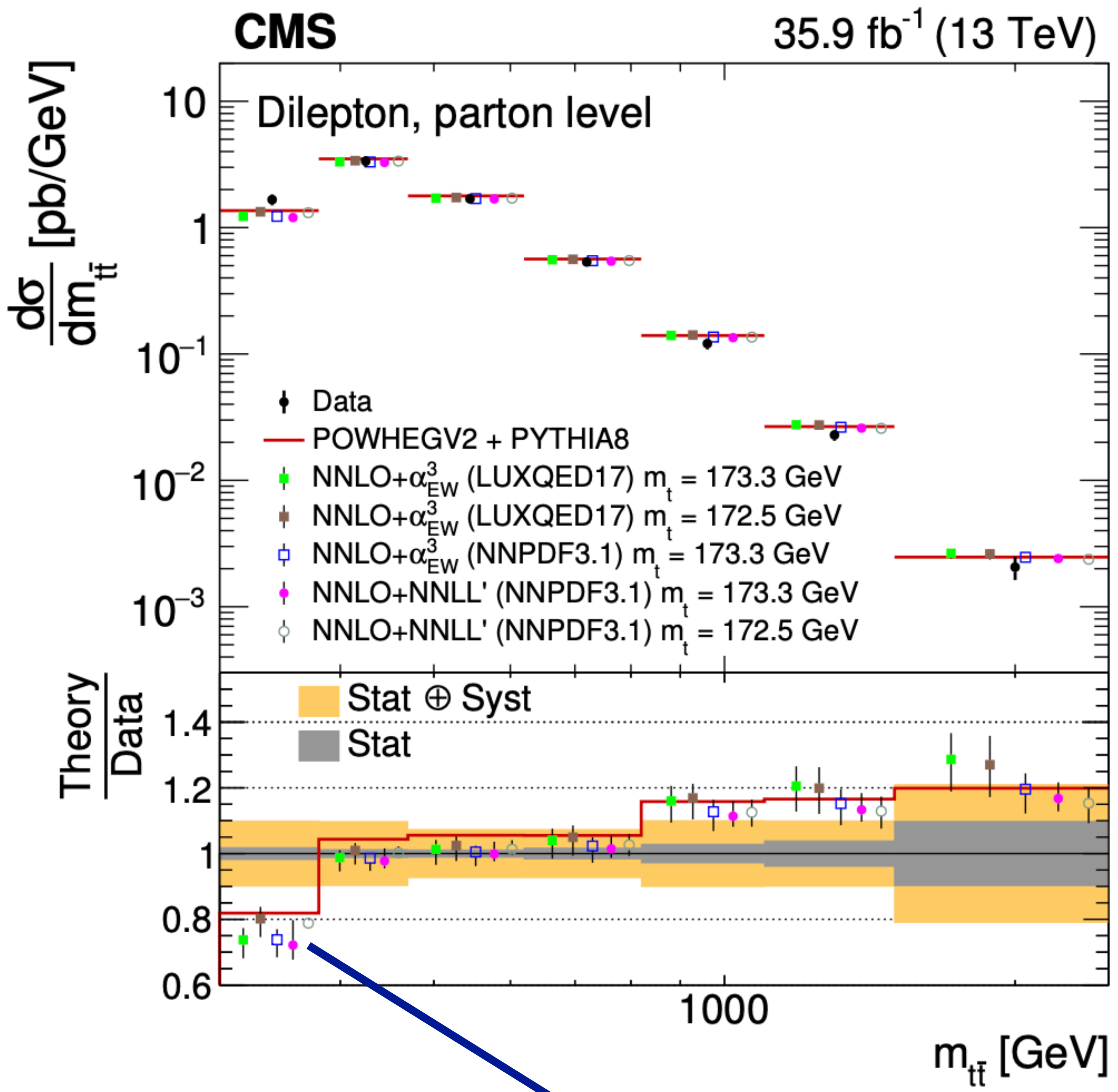

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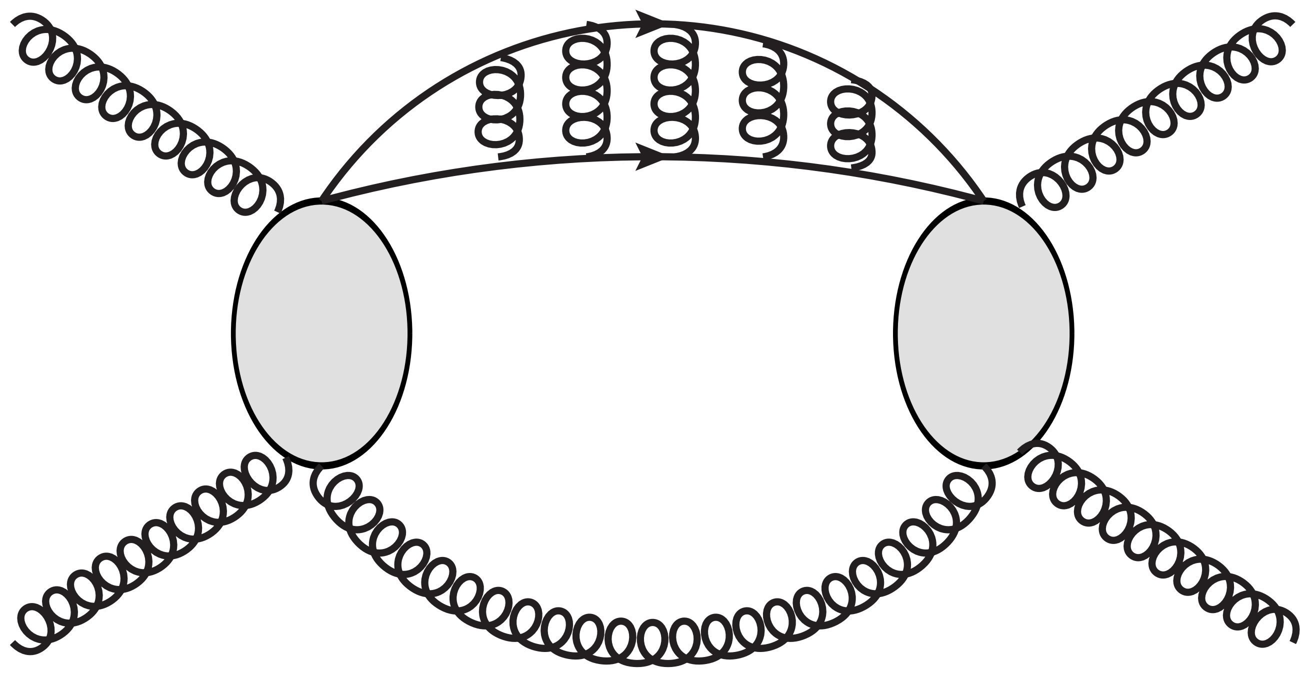
Higher order terms are important!

The "toponium"



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- Similar effects well-studied in quarkonium systems (hence the name "toponium")
- However, there are important differences!

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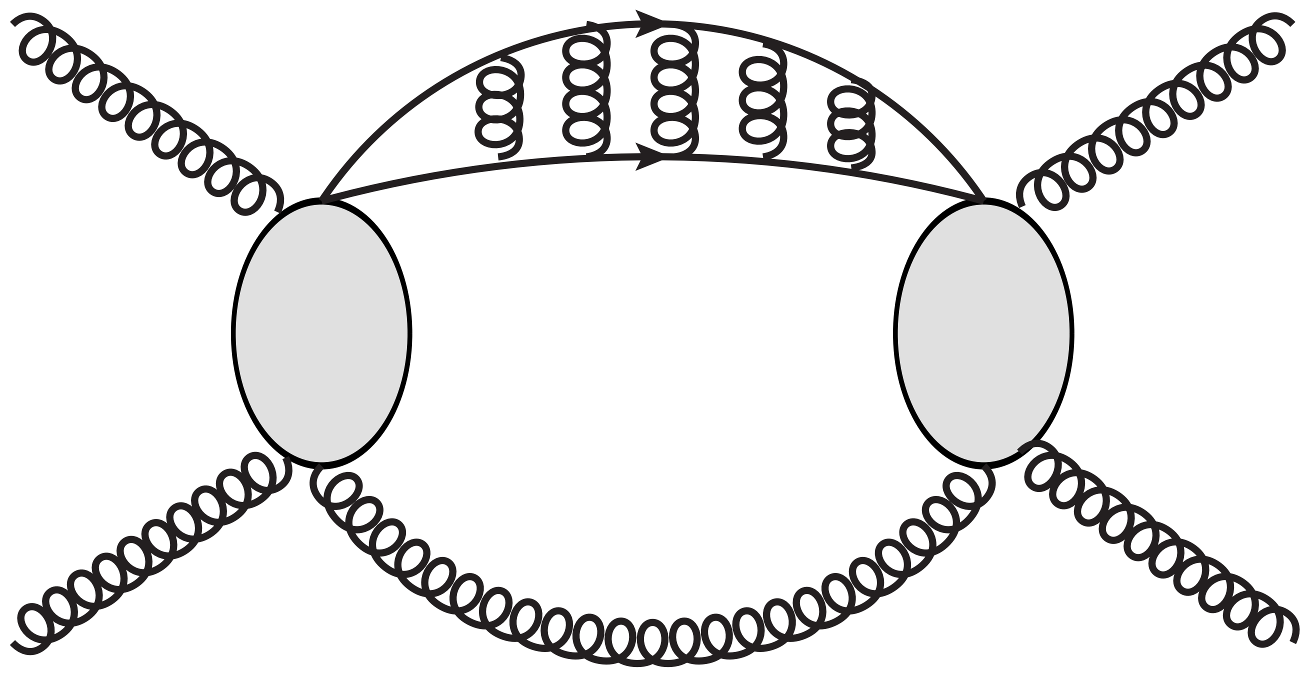

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η_c, η_b	Lifetime $\sim 10^{-22}$ s
$J/\psi, \Upsilon$	Lifetime $\sim 10^{-20}$ s
	Decay mainly through strong interactions (annihilation)
$t\bar{t}$	Lifetime $\sim 10^{-25}$ s
	Decay mainly through weak interactions (individually)

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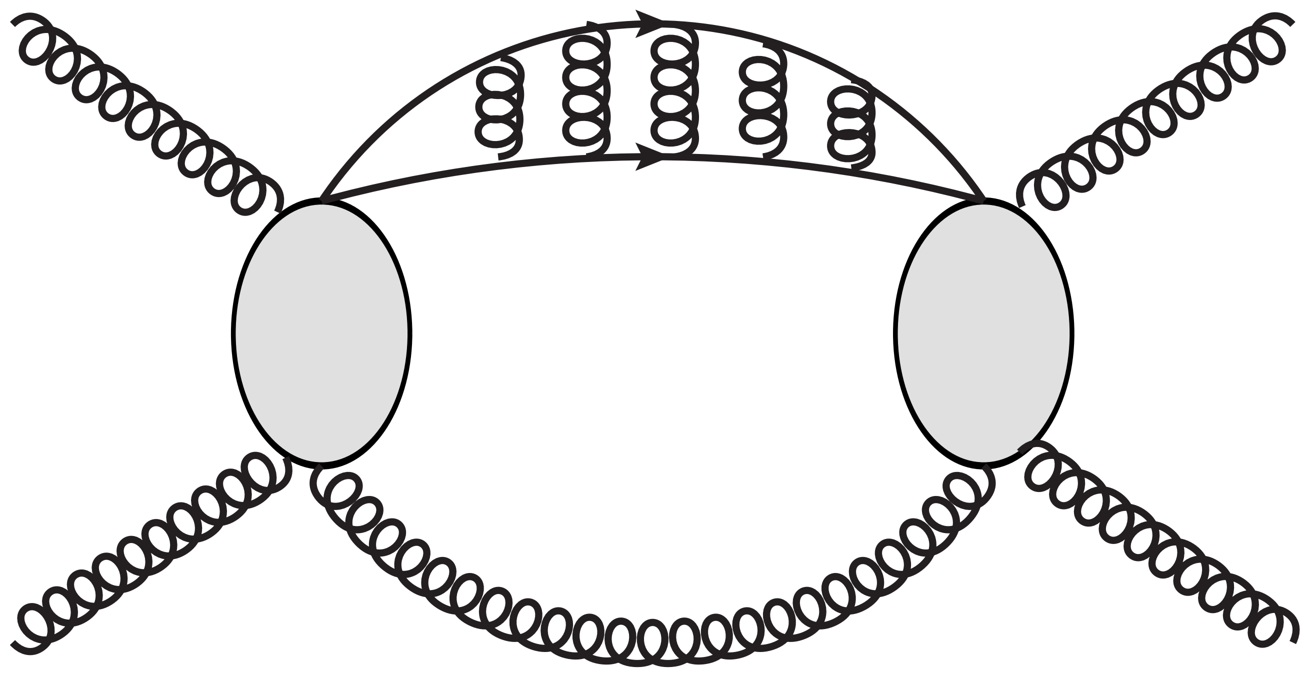

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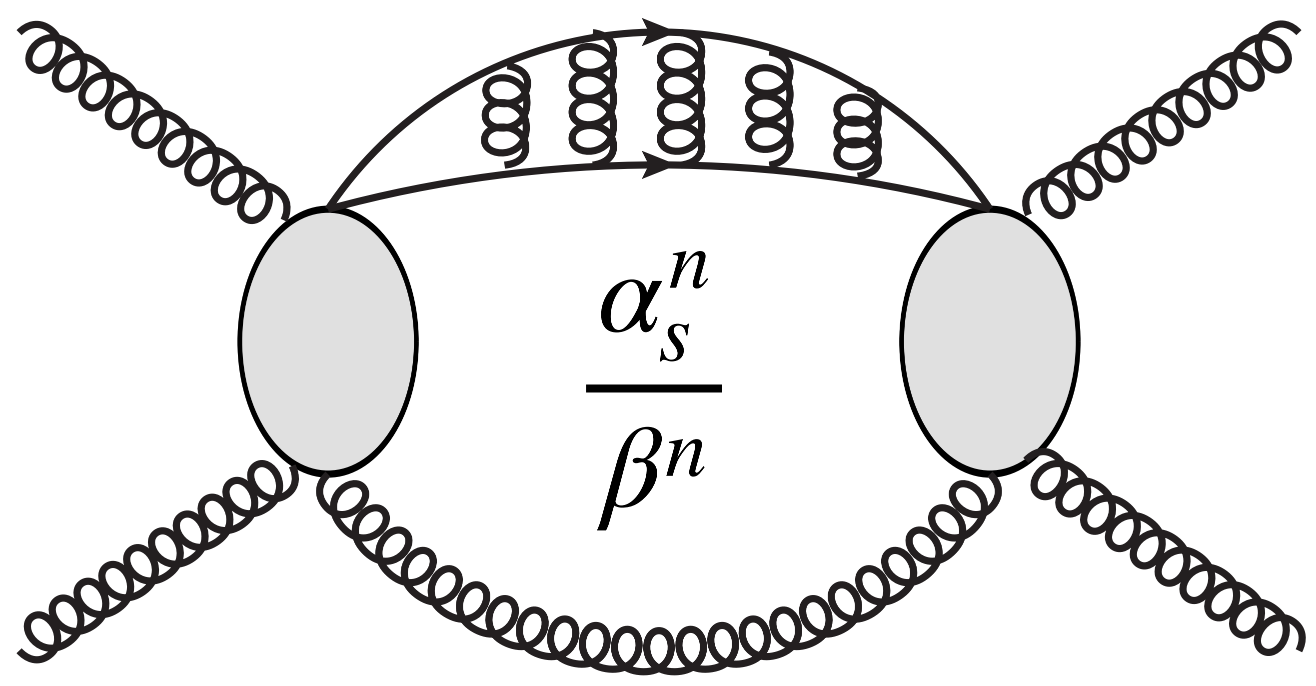
	"quasi-bound state"

Resummation of non-relativistic enhancement



See also:
Fadin et al.: 1987~1990
Kiyo et al.: 0812.0919
Sumino and Yokoya: 1007.0075
Beneke et al.: 1109.1536

Some further works:
Fuks et al.: 2102.11281, 2411.18962, 2509.03596
Garzelli et al.: 2412.16685
Wang et al.: 2411.17955
Fu et al.: 2412.11254, 2504.12634

Factorization in the $\beta \sim \alpha_s \ll 1$ limit

$$\frac{d\sigma}{dM_{t\bar{t}}d\Theta} \sim \int H \times J \times f \times f$$

Hard functions

Potential functions in NRQCD

PDFs

NNLO+NLP resummation

Ju, Wang, Wang, Xu, Xu, LLY: 1908.02179, 2004.03088

- Calculated NLO hard functions relevant for double differential distributions and dynamic scales
- Incorporated potential functions at NLP
- Combined with NNLO fixed-order results by matching
- First to point out this effect in Run 2 data

Resummation of non-relativistic enhancement

Ju, Wang, Wang, Xu, Xu, LLY:
1908.02179, 2004.03088

$$\frac{d\sigma^{\text{NNLO+NLP}}}{dM_{t\bar{t}}d\Theta} = \frac{d\sigma^{\text{NLP}}}{dM_{t\bar{t}}d\Theta} + \frac{d\sigma^{\text{NNLO}}}{dM_{t\bar{t}}d\Theta} - \frac{d\sigma^{\text{nnLO}}}{dM_{t\bar{t}}d\Theta}$$

NLP re-expanded to NNLO

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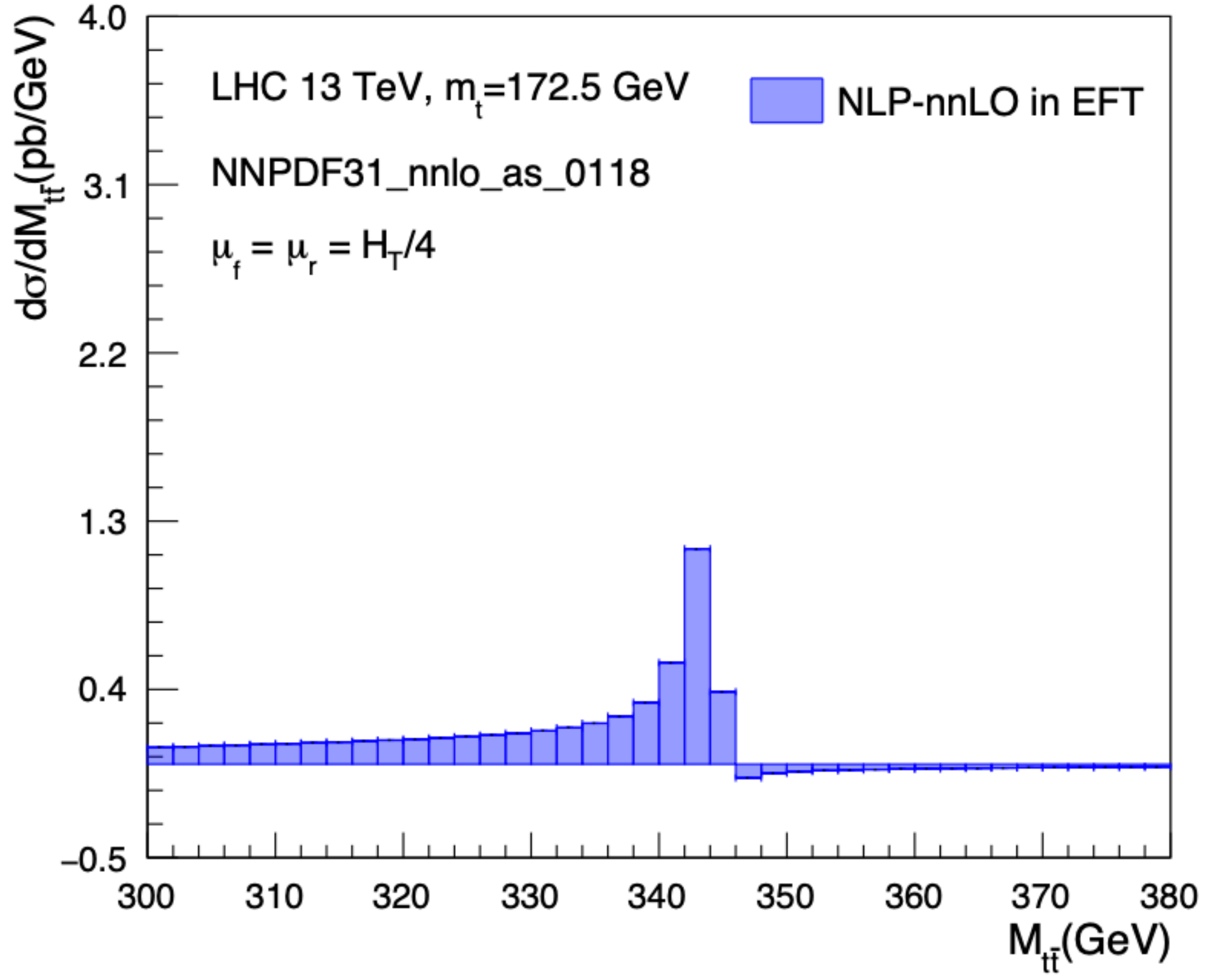

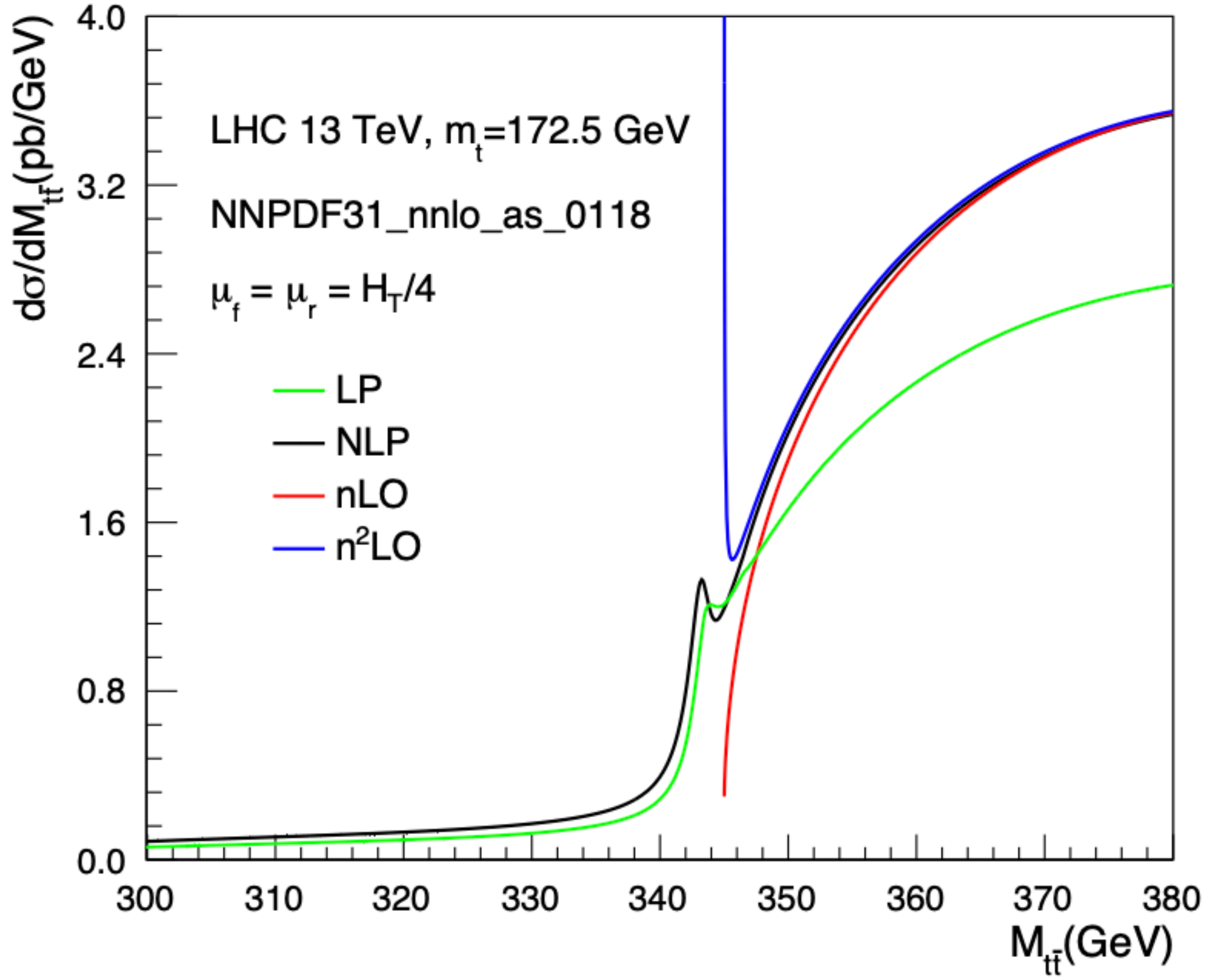

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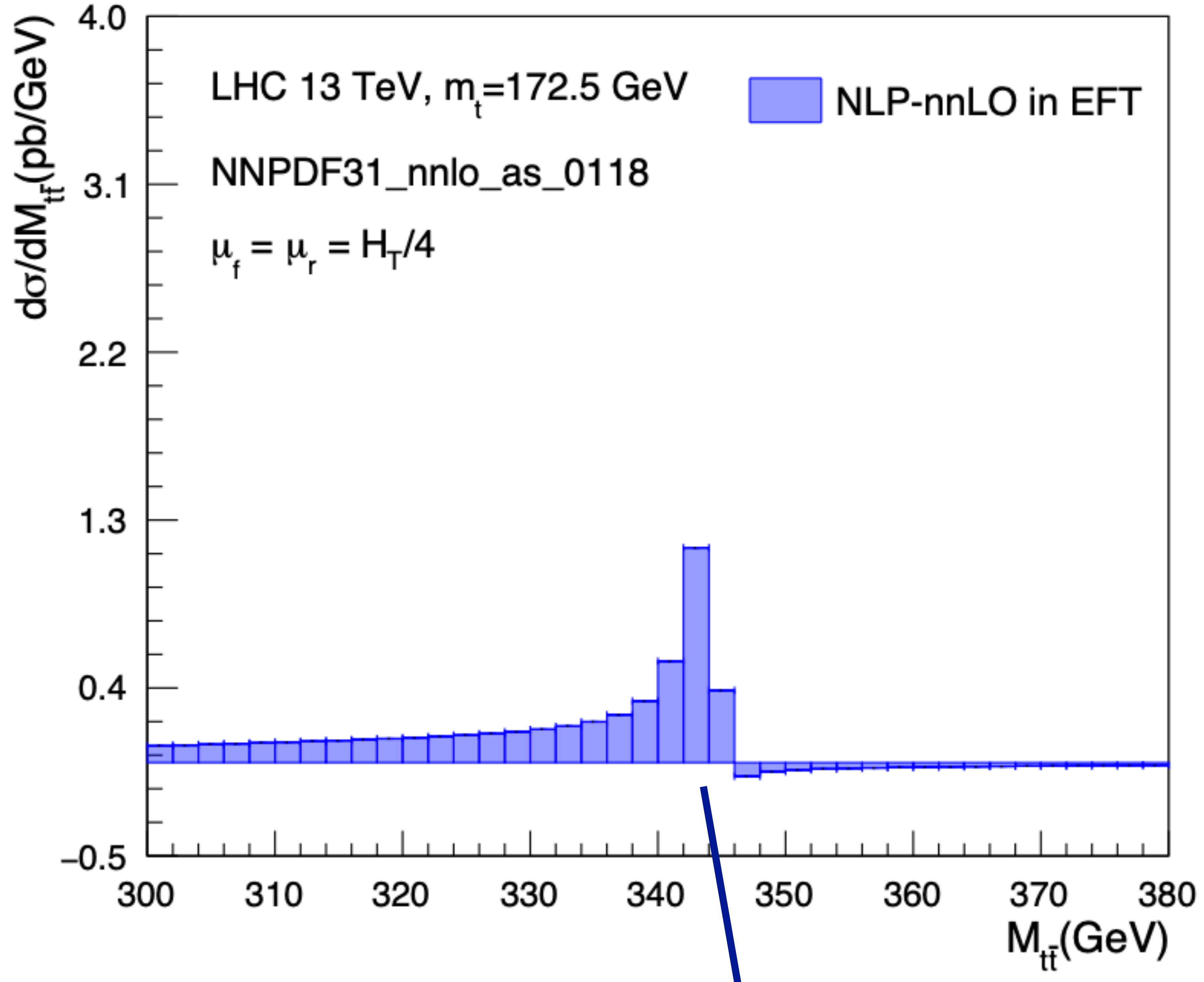

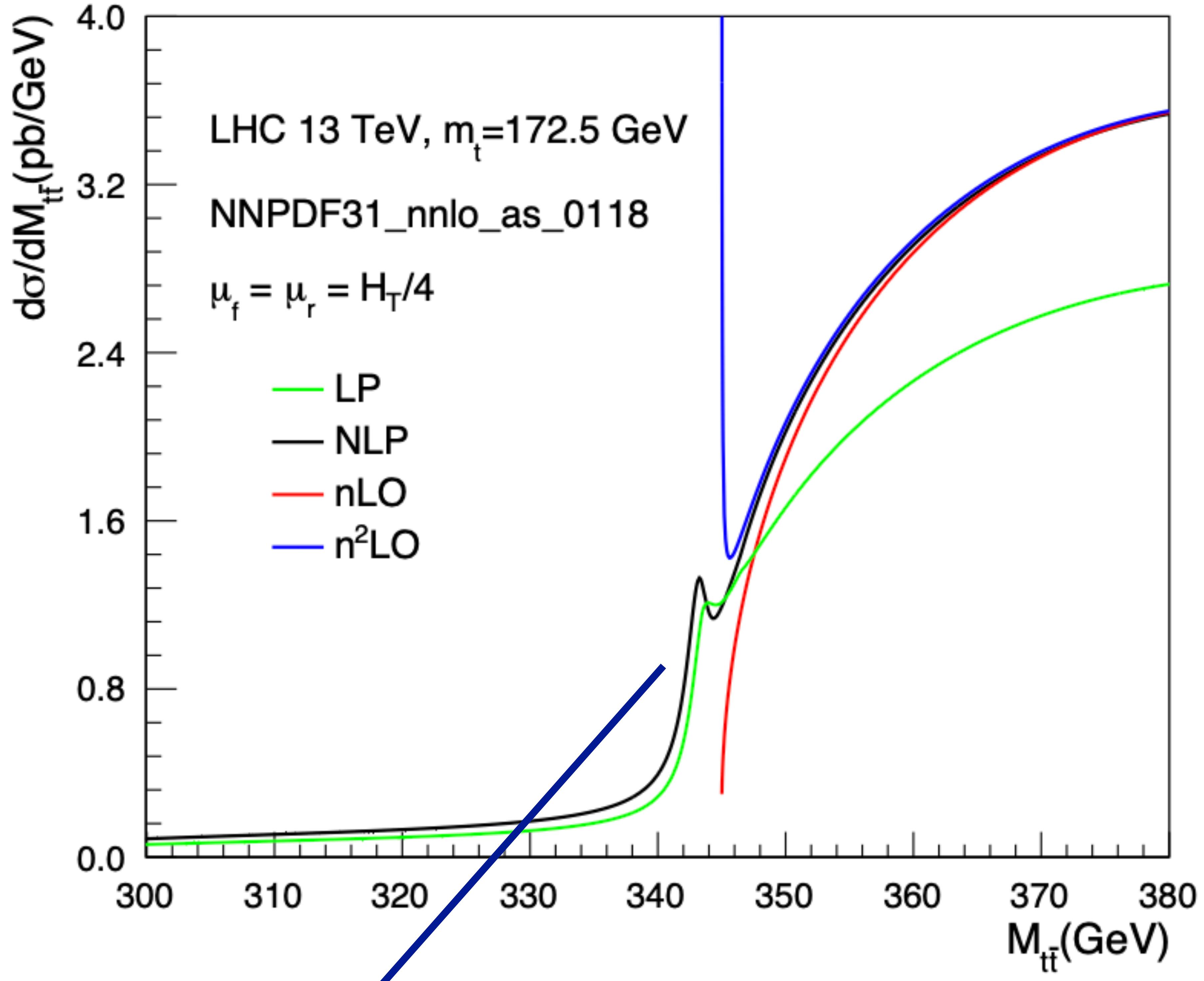

Enhancement near threshold

Resummation of non-relativistic enhancement

Ju, Wang, Wang, Xu, Xu, LLY: 1908.02179, 2004.03088

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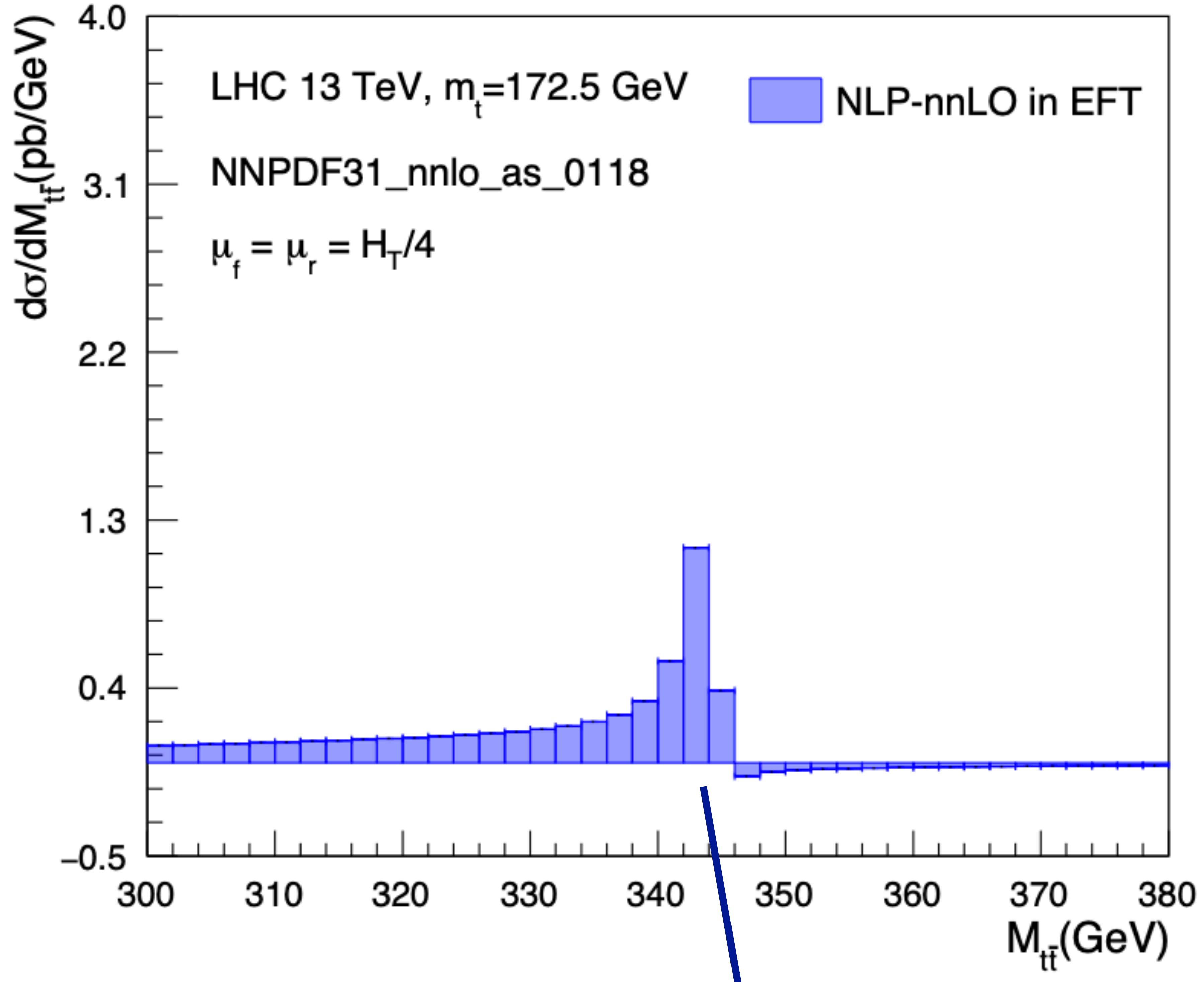

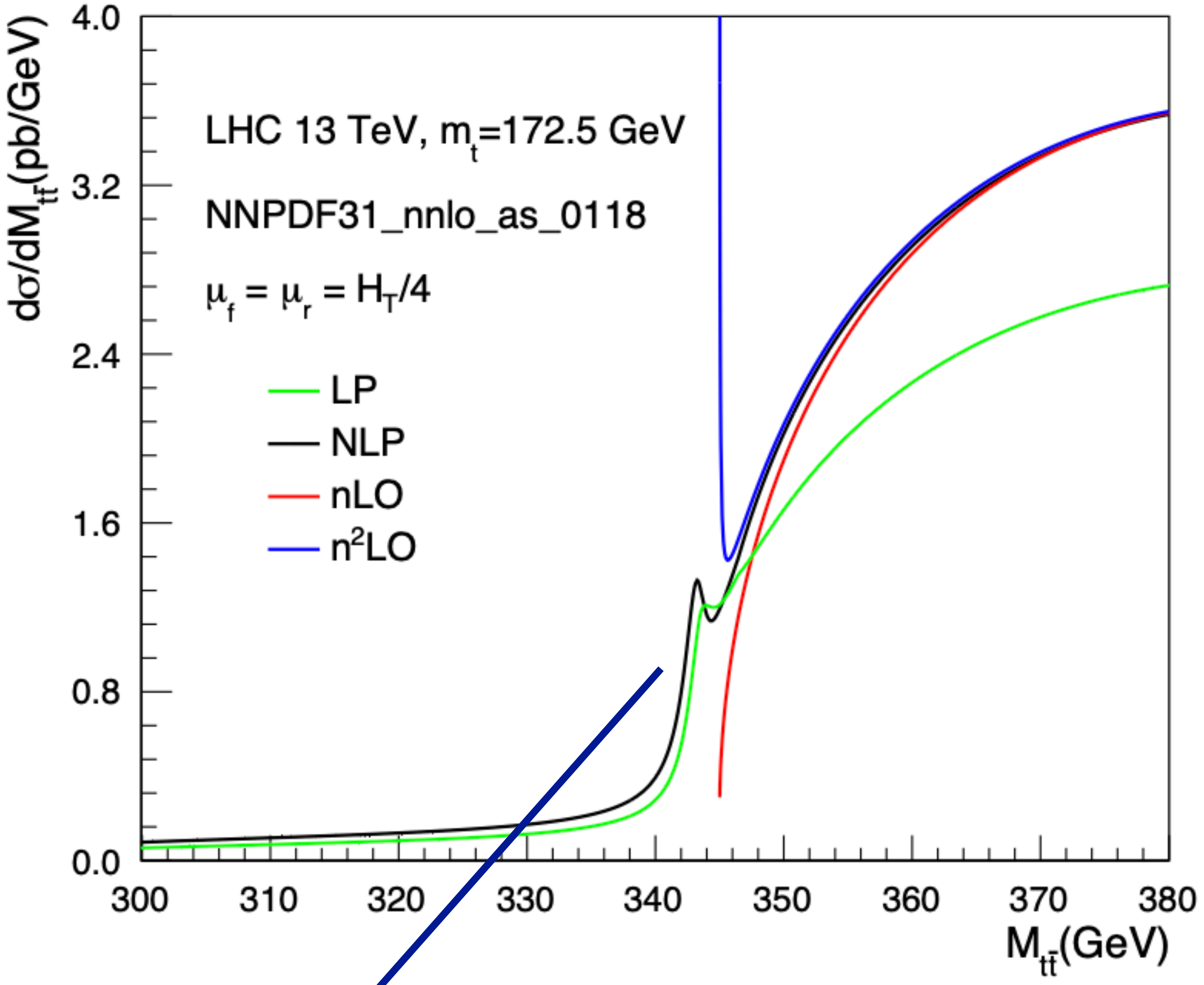

Enhancement near threshold

Note: include all color and spin configurations

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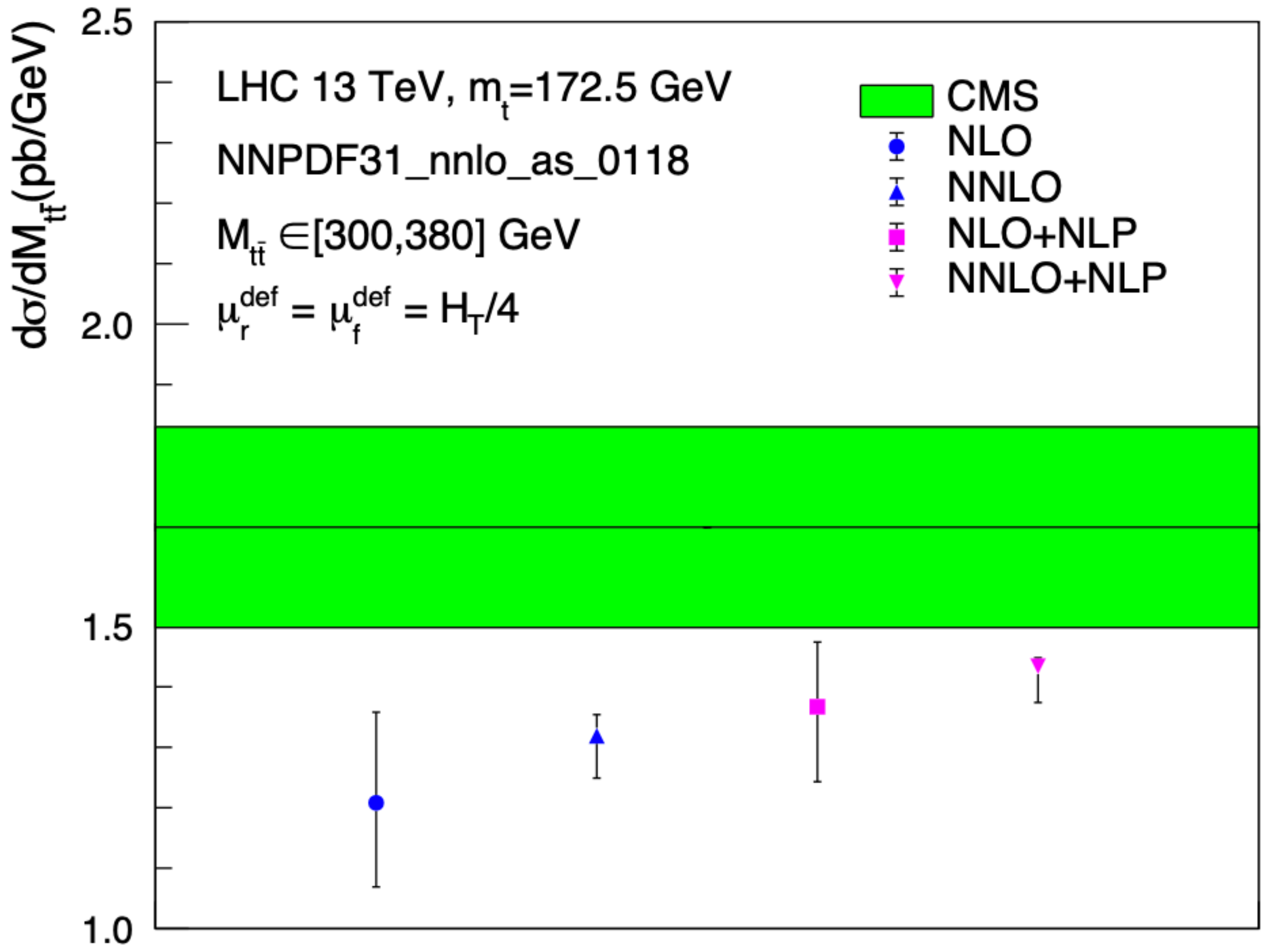
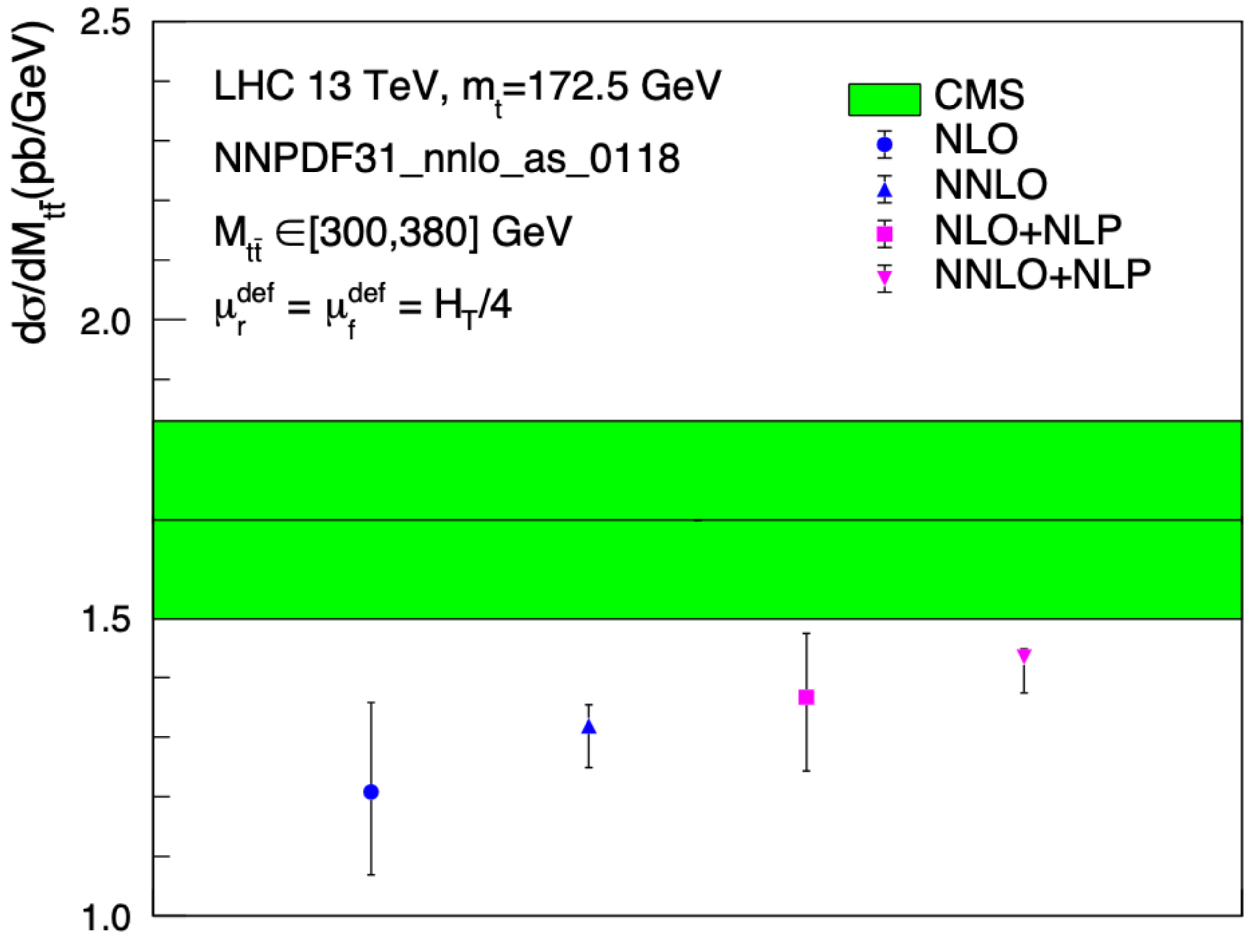


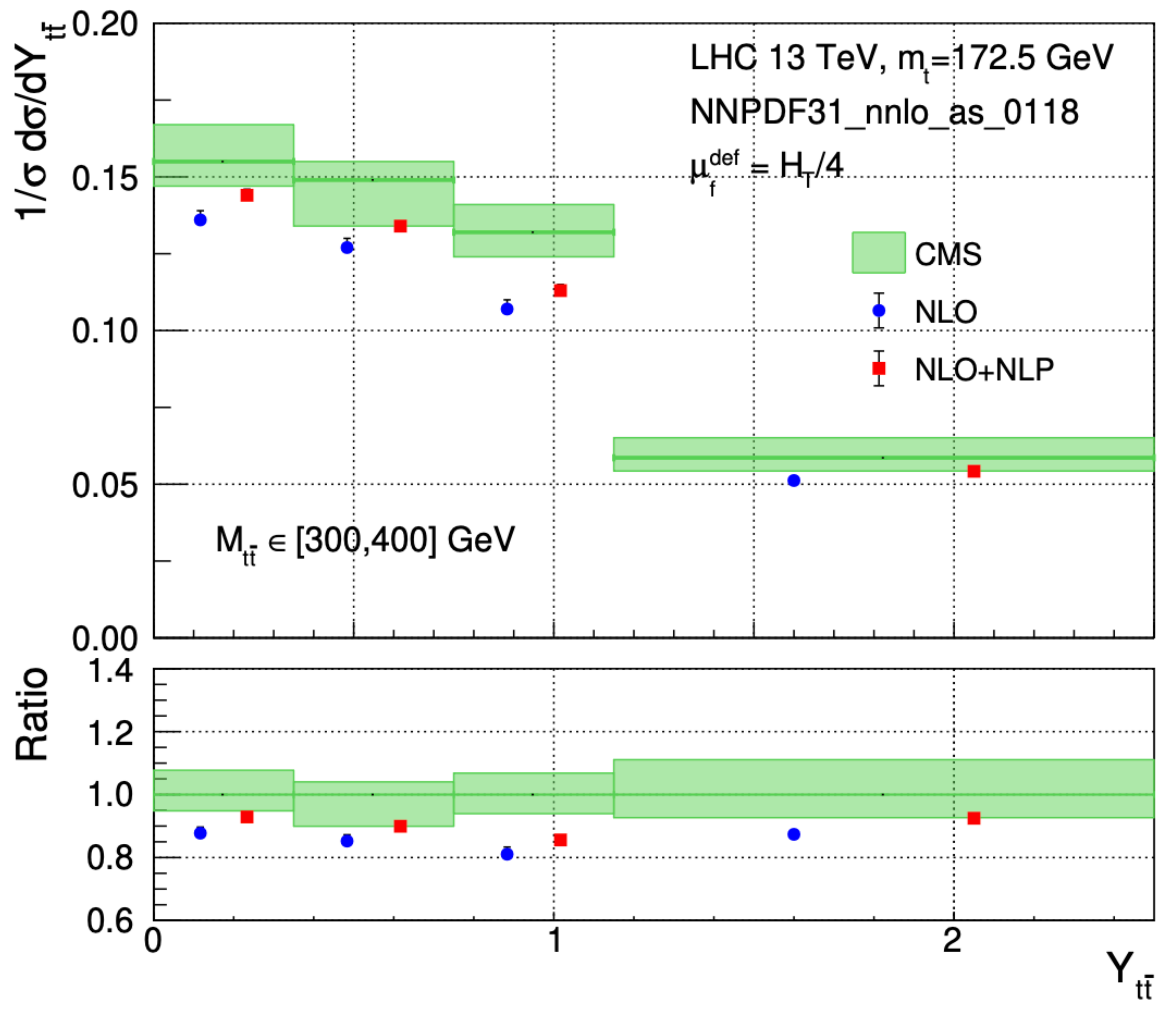
Enhancement ~ 10 pb

Resummation of non-relativistic enhancement

Ju, Wang, Wang, Xu, Xu, LLY: 1908.02179, 2004.03088



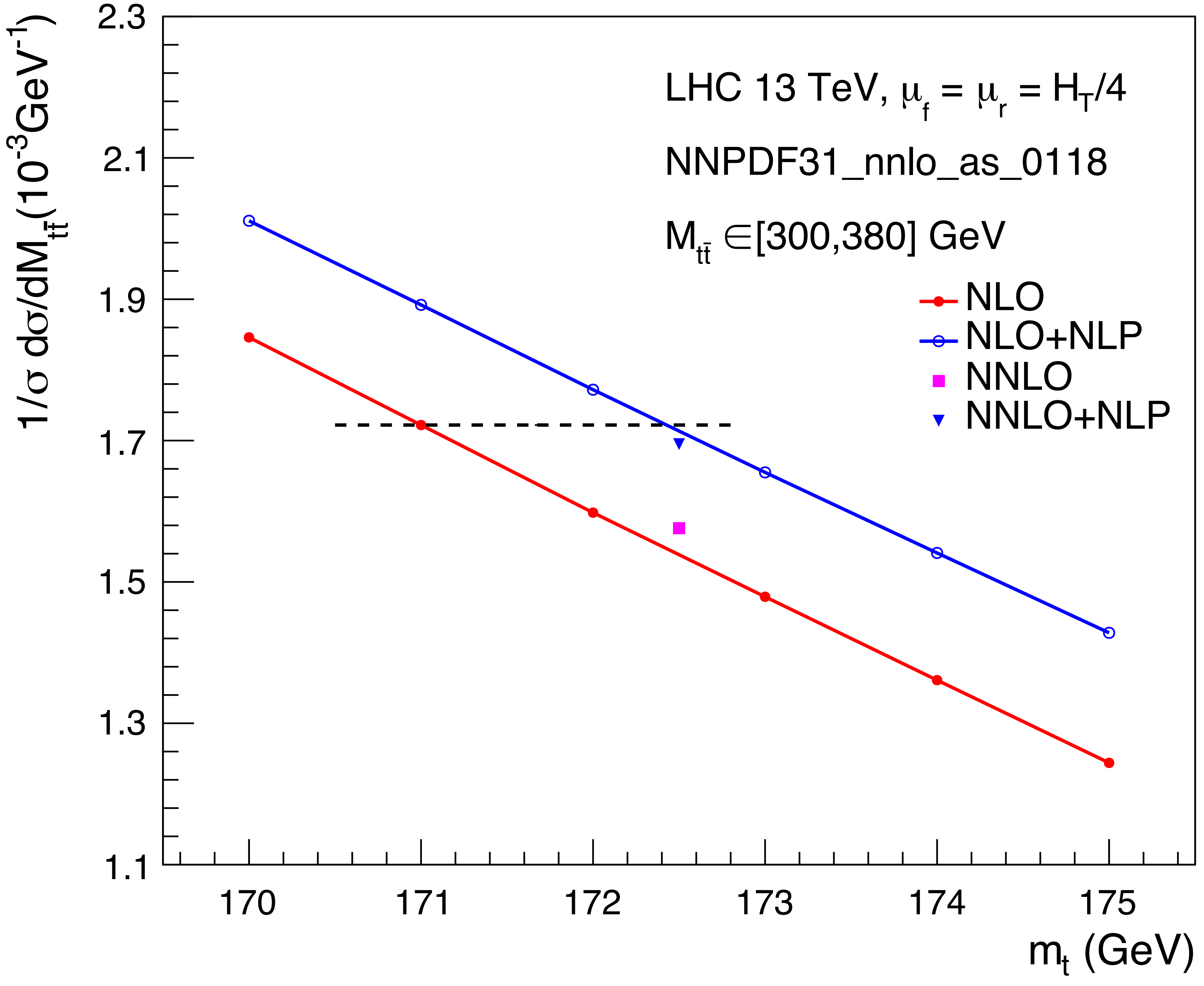
Enhancement ~ 10 pb



Similar effects in double distributions

Resummation of non-relativistic enhancement

Ju, Wang, Wang, Xu, Xu, LLY: 1908.02179, 2004.03088



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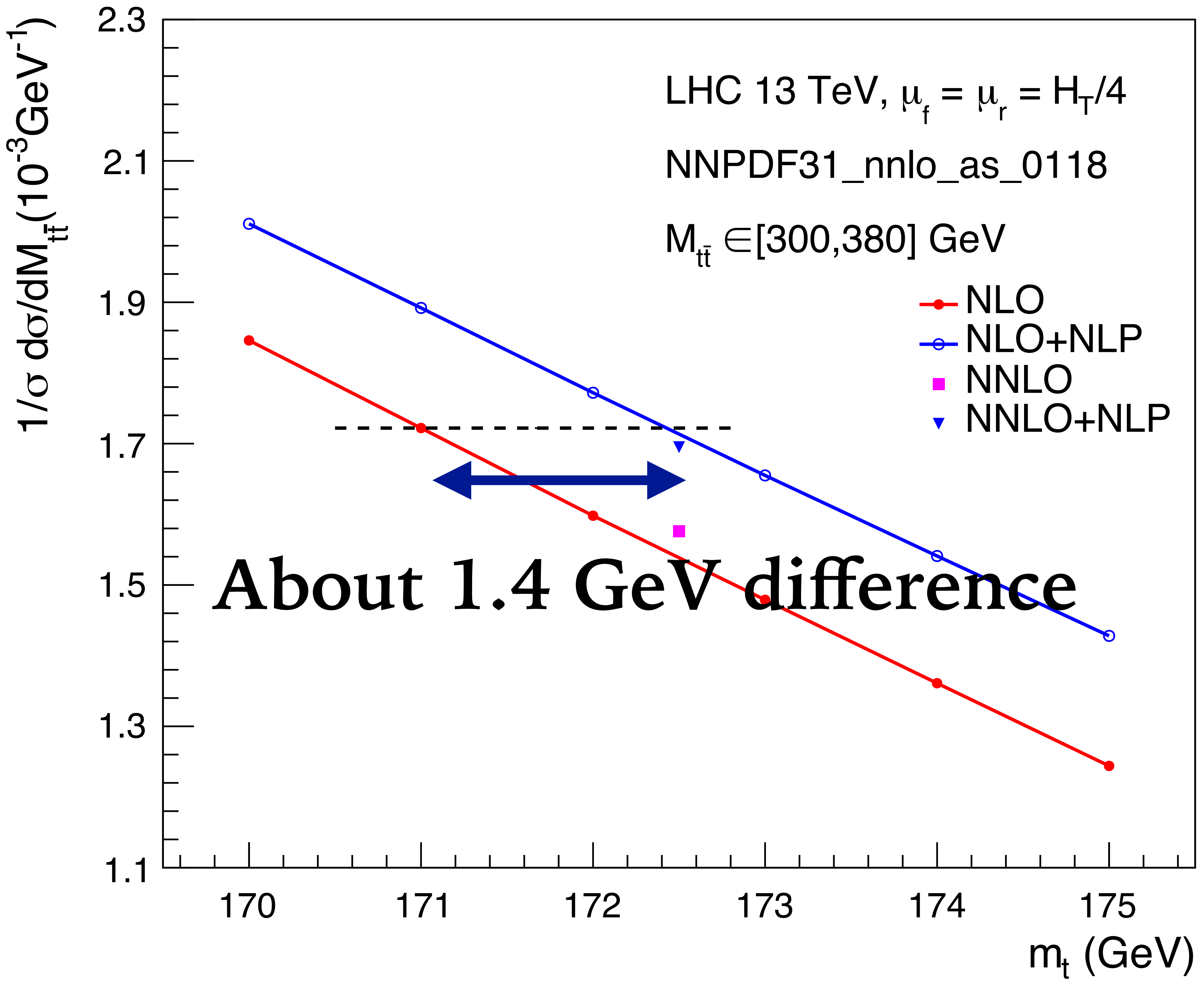
Ju, Wang, Wang, Xu, Xu, LLY: 1908.02179, 2004.03088



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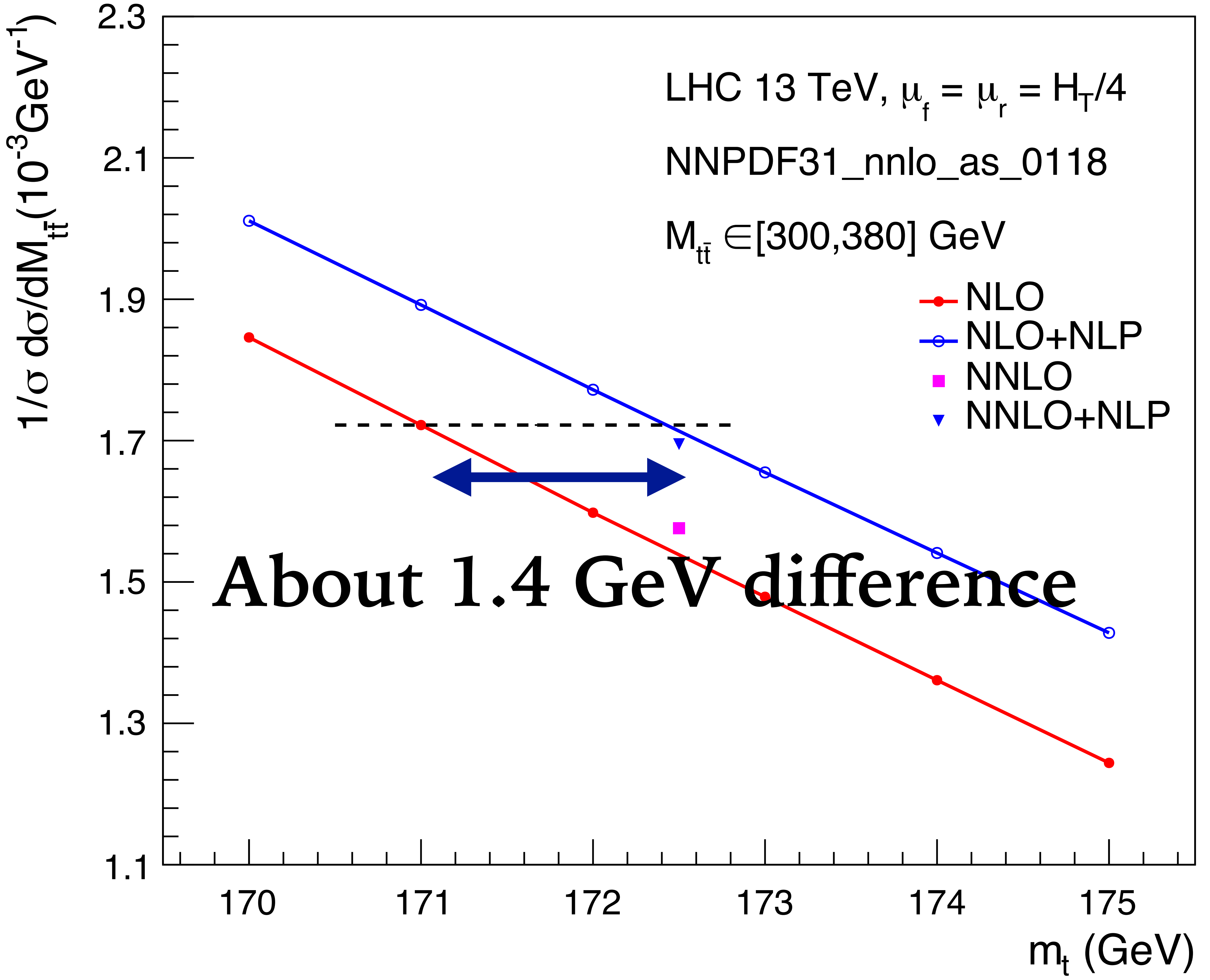
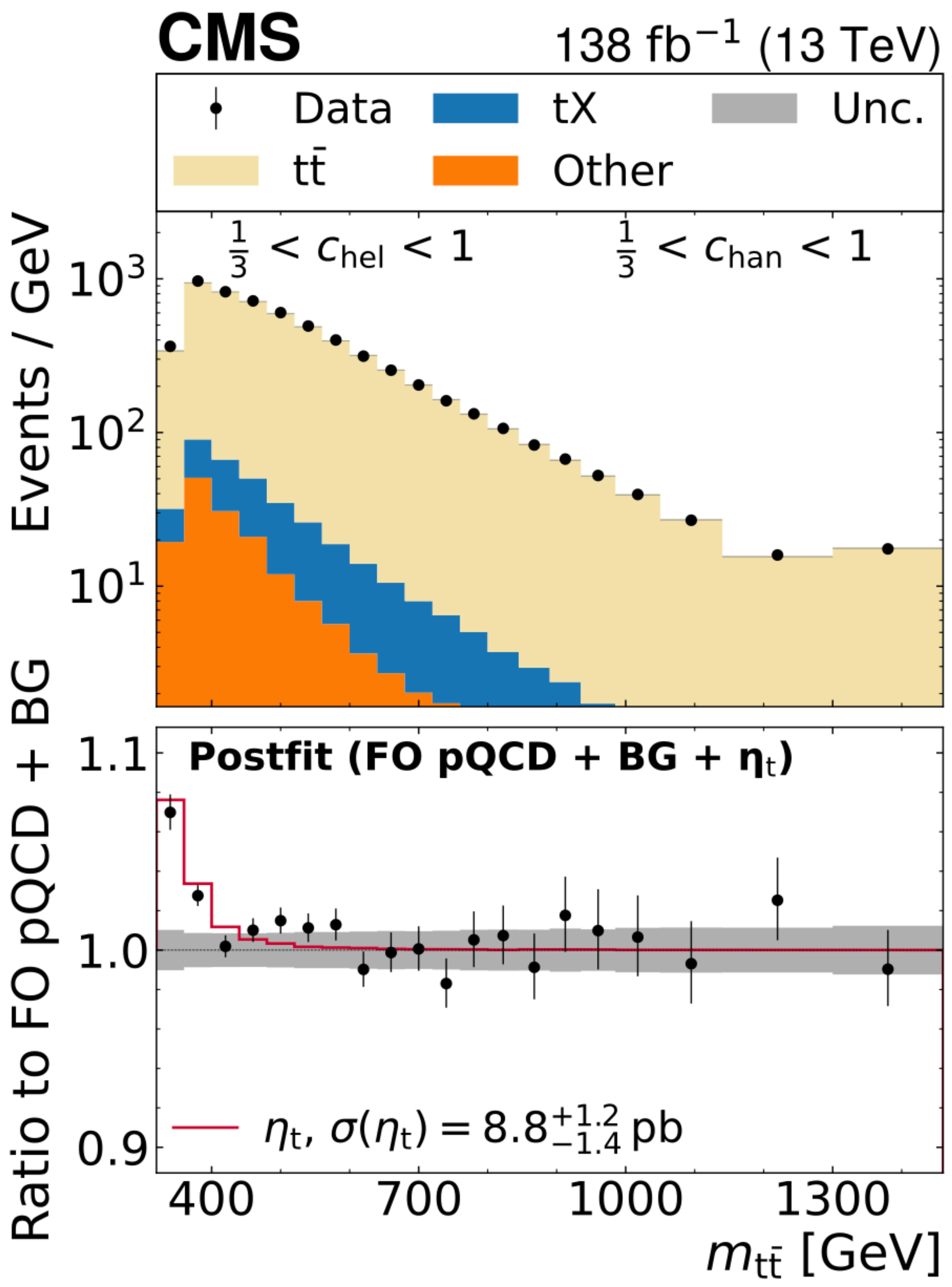
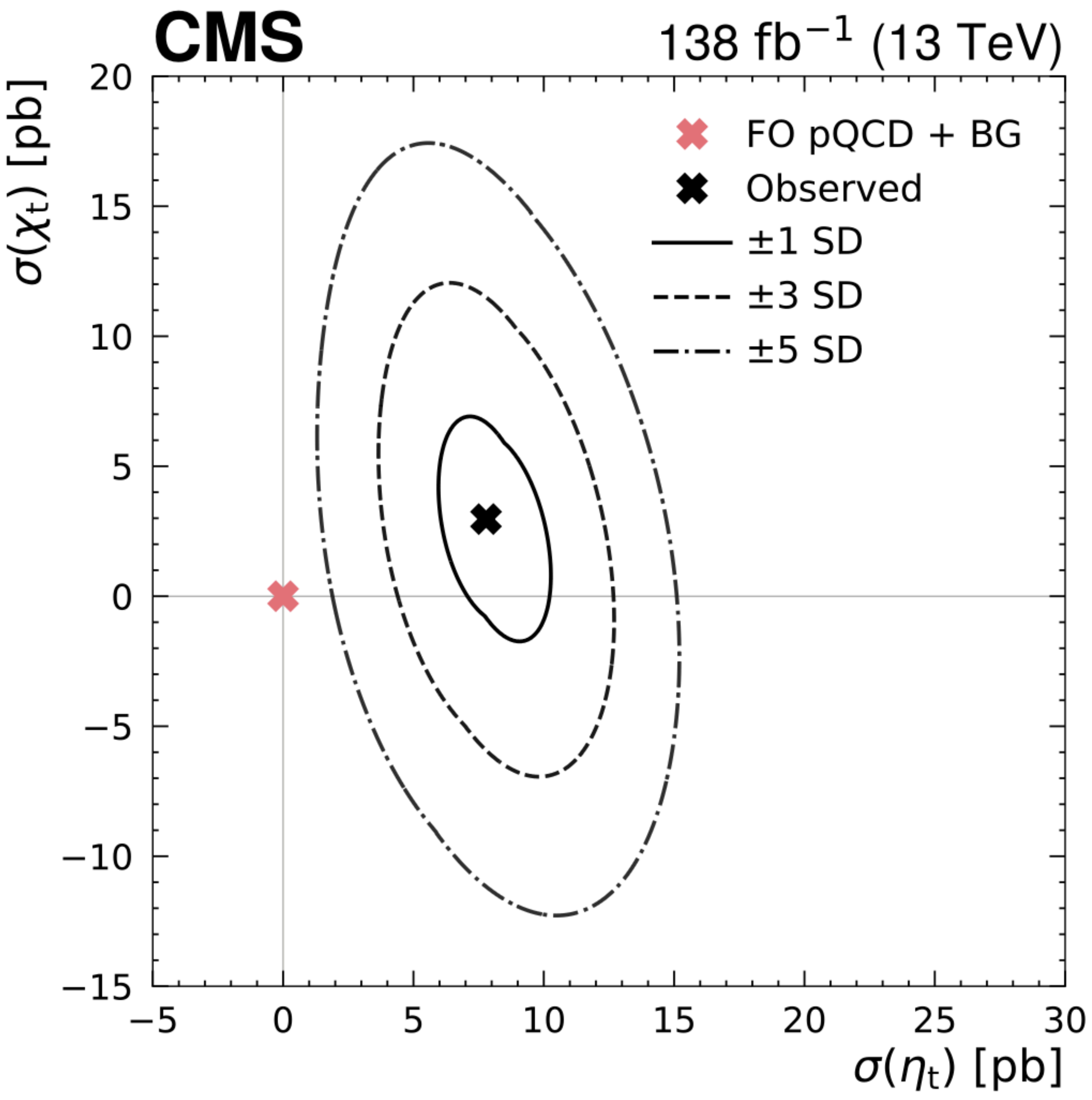


Account for most of the tension between direct and indirect measurements!

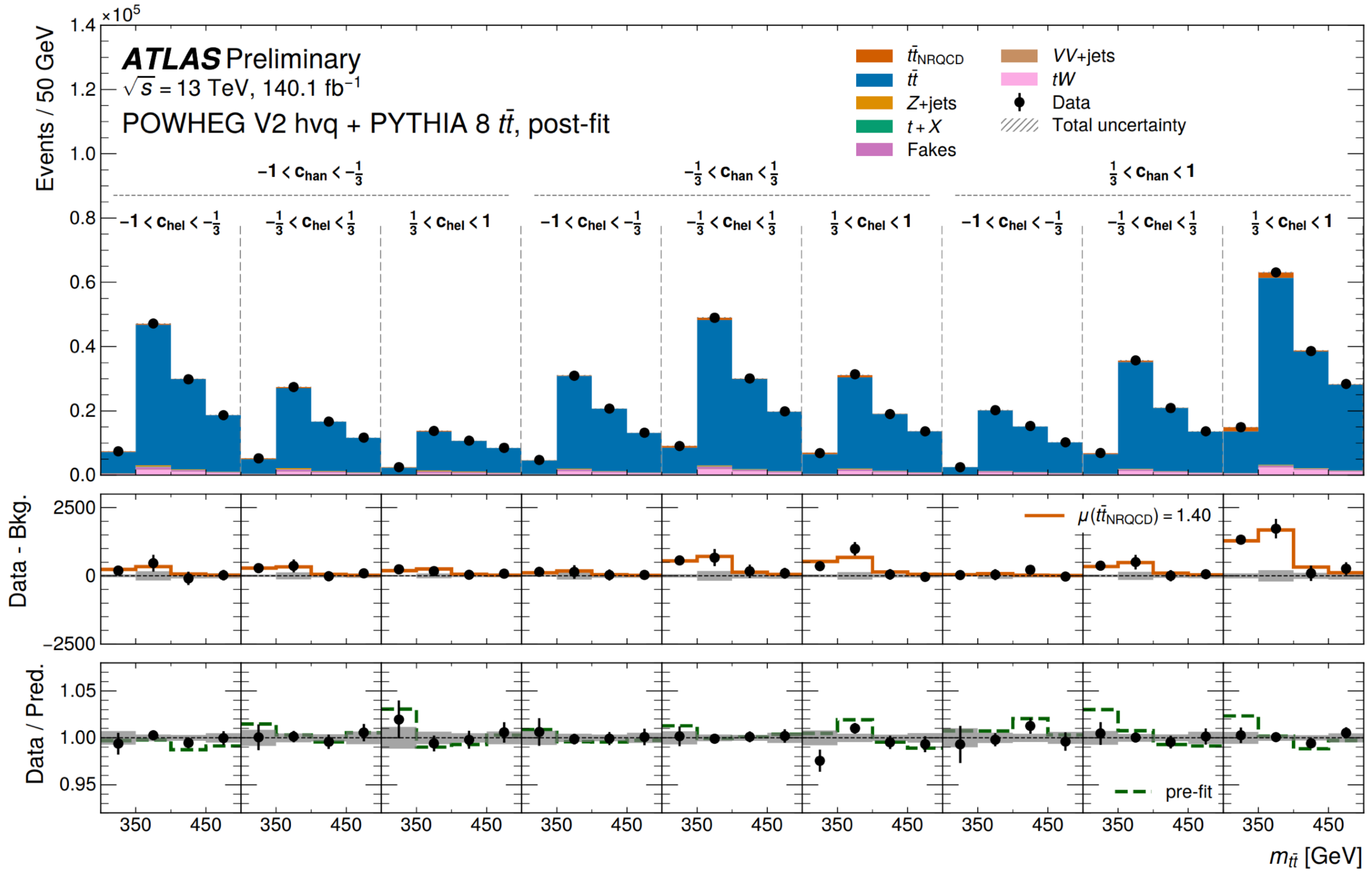
Recent CMS measurement





CMS: 2503.22382

Recent ATLAS measurement



See talk by Hai-Feng Li

ATLAS-CONF-2025-008

Further discussions

Recent works often focus on color-singlet pseudo-scalar configuration "η_t"

Signatures of toponium formation in LHC run 2 data

2102.11281

Benjamin Fuks,[1, 2, *] Kaoru Hagiwara,[3, †] Kai Ma,[4, 5, 6, ‡] and Ya-Juan Zheng[7, §]

[1] *Sorbonne Université, CNRS, Laboratoire de Physique Théorique et Hautes Énergies, LPTHE, F-75005 Paris, France*
[2] *Institut Universitaire de France, 103 boulevard Saint-Michel, 75005 Paris, France*
[3] *KEK Theory Center, Tsukuba 305-0801, Japan*
[4] *School of Fundamental Physics and Mathematical Science, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, Zhejiang, China*
[5] *International Centre for Theoretical Physics Asia-Pacific, Beijing/Hangzhou, China*
[6] *Department of Physics, Shaanxi University of Technology, Hanzhong 723000, Shaanxi, China*
[7] *Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045, U.S.A.*

Simulating toponium formation signals at the LHC

2411.18962

Benjamin Fuks [a,1], **Kaoru Hagiwara** [b,2], **Kai Ma** [c,3] **and Ya-Juan Zheng** [d,4]

[1] Laboratoire de Physique Théorique et Hautes Énergies (LPTHE), UMR 7589, Sorbonne Université et CNRS, 4 place Jussieu, 75252 Paris Cedex 05, France
[2] KEK Theory Center, Tsukuba 305-0801, Japan
[3] Faculty of Science, Xi'an University of Architecture and Technology, Xi'an, 710055, China
[4] Faculty of Education, Iwate University, Morioka, Iwate 020-8550, Japan

Toponium: Implementation of a toponium model in FeynRules

2504.12634

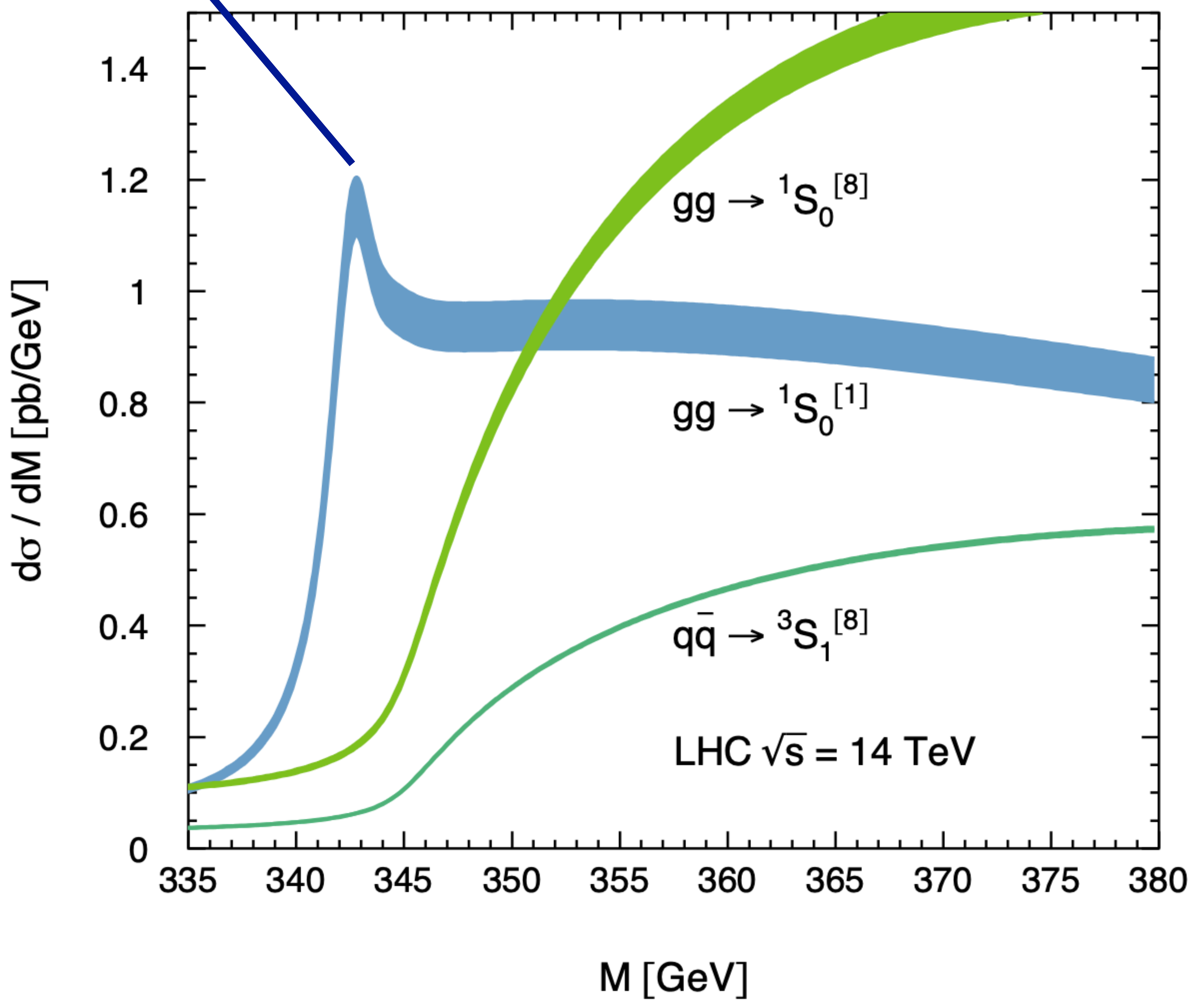
Jing-Hang Fu[a], Yu-Jie Zhang[a], Guang-Zhi Xu[b], Kui-Yong Liu[c,*]

[a] *School of Physics, Beihang University, Beijing, China.*
[b] *School of Physics, Liaoning University, Shenyang, China.*
[c] *School of Physics, Liaoning Normal University, Dalian, China.*

Further discussions

"η_t"

Color and spin configurations at LO



Kiyo et al.: 0812.0919

Further discussions

"η_t"

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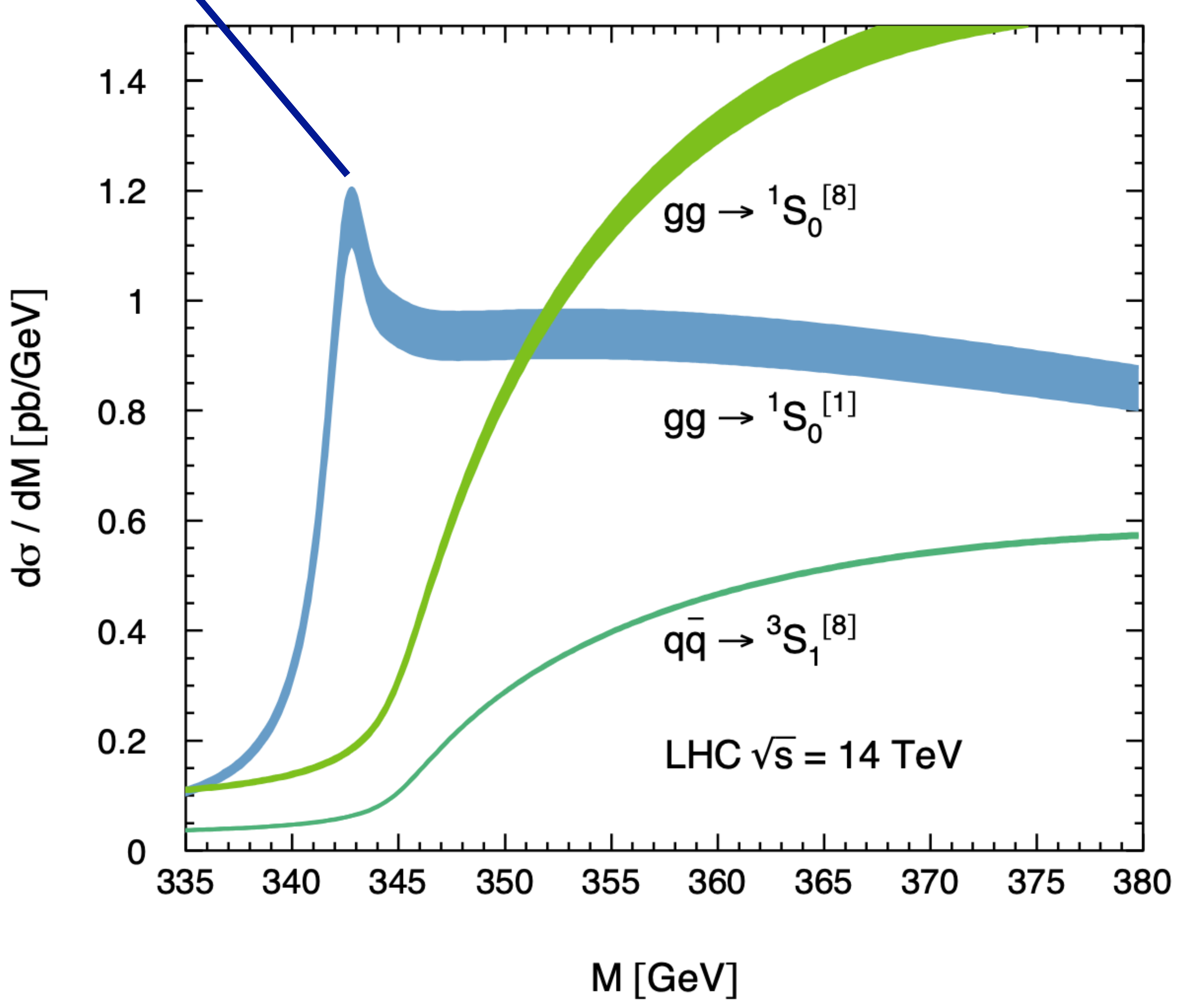
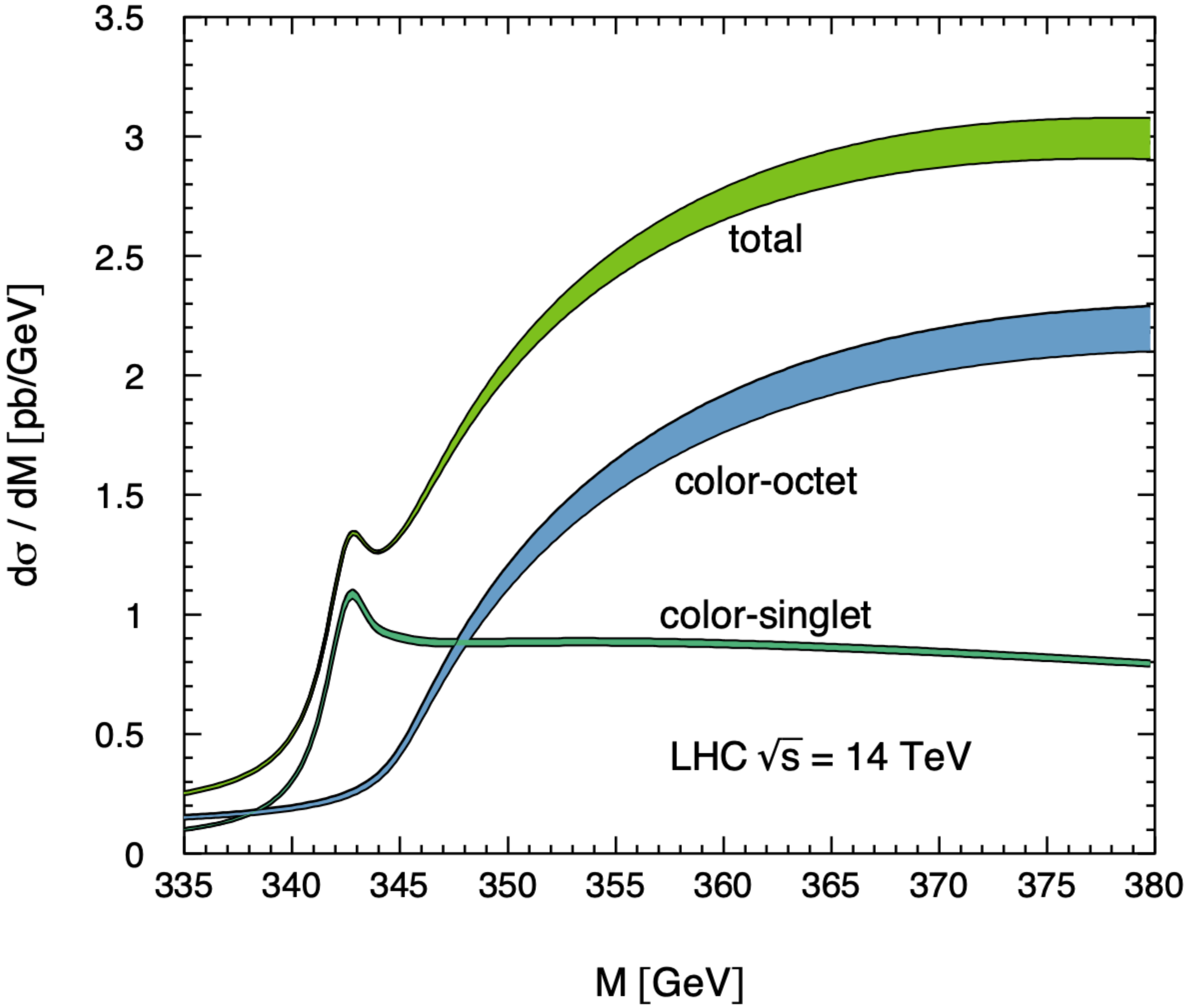


Kiyo et al.: 0812.0919

Color configurations at NLO



Further discussions

“η_t”

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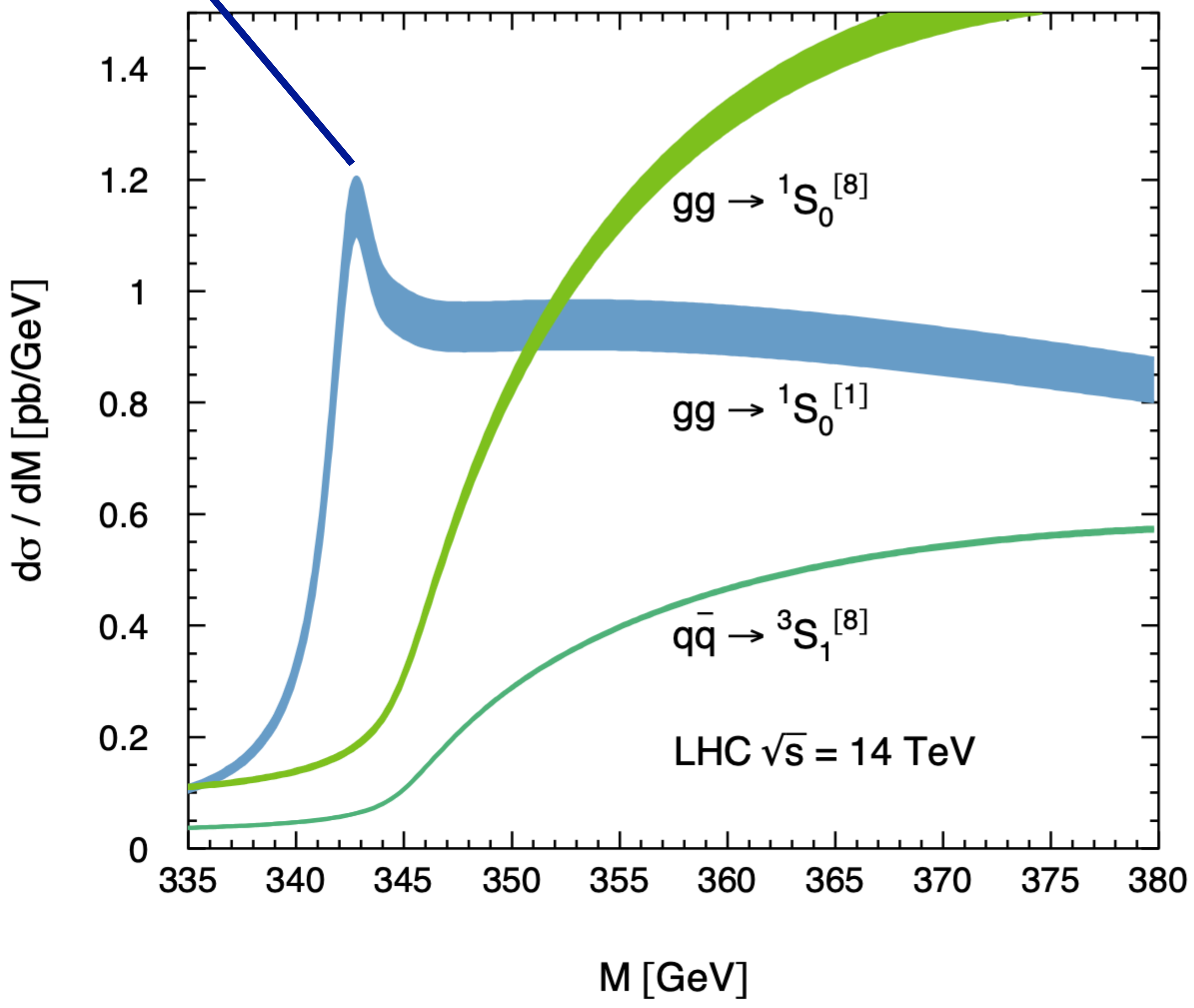

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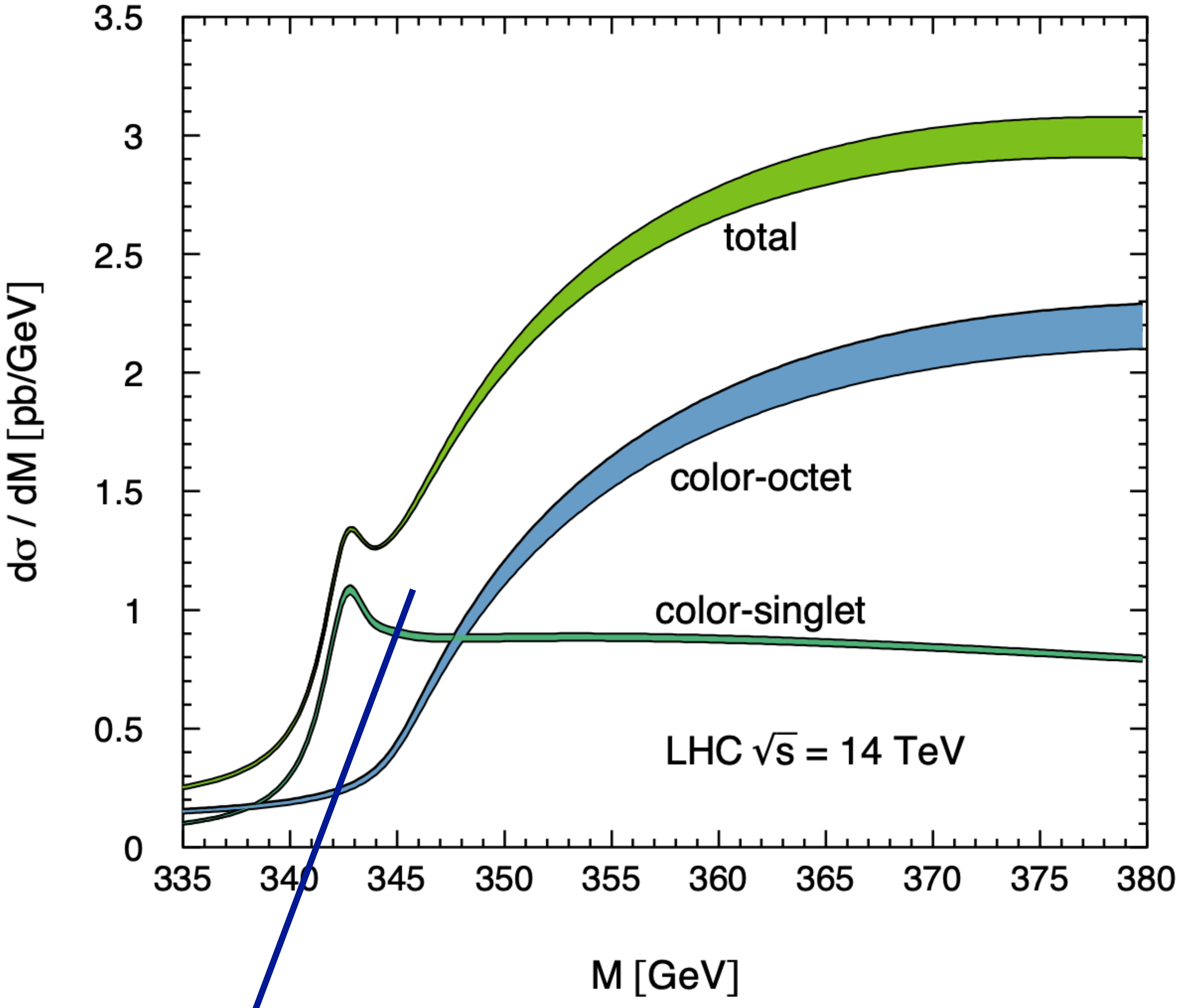
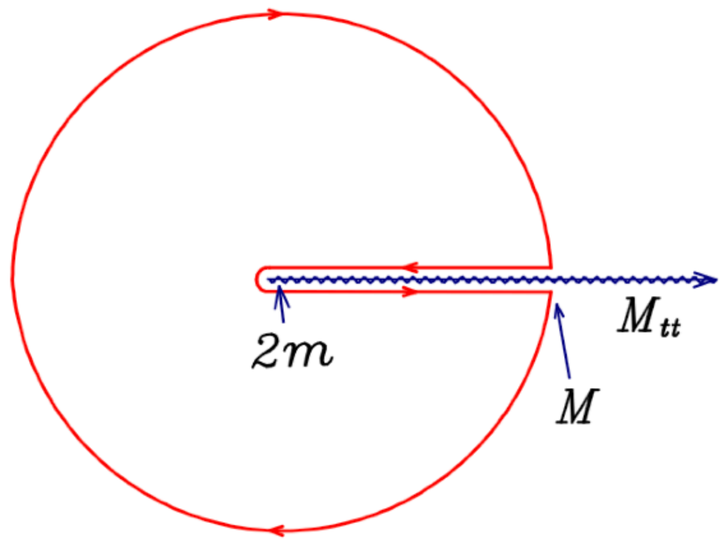


Color-octet contributions are non-negligible

Further discussions

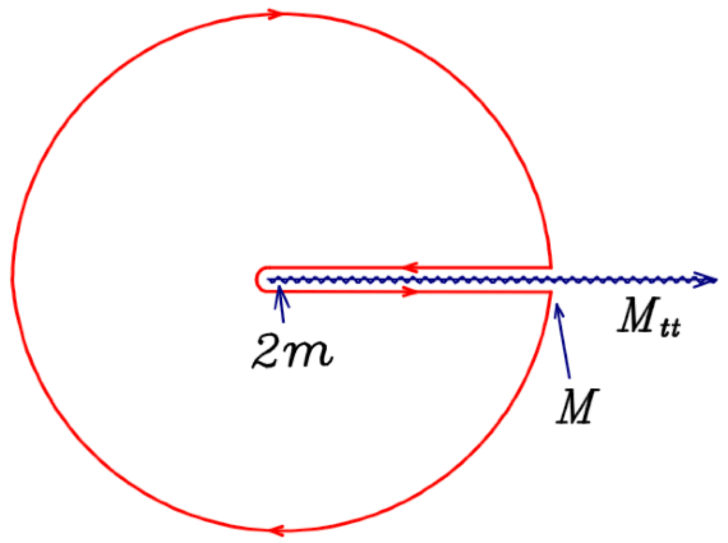


Typical value of β depends on the upper edge of the first bin

Nason et al.: 2505.00096

See also Beneke et al.: 1606.02434

Further discussions



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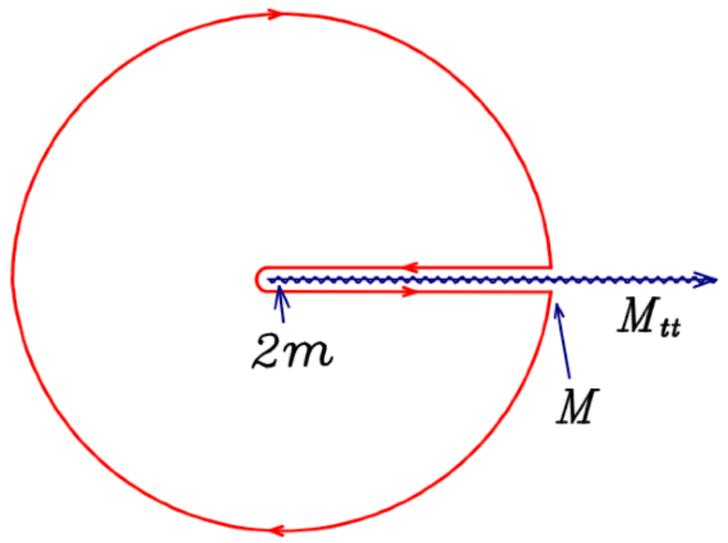
Fixed-order expansion is enough instead of a full resummation if $\beta \gg \alpha_s$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M_{t\bar{t}}\mathrm{d}\Theta} \sim \int H \times J \times f \times f \sim \beta \left(1 + \frac{\alpha_s}{\beta} + \frac{\alpha_s^2}{\beta^2} + \frac{\alpha_s^3}{\beta^3} + \cdots \right)$$

Further discussions



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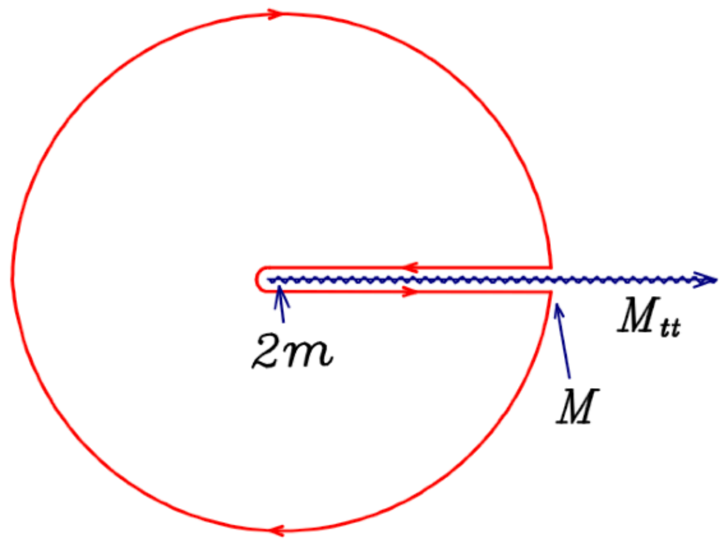
$M_{t\bar{t}}$ (GeV)	β
400	0.50
380	0.42
360	0.28
350	0.16

Further discussions



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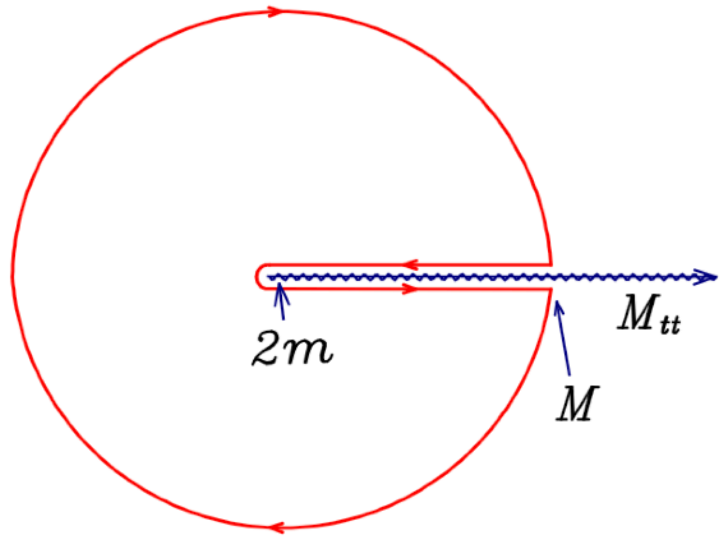
Resummation is needed for $M_{t\bar{t}} \lesssim 350$ GeV!

Further discussions



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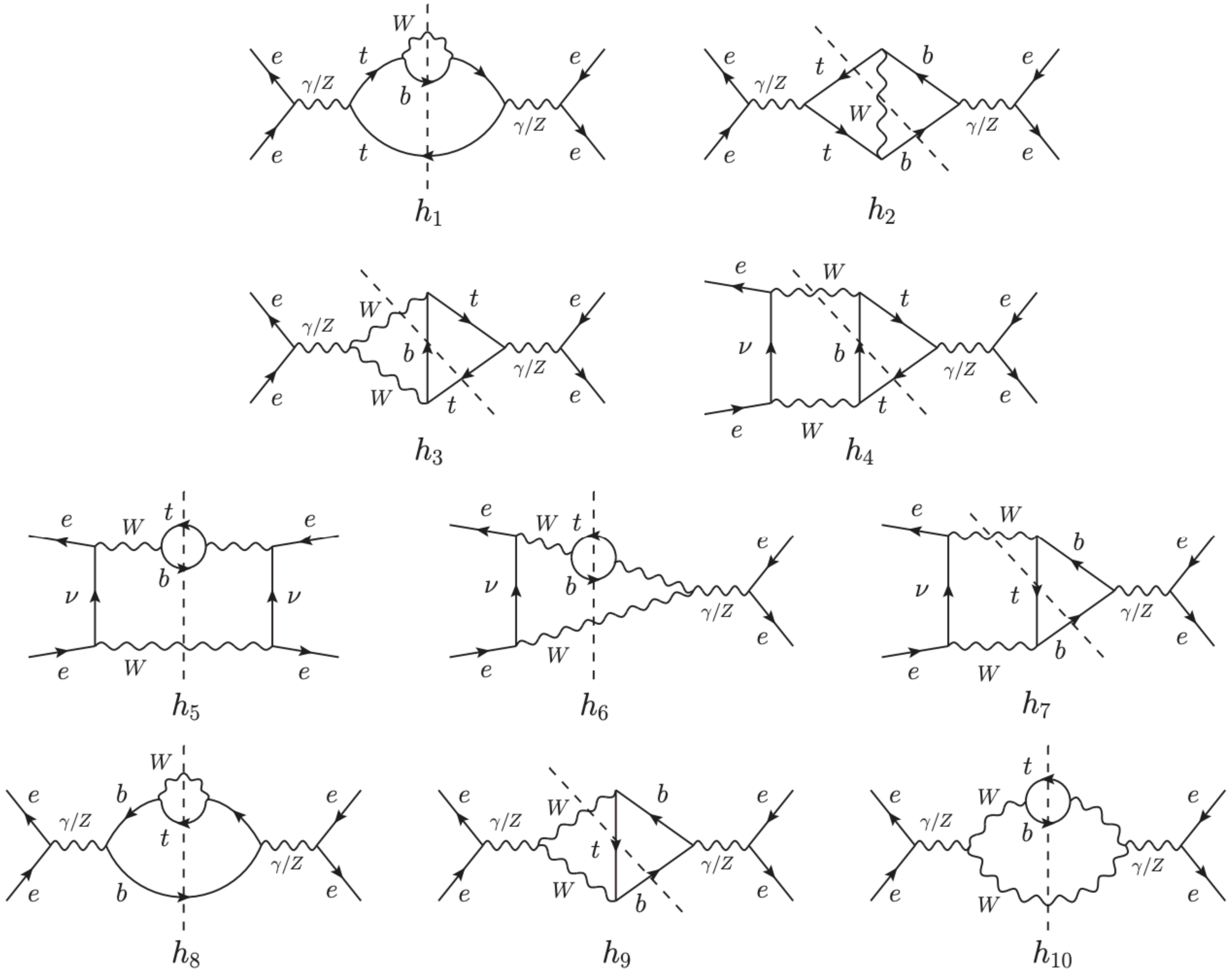
Note also: $M_{t\bar{t}}\beta^2$ is still a perturbative scale

$$E = M_{t\bar{t}} - 2m_t + i\Gamma_t \sim \frac{1}{2}M_{t\bar{t}}\beta^2 + i\Gamma_t$$

Further discussions



Top quark decay and non-resonant effects?

Existing studies only focus on lepton colliders

See, e.g., Beneke et al.: 1711.10429, Bach et al.: 1712.02220

Summary

- Near-threshold production of top quark pairs is phenomenologically important
  - Indirect top quark mass measurements from differential cross sections
  - Spin correlations and quantum entanglement (talk by Qiang Li)
- Indication of non-relativistic enhancement in LHC Run 2 data
  - Performed NLP resummation combined with NNLO fixed-order results for differential distributions
  - Enhancement $\sim$ 10 pb, bridging the discrepancy between indirect and direct top quark mass measurements
- Recent observations by both CMS and ATLAS collaborations

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Thank you!