Review of SM polarization theory

Bin Yan
Institute of High Energy Physics

TeV物理前沿专题研讨会暨第31届LHC Mini-Workshop
2025年10月10日-12日

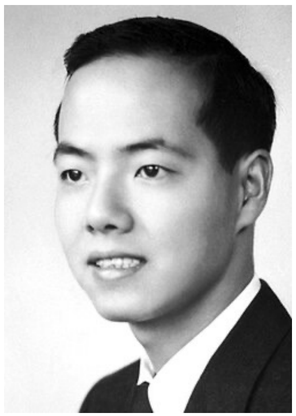
Outline

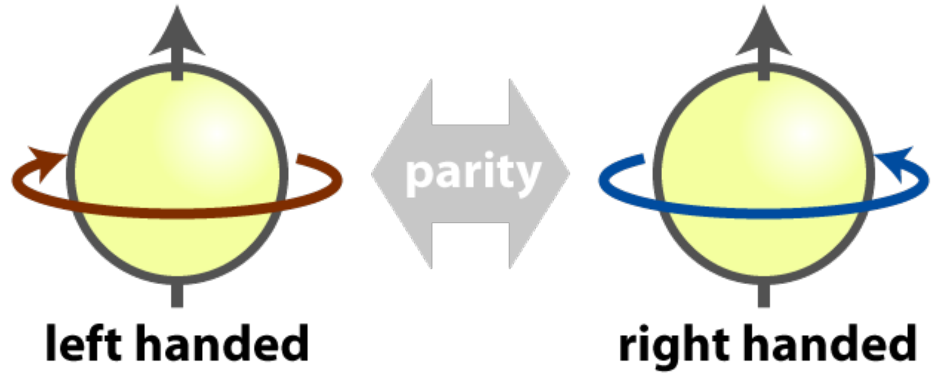
- Single particle polarization
  - Perturbative decay approach
  - Non-perturbative approach: QCD spin effects

- Spin correlation effects of particles
  - Perturbative decay approach
  - Non-perturbative approach: QCD spin effects

Topics are selected from a personal perspective and apologize if certain important works or results are not covered.

Parity and weak interactions



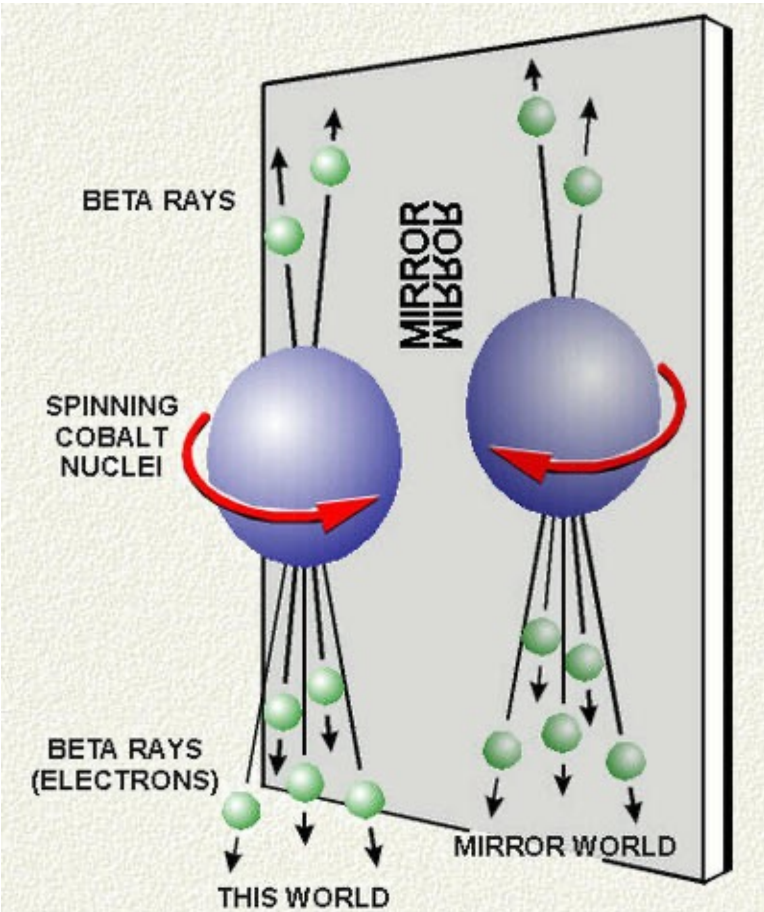


- Mid-1950s $\tau - \theta$ puzzle: two identical kaon

$$\theta^+ \to \pi^+ + \pi^0, \quad \tau^+ \to \pi^+ + \pi^+ + \pi^-$$

- One particle or two particles?
- Parity conservation never be confirmed in weak interaction

Wu experiment



- Gamma rays: determine the spin of cobalt-60
- Parity transformation:

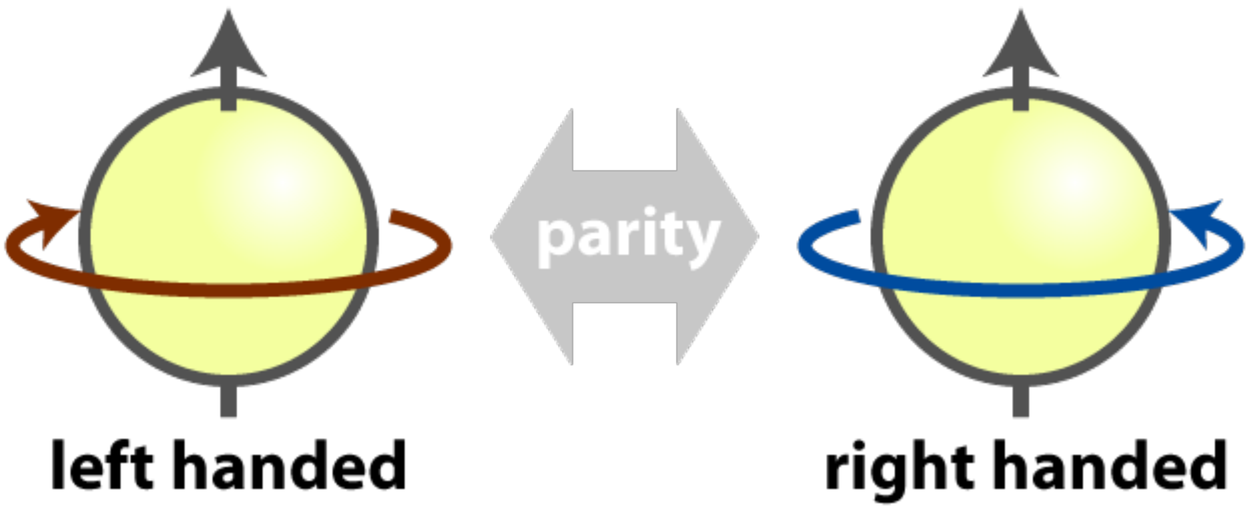
$$\overrightarrow{S} \rightarrow \overrightarrow{S} \qquad \overrightarrow{p} \rightarrow -\overrightarrow{p}$$

- Rate of emission of electrons along the magnetic field and in the opposite direction

$$^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + \mathrm{e}^- + \bar{\nu}_e + 2\gamma$$

1956: testing the conservation of parity

Spin effects and New Physics



- Parity violation: left-handed ≠ right-handed
- The particle would be polarized when involving the parity violation effects
- Polarization of particles: A tool to probe the interactions

Single particle polarization in SM and beyond

Example 1: Top quark polarization

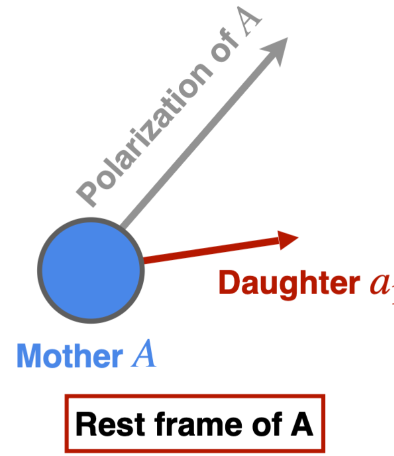
- Top quark polarization is reconstructed from decay distribution:

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_f} = \frac{1}{2}(1 + p\beta_f \cos\theta_f)$$

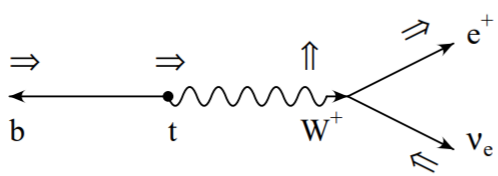
p: polarization of the ensemble

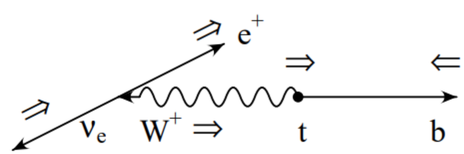
Spin analyzing power

$\ell^+, \bar{d}$: $\beta_f = 1$; $\bar{\upsilon}, u$: $\beta_f = -0.31$; b: $\beta_f = -0.41$

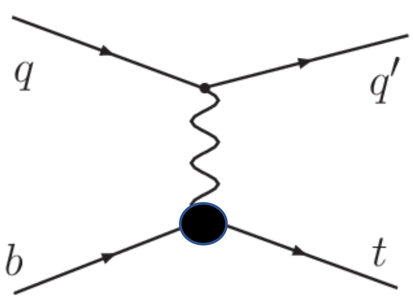


- Charged lepton and down-type quark tend to fly along top quark spin





- Polarization of the top quark: interaction information of the scattering



Top quark is polarized in single-top quark production: tbW anomalous couplings

Top quark polarization

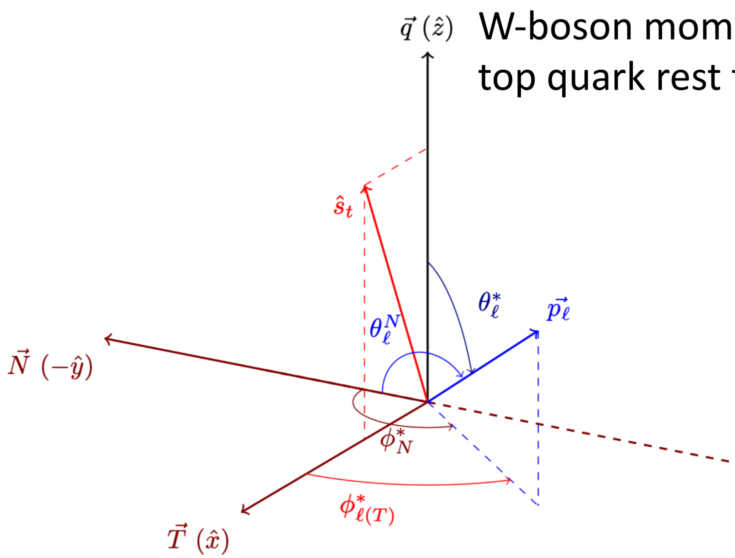
- General formalism for W-boson polarization in top quark decay

Depends on the W-boson polarization

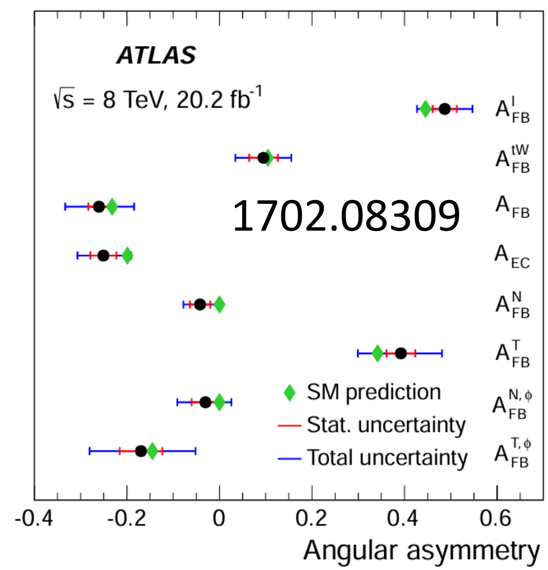
$$\frac{1}{\Gamma}\frac{d\Gamma}{d(\cos\theta_\ell^*)d\phi_\ell^*} = \frac{3}{8\pi}\left\{\frac{2}{3} + \frac{1}{\sqrt{6}}\langle T_0\rangle\left(3\cos^2\theta_\ell^* - 1\right) + \langle S_3\rangle\cos\theta_\ell^* \right.$$
$$+ \langle S_1\rangle\cos\phi_\ell^*\sin\theta_\ell^* + \langle S_2\rangle\sin\phi_\ell^*\sin\theta_\ell^*$$
$$\left. - \langle A_1\rangle\cos\phi_\ell^*\sin 2\theta_\ell^* - \langle A_2\rangle\sin\phi_\ell^*\sin 2\theta_\ell^* \right\}.$$

Depends on the top quark polarization

J. A. Aguilar-Saavedra and J. Bernabeu PRD 93 (2016) 011301

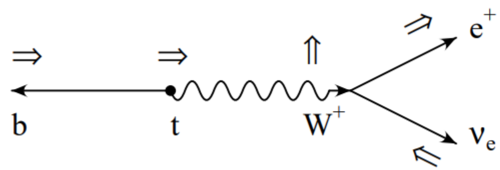


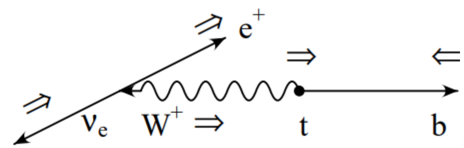
W-boson momentum in top quark rest frame

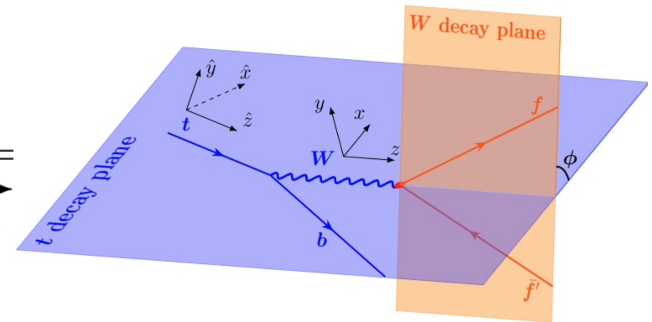


Top quark polarization

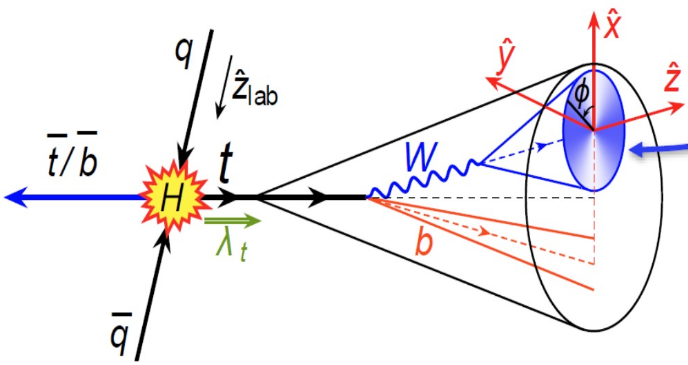
Zhite Yu, C.-P. Yuan, PRL 129 (2022) 11,11







W boson is not linearly polarized (+/- interference) in top quark rest frame



$$\frac{\mathrm{d}E}{\mathrm{d}\phi} = \frac{E_{\mathrm{tot}}}{2\pi}\left[1 + \xi \cos 2\phi\right]$$ **Infrared safe**

Boosted limit: $\xi = \xi(\lambda_t) = 0.145(\lambda_t - 1)$

[Assuming SM tbW coupling]

Azimuthal correlation

→ **Boosted top polarization**

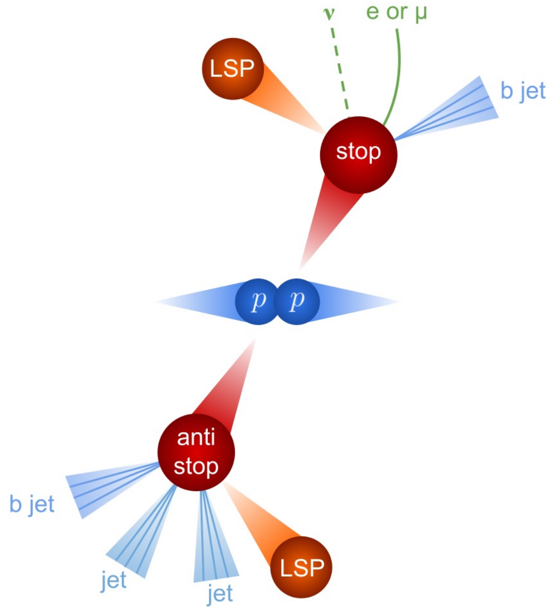
- Measuring longitudinal polarization of boosted top
- New top tagger against QCD jets

→ A new tool to probe the NP effects,
e.g. the CP violation in top quark decay

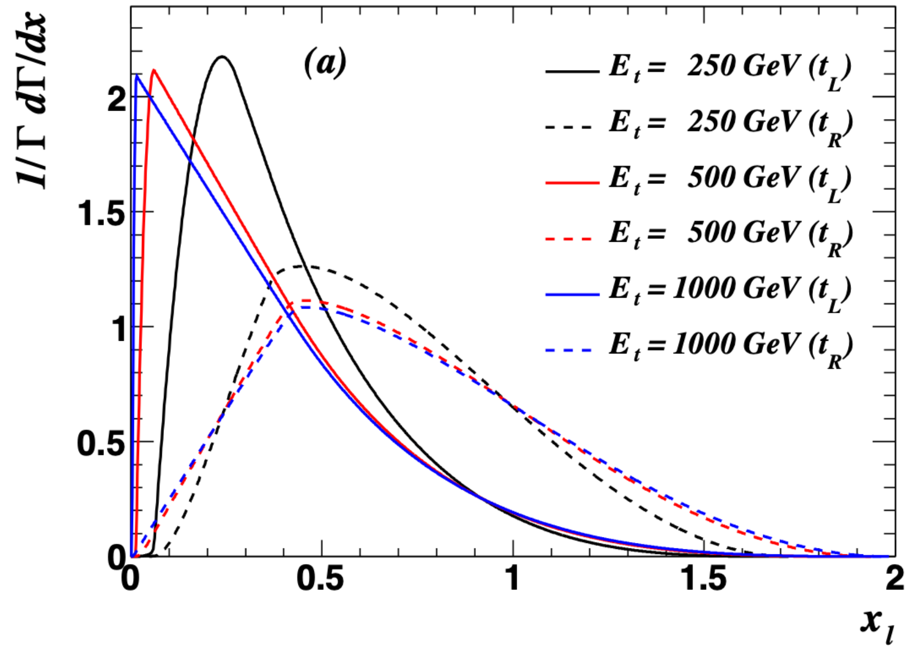
Qi Bi, Bin Yan, Zhite Yu,
working in progress

Top quark polarization

- Top quark polarization rely on the reconstruction of the top quark kinematics
- It is challenge to reconstruct the top quark kinematics for events with a top quark pair plus missing energy:



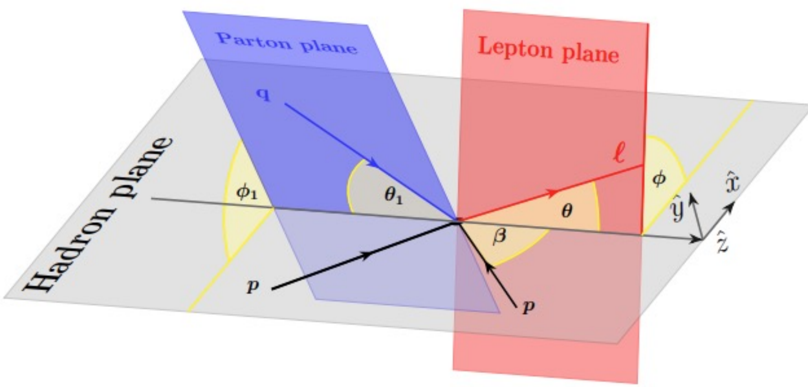
Charged lepton energy fractions



- Right-handed top produce more energetic leptons than left-handed

A. Czarnecki, M. Jezabek, J. H. Kuhn, NPB 351 (1991) 70
C. R. Schmidt, M. E. Peskin, PRL 69 (1992) 410
E. L. Berger, Q.-H. Cao, J.-H. Yu, H. Zhang, PRL 109 (2012) 152004

Example2: gauge boson polarization



Collins-Soper frame

$$\frac{d\sigma}{d^4q\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{\mathrm{unpol.}}}{d^4q}\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0\,(1-3\cos^2\theta) + A_1\,\sin(2\theta)\cos\phi + \frac{1}{2}A_2\,\sin^2\theta\,\cos(2\phi) + A_3\,\sin\theta\,\cos\phi + A_4\,\cos\theta + A_5\,\sin^2\theta\,\sin(2\phi) + A_6\,\sin(2\theta)\,\sin\phi + A_7\,\sin\theta\,\sin\phi\Big\},$$

$$\rho_{\lambda_Z\lambda'_Z} = \begin{pmatrix} \frac{1-\delta_L}{3}+\frac{J_3}{2} & \frac{J_1+2Q_{xz}-i(J_2+2Q_{yz})}{2\sqrt{2}} & \lambda_T - iQ_{xy} \\ \frac{J_1+2Q_{xz}+i(J_2+2Q_{yz})}{2\sqrt{2}} & \frac{1+2\delta_L}{3} & \frac{J_1-2Q_{xz}-i(J_2-2Q_{yz})}{2\sqrt{2}} \\ \lambda_T + iQ_{xy} & \frac{J_1-2Q_{xz}+i(J_2-2Q_{yz})}{2\sqrt{2}} & \frac{1-\delta_L}{3}-\frac{J_3}{2} \end{pmatrix}$$

$$\frac{\Gamma}{\Omega_f^*} \propto \frac{|B_+|^2+|B_-|^2}{2}\Big[\frac{2}{3}+\frac{\delta_L}{3}(1-3\cos^2\theta_f^*)+\lambda_T\sin^2\theta_f^*\cos 2\phi_f^* + Q_{yz}\sin 2\theta_f^*\sin\phi_f^* + Q_{xz}\sin 2\theta_f^*\cos\phi_f^* + Q_{xy}\sin^2\theta_f^*\sin 2\phi_f^*\Big] + \frac{|B_+|^2-|B_-|^2}{2}\left(J_1\sin\theta_f^*\cos\phi_f^* + J_2\sin\theta_f^*\sin\phi_f^* + J_3\cos\theta_f^*\right).$$

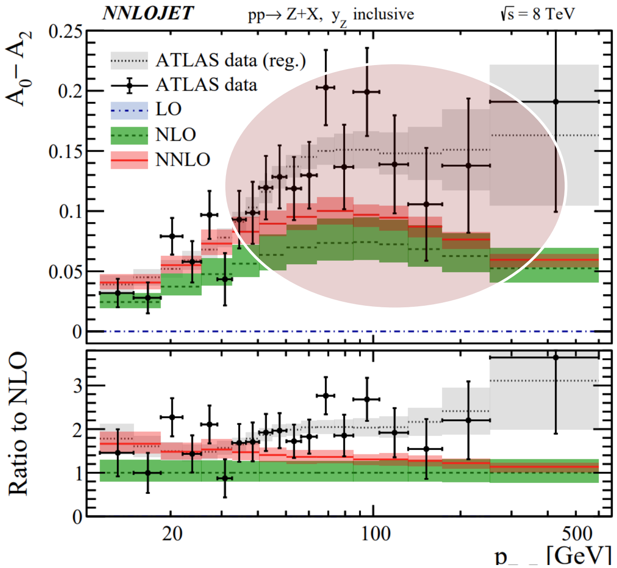
Lam-Tung relation: $A_0 = A_2$

Linear and Longitudinal polarization of Z boson

$A_0 \neq A_2$ @ NNLO in QCD
non-coplanarity between the hadron and parton planes

J.C. Peng et al, PLB 758,384 (2016)

Lam-Tung relation and polarization



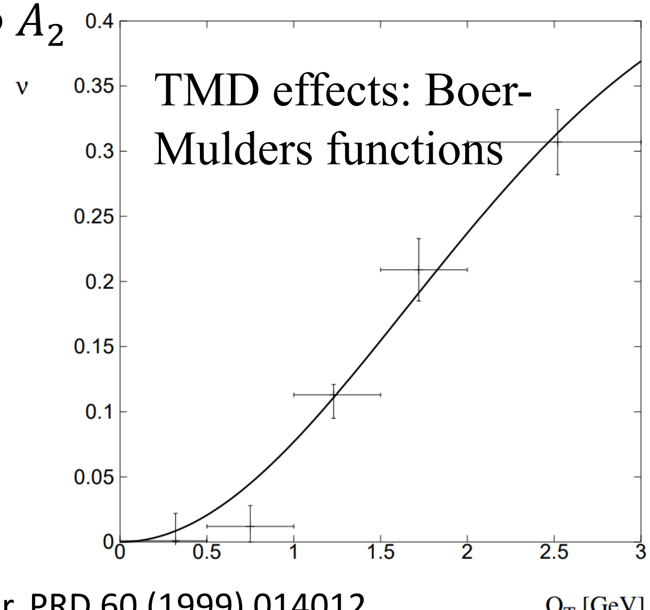
R. Gauld et al, JHEP 2017, N3LO

These results are confirmed by CMS and LHCb

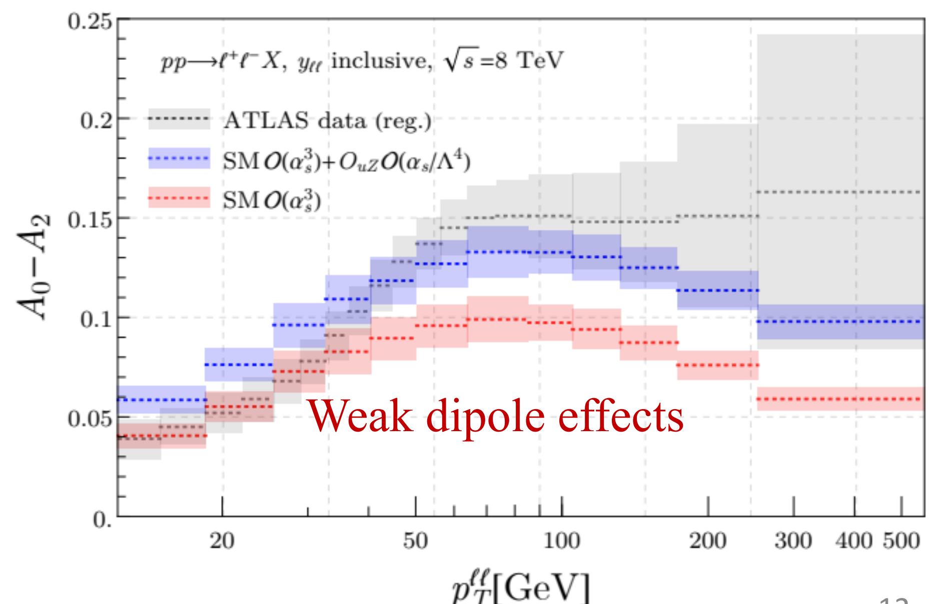
The discrepancy with the SM prediction
NP effects or non-perturbative effects ?

Xu Li, Bin Yan and C.-P. Yuan, PRD 111 (2025) 073007
Guanghui Li, Xu Li, Bin Yan, 2503.17663
S. Grossi, X. Li, L. Rolla, R. Torre, Bin Yan, working in progress

$\upsilon \propto A_2$

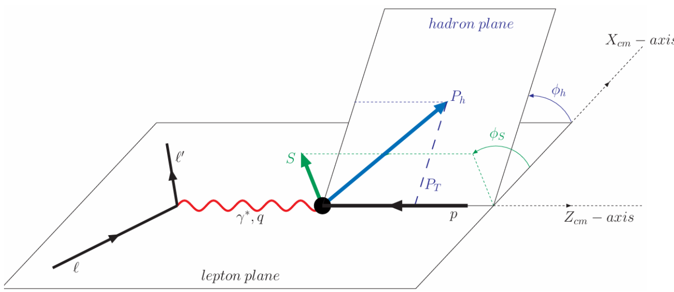


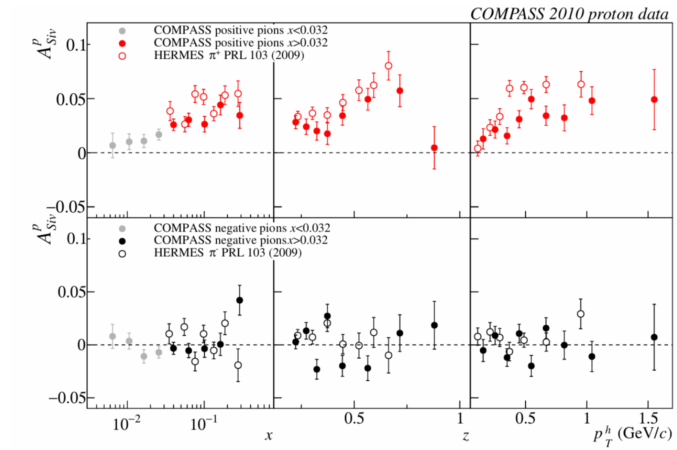
Boer, PRD 60 (1999) 014012



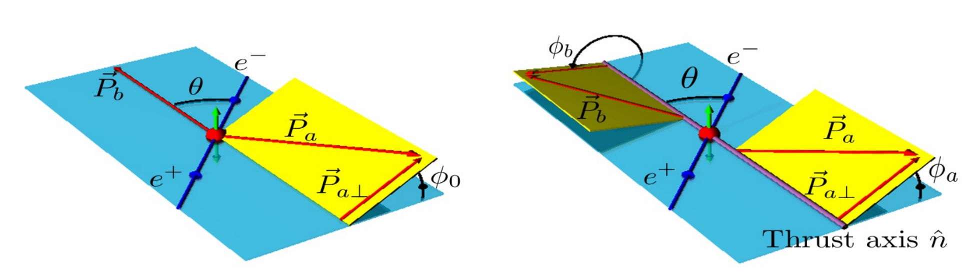
Example3: Spin effects in QCD

➢ Nucleon structure: PDFs

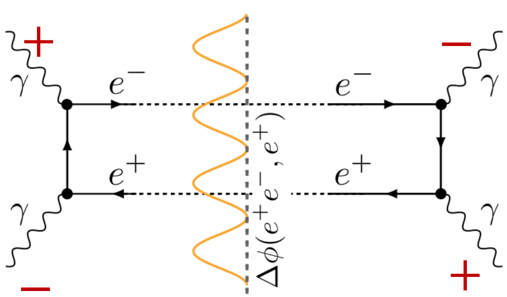


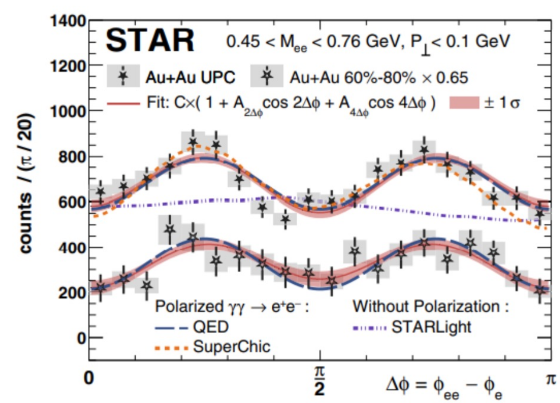


➢ Nucleon structure: FFs



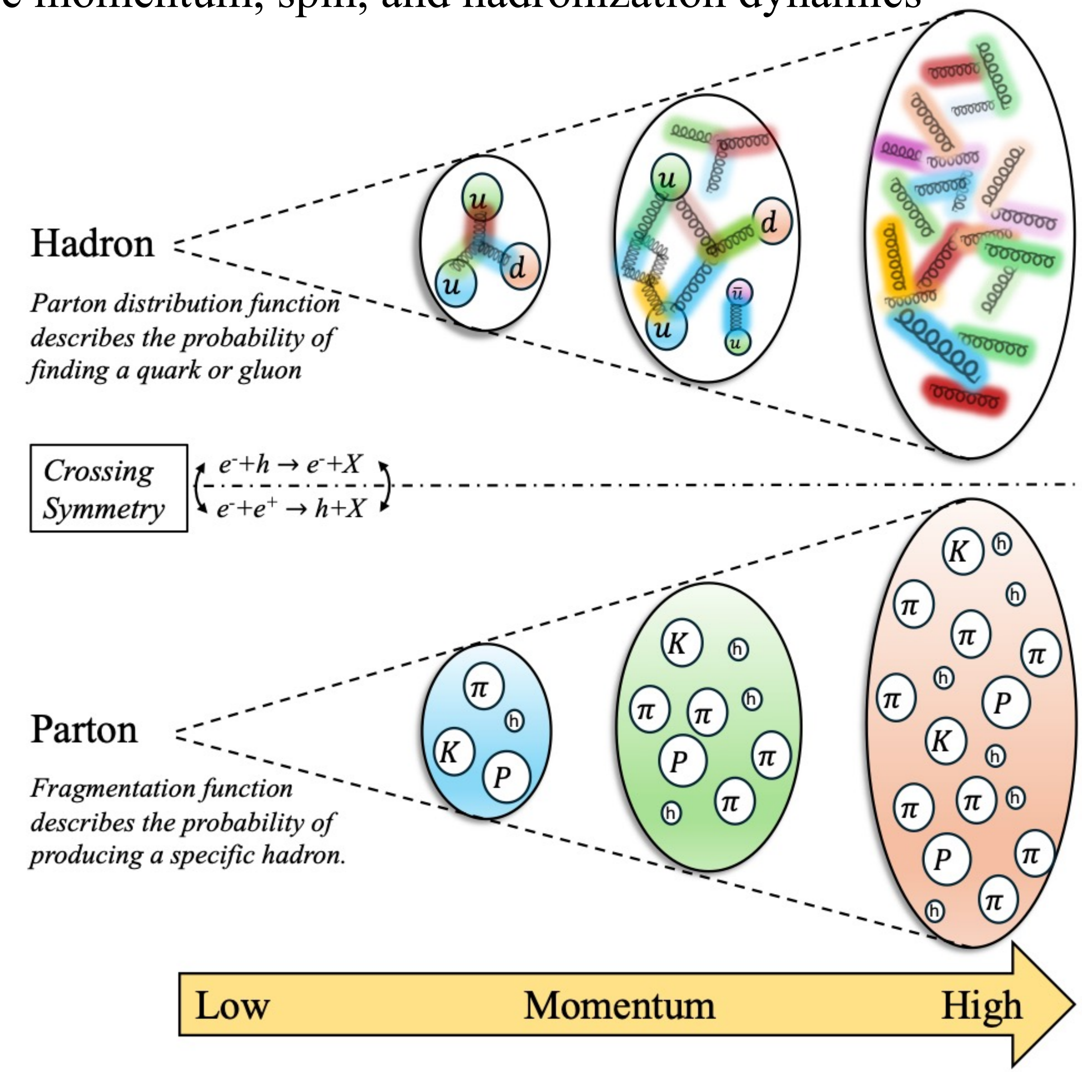
➢ UPCs





PDFs and FFs

- Spin phenomena in QCD arise from the intrinsic correlations between parton transverse momentum, spin, and hadronization dynamics

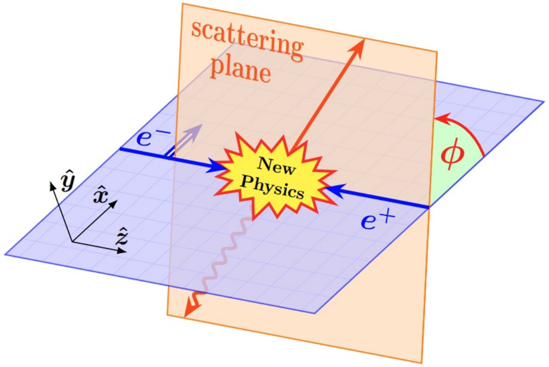


Transverse spin effects of electron

- Transversely polarized effect of beams @ lepton collider

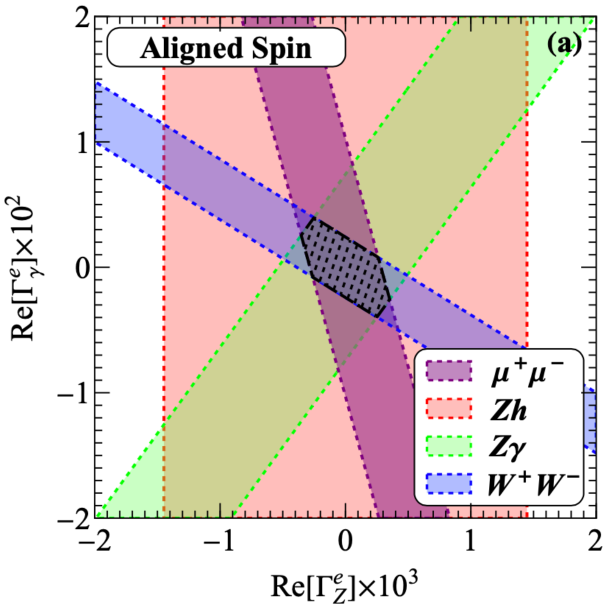
The interference between the different helicity states

$$\rho = \frac{1}{2}(1+\boldsymbol{\sigma}\cdot\boldsymbol{s}) = \frac{1}{2}\begin{pmatrix} 1+\lambda & b_{\rm T}e^{-i\phi_0} \\ b_{\rm T}e^{i\phi_0} & 1-\lambda \end{pmatrix}$$

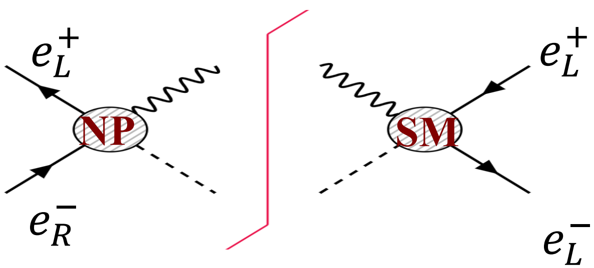


$\sqrt{s} = 250$ GeV, $\mathcal{L} = 5$ ab^{-1} $\quad (b_T, \bar{b}_T) = (0.8, 0.3)$

Xin-Kai Wen, Bin Yan, Zhite Yu, C.-P. Yuan, PRL 131 (2023) 241801



	U	L	T
U	$\lvert\mathcal{M}\rvert^2_{UU} \to 1$	$\lvert\mathcal{M}\rvert^2_{UL} \to 1$	$\lvert\mathcal{M}\rvert^2_{UT} \to \cos\phi, \sin\phi$
L	$\lvert\mathcal{M}\rvert^2_{LU} \to 1$	$\lvert\mathcal{M}\rvert^2_{LL} \to 1$	$\lvert\mathcal{M}\rvert^2_{LT} \to \cos\phi, \sin\phi$
T	$\lvert\mathcal{M}\rvert^2_{TU} \to \cos\phi, \sin\phi$	$\lvert\mathcal{M}\rvert^2_{TL} \to \cos\phi, \sin\phi$	$\lvert\mathcal{M}\rvert^2_{TT} \to 1, \cos 2\phi, \sin 2\phi$



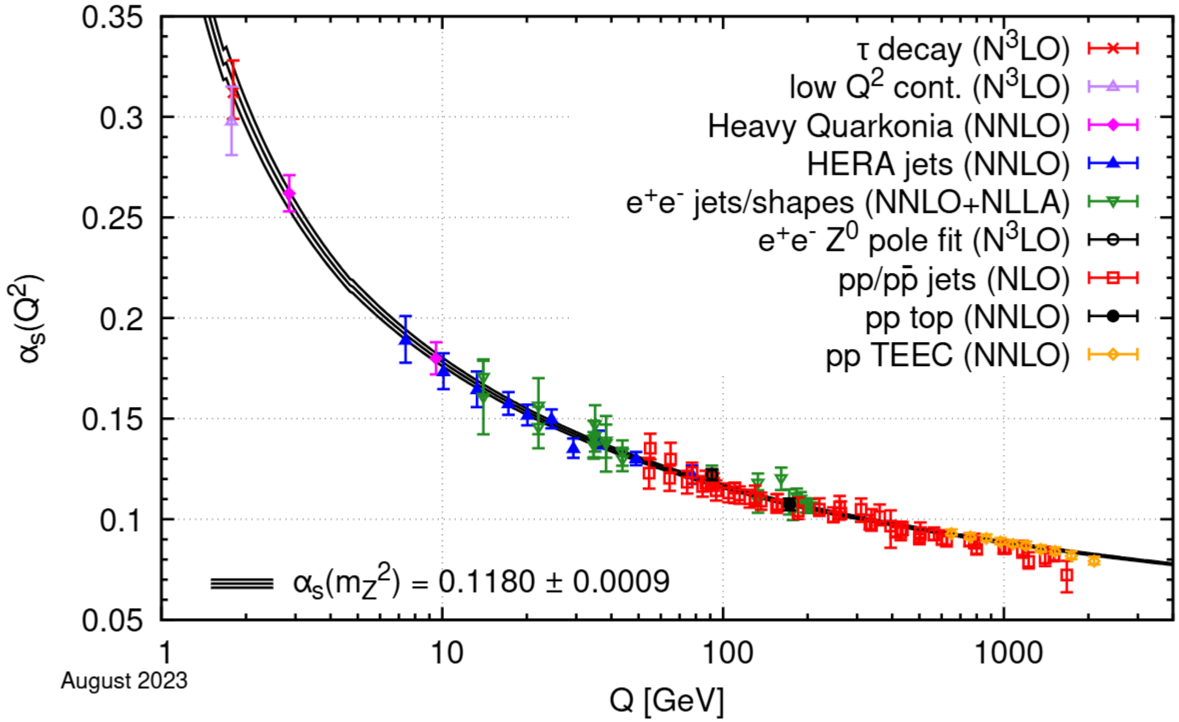
$\bar{e}_L\sigma_{\mu\nu}e_R A^{\mu\nu}, \bar{e}_L\sigma_{\mu\nu}e_R Z^{\mu\nu}$

Chiral-odd effects from dipole interactions

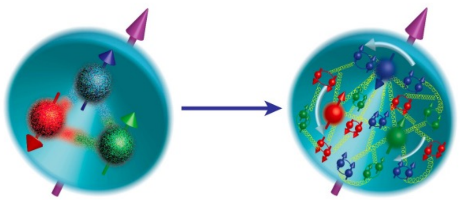
- Our bounds are much stronger than other approaches by 2 orders of magnitude
- Weak dipole coupling, SSA: 0.01%, LHC: 1%

Electroweak dipole moments of quarks

- The quark can not be a free particle due to the QCD confinement



Asymptotic freedom of QCD theory

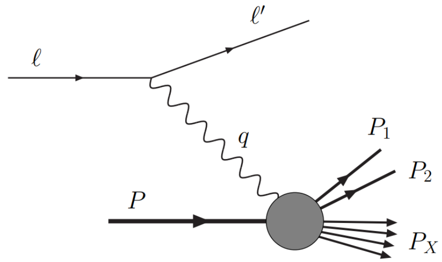


- How to probe the spin information of quarks?

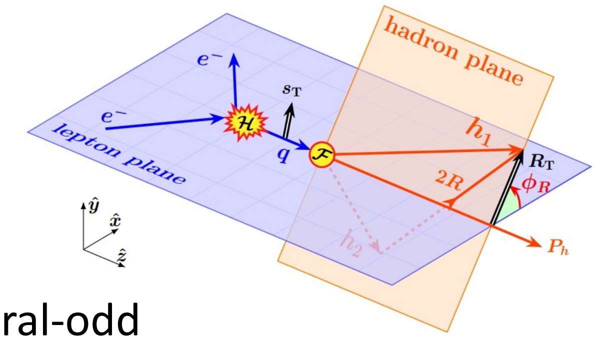
The non-perturbative functions, i.e., the parton distirbuion functions and the fragmentation functions

Transverse spin effects of quark @ EIC

- The transverse spin of quarks can be generated by the quark dipole moments



$$\bar{q}_L \sigma_{\mu\nu} q_R A^{\mu\nu}, \bar{q}_L \sigma_{\mu\nu} q_R Z^{\mu\nu}$$



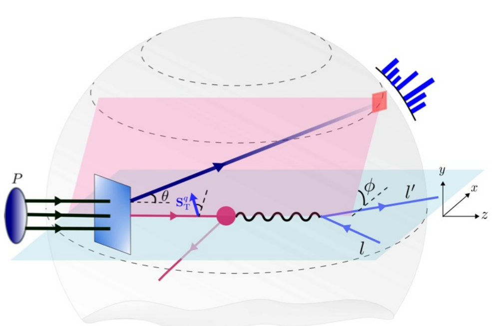
- The interference dihadron fragmentation function: chiral-odd

$$\frac{d\sigma}{dx\,dy\,dz\,dM_h\,d\phi_R} = \frac{N}{2\pi}\sum_q f_q(x,Q)\left[D_{h_1h_2/q}(z,M_h;Q)\right.$$
$$\left. - (\boldsymbol{s}_{T,q}(x,Q)\times \hat{\boldsymbol{R}}_T)^z H_{h_1h_2/q}(z,M_h;Q)\right]C_q(x,Q)$$

$$s_q^x = \frac{2}{C_q}\left(w_\gamma^q\ \mathrm{Re}\,\Gamma_\gamma^q + w_Z^q\ \mathrm{Re}\,\Gamma_Z^q\right)$$
$$s_q^y = \frac{2}{C_q}\left(w_\gamma^q\ \mathrm{Im}\,\Gamma_\gamma^q + w_Z^q\ \mathrm{Im}\,\Gamma_Z^q\right)$$

Xin-Kai Wen, Bin Yan, Zhite Yu, C.-P. Yuan, 2408.07255

- Nucleon energy correlator: related to Boer-Mulder quark TMD



$$\Sigma(\theta,\phi) = \sum_{i\in X}\int \mathrm{d}\sigma^{l+p\to l'+X}\frac{E_i}{E_N}\delta(\theta^2-\theta_i^2)\delta(\phi-\phi_i)$$

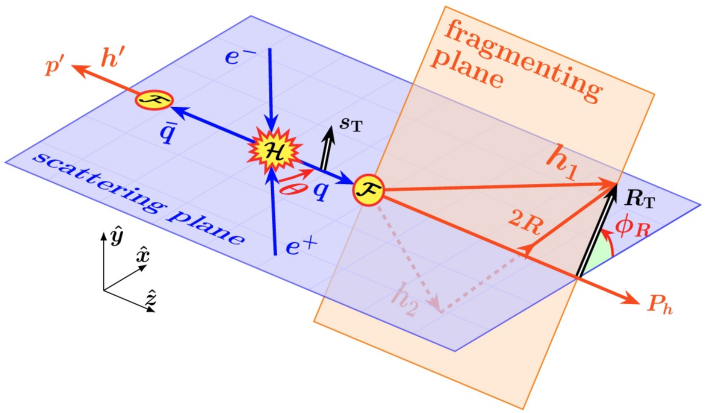
Yingsheng Huang, Xuan-Bo Tong, Hao-Lin Wang, 2508.08516

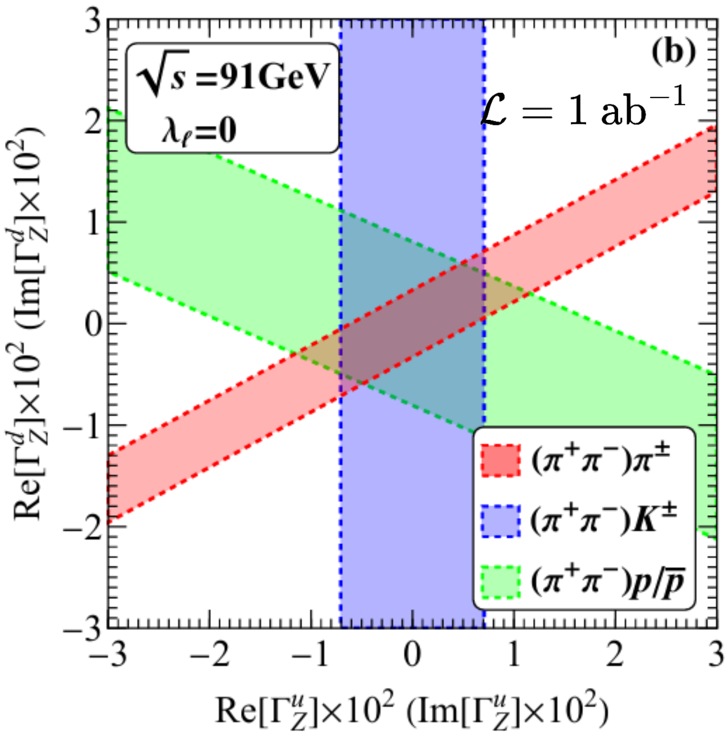
The flat direction in flavor space of dipole couplings?

Transverse spin effects of quark @ CEPC

Xin-Kai Wen, Bin Yan, Zhite Yu, C.-P. Yuan, PRD 112 (2025) 053004

$$\frac{d\sigma}{dy\,dz\,d\bar{z}\,dM_h\,d\phi_R} = \frac{1}{32\pi^2 s}\sum_{q,\,q\to\bar{q}} C_q(y)\,D_{\bar{q}}^{h'}(\bar{z})$$
$$\times\left[D_q^{h_1h_2}(z,M_h) - (\boldsymbol{s}_{T,q}(y)\times\hat{\boldsymbol{R}}_T)^z H_q^{h_1h_2}(z,M_h)\right]$$





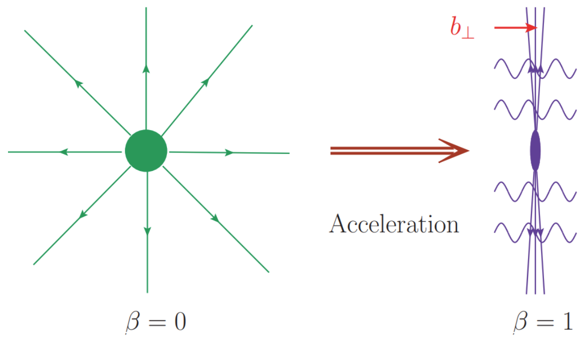
$$s_q^x = \frac{2}{C_q}\left(w_\gamma^q\ \mathrm{Re}\,\Gamma_\gamma^q + w_Z^q\ \mathrm{Re}\,\Gamma_Z^q\right)$$

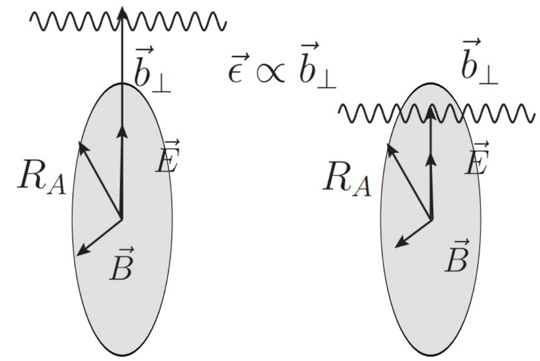
$$s_q^y = \frac{2}{C_q}\left(w_\gamma^q\ \mathrm{Im}\,\Gamma_\gamma^q + w_Z^q\ \mathrm{Im}\,\Gamma_Z^q\right)$$

$$\bar{q}_L\sigma_{\mu\nu}q_R A^{\mu\nu},\ \bar{q}_L\sigma_{\mu\nu}q_R Z^{\mu\nu}$$

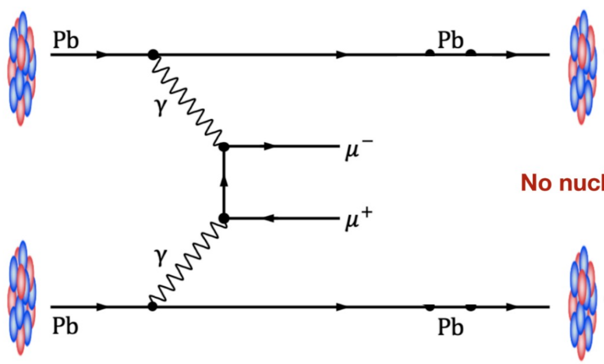
- The flat direction can be closed by combing more processes
- Z-boson dipole: O(0.001)

Linear polarization @ UPCs





C. Li, J. Zhou, Y. J. Zhou, Phys. Lett. B. 795, 576 (2019)



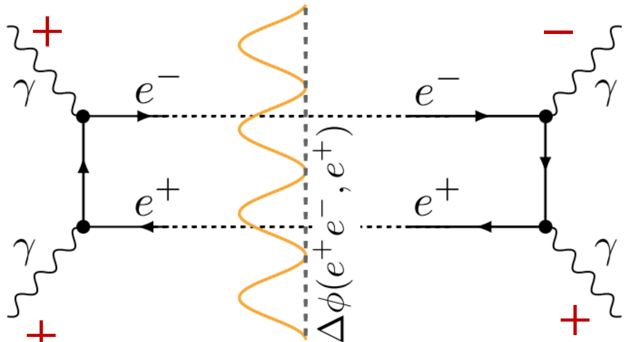
No nucleus breakup

- Ultra-relativistic charged nuclei produce highly Lorentz contracted electromagnetic field
- Weizsacker-Williams equivalent photon approximation
- Photons are linearly polarized
- Large quasi-real photon flux $\propto Z^2$
- The impact parameter $b_\perp > 2R_A$

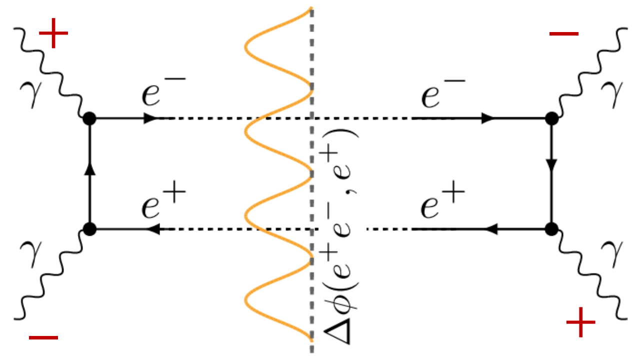
The linear polarization for gluons based on the NEEC:
Yuxun Guo, Xiaohui Liu, Feng Yuan, HuaXing Zhu, Research 2025 (2025) 0552
Xiao Lin Li, Xiaohui Liu, Feng Yuan, HuaXing Zhu, PRD 108 (2023) L091502

Linear polarization @ UPCs

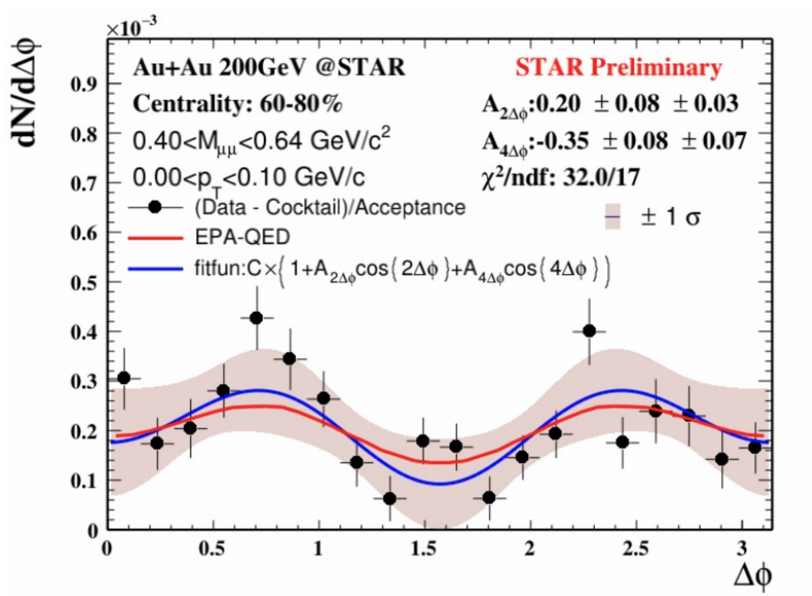
D. Y. Shao, C. Zhang, J. Zhou, Y. Zhou, PRD107 (2023) 3, 036020



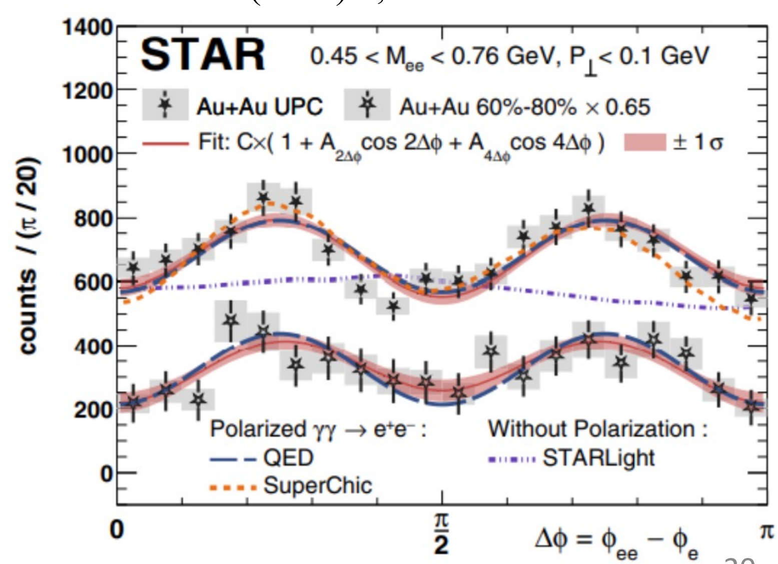
$$\frac{m_e^2}{p_T^2}\,cos2\Delta\phi$$



$$cos4\Delta\phi$$



PRL 127 (2021) 5, 052302



Linear polarization @ UPCs

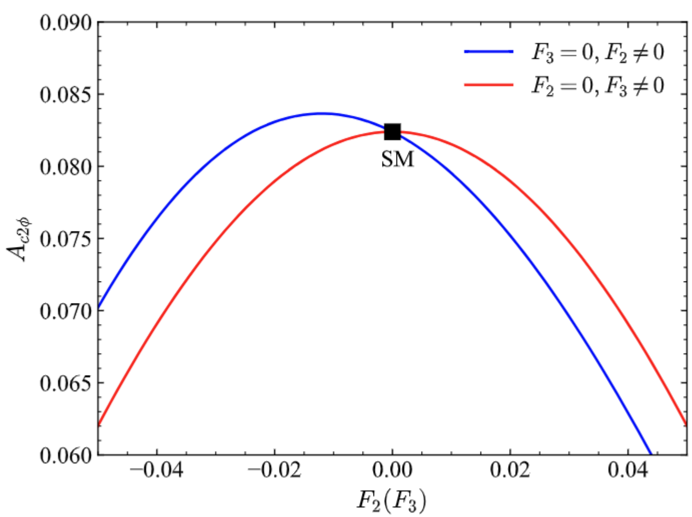
Dingyu Shao, Bin Yan, Shu-Ruan Yuan, Cheng Zhang,
Sci. China Phys. Mech. Astron. 67 (2024) 281062

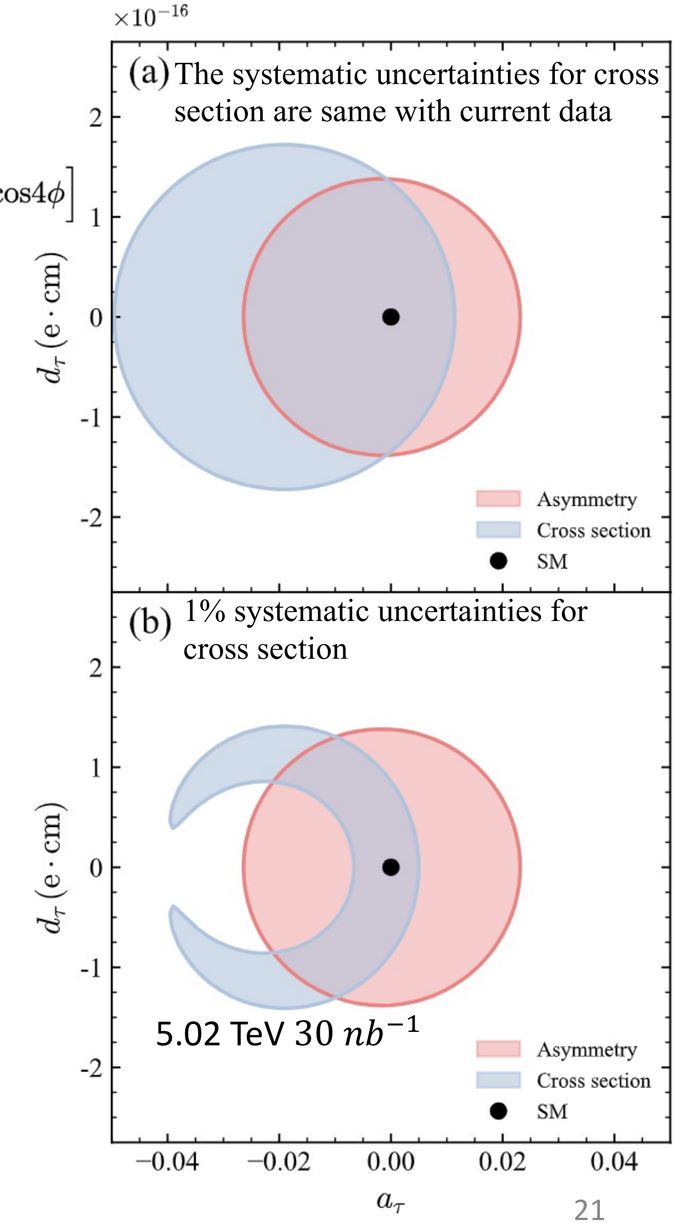
$$d\sigma \sim \left[A_0 + B_0^{(1)} F_2 + B_0^{(2)} F_2^2 + C_0^{(2)} F_3^2 + \left(A_2 + B_2^{(2)} F_2^2 + C_2^{(2)} F_3^2\right)\cos 2\phi + A_4 \cos 4\phi\right]$$

$$F_2(0) = a_\tau, \quad F_3(0) = 2\frac{m_\tau d_\tau}{e}$$

Suppressed by lepton mass

$$A_{c2\phi} = \frac{\sigma(\cos 2\phi > 0) - \sigma(\cos 2\phi < 0)}{\sigma(\cos 2\phi > 0) + \sigma(\cos 2\phi < 0)}$$





Linear polarization @ lepton colliders

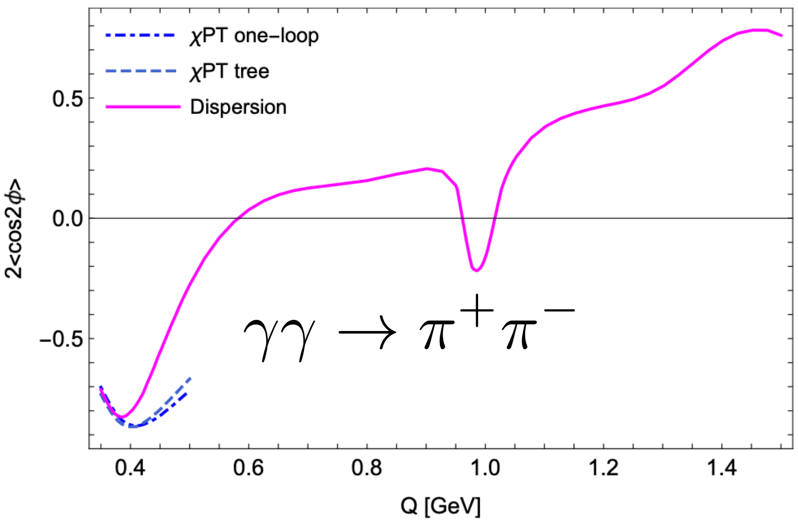
- Linear polarization of gauge bosons naturally emerges in TMD factorization

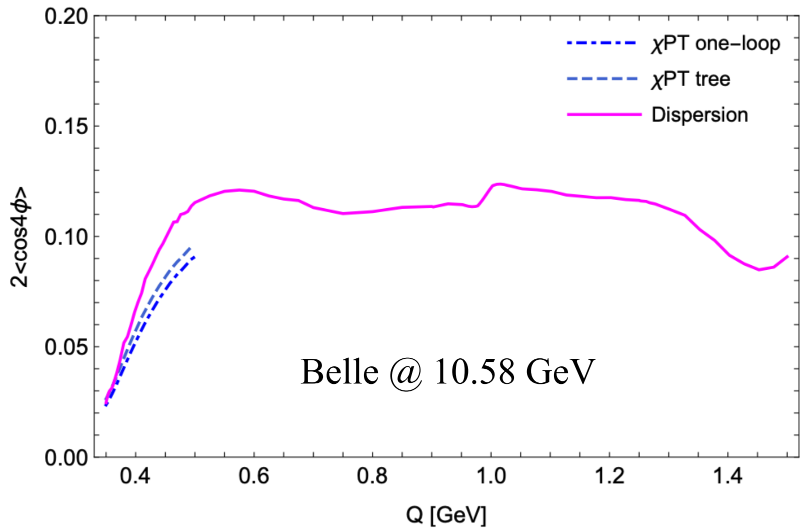
$$\int \frac{dy^- d^2 y_\perp}{P^+(2\pi)^3} e^{ik\cdot y} \langle e| F_+^\mu(0) F_+^\nu(y) |e\rangle \Big|_{y^+=0}$$
$$= \frac{\delta_\perp^{\mu\nu}}{2} x f(x, k_\perp^2) + \left(\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} - \frac{\delta_\perp^{\mu\nu}}{2} \right) x h_1^\perp(x, k_\perp^2)$$

Yu Jia, Jian Zhou, Ya-jin Zhou, PRL 134 (2025) 141901

Unpolarized photon TMD

Linearly polarized photon TMD





- Linear polarization: probing the tau lepton dipole moments

D. Y. Shao, H. Xiang, F. Xu, B. Yan and C. Zhang, 2506.15245

Spin correlation in SM and beyond

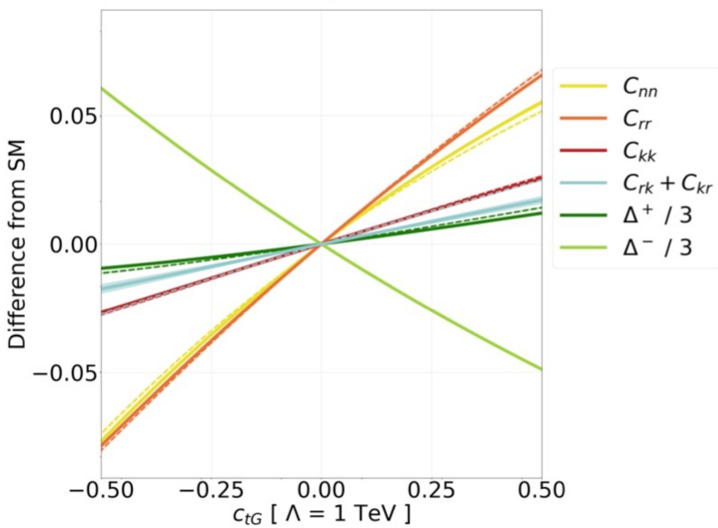
Example 1: Top quark pair production

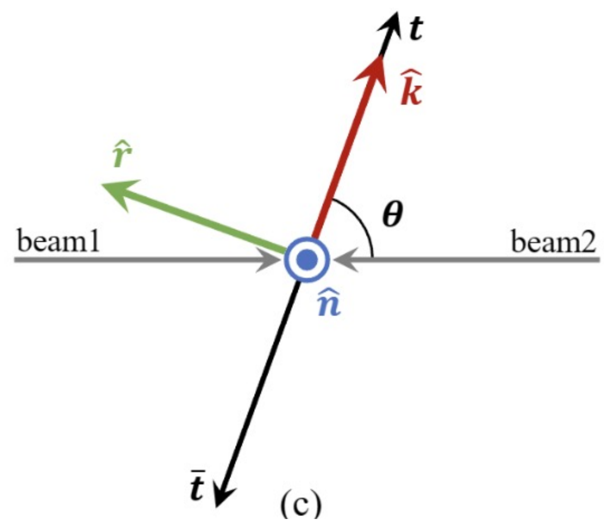
- The spin correlation of top quarks can be described by the general density matrix:

$$\rho = \frac{I_2 \otimes I_2 + B_i \sigma_i \otimes I_2 + \bar{B}_i I_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

  - $B_i, \bar{B}_i$: the polarization of each particle
  - C_{ij} : the spin correlation of fermion pair
  - Spin correlations are central to exploring quantum entanglement and Bell nonlocality
  - Spin correlations, entanglement and SMEFT effects

See tallks from Qing-Hong Cao, Qiang Li and Tian Jun Li



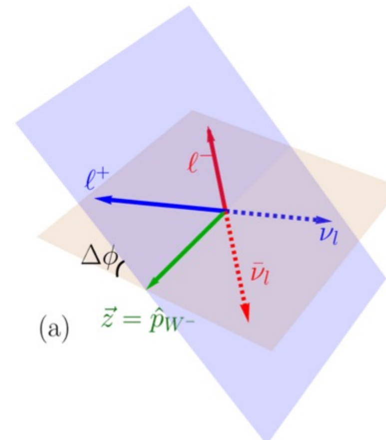


C. Severi and E. Vryonidou, JHEP 01 (2023)148
W. Bernreuther, Long Chen, Zong-Guo Si, PRD 109 (2024) 116016

Example 2: gauge boson pair production

- Gauge boson pair spin correlations provide a probe of CP symmetry

$$\frac{1}{\sigma}\frac{d\sigma}{d\Delta\phi} = \frac{1}{2\pi} + A\cos(\Delta\phi) + \tilde{A}\sin(\Delta\phi) + (\text{higher partial wave terms}).$$

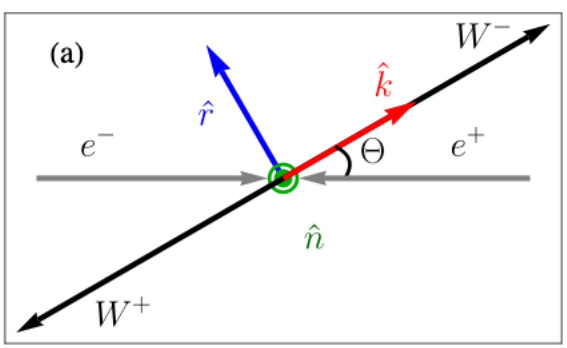


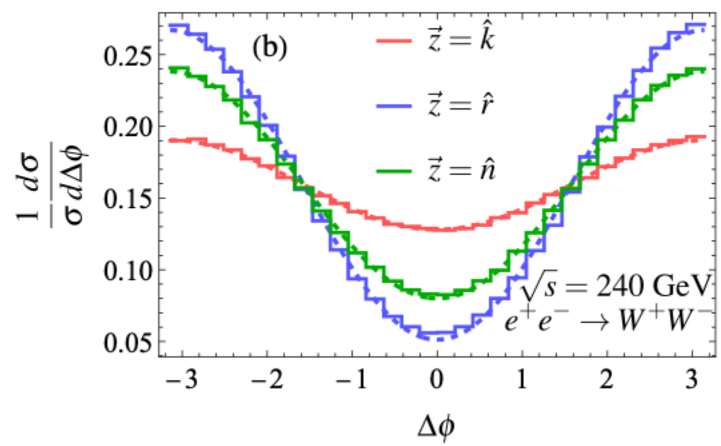
$$(\theta, \phi, \bar{\theta}, \bar{\phi}) \xrightarrow{\text{CP}} (\pi - \bar{\theta}, \pi + \bar{\phi}, \pi - \theta, \pi + \phi)$$

$$\Delta\phi \xrightarrow{\text{CP}} -\Delta\phi,$$

$$\mathcal{O}_{\widetilde{W}WW} = \text{Tr}\left[\widetilde{W}_{\mu\nu} W^{\nu\rho} W_{\rho}^{\mu}\right], \qquad \mathcal{O}_{\widetilde{W}} = (D_\mu \Phi)^\dagger \widetilde{W}^{\mu\nu} (D_\nu \Phi)$$

- The results are sensitive to the different rotational axis



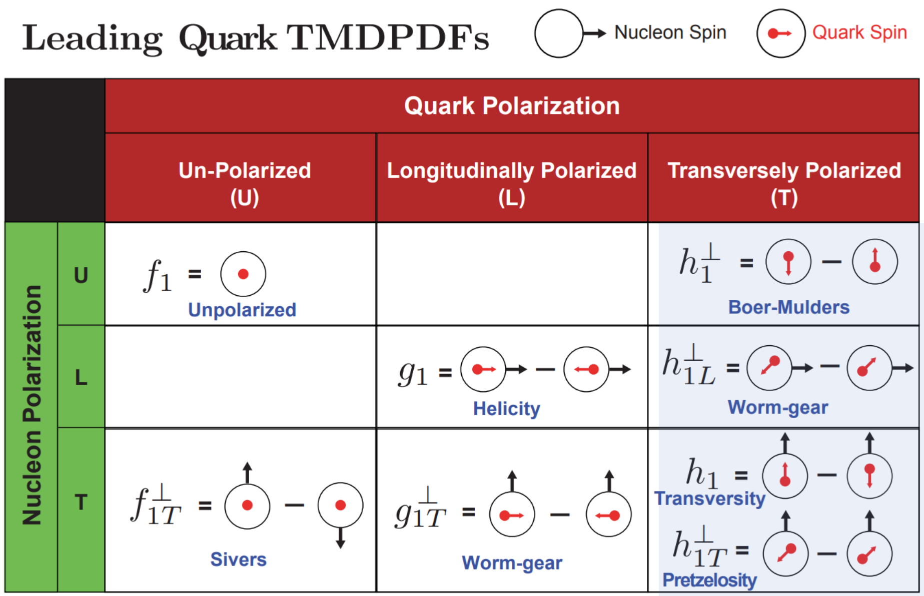


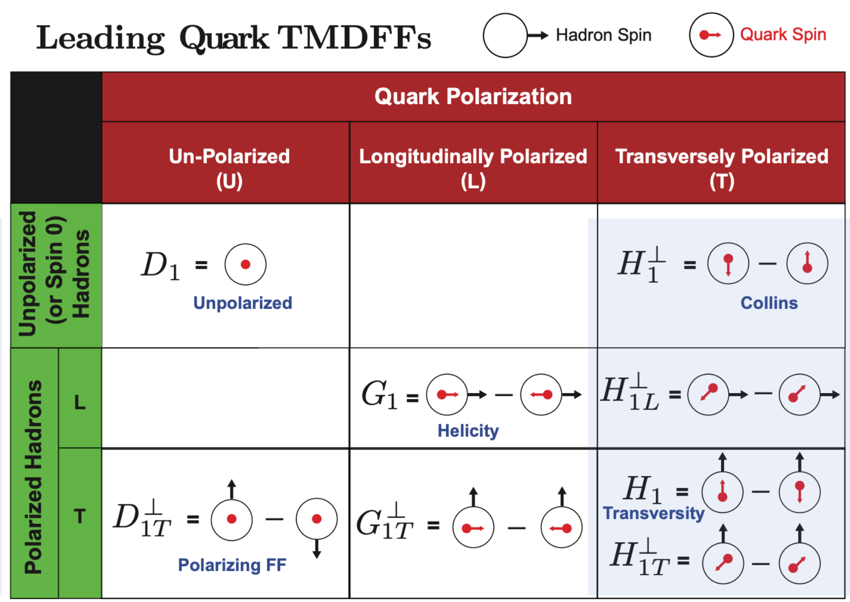
Kun Cheng, Yi-Jing Fang, Tao Han, Matthew Low, PRD 112 (2025) 013005

Example3: Spin effects in QCD

- Spin correlation effects are essential for accessing chiral-odd nonperturbative functions in the nucleon structure

See papers of Zuo-Tang Liang, Tianbo Liu, Shuyi Wei, Jian Zhou, et al

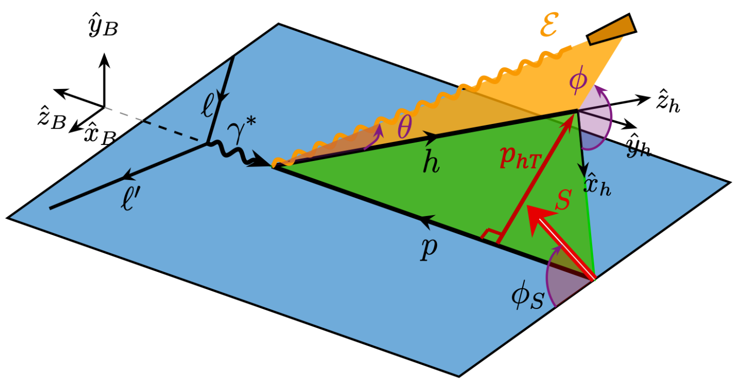




- Nonperturbative functions for transversely polarized quarks are chiral-odd.
- Physical observables must be chiral-even: require pairing of two chiral-odd functions
  - ☐ Collins Azimuthal Asymmetries in SIDIS, Collins function + Transversity function
  - ☐ Dihadron production in lepton colliders, Two Collins functions
  - ☐ Dihadron production in SIDIS, Interference dihadron fragmentation + Transversity function

Quark transverse spin in energy correlators

- Collins-type fragmentation energy correlator

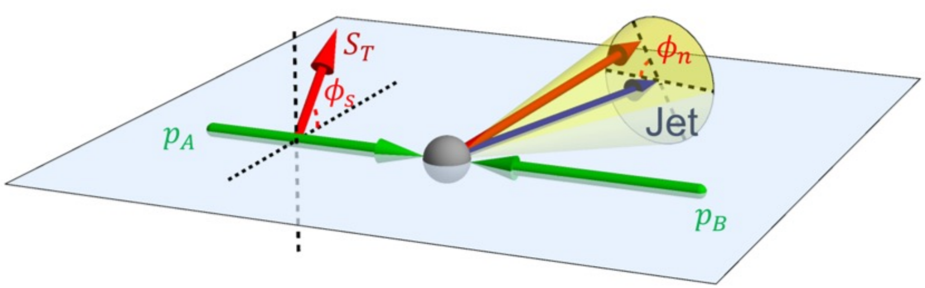


Transversity + Collins-type FFs
(Collinear factorization)

Q. H. Cao, Z. Yu, C.-P. Yuan, S. Zhang, H. X. Zhu, 2509.18892

See Hua-Xing's talk

- One-point energy correlator

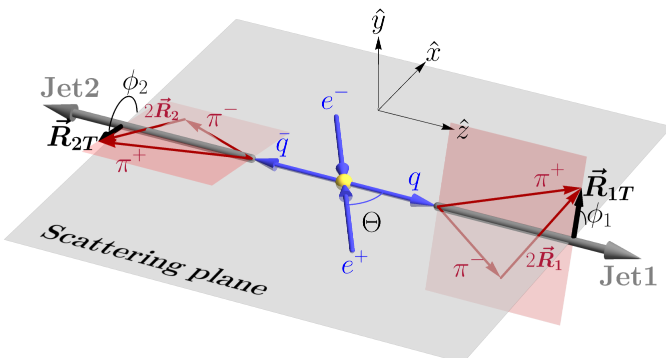


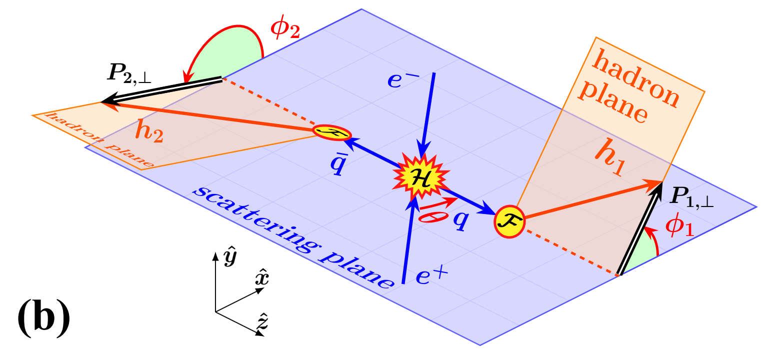
Transversity + transversely polarized fragmentation jet function (TMD)

M. S. Gao, Z. B. Kang, W. Li, D. Y. Shao, 2509.15809

Quantum entanglement of light quarks

- Quantum entanglement of quark systems can be studied through transverse spin correlations: dihadron interference FFs (collinear) and Collins functions (TMD)





K. Cheng and B. Yan, PRL 135 (2025) 011902

K. Cheng, T. Han, G. Li and B. Yan, 2510.xxxxx

See S. J. Lin, M. J. Liu, D. Y. Shao and S. Y. Wei, 2507. 15387 for $\Lambda\bar{\Lambda}$ production

- Light quark pair are 100% correlated in the central scattering region

$$C_{ij} = \mathrm{diag}\left(\frac{\sin^2\Theta}{1+\cos^2\Theta}, -\frac{\sin^2\Theta}{1+\cos^2\Theta}, 1\right)$$

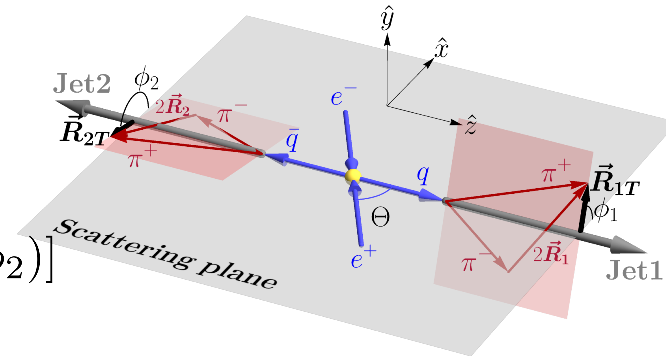
See Guanghui's talk

- The maximally entangled Bell state: Bell inequality violation effects

$$\mathcal{B}_{\pm} \equiv C_{xx} \pm C_{yy} \qquad \mathcal{B}_{+} = 0, \quad \mathcal{B}_{-} = \frac{2\sin^2\Theta}{1+\cos^2\Theta}.$$

New physics and entangled quarks

- The entangled triplet nature of the quark pair in SM:



$$\frac{d\sigma}{d\phi_1 d\phi_2} \propto C + \mathcal{F}[B_- \cos(\phi_1 + \phi_2) + B_+ \cos(\phi_1 - \phi_2)]$$

$$\mathcal{B}_\pm \equiv C_{xx} \pm C_{yy} \qquad C_{ij} = \mathrm{diag}\left(\frac{\sin^2\Theta}{1+\cos^2\Theta}, -\frac{\sin^2\Theta}{1+\cos^2\Theta}, 1\right)$$

$$\mathcal{B}_+ = 0, \quad \mathcal{B}_- = \frac{2\sin^2\Theta}{1+\cos^2\Theta}. \qquad |\psi\rangle = \frac{1}{\sqrt{2}}(|{\uparrow_y}{\downarrow_y}\rangle + |{\downarrow_y}{\uparrow_y}\rangle)$$

See Xin-Kai's talk

- $\mathcal{B}_+ = 0$ is a result of the chiral symmetry of SM (massless quarks)

- New spin structure $\mathcal{B}_+ \neq 0$ from chiral symmetry breaking interactions: dipole couplings

$$\mathcal{L}_{\rm eff} \supset \frac{v}{\sqrt{2}\Lambda^2}\left(C_\gamma^u \bar{u}_L \sigma^{\mu\nu} u_R + C_\gamma^d \bar{d}_L \sigma^{\mu\nu} d_R\right) A_{\mu\nu} + \mathrm{h.c.} \qquad \phi = \begin{cases} 0 & \text{CP even} \\ \pi & \text{CP odd} \end{cases}$$

- Entangled states for quark pair: spin triplet or singlet $\quad |\psi\rangle = \dfrac{1}{\sqrt{2}}(|{\uparrow_z}{\downarrow_z}\rangle + e^{i\phi}|{\downarrow_z}{\uparrow_z}\rangle)$

Qing-Hong Cao, Guanghui Li, Xin-Kai Wen and Bin Yan, 2509.18276

Summary

- Polarization and correlations are powerful tools for testing SM interactions and searching for new physics
- Perturbative decay approaches: polarization and spin correlations determine the properties of unstable particles (top quark, weak bosons)
- Nonperturbative methods with PDFs and FFs: access polarization and correlation effects of light quarks
- New opportunities in QCD spin physics: polarization and correlations as windows to new physics

Thank you