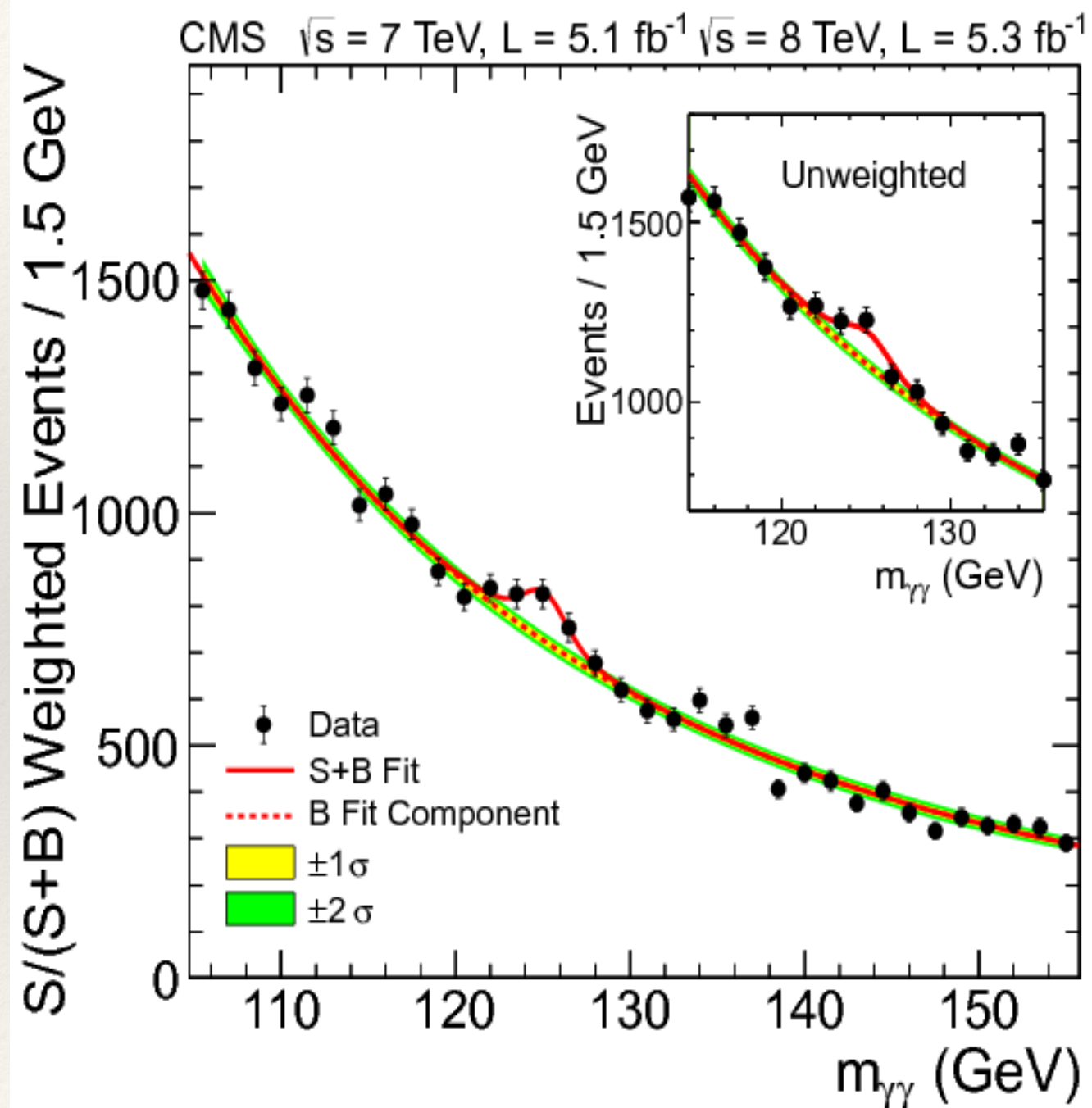
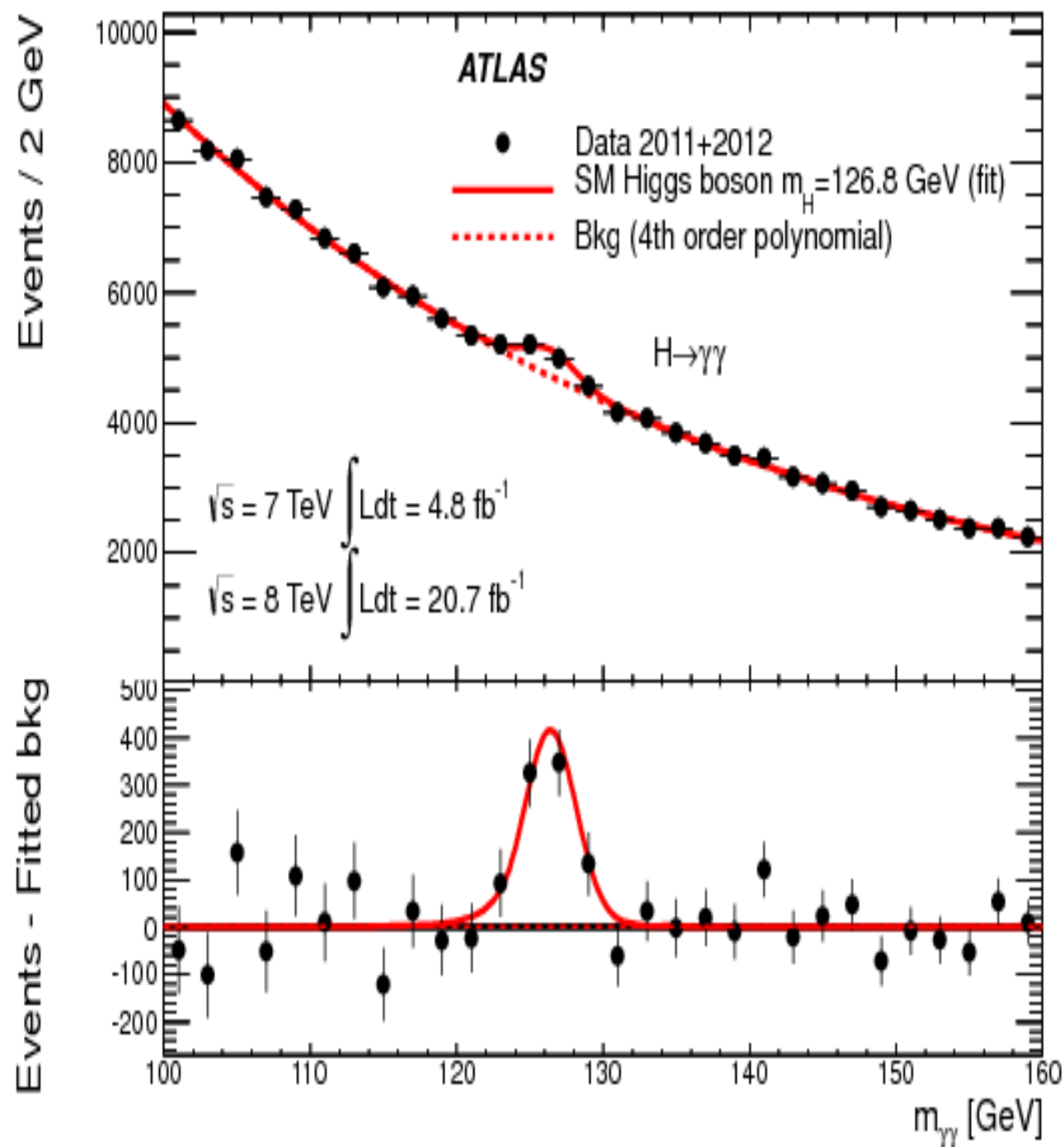


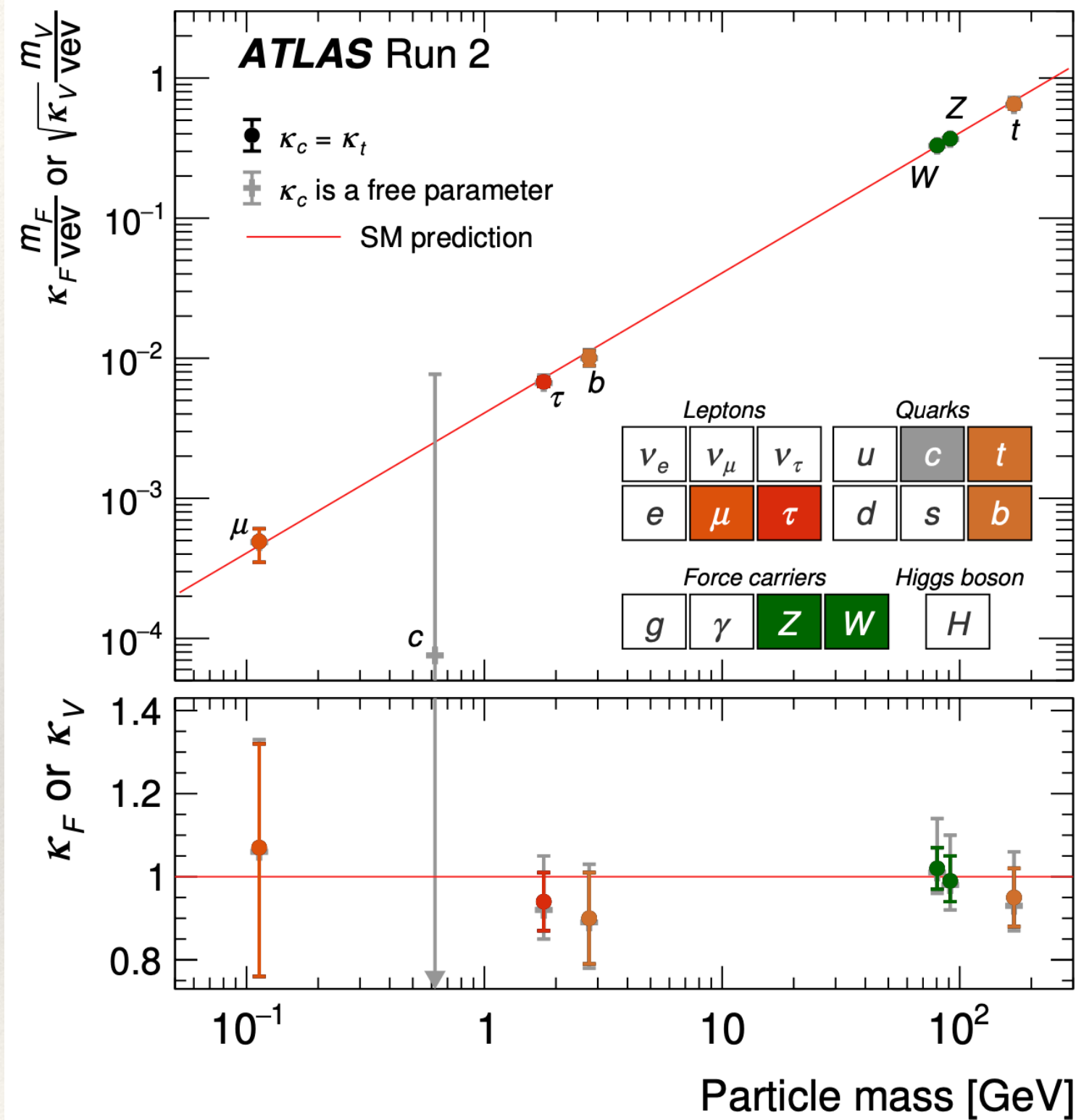
DiHiggs Theory

Jian Wang (王健)
Shandong University

TeV物理前沿专题研讨会暨第31届LHC Mini-Workshop
Qingdao, Oct 11, 2025

Discovery in 2012





So far, the H(125) properties are consistent with the SM expectation!

Its couplings with SM particles are proportional to their masses;
Higgs mechanism

existence of a “fifth force”
 different from gravity

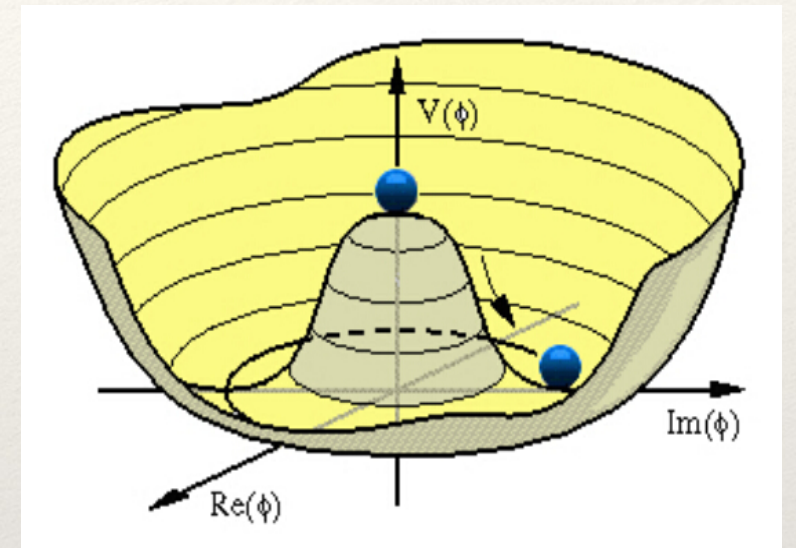
Higgs self-coupling in the SM

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$



$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$V(h) = \frac{1}{2} m_h^2 h^2 + \sqrt{\frac{\lambda}{2}} m_H h^3 + \frac{1}{4} \lambda h^4$$
$$\lambda = m_H^2 / 2v^2$$

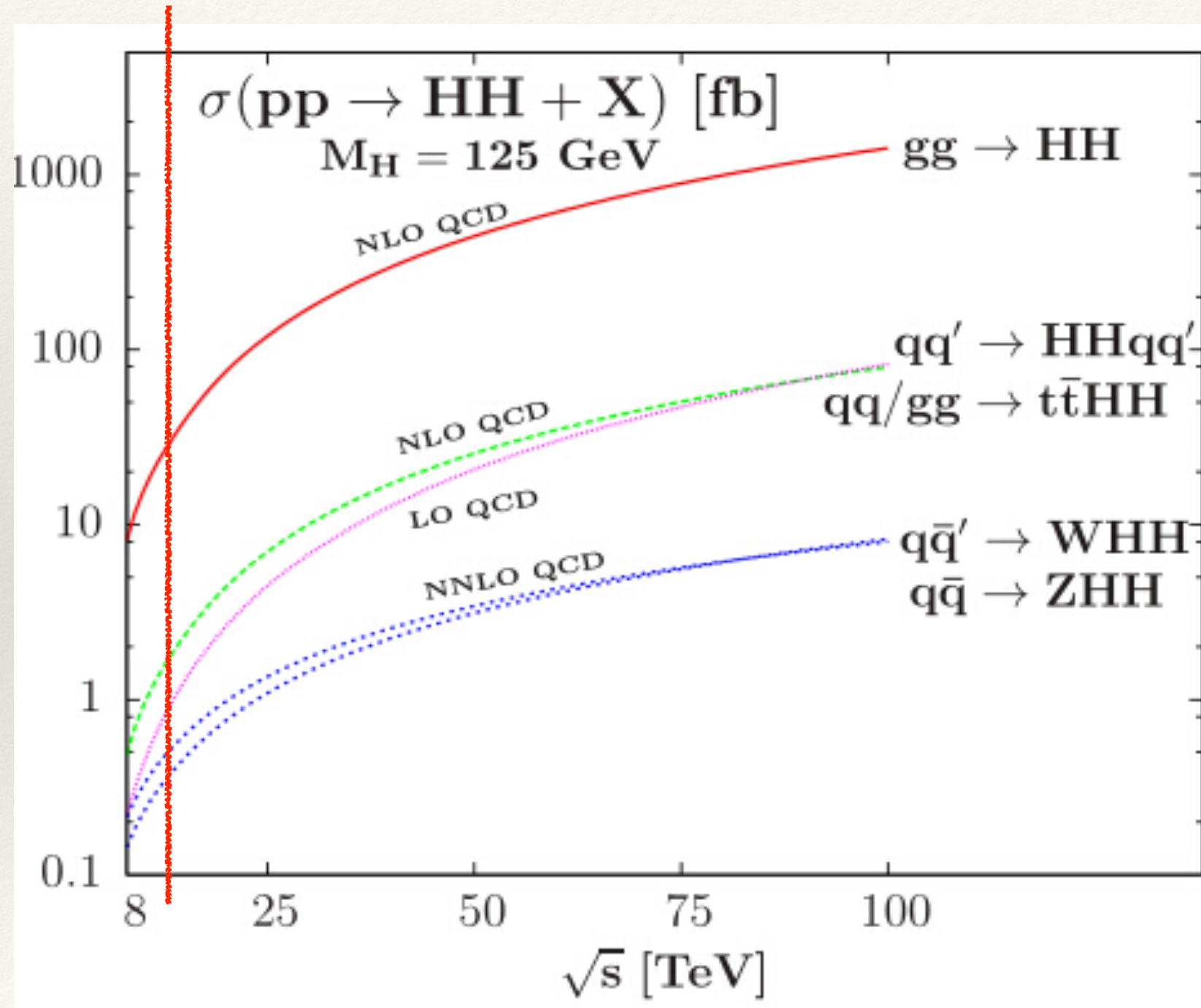


In some new physics models, the trilinear Higgs self-coupling may change by O(100)%, while the couplings with gauge bosons and fermions are still in agreement with SM.

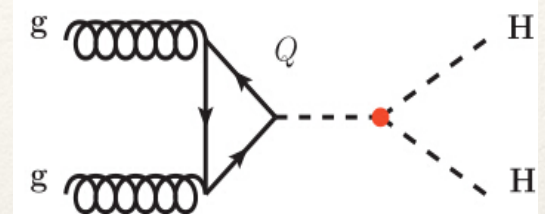
S.Kanemura, et al, PLB558,157

We need to measure the trilinear self coupling directly.

Higgs pair production at the LHC

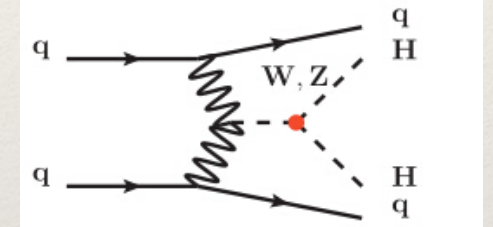


- gluon fusion



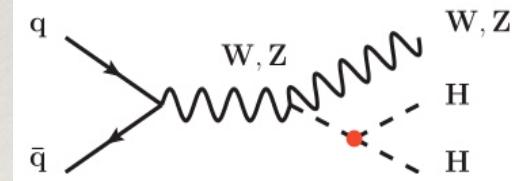
NNLO in QCD

- vector boson fusion



NNLO in QCD

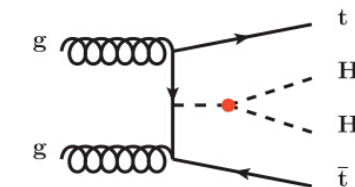
- double Higgs-strahlung



NNLO in QCD

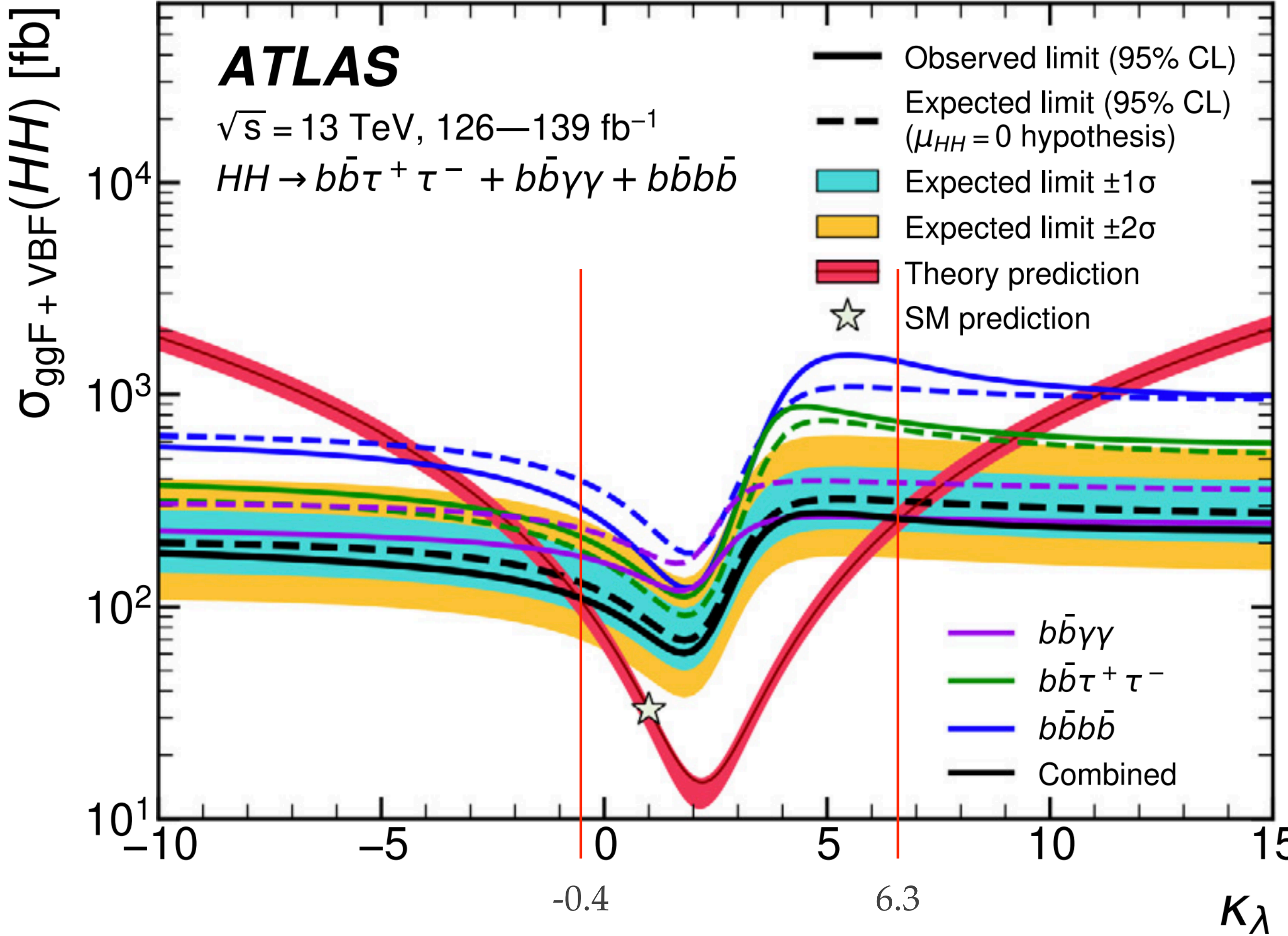
[J.B. *et al*, JHEP 1304 (2013) 151]

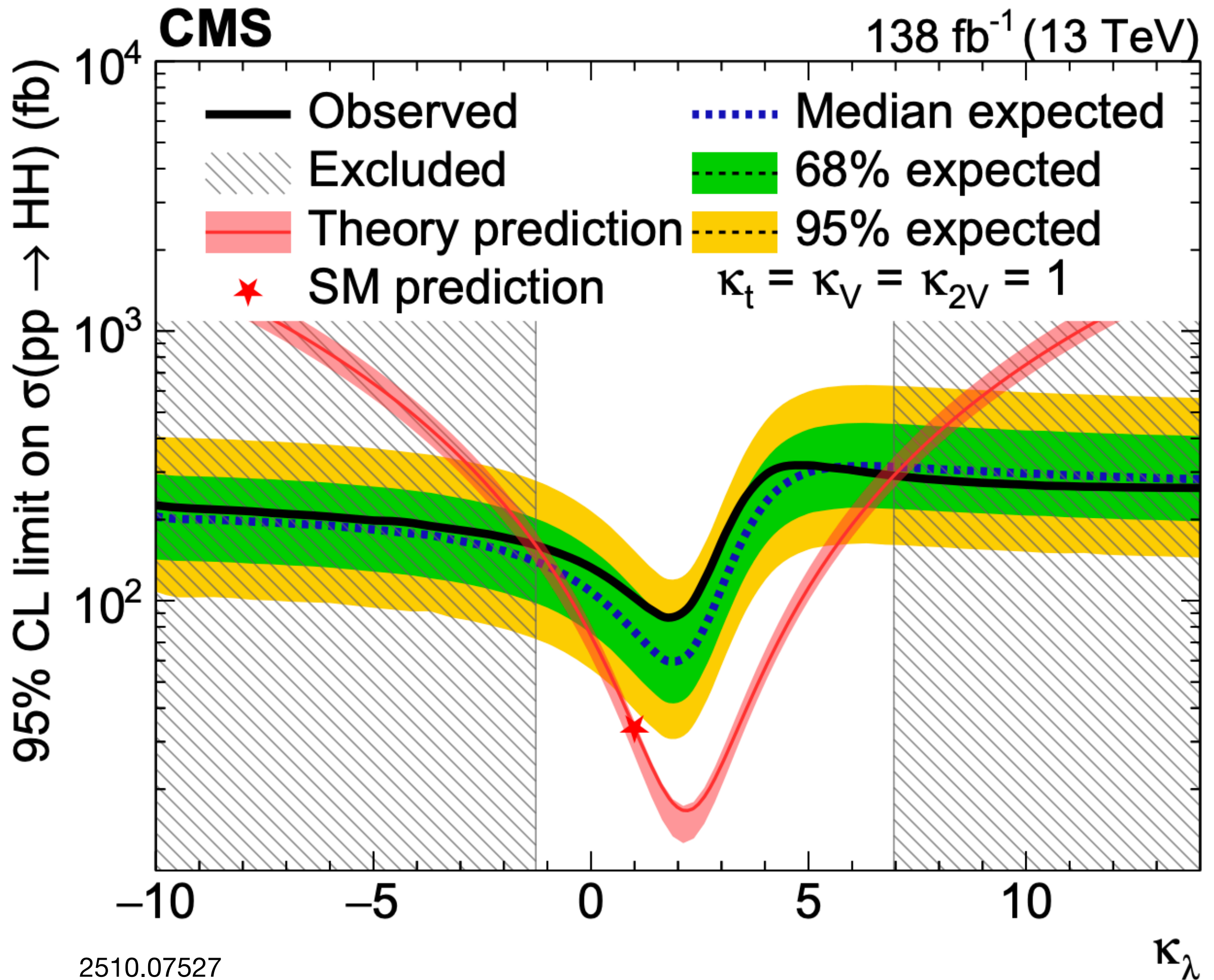
- associated production with top quark

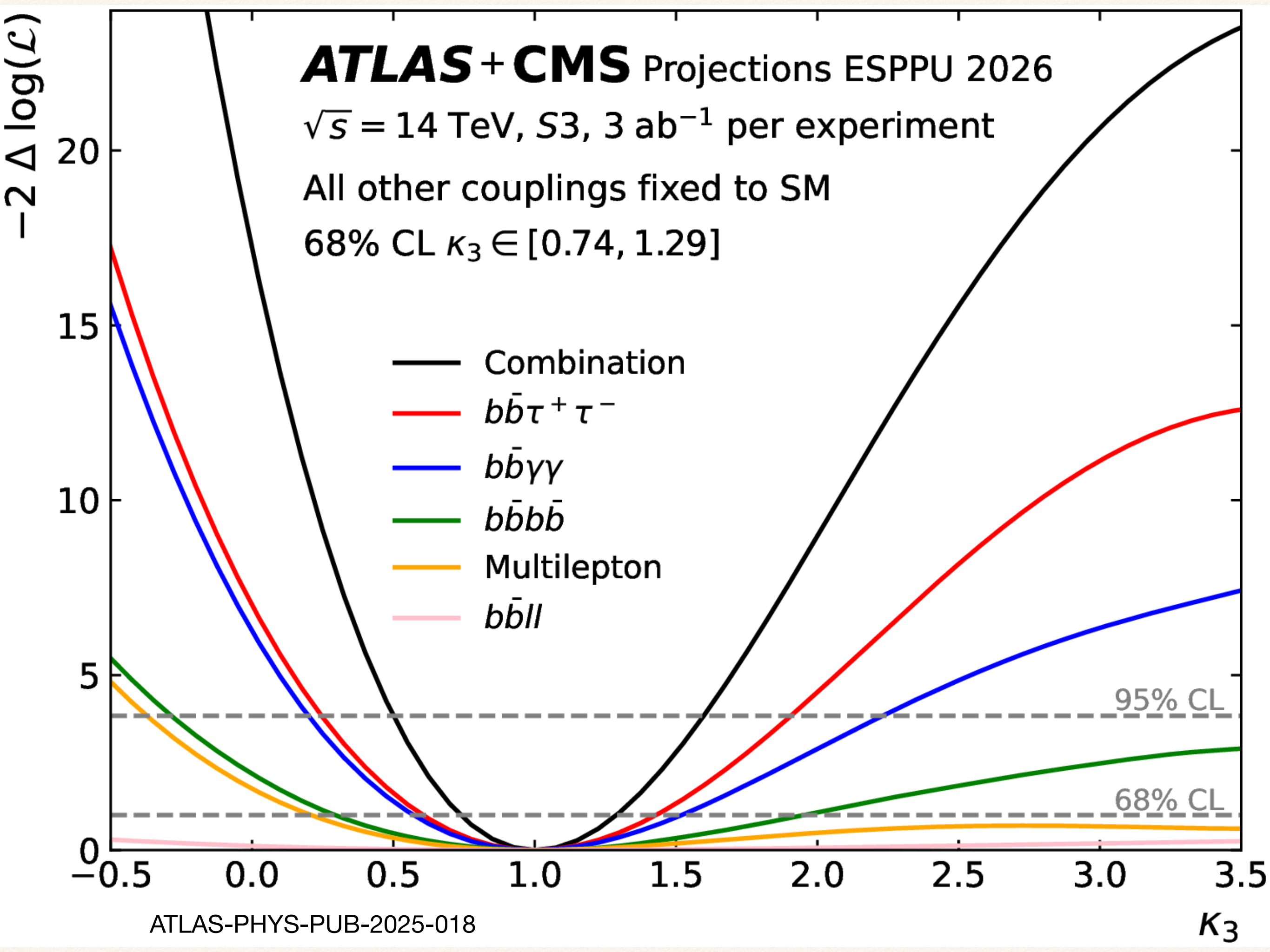


NLO in QCD

[Frederix *et al*, Phys.Lett. B732 (2014) 142]

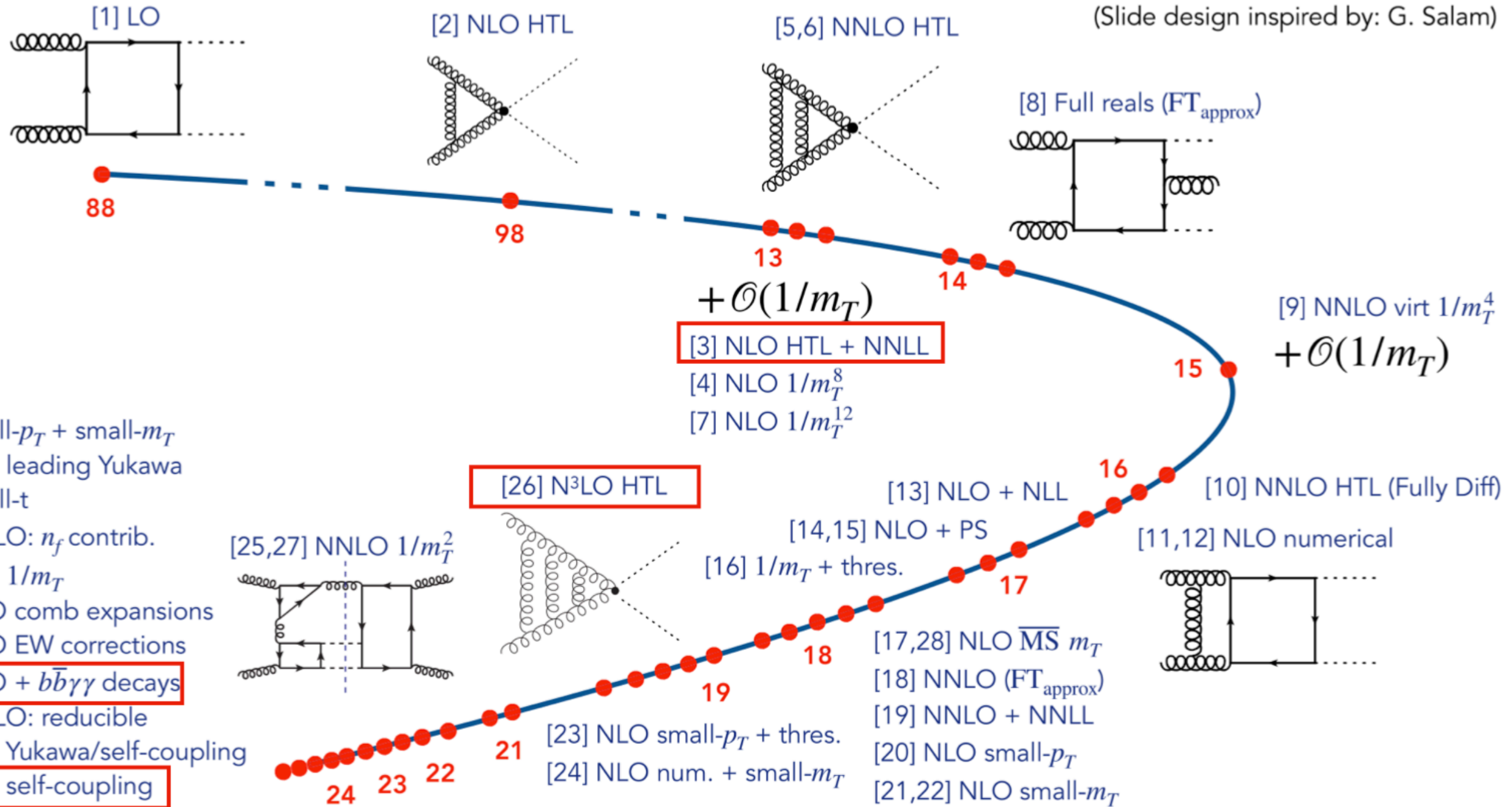






Perturbative corrections

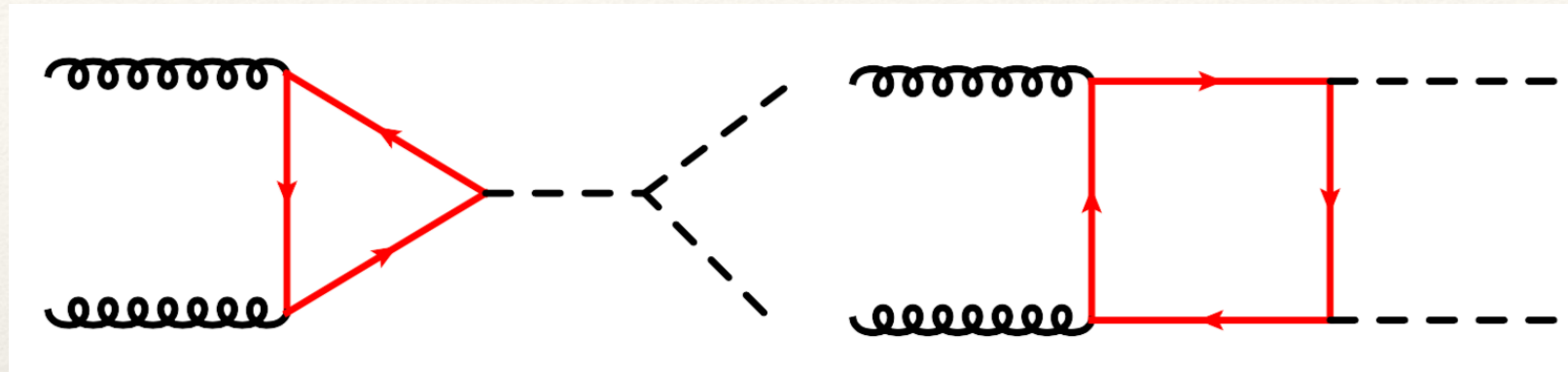
(Slide design inspired by: G. Salam)



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagnaschi, Degrassi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23; [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, SPJ, Kerner, Stone, Vestner; [39] Li, Si, Wang, Zhang, Zhao 24

taken from Spira's talk at Higgs 2024

gg → HH as a function of κ



$$\sigma_{HH} = A + B\kappa + C\kappa^2$$

computation	A [fb]	A/A(LO)	B [fb]	B/B(LO)	C [fb]	C/C(LO)
LO m_t fin	35.0		-23.0		4.73	
NLO m_t fin	62.6	1.79	-44.4	1.93	9.64	2.04
NLO m_t fin × NNLO SM FTApprox	70.0	2.00	-49.6	2.16	10.8	2.28
NNLO + NNLL $m_t \rightarrow \infty \times$						
NNLO+NLL SM (partial m_t fin)	71.3	2.04	-47.7	2.08	9.93	2.10

The meaning of κ

$$(\sigma \cdot \text{BR}) (gg \rightarrow H \rightarrow \gamma\gamma) = \sigma_{\text{SM}}(gg \rightarrow H) \cdot \text{BR}_{\text{SM}}(H \rightarrow \gamma\gamma) \cdot \frac{\kappa_g^2 \cdot \kappa_\gamma^2}{\kappa_H^2}$$

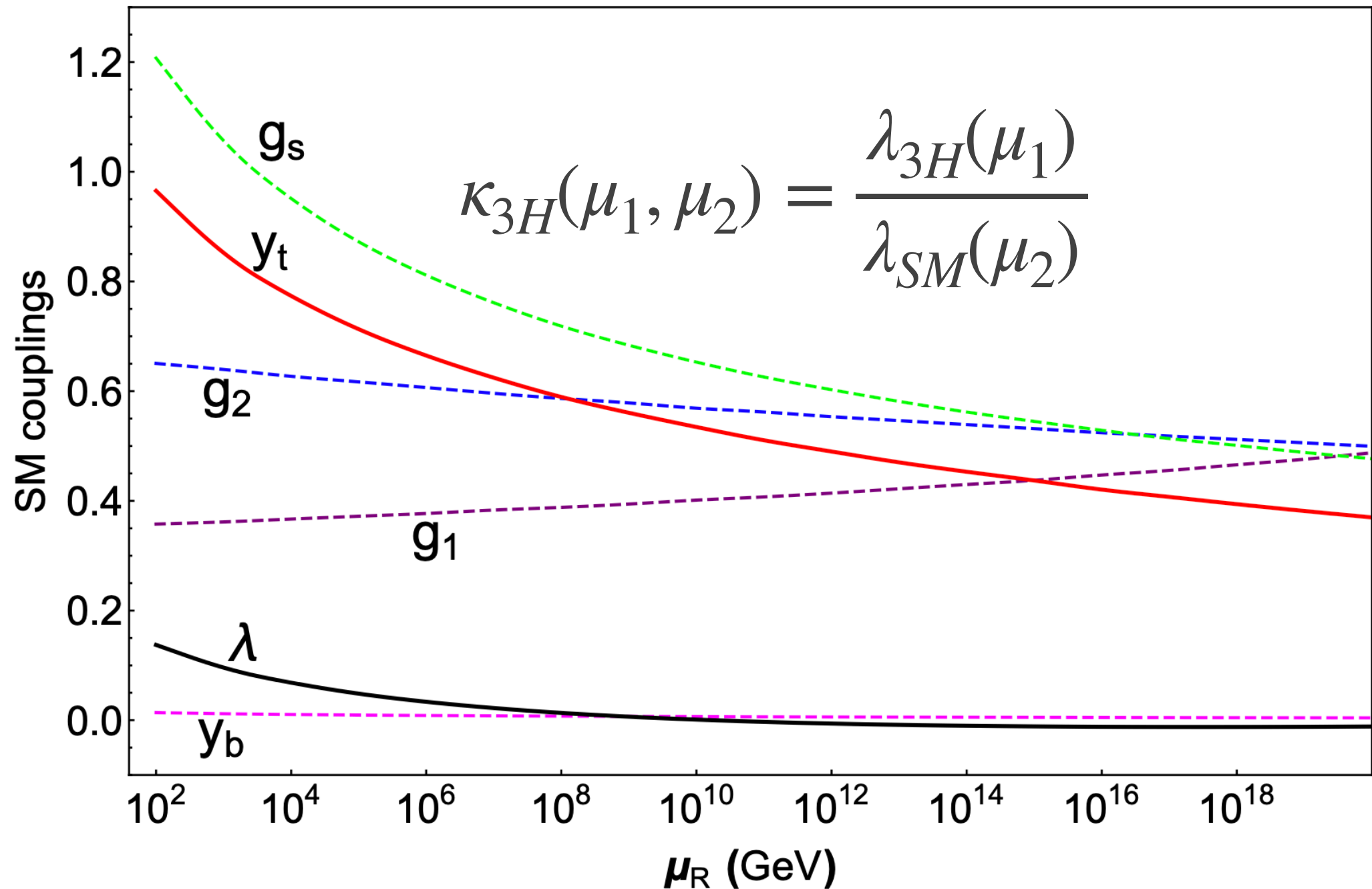
It can be considered as a rescaling factor of the corresponding parameter in the Lagrangian.

It is a ratio of two parameters in two Lagrangians ($\mathcal{L}_{NP}$ and $\mathcal{L}_{SM}$).

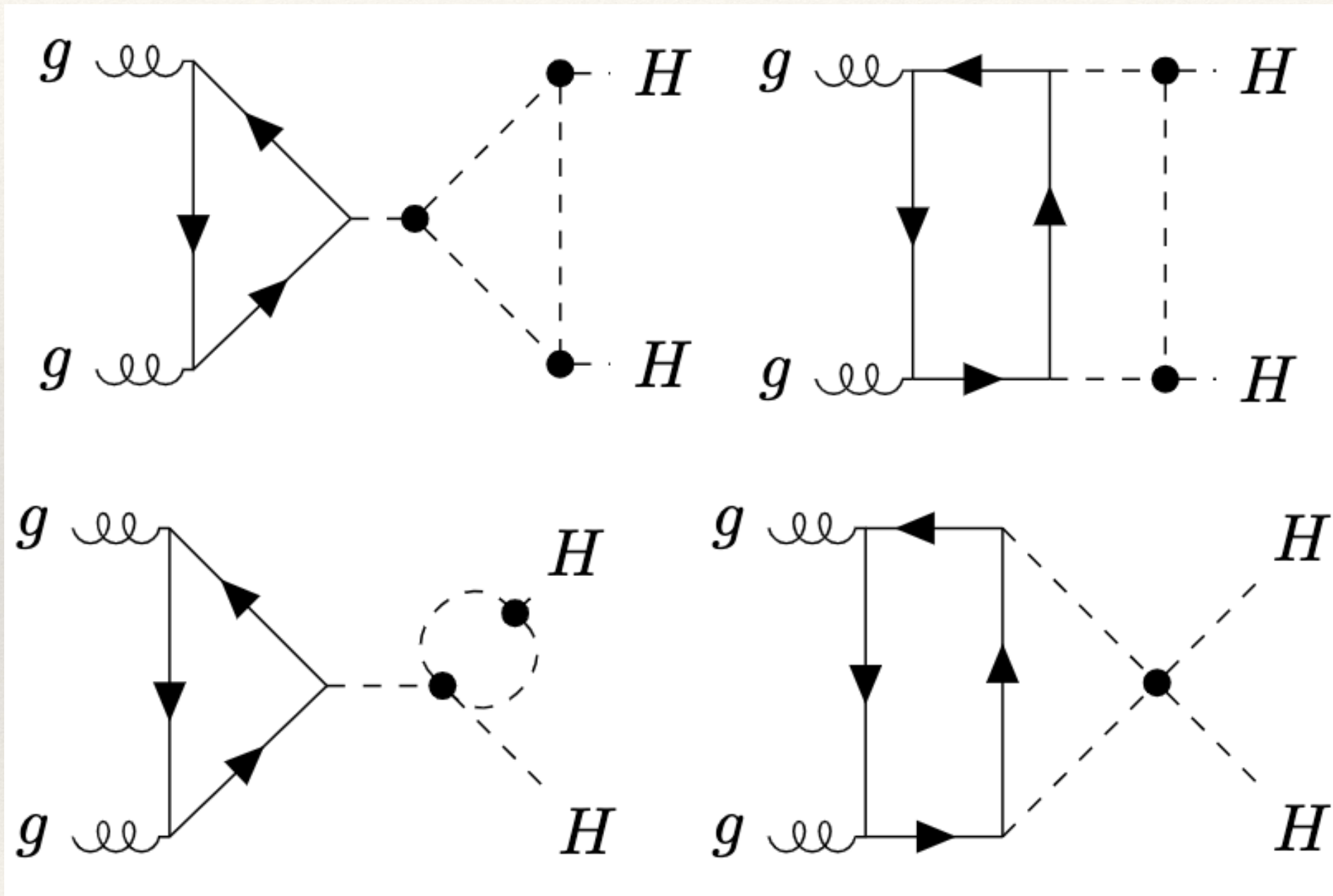
$$\kappa_{3H} = \frac{\lambda_{3H}}{\lambda_{SM}} \quad \kappa_{4H} = \frac{\lambda_{4H}}{\lambda_{SM}}$$

How to rescale the single parameter in $\mathcal{L}_{SM}$?

The meaning of κ



A more realistic function form



$$\sigma_{HH} = A + B\kappa_3 + C\kappa_3^2 + D\kappa_3^3 + E\kappa_3^4 + F\kappa_3^2\kappa_4 + G\kappa_3\kappa_4 + H\kappa_4$$

Non-trivial task

Input parameters of conventional EW calculations:

$$e, m_H, m_t, m_W, m_Z$$

If one takes the Higgs self-coupling λ as an input, the correction would be proportional to λ .

Performing rescaling $\lambda \rightarrow \kappa\lambda$ before or after substituting $m_H^2 = 2\lambda v^2$ gives different results.

EFT

- To describe the new physics beyond the SM, we often take a consistent effective field theory framework.
- SMEFT: Higgs field as a doublet, small Wilson coefficients
- HEFT: Higgs field as a singlet, non-linear, no explicit constraints on couplings

$$\mathcal{L}_{\text{HEFT}} = \mathcal{L}_2 + \mathcal{L}_4$$

$$\begin{aligned} \mathcal{L}_2 = & \frac{v^2}{4} \left(1 + 2a \frac{H}{v} + b \left(\frac{H}{v} \right)^2 + \dots \right) \text{Tr}[D_\mu U^\dagger D^\mu U] \\ & + \frac{1}{2} \partial_\mu H \partial^\mu H - V(H) - \frac{1}{2g^2} \text{Tr}[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}] \\ & - \frac{1}{2g'^2} \text{Tr}[\hat{B}_{\mu\nu} \hat{B}^{\mu\nu}] + \mathcal{L}_{GF} + \mathcal{L}_{FP}. \end{aligned}$$

$$U(\pi^a) = e^{i\pi^a \tau^a / v}$$

$$\begin{aligned} V(H) = & (-\mu^2 + \lambda v^2) v H + \frac{1}{2} (-\mu^2 + 3\lambda v^2) H^2 \\ & + \kappa_3 \lambda v H^3 + \kappa_4 \frac{\lambda}{4} H^4. \end{aligned}$$

EFT

$$\begin{aligned}
\mathcal{L}_4 = & -a_{dd\nu\nu 1} \frac{\partial^\mu H \partial^\nu H}{v^2} \text{Tr}[\mathcal{V}_\mu \mathcal{V}_\nu] - a_{dd\nu\nu 2} \frac{\partial^\mu H \partial_\mu H}{v^2} \text{Tr}[\mathcal{V}^\nu \mathcal{V}_\nu] \\
& + \left(a_{11} + a_{H11} \frac{H}{v} + a_{HH11} \frac{H^2}{v^2} \right) \text{Tr}[\mathcal{D}_\mu \mathcal{V}^\mu \mathcal{D}_\nu \mathcal{V}^\nu] - \frac{m_H^2}{4} \left(2a_{H\nu\nu} \frac{H}{v} + a_{HH\nu\nu} \frac{H^2}{v^2} \right) \text{Tr}[\mathcal{V}^\mu \mathcal{V}_\mu] \\
& - \left(a_{HWW} \frac{H}{v} + a_{HHWW} \frac{H^2}{v^2} \right) \text{Tr}[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}] + i \left(a_{d2} + a_{Hd2} \frac{H}{v} \right) \frac{\partial^\nu H}{v} \text{Tr}[\hat{W}_{\mu\nu} \mathcal{V}^\mu] \\
& + \left(a_{\square\nu\nu} + a_{H\square\nu\nu} \frac{H}{v} \right) \frac{\square H}{v} \text{Tr}[\mathcal{V}_\mu \mathcal{V}^\mu] + \left(a_{d3} + a_{Hd3} \frac{H}{v} \right) \frac{\partial^\nu H}{v} \text{Tr}[\mathcal{V}_\nu \mathcal{D}_\mu \mathcal{V}^\mu] \\
& + \left(a_{\square\square} + a_{H\square\square} \frac{H}{v} \right) \frac{\square H \square H}{v^2} + a_{dd\square} \frac{\partial^\mu H \partial_\mu H \square H}{v^3} \\
& + \left(a_{Hdd} \frac{m_H^2}{v^2} + a_{ddW} \frac{m_W^2}{v^2} + a_{ddZ} \frac{m_Z^2}{v^2} \right) \frac{H}{v} \partial^\mu H \partial_\mu H.
\end{aligned}$$

Brivio et al, JHEP, 1403, 024

Gavela, Kanshin, Machado, Saa, JHEP, 1503, 043

Renormalization:

- Background field method:
- Scattering amplitude:

Guo, Ruiz-Femenia, Sanz-Cillero, PRD92,074005(2015)

Buchalla, et al, NPB, 928,93 (2018), PRD104,076005(2021)

Herrero, Morales, PRD106,073008 (2022)

Renormalization

In the SM, the Lagrangian for the Higgs sector can be written as

$$\mathcal{L}_H = (D_\mu \phi_0)^\dagger (D^\mu \phi_0) + \mu_0^2 (\phi_0^\dagger \phi_0) - \lambda_0 (\phi_0^\dagger \phi_0)^2$$

where ϕ_0 denotes the bare Higgs doublet and D_μ is the covariant derivative. The relations between the bare fields and couplings, and their renormalized counterparts, are given by $\phi_0 = Z_\phi^{1/2} \phi$, $\mu_0^2 = Z_\mu^2 \mu^2$, and $\lambda_0 = Z_\lambda \lambda$.

The EW gauge symmetry is spontaneously broken once the Higgs field develops a non-vanishing vacuum expectation value v . Taking the unitary gauge, we write the Higgs field as

$$\phi = \frac{1}{\sqrt{2}} (0, Z_v v + H)^T$$

Our strategy is equivalent to the application of HEFT in Higgs boson pair production.

Renormalization

The renormalized Lagrangian in the κ framework after EW gauge symmetry breaking:

$$\begin{aligned} \mathcal{L}_H^\kappa = & \frac{1}{2}Z_\phi(\partial_\mu H)^2 - \left(-\frac{1}{2}Z_{\mu^2}Z_\phi Z_\nu^2\mu^2v^2 + \frac{1}{4}Z_\lambda Z_\phi^2 Z_\nu^4\lambda v^4 \right) - (Z_\lambda Z_\phi^2 Z_\nu^3\lambda v^3 - Z_{\mu^2}Z_\phi Z_\nu\mu^2v)H \\ & - \left(\frac{3}{2}Z_\lambda Z_\phi^2 Z_\nu^2\lambda v^2 - \frac{1}{2}Z_{\mu^2}Z_\phi\mu^2 \right) H^2 - Z_{\kappa_{3H}}Z_\lambda Z_\phi^2 Z_\nu\lambda_{3H}vH^3 - \frac{1}{4}Z_{\kappa_{4H}}Z_\lambda Z_\phi^2\lambda_{4H}H^4 + \dots \end{aligned}$$

The linear term is

$$(\mu^2v - \lambda v^3)H + [(\delta Z_{\mu^2} + \delta Z_\phi + \delta Z_\nu)\mu^2v - (\delta Z_\lambda + 2\delta Z_\phi + 3\delta Z_\nu)\lambda v^3]H$$

We choose the renormalization scheme in which there is no tadpole contributions.

$\mu^2 = \lambda v^2$ and $(\delta Z_{\mu^2} - \delta Z_\lambda - \delta Z_\phi - 2\delta Z_\nu)\mu^2v + T = 0$ with T the one-loop diagrams.

$$T = \frac{3\lambda_{3H}v}{16\pi^2}m_H^2 \left(\frac{1}{\epsilon} + \ln \frac{\mu_R^2}{m_H^2} + 1 \right)$$

Renormalization

The quadratic term is

$$\begin{aligned} & \frac{1}{2}(\partial_\mu H)^2 - \mu^2 H^2 + \frac{1}{2}\delta Z_\phi(\partial_\mu H)^2 - \left(\frac{3}{2}\delta Z_\lambda + \frac{5}{2}\delta Z_\phi - \frac{1}{2}\delta Z_{\mu^2} + 3\delta Z_\nu \right) \mu^2 H^2 \\ & \equiv \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}m_H^2 H^2 + \frac{1}{2}\delta Z_\phi(\partial_\mu H)^2 - \frac{1}{2}(\delta Z_{m_H^2} + \delta Z_\phi)m_H^2 H^2 \end{aligned}$$

We choose the on-shell renormalization scheme.

$$\begin{aligned} \delta Z_{m_H^2} &= \frac{3\lambda_{4H}}{16\pi^2} \left(\frac{1}{\epsilon} + \ln \frac{\mu_R^2}{m_H^2} + 1 \right) + \frac{9\lambda_{3H}^2 v^2}{m_H^2} \frac{1}{8\pi^2} \left(\frac{1}{\epsilon} + \ln \frac{\mu_R^2}{m_H^2} + 2 - \frac{\pi}{\sqrt{3}} \right) \\ \delta Z_\phi &= \frac{9\lambda_{3H}^2 v^2}{8\pi^2} \frac{\sqrt{3} - 2\pi/3}{\sqrt{3}m_H^2} \end{aligned}$$

Since we focus on the corrections induced by the Higgs self-couplings, we can simply take

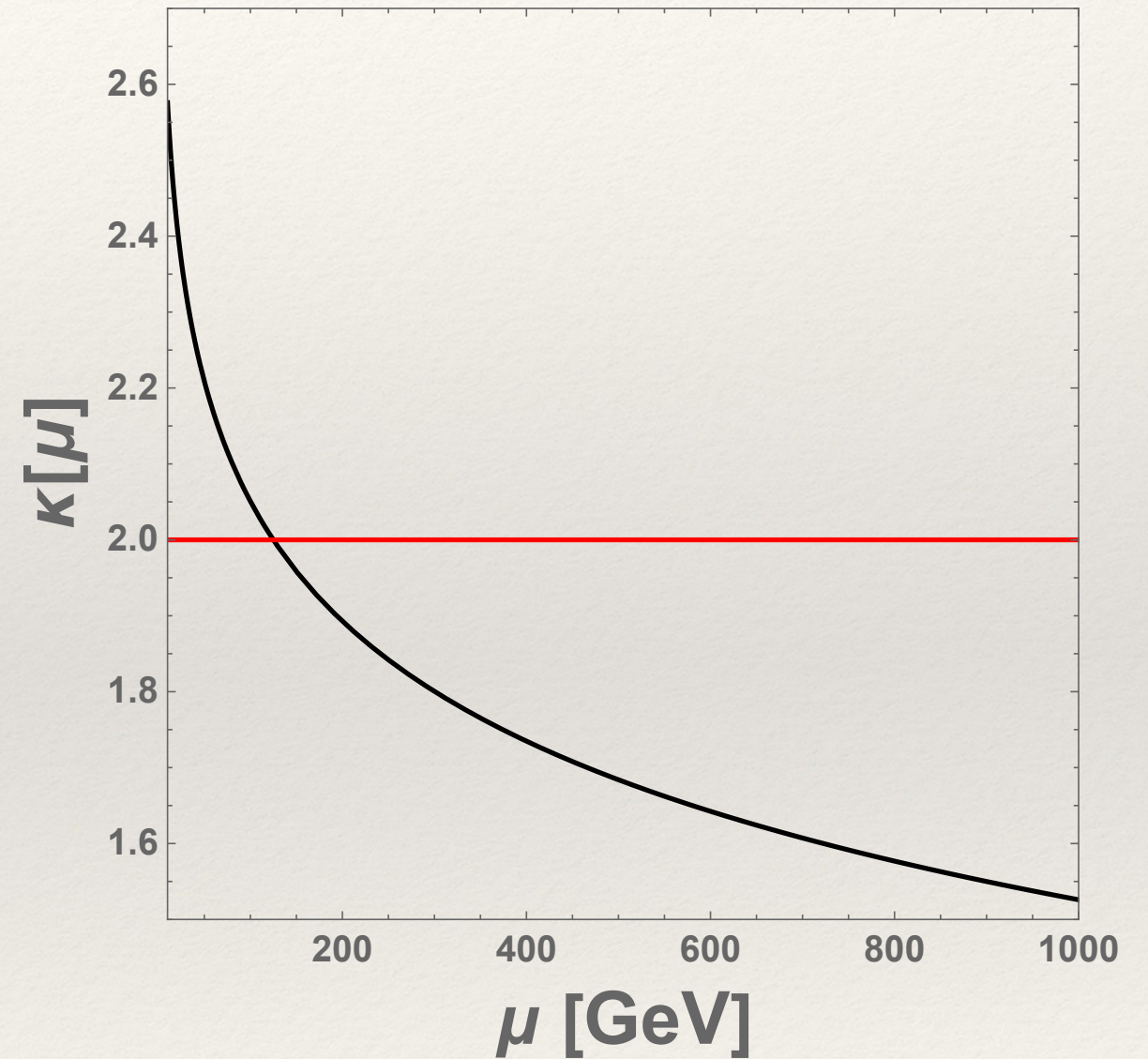
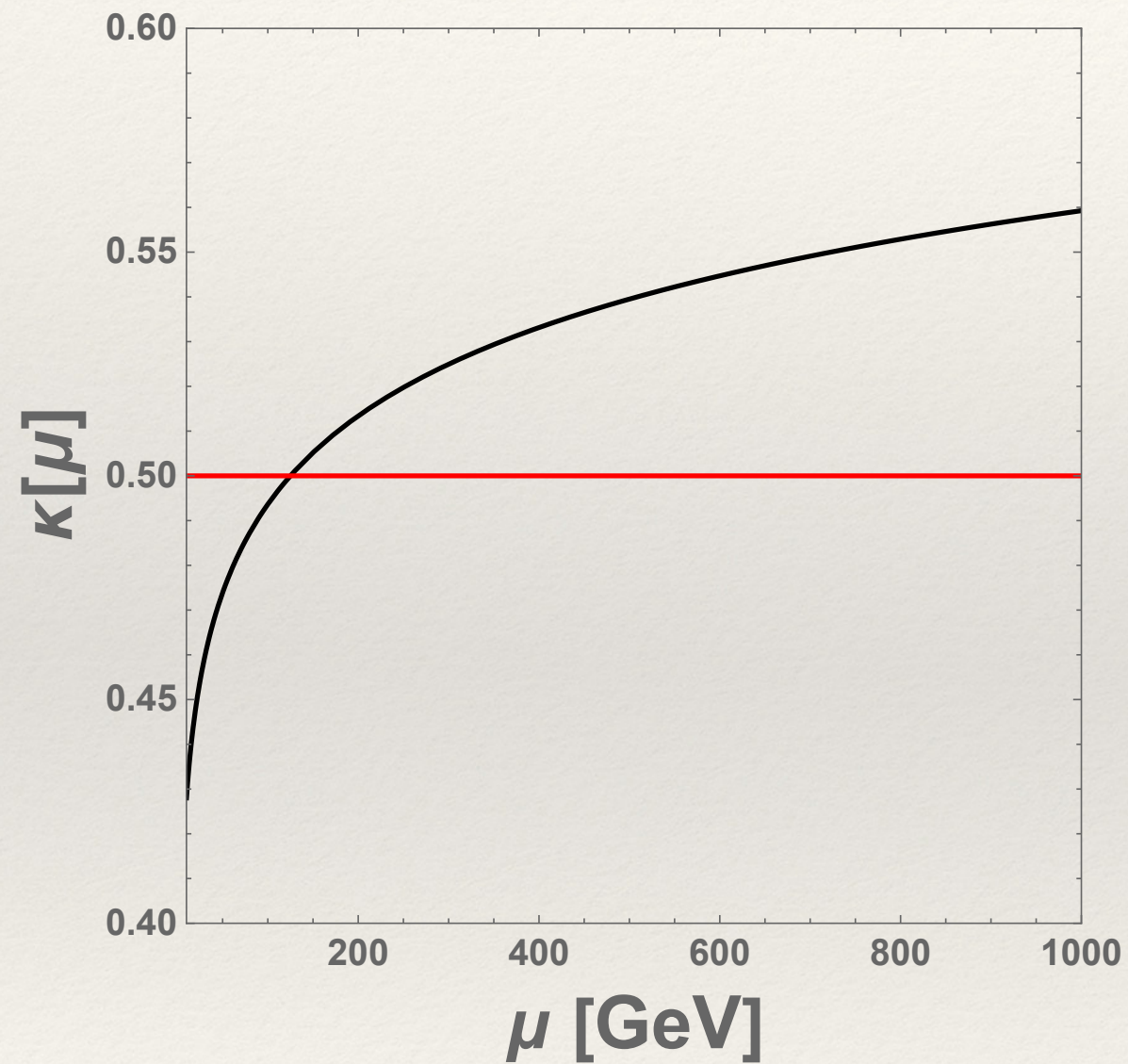
$$\delta Z_\nu + \delta Z_\phi/2 = 0$$

Renormalization

The result of one-particle reducible diagrams and counter-terms:

$$\begin{aligned} \mathcal{M}_{gg \rightarrow H^* \rightarrow HH}^{\text{LO}} \times & \left\{ \frac{3}{16\pi^2} \frac{1}{\epsilon} \left(-2\lambda_{4H} - \lambda_{3H} + 6\lambda_{3H}^2 \frac{v^2}{m_H^2} \right) + \delta Z_{\kappa_{3H}} \right. \\ & + \frac{3}{16\pi^2} \ln \frac{\mu_R^2}{m_H^2} \left[-2\lambda_{4H} - \lambda_{3H} + 6\lambda_{3H}^2 \frac{v^2}{m_H^2} \right] \\ & - \frac{9\lambda_{3H}^2}{8\pi^2} \frac{v^2}{s - m_H^2} \left[\beta \left(\ln \left(\frac{1 - \beta}{1 + \beta} \right) + i\pi \right) + \frac{s}{m_H^2} \left(1 - \frac{2\pi}{3\sqrt{3}} \right) + \frac{5\pi}{3\sqrt{3}} - 1 \right] \\ & + \frac{3\lambda_{3H}^2}{8\pi^2} \frac{v^2}{m_H^2} \left(12 - \frac{7\pi}{\sqrt{3}} \right) - \frac{9\lambda_{3H}^2 v^2}{4\pi^2} C_0[m_H^2, m_H^2, s, m_H^2, m_H^2, m_H^2] \\ & \left. - \frac{3\lambda_{4H}}{16\pi^2} \left[\beta \left(\ln \left(\frac{1 - \beta}{1 + \beta} \right) + i\pi \right) + 5 - \frac{2\pi}{\sqrt{3}} \right] - \frac{3\lambda_{3H}}{16\pi^2} \right\}, \end{aligned}$$

Running of κ



Updated function forms

The λ dependent correction is

$$\delta\sigma_{\text{ggF,EW}}^{\kappa_\lambda} = (0.075\kappa_{\lambda_{3H}}^4 - 0.158\kappa_{\lambda_{3H}}^3 - 0.006\kappa_{\lambda_{3H}}^2 \kappa_{\lambda_{4H}} - 0.058\kappa_{\lambda_{3H}}^2 + 0.070\kappa_{\lambda_{3H}} \kappa_{\lambda_{4H}} - 0.149\kappa_{\lambda_{4H}}) \text{ fb}$$

$$\delta\sigma_{\text{VBF,EW}}^{\kappa_\lambda} = (0.0215\kappa_{\lambda_{3H}}^4 - 0.0324\kappa_{\lambda_{3H}}^3 - 0.0019\kappa_{\lambda_{3H}}^2 \kappa_{\lambda_{4H}} - 0.0043\kappa_{\lambda_{3H}}^2 + 0.0151\kappa_{\lambda_{3H}} \kappa_{\lambda_{4H}} - 0.0211\kappa_{\lambda_{4H}}) \text{ fb}$$

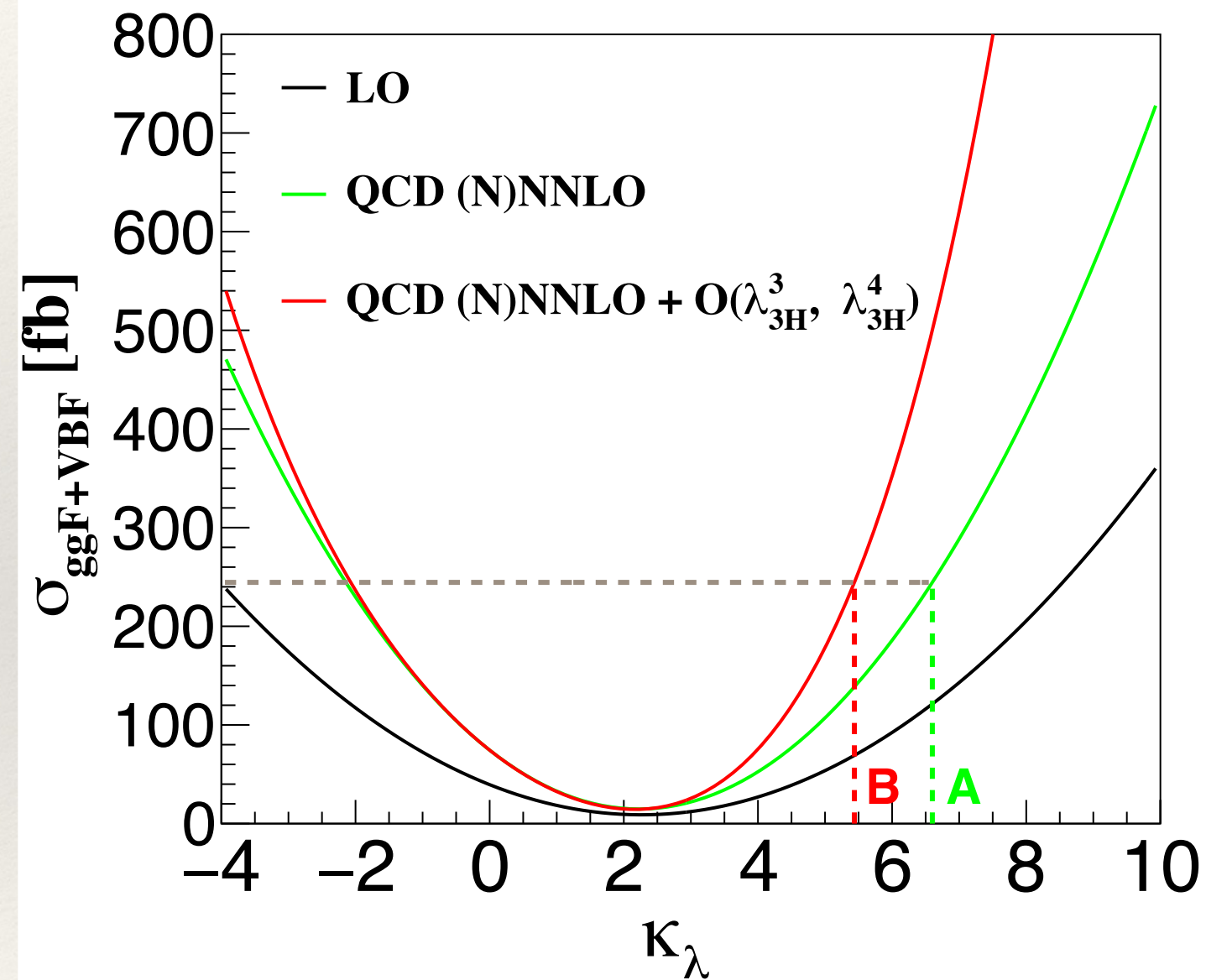
$\kappa_{\lambda_{3H}}$	$\kappa_{\lambda_{4H}}$	ggF			VBF		
		$\sigma_{\text{LO}}^{\kappa_\lambda}$	$\sigma_{\text{NNLO-FT}}^{\kappa_\lambda}$	$\delta\sigma_{\text{EW}}^{\kappa_\lambda}$	$\sigma_{\text{LO}}^{\kappa_\lambda}$	$\sigma_{\text{NNLO}}^{\kappa_\lambda}$	$\delta\sigma_{\text{EW}}^{\kappa_\lambda}$
1	1	16.7	31.2	-0.225	1.71	1.69	-2.30×10^{-2}
3	1	8.59	18.4	1.28	3.59	3.53	8.35×10^{-1}
6	1	67.3	161	60.6	25.1	24.6	20.7
1	3	16.7	31.2	-0.393	1.71	1.69	-3.89×10^{-2}
1	6	16.7	31.2	-0.646	1.71	1.69	-6.27×10^{-2}
3	3	8.59	18.4	1.30	3.59	3.53	8.50×10^{-1}
6	6	67.3	161	61.0	25.1	24.6	20.7

The QCD corrections are significant in ggF, but not sensitive to κ_{3H} .

The EW corrections are 91% (82%) in ggF (VBF) for $\kappa_{3H} = 6$.

The dependence on λ_{4H} is weak.

More stringent constraint

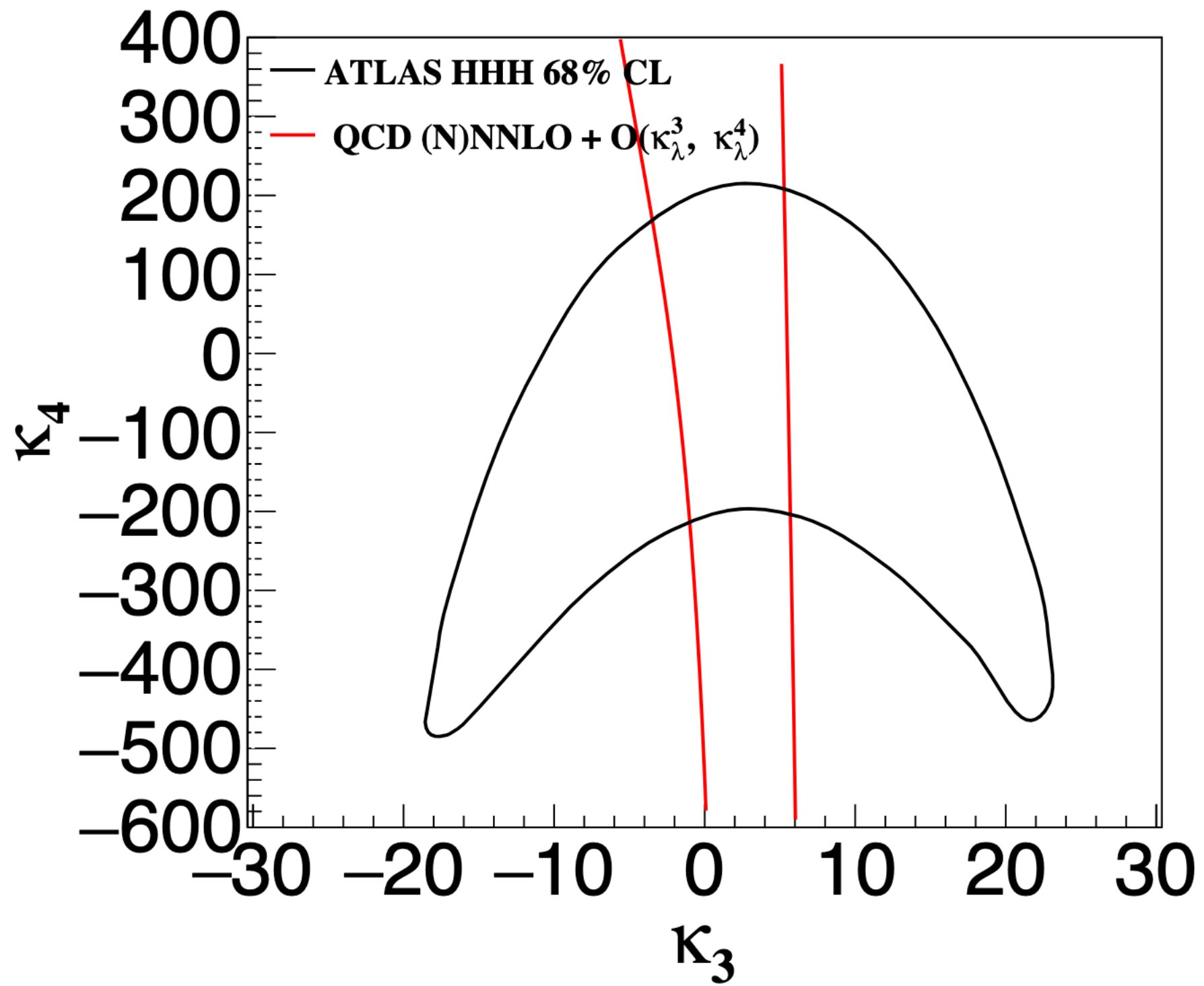


ATLAS (CMS) limit

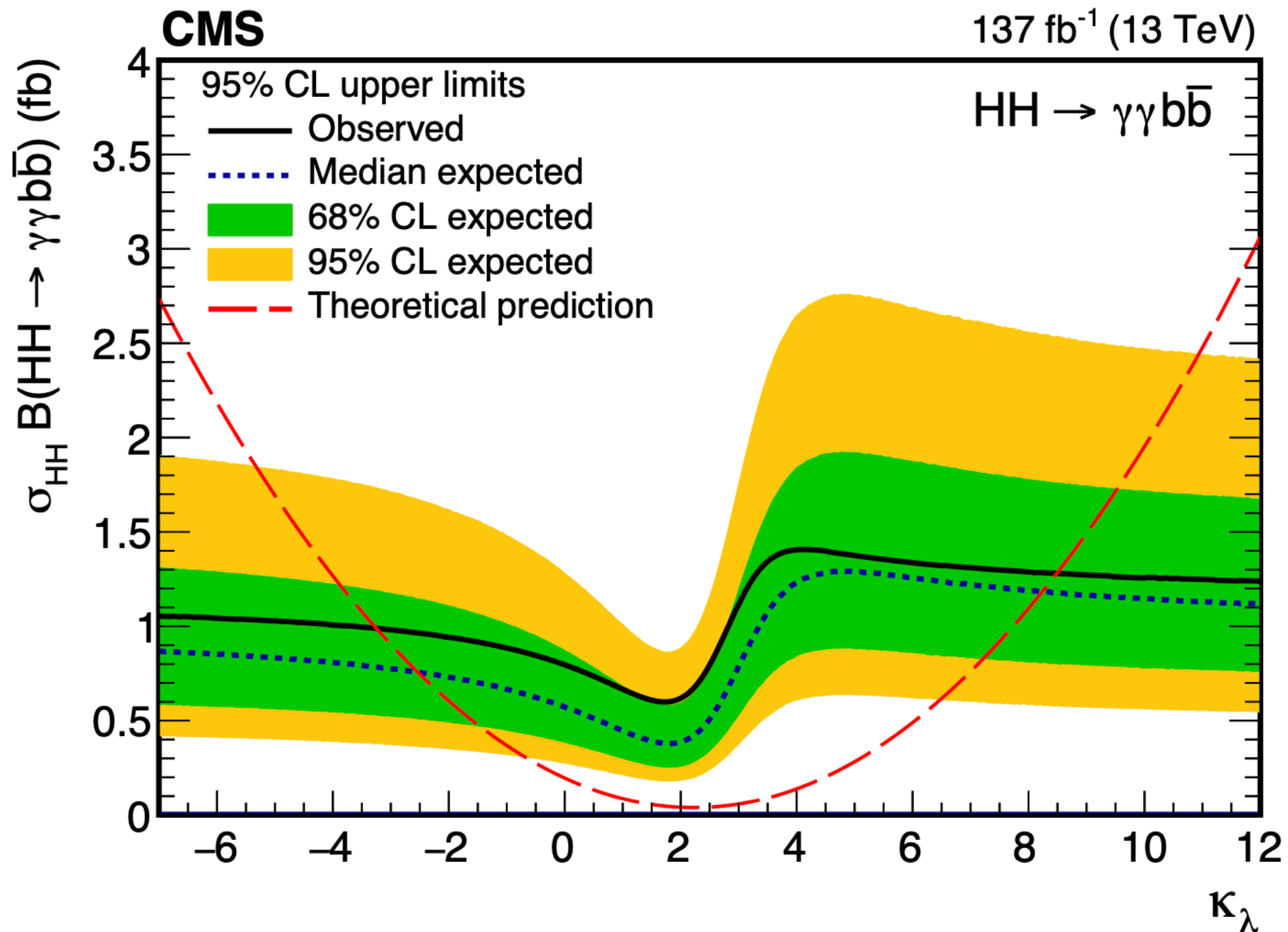
6.6 (6.49)



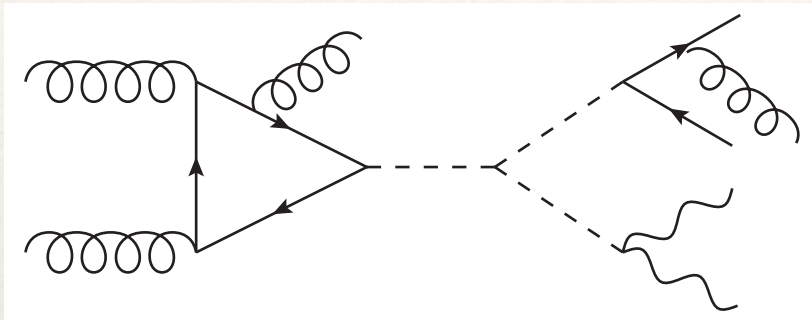
5.4 (5.37)



Why does the exp. bound depend κ_λ ?



$$gg \rightarrow HH \rightarrow b\bar{b}\gamma\gamma$$



$$p_T \geq 25 \text{ GeV}, \quad |\eta| \leq 2.5,$$

$$90 \text{ GeV} \leq m_{jj} \leq 190 \text{ GeV}$$

$$\sigma_{\text{pro+dec}(b\bar{b},\gamma\gamma)}^{\text{dec}(1)} = \sigma_{\text{pro}}^{(0)} \frac{1}{\Gamma_{H_1 \rightarrow b\bar{b}}^{(0)}} \frac{1}{\Gamma_{H_2 \rightarrow \gamma\gamma}^{(0)}} \Gamma_{\text{dec}(b\bar{b},\gamma\gamma)}^{(0)} \left(\frac{\int d\Gamma_{H_1 \rightarrow b\bar{b}}^{(1)} F_J}{\int d\Gamma_{H_1 \rightarrow b\bar{b}}^{(0)} F_J} - \frac{\Gamma_{H_1 \rightarrow b\bar{b}}^{(1)}}{\Gamma_{H_1 \rightarrow b\bar{b}}^{(0)}} \right) \times R(H_1 \rightarrow b\bar{b}) R(H_2 \rightarrow \gamma\gamma).$$

F_J is the measurement function.

For inclusive observables, no QCD corrections in decay are needed.

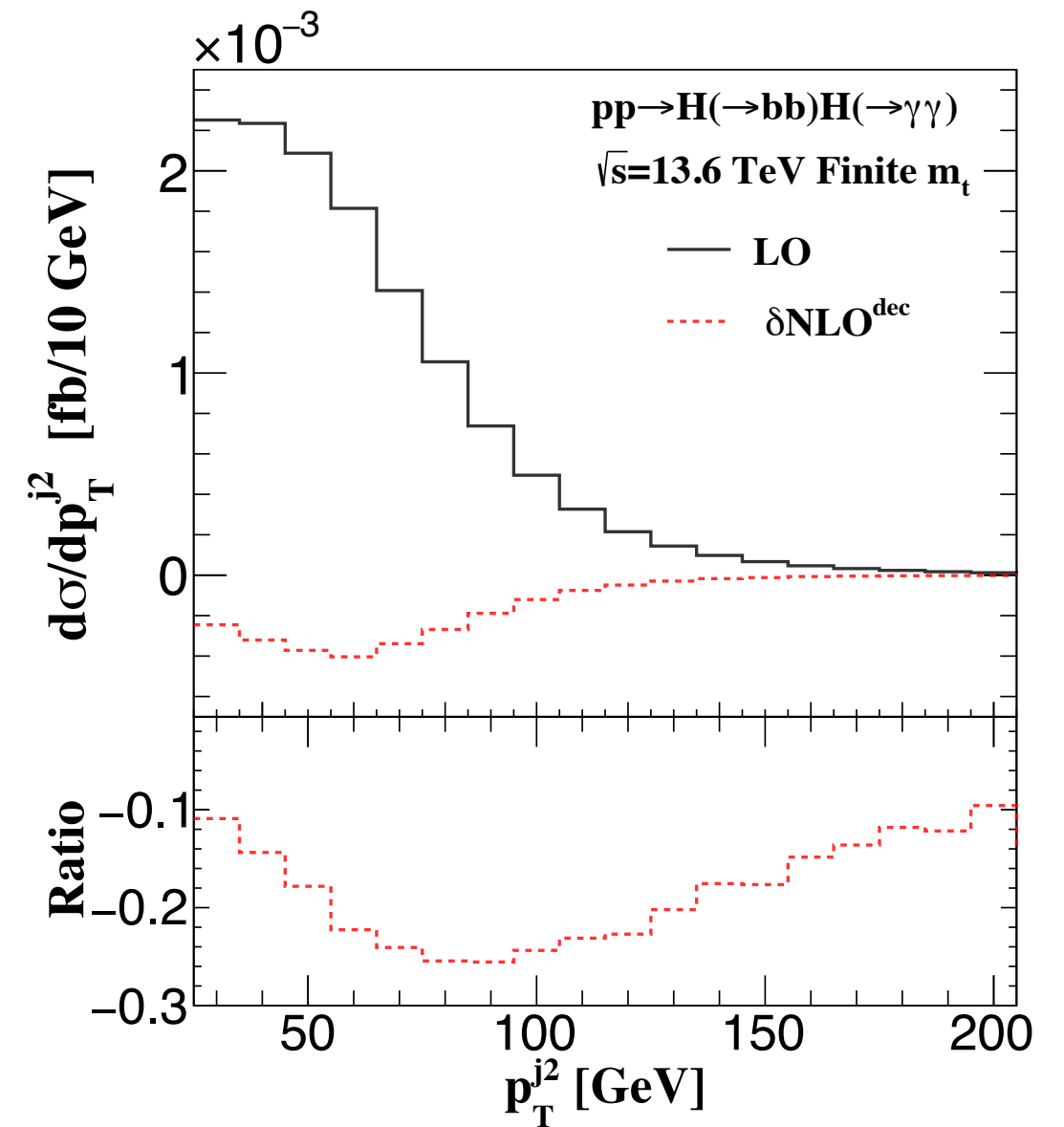
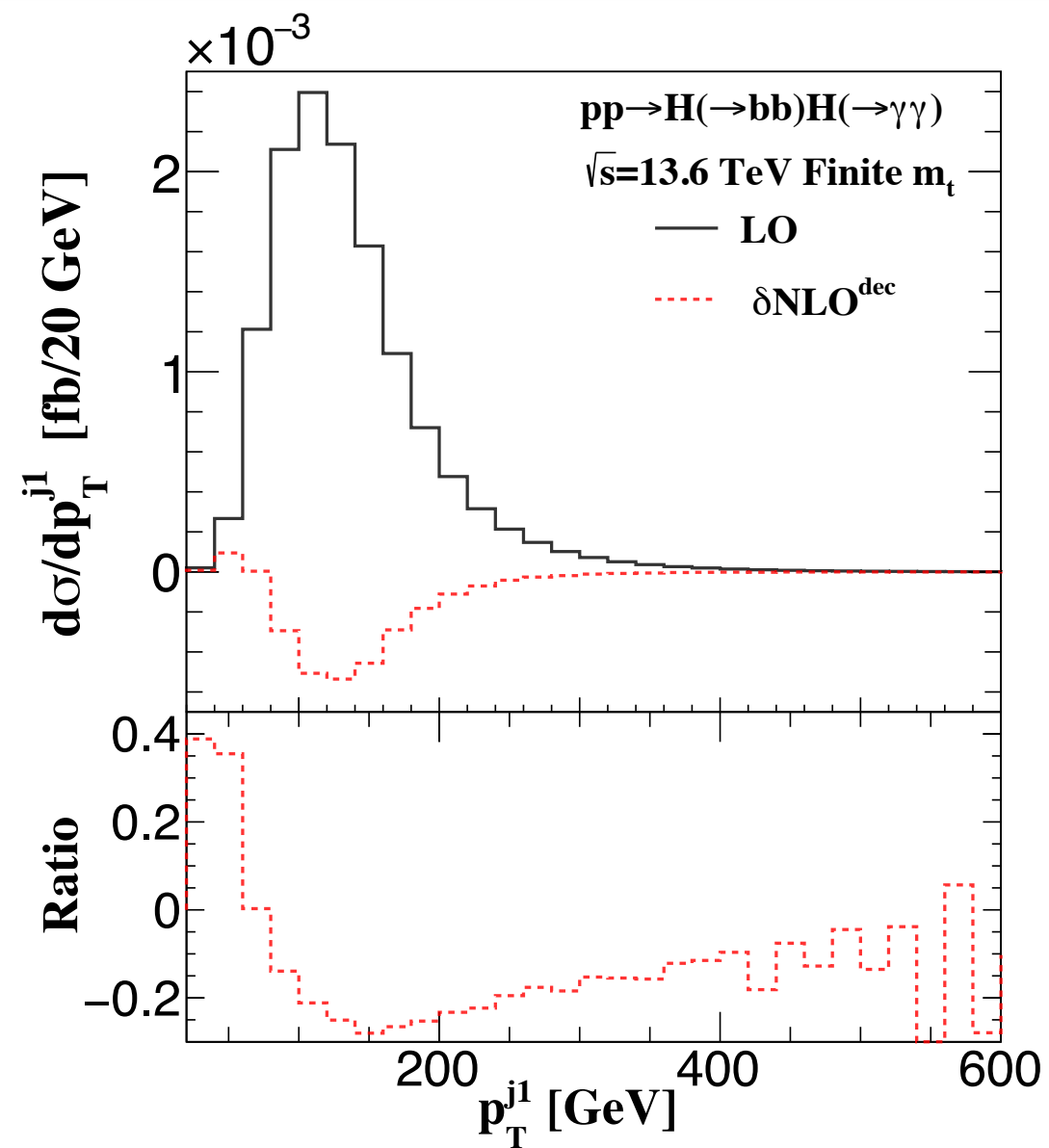
$$R(H \rightarrow b\bar{b}) = 58.2 \%, \quad R(H \rightarrow \gamma\gamma) = 0.23 \%$$

$$gg \rightarrow HH \rightarrow b\bar{b}\gamma\gamma$$

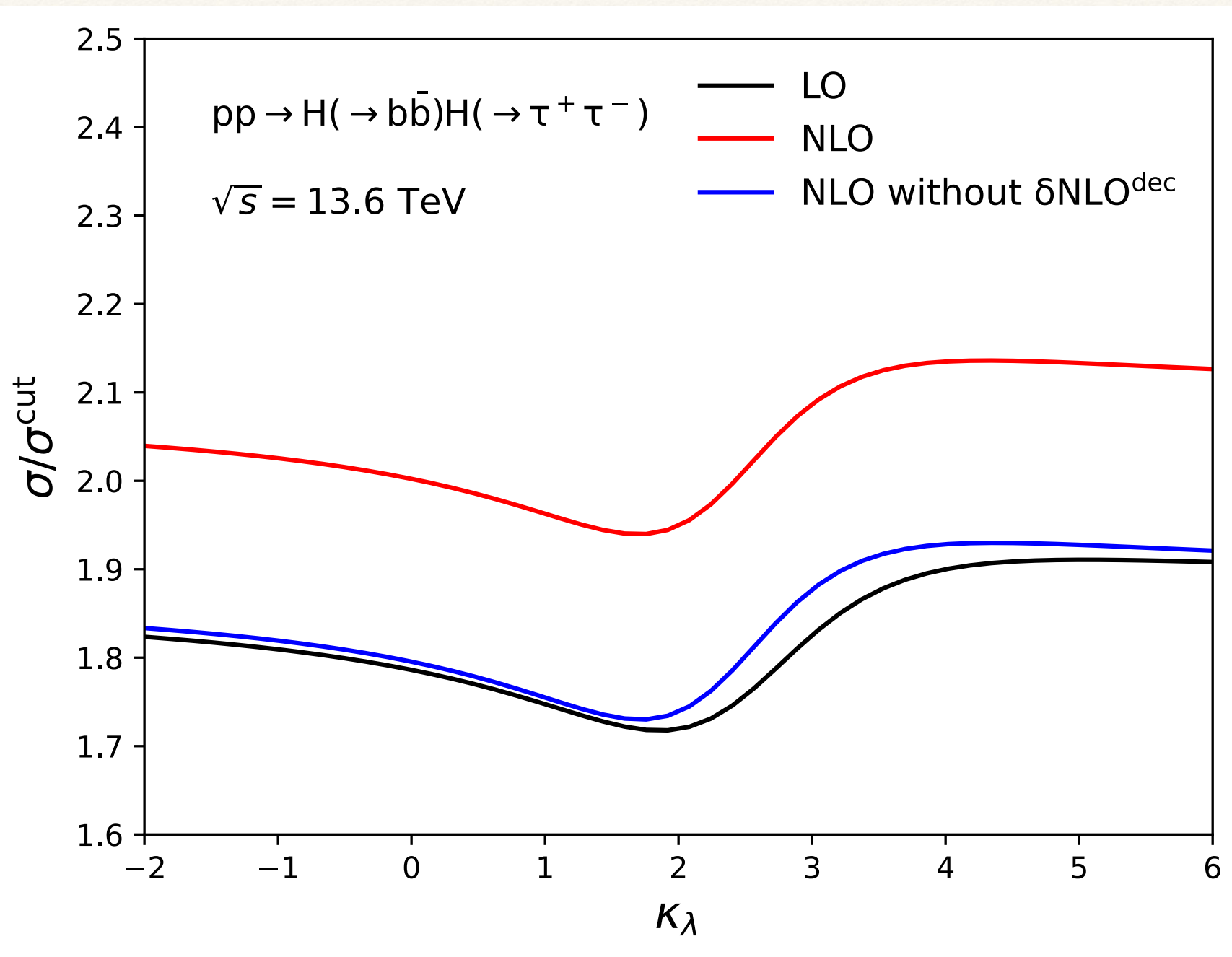
	without decays	with decays but no cuts		with decays and cuts	
		LO ^{dec}	δ NLO ^{dec}	LO ^{dec}	δ NLO ^{dec}
LO _{∞} ^{pro}	17.07 ^{+31%} _{-22%}	0.02257 ^{+31%} _{-22%}	0	0.01257 ^{+30%} _{-22%}	-0.00175 ^{+42%} _{-28%}
LO _{m_t} ^{pro}	19.85 ^{+28%} _{-20%}	0.02624 ^{+28%} _{-20%}	0	0.01395 ^{+27%} _{-20%}	-0.00261 ^{+39%} _{-27%}
δ NLO _{∞} ^{pro}	14.86 ^{+6%} _{-7%}	0.01964 ^{+6%} _{-7%}	—	0.01064 ^{+6%} _{-7%}	—
δ NLO _{m_t} ^{pro}	13.08 ^{+4%} _{-8%}	0.01729 ^{+4%} _{-8%}	—	0.00914 ^{+4%} _{-8%}	—
Full NLO result					
NLO _{∞}	31.93 ^{+18%} _{-15%}	0.04221 ^{+18%} _{-15%}		0.02146 ^{+15%} _{-14%}	
NLO _{m_t}	32.93 ^{+14%} _{-13%}	0.04354 ^{+14%} _{-13%}		0.02047 ^{+10%} _{-11%}	

- Only 53% events pass the cuts!
- QCD corrections reduce the cross section by 19%!

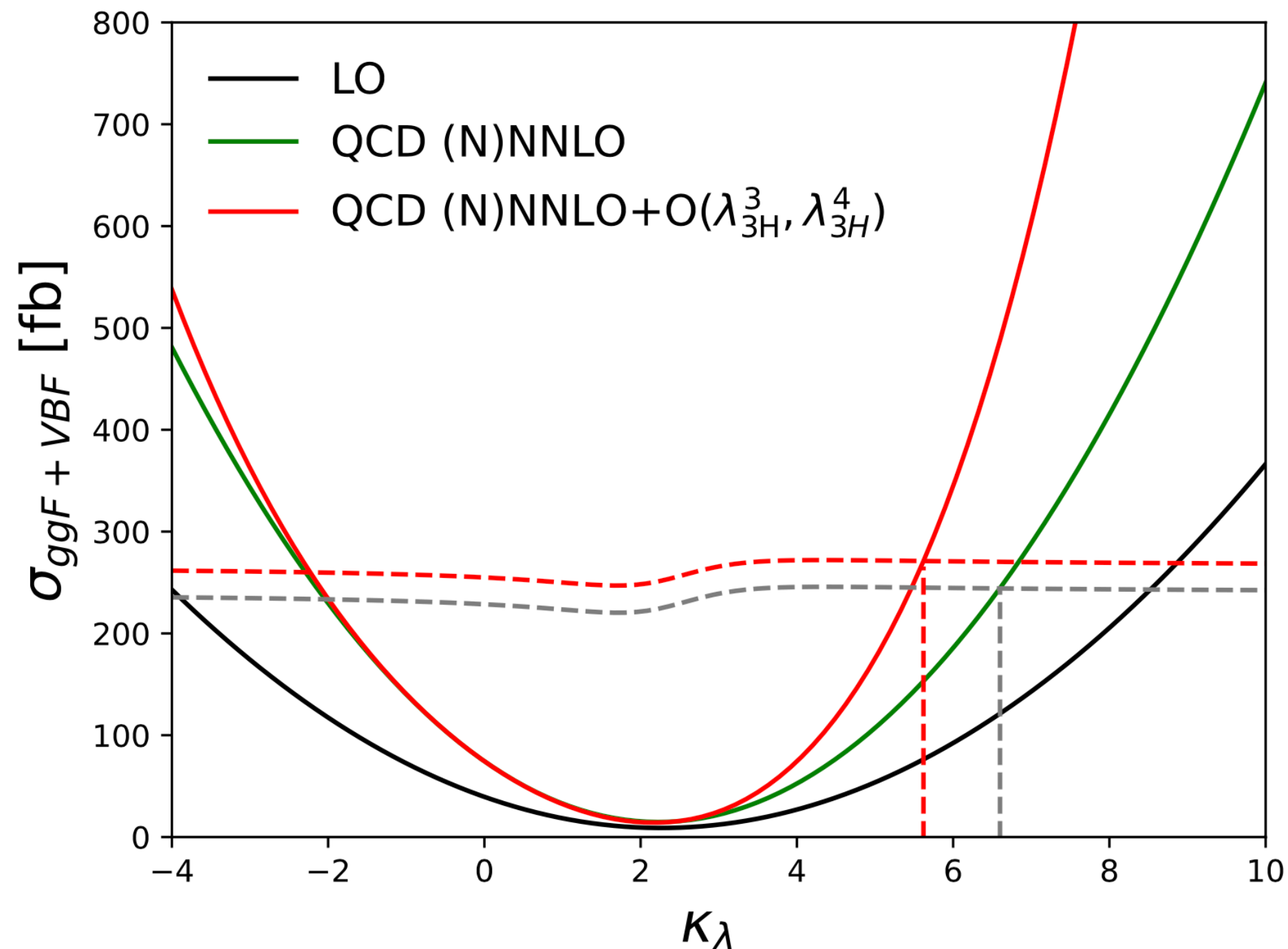
$$gg \rightarrow HH \rightarrow b\bar{b}\gamma\gamma$$



Signal acceptance

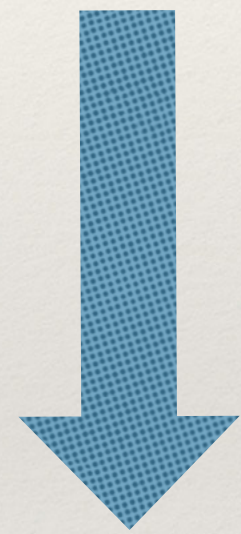


Imposing constraint



ATLAS limit

6.6



5.6

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2402.00401, 2503.22001

Summary

The SM is a master piece in human history. It has been tested by a lot of experiments at very high precision level.

However, the Higgs sector still needs more precise comparison between theories and experiments.

Higher-order corrections provide more precise estimate of the dependence on Higgs self-couplings.

Thanks a lot for your attention!