Quantum Entanglement of Light Quarks Probed by TMD Fragmentation

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TeV物理前沿专题研讨会暨第31届LHC Mini-Workshop, 12 October, 2025

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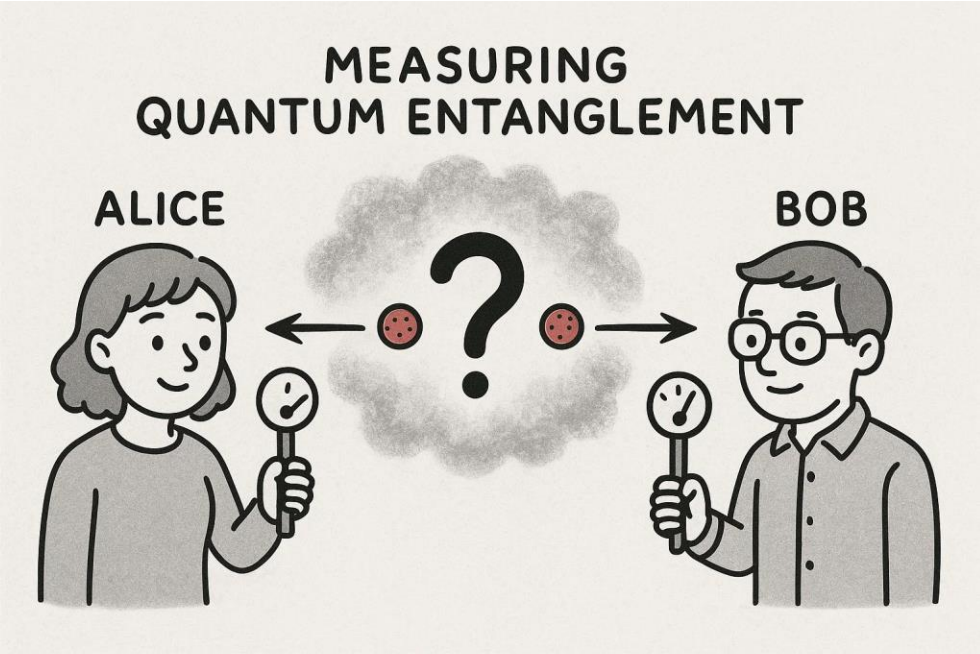
OVERVIEW

Quantum Entanglement

Quantum Entanglement : nonlocal correlation between quantum systems



O_A $\qquad$ O_B

$$\langle O_A \otimes O_B \rangle \stackrel{?}{=} \sum_i p_i \langle O_A \rangle_i \langle O_B \rangle_i$$

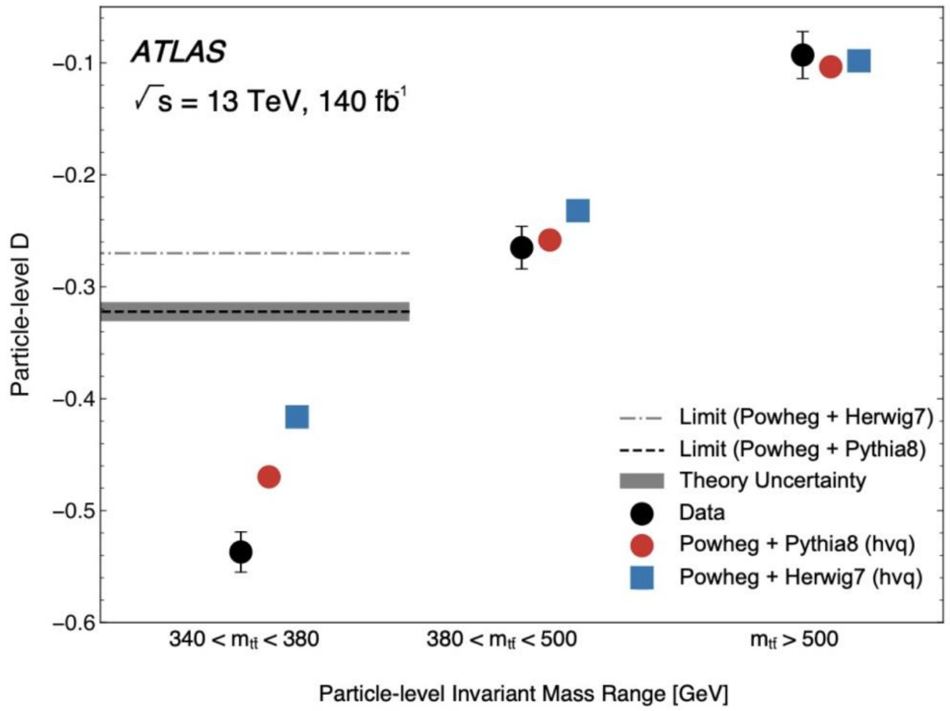
Classical Probability Theory: ✔

Quantum Theory: $\begin{cases} \rho = \sum_i p_i \, \rho_i^A \otimes \rho_i^B & \text{✔ (separable)} \\ \text{otherwise} & \text{✘ (entangled)} \end{cases}$

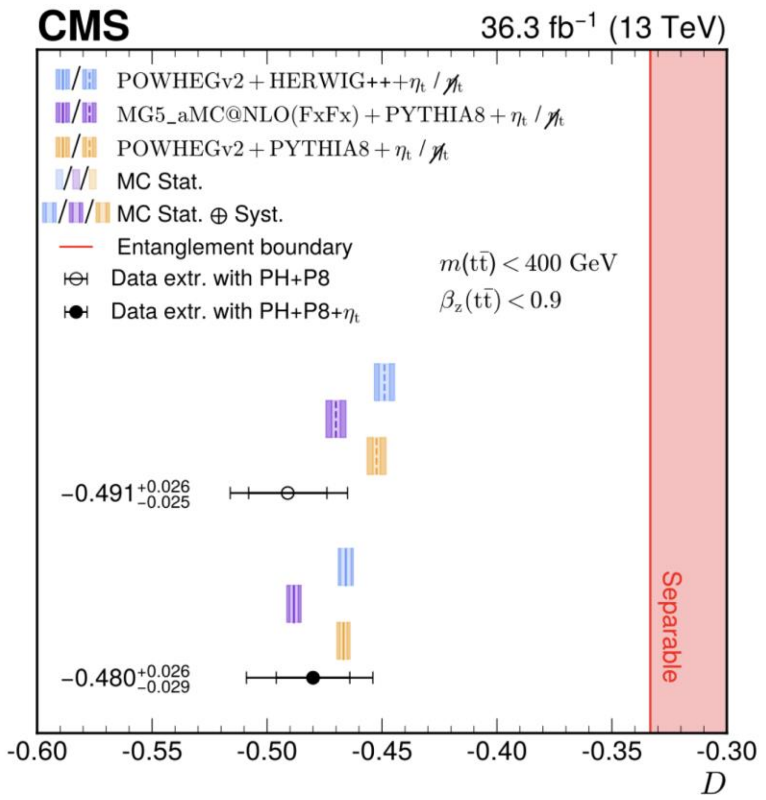
Entanglement is one of the most fundamental and non-classical features of quantum mechanics.

Quantum Entanglement at Colliders

ATLAS, Nature 633, 542 (2024)



CMS, Rept. Prog. Phys. 87 (2024) 117801



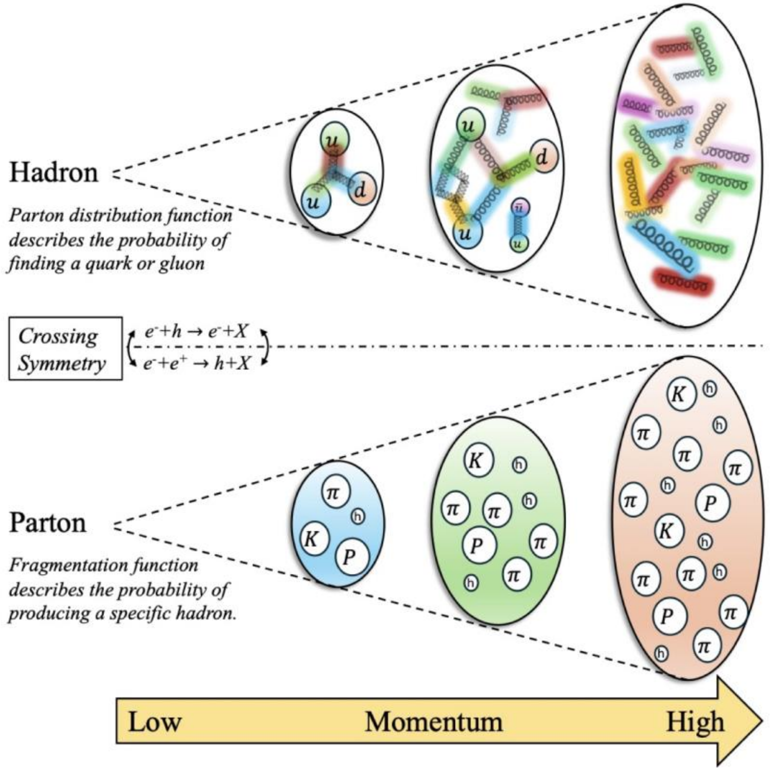
The LHC probed the entanglement of top quark pairs via their decay:

$$\mathrm{d}\sigma_{i\to t\bar{t}\to f} \propto \sum_{ss',\bar{s}\bar{s}'} \rho^{t\bar{t}}_{ss',\bar{s}\bar{s}'} \Gamma^{t}_{ss'} \Gamma^{\bar{t}}_{\bar{s}\bar{s}'}, \qquad \text{where} \quad \Gamma^{t}_{ss'} = \mathcal{M}_{t_s\to f}\mathcal{M}^{*}_{t_{s'}\to f}$$

This decay approach is only available for unstable particles ...

TMD Fragmentation and Quark Polarization

PDF and FF



Leading Quark TMDFFs

Hadron Spin — Quark Spin

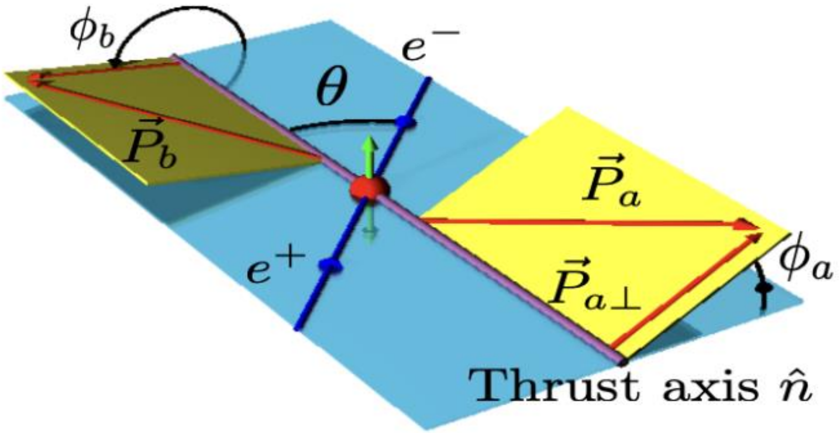
		Quark Polarization		
		Un-Polarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
Unpolarized (or Spin 0) Hadrons		D_1 = Unpolarized		$H_1^\perp$ = − Collins
Polarized Hadrons	**L**		G_1 = − Helicity	$H_{1L}^\perp$ = −
	T	$D_{1T}^\perp$ = − Polarizing FF	$G_{1T}^\perp$ = −	H_1 = − Transversity $H_{1T}^\perp$ = −

The spin information of light quarks can transfer to the hadrons: e.g. Collins functions

SIA with TMD Fragmentation

Consider the SIA process with transverse momentum:

$$e^+e^- \to \gamma^* \to q\bar{q} \to h_a + h_b + X$$



Collins Function

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_a \mathrm{d}\Omega_b} \sim 1 + \left(\frac{C_{xx} - C_{yy}}{2} \times \mathcal{F}_h \frac{H_1^{\perp,q} H_1^{\perp,\bar{q}}}{D_1^q D_1^{\bar{q}}}\right) \cos(\phi_a + \phi_b)}$$

Unpolarized Function

where, $\mathcal{F}_h = \dfrac{P_{a\perp} P_{b\perp}}{z_a z_b M_{h_a} M_{h_b}}$

SIA with Di-Hadrons Fragmentation

Similar discussion has been conducted for di-hadron fragmentation case.

$$e^+e^- \to \gamma^* \to q\bar{q} \to (\pi^+\pi^-) + (\pi^+\pi^-) + X$$

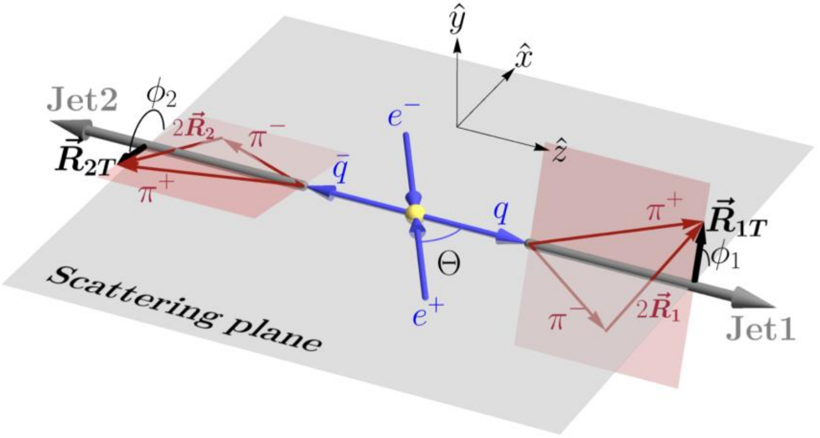
Kun Cheng, Bin Yan, Phys. Rev. Lett. 135 (2025) 1, 1

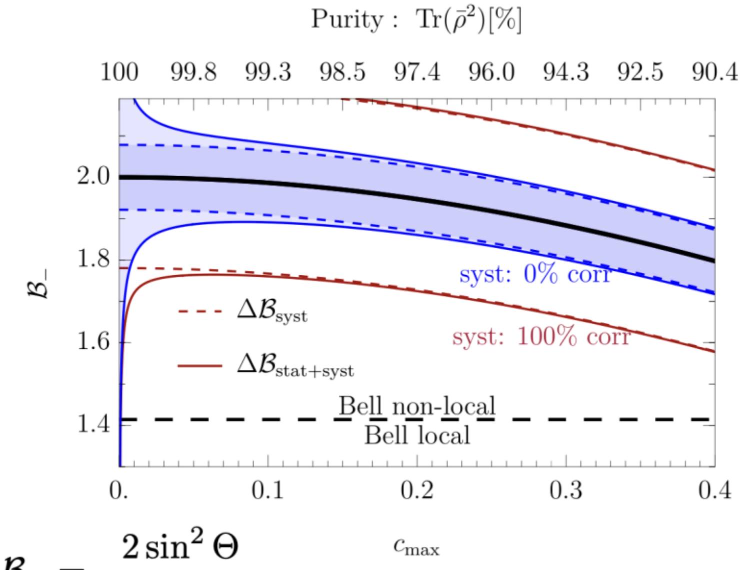
$$\frac{d\sigma}{dz_1 dz_2 dM_1 dM_2 d\phi_1 d\phi_2} = \sigma_{\text{hard}} \Bigg[\sum_q e_q^2 D_1^q(z_1, M_1) D_1^{\bar{q}}(z_2, M_2) + \frac{1}{2} \sum_q e_q^2 H_1^{\sphericalangle, q}(z_1, M_1) H_1^{\sphericalangle, \bar{q}}(z_2, M_2) \Big(\mathcal{B}_- \cos(\phi_1 + \phi_2) - \mathcal{B}_+ \cos(\phi_1 - \phi_2) \Big) \Bigg]$$

Unpolarized diFF

Transverse polarized diFF

$$\mathcal{B}_\pm \equiv C_{xx} \pm C_{yy}$$





$$\mathcal{B}_- = \frac{2\sin^2\Theta}{1+\cos^2\Theta}.$$

METHODS

Quantum Information in Spin Space

The complete information of a quantum system is described by its density matrix:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \qquad p_i > 0, \quad \sum_i p_i = 1$$

For spin-1/2 system, the spin density matrix can be parameterized as:

$$\rho = \frac{1}{2}\left(\mathbb{I}_2 + \sum_i S_i \sigma_i\right), \qquad S_i = \langle\sigma_i\rangle = \mathrm{Tr}(\rho\sigma_i)$$

For two-particle system:

$$\rho = \frac{1}{4}\left(\mathbb{I}_4 + \sum_i S_i^+ \, \sigma_i \otimes \mathbb{I}_2 + \sum_i S_i^- \, \mathbb{I}_2 \otimes \sigma_i + \sum_{i,j} C_{ij} \, \sigma_i \otimes \sigma_j\right),$$

$$S_i^+ = \mathrm{Tr}(\rho \, \sigma_i \otimes \mathbb{I}_2) \qquad S_i^- = \mathrm{Tr}(\rho \, \mathbb{I}_2 \otimes \sigma_i) \qquad C_{ij} = \mathrm{Tr}(\rho \, \sigma_i \otimes \sigma_j)$$

Quantum Information in Spin Space

The quantum entanglement can be confirmed by one of its sufficient conditions: the violation of the Bell (CHSH) inequality.

John F. Clauser et al. Phys. Rev. Lett. 1969, 23: 880–884.

$$\mathcal{B}_{\mathrm{CHSH}}(A_1, A_2, B_1, B_2) = A_1 B_1 - A_1 B_2 + A_2 B_1 + A_2 B_2$$

$$A_k = \vec{a}_k \cdot \vec{\sigma}, \qquad B_k = \vec{b}_k \cdot \vec{\sigma}$$

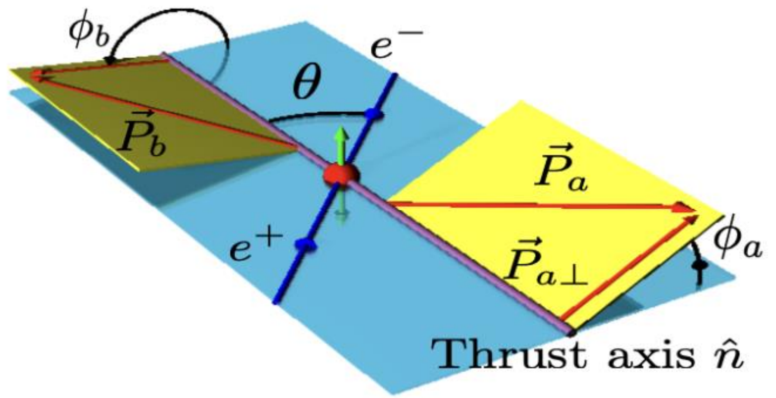
$$\text{separable} \Longrightarrow \forall \vec{a}_k, \vec{b}_k, \;\; -2 < \langle \mathcal{B}_{\mathrm{CHSH}} \rangle < 2$$

With the parameterization in the last page,

$$\langle \mathcal{B}_{\mathrm{CHSH}} \rangle = \mathrm{Tr}(\rho \mathcal{B}_{\mathrm{CHSH}}) \;= \vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2),$$

Quantum Tomography via TMD Fragmentation

Consider the SIA process with transverse momentum:



$$e^+e^- \to \gamma^* \to q\bar{q} \to h_a + h_b + X$$

$$\mathrm{d}\sigma = \sum_q \sum_{s_z s'_z \bar{s}_z \bar{s}_z{}'} \mathrm{d}\hat{\sigma}^q_{s_z s'_z, \bar{s}_z \bar{s}'_z} \mathcal{D}^{s'_z s_z}_{h_a/q}(z_a, \boldsymbol{P}_{a\perp}) \mathcal{D}^{\bar{s}'_z \bar{s}_z}_{h_b/\bar{q}}(z_b, \boldsymbol{P}_{b\perp}) \mathrm{d}z_a \mathrm{d}^2\boldsymbol{P}_{a\perp} \mathrm{d}z_b \mathrm{d}^2\boldsymbol{P}_{b\perp}$$

At leading twist:

$$\mathcal{D}^{s'_z s_z}_{h_a/q}(z_a, \boldsymbol{P}_{a\perp}) = \begin{pmatrix} D_1^q & \frac{-\mathrm{i}P^x_{a\perp} - P^y_{a\perp}}{z_a M_a} H_1^{\perp,q} \\ \frac{\mathrm{i}P^x_{a\perp} - P^y_{a\perp}}{z_a M_a} H_1^{\perp,q} & D_1^q \end{pmatrix}$$

Quantum Tomography via TMD Fragmentation

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}z_a\mathrm{d}z_b\mathrm{d}^2\boldsymbol{P}_{a\perp}\mathrm{d}^2\boldsymbol{P}_{b\perp}\mathrm{d}\cos\theta} = (1+\cos^2\theta)\Big[\sum_q Q_q^2 D_1^q D_1^{\bar{q}} + \sum_q Q_q^2 \mathcal{F}_h H_1^{\perp,q} H_1^{\perp,\bar{q}}\Big(\frac{C_{xx}-C_{yy}}{2}\cos(\phi_a+\phi_b) - \frac{C_{xx}+C_{yy}}{2}\cos(\phi_a-\phi_b) + \frac{C_{xy}+C_{yx}}{2}\sin(\phi_a+\phi_b) + \frac{C_{xy}-C_{yx}}{2}\sin(\phi_a-\phi_b)\Big)\Big]$$

where, $\sigma_0 = \dfrac{3\pi\alpha^2}{2s}$

The tree level calculation under SM gives:

$$C_{ij} = \begin{pmatrix} \frac{\sin^2\theta}{1+\cos^2\theta} & 0 & 0 \\ 0 & -\frac{\sin^2\theta}{1+\cos^2\theta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

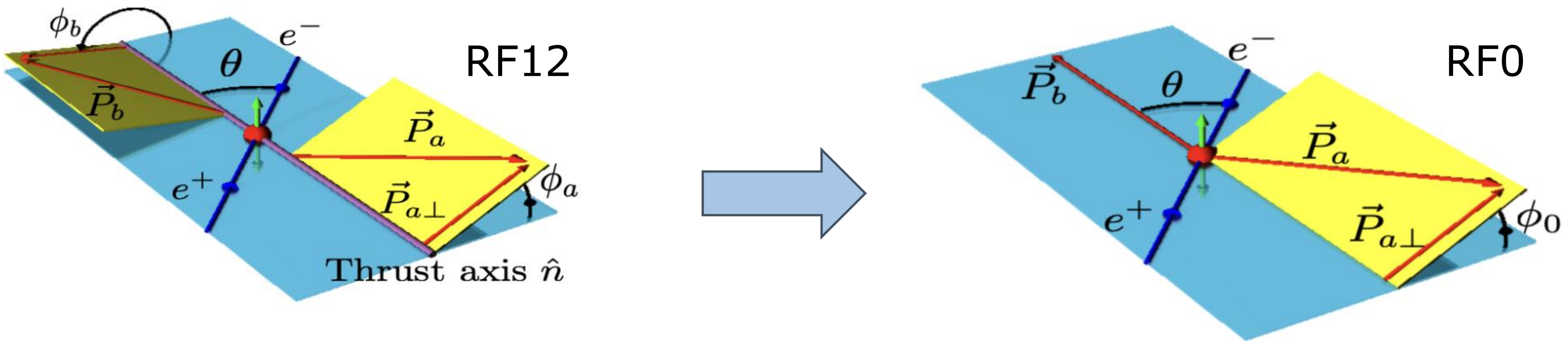
only $\dfrac{C_{xx}-C_{yy}}{2}$ term survives, which corresponds to a particular Bell observable as

$$C_{xx}-C_{yy} = \frac{1}{\sqrt{2}}\langle\mathcal{B}_{\mathrm{CHSH}}\rangle\Big|_{\vec{a}_1=\hat{x},\ \vec{a}_2=-\hat{y},\ \vec{b}_1-\vec{b}_2=\sqrt{2}\hat{x},\ \vec{b}_1+\vec{b}_2=\sqrt{2}\hat{y}}$$

violating the CHSH inequality when $\mathcal{B}^- \equiv C_{xx}-C_{yy} > \sqrt{2}$

INPUT OBSERVABLES
&
OUTPUT RESULTS

TMD Observables



Perform a convolution to eliminate the dependence of thrust axis:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}z_a\mathrm{d}z_b\mathrm{d}^2\mathbf{P}_\perp\mathrm{d}\cos\theta} = z_b^2(1+\cos^2\theta)\times \sum_q Q_q^2\Big[\mathcal{C}(DD) + \frac{1}{2}(C_{xx}-C_{yy})\,\mathcal{C}\left(\frac{(\hat{\mathbf{h}}\cdot\mathbf{P}_{a\perp})(\hat{\mathbf{h}}\cdot\mathbf{P}_{b\perp})-(\hat{\mathbf{t}}\cdot\mathbf{P}_{a\perp})(\hat{\mathbf{t}}\cdot\mathbf{P}_{b\perp})}{z_a z_b M_{h_a} M_{h_b}} HH\right)\cos 2\phi_0 + (q\leftrightarrow\bar{q}\text{ for }D\text{ and }H)\Big].$$

where, $\mathcal{C}(wDD) = \int \frac{\mathrm{d}^2\mathbf{P}_{a\perp}}{z_a^2}\frac{\mathrm{d}^2\mathbf{P}_{b\perp}}{z_b^2}\delta^{(2)}\left(\frac{\mathbf{P}_\perp}{z_a}-\frac{\mathbf{P}_{a\perp}}{z_a}-\frac{\mathbf{P}_{b\perp}}{z_b}\right)\times w(\mathbf{P}_{a\perp},\mathbf{P}_{b\perp})D_{h_a/q}(z_a,\mathbf{P}_{a\perp}^2)D_{h_b/\bar{q}}(z_b,\mathbf{P}_{b\perp}^2).$

$\hat{\mathbf{h}}$, $\hat{\mathbf{t}}$ are a couple of orthogonal unit vectors perpendicular to $\mathbf{P}_b$, with $\hat{\mathbf{h}}$ towards $\mathbf{P}_\perp$.

TMD Observables

In the experiments, the events are divided into two categories: "like" and "unlike" events:

$$\mathrm{d}\sigma^{\mathrm{L}} := \mathrm{d}\sigma_{\pi^+\pi^+} + \mathrm{d}\sigma_{\pi^-\pi^-}, \qquad \mathrm{d}\sigma^{\mathrm{U}} := \mathrm{d}\sigma_{\pi^+\pi^-} + \mathrm{d}\sigma_{\pi^-\pi^+},$$

and their sum "charged" events: $\mathrm{d}\sigma^{\mathrm{C}} := \mathrm{d}\sigma^{\mathrm{L}} + \mathrm{d}\sigma^{\mathrm{U}}$

The observable is obtained through the ratio of their normalized ϕ_0 distribution:

$$\frac{R^{\mathrm{U}}}{R^{\mathrm{L}}} \approx 1 + \frac{\mathcal{B}^-}{2}\left(\frac{\sum_{\mathrm{U}} \mathcal{C}(w_1 HH)}{\sum_{\mathrm{U}} \mathcal{C}(DD)} - \frac{\sum_{\mathrm{L}} \mathcal{C}(w_1 HH)}{\sum_{\mathrm{L}} \mathcal{C}(DD)}\right)\cos(2\phi_0)$$

$$= 1 + A_0^{\mathrm{UL}}\cos(2\phi_0)$$

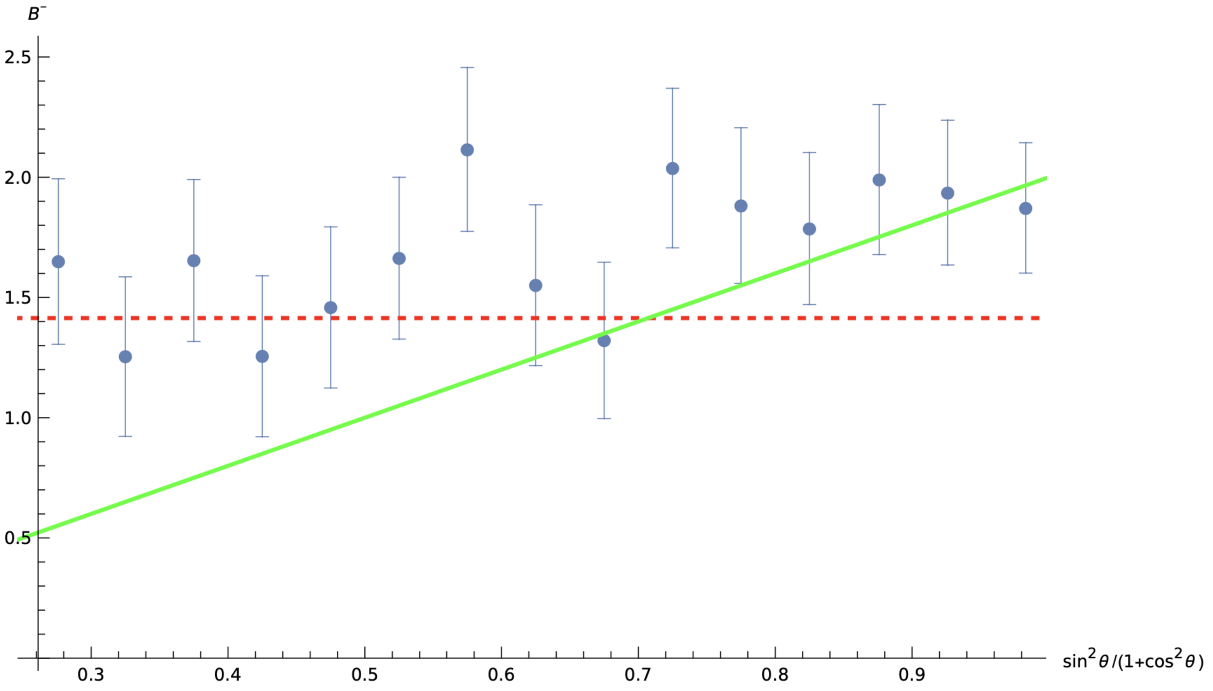
where, $w_1 = \dfrac{(\hat{\mathbf{h}}\cdot\mathbf{P}_{a\perp})(\hat{\mathbf{h}}\cdot\mathbf{P}_{b\perp}) - (\hat{\mathbf{t}}\cdot\mathbf{P}_{a\perp})(\hat{\mathbf{t}}\cdot\mathbf{P}_{b\perp})}{z_a z_b M_{h_a} M_{h_b}}$

the observable for UC is in the same form.

CHSH Inequality Violating

BABAR, Phys. Rev. D, 90(5):052003, 2014

We employed the methods on the data from BABAR experiment, with the TMD FFs fitted by Chunhua Zeng, Hongxin Dong, Tianbo Liu, Peng Sun, Yuxiang Zhao, Phys. Rev. D 109 (2024) 5, 056002; Ignazio Scimemi, Alexey Vladimirov, JHEP 06 (2020) 137



Combine the last six bins. When the bins are independent, the results are

$$B^- = 1.91 \pm 0.13 \text{ (from } A_0^{\mathrm{UL}},\ 3.8\sigma), \qquad B^- = 1.80 \pm 0.16 \text{ (from } A_0^{\mathrm{UC}},\ 2.4\sigma).$$

When the bins are fully correlated, the results are

$$B^- = 1.91 \pm 0.30 \text{ (from } A_0^{\mathrm{UL}},\ 1.6\sigma), \qquad B^- = 1.80 \pm 0.0.40 \text{ (from } A_0^{\mathrm{UC}},\ 1.0\sigma).$$

SUMMARY

- We have proposed a novel approach to probe quantum entanglement of **light quarks** through **TMD fragmentation** in e^+e^- annihilation
- The spin information of quarks can be transferred to the produced hadrons via **Collins fragmentation functions**.
- By analyzing **the ϕ_0 distribution**, the density matrix of the quark pair can be extracted, enabling **tests of Bell-type inequalities** at the parton level.
- The method has been validated using **BABAR experimental data**, showing the significant indication of entanglement at the level of **1.6~3.8** σ.