SMEFT, HEFT and UV models

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2412.00355(*JHEP* 06 (2025) 021), 2503.00707(*JHEP* 06 (2025) 249), in progress

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TeV物理前沿专题研讨会暨第31届LHC Mini-Workshop

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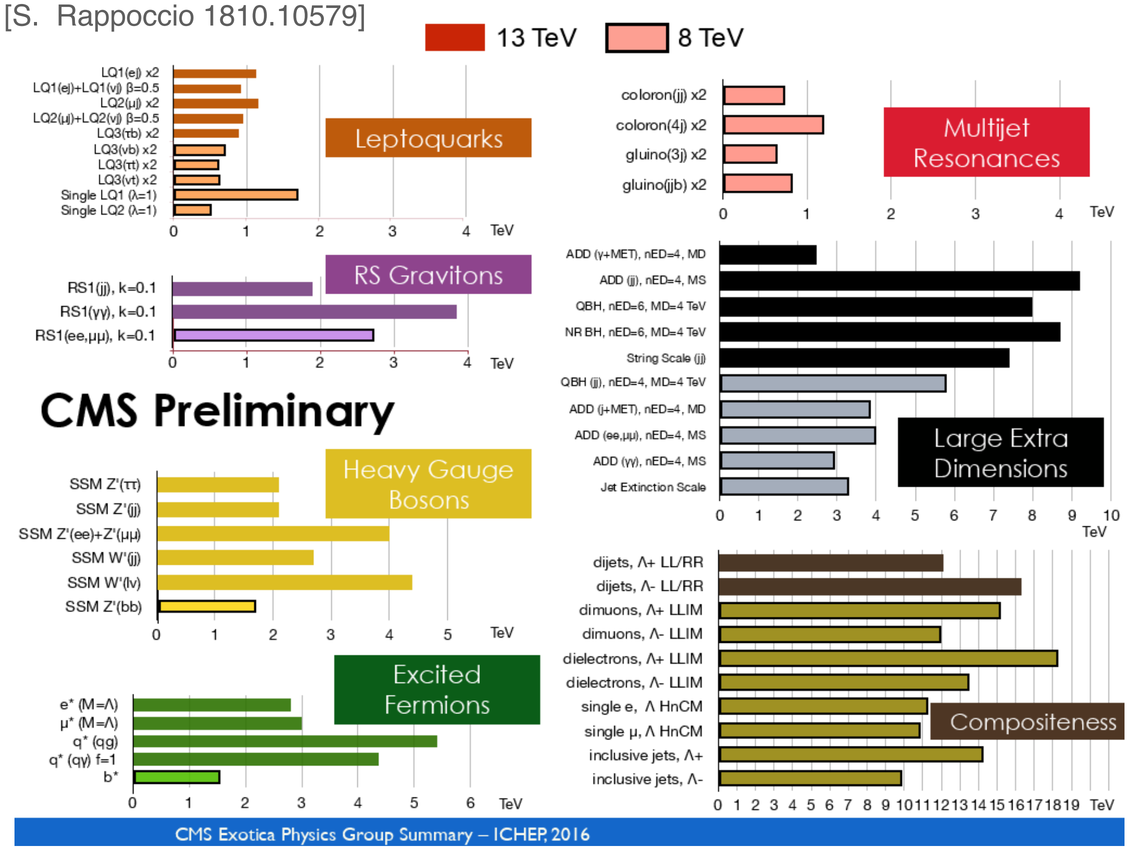
Outline

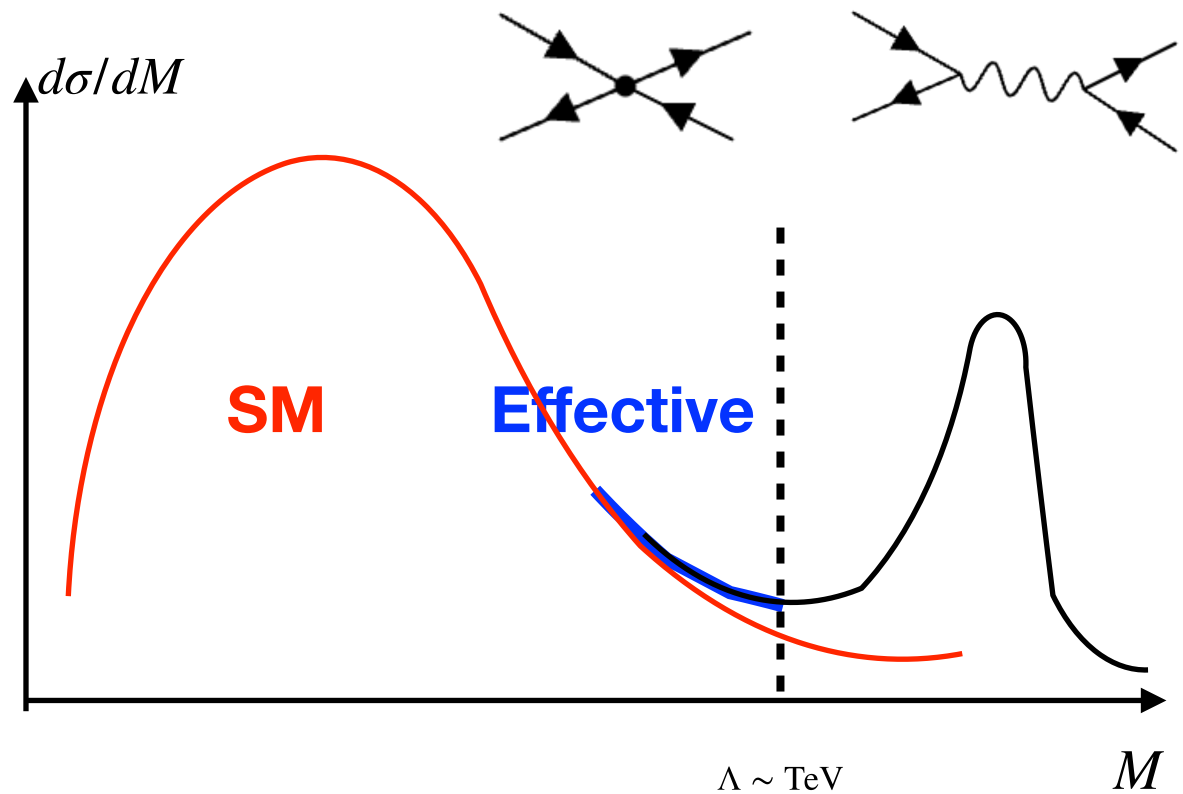
- Introduction: SMEFT, HEFT and Matching.
- A non-linear framework for matching.
- Matching Higgs Triplet Model to HEFT in decoupling regime.
- Non-decoupling regime and one-loop matching.
- Summary.

Why EFT

- In **direct** search of new particles, we get constraints.

[S. Rappoccio 1810.10579]





New Particle is heavy, so it is better to study the **indirect** effects?

- UV theory is not known, use most general EFT operators to parameterize NP. (Bottom-up approach).
- UV theory is known, integrate out the heavy particles and match to EFT. (Top-down approach).

Two EFTs: SMEFT and HEFT

Both are invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetry and contains SM fields.

- SMEFT, linear realization of the Higgs and Goldstones, canonical dimension

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ \\ v + h + iG^0 \end{pmatrix}, \quad \mathscr{L}_{\text{SMEFT}} \supset \frac{1}{2} D_\mu H^\dagger D^\mu H + \frac{m^2}{2} H^\dagger H - \lambda (H^\dagger H)^2 + \frac{C_H}{\Lambda^2} (H^\dagger H)^3 + \cdots$$

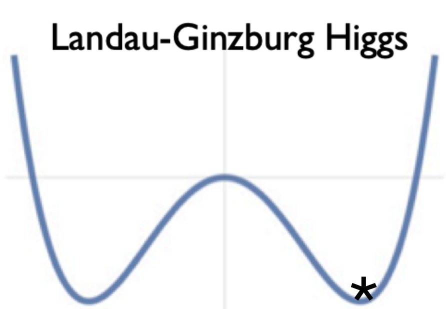
- HEFT, nonlinear realization, chiral dimension

$$h, U \equiv \exp\left(\frac{i\pi_i \sigma_i}{v_{\text{EW}}} \right), \quad \mathscr{L}^{\text{LO}}_{\text{HEFT}} \supset \frac{1}{2} D_\mu h D^\mu h - V(h) + \frac{v_{\text{EW}}^2}{4} F(h) \text{Tr}(D_\mu U^\dagger D^\mu U) + \cdots$$

See Jian Wang,
Lei Zhang, Nan Lu's talks

$$V(h) = \frac{1}{2} m_h^2 h^2 \left[1 + (1 + \Delta\kappa_3) \frac{h}{v_{\text{EW}}} + \cdots \right], \quad F(h) = 1 + 2(1 + \Delta a) \frac{h}{v_{\text{EW}}} + \cdots$$

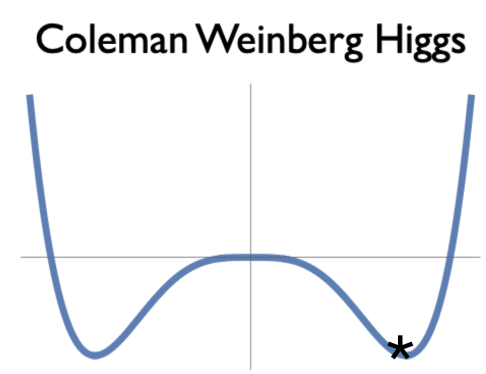
[H. Sun, M.-L. Xiao, and J.-H. Yu, 2206.07722]



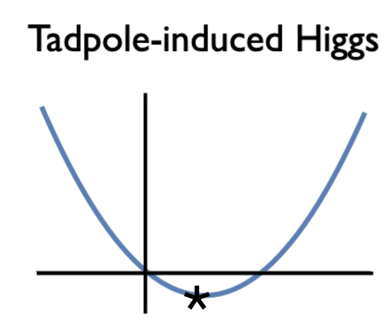
$$V(\phi) = -m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$V(\phi) = -a \sin^2(\phi/f) + b \sin^4(\phi/f)$$

SMEFT/HEFT



$$V(\phi) = \lambda (\phi^\dagger \phi)^2 + \epsilon (\phi^\dagger \phi)^2 \log \frac{\phi^\dagger \phi}{\mu^2}$$



$$V(\phi) = -\mu^3 \sqrt{\phi^\dagger \phi} + m^2 \phi^\dagger \phi$$

HEFT

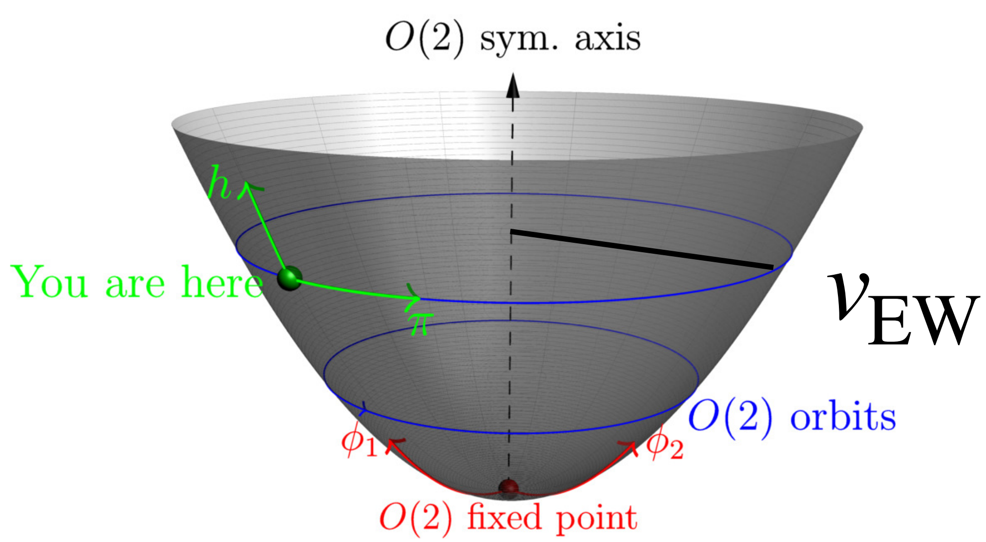
Pankaj Agrawal, Debashis Saha, Ling-Xiao Xu,, Jiang-Hao Yu, C.-P. Yuan,1907.02078

A Geometric Picture

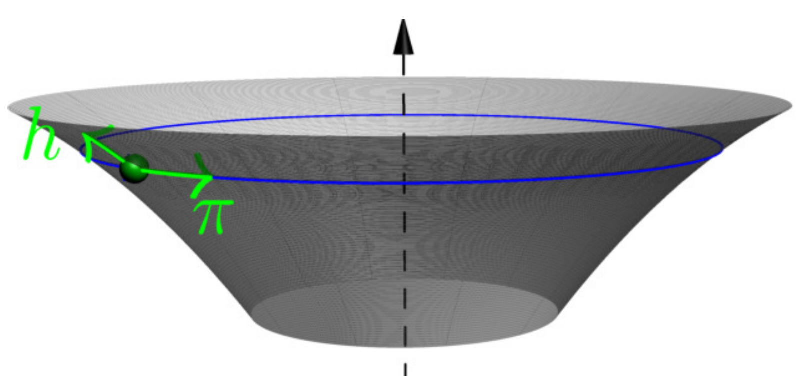
SM doublet

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix} \longrightarrow \quad \vec{\phi} = \begin{pmatrix}\phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4\end{pmatrix}, \quad \vec{\phi} \to O\vec{\phi}, \text{ where } \; O \in O(4) \supset SU(2) \times U(1)$$

$$\mathcal{L}_{\mathrm{SMEFT}} = \frac{1}{2}A(\vec{\phi}\cdot\vec{\phi})(\partial\vec{\phi}\cdot\partial\vec{\phi}) + \frac{1}{2}B\left(\vec{\phi}\cdot\vec{\phi}\right)(\vec{\phi}\cdot\partial\vec{\phi})^2 - V\left(\vec{\phi}\cdot\vec{\phi}\right) + \mathcal{O}\left(\partial^4\right)$$



SMEFT/HEFT

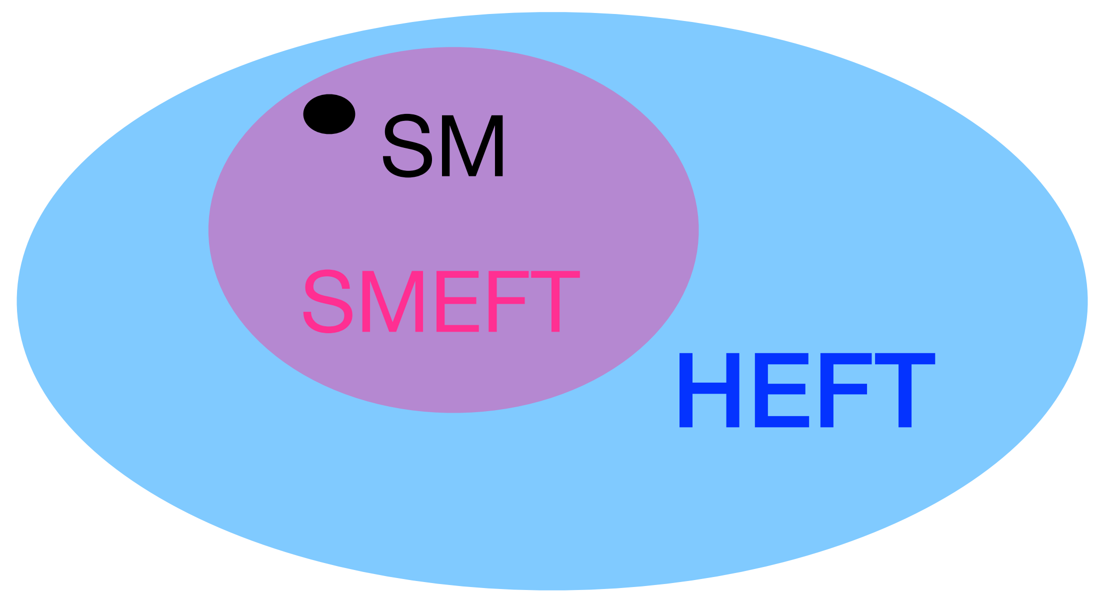


from N. Craig, HEFT2021

only HEFT

HEFT encompasses SMEFT

R. Alonso, E. Jenkins, A. Manohar [1511.00724,1605.03602]



A Geometric Picture

SM doublet

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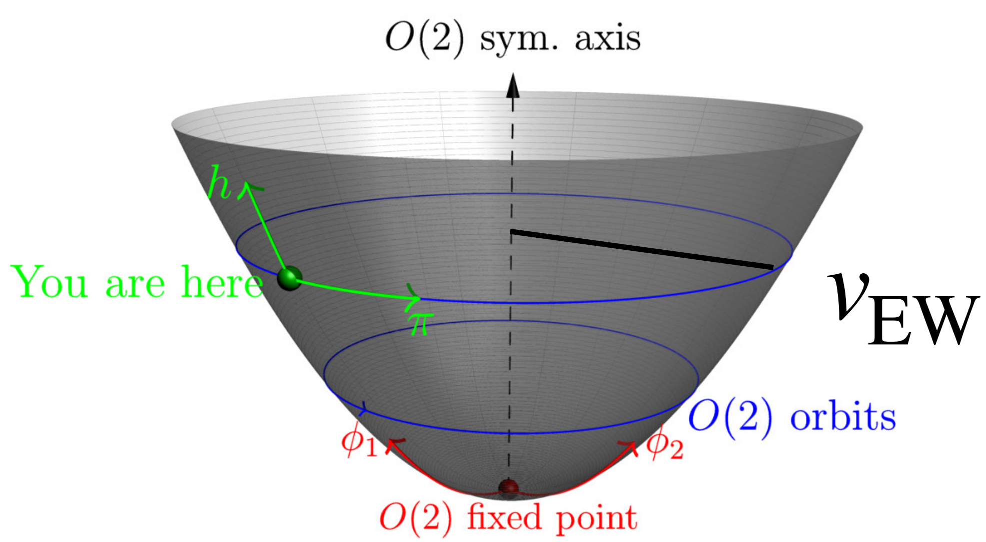
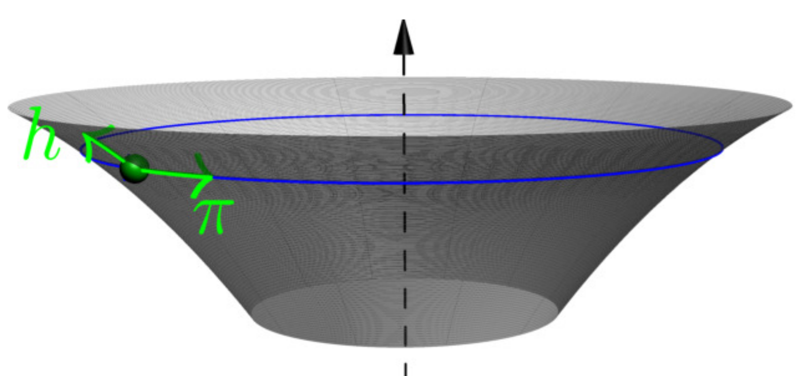


SMEFT/HEFT



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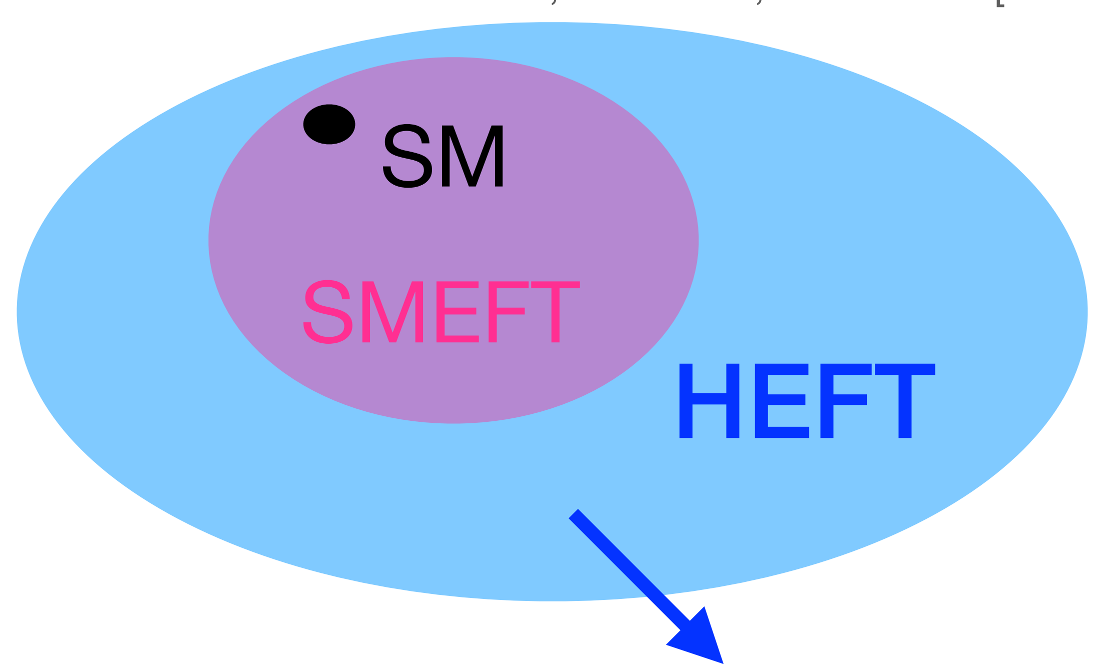
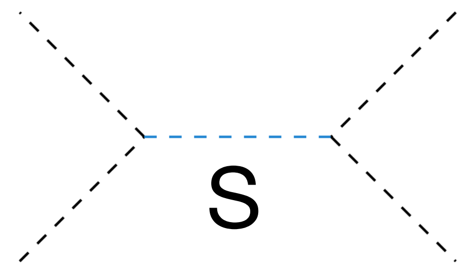


BSM Non-decoupling Particles

Non-decoupling Particles

Decoupling: While the mass of a particle goes infinity, its contribution vanishes.

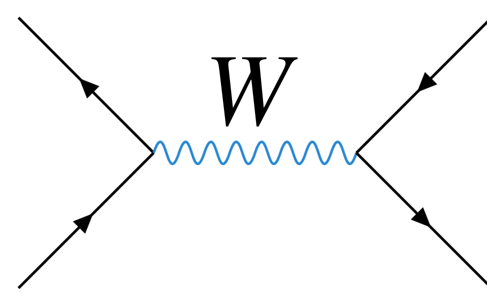
E.g. $\mathscr{L} \supset -\frac{1}{2}M_S^2 S^2 - \kappa S|H|^2$.



$\sim \kappa^2/M_S^2 \to 0$ (if $M_S \to \infty$)

- Non-decoupling effects at tree level

E.g. $\mu^- \to e^- \nu_\mu \bar{\nu}_e$



$\sim \frac{g_2^2}{M_W^2} \to \frac{1}{v_{\rm EW}^2}$ (if $M_W \to \infty$)

Due to Higgs mechanism, M_W is not a free parameter.

$$\boxed{M_W = g_2 v_{\rm EW}/2 \ \to g_2 = 2M_W/v_{\rm EW}}$$

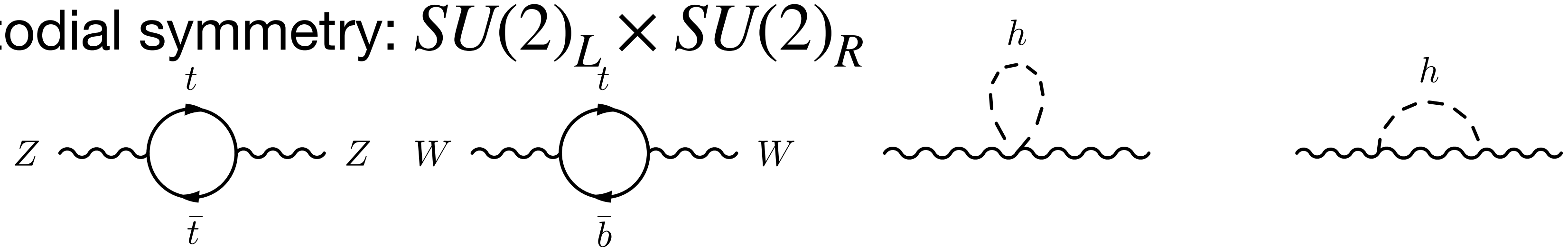
Non-decoupling Particles

- Non-decoupling effects at one-loop level

Ilaria Brivio and Michael Trott, 1706.08945

$$\rho \simeq \frac{\bar{g}_Z^2\,\bar{m}_W^2}{\bar{g}_2^2\,\bar{m}_Z^2} + \frac{N_c\,\hat{G}_F}{8\,\pi^2\sqrt{2}}\left(\boxed{m_t^2} + m_b^2 - 2\frac{m_t^2\,m_b^2}{m_t^2 - m_b^2}\log\left(\frac{m_t^2}{m_b^2}\right)\right) - \frac{11\hat{G}_F\,\hat{M}_Z^2 s_{\hat{\theta}}^2}{24\sqrt{2}\pi^2}\log\left(\boxed{\frac{m_h^2}{m_Z^2}}\right)$$

≈ 1 Custodial symmetry: $SU(2)_L \times SU(2)_R$



As $m_t \to \infty$, or $m_h \to \infty$, $\Delta\rho \nrightarrow 0$

$$\boxed{m_t = y_t v_{\rm EW}/2 \ \to y_t = m_t/v_{\rm EW}}$$

BSM non-decoupling effects

Timothy Cohen,Nathaniel Craig, Xiaochuan Lu, and Dave Sutherland, 2008.08597

1. The new particle derive their masses majorly from the Higgs mechanism. or
2. There are additional sources of electroweak symmetry breaking.

Generally they are more easier to be found in HL-LHC or CEPC.

Matching UV Models to HEFT

Through matching we would like to study,

- Could a same UV model match to both SMEFT and HEFT? When will the HEFT be **needed**?

- The SMEFT matching is mature at one-loop level (diagrammatic method and functional method). How to make the HEFT matching **programmable**?

Covariant Derivative Expansion (CDE)

How to use the Standard Model effective field theory, Brian Henning, Xiaochuan Lu, and Hitoshi Murayama, 1412.1837

Universal One-loop Action

The Universal One-Loop Effective Action, Aleksandra Drozd, John Ellis, J´er´emi Quevillon and Tevong You, 1512.03003

Automation

STrEAMlining EFT Matching, Timothy Cohen,[1] Xiaochuan Lu,[1] and Zhengkang Zhang, 2012.07851

One-Loop UV-SMEFT dictionary

From the EFT to the UV: the complete SMEFT one-loop dictionary, Guilherme Guedes, Pablo Olgoso, 2412.14253

Linear Standard Model extensions in the SMEFT at one loop and Tera-Z,

John Gargalionis, Jérémie Quevillon, Pham Ngoc Hoa Vuong, Tevong You, 2412.01759

- A non-linear framework of UV models (2412.00355)
- Matching Real Triplet Higgs Model to HEFT in the decoupling regime (2503.00707)
- Matching in non-decoupling regime (in progress),
- Matching at one-loop (in progress)

Real Higgs Triplet Extension of the SM (RHTE)

- A singlet extension, a second doublet extension (2HDM), next is triplet.
  [G. Buchala et al, 1608.03564, 2312.13885], [S. Dawson et al, 2305.07689, 2311.16897],[F. Arco et al, 2307.15693]

- The custodial violation appears at tree level with a non-zero VEV.

The Model: the SM plus a real $SU(2)_L$ triplet with $Y = 0$

Linear form

$$\mathrm{H} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v_\mathrm{H} + h + iG^0\right) \end{pmatrix}, \qquad \Sigma = \frac{1}{2}\Sigma_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -v_\Sigma - \Sigma^0 \end{pmatrix}, i = 1, 2, 3,$$

$$\mathscr{L}_\mathrm{RHTE}(\mathrm{H}, \Sigma) \supset D_\mu \mathrm{H}^\dagger D^\mu \mathrm{H} + \langle D_\mu \Sigma^\dagger D^\mu \Sigma \rangle - V(H, \Sigma),$$

$$V(\mathrm{H}, \Sigma) = Y_1^2 \mathrm{H}^\dagger \mathrm{H} + Z_1 (\mathrm{H}^\dagger \mathrm{H})^2 + Y_2^2 \langle \Sigma^\dagger \Sigma \rangle + Z_2 \langle \Sigma^\dagger \Sigma \rangle^2 + Z_3 \mathrm{H}^\dagger \mathrm{H} \langle \Sigma^\dagger \Sigma \rangle + 2 Y_3 \mathrm{H}^\dagger \Sigma \mathrm{H}$$

$Z_i s$ are dimensionless, $Y_i s$ are dimensional

$< \ldots >$ denotes trace

Matching RHTE to SMEFT

$$\mathscr{L}_{\rm RHTE}(\mathrm{H},\Sigma) \supset D_\mu \mathrm{H}^\dagger D^\mu \mathrm{H} + \langle D_\mu \Sigma^\dagger D^\mu \Sigma\rangle - V(H,\Sigma),$$

$$V(\mathrm{H},\Sigma) = Y_1^2 \mathrm{H}^\dagger \mathrm{H} + Z_1 (\mathrm{H}^\dagger \mathrm{H})^2 + \underbrace{Y_2^2}_{\text{mass}} \langle \Sigma^\dagger \Sigma\rangle + Z_2 \langle \Sigma^\dagger \Sigma\rangle^2 + Z_3 \mathrm{H}^\dagger \mathrm{H} \langle \Sigma^\dagger \Sigma \rangle + 2 Y_3 \mathrm{H}^\dagger \Sigma \mathrm{H}$$

$$\mathscr{L}^\Sigma = \frac{1}{2} \vec{\Sigma}^T (-D_\mu D^\mu - Y_2^2 - Z_3 \mathrm{H}^\dagger \mathrm{H}) \vec{\Sigma} + Y_3 \vec{\Sigma} \cdot \mathrm{H}^\dagger \vec{\sigma} \mathrm{H} - \frac{1}{4} Z_2 (\vec{\Sigma} \cdot \vec{\Sigma})^2$$

EoM of Σ:

$$(-D_\mu D^\mu - Y_2^2 - Z_3 \mathrm{H}^\dagger \mathrm{H}) \vec{\Sigma}_c = - Y_3 \mathrm{H}^\dagger \vec{\sigma} \mathrm{H} + Z_2 (\vec{\Sigma}_c \cdot \vec{\Sigma}_c) \vec{\Sigma}_c$$

$$\vec{\Sigma}_c = - \frac{1}{-D_\mu D^\mu - Y_2^2 - Z_3 \mathrm{H}^+ \mathrm{H}} Y_3 \mathrm{H}^+ \vec{\sigma} \mathrm{H} + \frac{1}{-D_\mu D^\mu - Y_2^2 - Z_3 \mathrm{H}^+ \mathrm{H}} Z_2 (\vec{\Sigma}_c \cdot \vec{\Sigma}_c) \vec{\Sigma}_c$$

Expansion with $1/Y_2^2$ (if $Y_2^2 \gg v_{\rm EW}^2$)

$$\mathscr{L}_{\rm SMEFT} = \frac{1}{2Y_2^2} Y_3^2 \mathrm{H}^\dagger \vec{\sigma} H \cdot \mathrm{H}^\dagger \vec{\sigma} \mathrm{H} + \frac{1}{2} (\mathrm{H}^\dagger \vec{\sigma} H)^T \frac{1}{Y_2^2} (-D_\mu D^\mu - Z_3 H^\dagger H) \frac{1}{Y_2^2} \mathrm{H}^\dagger \vec{\sigma} H + \cdots$$

T. Corbett, A. Helset, A. Martin, M. Trott, [2102.02819]
J. Ellis, K. Mimasu, F. Zamperdri, [2304.06663]

Matching RHTE to HEFT

$$h, U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{\rm EW}}\right), \quad \mathscr{L}^{\rm LO}_{\rm HEFT} \supset \frac{1}{2}D_\mu h D^\mu h - V(h) + \frac{v_{\rm EW}^2}{4}F(h){\rm Tr}(D_\mu U^\dagger D^\mu U) + \cdots$$

$$V(h) = \frac{1}{2}m_h^2 h^2\left[1 + (1+\Delta\kappa_3)\frac{h}{v_{\rm EW}} + \cdots\right], \quad F(h) = 1 + 2(1+\Delta a)\frac{h}{v_{\rm EW}} + \cdots$$

RHTE in linear form

$$\mathrm{H} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v_{\rm H} + h + iG^0\right) \end{pmatrix}, \quad \Sigma = \frac{1}{2}\Sigma_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -v_\Sigma - \Sigma^0 \end{pmatrix}, i = 1,2,3$$

$$\boxed{v_{\rm EW}^2 = v_{\rm H}^2 + 4v_\Sigma^2}$$

$$\begin{pmatrix} G^+_{\rm EW} \\ H^+ \end{pmatrix} = \begin{pmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{pmatrix}\begin{pmatrix} G^+ \\ \Sigma^+ \end{pmatrix} \qquad \begin{pmatrix} h \\ K \end{pmatrix} = \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix}\begin{pmatrix} h^0 \\ \Sigma^0 \end{pmatrix}$$

$$\tan\delta = 2v_\Sigma/v_{\rm H}.$$

1. Solve EoMs of $H^\pm, K$.
2. Embed $G^\pm_{\rm EW}, G^0$ into an exponential matrix form. **?! It must be complicated.**

Find a non-linear representation

SM:

$$H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_{\rm EW} + h + iG^0) \end{pmatrix} \longrightarrow \boxed{\mathrm{H} = U\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_{\rm EW} + h^0 \end{pmatrix}, \quad U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{\rm EW}}\right)}$$

2HDM

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix},$$

Higgs Basis:

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_{\rm EW} + h_1^H + iG^0) \end{pmatrix}, H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2^H + iA) \end{pmatrix}$$

$$\longrightarrow \boxed{\mathscr{H}_1 = U\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_{\rm EW} + h_1^H \end{pmatrix}, \qquad \mathscr{H}_2 = U\begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2^H + iA) \end{pmatrix}}$$

U could be considered as a rotation.

S. Dawson et al, 2305.07689

Triplet

$$\mathrm{H} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_{\rm H} + h + iG^0) \end{pmatrix}, \quad \Sigma = \frac{1}{2}\Sigma_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -v_\Sigma - \Sigma^0 \end{pmatrix}, i = 1,2,3$$

$$v_{\rm EW}^2 = v_{\rm H}^2 + 4v_\Sigma^2$$

$$\longrightarrow \boxed{\mathrm{H} = U\frac{1}{\sqrt{2}}\begin{pmatrix} 2\frac{v_\Sigma}{v_{\rm H}}\phi^\pm \\ v_{\rm H} + h^0 \end{pmatrix}, \qquad \Sigma = U\Phi U^\dagger,\ \Phi = \frac{1}{2}\phi_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \phi^0 & \sqrt{2}\phi^+ \\ \sqrt{2}\phi^- & -v_\Sigma - \phi^0 \end{pmatrix}}$$

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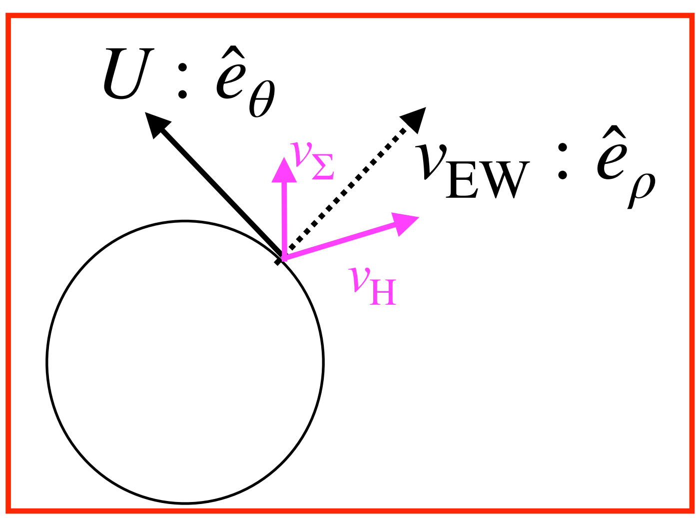
Kinetic mixing cancels

$$D_\mu H^\dagger D^\mu H \supset 2\frac{v_\Sigma}{v_H} \times v_\mathrm{H}\epsilon_{3jk}D_\mu\phi_j D^\mu\pi_k/(2v_\mathrm{EW}) \qquad \langle D_\mu\Sigma^\dagger D^\mu\Sigma\rangle \supset -v_\Sigma\epsilon_{3jk}D_\mu\phi_j D^\mu\pi_k/v_\mathrm{EW}$$

U does not appear in potential, no mass mixing

$$V(\mathrm{H},\Sigma) = Y_1^2\mathrm{H}^\dagger\mathrm{H} + Z_1(\mathrm{H}^\dagger\mathrm{H})^2 + Y_2^2\langle\Sigma^\dagger\Sigma\rangle + Z_2\langle\Sigma^\dagger\Sigma\rangle^2 + Z_3\mathrm{H}^\dagger\mathrm{H}\langle\Sigma^\dagger\Sigma\rangle + 2Y_3\mathrm{H}^\dagger\Sigma\mathrm{H}$$



Does these two rules suitable for a general $SU(2)$ representations? E.g. a quadruplet, a quintet.

Quadruplet with $Y = 3/2$

$$\begin{pmatrix} \Theta_{111} \\ \sqrt{3}\Theta_{112} \\ \sqrt{3}\Theta_{122} \\ \Theta_{222} \end{pmatrix} \to \begin{pmatrix} \Theta^{3+} \\ \Theta^{++} \\ \boxed{\begin{matrix}\Theta^{+} \\ \Theta^{0}\end{matrix}} \end{pmatrix}$$

$$\mathrm{H}_i = \underline{U_i^j}\mathfrak{h}_j, \quad \mathfrak{h} = \begin{pmatrix} \chi^+ \\ \frac{1}{\sqrt{2}}\left(v_{\mathrm{H}} + h^0 + i\chi^0\right) \end{pmatrix}$$

$$\Theta_{ijk} = \underline{U_i^l U_j^m U_k^n}\phi_{lmn}$$

$$\langle \phi_{222} \rangle = \langle \phi^{*222} \rangle = v_\Theta/\sqrt{2}, \mathrm{Im}(\phi_{222}) = \eta_4/\sqrt{2}, \phi_{122} = \phi^+/\sqrt{3}$$

$v_H/\sqrt{2}$

$$\mathscr{L}_{\mathrm{H}}^{\text{mix}} = \langle \mathfrak{h}_2 \rangle \Big((U^\dagger D_\mu U)_1^2 D^\mu \mathfrak{h}^{*1} - (U^\dagger D_\mu U)_2^1 D^\mu \mathfrak{h}_1 + (U^\dagger D_\mu U)_2^2 (D^\mu \mathfrak{h}^{*2} - D^\mu \mathfrak{h}_2) \Big),$$

$$\mathscr{L}_{\Theta}^{\text{mix}} = 3\langle \phi_{222} \rangle \Big((U^\dagger D_\mu U)_1^2 D^\mu \phi^{*122} - (U^\dagger D_\mu U)_2^1 D^\mu \phi_{122} + (U^\dagger D_\mu U)_2^2 (D^\mu \phi^{*222} - D^\mu \phi_{222}) \Big),$$

$v_\Theta/\sqrt{2}$

$$\chi^+ = -\frac{3v_\Theta}{v_{\mathrm{H}}}\phi_{122} = -\frac{\sqrt{3}v_\Theta}{v_{\mathrm{H}}}\phi^+, \quad \chi^0 = -\frac{3v_\Theta}{v_H}\eta_4$$

General Scalar Extensions

$$\Phi_{ijklm\cdots} = U^i_{i_1} U^j_{j_1} U^k_{k_1} U^l_{l_1} U^k_{k_1} U^m_{m_1} \cdots \phi_{i_1 j_1 k_1 l_1 m_1 \cdots}$$

$\underbrace{1\cdots1}_{j-y+1}\underbrace{2\cdots2}_{j+y-1}$ for positive charge

$\underbrace{1\cdots1}_{j-y-1}\underbrace{2\cdots2}_{j+y+1}$ for negative charge

$\underbrace{1\cdots1}_{j-y}\underbrace{2\cdots2}_{j+y}$ Neutral

$$\begin{aligned}
&(D\Phi^{*i_1i_2i_3i_4i_5\cdots})(D\Phi_{i_1i_2i_3i_4i_5\cdots})\\
=&(D\phi^{*i_1i_2i_3i_4i_5\cdots})(D\phi_{i_1i_2i_3i_4i_5\cdots}) + (DU^{*i_n}_{k_n} DU^{j_n}_{i_n})\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots}\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots}\\
&+ (U^{*i_n}_{k_n} DU^{j_n}_{i_n} D\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots}\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots} + DU^{*i_n}_{k_n} U^{j_n}_{i_n}\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots} D\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots})\\
&+ (U^{*i_m}_{k_m} DU^{j_m}_{i_m} DU^{*i_n}_{k_n} U^{j_n}_{i_n} + DU^{*i_m}_{k_m} U^{j_m}_{i_m} U^{*i_n}_{k_n} DU^{j_n}_{i_n})\\
&\quad \phi^{*\cdots i_{m-1}k_m i_{m+1}\cdots i_{n-1}k_n i_{n+1}\cdots}\phi_{\cdots i_{m-1}j_m i_{m+1}\cdots i_{n-1}j_n i_{n+1}\cdots} \qquad (4.12)\\
\supset&\frac{v_\phi}{\sqrt{2}}\left[(j+y)(U^\dagger DU)^2_2 + (j-y)(U^\dagger DU)^1_1\right] D\left(\phi^{0*}-\phi^0\right)\\
&+ v_\phi/\sqrt{2}(U^\dagger DU)^2_1\left[\sqrt{(j+y)(j-y+1)}D\phi^{+*} - \sqrt{(j-y)(j+y+1)}D\phi^-\right]\\
&\quad + v_\phi/\sqrt{2}(U^\dagger DU)^1_2\left[\sqrt{(j-y)(j+y+1)}D\phi^{-*} - \sqrt{(j+y)(j-y+1)}D\phi^+\right]
\end{aligned}$$

$$\chi^+ = \frac{v_\phi}{v_H}\left(\sqrt{(j-y)(j+y+1)}\phi^{-*} - \sqrt{(j+y)(j-y+1)}\phi^+\right) \quad \chi^0 = -\frac{2yv_\phi}{v_H}\eta^0$$

In this non-linear representation, U and heavy states are separate. As HEFT matching is to “integrate out” heavy states and leave Goldstones in U form, under this representation the matching become straight and simple, further programmable.

HEFT Matching of the real Higgs triplet extension

Non-linear representation

$$\boxed{\mathrm{H} = U\frac{1}{\sqrt{2}}\begin{pmatrix} 2\frac{v_\Sigma}{v_\mathrm{H}}\phi^\pm \\ v_\mathrm{H}+h^0 \end{pmatrix}, \qquad \Sigma = U\Phi U^\dagger,\ \Phi = \frac{1}{2}\phi_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma+\phi^0 & \sqrt{2}\phi^+ \\ \sqrt{2}\phi^- & -v_\Sigma-\phi^0 \end{pmatrix}}$$

$$\mathcal{L}_{\mathrm{RHTE}}(\mathrm{H},\Sigma) \supset (D_\mu \mathrm{H})^\dagger (D^\mu \mathrm{H}) + \langle D_\mu \Sigma^\dagger D^\mu \Sigma\rangle - V(H,\Sigma),$$

$$V(\mathrm{H},\Sigma) = Y_1^2 \mathrm{H}^\dagger\mathrm{H} + Z_1(\mathrm{H}^\dagger\mathrm{H})^2 + Y_2^2\langle\Sigma^\dagger\Sigma\rangle + Z_2\langle\Sigma^\dagger\Sigma\rangle^2 + Z_3\mathrm{H}^\dagger\mathrm{H}\langle\Sigma^\dagger\Sigma\rangle + 2Y_3\mathrm{H}^\dagger\Sigma\mathrm{H}$$

Minimum condition $\quad Y_1^2 = -Z_1 v_\mathrm{H}^2 - Z_3 v_\Sigma^2/2 + Y_3 v_\Sigma, \quad Y_2^2 = -Z_3 v^2/2 - Z_2 v_\Sigma^2 + \dfrac{Y_3 v^2}{2v_\Sigma}.$

Theoretical constraints $\quad Z_1, Z_2 \geq 0, \quad |Z_3| \geq -2\sqrt{Z_1 Z_2}$

$$\max(0, Y_3^-) < Y_3 < Y_3^+ \qquad Y_3^\pm = \frac{1}{2v_\Sigma}\left(Z_1 v_\mathrm{H}^2 + 2Z_3 v_\Sigma^2 \pm \sqrt{Z_1^2 v_\mathrm{H}^4 + 4Z_1 Z_3 v_\mathrm{H}^2 v_\Sigma^2 + 16 Z_1 Z_2 v_\Sigma^4}\right)$$

Experiment constraints

$$\xi \equiv \frac{v_\Sigma}{v_\mathrm{H}} \lesssim 0.02\ (\Delta\rho \leq 0.0005)\ , v_\mathrm{EW} = 246\ \mathrm{GeV},\ m_h = 125\ \mathrm{GeV}$$

Parameter set

$$(Z_1, Z_2, Z_3, Y_3, v_\mathrm{EW}, \xi)$$

Power Counting

$$\xi \equiv \frac{v_\Sigma}{v_\mathrm{H}}$$

EoMs

$$\mathcal{L} = \mathcal{L}_{\mathrm{kin}}(\phi_1,\phi_2,K,h,U) - V(Z_1,Z_2,Z_3,Y_3,v,\xi;\phi_1,\phi_2,K,h,U),$$

$$\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu H^a)}\right] - \frac{\partial\mathcal{L}}{\partial H^a} = 0\,, \quad H^a = (K,\phi_1,\phi_2).$$

$$K = K_0 + \xi K_1 + \xi^2 K_2 + \ldots$$

$$\phi_1 = \phi_{10} + \xi\phi_{11} + \xi^2\phi_{12} + \ldots$$

$$\phi_2 = \phi_{20} + \xi\phi_{21} + \xi^2\phi_{22} + \ldots,$$

The HEFT (LO and NLO)

$$\mathscr{L}(\xi^0) = \frac{1}{2}D_\mu h D^\mu h + \frac{1}{4}v_H^4 Z_1 - h^2 v_H^2 Z_1 - h^3 v_H Z_1 - \frac{1}{4}h^4 Z_1 - \frac{1}{4}\left(v_H^2 + 2hv_H + h^2\right)\langle V_\mu V^\mu\rangle$$

$$\mathscr{L}(\xi^1) = \frac{\xi Y_3}{4v_H}\left(-v_H^4 + 4h^2 v_H^2 + 4h^3 v_H + h^4\right)$$

$$V_\mu = U^\dagger D_\mu U$$

$$\mathscr{L}(\xi^2) = \frac{\xi^2}{4v_H^2}\left\{8h^2 D_\mu h D^\mu h + v_H^6 Z_3 + 8h^2 v_H^4(2Z_1 - Z_3) + 8h^3 v_H^3(5Z_1 - 2Z_3)\right.$$
$$+2h^4 v_H^2(16Z_1 - 7Z_3) + 2h^5 v_H(4Z_1 - 3Z_3) - h^6 Z_3$$
$$-4\left(v_H^4 + 3hv_H^3 + 4h^2 v_H^2 + 3h^3 v_H + h^4\right)\langle V_\mu V^\mu\rangle$$
$$\left.+2\left(v_H^4 + 4hv_H^3 + 6h^2 v_H^2 + 4h^3 v_H + h^4\right)\underline{\langle V_\mu \sigma_3\rangle\langle V^\mu \sigma_3\rangle}\right\}$$

Custodial symmetry breaking

Operator	$P(h)/\left[\xi^3/(Y_3 v^3)\right]$
$\langle V_\mu V^\mu\rangle\langle V_\nu V^\nu\rangle$	$(h+v)^4$
$\langle V_\mu \sigma_3\rangle\langle V^\mu \sigma_3\rangle\langle V_\nu V^\nu\rangle$	$-2(h+v)^4$
$\langle V_\mu \sigma_3\rangle\langle V_\nu \sigma_3\rangle\langle V^\mu V^\nu\rangle$	$2(h+v)^4$
$\langle V_\mu V_\nu \sigma_3\rangle\langle V^\mu \sigma_3\rangle D^\nu h$	$-4(h+v)^3$
$\langle V_\mu V_\nu \sigma_3\rangle\langle V^\nu \sigma_3\rangle D^\mu h$	$4(h+v)^3$
$\langle V_\mu V^\mu\rangle D_\nu h D^\nu h$	$4(h+v)^2$
$\langle V_\mu \sigma_3\rangle\langle V^\mu \sigma_3\rangle D_\nu h D^\nu h$	$-4(h+v)^2$
$\langle V_\mu V_\nu\rangle D^\mu h D^\nu h$	$-8(h+v)^2$
$\langle V_\mu \sigma_3\rangle\langle V_\nu \sigma_3\rangle D^\mu h D^\nu h$	$4(h+v)^2$
$D_\mu h D^\mu h D_\nu h D^\nu h$	4

p^4

$$V_\mu = U^\dagger D_\mu U$$

Identities

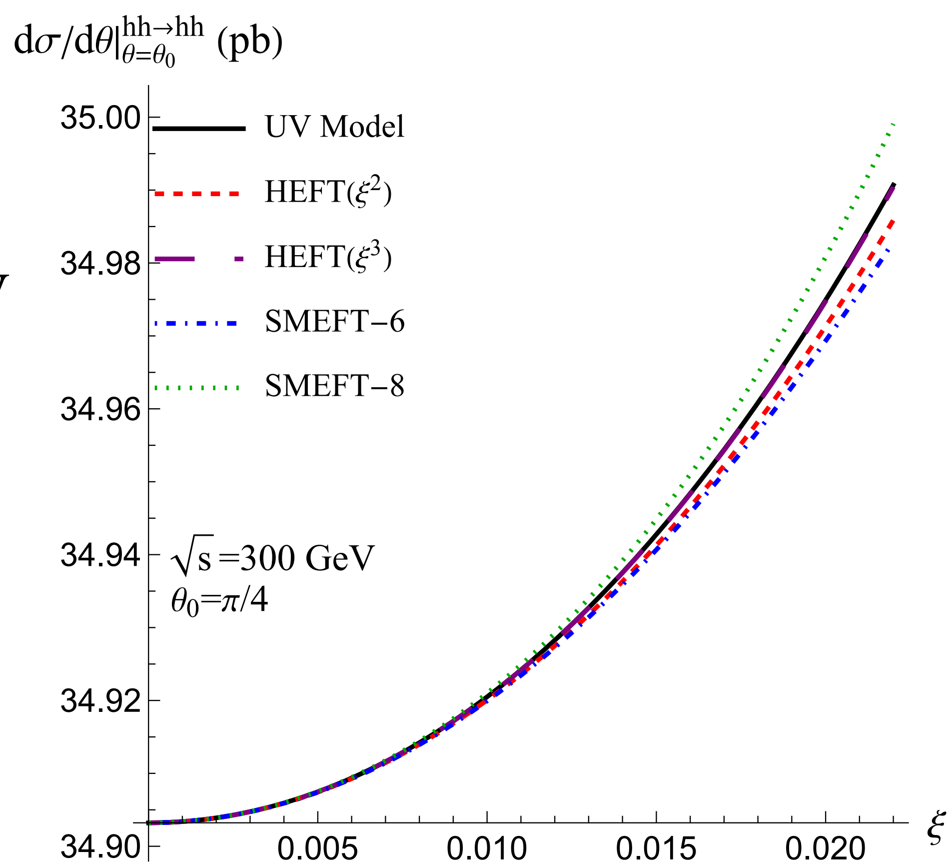
$$2\langle V_\mu V^\mu\rangle = \langle V_\mu \sigma_i\rangle\langle V^\mu \sigma_i\rangle,$$

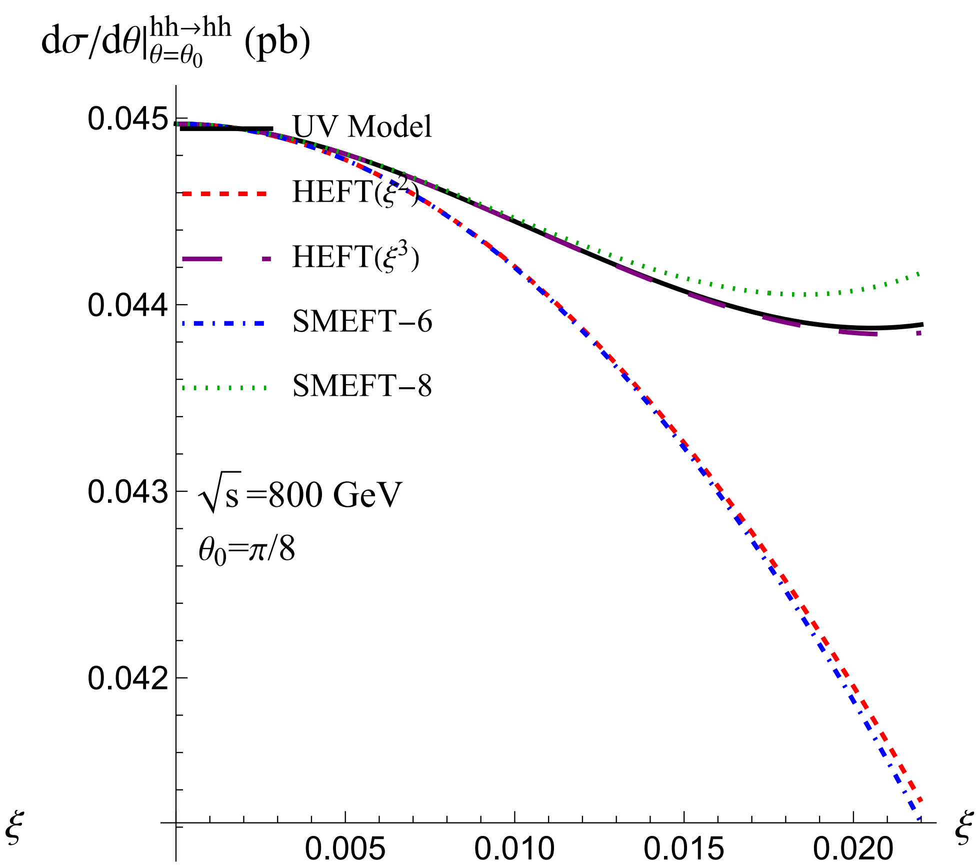
$$\langle V_\mu \sigma_1\rangle\langle V_\nu \sigma_2\rangle - \langle V_\nu \sigma_1\rangle\langle V_\mu \sigma_2\rangle = -2i\langle V_\mu V_\nu \sigma_3\rangle$$

$$\frac{\partial\langle V_\mu V^\mu\rangle}{\partial U} = 2U^\dagger D_\mu U D^\mu U^\dagger$$

SMEFT Vs. HEFT

$Y_2 > 2\text{TeV}$





Similar results also from 2504.02580,
Yi Liao, Xiao-Dong Ma, Yoshiki Uchida

HEFT converges faster, which is same as in $WW \to hh, ZZ \to hh$ process

Where is non-decoupling

Parameter set 1 $\quad (Z_1, Z_2, Z_3, Y_3, v_{\rm EW}, \xi)$

Power Counting $\quad \xi \equiv \dfrac{v_\Sigma}{v_{\rm H}}$

$$m_h^2 = 2Z_1 v_H^2 - 2\xi Y_3 v_H - 4\xi^2 v_H^2 (2Z_1 - Z_3) + O(\xi^3)$$

$$m_K^2 = \frac{Y_3 v_H}{2\xi} + 2\xi Y_3 v_H + 4\xi^2 v_H^2 (2Z_1 - Z_3) + 2\xi^2 v_H^2 Z_2 + O(\xi^3)\ ,$$

$$m_{\phi^\pm}^2 = \frac{Y_3 v_H}{2\xi} + 2\xi Y_3 v_H$$

$\xi \to 0$ corresponds to a decoupling limit.

Parameter set $(m_h, m_{\phi^\pm}^2, m_K^2, \sin\gamma, v_H, \xi)$

$$Y_3 = \frac{2v_\Sigma}{v_H^2 + 4v_\Sigma^2} m_{\phi^\pm}^2$$

$$Z_1 = \frac{1}{2v_H^2}\left(c_\gamma^2 m_h^2 + s_\gamma^2 m_K^2\right)$$

$$Z_2 = \frac{1}{2v_\Sigma^2}\left(s_\gamma^2 m_h^2 + c_\gamma^2 m_K^2 - \frac{v_H^2}{v_H^2 + 4v_\Sigma^2} m_{\phi^\pm}^2\right)$$

$$Z_3 = \frac{1}{v_H v_\Sigma}\left[s_\gamma c_\gamma \left(m_K^2 - m_h^2\right) + \frac{2v_H v_\Sigma}{v_H^2 + 4v_\Sigma^2} m_{\phi^\pm}^2\right]$$

$$\begin{pmatrix} h \\ K \end{pmatrix} = \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} h^0 \\ \phi^0 \end{pmatrix}$$

$$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu H^a)}\right] - \frac{\partial \mathcal{L}}{\partial H^a} = 0\ , \quad H^a = (K, \phi_1, \phi_2).$$

$$t \equiv v_H^2 / m_{\phi^\pm}^2, \qquad \begin{aligned} K &= K_0 + tK_1 + t^2 K_2 + \cdots\ , \\ \phi_1 &= \phi_{10} + t\phi_{11} + t^2\phi_{12} + \cdots\ , \\ \phi_2 &= \phi_{20} + t\phi_{21} + t^2\phi_{22} + \cdots\ , \end{aligned}$$

Where is non-decoupling

Parameter set 1 $(Z_1, Z_2, Z_3, Y_3, v_{\rm EW}, \xi)$

Power Counting $\xi \equiv \dfrac{v_\Sigma}{v_{\rm H}}$

$$m_h^2 = 2Z_1 v_H^2 - 2\xi Y_3 v_H - 4\xi^2 v_H^2 (2Z_1 - Z_3) + O(\xi^3)$$

$$m_K^2 = \frac{Y_3 v_H}{2\xi} + 2\xi Y_3 v_H + 4\xi^2 v_H^2 (2Z_1 - Z_3) + 2\xi^2 v_H^2 Z_2 + O(\xi^3) \ ,$$

$$m_{\phi^\pm}^2 = \frac{Y_3 v_H}{2\xi} + 2\xi Y_3 v_H$$

$\xi \to 0$ corresponds to a decoupling limit.

Parameter set $(m_h, m_{\phi^\pm}^2, m_K^2, \sin\gamma, v_H, \xi)$

$$\begin{pmatrix} h \\ K \end{pmatrix} = \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} h^0 \\ \phi^0 \end{pmatrix}$$

$$Y_3 = \frac{2v_\Sigma}{v_H^2 + 4v_\Sigma^2} m_{\phi^\pm}^2$$

$$Z_1 = \frac{1}{2v_H^2}\left(c_\gamma^2 m_h^2 + s_\gamma^2 m_K^2\right)$$

$$Z_2 = \frac{1}{2v_\Sigma^2}\left(s_\gamma^2 m_h^2 + c_\gamma^2 m_K^2 - \frac{v_H^2}{v_H^2 + 4v_\Sigma^2} m_{\phi^\pm}^2\right)$$

$$Z_3 = \frac{1}{v_H v_\Sigma}\left[s_\gamma c_\gamma \left(m_K^2 - m_h^2\right) + \frac{2v_H v_\Sigma}{v_H^2 + 4v_\Sigma^2} m_{\phi^\pm}^2\right]$$

[S. Dawson et al, 2311.16897],

Scaling:

if $m_{\phi^\pm}^2 \sim \mathcal{O}(t^{-1}), \quad m_K^2 \sim \mathcal{O}(t^{-1}), \quad \sin\gamma \sim \mathcal{O}(t), \quad \xi \sim \mathcal{O}(t)$

Decoupling

if $m_{\phi^\pm}^2 \sim \mathcal{O}(t^{-1}), \quad m_K^2 \sim \mathcal{O}(t^{-1})$ Non-decoupling

HEFT in non-decoupling regime

$$\mathcal{L}(t^{-1}) = \frac{v_H^2\left[(4\xi^2+1)m_K^2(s_\gamma+\xi c_\gamma)^2-\xi^2 m_\phi^2\right]}{8(4\xi^2+1)} - \frac{h^3 m_\phi^2 s_\gamma^2(2\xi c_\gamma+s_\gamma)}{2\xi(4\xi^2+1)v_H}$$
$$-\frac{h^4}{8\xi^2(4\xi^2+1)^2 v_H^2 m_K^2}\Big\{m_\phi^2 s_\gamma^2\left[(4\xi^2+1)m_K^2(6(2\xi^2-1)c_\gamma^4+7c_\gamma^2+18\xi c_\gamma^3 s_\gamma-4\xi c_\gamma s_\gamma-1)\right.$$

$$\mathcal{L}(t^0) = \frac{1}{2}\langle V_\mu\sigma_3\rangle\langle V^\mu\sigma_3\rangle\left\{h^2 s_\gamma\left[-\xi c_\gamma^3+\frac{c_\gamma m_\phi^2(3s_\gamma(c_\gamma-2\xi s_\gamma)+4\xi)}{(4\xi^2+1)m_K^2}+s_\gamma^3\right]-2h\xi v_H s_\gamma+\xi^2 v_H^2\right\}$$
$$+\frac{1}{4}\langle V_\mu V^\mu\rangle\left\{\frac{h^2}{\xi}\left[\frac{m_\phi^2 s_\gamma(c_\gamma(3(8\xi^2-1)s_\gamma^2-16\xi^2)+2\xi(9s_\gamma^2-8)s_\gamma)}{(4\xi^2+1)m_K^2}+c_\gamma s_\gamma(4\xi^2+(1-4\xi^2)s_\gamma^2)\right.\right.$$
$$\left.\left.+\xi(-5s_\gamma^4+2s_\gamma^2-1)\right]-2hv_H(c_\gamma-4\xi s_\gamma)-(4\xi^2+1)v_H^2\right\}$$

$L(t^1)$ Op: $\langle V_\mu V^\mu\rangle\langle V_\nu V^\nu\rangle$

$$\frac{v_H^2(4\xi c_\gamma+s_\gamma)^2}{8m_K^2}$$
$$-\frac{hv_H(4\xi c_\gamma+s_\gamma)}{8\xi(4\xi^2+1)m_K^4}\Big\{s_\gamma^2\left[c_\gamma^2(4(16\xi^4-1)m_K^2+3(1-16\xi^2)m_\phi^2)+16\xi^2 m_\phi^2\right]$$
$$+2\xi c_\gamma s_\gamma\left[(27c_\gamma^2-16)m_\phi^2-2(4\xi^2+1)(5c_\gamma^2-4)m_K^2\right]+2c_\gamma^2 m_\phi^2\left[3(4\xi^2-1)c_\gamma^2-8\xi^2+3\right]\Big\}$$

The comparison between HEFTs and SMEFT is progress.

HEFT at 1-loop

$$\frac{\delta^2 S}{\delta\phi_i(x)\delta\phi_j(y)} = (1+2\xi^2)\langle D_\mu U^\dagger D^\mu U\rangle\,\delta_{ij}\delta^{(4)}(x-y) + \langle U^\dagger D_\mu U\sigma_i\rangle\,\langle U^\dagger D_\mu U\sigma_j\rangle\,\delta^{(4)}(x-y) - \langle (U^\dagger D_\mu U)^2\rangle\,\delta_{ij}\delta^{(4)}(x-y)$$
$$+\,\mathrm{i}\,\langle U^\dagger D_\mu U\sigma_l\rangle\,(\varepsilon_{lij}+2\xi^2\delta_{l3}\varepsilon_{3ij})\,D^\mu\delta^{(4)}(x-y)$$
$$-\,(1+4\xi^2)\delta_{ij}D^2\delta^{(4)}(x-y) - \mathrm{i}D^\mu\Big[\langle U^\dagger D_\mu U\sigma_l\rangle\,(\varepsilon_{lji}+2\xi^2\delta_{l3}\varepsilon_{3ji})\delta^{(4)}(x-y)\Big]$$

$$V^i_\mu = \langle U^\dagger D_\mu U\sigma_i\rangle$$

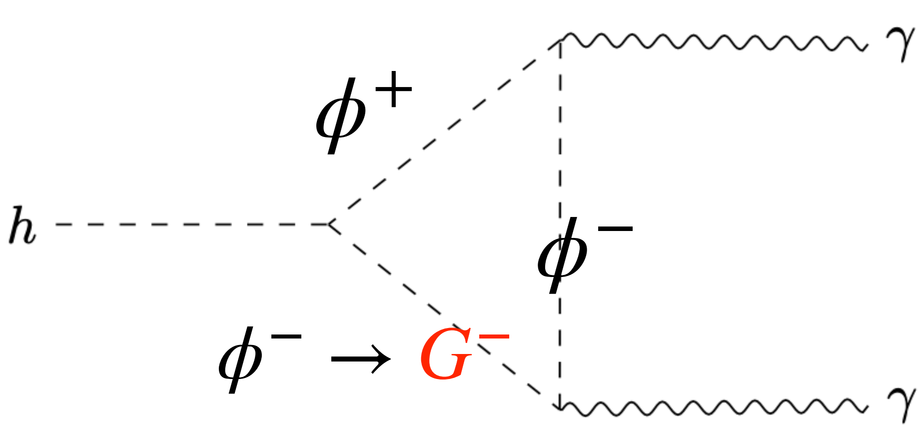
$\boxed{\text{Universal One-loop Action}}$

Brian Henning, Xiaochuan Lu, and Hitoshi Murayama, 1412.1837

$$32\pi^2\Delta\mathcal{L}^{\text{1-loop}}_{\text{EFT}}$$
$$\supset -h\left(2a\,\langle V_\mu V^\mu\rangle - bV^1_\mu V^\mu_1 - bV^2_\mu V^\mu_2\right) d\log\frac{M^2}{\mu^2}$$
$$+\frac{d}{6M^2}\partial_\mu h D^\mu\left[2a\,\langle V_\nu V^\nu\rangle - b\left(V^1_\nu V^\nu_1 + V^2_\nu V^\nu_2\right)\right] + \frac{g'^2 d}{6M^2}hB_{\mu\nu}B^{\mu\nu}$$

$$V_\mu \equiv U^\dagger D_\mu U \xrightarrow[\text{gauge}]{\text{unitary}} W^\pm_\mu, Z_\mu$$

Heavy-particle only



1. Heavy-light mixing contribution?
2. How to deal with Goldstones?

HEFT at 1-loop

$$W^\mu \to \hat{W}^\mu + W^\mu, \quad B^\mu \to \hat{B}^\mu + B^\mu, \quad H \to \hat{H} + H, \quad U \to \hat{U}U,$$

Stueckelberg Transformation (get unitary gauge)

$$\hat{W}^\mu \to \hat{U}\hat{W}^\mu\hat{U}^\dagger + \frac{i}{g_2}\hat{U}\partial^\mu\hat{U}^\dagger \qquad W^\mu \to \hat{U}W^\mu\hat{U}^\dagger,$$

$$U = 1 + i\frac{\pi}{v_{\rm EW}} - \frac{1}{2}\frac{\pi^2}{v_{\rm EW}^2} + \mathcal{O}(\pi^3), \quad \pi = \pi_i\sigma_i,$$

$$\mathrm{tr}(D_\mu U^\dagger D^\mu U) \to \quad -4\ \mathrm{tr}\left\{\pi\left(\frac{1}{v^2}\hat{D}_\mu\hat{D}^\mu + g_2^2\frac{1}{v^2}\hat{C}_\mu\hat{C}^\mu\right)\pi\right\}$$

$$\hat{C}^\mu = \hat{W}_1^\mu\sigma_1 + \hat{W}_2^\mu\sigma_2 + \frac{1}{c_{\rm W}}\hat{Z}^\mu\sigma_3$$

In progress

Summary

- HEFT is more general than SMEFT. To study non-decoupling effects we need develop HEFT tools.

- We get a non-linear framework of UV models (with general scalar extension) which encapsulate the Goldstones in its exponential form. Under this framework, matching UV models to HEFT become straightforward.

- We match the triplet Higgs model to HEFT in the decoupling regime by functional method and show its numerical results.

- For non-decoupling regime, we get the HEFTs by scaling method.

- For one-loop matching, we are trying to embed non-linear Goldstones into the universal one-loop actions (developed in SMEFT matching).

Thanks for your attention!