第三章:实验统计分析工具ROOT

授课人: 董燎原

中国科学院高能物理研究所

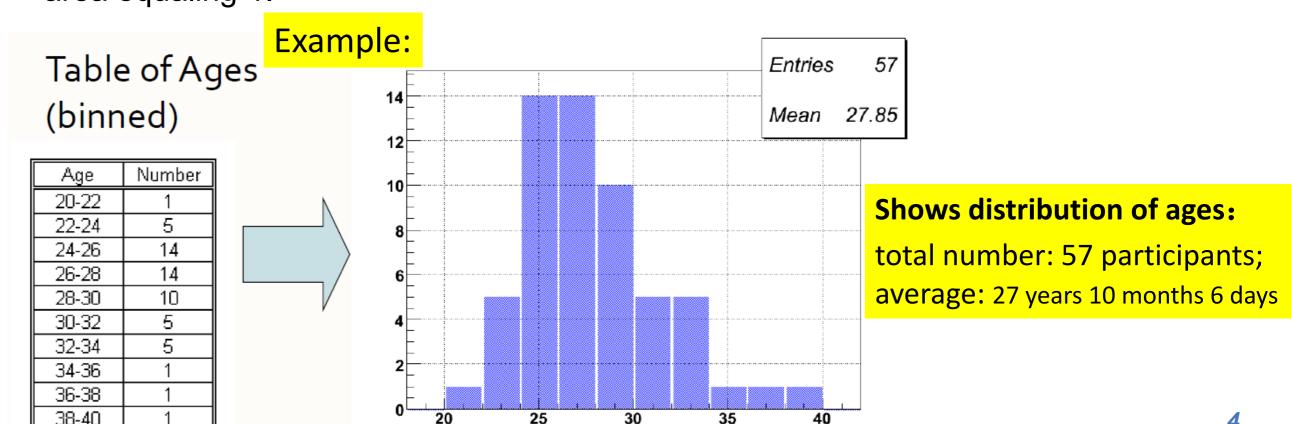
Histograms 直方图

Outline

- We have seen:
 - what is a histogram;
 - how to create a one-dimensional histogram;
 - how to draw the histogram
- We will see and work on with the exercises
 - how to extract information from an histogram
 - how to manipulate histograms
 - histograms in multi-dimension
 - what is a profile histogram
 - weighted histograms
 - sparse histograms
- More detailed on the ROOT Graphics Pad and the Canvas
- Visualization techniques in ROOT

What is Histogram

- In statistics, a graphical representation, showing a visual impression of the distribution of data.
- An estimate of the probability distribution of a variable
- Each rectangle erected over discrete intervals (bins)
- Area of a rectangle equals to the frequency of the observations in the interval (bin).
- Height of a rectangle also equals to the frequency density of the interval, i.e., the frequency divided by the width of the interval.
- The total area of the histogram is equal to the number of data.
- A histogram may also be normalized displaying relative frequencies, with the total area equaling 1.



What Histogram Tells

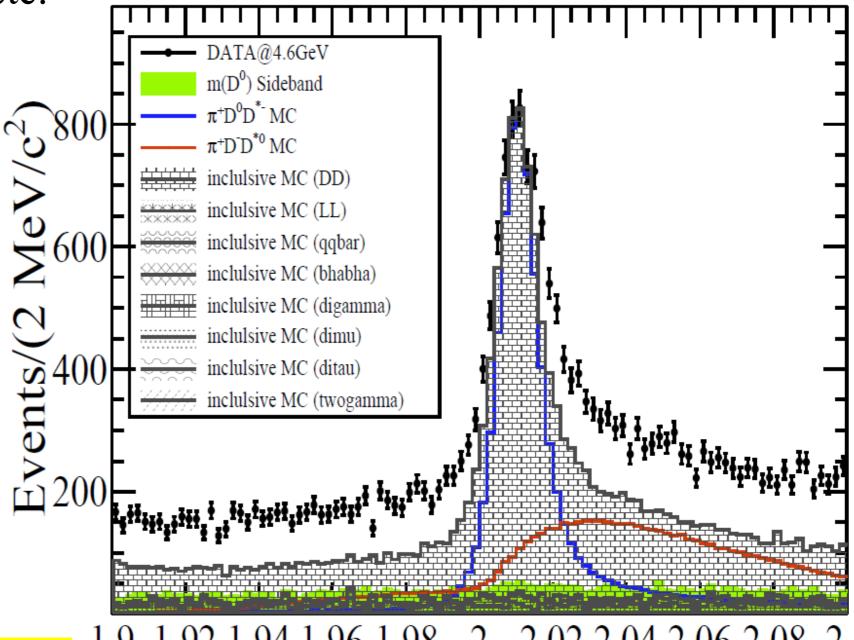
• Presenting data to determine which causes dominate

• Understanding the distribution of occurrences of different problems,

causes, consequences, etc.

• What is the most common system response?

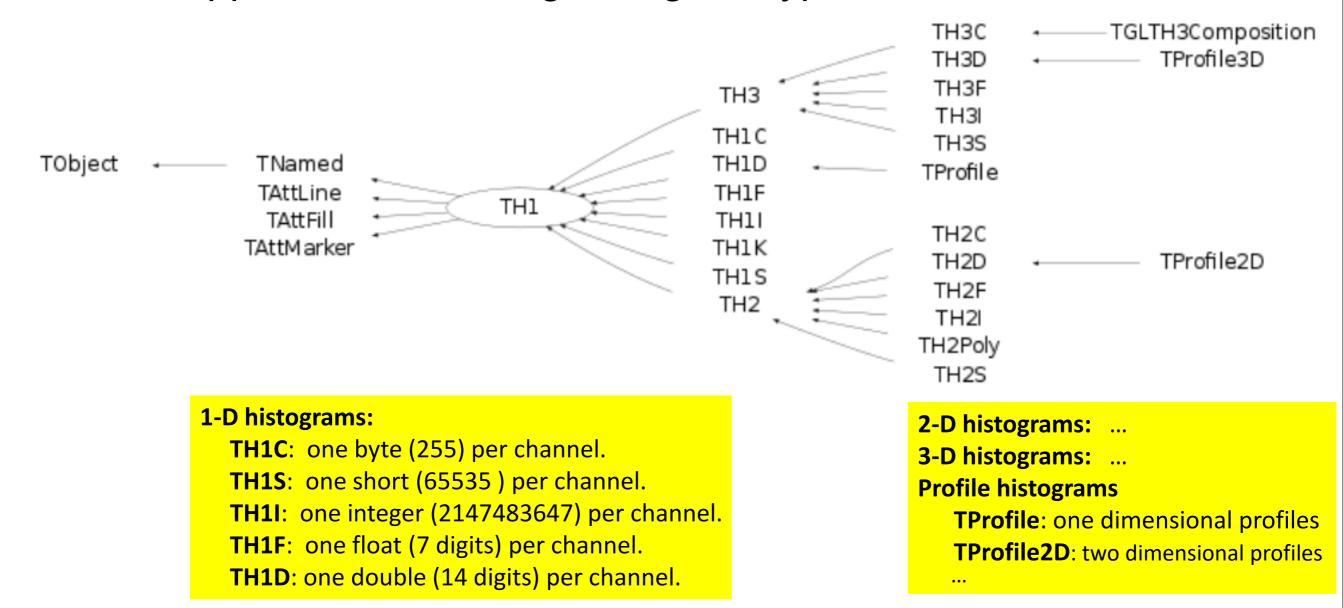
- What distribution
 (center, variation and shape) does the data have?
- Does the data look symmetric or is it skewed to the left or right?



右图有什么地方需要改进?

Histogram Classes

ROOT supports the following histogram types:



- TH1 is the base class for all Histogram classes.
- TH1,TH2,TH3 are generic. Specialized should be used for constructing the objects.
- Most used classes are TH1(2,3)F and TH1(2,3)D.

Creating Histograms

TH1 constructor: the name of the histogram (name), the title (title), the number of bins (nbinsx), the x minimum (xlow), x maximum (xup) and array of low-edges for each bin (xbins).

```
TH1F();
TH1F(const TVectorF& v);
TH1F(const TH1F& h1f);
TH1F(const char* name, const char* title, Int_t nbinsx,
const Float_t* xbins);
TH1F(const char* name, const char* title, Int_t nbinsx,
const Double_t* xbins);
TH1F(const char* name, const char* title, Int_t nbinsx,
Double_t xlow, Double_t xup);
virtual ~TH1F();
```

For equidistant bins:

```
TH1F * h1 = new TH1F("h1","h1 title",100,0,4.4);
TH2F * h2 = new TH2F("h2","h2 title",40,0,4,30,-3,3);
```

For non-equidistant bins:

```
TH1D * h1 = new TH1D("h1","Example histogram",nbins,xbins);
```

• xbins: array of (nbins+1) values with the bin-edges

Filling Histograms

A histogram is typically filled with statements like:

```
h1 \rightarrow Fill(x);
 h1->Fill(x,w); //with weight
                                                               Histo from a Gaussian
                                                                                                          h1
 h2 \rightarrow Fill(x, y);
                                                                                                      Entries
                                                                                                             5000
                                                                                                      Mean 0.008066
 h2 \rightarrow Fill(x, y, w);
                                                               140
                                                                                                            0.9996
 h3 \rightarrow Fill(x, y, z);
                                                               120
 h3 \rightarrow Fill(x, y, z, w);
                                                               100
 root[] TH1F h1("h1", "Histo from a Gaussian", 100, -3, 3);
 root[] h1.FillRandom("gaus", 5000);
 root[] h1.Draw();
Increment a bin number by calling
                                                                20
     TH1::AddBinContent()
```

replace the existing content via

TH1::SetBinContent()

access the bin content of a given bin:

TH1::GetBinContent()

```
Double_t binContent = h->GetBinContent(bin);
```

Drawing Histograms

- call the Draw method of a histogram (TH1::Draw) to draw the histogram
- it creates a THistPainter object and saves a pointer to painter as a data member of the histogram
- By default, the TH1::Draw clears the pad before drawing the new image of the histogram. You can use the "SAME" option to leave the previous display in tact and superimpose the new histogram.
- The same histogram can be drawn with different graphics options
- Most options can be concatenated without spaces or commas
- The options are not case sensitive
- You can also set the default drawing option with TH1::SetOption. To see the current option use TH1::GetOption.

Try examples in tutorials: *hist/draw2dopt.C*

Axis and Bins

The histogram class has an axis class which contains the bins

```
TAxis * axis = h1->GetXaxis();
```

– one can query the number of bins, lower/upper bin edge from the axis:

```
axis->GetNbins();
axis->GetBinLowEdge(bin_number);
axis->GetBinCenter(bin_number);
axis->GetBinUpEdge(bin_number);
```

for the corresponding bin number given x value use

```
bin_number = axis->FindBin(x_value);
```

one can set the axis range (e.g. for zooming the histogram)

```
axis->SetRange(firstbin,lastbin);
```

- N.B.: axis bin number starts from 1
 - bin number 0 is the Underflowbin
 - bin number nbin+1 is the Overflowbin

Giving Titles to the X, Y and Z Axis

Because the axis title is an attribute of the axis, you have to get the axis first and then call **TAxis**::SetTitle.

```
h->GetXaxis()->SetTitle("X axis title");
h->GetYaxis()->SetTitle("Y axis title");
```

The histogram title and the axis titles can be any **TLatex** string.

```
h->GetXaxis()->SetTitle("E_{T}");
```

specify the histogram title and the axis titles at creation time given in the "title" parameter. They must be separated by ";":

```
TH1F* h=new TH1F("h","Histogram title; X Axis; Y Axis; Z Axis",100,0,1);
TH1F* h=new TH1F("h","Histogram title; Y Axis",100,0,1);
TH1F* h=new TH1F("h","; Y Axis",100,0,1);
h->SetTitle("Histogram title; Another X title Axis");
```

Setting Axis Attributes

Use TH1::GetXaxis() to get the pointer to the TAxis object to set the Axis attributes

See Methods in **Taxis** class

```
TAxis* TH1::GetXaxis() const
TAxis* TH1::GetYaxis() const
TAxis* TH1::GetZaxis() const
```

Getting Data From an Histogram

 To extract content and error from bin number ibin of the histogram h1:

- By default histograms have a bin error equal to √N, where N is the bin content.
 - It is assumed that the observed bin content follows a Poisson distribution.
- One can set a different error for each bin:

```
root [1] h1->SetBinError(ibin,error)
```

 N.B: when setting the error in each bins, the histogram will store them in an internal array and will change its default style when drawn

Histogram Operations (1)

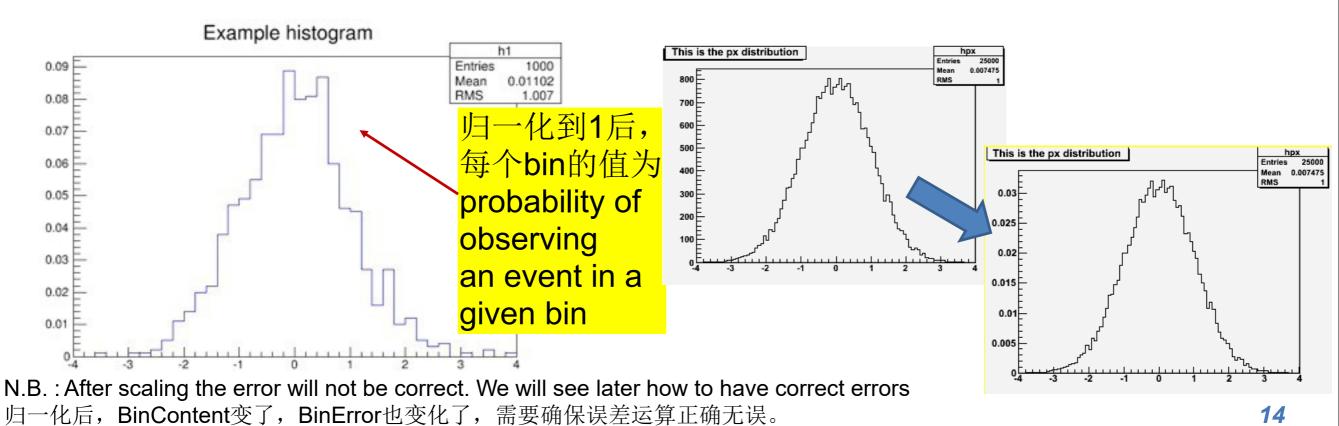
- Histogram scaling (normalization) 直方图的归一化
 - -useful for plotting histograms in the same pad (归一化到同一积分可直观比较两种分布之间的差别)
 - -useful for seeing histogram as an estimate of a probability density function (PDF) (可直观估计几率密度函数)

Method 1:

Double_t scale = norm/h->Integral(); h->Scale(scale); //归一化到norm(=1.0)

Method 2:

h->DrawNormalized("option_t", norm);



Histogram Operations (2)

- Add Histograms: 直方图的相加和相减
 - -merge two histograms which have same axis:

```
TH1::SetDefaultSumw2();
TH1D *h3 = new TH1D("h3","h1+h2",nbin,xmin,xmax);
h3->Add(h1,h2,a,b); //h3 = a*h1+b*h2 (一般a=b=1)
```

相加:常用于实验数据的叠加。

can also be used to subtract histograms

```
h3->Add(h1,h2,a,-b); //h3 = a*h1-b*h2 (一般a=b=1)
```

相减: 常用于实验中本底的扣除。

注意:

- 1. 两个直方图的区间大小和区间数必须相同,才能操作。
- 2. 在归一化和加减乘除中,要正确得到统计误差,需要调用: TH1::SetDefaultSumw2(); 或对每个直方图(如his)调用 his->Sumw2();

静态函数SetDefaultSumw2中的注释:

```
// static function. When this static function is called with sumw2=kTRUE,
```

// all new histograms will automatically activate the storage of the sum of squares of errors,

// ie TH1::Sumw2 is automatically called.

Histogram Operations (3)

Divide Histograms: 直方图相除和相乘

```
TH1D *h3 = new TH1D("h3","h1/h2",nbin,xmin,xmax);
h3->Sumw2();
h3->Divide(h1,h2,a,b);
H3->Multiply(h1,h2,a,b);
```

相除: 常用于实验中的效率的估计。

相乘: 常用于实验中的对分布的修正(如效率修正等)。

-if h1 is a subset of h2, the bin content of h3 is binomially distributed ■use option "B" to get the correct bin errors (h1和h2不独立:如h1包含于h2)

```
h3->Divide(h1,h2,a,b,"B");
```

二项分布误差:

$$\sigma = \sqrt{\frac{\frac{n_1}{n_2}(1 - \frac{n_1}{n_2})}{n_2}}$$

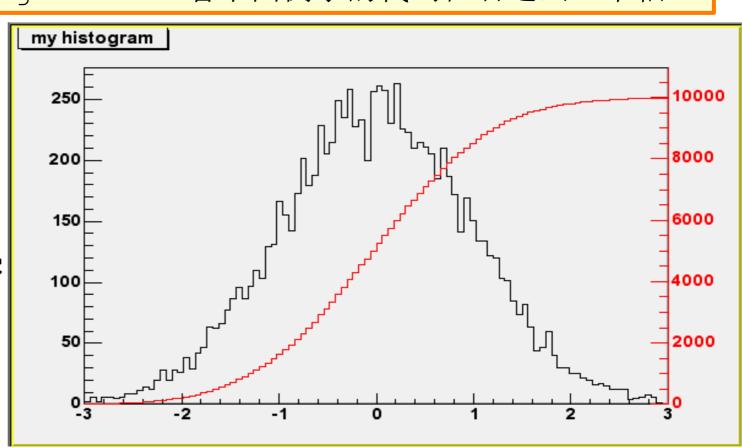
Miscellaneous Operations

```
TH1:: Smooth () - smoothes the bin contents of a 1D histogram.
TH1:: Integral (Option t *opt) - returns the integral of bin contents in a
       given bin range. If the option "width" is specified, the integral is the sum
       of the bin contents multiplied by the bin width in x.
TH1::GetMean(int axis) - returns the mean value along axis.
TH1::GetRMS (int axis) - returns the Root Mean Square along axis.
TH1::GetEntries() - returns the number of entries.
TH1::Add(const TH1* h1, Double t c1 = 1)
TH1::Add(TF1* h1, Double t c1 = 1, Option t* option = "")
TH1::Add(const TH1* h1, const TH1* h2, Double t c1 = 1,
Double t c2 = 1) - add histograms weighted by c1 and c2.
```

Superimposing Histograms with Different Scales

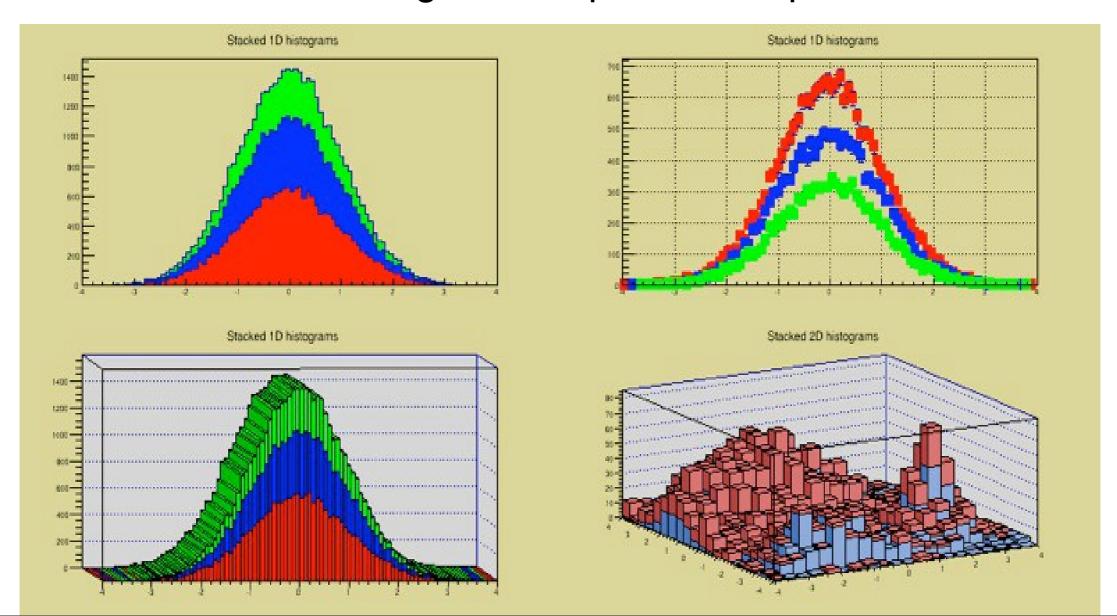
```
TH1F *h1 = new TH1F("h1","my histogram",100,-3,3);
... // (填高斯分布)
h1->Draw();
TH1F *hint1 = new TH1F("hint1","my histogram",100,-3,3);
... // (填积分值)
//scale hint1 to the pad coordinates
Float_t rightmax = 1.1*hint1->GetMaximum();
Float_t scale = gPad->GetUymax()/rightmax;
hint1->SetLineColor(kRed);
hint1->Scale(scale);
hint1->Draw("same");
... // (draw an axis on the right side 看下面例子的代码在右边画一个轴)
```

example:
\$ROOTSYS/tutorials/hist/twoscales.C



Histogram Stacks

- THStack
 - Collection of 1D or 2D histograms.
 - Allow to draw several histograms in one go, on top of each other.
 - The frame is computed automatically.
 - Often better than doing several plots with option "SAME"



Histogram Stacks

A THStack is a collection of TH1 (or derived) objects. Use THStack::Add(TH1 *h) to add a histogram to the stack.

```
TCanvas c1("c1", "stacked hists", 10, 10, 700, 900);
THStack hs ("hs", "test stacked histograms");
hs.Add(h1); \\red
hs.Add(h2); \\blue
hs.Add(h3); \\green
c1.Divide (1,2);
                          1400
c1.cd(1);
                                                     600
                          1200
hs.Draw();
                                                     500
c1.cd(2);
                          1000F
                                                     400
hs->Draw("nostack");
                          800
                                                     300E
                           600E
                                                     200
                           400F
                                                     100
                           200
```

常用于实验数据中不同类型本底的叠加。

example: \$ROOTSYS/tutorials/hist/hstack.C

Histogram Stacks: example

一个显示本底成分的例子:

```
► Data
                                              Events/(10.0 MeV/c
                                                                         other bg
                                                 100
 Hist data->Draw("E1");
                                                                         ggbar bg
 Hist others->SetFillColor(kBlue);
                                                                         opencharm bg
 Hist opencharm->SetFillColor(kGreen);
                                                                         Hist qqbar->SetFillColor(kRed);
 Hist DsSTDs DsDs->SetFillColor(7);
                                                  50
 THStack *hs = new THStack("hs", "Stacked");
hs->Add(Hist others);
 hs->Add(Hist qqbar);
hs->Add(Hist opencharm);
hs->Add(Hist DsSTDs DsDs);
hs->Draw("same");
                                                              M_{\pi^+\pi^-\pi^0}\,(GeV/c^2)
 Hist_data->Draw("sameE1");	──为何还要再画一次数据的图
leg = new TLegend(0.55, 0.60, 0.85, 0.9);
   leg->AddEntry(Hist data, "Data", "lep");
   leg->AddEntry(Hist others, "other bg", "f");
   leg->AddEntry(Hist qqbar, "qqbar bg", "f");
   leq->AddEntry(Hist opencharm, "opencharm bg", "f");
   leg->AddEntry(Hist DsSTDs DsDs,"D* {s}D {s}&D {s}D_{s}D_{s}, "f");
   leg->SetTextSize(0.06);
   leg->SetLineColor(10);
   leg->SetFillColor(10);
   leq->Draw();
```

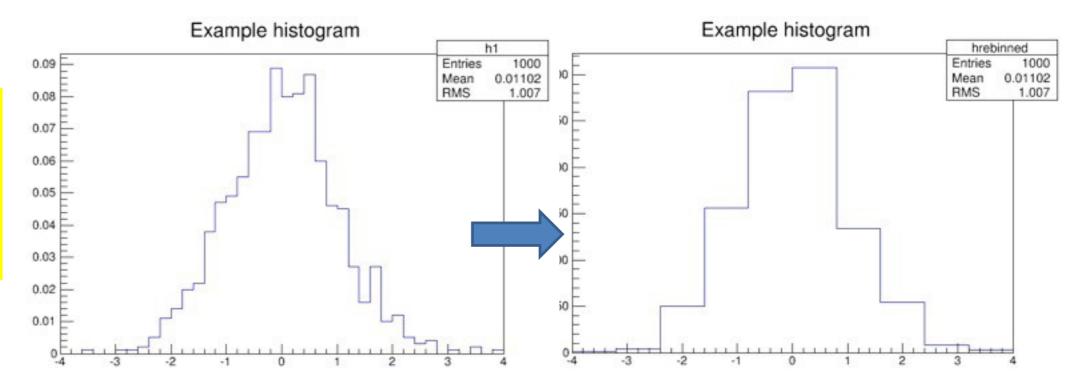
Histogram Re-Binning

- Rebin: merge bins together.
 - Possible to merge adjacent bins in a given group,
 - e.g. an histogram with 100 bins can be re-binned in a histogram with 25 bins .
 - Possible to merge bins using new bin edges provided by the user, i.e making a new a variable bin histograms.
 - the new bin edges must corresponds to existing bin edges. It is not possible to split bins.

 Histogram errors and statistics are re-computed according to the new binning.

TH1 * hrb = h1->Rebin(4,"hrebinned");

Rebin original
histogram (40 bins)
in a new one with
10 bins grouping 4
bins together
(ngroup =4)



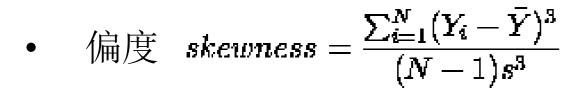
Statistics Display

By default, a histogram drawing includes the statistics box. Use TH1::SetStats(kFALSE) to eliminate the statistics box. gStyle->SetOptStat(mode) allow you to set displayed information.

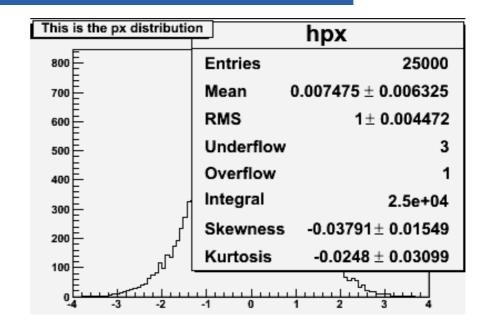
The parameter mode has up to nine digits that can be set OFF (0) or ON as follows: mode = ksiourmen (default = 000001111 or nemr)

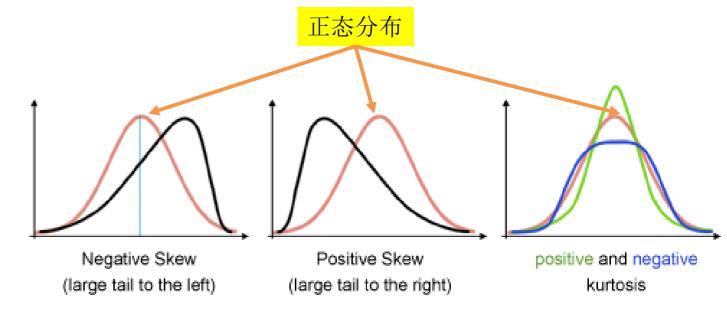
Statistical Issues in Histogram

- Mean
- Minimum / Maximum
- Standard Deviation (RMS)
- Bin width / Bin number



• 峰度:
$$kurtosis = \frac{\sum_{i=1}^{N} (Y_i - \bar{Y})^4}{(N-1)s^4} - 3$$





The binning issue: no "best" number of bins, and different bin sizes can reveal different features of the data. It requires some judgment, and perhaps some experimentation, based on the analyst's experience.

2D Histograms

- Frequency distribution of (X,Y) observations.
 - Construct 2D histogram specifying the number of bins in X axis, the minimum and maximum of axis range and the same for the Y axis

```
TH2D * h2 = new TH2D("h2","Example 2D Hist", 40, -4., 4., 40, -4., 4);
```

- fill 2D histogram with 10000 x,y normal data

```
for (int i = 0; i<10000; ++i) {
  double x = gRandom->Gaus(0,1);
  double y = gRandom->Gaus(1,2);
  h2->Fill(x,y);
}
```

- use h2->Draw() for drawing, several options are available
 - Color, Contour, lego, surface and box plots

Drawing 2D histograms

Color plots:

```
h2->Draw("COLZ");
```

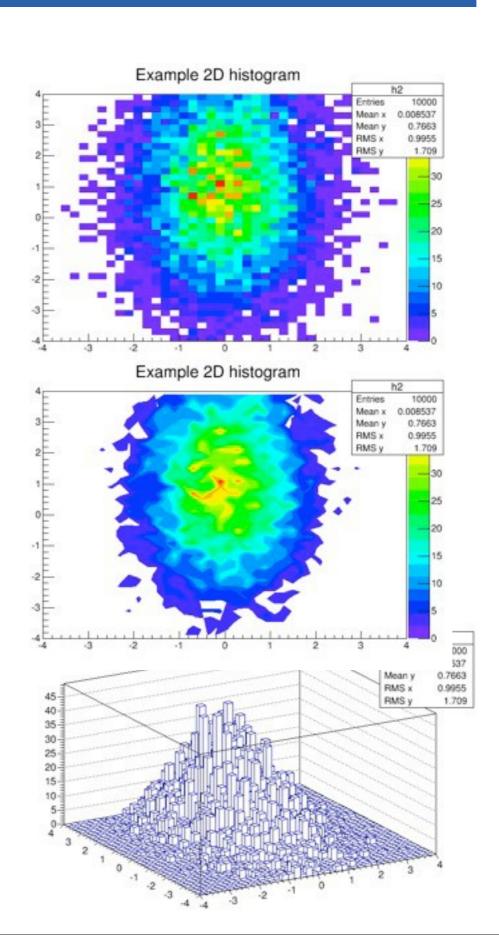
"Z" means drawing the color palette for the bin content (i.e. the Z axis)

Contour plots:

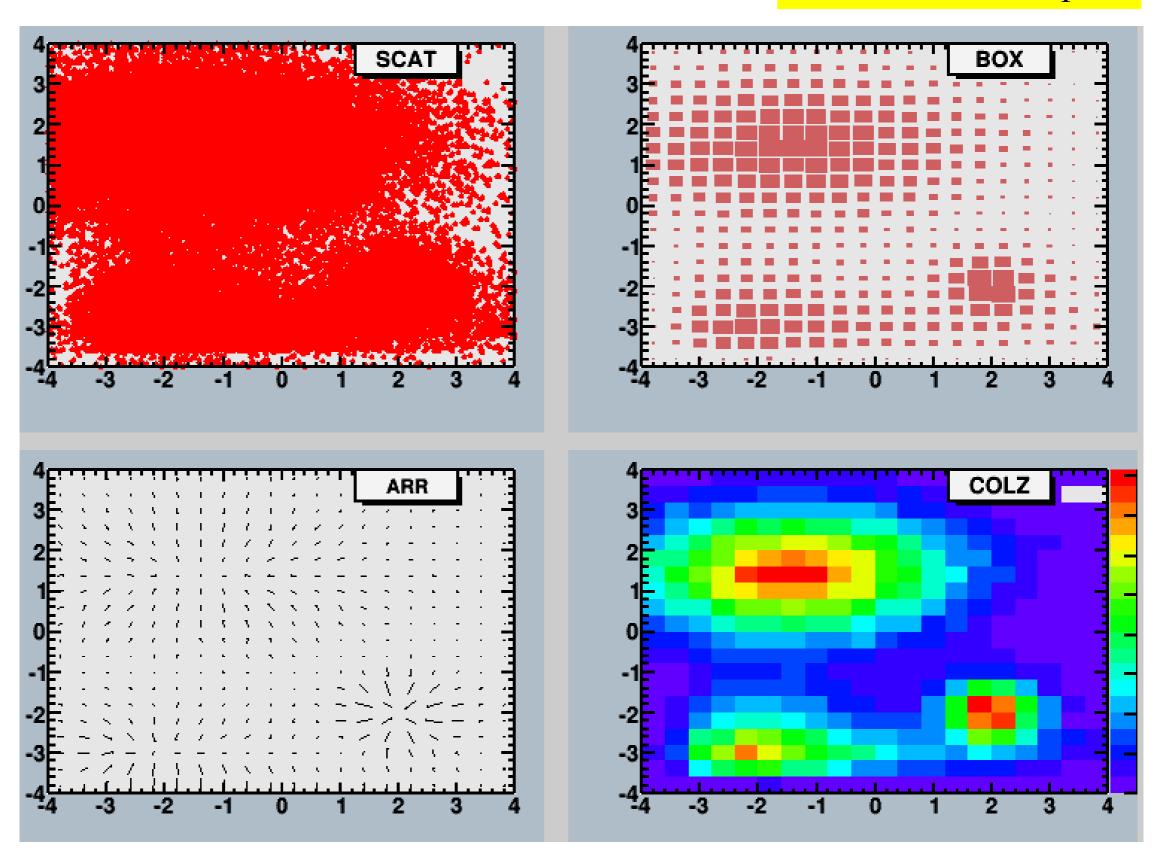
```
h2->Draw("CONTZ");
```

Lego plots:

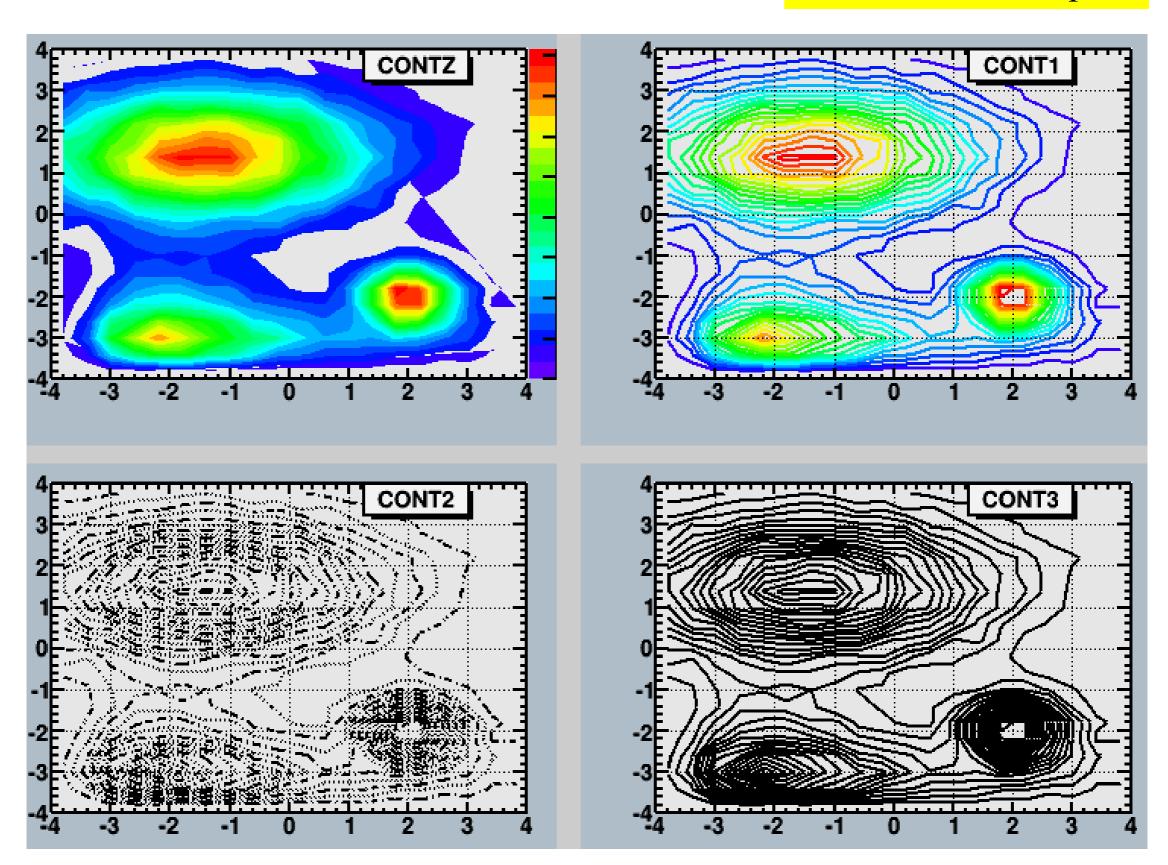
```
h2->Draw("LEGO");
```



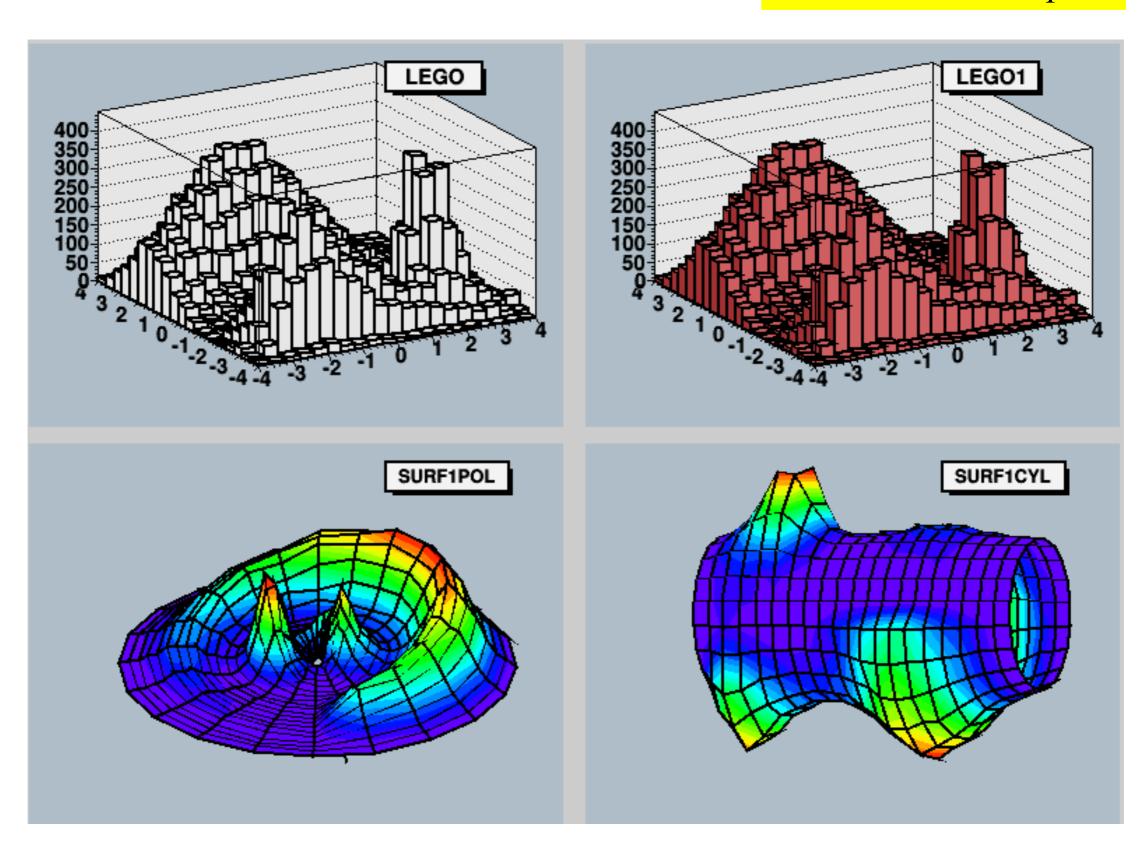
2-D drawing options (hist/draw2dopt.C)



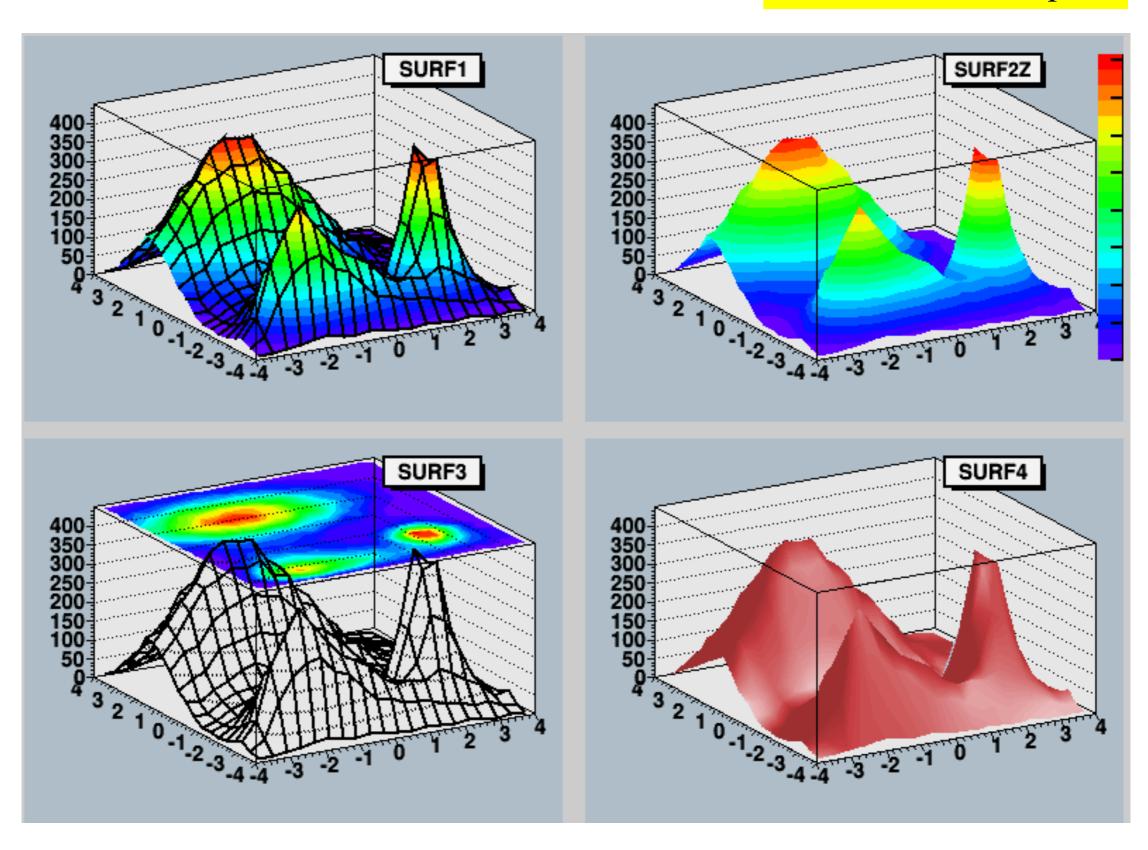
2-D drawing options (hist/draw2dopt.C) (2)



2-D drawing options (hist/draw2dopt.C) (3)

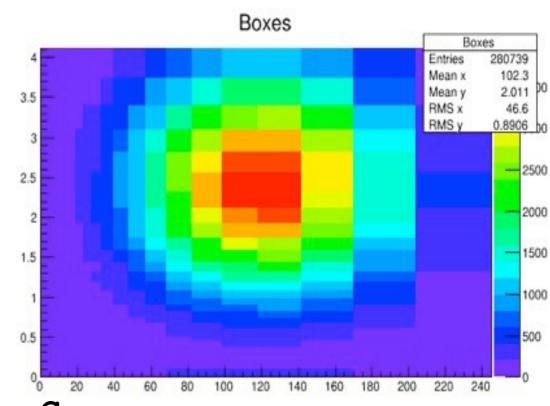


2-D drawing options (hist/draw2dopt.C) (4)



Other Histogram Types

- THnSparse
 - multi-dimensional sparse histogram useful to save memory only when a small fractions of the bins are filled
 - often more effective than using a TH3
- THn
 - –non-sparse multi-dimensional histogram (NDim > 3)
- TH2Poly
 - -2D histogramwith polygons shapes



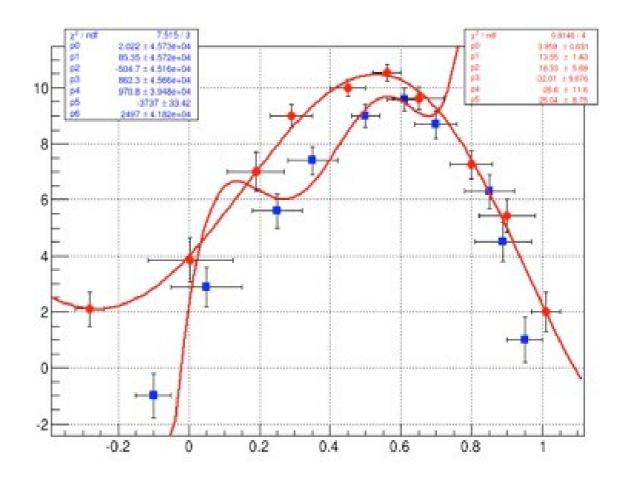
example:

\$ROOTSYS/tutorials/hist/th2polyBoxes.C \$ROOTSYS/tutorials/hist/sparsehist.C

More on Graphs

TMultiGraph

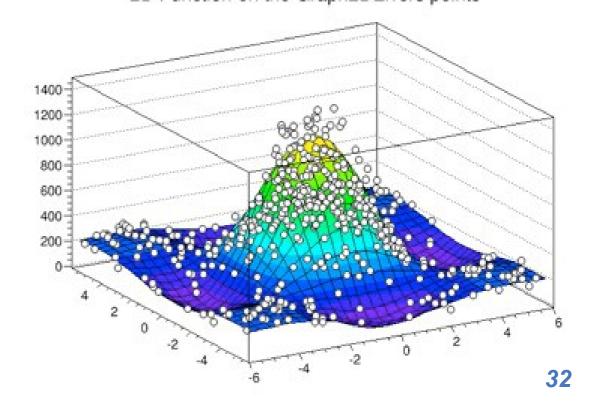
- Graphs collection
- Equivalent to THStack but for graphs
- Allow to Draw several graphs in one go



2-Dimensional Graphs

- -TGraph2D, TGraph2DErrors
 - classes for storing and drawing (X,Y,Z)
 data and (for TGraph2DErrors) with
 error in both X,Y and Z
 - X,Y are independent variables and Z the dependent variable

2D Function on the Graph2DErrors points



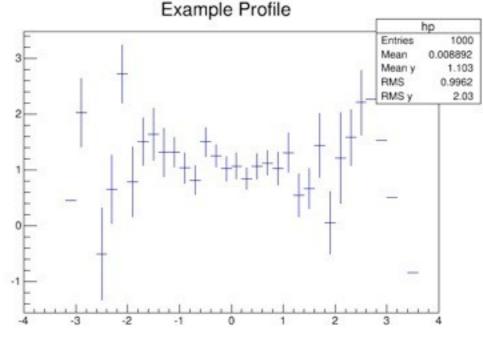
Profile Histograms

A 2D Histogram can be projected into a 1D histogram

```
TH1 * hX = h2->ProjectionX();
```

- when projecting on the X axis the bins with the same x value are all summed together.
- A Profile histogram is a different way of projecting 2D data.
 - for each bin in x the sample mean of the y values is plotted

```
TProfile * hp = new TProfile("hp","hp",40,-4.0,4.0);
for (int i = 0; i<10000; ++i) {
   double x = gRandom->Gaus(0,1);
   double y = gRandom->Gaus(1,2);
   hp->Fill(x,y);
}
hp->Draw();
```



- various options for displaying errors
- default uses error in the sample mean (RMS/ \sqrt{N})

Projections

- One can make:
 - a 1-D projection of a 2-D histogram or profile.
 See

TH2::ProfileX, TH2::ProfileY, TProfile::ProjectionX, TProfile2D::ProjectionXY, TH2::ProjectionX, TH2::ProjectionY.

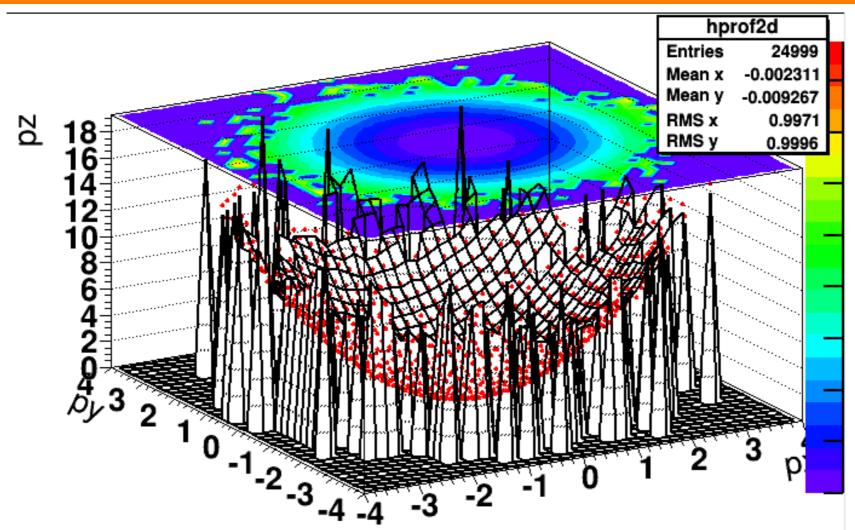
- a 1-D, 2-D or profile out of a 3-D histogram see TH3::ProjectionZ, TH3::Project3D.
- These projections can be fit via:
 TH2::FitSlicesX, TH2::FitSlicesY, TH3::FitSlicesZ.

hist/DynamicSlice.C

TProfile2D

root Profile2D.C

```
Profile2D() {
    gStyle->SetPalette(1);
    TCanvas *c1 = new TCanvas("c1","Profile histogram example",200,10,700,500);
    hprof2d = new TProfile2D("hprof2d","TProfile2D;px;py;pz",40,-4,4,40,-4,4,0,20);
    Float_t px, py, pz;
    for ( Int_t i=0; i<25000; i++) {
        gRandom->Rannor(px,py);
        pz = px*px + py*py;
        hprof2d->Fill(px,py,pz,1);
    }
    Return 2 numbers distributed following
    a gaussian with mean=0 and sigma=1
}
hprof2d->Draw("SURF3ZE");
}
```



Weighted Histograms

- Histogram can be filled with a "weight"
 - observations do not contribute equally, but some of them contribute more or less than others;
 - the weight expresses how much an observation contributes.
 - Filling a 1D histogram with observation x and weight w:

$$h1->Fill(x,w);$$

• Filling a 2D histogram with observation x,y and weight w:

$$h2->Fill(x,y,w);$$

- -A weighted histogram will have and display as:
 - bin content = sum of all the weights accumulated in the bin;
 - bin error = $\sqrt{W2}$: W2 = sum of the weight square in the bin. (in ROOT versions <= 5.34, if one has called TH1::Sumw2() before filling the histogram)

Histograms in Files

The following statements create a ROOT file and store a histogram on the file

```
TFile f("histos.root", "new");
TH1F h1("hgaus", "histo from a gaussian", 100, -3, 3);
h1.FillRandom("gaus", 10000);
h1->Write();
```

To read this histogram in another ROOT session, do:

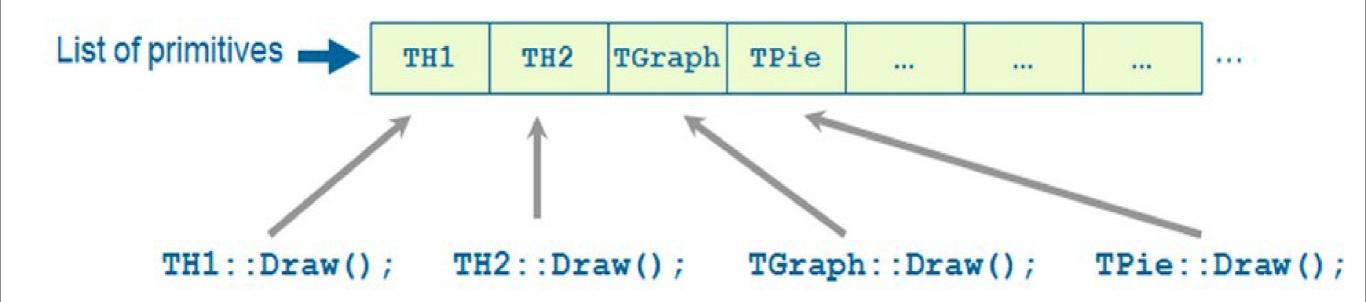
```
TFile f("histos.root");
TH1F *h = (TH1F*)f.Get("hgaus");
```

One can save all histograms in memory to the file by:

```
file->Write();
```

The Graphics Pad (1/2)

The ROOT graphics is build around the *Graphics Pad* concept (class TPad). A Graphics *Pad* is a linked list of primitives of any type (graphs, histograms, shapes, tracks, etc.). It is a kind of *display list* as shown on the following picture:



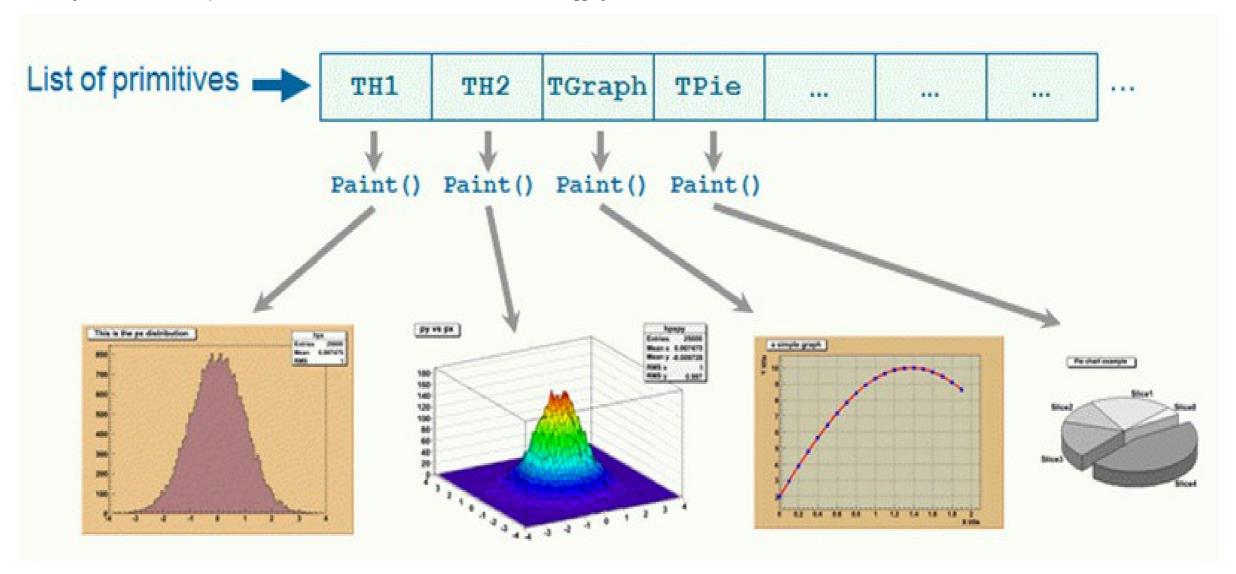
Adding an element into a Graphics Pad is done by the Draw() method of each classes.

On the previous picture the *Draw()* method has been called on: a 1D histogram, a 2D histogram, a graph and finally a pie chart.

All these objects are now stored in the Graphics Pad's display list.

The Graphics Pad (2/2)

A Graphics Pad is painted by calling sequentially the *Paint()* method of each object in the list of primitives, as shown on the following picture:



The Graphics Pad's (re)painting does not need to be done explicitly by the ROOT user. It is done automatically at the end of a macro execution or when a Graphics Pad has been modified.

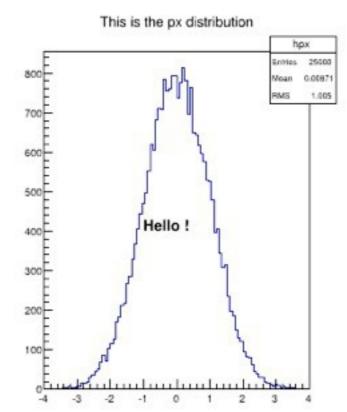
In some cases a pad need to be painted during a macro execution. To force the pad painting gPad->Update() should be performed.

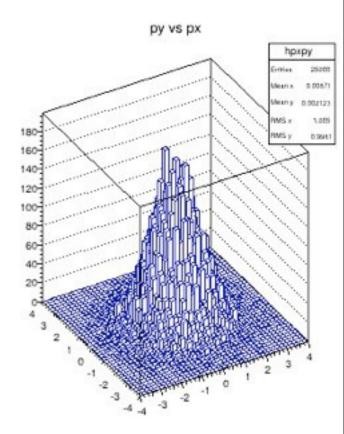
The Graphics Canvas

A canvas is the graphics window in which the ROOT graphics will be displayed. It can be created using the class TCanvas. One can create as many canvases as needed during a ROOT session.

A TCanvas usually contains at least one TPad. Most of the time it contains several, each of them having its own coordinate system. A simple way to quickly make several pads in a canvas is to use the method Divide() like in the following example:

```
TCanvas *c = new TCanvas("c","my canvas",
600,400);
c->Divide(2,1);
c->cd(1);
hpx->Draw();
c->cd(2);
hpxpy->Draw("lego");
c->cd(1);
TText *T = new TText(-1.,400.,"Hello !");
T->Draw();
```





Visualization Techniques in ROOT

How ROOT visualize 2, 3, 4 and N data variables

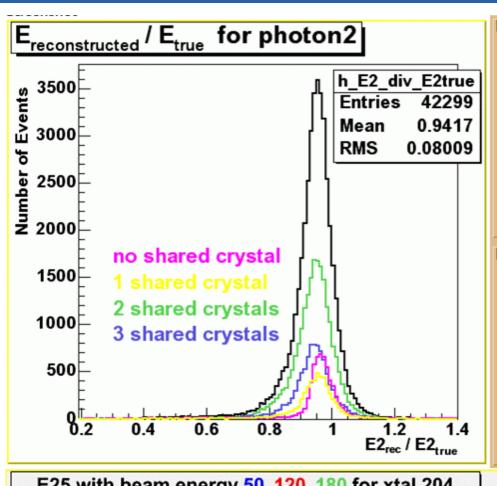
Visualization Techniques

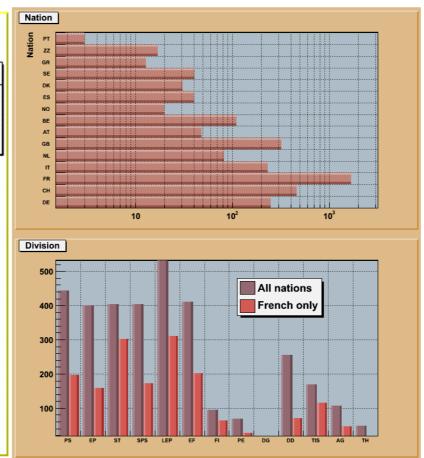
The ROOT framework provides many techniques to visualize multi-variable data sets from 2 until N variables.

- -2 variables visualization techniques are used to display Trees, Ntuples, 1D histograms, functions y=f(x), graphs.
- -3 variables visualization techniques are used to display Trees, Ntuples, 2D histograms, 2D Graphs, 2D functions ...
- -4 variables visualization techniques are used to display Trees, Ntuples, 3D histograms, 3D functions ...
- -N variables visualization techniques are used to display Trees and Ntuples ...

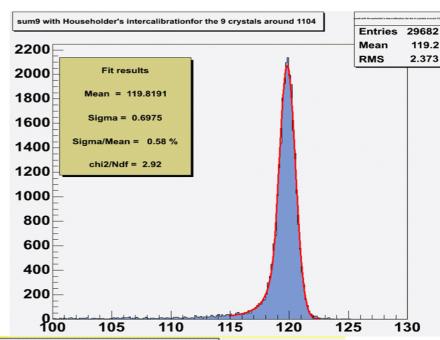
The next slides present them all, highlighting the best use one can do of each of them.

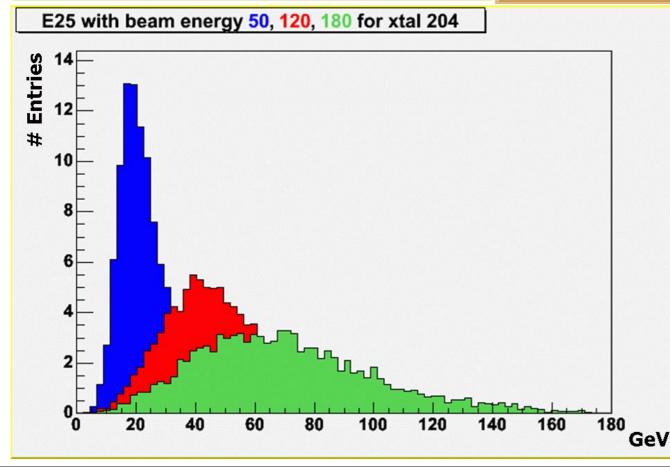
2 Variables Techniques (1/3)

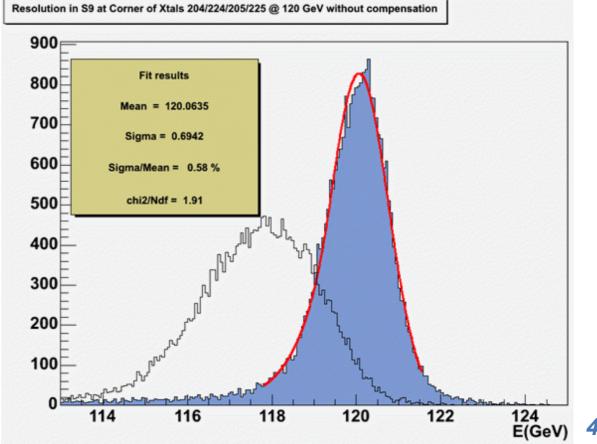




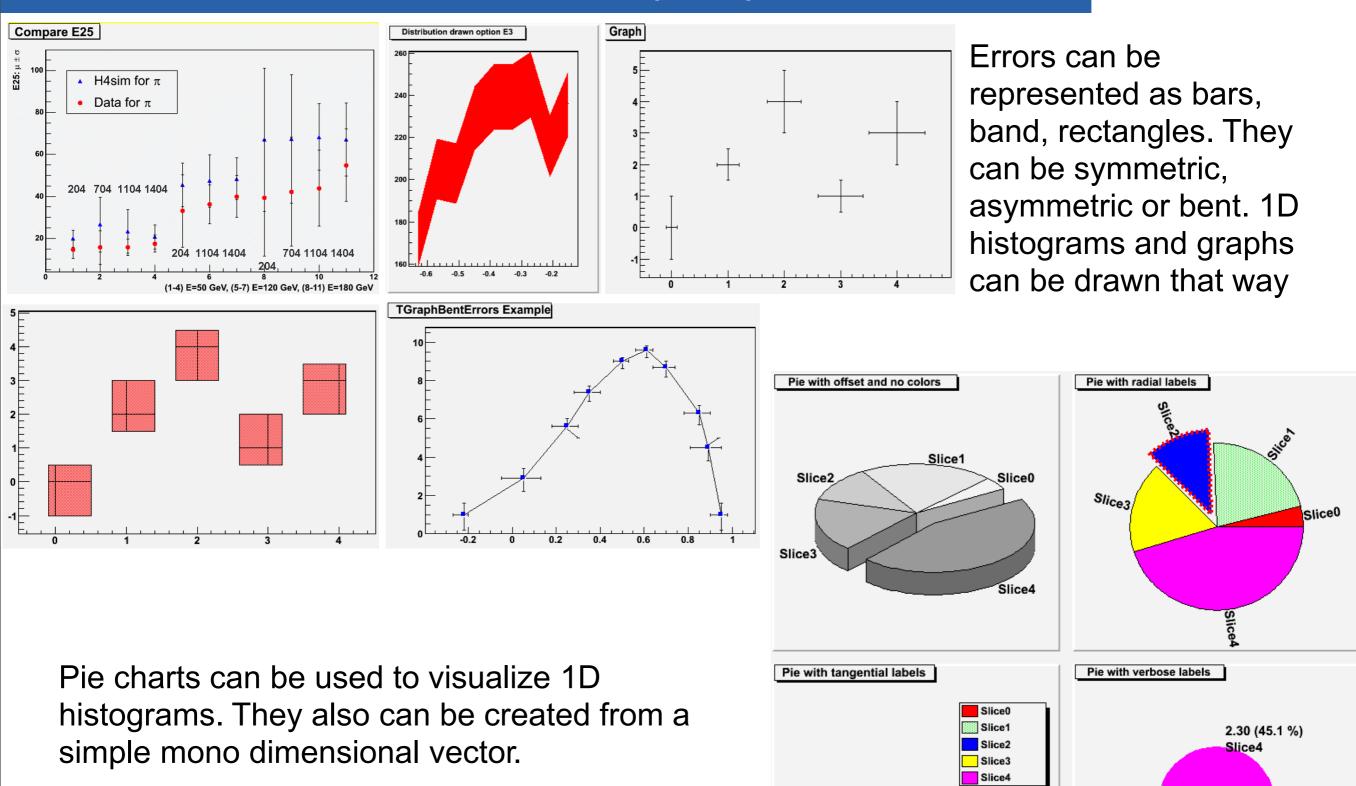
Bar charts and lines are a common way to represent 1D histograms.







2 Variables Techniques (2/3)



1.10 (21.6 %

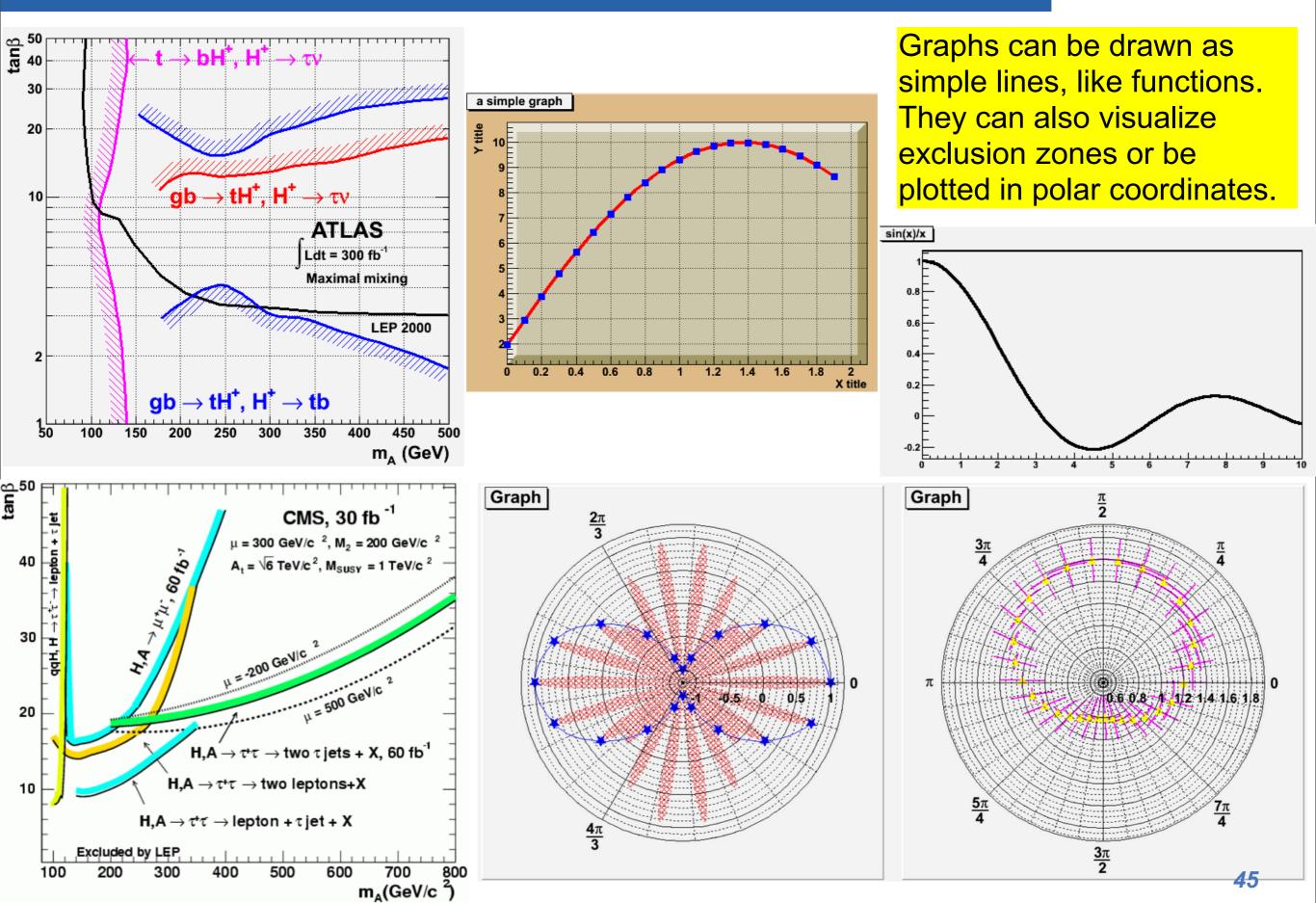
Slice1

Slice4

Slice0 0.60 (11.8 %)

0.90 (17.5 %)

2 Variables Techniques (3/3)



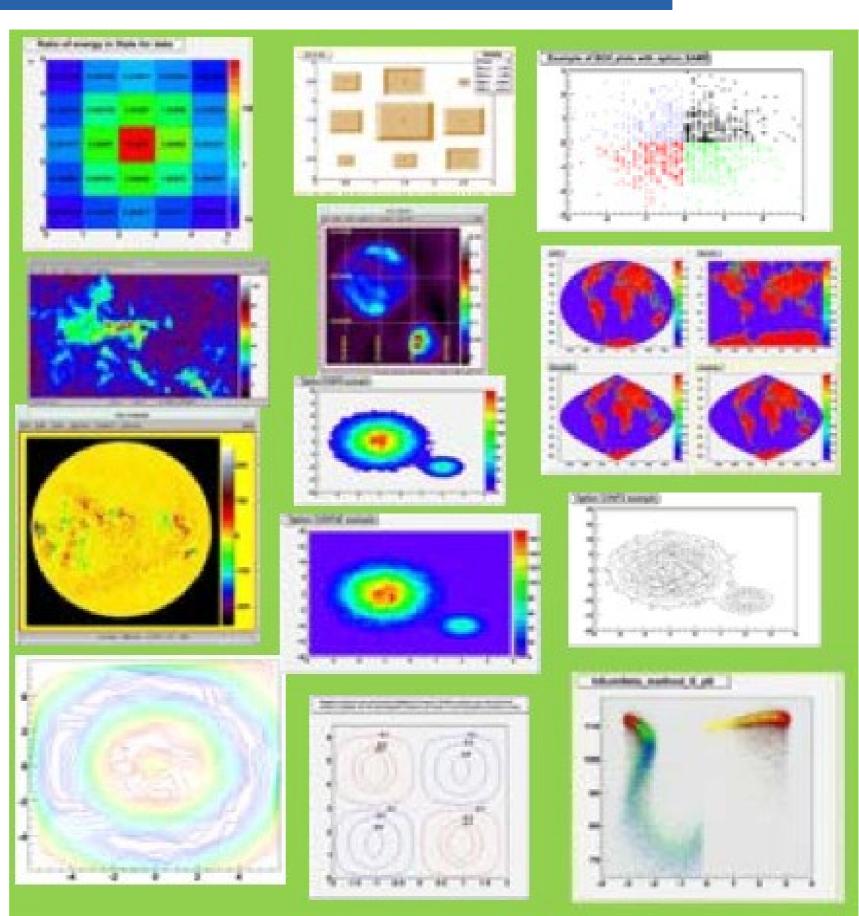
3 Variables Techniques (1/2)

Several techniques are available to visualize 3 variables data sets in 2D.

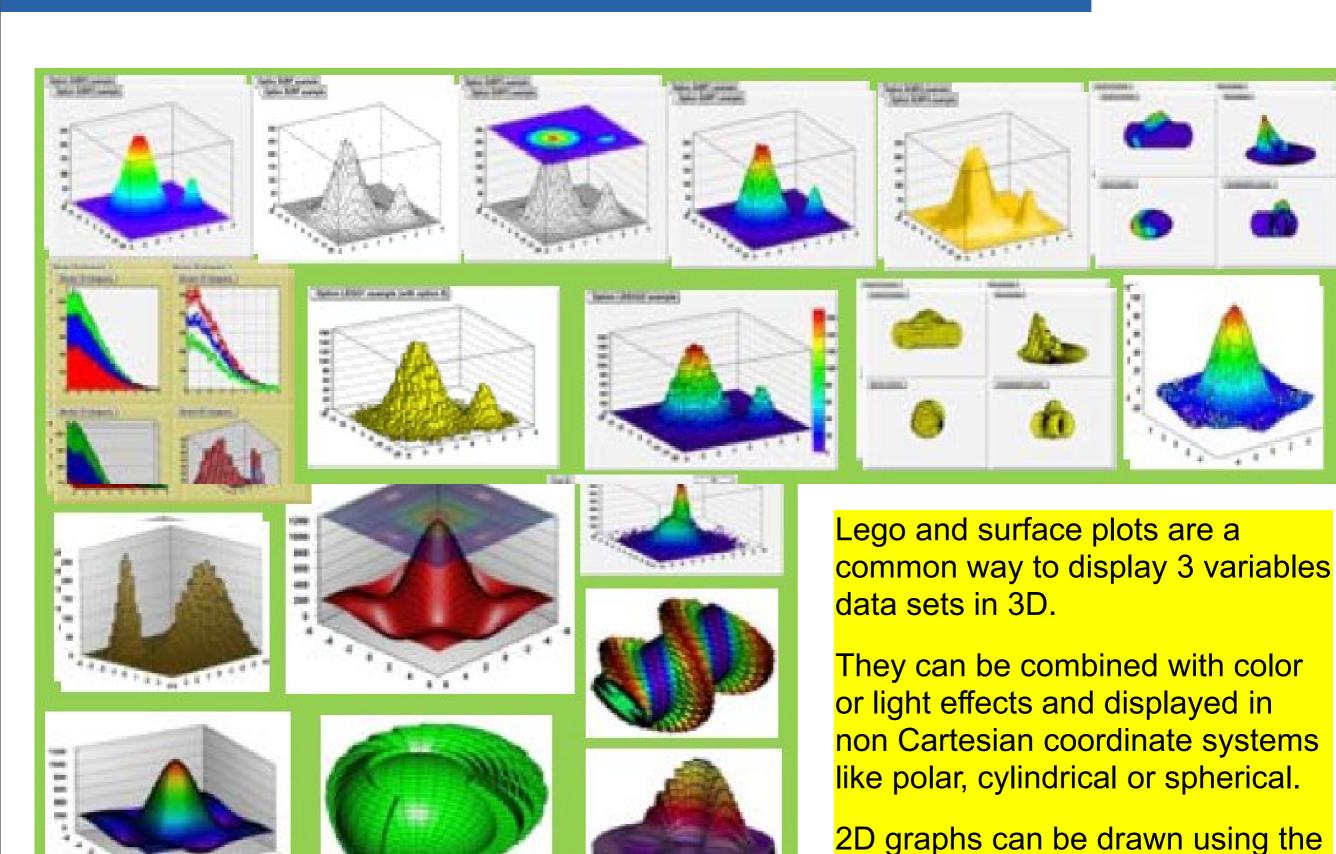
Two variables are mapped on the X and Y axis and the 3rd one on some graphical attributes like the color or the size of a box, a density of points (scatter plot) or simply by writing the value of the bin content.

The 3rd variable can also be represented using contour plots.

Some special projections (like Aitoff) are available to display such contours.



3 Variables Techniques (2/2)

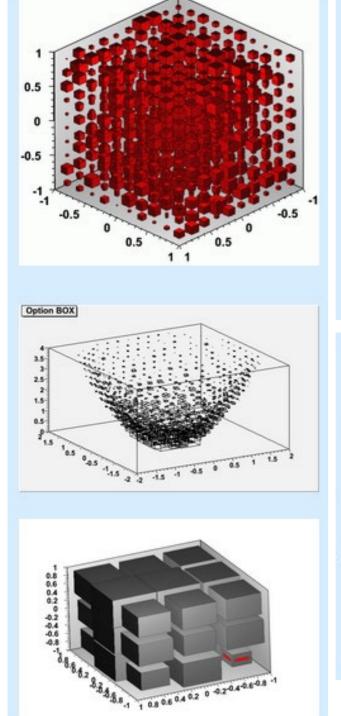


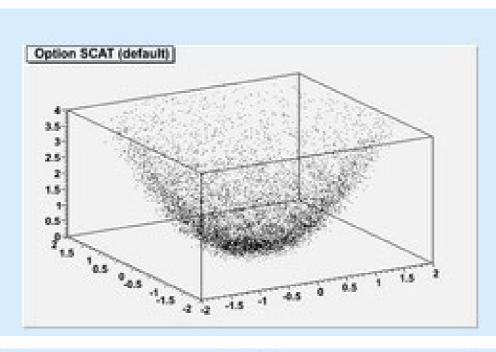
47

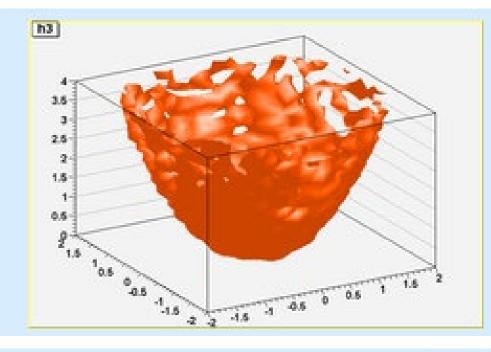
Delaunay triangulation technique.

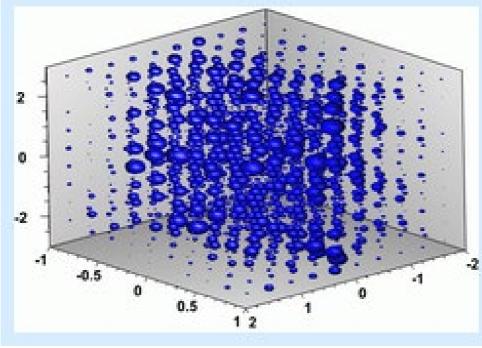
4 Variables Techniques (1/2)

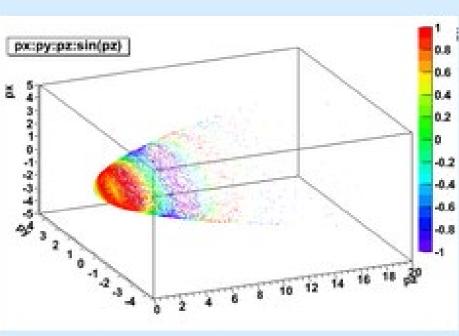
The 4 variables data set representations are extrapolations of the 3 variables ones. Rectangles become boxes or spheres, contour plots become iso-surfaces. The scatter plots (density plots) are drawn in boxes instead of rectangles. The 4th variable can also be mapped on colors. The use of OpenGL allows to enhance the plots' quality and the interactivity.



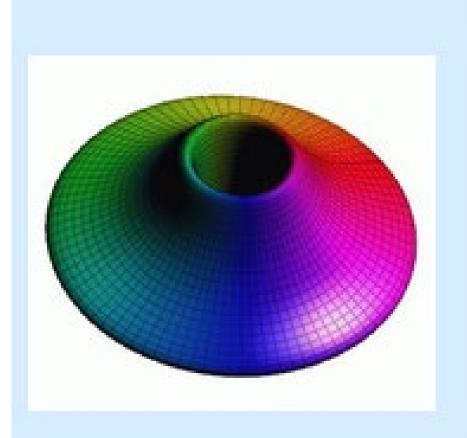


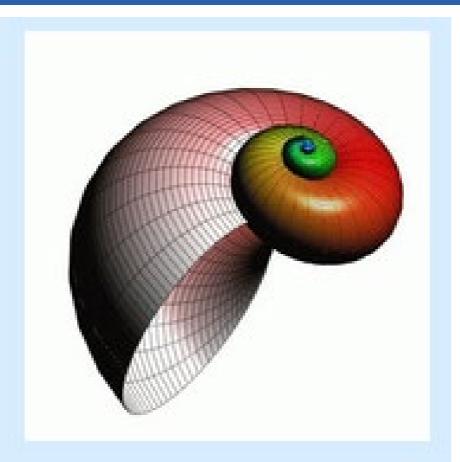


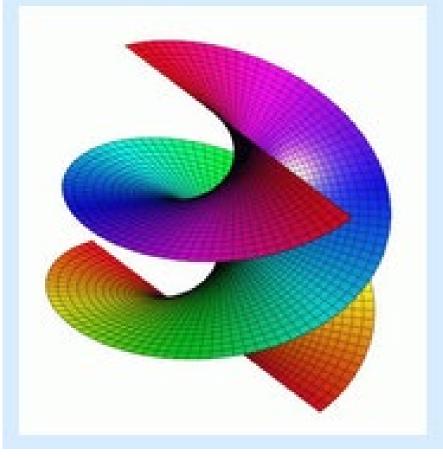


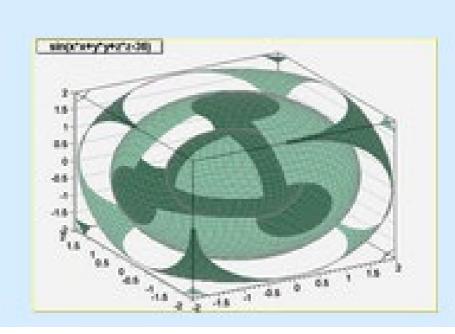


4 Variables Techniques (2/2)









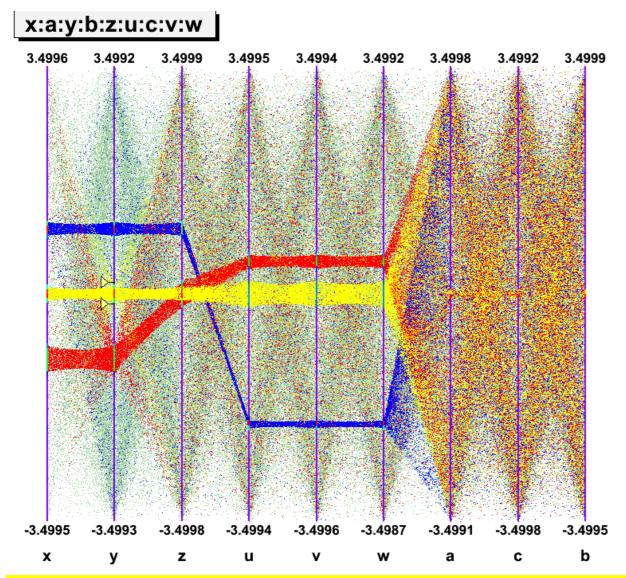
Functions like t= f(x,y,z) and 3D histograms are 4 variables objects.

ROOT can render using OpenGL.

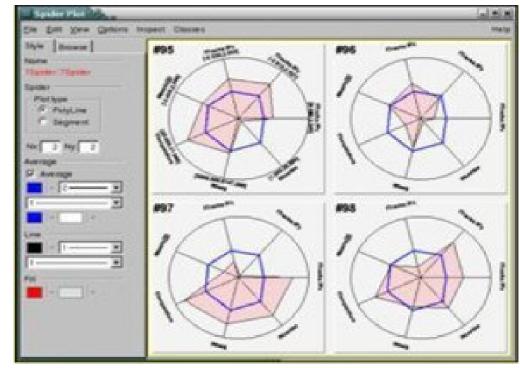
It allows to enhance the plots' quality and the interactivity.

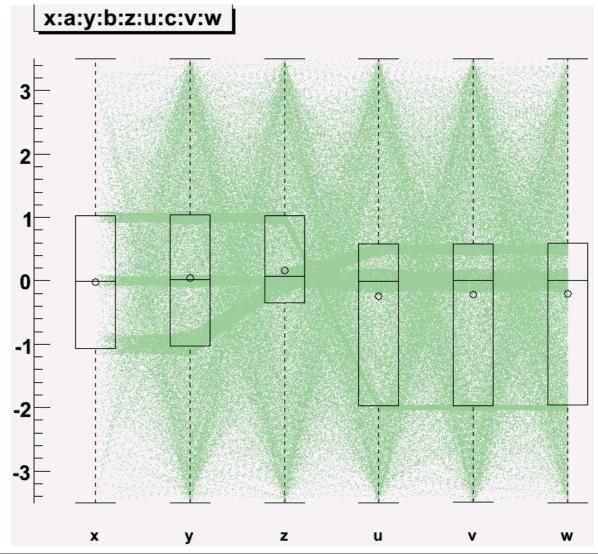
Cutting planes, projection and zoom allow to better understand the data set or function.

N Variables Techniques

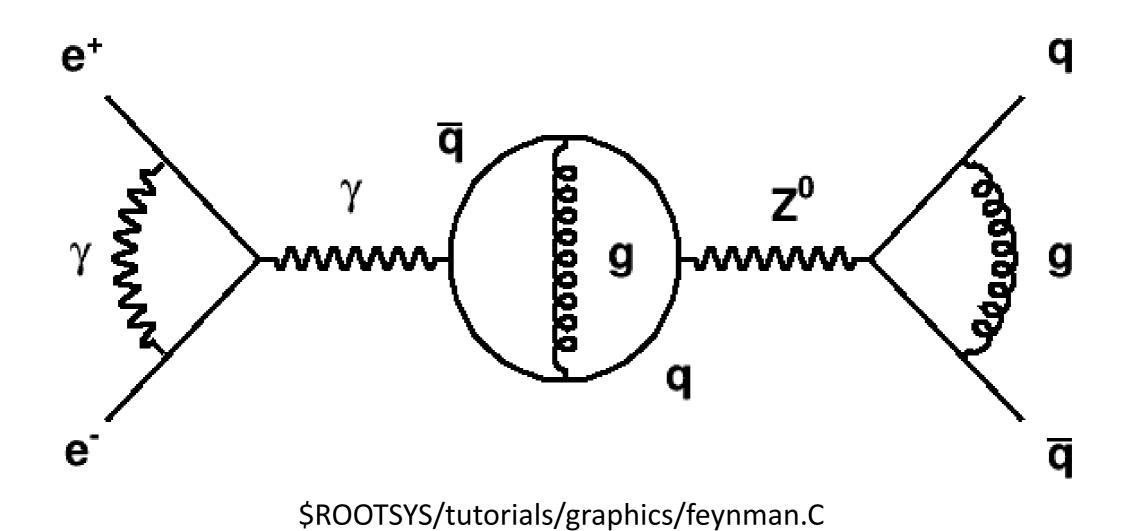


Above 4 variables more specific visualization techniques are required; ROOT provides three of them. The parallel coordinates (above) the candle plots (right) which can be combined with the parallel coordinates. And the spider plot (top right). These three techniques, and in particular the parallel coordinates, require a high level of interactivity to be fully efficient.





Feynman diagram



Summary

- We have learned more about ROOT Histograms
 - how to access their information
 - how to perform operations on histograms
- Multi-dimensional histograms and projections
 - what is a profile histogram
- Remember:
 - all graphics options for plotting all histogram types are documented in the THistPainter class:
 - http://root.cern.ch/root/html/THistPainter.html
- Next we will look on
 - what is fitting
 - how to fit histograms (and graphs) in ROOT

Dig \$ROOTSYS/tutorials/hist

- Run the example macros
- Try to understand the codes of the example macros:
 references: http://root.cern.ch/drupal/content/reference-guide
 class header descriptions
- "grep" those macros to find what you need