

2 → n 过程的数量:

3n-5 个独立变量, $C_n^2 + n$ 个 Lorentz 不变量

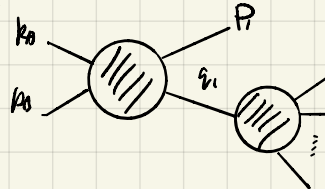
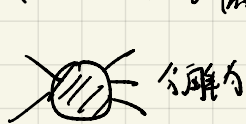
因此有 $\frac{n^2-n+10}{2}$ 个独立不变量

for $n=3$, 有 2 个独立不变量,

$$m_s^2 = k_0^2 = (k_0 - p_1 - p_2 - p_3)^2$$

1 将 2 → n 过程,

$$d\Gamma_n = (2\pi)^4 \delta^4(\dots) \prod_i \frac{d^3\vec{p}_i}{(2\pi)^3 2E_i}$$



$$\delta^4(q_1 - \sum p_i) d^4q_1 \delta(q_1^2 - s_1) ds_1$$

$$d\Gamma_n = (2\pi)^4 \delta^4(k_0 + k_0 - q_1 - p_1) \frac{d^3\vec{p}_1}{(2\pi)^3 2p_1} \frac{d^4q_1}{(2\pi)^4 2q_1} ds_1 \delta^4(q_1 - \sum_{i=2}^n p_i) \prod_{i=2}^n \frac{d^3\vec{p}_i}{(2\pi)^3 2p_i}$$

$$= \frac{p_1}{8\pi\sqrt{s}} d\cos\theta \frac{2\sqrt{s} ds_1}{22} \frac{1}{2\sqrt{s_1}} (2\pi)^4 \delta^4(q_1 - \sum_{i=2}^n p_i) \prod_{i=2}^n \frac{d^3\vec{p}_i}{(2\pi)^3 2p_i}$$

$$\sqrt{s_1} \max = \sqrt{s} - m^2$$

$$\sqrt{s_1} \min = \sum_{i=2}^n m_i$$

相差值 $q_2, \dots$

作积分:

$$p_{1\mu} = (\sqrt{m^2 + p_1^2}, p_1 \sin\theta_1, 0, p_1 \cos\theta_1) \text{ take } \varphi=0$$

$$p_{3\mu} = (\sqrt{m^2 + p_3^2}, p_3 \sin\theta_2 \cos\varphi, p_3 \sin\theta_2 \sin\varphi, p_3 \cos\theta_2) \leftarrow \text{在 } q_1 \text{ 静止系.}$$

$$\text{Grammer: } M_{ij} = p_i \cdot p_j, |M| \equiv 0.$$

for 2 → n 个粒子:

3n 个积分变量, 能守恒守恒 4 个恒等式.

$$3n - 4 - 1 = 3n - 5$$

$C_n^2 + n$: 积分变量.

2 个约束. 共

$$C_n^2 + n - 2 = \frac{n(n-1)}{2} + n - 2$$

n=2

1

1

n=3

4

4

n=4

7

8 ← Grammer 恒等式
7 → 8-1=7

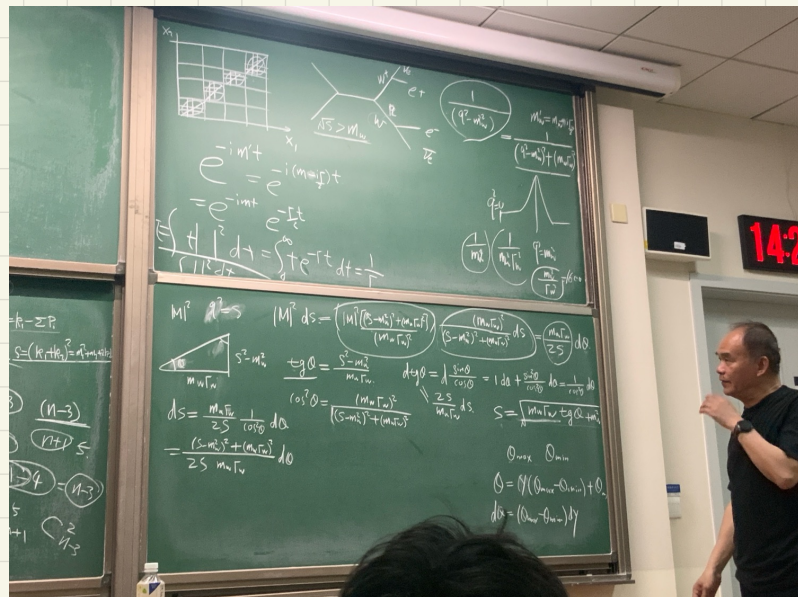
n=5

10

13.

2 个 Grammer 恒等式. 独立变量与 Lorentz 不变量一样.

$$C_{n-3}^2 \rightarrow \frac{(n-3)(n-4)}{2} + (n-3) \text{ 个约束.}$$



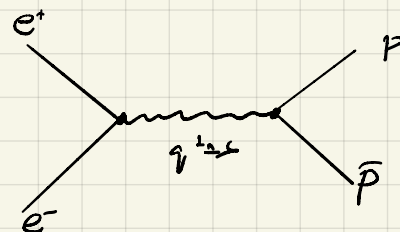
量子抽样

使用 BW 分布作量子抽样拟合

九讲. 强相互作用的有效拉氏量的构造和分解.

证:

$$\epsilon_\mu^{\lambda} \bar{U}(q) [F_1(q^2) \gamma^\mu + F_2(q^2) \sigma^{\mu\nu} q_\nu] U(q)$$

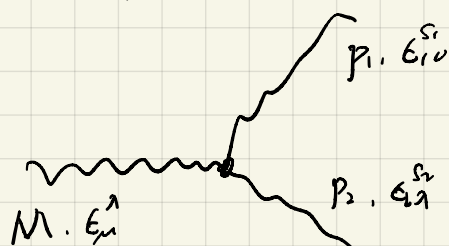


$$F_1(q^2=0, q \rightarrow 0) = -ie$$

$$F_2(q^2=0, q \rightarrow 0) = 0$$

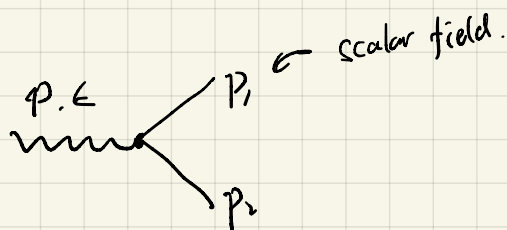
Landau - Yang

Theorem.



HW IV

for



initial state is massive vector boson.

$$\begin{aligned} M &= \epsilon_\mu(\lambda) (a p_1^\mu + b p_2^\mu) \\ &= \epsilon_\mu(\lambda) (a p_1^\mu + b p_2^\mu) \quad \forall \lambda \Rightarrow a=b \\ &= a \epsilon_\mu (p_1 + p_2)^\mu = a \epsilon_\mu p^\mu = 0. \quad \text{禁戒} \end{aligned}$$

分波分析基础: 强有效拉氏量.

⑧. 强相互作用有效拉氏量构造与分解分析

1. 标量场 $\phi(x)$, $J=0$
2. 费米场 $\psi(x)$, $J=\frac{1}{2}$
3. 矢量场 $A_\mu(x)$, $J=1$

1. $(\partial^2 + m^2)\phi = 0$

$$\phi = \int \frac{d^3k}{(2\pi)^3 2E_k} [a_k e^{-ikx} + a_k^\dagger e^{ikx}] \quad \text{Real scalar field.}$$

$$\phi = \int \frac{d^3k}{(2\pi)^3 2E_k} [a_k^\dagger e^{ikx} + b_k e^{-ikx}] \quad \text{complex scalar field.}$$

$$\begin{cases} P \phi(x) P^\dagger = \pm \phi(\tilde{x}), & \tilde{x} = (t, -\vec{x}) \\ C \phi C^\dagger = \pm \phi \text{ for Real.} & C^\dagger \phi C = \alpha \phi^\dagger, \text{ for complex} \end{cases}$$

2. 矢量场 A_μ : $J^\mu = 1^-$

$$\begin{cases} P A_\mu(x) P^\dagger = A_\mu(\tilde{x}) & \hat{P} |k, \lambda\rangle = -|\tilde{k}, \lambda\rangle \\ C A_\mu(x) C^\dagger = -A_\mu(x) & \hat{C} |k, \lambda\rangle = -|k, \lambda\rangle \end{cases}$$

$$A_\mu(x) = \int \frac{d^3k}{(2\pi)^3 2E_k} (e^{ikx} \epsilon_\mu(k, \lambda) a_{k, \lambda} + e^{-ikx} \epsilon_\mu^*(k, \lambda) a_{k, \lambda}^\dagger)$$

$$\begin{cases} P a_{k, \lambda} P^\dagger = -a_{\tilde{k}, \lambda}^\dagger \\ C a_{k, \lambda}^\dagger C^\dagger = -a_{k, \lambda}^\dagger \end{cases}$$

3. photon $J^\mu = \frac{1}{2}^+$

$$P |k, s\rangle = |\tilde{k}, s\rangle$$

$$\psi(x) = \int \frac{d^3k}{(2\pi)^3 2E_k} \sum_s [e^{ikx} u(k, s) a_{k, s} + e^{-ikx} \bar{u}(k, s) a_{k, s}^\dagger]$$

$$\Rightarrow \begin{cases} P a_{k, s} P^\dagger = a_{\tilde{k}, s}^\dagger \\ P b_{k, s}^\dagger P^\dagger = -b_{\tilde{k}, s}^\dagger \\ C a_{k, s} C^\dagger = b_{k, s}^\dagger \\ C b_{k, s}^\dagger C^\dagger = a_{k, s}^\dagger \end{cases} \quad \begin{cases} P \psi(x) P^\dagger = \gamma^0 \psi(\tilde{x}) \\ C \psi(x) C^\dagger = \psi(x) \gamma^0 \end{cases}$$

4. 高自旋玻色场

$$\phi_{\mu\nu}(x), J^{\mu\nu} = J^{sp, 1}$$

$$\phi_{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^3 2E_k} [e^{ikx} \epsilon_{\mu\nu}^*(k, \lambda) a_{k, \lambda}^\dagger + e^{-ikx} \epsilon_{\mu\nu}(k, \lambda) a_{k, \lambda}]$$

$$P \phi_{\mu\nu}(x) P^\dagger = (-1)^J \eta_p \phi^{\mu\nu}(\tilde{x})$$

$$P a_{k,\lambda} |0\rangle = P |k,\lambda\rangle = \eta_p |k,\lambda\rangle$$

$$P a_{k,\lambda}^\dagger P^\dagger = \eta_p a_{k,\lambda}^\dagger$$

$$P b_{k,\lambda} P^\dagger = \eta_p b_{k,\lambda}$$

$$C \phi_{\mu\nu}(x) C^\dagger = \begin{cases} \eta_c \phi_{\mu\nu}(x) & \text{Real} \\ \phi_{\mu\nu}^\dagger(x) & \text{Complex} \end{cases}$$

5. 高自旋粒子. $J^2 = (n+1/2) \hbar^2$

$$\psi_{\mu\nu}(x) = \int \frac{d^4k}{(2\pi)^4 2k^0} \sum_{\lambda} [e^{ikx} \mathcal{X}_{\mu\nu}(k,\lambda) a_{k,\lambda}^\dagger + e^{-ikx} \mathcal{X}_{\mu\nu}(k,\lambda) a_{k,\lambda}]$$

$\mathcal{X}_{\mu\nu}(k,\lambda)$ suppose to be a presentation of: $\Lambda \otimes \frac{1}{2}$

$$P \psi_{\mu\nu}(x) P^\dagger = (-1)^{J-\frac{1}{2}} \eta_p \gamma^0 \psi^{\mu\nu}(\tilde{x})$$

$$P \bar{\psi}_{\mu\nu}(x) P^\dagger = (-1)^{J-\frac{1}{2}} \eta_p \bar{\psi}^{\mu\nu} \gamma^0$$

$$P a_{k,\lambda}^\dagger P^\dagger = \eta_p a_{k,\lambda}^\dagger$$

$$P b_{k,\lambda} P^\dagger = -\eta_p b_{k,\lambda}$$

$$C \psi_{\mu\nu} C^\dagger = \psi_{\mu\nu}^c = C \bar{\psi}^T, \quad C C^\dagger = -1$$

$$C \bar{\psi}_{\mu\nu} C^\dagger = -\psi_{\mu\nu}^c C^\dagger = \psi_{\mu\nu}^T C$$

$$\psi^c = \psi, \quad \text{Dirac 粒子和反粒子}$$

强相互作用的 1-2 顶点,

$$\frac{J}{4} \psi \rightarrow N^*(1600) + \bar{P}$$

$$H_2 = \int d^4x \bar{\psi}_{A,\mu,\nu} \Gamma(\hat{p}_0, \hat{p}_1, \hat{p}_2) \psi_{B,\mu,\nu}(x) \phi_{C,\alpha,\beta}(x)$$

with P-parity

$$P H_2 P^\dagger = H_2, \quad H_2^\dagger = H_2$$

$$\begin{aligned} P H_2 P^\dagger &= \int d^4x P \bar{\psi}_{A,\mu,\nu} P^\dagger \Gamma P^\dagger, P \psi_{B,\mu,\nu} P^\dagger P \phi_{C,\alpha,\beta} P^\dagger \\ &= (-1)^{J_A - \frac{1}{2} + J_B - \frac{1}{2} + J_C} \eta_A \eta_B \eta_C \int d^4x \bar{\psi}_{A,\mu,\nu} \gamma^0 \Gamma \gamma^0 \psi_{B,\mu,\nu} \phi_{C,\alpha,\beta} \\ &= \eta \int d^4x \bar{\psi}_{A,\mu,\nu} \gamma^0 \Gamma \gamma^0 \psi_{B,\mu,\nu} \phi_{C,\alpha,\beta} = H_2 \end{aligned}$$

$$\Rightarrow \eta \gamma^0 \Gamma \gamma^0 \gamma^{\mu_1 \dots \mu_n} (\hat{p}_0, \hat{p}_1, \hat{p}_2) \gamma^0 = \Gamma_{\mu_1 \dots \mu_n} (\hat{p}_0, \hat{p}_1, \hat{p}_2)$$

$$② H_2 = \int d^4x \phi_{A\alpha}(x) \phi_{B\alpha}(x) \phi_{C\alpha}(x) \Gamma^{\mu\nu\lambda}(p_A, p_B, p_C)$$

$$P H_2 P^{-1} = \underbrace{(-1)^{j_A+j_B+j_C}}_{\downarrow \kappa} n_A n_B n_C \phi_{A\alpha}^*(x) \phi_{B\alpha}^*(x) \phi_{C\alpha}^*(x) \Gamma^{\mu\nu\lambda}(\tilde{p}_A, \tilde{p}_B, \tilde{p}_C)$$

$$\Rightarrow n \Gamma^{\mu\nu\lambda}(\tilde{p}_A, \tilde{p}_B, \tilde{p}_C) = \Gamma^{\mu\nu\lambda}(p_A, p_B, p_C)$$

C-parity.

complex scalar:

$$H_2 = H_2 + H_2^\dagger$$

$$H_2^\dagger = \int d^4x \bar{\psi}_A(x) \gamma^0 \Gamma^{\mu\nu\lambda}(p_A, p_B, p_C) \gamma^0 \psi_B(x) \phi_{C\alpha}^\dagger(x)$$

$$\text{for } H_2^C = C H_2 C^{-1} = H_2$$

$$\Rightarrow \gamma^0 \Gamma^{\mu\nu\lambda} \gamma^0 = [C \Gamma^{\mu\nu\lambda} C^{-1}]^\dagger$$

$$J/\psi \rightarrow \bar{N}^* + p$$

$1^- \quad \quad \quad \frac{1}{2}^- \quad \frac{1}{2}^+$

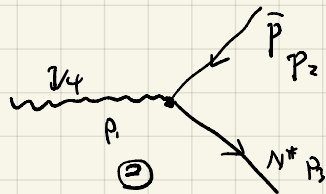
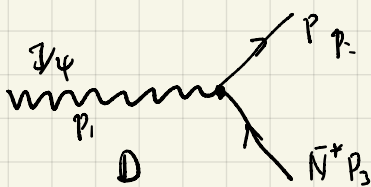
$$H_2 = \bar{\psi}_p(x) (ia\gamma_\mu \gamma_5 + b\gamma_\mu \gamma_5 p_\mu) \psi_{N^*}(x) \phi^*(x)$$

$$J/\psi \rightarrow N^* + \bar{p}$$

$$H_2^C = \bar{\psi}_{N^*}(x) (-ia\gamma_\mu \gamma_5 + b\gamma_\mu \gamma_5 p_\mu) \psi_p(x) \phi^*(x)$$

在 charge conjugate 中.
 $\hat{p}$ as operators, 出 -T 的号

最终搞的一致.



$$M_1 = \bar{u}(p_2, s_2) \Gamma_\mu v(p_1, s_1) \epsilon^\mu(\lambda)$$

$$M_2 = \bar{u}(p_1, s_1) \Gamma_\mu v(p_2, s_2) \epsilon^\mu(\lambda)$$

$$C^T = -C$$

$$\begin{aligned} M_1 &= \bar{u}(p_2, s_2) \Gamma_\mu v(p_1, s_1) \epsilon^\mu(\lambda) & C^\dagger V(p, s) &= \bar{u}^\dagger(p, s) \\ M_2 &= \bar{u}(p_1, s_1) \Gamma_\mu^C v(p_2, s_2) \epsilon^\mu(\lambda) & u(p, s) &= C \bar{v}^\dagger(p, s) = C (v^\dagger p)^\dagger \\ &= M_2^T = \bar{v}^\dagger(p_2, s_2) \Gamma_\mu^T \bar{u}(p_1, s_1) \epsilon^\mu(\lambda) & \bar{v}^\dagger(p, s) &= \bar{u}(p, s) C^\dagger \\ &= \bar{u}(p_1, s_1) C^T \Gamma_\mu^T C v(p_2, s_2) \epsilon^\mu(\lambda) & \bar{u}^\dagger &= (u^\dagger p)^\dagger = p^\dagger u^\dagger \\ & & u^\dagger &= C^* \end{aligned}$$

$$\Gamma_\mu = C^T \Gamma_\mu^T C^{-1} = -C (V_\mu^\dagger \Gamma_\mu^\dagger V_\mu) C^{-1}$$

$$C^T = (-iY_1, Y_0)^T = -iY_0^T Y_2^T = -iY_0^T Y_2 = -C \quad H_I = H_I^* \quad (Y \bar{Y}) \quad (Y e^e)$$

$$Y^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad Y_1 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad 2. \text{ 证明}$$

$$H_I = \bar{\Psi}_A \Gamma^T \Psi_B \phi_c \dots \quad H_I = H_{I1} + H_{I2} \Rightarrow H_I^c = H_I$$

$$H_I^c = -\bar{\Psi}_A^T (\quad) \Psi_B^T C \phi^* \dots \quad \Gamma = C \Gamma C (Y^0 \Gamma^T Y^0)^T C^{-1} \quad \phi_c \in C \text{ 表示本征态 } C \phi_c$$

$$\Gamma = C \Gamma C (Y^0 \Gamma^T Y^0)^T C^{-1} \quad Y_A = -i C (Y^0 Y_A^* Y^0)^T C^{-1} = -C Y_A^T C^{-1}$$

$$\textcircled{-} \bar{\Psi}_B (\quad)^T \Psi_A C \phi^* \dots$$

$$H_2^c = -\bar{\Psi}_A^T (\quad) \Psi_B^T C \phi^* \dots - (-\bar{\Psi}_B (\quad)^T \Psi_A C \phi^* \dots)$$

2. 证明.

$$\textcircled{1} H_{I2} = \bar{\Psi}_A \Gamma^T \Psi_B \phi_c \rightarrow \phi_c \text{ 为 } C \text{ 本征态.} \quad H_2 = H_{I2} + H_2^c \Rightarrow H_2^c = H_2$$

$$\Gamma = C \Gamma C (Y^0 \Gamma^T Y^0)^T C^{-1}$$

$$Y_A = -i C (Y^0 Y_A^* Y^0)^T C^{-1} = -C Y_A^T C^{-1}$$

$$\textcircled{2} H_{I2} = \phi_0 \phi_0 \phi_c \Gamma \dots$$

$$\textcircled{2.1} \text{ 都为 } C \text{ 本征态:} \quad n = C \phi_0 C \phi_0 C \phi_c$$

$$n_c \Gamma^T (\hat{p}_0 \hat{p}_0 \hat{p}_c) = \Gamma^T (\hat{p}_0 \hat{p}_0 \hat{p}_c)$$

③ 计算.

$$\eta_p = \tau^{(j_1, j_2, j_3)} n_0 n_1 n_2$$

$$\Gamma \cdot (\hat{p}_0 \hat{p}_0 \hat{p}_c) = \eta_p \Gamma' \cdot (\hat{p}_0 \hat{p}_0 \hat{p}_c) \Gamma'$$

强子八重模型

for $SU(3)$. $U = e^{i\theta_j T_j}$

$$\text{Tr}[T_i, T_j] = \frac{1}{2} \delta_{ij} \quad [T_i, T_j] = if_{ijk} T_k \quad f_{ijk} = -f_{jik}$$

$$\{T_i, T_j\} = \frac{1}{3} \delta_{ij} + d_{ijk} T_k \quad d_{ijk} = d_{jik}$$

① $q^i = U^i_j q^j \quad \bar{q}_i = (U^\dagger)_i^j \bar{q}_j$

② $\delta_{ij}, \epsilon^{ijk}, \epsilon_{ijk}$ 在 $SU(3)$ 下均为不变张量

不可约张量, 不能因 δ, ϵ 而降阶为非0张量

八重态, 张量构成d维表示

$$3 \otimes 3 = \bar{3} \oplus 6$$

介子八重态: $J^P = 0^-$

(π^-, π^0, π^+) , $I=1$, 三重态

$(K^+, K^0), (\bar{K}^0, K^-)$, $I=1/2$ 二重态

η, η' $I=0$

	I	I_3	B	S	Q	Y
u	$1/2$	$1/2$	$1/3$	0	$2/3$	$1/3$
d	$1/2$	$-1/2$	$1/3$	0	$-1/3$	$1/3$
s	0	0	$1/3$	-1	$-1/3$	$-2/3$

$$\begin{cases} Y = B + S \\ Q = I_3 + \frac{Y}{2} \end{cases}$$

for 八重态: $B^i_j = q^i \bar{q}_j - \frac{1}{3} q^k \bar{q}_k \delta^i_j$

$$B^i_j = \begin{pmatrix} u\bar{u} - \frac{1}{3}(u\bar{u} + d\bar{d} + s\bar{s}) & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} - \frac{1}{3}(u\bar{u} + d\bar{d} + s\bar{s}) & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} - \frac{1}{3}(u\bar{u} + d\bar{d} + s\bar{s}) \end{pmatrix}$$

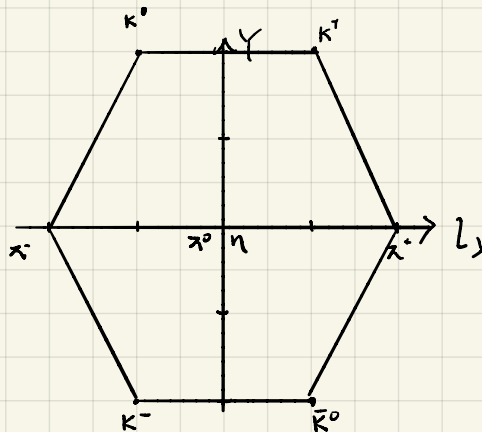
for $\eta_8 = \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s})$, $\pi^0 = \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d})$

$$B^i_j = \begin{pmatrix} \frac{\eta_8}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ -\pi^- & \frac{\eta_8}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & K^0 \\ -K^0 & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta_8 \end{pmatrix}$$

$$\eta_1 = \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s})$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \theta_1 & \cos \theta_1 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}, \quad \eta_8 \rightarrow \eta, \text{ 混合:}$$

$$\theta_1 = \begin{cases} -11.5^\circ \\ -24.8^\circ \end{cases}$$



$J^P = 1^-$ 介子八重态:

$(\rho^-, \rho^0, \rho^+) (K^{*-}, K^{*0}) (\bar{K}^{*0}, K^{*-}) \psi \phi$

$$\begin{pmatrix} \psi \\ \phi \end{pmatrix} = \begin{pmatrix} \cos \theta_4 & -\sin \theta_4 \\ \sin \theta_4 & \cos \theta_4 \end{pmatrix} \begin{pmatrix} \psi_8 \\ \psi_1 \end{pmatrix}$$

$$\theta_4 = \begin{cases} 35.7^\circ \\ 36^\circ \end{cases}$$

理想混合:

量子:

全对称/全反对称必为不同的 K^2 :

$$T^{ijk} = g^i g^j g^k, \quad A_i^k = T^{ijk} \epsilon_{ij}$$

$$A_i^k = A_i^k - \frac{1}{3} A_i^j \delta_j^k \quad 8 \text{ 维}$$

$$D^{ijk} = \sum T^{ijk} \quad 10 \text{ 维}$$

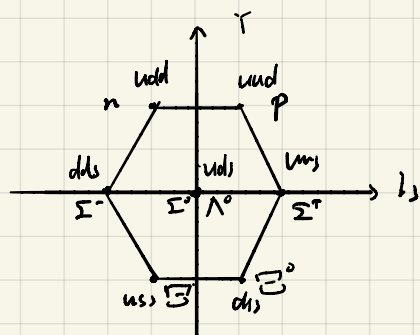
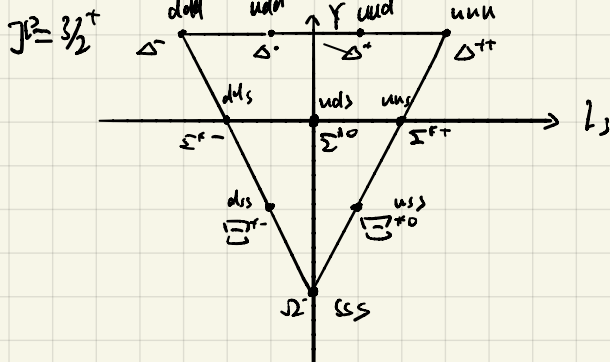
$$T^{ijk} \epsilon_{ijk} \quad 1 \text{ 维}$$

因子: $\bar{3} \otimes \bar{3} \otimes \bar{3} = 1 \oplus 8 \oplus 8 \oplus 10$

量子: $J^P = \frac{1}{2}^+$: $(\Sigma^+, \Sigma^0, \Sigma^-) (p, n), (\Xi^0, \Xi^-) \Lambda^0$

$J^P = \frac{3}{2}^+$: $(\Delta^{++}, \Delta^+, \Delta^0, \Delta^-) (\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}) (\Xi^{*0}, \Xi^{*-}) \Omega^-$

$J^P = \frac{1}{2}^+$:



色荷有反

张子良讲, 张公式

$$m = m_0 + a\gamma + b[(1+\gamma) - \frac{1}{c}\gamma^2]$$

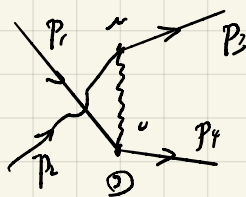
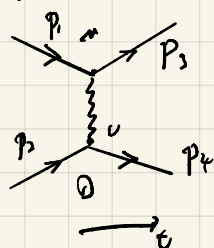
chapt 6. 电弱相互作用.

点粒子的导数无穷大.

QED:

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - m^2 \bar{\psi} \psi$$

Moller 散射



$$\text{wavy line } \frac{-ig_{\mu\nu}}{p^2} \quad \text{fermion line } \frac{i}{\not{p} - m}$$

$$\text{vertex } = iQe\gamma_\mu$$

$$M_1 = \bar{u}(p_3, s_3) iQe\gamma_\mu u(p_1, s_1) \frac{-ig_{\mu\nu}}{p^2} \bar{u}(p_2, s_2) iQe\gamma_\nu u(p_4, s_4)$$

$$= L_{\mu} g^{\mu\nu} L_{\nu} \frac{-i}{(p_1 - p_3)^2}$$

$$M_2(p_1, p_2, p_3, p_4) = M_1(p_1, p_4, p_3, p_2) \text{ for } p_1, p_2 \text{ 交换位置.}$$

1 → 3
2 → 2

$$A = \frac{1}{4} \sum_{s_i} |M_1 + M_2|^2 = \frac{1}{4} \sum_{s_i} [M_1 M_1^* + M_2 M_2^* + 2\text{Re}(M_1 M_2^*)]$$

$$\sum_{s_i} M_i M_i^* = \sum_{s_i} \frac{Q^4 e^4}{(p_1 - p_3)^4} (l_{\mu} l_{\nu}) (l_{\mu} l_{\nu})^* = \frac{Q^4 e^4}{(p_1 - p_3)^4} L_{\mu\nu} L^{\mu\nu} \quad \left\{ \begin{array}{l} L_{1\mu\nu} = \sum_{s_i} l_{\mu} l_{\nu} \\ L_{2\mu\nu} = \sum_{s_i} l_{2\mu} l_{2\nu} \end{array} \right.$$

$$\begin{aligned} L_{\mu\nu} &= \sum_{s_1, s_3} [\bar{u}(p_3, s_3) \gamma_\mu u(p_1, s_1) \bar{u}(p_1, s_1) \gamma_\nu u(p_3, s_3)] \\ &= \sum_{s_1, s_3} [\gamma_\mu u(p_1, s_1) \bar{u}(p_1, s_1) \gamma_\nu u(p_3, s_3) \bar{u}(p_3, s_3)] \\ &= \sum_{s_1, s_3} \text{Tr}[(\not{p}_3 + m) \gamma_\mu (\not{p}_1 + m) \gamma_\nu] \end{aligned}$$

$$\sum_{s_i} u(p, s_i) \bar{u}(p, s_i) = \not{p} + m$$

$$L_{2\mu\nu} = \sum_{s_2, s_4} \text{Tr}[(\not{p}_4 + m) \gamma_\mu (\not{p}_2 + m) \gamma_\nu]$$

$$L_{1\mu\nu} = \text{Tr}[\not{p}_3 \gamma_\mu \not{p}_1 \gamma_\nu + m^2 \gamma_\mu \gamma_\nu]$$

$$= 4m^2 g_{\mu\nu} + 4P_{3\mu} P_{1\nu} - 4P_1 \cdot P_3 g_{\mu\nu} + 4P_{1\mu} P_{3\nu}$$

Gamma Matrix Trace

$$L_{2\mu\nu} = 4m^2 g_{\mu\nu} + 4P_{4\mu} P_{2\nu} - 4P_2 \cdot P_4 g_{\mu\nu} + 4P_{2\mu} P_{4\nu}$$

$$\begin{aligned} \frac{1}{16} L_{\mu\nu} L^{\mu\nu} &= [(m^2 - P_1 \cdot P_3) g_{\mu\nu} + P_{1\mu} P_{3\nu} + P_{1\nu} P_{3\mu}] [(m^2 - P_2 \cdot P_4) g^{\mu\nu} + P_2^\mu P_4^\nu + P_2^\nu P_4^\mu] \\ &= 4(m^2 - P_1 \cdot P_3)(m^2 - P_2 \cdot P_4) + \dots \\ &= 2[-P_1 \cdot P_3 P_2 \cdot P_4 + P_1 \cdot P_4 P_2 \cdot P_3 + P_1 \cdot P_2 P_3 \cdot P_4] \rightarrow m=0 \end{aligned}$$

$$A_1 = \frac{1}{4} \sum |M_i|^2 = \frac{2Q^4 e^4}{(m^2 - P_1 \cdot P_3)^2} (P_1 \cdot P_3 P_2 \cdot P_4 - P_1 \cdot P_2 P_3 \cdot P_4 + P_1 \cdot P_4 P_2 \cdot P_3)$$

$$A_2 = \frac{1}{4} \sum |M_i|^2 = A(P_3 \leftrightarrow P_4) = \frac{2Q^4 e^4}{(m^2 - P_1 \cdot P_4)^2} (P_1 \cdot P_2 P_3 \cdot P_4 - P_1 \cdot P_4 P_2 \cdot P_3 + P_1 \cdot P_3 P_2 \cdot P_4)$$

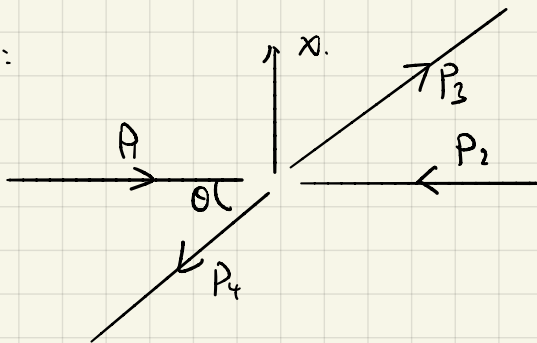
$$A_{12} = \frac{2}{4} \text{Re} \sum M_i \cdot M_i^\dagger = -\frac{1}{4} \frac{2Q^4 e^4}{(m^2 - P_1 \cdot P_3)(m^2 - P_1 \cdot P_4)} \frac{1}{4} \text{Tr}[P_1 \not{P}_3 P_4 \not{P}_2 P_4 \not{P}_2 P_3 \not{P}_1]$$

$$= \frac{2}{4} \text{Tr}[P_1 (-P_3 \not{P}_2 P_4 + P_4 \not{P}_2 P_3 + P_2 \not{P}_3 P_4 - P_4 \not{P}_3 P_2) P_3 P_1]$$

$$= \frac{2}{4} \text{Tr}[P_1 (2P_4 P_3 P_2 + 2P_4 P_2 P_3 + 2P_3 P_2 P_4 - 2P_2 P_3 P_4)]$$

?

Consider :



$$P_1 = \left(\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2}\right)$$

$$P_2 = \left(\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2}\right)$$

$$P_3 = \left(\frac{\sqrt{s}}{2}, \frac{\sqrt{s}}{2} \sin \theta, 0, \frac{\sqrt{s}}{2} \cos \theta\right)$$

$$P_4 = \left(\frac{\sqrt{s}}{2}, \frac{\sqrt{s}}{2} \cos \theta, 0, -\frac{\sqrt{s}}{2} \sin \theta\right)$$

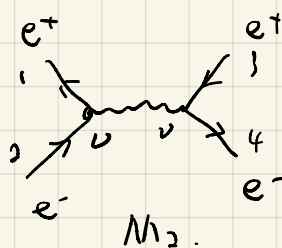
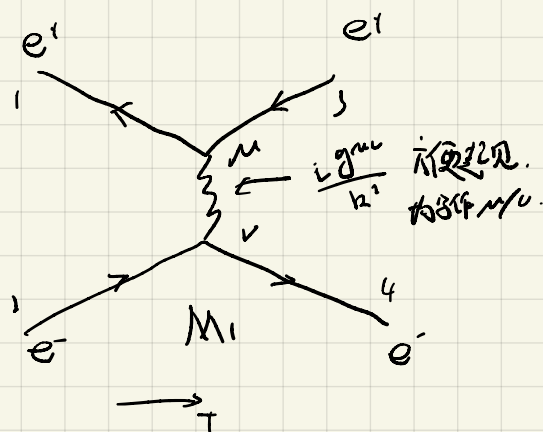
$$d\sigma_{e^+e^- \rightarrow e^+e^-} = \frac{Q^2}{2s} \delta^4(p_1 + p_2 - p_3 - p_4) \frac{d^3\vec{p}_3}{(2\pi)^3 2p_3^0} \frac{d^3\vec{p}_4}{(2\pi)^3 2p_4^0} A$$

$$= \frac{1}{(2\pi)^4 p_1^0 p_2^0 p_3^0 p_4^0 2s} \delta(\sqrt{s} - 2p_+^-) \cdot p_+^2 p_-^2 d\cos\theta \, 2A$$

$$= \frac{1}{(2\pi)^4 s^2} d\cos\theta \, A = \frac{d\cos\theta}{22s} A \frac{d\varphi}{2\pi}$$

$$= \frac{s^2}{8z^2} \left[\frac{1 + \cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}} + \frac{1 + \sin^2\frac{\theta}{2}}{\cos^4\frac{\theta}{2}} + \frac{2}{\sin^2\frac{\theta}{2} \cos^2\frac{\theta}{2}} \right] d\Omega$$

QED图:



$$M = M_1 - M_2$$

$$\bar{\psi}_1 \psi_2 \bar{\psi}_3 \psi_4 \quad \bar{\psi}_x \gamma^\mu \psi_x \quad \bar{\psi}_y \gamma^\mu \psi_y$$

$$\text{for } M_1: ? (-1)$$

$$\text{for } M_2: ? (+1)$$

W 因子整理:

$$\sum_s u(p,s) \bar{u}(p,s) = \not{p} + m \quad \leftrightarrow \quad \frac{d^3\vec{p}}{(2\pi)^3 2p^0}$$

$$\epsilon^{\mu\nu\lambda\sigma} = 1$$

$$e_{\mu\nu} = \bar{u}(p_1) \gamma_\mu u(p_3)$$

$$e_{\mu\nu}^\dagger = \bar{u}(p_2) \gamma^\mu \gamma^\nu \gamma^\mu u(p_4)$$

$$|M_1|^2 \text{ 计算}$$

一个圈是个 trace.

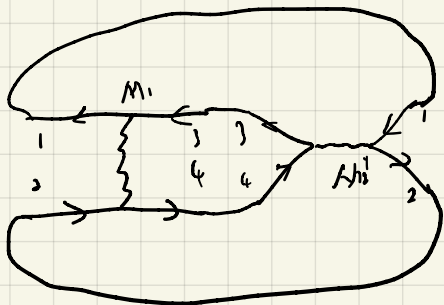
$$\sum |M_1|^2 = \text{Tr}[\not{p}_1 \gamma^\mu \not{p}_3 \gamma^\nu] |Qe|^4$$

$$\text{Tr}[\not{p}_2 \gamma^\mu \not{p}_4 \gamma^\nu] \cdot \frac{1}{p^4}$$

for $|M_2|^2$: 类似.

for $\text{Re}[M, M^\dagger]$

$$2 \text{Re}[M, M^\dagger] = 2 \text{Tr}[\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu \not{p}_4 \gamma_\nu \not{p}_3 \gamma_\mu] \cdot \frac{(Q_e)^4}{k^2 \cdot k'^2}$$



$$\frac{d\sigma_{ee \rightarrow ee}}{d\Omega} = \frac{\alpha^2}{8E^2} \left[\frac{1 + \cos^2\theta}{\sin^4\theta_2} - \frac{2\cos^2\theta}{\sin^2\theta_2} + \frac{1 + \cos^2\theta}{2} \right]$$

Diagram showing momenta p_1 and p_3 at an angle θ .

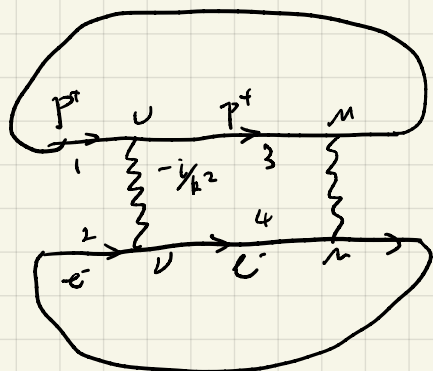
$$p_1 = \left(\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2}\right) \quad p_3 = \left(\frac{\sqrt{s}}{2}, \frac{\sqrt{s}}{2} \sin\theta, 0, \frac{\sqrt{s}}{2} \cos\theta\right)$$

$$p_2 = \left(\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2}\right) \quad p_4 = \left(\frac{\sqrt{s}}{2}, -\frac{\sqrt{s}}{2} \sin\theta, 0, -\frac{\sqrt{s}}{2} \cos\theta\right)$$

6.4 核子的电磁相互作用. $Q_e = -1, Q_p = +1$

$$m_e = 0.51 \text{ MeV} \quad m_p = 938 \text{ MeV}$$

$p^+ + e^- \rightarrow p^+ + e^-$ $iQ_e e \gamma^\mu$



$$\Sigma|M|^2 = \text{Tr}[(\not{p}_3 + m) \gamma^\mu (\not{p}_1 + m) \gamma^\nu] \text{Tr}[\not{p}_4 \gamma_\nu \not{p}_2 \gamma_\mu] \frac{(Q_e Q_p e^2)^2}{k^4}$$

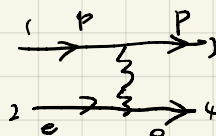
$$p_1 = (r, 0, 0, 0) \quad p_2 = (p_1 + p_2 - p_4)$$

$$p_3 = (Z, 0, 0, Z) \quad p_4 = (Z', Z' \sin\theta, 0, Z' \cos\theta)$$

$$Z' = \frac{Z}{1 + 2 \frac{Z}{m_p} \sin^2\theta_2}$$

$$\frac{d\sigma_{p+e}}{d\Omega} = \frac{\alpha^2 \cos^2\theta}{4E^2 \sin^4\theta_2} \frac{1}{1 + 2 \frac{Z}{m_p} \sin^2\theta_2} \left[1 + \frac{Q^2}{2m_p^2} \tan^2\theta_2 \right]$$

若对于非点粒子, 其顶点可能有更复杂形式



$$\bar{u}(p_3, s_3) \Gamma^\mu u(p_1, s_1) \quad , p_1 = p_2 + k$$

$$\Gamma^\mu(p, k) = 2 \cdot (a_1 \cancel{\not{p}} + a_2 \not{p}^\mu) + a_3 \not{p}^\mu + a_4 \sigma^{\mu\nu} p_\nu + a_5 \sigma^{\mu\nu} p_{1\nu}$$

$$\Rightarrow a_4 [\sigma^{\mu\nu} p_\nu + \sigma^{\mu\nu} p_{1\nu}] = a_4 \sigma^{\mu\nu} k_\nu k_{1\nu}$$

for $(p_3 - m) u(p_3, s_3) = 0$.

$\bar{u}(p_3, s_3) \hat{p}_3 = m \bar{u}(p_3)$

$\bar{u}(p_3) \sigma^{\mu\nu} \hat{p}_3 u(p) \propto \bar{u}(p) \left[\frac{p_3^\mu \hat{p}_3^\nu - p_3^\nu \hat{p}_3^\mu}{2} \right] u(p)$

↓
 $\hat{p}_3^\mu \hat{p}_3^\nu + 2 B^{\mu\nu}$
 ↓
 $2 p_3^\mu - 2 m \gamma^\mu$

流守恒: $\partial_\mu \Gamma^\mu = 0$

假设: $\Gamma^\mu = F(k') \gamma^\mu + i F_2(k') \sigma^{\mu\nu} k_\nu$

finally.

$\frac{d\sigma_{ep \rightarrow ep}}{d\Omega} = \frac{\alpha^2 \cos^2 \theta_2}{4 z^2 \sin^4 \theta_1} \frac{1}{1 + 2 \frac{z}{m_p} \sin^2 \theta_1} \left[F_1^2(k') + Q^2 F_2^2(k') + \frac{Q^2}{2 m_p} (F_1 + 2 m_p F_2) \tan^2 \frac{\theta_2}{2} \right]$

↗ $\alpha^2 z k^2$
 ↘ 并(取同0)

→ Mott公式

求核子核矩.

if proton 半径为0. 假设质子是点粒子 其截面为上式. (Mott)公式

$G_2(Q^2) = F_1 - \frac{Q^2}{2 m_p} F_2$
 $G_M(Q^2) = F_1 - 2 m_p F_2$

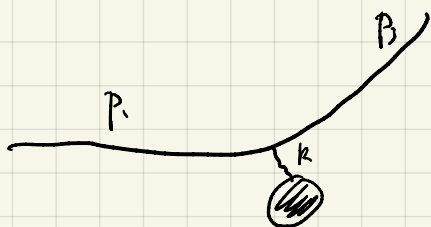
$\frac{d\sigma_{ep \rightarrow ep}}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{Mott} \left[\frac{G_E^2 + 2 G_M^2}{1 + \tau} + 2 \tau G_M^2 \tan^2 \frac{\theta_2}{2} \right]$

$\nabla u = -\frac{p}{\epsilon_0} = -4\pi\rho$

$\langle r^2 \rangle \propto -6 \frac{dG_E}{dQ^2}$

$\langle r^2 \rangle = -6 \frac{dG_E}{dQ^2} \Big|_{Q^2=0} \approx 0.64 \text{ fm}^2$

$\langle r^2 \rangle = -6 \frac{dG_M}{dQ^2} \Big|_{Q^2=0} \approx 0.64 \text{ fm}^2$



$\frac{d\sigma}{d\Omega} = \frac{m_e^2}{4\pi} \left| -e \int d^3r e^{i(\vec{p} - \vec{p}') \cdot \vec{r}} \frac{d^3r}{r} \right|^2$
 $= \frac{m_e^2 e^4}{4(4\pi\epsilon_0)^2} \left| \frac{1}{\epsilon} \int d^3r \rho(r) e^{i\vec{k} \cdot \vec{r}} d^3r \right|^2$

$\downarrow$
 $1 - \frac{\vec{k}_0 \cdot \vec{p}_1}{p} \int \rho(r) \frac{1}{r} r, d^3r$

$\rho(r) = \frac{m^2}{8\pi} e^{-mr}$

$\sum \frac{d\sigma_{ep \rightarrow ep}}{d\Omega} = \sum \frac{(4\pi e^2)^2}{k^4} L_{\mu\nu} L^{\mu\nu} \left(\frac{d^3p_3}{(2\pi)^3 2E_3} \right) \delta^4(\dots)$

$M = \int d^4x \bar{\psi} \gamma^\mu \psi A_\mu = \int d^4x \bar{\psi} \gamma^\mu \psi \int \frac{d^4k}{(2\pi)^4} \tilde{A}_\mu(k) e^{ikx}$

$\bar{\psi} \gamma^\mu \psi = \frac{1}{(2\pi)^4} \int d^4k \tilde{\psi} \gamma^\mu \psi = \frac{1}{(2\pi)^4} \int d^4k \tilde{\psi} \gamma^\mu \psi = \frac{1}{(2\pi)^4} \int d^4k \tilde{\psi} \gamma^\mu \psi$

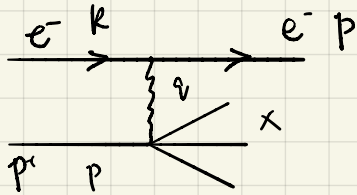
$T^{\mu\nu}(P, k) = a_1 g^{\mu\nu} + a_2 P^\mu P^\nu + a_3 k^\mu k^\nu + a_4 P^\mu k^\nu + a_5 P^\nu k^\mu + a_6 \epsilon^{\mu\nu\rho\sigma} P_\rho k_\sigma$

$k_\mu T^{\mu\nu} = 0 \Rightarrow a_1 k^\nu + a_2 P k P^\nu + a_3 k^2 k^\nu + a_4 P k P^\nu + a_5 P k P^\nu + a_6 \epsilon^{\mu\nu\rho\sigma} P_\rho k_\sigma = 0$

$k_\nu T^{\mu\nu} = 0 \Rightarrow T^{\mu\nu}(P, k) = \left(P^\mu - \frac{P \cdot k}{k^2} k^\mu \right) \left(P^\nu - \frac{P \cdot k}{k^2} k^\nu \right) \frac{W_2}{m_p} + \left(\frac{k^\mu k^\nu}{k^2} - g^{\mu\nu} \right) W_1$

$$\frac{d^2 \Sigma(k)}{d\Omega d^2} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} [W_1(\nu, k^2) \cos^2 \frac{\theta}{2} + 2W_2(\nu, k^2) \sin^2 \frac{\theta}{2}] \quad \leftarrow \text{有关不变性解为 P 内部有 } 1/2 \text{ 的核, 也即核的夸克模型}$$

1.5 电子-核子的非弹性散射的费曼图



单举过程 inclusive process
通举过程 exclusive process

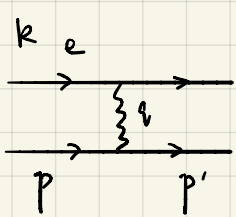
$$\frac{d^2 \Sigma_{ep \rightarrow eX}}{d\Omega d^2} = \sum_X \frac{(2\pi)^4 \delta^4}{2} \frac{1}{S_X} |M|^2 \frac{1}{(2\pi)^3 2E'} [dx] \quad \text{take } q^2 = Q^2, p^2 = M_N^2, \frac{p \cdot q}{2M_N} = \nu$$

$$Q^2 = q^2 = (k - k')^2$$

$$= \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} [W_1(\nu, Q^2) \cos^2 \frac{\theta}{2} + 2W_2(\nu, Q^2) \sin^2 \frac{\theta}{2}]$$

当 Q^2 很大时, $W_1, \nu W_2$ 是 $\frac{Q^2}{\nu}$ 的函数, 与 Q^2 无关.

电子-点粒子 (spin 1/2) 的弹性散射截面.



$$\frac{d^2 \Sigma}{d\Omega d^2} = \frac{\alpha^2 Q_i^2}{4E^2 \sin^4 \frac{\theta}{2}} \delta(\nu - \frac{Q^2}{2m_N}) \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} \cos^2 \frac{\theta}{2} + \tau G_M^2 \sin^2 \frac{\theta}{2} \right]$$

点粒子: $G_E = 1, G_M = 1$

$$[\cos^2 \frac{\theta}{2} + \tau \sin^2 \frac{\theta}{2}]$$

$$W_2 = Q_i^2 \delta(\nu - \frac{Q^2}{2m_N}) \quad W_1 = \frac{Q_i^2 Q^2}{4m_N} \delta(\nu - \frac{Q^2}{2m_N})$$

各种内部粒子, Q_i, x_i

$$\vec{p}_i = x_i \vec{p} \quad \sum x_i = 1, \quad \vec{p} = \infty, \text{ 核子大质量子, take } m_i = x_i M_N = 0$$

$$\vec{p}_i = x_i \vec{p}$$

$$\nu_i = \frac{p_i \cdot q}{2m_N} = \frac{x_i p \cdot q}{2m_N} = x_i \nu$$

$$\nu W_2 = \int \sum_i Q_i^2 \delta(x_i - \frac{Q^2}{2m_N \nu}) f_i(x_i) dx_i \quad W_1 = \int \sum_i \frac{Q_i^2 Q^2}{4m_N \nu} \delta(x_i - \frac{Q^2}{2m_N \nu}) f_i(x_i) dx_i$$

$$x_i = \frac{Q^2}{2m_N \nu}$$

$$\left. \begin{aligned} F_2 &\equiv 2W_2 = \sum_i Q_i^2 f_i(x) \\ F_1 &\equiv 2m_W W_1 = \sum_i Q_i^2 f_i(x) x \end{aligned} \right\} \Rightarrow F_i(x) = x F_2(x)$$

for $i = u, d, s, \bar{u}, \bar{d}, \bar{s}$

$$P: F_2^P(x) = \sum_i Q_i^2 f_i^P(x) = \frac{4}{9} f_u^P(x) + \frac{1}{9} f_d^P(x) + \frac{1}{9} f_s^P(x) + \dots$$

$$\int_0^1 (f_u^P(x) - f_{\bar{u}}^P(x)) dx = 2, \quad \int_0^1 (f_d^P(x) - f_{\bar{d}}^P(x)) dx = 1, \quad \int_0^1 (f_s^P(x) - f_{\bar{s}}^P(x)) dx = 0.$$

$$\left\{ \begin{aligned} P_u &= \int x (f_u^P(x) + f_{\bar{u}}^P(x)) dx = 0.36P \\ P_d &= \int x (f_d^P(x) + f_{\bar{d}}^P(x)) dx = 0.18P \\ P_s &= \int x (f_s^P(x) + f_{\bar{s}}^P(x)) dx \end{aligned} \right.$$

$$\left. \begin{aligned} \frac{\int [f_u^P(x) + f_{\bar{u}}^P(x) + f_d^P(x) + f_{\bar{d}}^P(x)] dx}{\int [f_u^P(x) + f_{\bar{u}}^P(x)] dx} &\approx 0.15 \\ \frac{\int f_s^P(x) dx}{\int [f_u^P(x) + f_{\bar{u}}^P(x)] dx} &\approx 0.03 \end{aligned} \right\} \Rightarrow \left\{ \begin{aligned} \int F_2^P(x) dx &\approx 0.16 \\ \int F_2^N(x) dx &\approx 0.12 \end{aligned} \right.$$

价约只有 0.5P 左右. 剩下约由胶子贡献

$$Z_L = \sum_i Z_i = \sum_i \sqrt{(x_i \vec{p})^2 + p_{Li}^2 + m_i^2} \quad p_i = (Z_i, x_i \vec{p}, \vec{p}_{Li})$$

$$\begin{aligned} &= \sum_i x_i |\vec{p}| \sqrt{1 + \frac{p_{Li}^2 + m_i^2}{(x_i |\vec{p}|)^2}} \\ &\approx \sum_i \left(x_i |\vec{p}| + \frac{p_{Li}^2 + m_i^2}{2 x_i |\vec{p}|} \right) \end{aligned}$$

$$\begin{aligned} Z_p &= \sqrt{|\vec{p}|^2 + m_N^2} = |\vec{p}| \left(1 + \frac{m_N^2}{|\vec{p}|^2} \right)^{1/2} \\ &= |\vec{p}| + \frac{m_N^2}{2|\vec{p}|} \end{aligned}$$

$$\Delta E = Z_L \cdot Z_p = \frac{1}{2|\vec{p}|} \left(\sum_i \frac{p_{Li}^2 + m_i^2}{x_i^2} - m_N^2 \right)$$

由 $\Delta E \Delta t \sim \hbar$.

$$\tau = \frac{\hbar}{\Delta E}, \quad \text{Lifetime } \tau \approx \hbar.$$

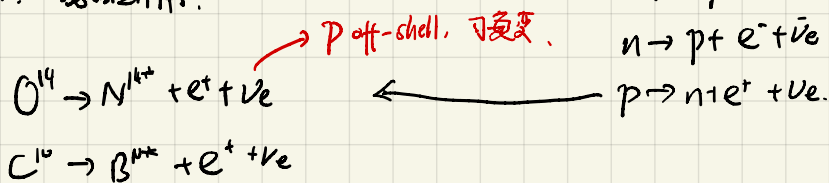
$$\frac{q'_{lab}}{q^0} = \frac{m_N}{\sqrt{|\vec{p}|^2 + m_N^2}}, \quad r = \frac{1}{1 - \frac{E}{E_0}}, \quad q' = \frac{\sqrt{|\vec{p}|^2 + m_N^2}}{m_N} q'_{lab}, \quad Z_{int} = \frac{\sqrt{|\vec{p}|^2 + m_N^2}}{m_N} \frac{\hbar}{q'_{lab}}$$

$$\frac{Z_{int}}{\tau} = \frac{m_N}{\sqrt{|\vec{p}|^2 + m_N^2}} \frac{1}{2|\vec{p}|} \left[\sum_i \frac{p_{Li}^2 + m_i^2}{x_i^2} - m_N^2 \right] \frac{1}{2\omega} \ll 1, \quad Z_{int} \ll \tau.$$

无夸大的说, 相互作用时间远小于核的寿命.

Chap 7. 弱相互作用.

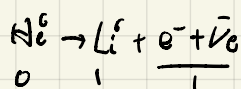
$m_n > m_p$.



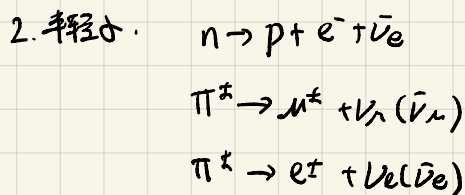
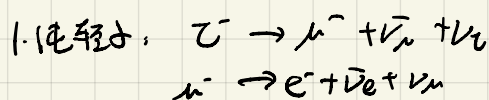
$$H = \frac{G}{\sqrt{2}} (\bar{\psi}_p \gamma^\mu \psi_n) (\bar{\psi}_e \gamma_\mu \psi_{\nu_e})$$

→ 推广变宽.

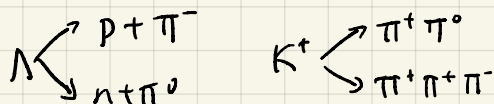
什默夫-泰勒型.



弱衰变分类:



3. 非轻子.

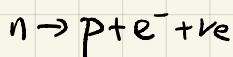
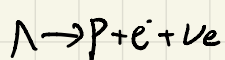


7.2 V-A理论.

$$\bar{\psi}_A \Gamma_{AB} \psi_B \quad \bar{\psi}_C \Gamma_{CD} \psi_D \quad T_{AB} = \gamma_\mu (1 - \gamma_5), \quad T_{CD} = \gamma_\mu (1 + \gamma_5)$$

$$H = \sum \frac{G_{ij}}{\sqrt{2}} \bar{l}_i \gamma_\mu (1 - \gamma_5) \nu_i \quad \bar{\nu}_j \gamma^\mu (1 - \gamma_5) l_j + h.c.$$

$$G_{ij} = G, \text{ universal}$$



$$H = \frac{G'}{\sqrt{2}} \bar{\psi}_p \gamma_\mu (1 - \gamma_5) \psi_n \quad \bar{\psi}_e \gamma^\mu (1 - \gamma_5) \psi_{\nu_e}$$

$$H = \frac{G}{\sqrt{2}} \bar{\psi}_p \gamma_\mu (a + b \gamma_5) \psi_n \dots$$

$$\begin{matrix} a \approx 1 \\ b \approx 1.25 \end{matrix}$$

$$G' = \frac{1}{5} G$$



but why? → Cabibbo 角

7.3 Cabibbo 理论 $\rightarrow$ GIM 机制

$$J_{\mu}^+ = \cos\theta_c J_{\mu}^{+\Delta S=0} + \sin\theta_c J_{\mu}^{+\Delta S=1}$$

$$\begin{aligned}\sin\theta_c &= V_{cs} \\ \cos\theta_c &= V_{cd}/V_s\end{aligned}$$

$$\Delta S=0 \quad J_{\mu}^{+\Delta S=0} = \bar{u} \gamma_{\mu} (1-\gamma_5) d$$

$$\Delta S=1 \quad J_{\mu}^{+\Delta S=1} = \bar{u} \gamma_{\mu} (1-\gamma_5) s$$

$$J_{\mu}^+ = \cos\theta_c \bar{u} \gamma_{\mu} (1-\gamma_5) d + \sin\theta_c \bar{u} \gamma_{\mu} (1-\gamma_5) s$$

$$= \bar{u} \gamma_{\mu} (1-\gamma_5) [\cos\theta_c d + \sin\theta_c s]$$

$$\equiv \bar{u} \gamma_{\mu} (1-\gamma_5) d' \quad \text{弱相互作用本征态.}$$

$$\text{即之有: } \begin{cases} d' = \cos\theta_c d + \sin\theta_c s \\ s' = -\sin\theta_c d + \cos\theta_c s \end{cases}$$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

GIM 机制.

$\leftarrow$ 质量本征态.

$$J_{\mu}^+ = (\bar{u} \bar{c})' \gamma_{\mu} (1-\gamma_5) \begin{pmatrix} d' \\ s' \end{pmatrix}'$$

$$\begin{pmatrix} d' \\ s' \end{pmatrix}' = A \begin{pmatrix} d \\ s \end{pmatrix} \quad \begin{pmatrix} u \\ c \end{pmatrix}' = B \begin{pmatrix} u \\ c \end{pmatrix}$$

$$(\bar{u} \bar{c})' = (\bar{u} \bar{c})' B^{\dagger}$$

$$J_{\mu}^+ = (\bar{u} \bar{c}) B^{\dagger} A \gamma_{\mu} (1-\gamma_5) \begin{pmatrix} d \\ s \end{pmatrix}$$

$$\downarrow \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

$$J_{\mu}^+ = \bar{u} \gamma_{\mu} (1-\gamma_5) [\cos\theta_c d + \sin\theta_c s] \xrightarrow{d'} + \bar{c} \gamma_{\mu} (1-\gamma_5) [-\sin\theta_c d + \cos\theta_c s] \xrightarrow{s'}$$

s' 已耦合.

7.4. K-M模型

$$J_\mu^+ = (\bar{u} \bar{c} \bar{t})' V_\mu (1-\gamma_5) \begin{pmatrix} d \\ s \\ b \end{pmatrix}' = (\bar{u} \bar{c} \bar{t}) V_\mu (1-\gamma_5) \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad j_\mu^+ = \frac{g}{2\sqrt{2}} J_\mu^+$$

$VV^\dagger = 1$, 共4个矩阵.

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{12} & S_{12} \\ 0 & -S_{12} & C_{12} \end{pmatrix} \begin{pmatrix} C_{13} & 0 & S_{13} e^{i\delta_{13}} \\ 0 & 0 & 0 \\ -S_{13} e^{-i\delta_{13}} & 0 & 0 \end{pmatrix} \begin{pmatrix} C_{11} & S_{11} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$C_{ij} = \cos \theta_{ij}$$

$$S_{ij} = \sin \theta_{ij}$$

$$j_\mu^0 = \frac{g}{2\sqrt{2}} (\bar{u} \bar{c} \bar{t})' V_\mu (a+b\gamma_5) \begin{pmatrix} u \\ c \\ t \end{pmatrix}' = \frac{g}{2\sqrt{2}} (\bar{u} \bar{c} \bar{t}) V_\mu (1-\gamma_5) \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$$J_\mu^0 = (\bar{u} \bar{c} \bar{t})' V_\mu (1-\gamma_5) \begin{pmatrix} u \\ c \\ t \end{pmatrix}' = (\bar{u} \bar{c} \bar{t}) b^\dagger b V_\mu (1-\gamma_5) \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

对角的

disb同理. 因此 tree-level 无味改变中性流

for lepton:

$$j_\mu^+ = \frac{g}{2\sqrt{2}} (\bar{\nu}_\mu \bar{\nu}_\tau)' V_\mu V_\mu (1-\gamma_5) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$\sigma(\nu_\mu + e^- \rightarrow \mu^- + \nu_e) = \frac{8}{s} (G_F K)^2$$

电子3动量.

for s.e.

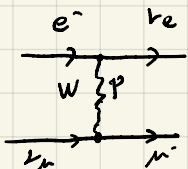
$$\sigma_c \leq \frac{\pi}{k^2}$$

$$k \leq \left(\frac{\pi}{\sqrt{2} G_F} \right)^{1/2} \sim 30 \text{ GeV. } \leftarrow \text{上限. 这相互作用模型为树图, 因此内部线有15打 (类似 string).$$

$W^\pm$ m_W 中间玻色子

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} (\bar{u} \bar{c} \bar{t}) V_\mu (1-\gamma_5) \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^\mu + h.c. + \frac{g}{2\sqrt{2}} (\bar{u} \bar{c} \bar{t}) V_\mu (a+b\gamma_5) \begin{pmatrix} u \\ c \\ t \end{pmatrix} + (e\mu\tau) \dots + h.c.$$

$$\frac{g}{2\sqrt{2}} (\bar{\nu}_\mu \bar{\nu}_\tau) V_\mu V_\mu (1-\gamma_5) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} W^\mu + h.c. + \text{中微子}.$$



$$M \propto \frac{1}{p^2 - m_W^2} \frac{g^2}{8} \dots = \frac{g^2}{8 m_W^2}$$

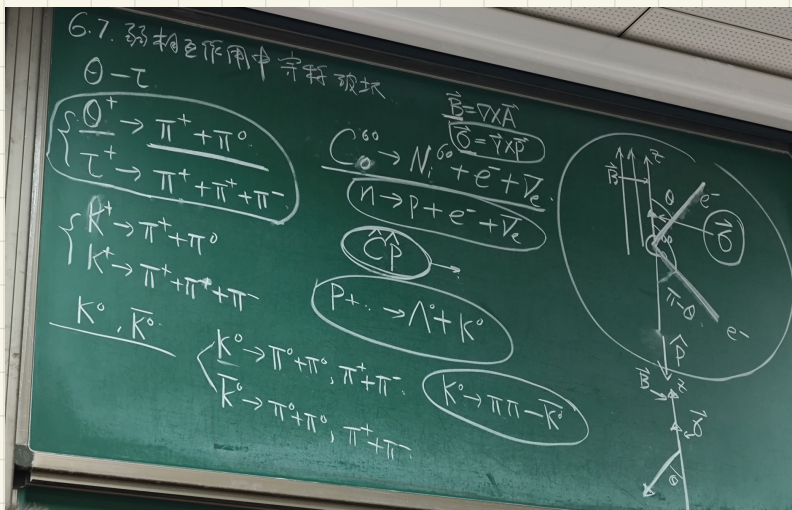
Therefore: Feynman Rule:

$$\text{wavy line } \nu \quad \frac{-i(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})}{p^2 - m^2}$$

$$\text{wavy line } \nu \quad \frac{g}{2f_\pi} \delta_{ij} \gamma_\mu (1-\gamma_5)$$

$$\text{wavy line } \nu \quad \frac{g}{2f_\pi} V_{ij} \gamma_\mu (1-\gamma_5) \quad \begin{cases} i = u, c \\ j = d, s, b \end{cases}$$

6.7 弱相互作用中的宇称、电荷



K^0 与 $\bar{K}^0$ 有混合。

$$\begin{cases} |K_1^0\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle) \\ |K_2^0\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle) \end{cases}$$

$$\begin{cases} \hat{P}|K_1^0\rangle = -|K_1^0\rangle \\ \hat{P}|K_2^0\rangle = -|K_2^0\rangle \end{cases}$$

$$\begin{cases} \hat{C}|K_1^0\rangle = e^{i\theta} |\bar{K}_1^0\rangle \\ \hat{C}|K_2^0\rangle = e^{-i\theta} |\bar{K}_2^0\rangle \end{cases}$$

$\theta = \pi$

$$\begin{cases} \hat{CP}|K_1^0\rangle = |K_1^0\rangle \\ \hat{CP}|K_2^0\rangle = -|K_2^0\rangle \end{cases}$$

$$\text{总结: } \begin{cases} \pi^+\pi^+ \\ \pi^+\pi^- \\ \pi^-\pi^-\pi^0 \end{cases} \quad \hat{P}: \quad \begin{cases} P_1 P_2 (1)^{L=0} = 1 \\ P_1 P_2 P_3 (1)^{L=0} (1)^{L=0} = -1 \end{cases} \quad \begin{cases} \hat{C}: C^L = 1 \\ \hat{C}: (-1)^L G = 1 \end{cases} \quad \begin{cases} CP: 1 \\ CP: -1 \end{cases}$$

$$\begin{cases} 470 \text{ MeV} & 270 \text{ MeV} \\ K_1^0 \rightarrow \pi\pi & K_S \\ 470 \text{ MeV} & 440 \text{ MeV} \\ K_2^0 \rightarrow \pi\pi\pi & K_L \end{cases}$$

$$\tau_S = 0.9 \times 10^{-10} \text{ s}$$

$$\tau_L = 5 \times 10^{-8} \text{ s}$$

6.7. 弱相互作用中宇称破坏

$$|K_1^0(t)\rangle = \frac{1}{\sqrt{2}} e^{-i(m_1 - i\frac{\Gamma}{2})t} |\psi_1(t)\rangle = \frac{1}{\sqrt{2}} e^{-im_1 t - \frac{\Gamma}{2}t} |K_1^0(t)\rangle$$

$$|K_2^0(t)\rangle = \frac{1}{\sqrt{2}} e^{-i(m_2 - i\frac{\Gamma}{2})t} |\psi_2(t)\rangle = \frac{1}{\sqrt{2}} e^{-im_2 t - \frac{\Gamma}{2}t} |K_2^0(t)\rangle$$

$$|K^0\rangle = \frac{1}{\sqrt{2}} [e^{-im_1 t - \frac{\Gamma}{2}t} |K_1^0(t)\rangle + e^{-im_2 t - \frac{\Gamma}{2}t} |K_2^0(t)\rangle]$$

$$= \frac{1}{2} [(e^{-im_1 t - \frac{\Gamma}{2}t} + e^{-im_2 t - \frac{\Gamma}{2}t}) |K_0\rangle + (e^{-im_1 t} - e^{-im_2 t}) |K_0\rangle]$$

$$P_{K^0} = \frac{1}{4} [e^{-\Gamma t} + e^{-\Gamma t} + 2e^{-\frac{1}{2}(\Gamma_1 + \Gamma_2)t} \cos \Delta m t] \quad \Delta m = m_2 - m_1 = (3.489 \pm 0.008) \times 10^{12} \text{ MeV}$$

$$P_{\bar{K}^0}(t) = \frac{1}{4} [e^{-\Gamma t} + e^{-\Gamma t} - 2e^{-\frac{1}{2}(\Gamma_1 + \Gamma_2)t} \cos \Delta m t]$$

中性K介子衰变中 $|\Delta I| = \frac{1}{2}$ 规则

$$\Gamma \rightarrow \frac{1}{2} = \Gamma \quad \Gamma_i : \Gamma = \sum \Gamma_i$$

$$Br_i = \frac{\Gamma_i}{\Gamma} \quad \frac{Br(K_i^0 \rightarrow \pi^+ \pi^-)}{Br(K_i^0 \rightarrow \pi \pi)} = \frac{(68.61 \pm 0.28)\%}{(31.39 \pm 0.28)\%} = 2.17$$

$$|\pi^+ \pi^-\rangle = \frac{1}{\sqrt{2}} |2,0\rangle + \frac{1}{\sqrt{2}} |0,0\rangle \quad \text{for } |\Delta I| = \frac{1}{2} \quad \text{for } |\Delta I| = \frac{1}{2} \quad |2,0\rangle \text{ 替代}$$

$$|\pi^- \pi^-\rangle = \frac{1}{\sqrt{2}} |2,0\rangle - \frac{1}{\sqrt{2}} |0,0\rangle$$

与实验 match.

CP破坏:

$$\begin{cases} K_L^0 = \frac{1}{\sqrt{2}} (\psi_1 |K_1^0\rangle + \psi_2 |K_2^0\rangle) \\ K_S^0 = \frac{1}{\sqrt{2}} (\psi_1 |K_1^0\rangle - \psi_2 |K_2^0\rangle) \end{cases}$$

混合的 CP 破坏:

$$\begin{cases} \eta_+ = \frac{A(K_L^0 \rightarrow \pi^+ \pi^-)}{A(K_S^0 \rightarrow \pi^+ \pi^-)} = |\eta_+| e^{i\phi_+} \\ \eta_- = \frac{A(K_L^0 \rightarrow \pi^- \pi^-)}{A(K_S^0 \rightarrow \pi^- \pi^-)} = |\eta_-| e^{i\phi_-} \end{cases}$$

非完全由混合引起

$$\eta_+ = \eta_- = \epsilon \quad \text{但实验不相符}$$

因此存在直接的 CP 破坏.

不满足 $|\Delta I| = \frac{1}{2}$, 破坏 CP

$$A(K_L^0 \rightarrow \pi^+ \pi^-) = \frac{1}{\sqrt{2}} A_2(K_L^0) + \frac{1}{\sqrt{2}} A_0(K_L^0)$$

$$A(K_S^0 \rightarrow \pi^+ \pi^-) = \frac{1}{\sqrt{2}} A_2(K_S^0) + \frac{1}{\sqrt{2}} A_0(K_S^0)$$

$$A(K_L^0 \rightarrow \pi^- \pi^-) = \frac{1}{\sqrt{2}} A_2(K_L^0) - \frac{1}{\sqrt{2}} A_0(K_L^0)$$

$$A(K_S^0 \rightarrow \pi^- \pi^-) = \frac{1}{\sqrt{2}} A_2(K_S^0) - \frac{1}{\sqrt{2}} A_0(K_S^0)$$

$$\Rightarrow \eta_{+-} = \frac{\epsilon + \frac{A_2(K_L^0)}{\sqrt{2} A_0(K_L^0)}}{1 - \epsilon \frac{A_2(K_S^0)}{\sqrt{2} A_0(K_S^0)}} \Rightarrow \epsilon'$$

for ϵ, ϵ' 为小量

$$\eta_{+-} = \epsilon + \epsilon'$$

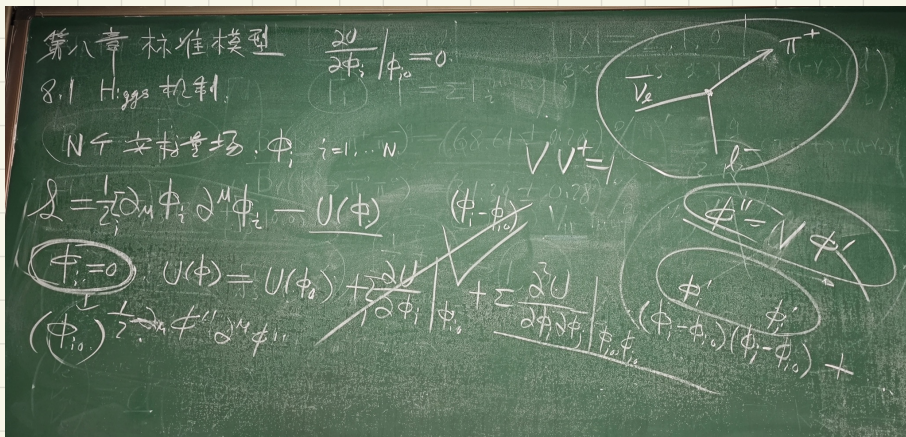
$$\eta_{00} = \epsilon - 2\epsilon'$$

$$\epsilon = (2.271 \pm 0.017) \times 10^{-3}$$

$$Re(\epsilon'/\epsilon) = \frac{\epsilon'}{\epsilon} = (2.1 \pm 0.5) \times 10^{-3}$$

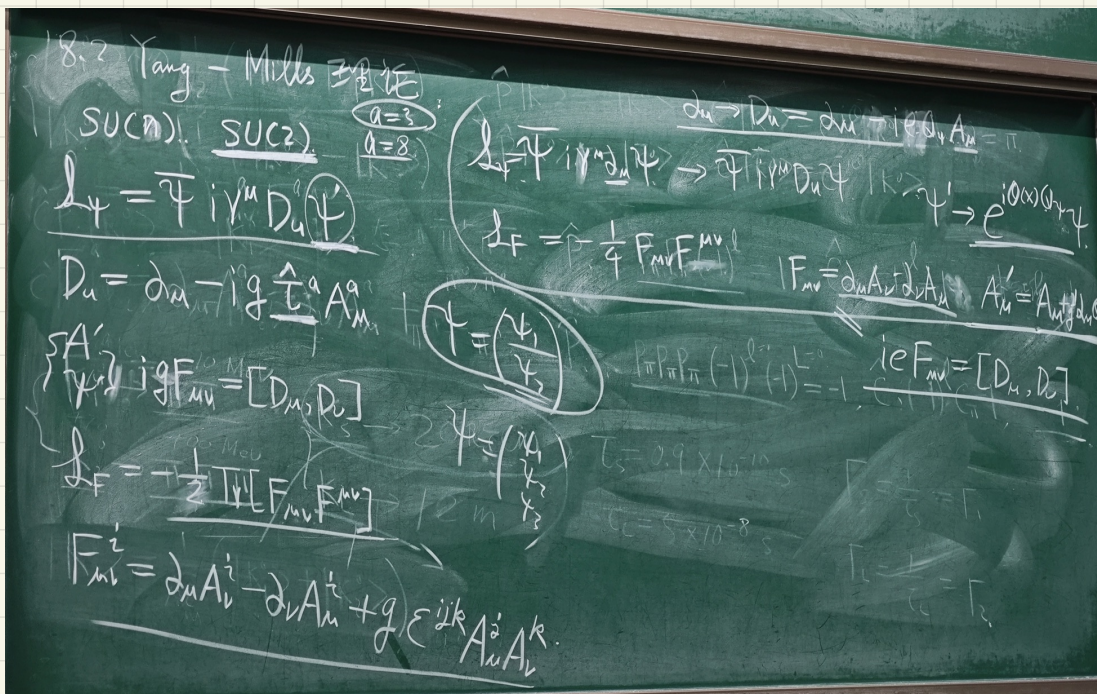
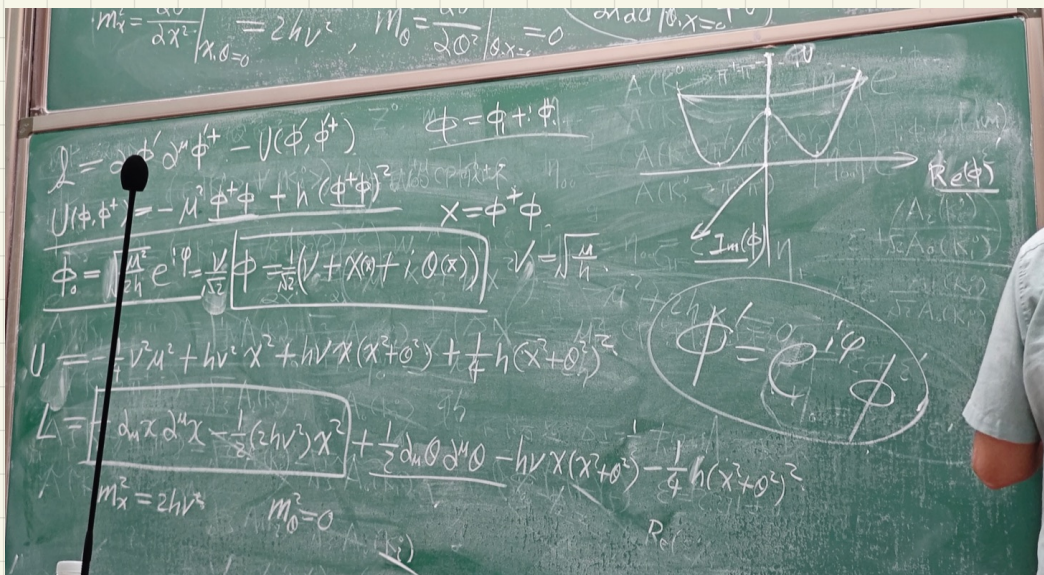
第八章 Standard model.

8.1 Higgs 机制



$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^\dagger - U(\phi, \phi^\dagger)$$

$$U(\phi, \phi') = \mu^2 \phi^\dagger \phi + h (\phi^\dagger \phi)^2$$



$$SU(2) \quad \tau^1 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau^2 = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau^3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}' = U \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad U = e^{i \alpha^i \tau^i}$$

$$U^\dagger U = 1.$$

$$8.3 \text{ standard model} \quad SU_L(2) \otimes U_Y(1) \quad R = \frac{1}{2}(1 + \tau_3) \quad L = \frac{1}{2}(1 - \tau_3) \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad e_R \quad \mu_R \quad \tau_R$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \quad u_R \quad d_R \quad c_R \quad s_R \quad t_R \quad b_R.$$

$$\psi = \psi_R + \psi_L = (R + L)\psi.$$

$$\bar{\psi}_R = \bar{\psi}_L$$

$$\bar{\psi}_L = \bar{\psi}_R$$

$$m \bar{\psi} \psi = m \bar{\psi} (R + L) (R + L) \psi = m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

$$U(1): \quad \psi(x) = e^{i Q e \theta(x)} \psi(x).$$

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + i e Q A_\mu$$

$$A'_\mu = A_\mu + \partial_\mu \theta(x).$$

$$SU(2): \quad \partial_\mu \rightarrow D_\mu = \partial_\mu + i \vec{\tau} \cdot \vec{A}_\mu$$

$$\psi' = U \psi \quad U = e^{i \alpha(x) \tau^i}$$

$$Q = \tau^3 + Y_2$$

$$\begin{cases} Y(l_L) = -1, \\ Y(q_L) = \frac{1}{3}. \end{cases}$$

$$\begin{cases} Y(l_R) = -2, \\ Y(u_R) = \frac{2}{3} \\ Y(d_R) = -\frac{2}{3} \end{cases}$$

怎么定下来的?

$$H = \begin{pmatrix} \Phi^+ \\ \phi^0 \end{pmatrix}$$

$$H' = U H, \quad Y(H) = 1 \quad (\text{无电荷})$$

因为 $U(1)$ 不变, 取 $Y(H) = 0$. 有如下形式.

$$\mathcal{L}_Y = \bar{l}_L^i H^\dagger l_L^j Y_{ij}^e + \bar{d}_L^i H^\dagger q_L^j Y_{ij}^d + \bar{u}_L^i \tilde{H}^\dagger q_L^j Y_{ij}^u$$

th.c.

$$\bar{\psi} = (i\sigma_2) \psi^* = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \begin{pmatrix} \chi \\ \phi \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{\text{fermi}} &= \bar{l}_L^i \not{\partial} l_L^i + \bar{l}_R^i \not{\partial} l_R^i + \bar{q}_L^i \not{\partial} q_L^i + \bar{u}_R^i \not{\partial} u_R^i + \bar{d}_R^i \not{\partial} d_R^i \\ &= \bar{l}_L^i i \gamma^\mu (\partial_\mu + i g' A_\mu + i g A_\mu^a) l_L^i + \bar{l}_R^i i \gamma^\mu (\partial_\mu + i g' A_\mu) l_R^i \end{aligned}$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H^\dagger)(D^\mu H) - V(H)$$

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$\text{规范: } H = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + h(x) + i\chi(x)) \end{pmatrix}, v = \sqrt{\frac{\mu}{\lambda}}$$

$$H' = U_0 H = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h(x)) \end{pmatrix}$$

$$\mathcal{L}_{\text{Higgs}} = \left[(\partial_\mu + i g' A_\mu + i g A_\mu^a) \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h) \end{pmatrix} \right]^\dagger \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h) \end{pmatrix}$$

.....

规范场相互作用

$$\begin{cases} w^\dagger w^\dagger f \\ w^\dagger w^\dagger z \end{cases}$$

$$\begin{cases} w w^\dagger z^\dagger \\ w w^\dagger f^\dagger \\ w^\dagger w^\dagger z^\dagger f \end{cases}$$

给物质场. 在对称群里的态, 写出低能

相空间计算

Muller 效应

bar bar 效应

几何效应 (部分对称贡献)