

拓扑图和粲重子CP

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PRD 112, 054022 (2025)

华中师范大学 2025.9.13

OUTLINE

- Introduction
- Topological diagrams
- Final-state rescattering
- CP violation
- Summary

CP VIOLATION IN BOTTOMED BARYON

- A new milestone in direct CP violation of baryons.

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)



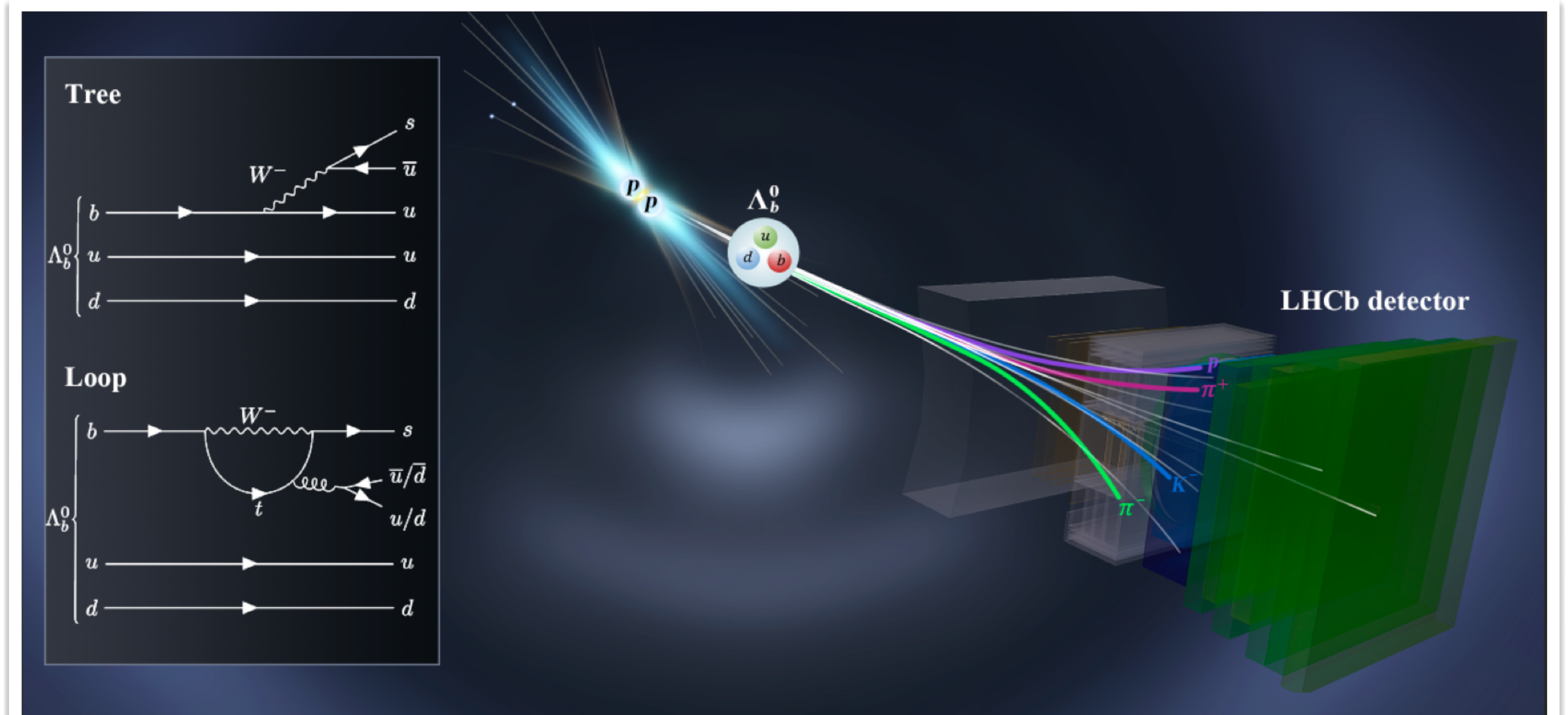
CERN-EP-2025-031
LHCb-PAPER-2024-054
March 21, 2025

Observation of charge-parity symmetry breaking in baryon decays

LHCb collaboration[†]

LHCb, 2503.16954, Nature 643 (2025) 8074, 1223-1228

$$\mathcal{A}_{CP} \equiv \frac{\Gamma(\Lambda_b^0 \rightarrow p K^- \pi^+ \pi^-) - \Gamma(\bar{\Lambda}_b^0 \rightarrow \bar{p} K^+ \pi^- \pi^+)}{\Gamma(\Lambda_b^0 \rightarrow p K^- \pi^+ \pi^-) + \Gamma(\bar{\Lambda}_b^0 \rightarrow \bar{p} K^+ \pi^- \pi^+)}$$



$$\mathcal{A}_{CP} = (2.45 \pm 0.46 \pm 0.10)\%$$

The first-ever observation of CP violation in baryon decays!

CHARM CP VIOLATION

- Charm CP violation also provides a window to explore fundamental physics.
- Charmed meson CP violation has been observed and explained.

PRL 108, 111602 (2012) Selected for a Viewpoint in Physics
PHYSICAL REVIEW LETTERS week ending
16 MARCH 2012

Evidence for CP Violation in Time-Integrated $D^0 \rightarrow h^- h^+$ Decay Rates

R. Aaij *et al.**
(LHCb Collaboration)

(Received 6 December 2011; published 12 March 2012; publisher error corrected 12 March 2012)

A search for time-integrated CP violation in $D^0 \rightarrow h^- h^+$ ($h = K, \pi$) decays is presented using 0.62 fb^{-1} of data collected by LHCb in 2011. The flavor of the charm meson is determined by the charge of the slow pion in the $D^{*+} \rightarrow D^0 \pi^+$ and $D^{*-} \rightarrow \bar{D}^0 \pi^-$ decay chains. The difference in CP asymmetry between $D^0 \rightarrow K^- K^+$ and $D^0 \rightarrow \pi^- \pi^+$, $\Delta A_{CP} \equiv A_{CP}(K^- K^+) - A_{CP}(\pi^- \pi^+)$, is measured to be $[-0.82 \pm 0.21(\text{stat}) \pm 0.11(\text{syst})]\%$. This differs from the hypothesis of CP conservation by 3.5 standard deviations.



PHYSICAL REVIEW LETTERS 122, 211803 (2019)

Editors' Suggestion Featured in Physics

Observation of CP Violation in Charm Decays

R. Aaij *et al.**
(LHCb Collaboration)

(Received 21 March 2019; revised manuscript received 2 May 2019; published 29 May 2019)

A search for charge-parity (CP) violation in $D^0 \rightarrow K^- K^+$ and $D^0 \rightarrow \pi^- \pi^+$ decays is reported, using pp collision data corresponding to an integrated luminosity of 5.9 fb^{-1} collected at a center-of-mass energy of 13 TeV with the LHCb detector. The flavor of the charm meson is inferred from the charge of the pion in $D^{*+}(2010)^+ \rightarrow D^0 \pi^+$ decays or from the charge of the muon in $\bar{B} \rightarrow D^0 \mu^- \bar{\nu}_\mu X$ decays. The difference between the CP asymmetries in $D^0 \rightarrow K^- K^+$ and $D^0 \rightarrow \pi^- \pi^+$ decays is measured to be $\Delta A_{CP} = [-18.2 \pm 3.2(\text{stat}) \pm 0.9(\text{syst})] \times 10^{-4}$ for π -tagged and $\Delta A_{CP} = [-9 \pm 8(\text{stat}) \pm 5(\text{syst})] \times 10^{-4}$ for μ -tagged D^0 mesons. Combining these with previous LHCb results leads to $\Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4}$, where the uncertainty includes both statistical and systematic contributions. The measured value differs from zero by more than 5 standard deviations. This is the first observation of CP violation in the decay of charm hadrons.



$$\Delta \mathcal{A}_{CP} = (-1.54 \pm 0.29) \times 10^{-3}$$

$$\begin{aligned} \Delta \mathcal{A}_{CP} &= -\frac{4\text{Im}[(\lambda_s - \lambda_d)\lambda_b^*]}{|\lambda_s - \lambda_d|^2} \left(\left| \frac{P}{T+E} \right|_{KK} \sin \theta_{KK} + \left| \frac{P}{T+E} \right|_{\pi\pi} \sin \theta_{\pi\pi} \right) \\ &= -1.31 \times 10^{-3} \left(\left| \frac{P}{T+E} \right|_{KK} \sin \theta_{KK} + \left| \frac{P}{T+E} \right|_{\pi\pi} \sin \theta_{\pi\pi} \right), \end{aligned}$$

$|P/T|$ naively expected to be $(\alpha_s(\mu_c)/\pi) \sim \mathcal{O}(0.1)$

$\rightarrow \Delta A_{CP} \sim 10^{-4}$

$P^{\text{LD}} = E^{\text{LD}} \approx E \rightarrow \Delta \mathcal{A}_{CP} = (-0.151 \pm 0.004)\%$

- Charmed baryon CP violation: a new frontier.

H.Y. Cheng, C.W. Chiang, PRD 86, 014014 (2012)

CP VIOLATION IN CHARMED BARYON

- Naive expectation

$$\mathcal{A}_{CP} \equiv \frac{\Gamma(\mathcal{B}_c \rightarrow \mathcal{B}P) - \Gamma(\bar{\mathcal{B}}_c \rightarrow \bar{\mathcal{B}}\bar{P})}{\Gamma(\mathcal{B}_c \rightarrow \mathcal{B}P) + \Gamma(\bar{\mathcal{B}}_c \rightarrow \bar{\mathcal{B}}\bar{P})},$$

SCS process: $A = \lambda_d A_d + \lambda_s A_s$

$$\mathcal{A}_{CP} = \frac{2\text{Im}(\lambda_d \lambda_s^*)}{|\lambda_d|^2} \frac{\text{Im}(A_d A_s^*)}{|A_d - A_s|^2} = 1.31 \times 10^{-3} \frac{|A_d A_s|}{|A_d - A_s|^2} \sin \delta_{ds},$$

strong phase

$\mathcal{O}(10^{-3})$ is expectable

- Sizable strong phase observed:

PHYSICAL REVIEW LETTERS **132**, 031801 (2024)

**First Measurement of the Decay Asymmetry
in the Pure W-Boson-Exchange Decay $\Lambda_c^+ \rightarrow \Xi^0 K^+$**

M. Ablikim *et al.*^{*}
(BESIII Collaboration)

 (Received 6 September 2023; accepted 30 November 2023; published 17 January 2024)

$$\alpha_{\Xi^0 K^+} = 0.01 \pm 0.16(\text{stat}) \pm 0.03(\text{syst})$$

$$\delta_p - \delta_s = -1.55 \pm 0.25(\text{stat}) \pm 0.05(\text{syst}) \text{ rad}$$

$$1.59 \pm 0.25(\text{stat}) \pm 0.05(\text{syst}) \text{ rad.}$$



Now is the time to theoretically explore charmed baryon CPV!

THE LATEST EXPERIMENTAL PROGRESS

Charm CPV: Charmed baryons

Science Bulletin 68 (2023) 583

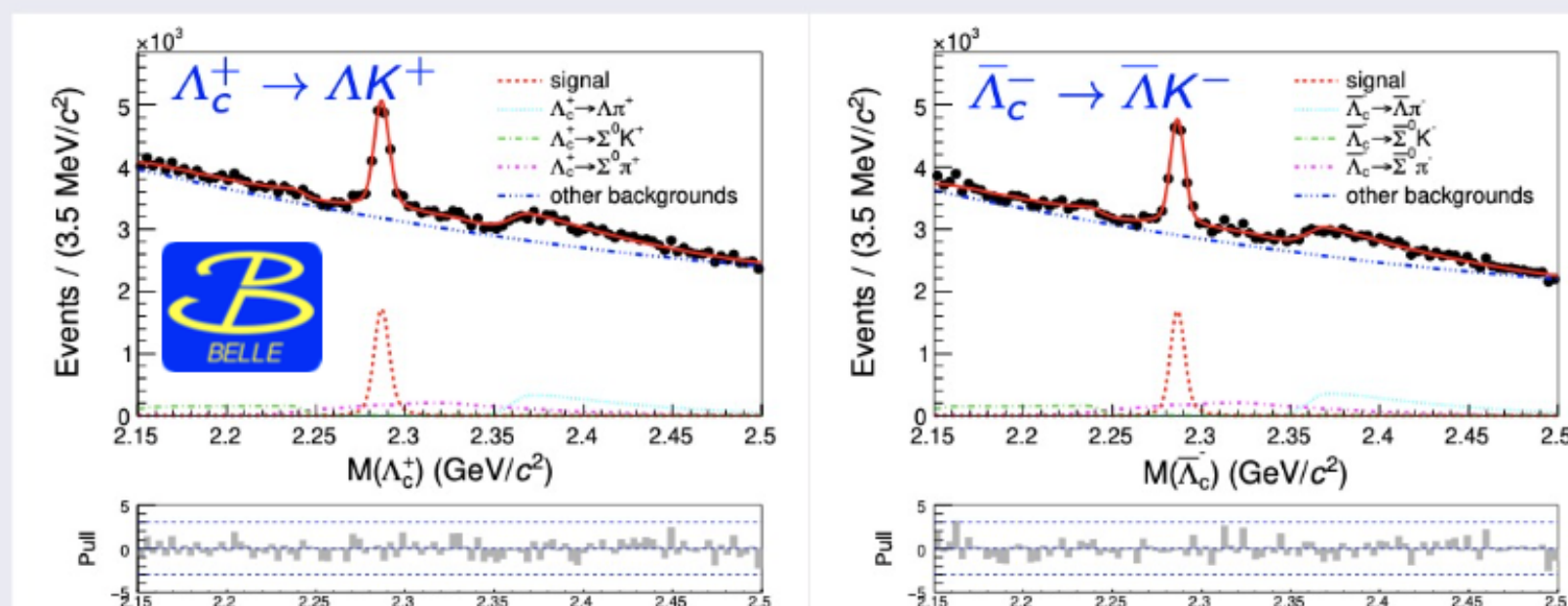


direct CPV in $\Lambda_c^+ \rightarrow \Lambda K^+, \Sigma^0 K^+$

- The sources of raw asymmetry of $\Lambda_c^+ \rightarrow \Lambda h^+$:

$$A_{\text{raw}}(\Lambda_c^+ \rightarrow \Lambda K^+) \approx \mathcal{A}_{CP}^{\Lambda_c^+ \rightarrow \Lambda K^+} + \mathcal{A}_{CP}^{\Lambda_c^+ \rightarrow p\pi^-} + A_\epsilon^\Lambda + A_\epsilon^{K^+} + A_{FB}^{\Lambda_c^+}$$

- Using (CF) $\Lambda_c^+ \rightarrow \Lambda\pi^+, \Sigma^0\pi^+$ to remove common sources.



- $\mathcal{A}_{CP}^{\text{dir}}(\Lambda_c^+ \rightarrow \Lambda K^+) = (+2.1 \pm 2.6 \pm 0.1)\%$

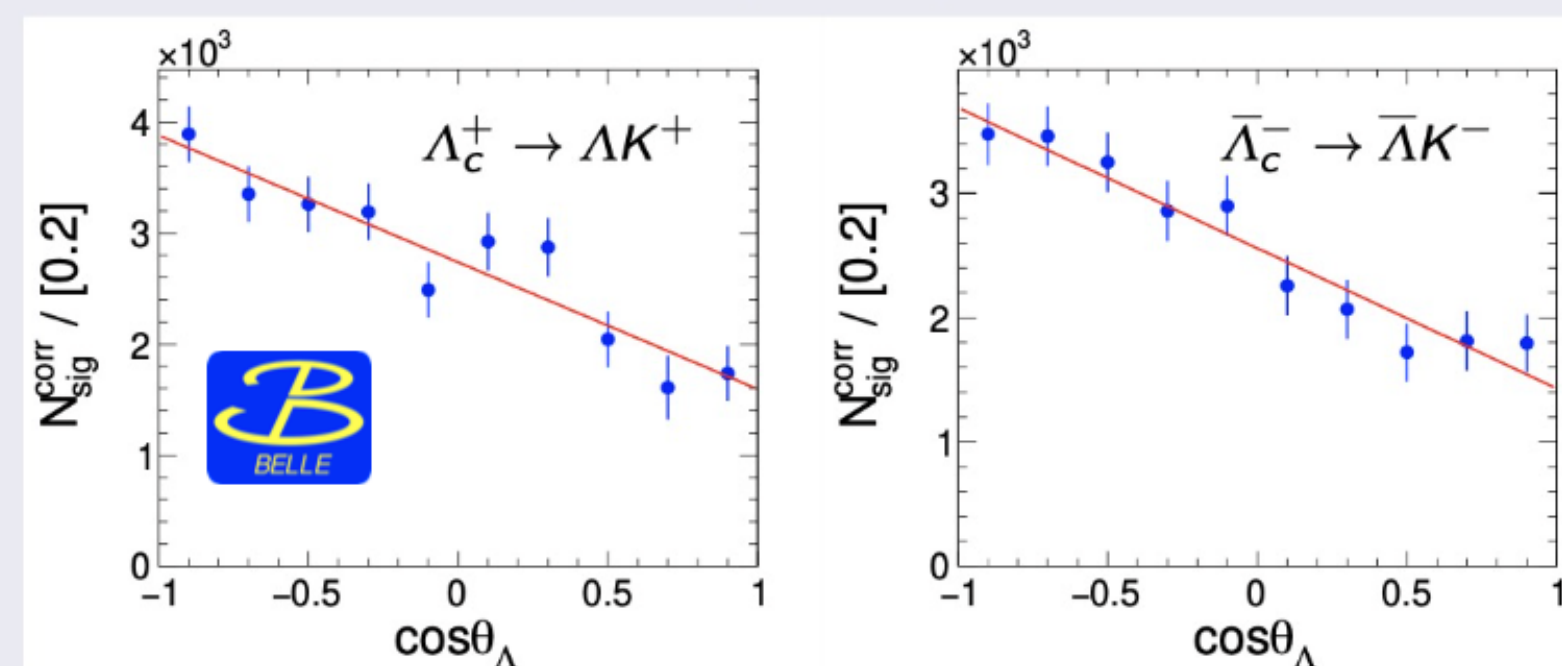
- $\mathcal{A}_{CP}^{\text{dir}}(\Lambda_c^+ \rightarrow \Sigma^0 K^+) = (+2.5 \pm 5.4 \pm 0.4)\%$

First $\mathcal{A}_{CP}^{\text{dir}}$ for SCS two-body decays of charmed baryons.

α -induced CPV in $\Lambda_c^+ \rightarrow \Lambda K^+, \Sigma^0 K^+$

- Measure $\alpha/\bar{\alpha}$ for the separate $\Lambda_c^+/\bar{\Lambda}_c^-$ samples.

- Calculate $\mathcal{A}_{CP}^\alpha \equiv (\alpha_{\Lambda_c^+} + \alpha_{\bar{\Lambda}_c^-})/(\alpha_{\Lambda_c^+} - \alpha_{\bar{\Lambda}_c^-})$.



- $\mathcal{A}_{CP}^\alpha(\Lambda_c^+ \rightarrow \Lambda K^+) = -0.023 \pm 0.086 \pm 0.071$

- $\mathcal{A}_{CP}^\alpha(\Lambda_c^+ \rightarrow \Sigma^0 K^+) = +0.08 \pm 0.35 \pm 0.14$

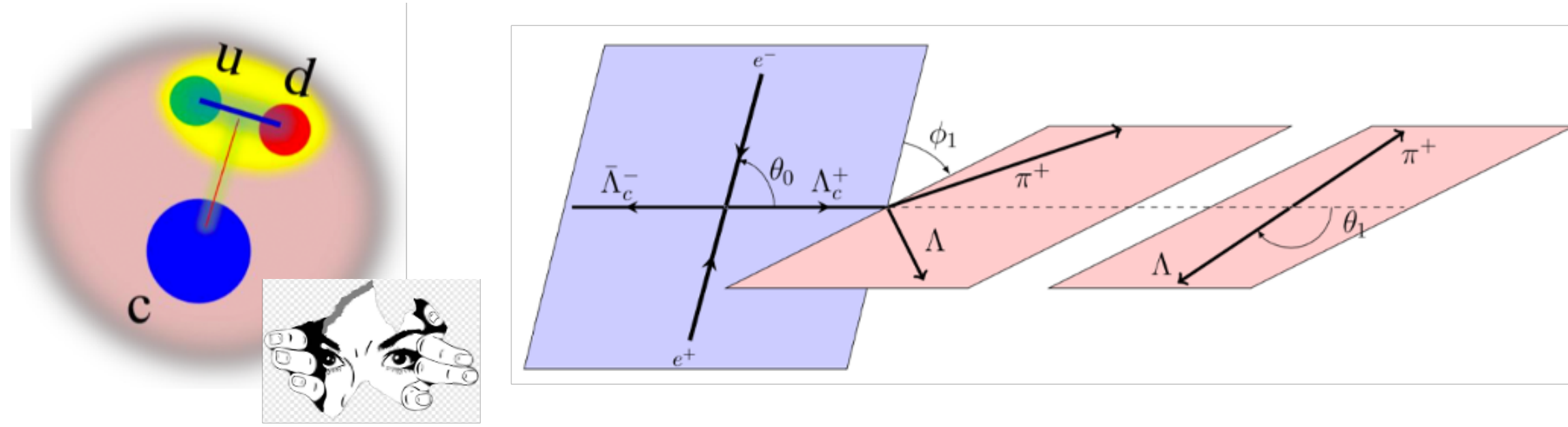
First $\mathcal{A}_{CP}^\alpha$ results for SCS decays of charmed baryons.

- No evidence of CPV is found.

李龙科 (湖南师大) 为主导

Belle II 粲重子 CPV 的第一个结果：翘首以盼，终于出来了。

KINEMATICS OF CHARMED BARYON DECAY



Branching Fractions



Decay Asymmetries (longitudinal, transverse)

$$M(\mathcal{B}_i \rightarrow \mathcal{B}_f + P) = i\bar{u}_f (A - B\gamma_5) u_i,$$

$$\Gamma = \frac{p_c}{8\pi} \frac{(m_i + m_f)^2 - m_P^2}{m_i^2} (|A|^2 + \kappa^2 |B|^2),$$

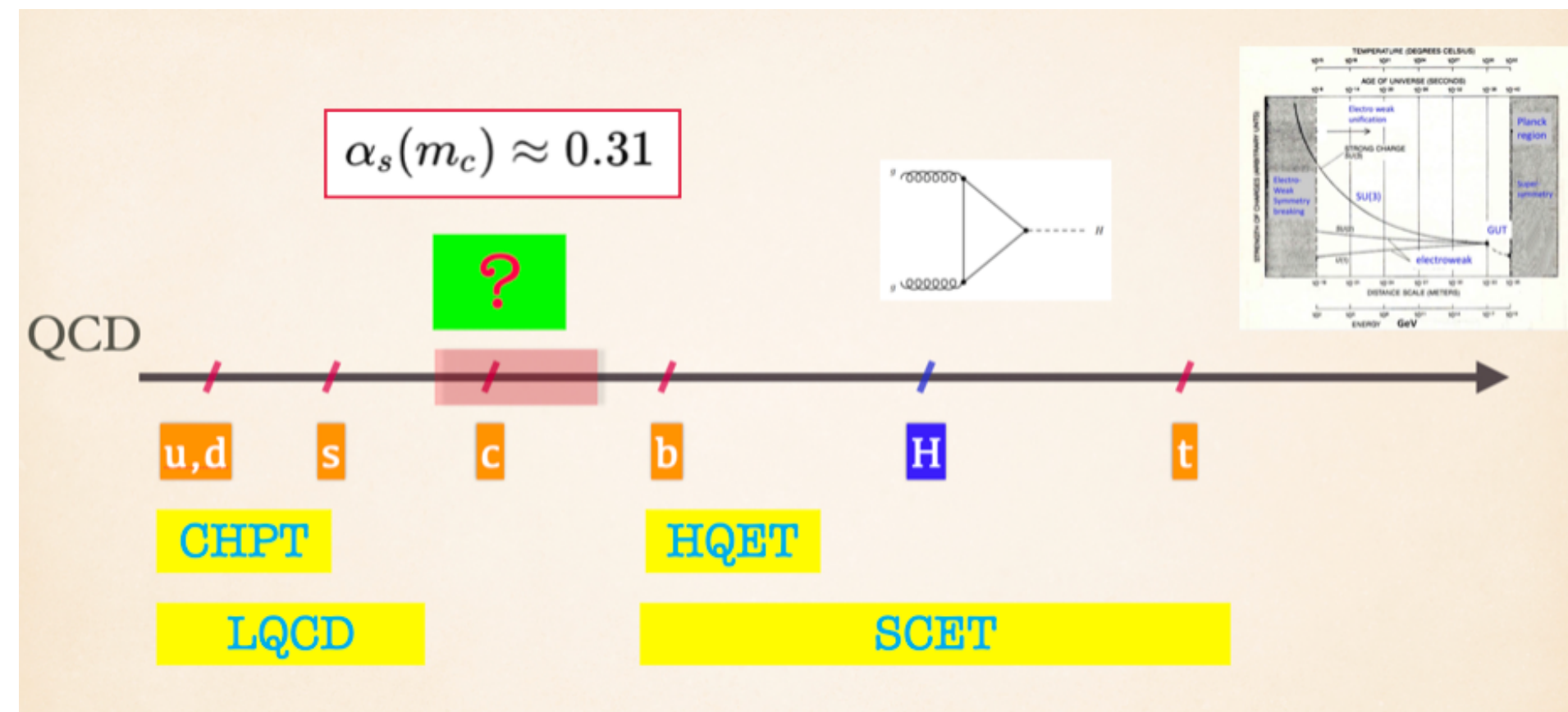
$$\alpha = \frac{2\kappa |A^* B| \cos(\delta_P - \delta_S)}{|A|^2 + \kappa^2 |B|^2}, \quad \beta = \frac{2\kappa |A^* B| \sin(\delta_P - \delta_S)}{|A|^2 + \kappa^2 |B|^2},$$

$$\gamma = \frac{|A|^2 - \kappa^2 |B|^2}{|A|^2 + \kappa^2 |B|^2},$$

$$\mathcal{A}_{CP} \equiv \frac{\Gamma(\mathcal{B}_c \rightarrow \mathcal{B}P) - \Gamma(\bar{\mathcal{B}}_c \rightarrow \bar{\mathcal{B}}\bar{P})}{\Gamma(\mathcal{B}_c \rightarrow \mathcal{B}P) + \Gamma(\bar{\mathcal{B}}_c \rightarrow \bar{\mathcal{B}}\bar{P})},$$

UNDERLYING DYNAMICS: TOPOLOGICAL DIAGRAMMS

- Charm system: challenging and charming



- Topological diagrams provide a way to underlying dynamics

PHYSICAL REVIEW D

VOLUME 44, NUMBER 9

1 NOVEMBER 1991

Quark-diagram analysis of charmed-baryon decays

Yoji Kohara

Nihon University at Fujisawa, Fujisawa, Kanagawa 252, Japan

(Received 29 May 1991)

The Cabibbo-allowed two-body nonleptonic decays of charmed baryons to a SU(3)-octet (or -decuplet) baryon and a pseudoscalar meson are examined on the basis of the quark-diagram scheme. Some relations among the decay amplitudes or rates of various decay modes are derived. The decays of Ξ_c^+ to a decuplet baryon are forbidden.

PHYSICAL REVIEW D

VOLUME 54, NUMBER 3

1 AUGUST 1996

Analysis of two-body decays of charmed baryons using the quark-diagram scheme

Ling-Lie Chau

Physics Department, University of California at Davis, California 95616

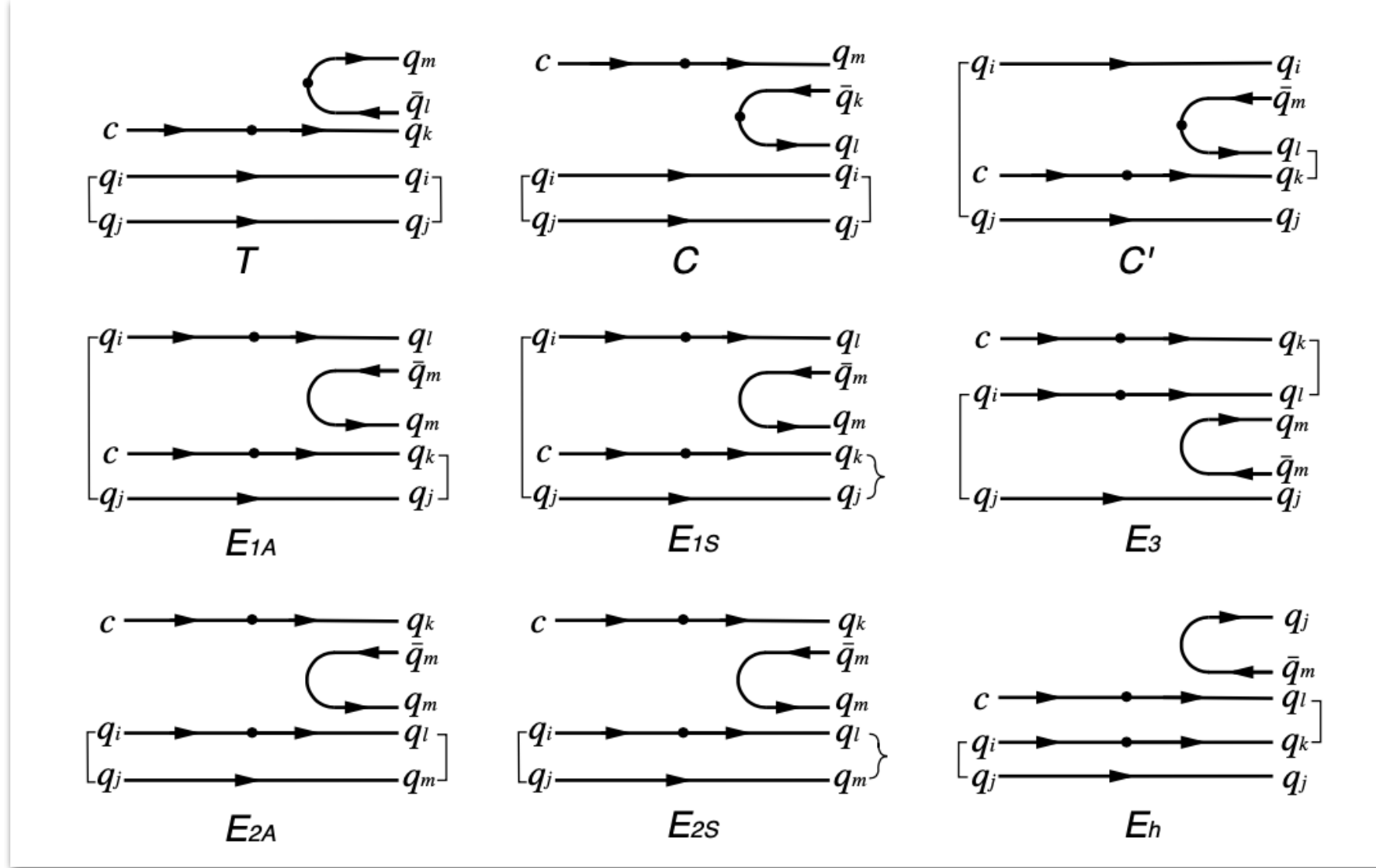
Hai-Yang Cheng and B. Tseng*

Institute of Physics, Academia Sinica, Taipei, Taiwan 115

(Received 25 August 1995)

TOPOLOGICAL DIAGRAM @ TREE

H. L. Zhong, FX, H.Y. Cheng, PRD 109 (2024), 114027; 2401.15926



$$\begin{aligned} \mathcal{A}_{\text{TDA}} = & T(\mathcal{B}_c)^{ij} H_l^{km} (\bar{\mathcal{B}}_8)_{ijk} (P^\dagger)_m^l \\ & + C(\mathcal{B}_c)^{ij} H_k^{ml} (\bar{\mathcal{B}}_8)_{ijl} (P^\dagger)_m^k + C'(\mathcal{B}_c)^{ij} H_m^{kl} (\bar{\mathcal{B}}_8)_{klj} (P^\dagger)_i^m \\ & + E_{1A}(\mathcal{B}_c)^{ij} H_i^{kl} (\bar{\mathcal{B}}_8)_{jkm} (P^\dagger)_l^m + E_{1S}(\mathcal{B}_c)^{ij} H_i^{kl} (P^\dagger)_l^m \left[(\bar{\mathcal{B}}_8)_{jmk} + (\bar{\mathcal{B}}_8)_{kmj} \right] \\ & + E_{2A}(\mathcal{B}_c)^{ij} H_i^{kl} (\bar{\mathcal{B}}_8)_{jlm} (P^\dagger)_k^m + E_{2S}(\mathcal{B}_c)^{ij} H_i^{kl} (P^\dagger)_k^m \left[(\bar{\mathcal{B}}_8)_{jml} + (\bar{\mathcal{B}}_8)_{lmj} \right] \\ & + E_3(\mathcal{B}_c)^{ij} H_i^{kl} (\bar{\mathcal{B}}_8)_{klm} (P^\dagger)_j^m + E_h(\mathcal{B}_c)^{ij} H_i^{kl} (\bar{\mathcal{B}}_8)_{klj} (P^\dagger)_m^m \end{aligned}$$

$$H_m^{kl} (\bar{q}_k c) (\bar{q}_l q^m) \quad \text{Weak interaction}$$

$$(\mathcal{B}_c)^{ij} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix} \quad P_j^i = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}$$

$$(\mathcal{B}_8)_j^i = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix} \quad (\mathcal{B}_8)_{ijk} = \epsilon_{ijl} (\mathcal{B}_8)_k^l$$

- reduce independent diagrams to 7

$$E_{2A} = -E_{1A}, \quad E_{2S} = -E_{1S}. \quad (\text{KPW theorem})$$

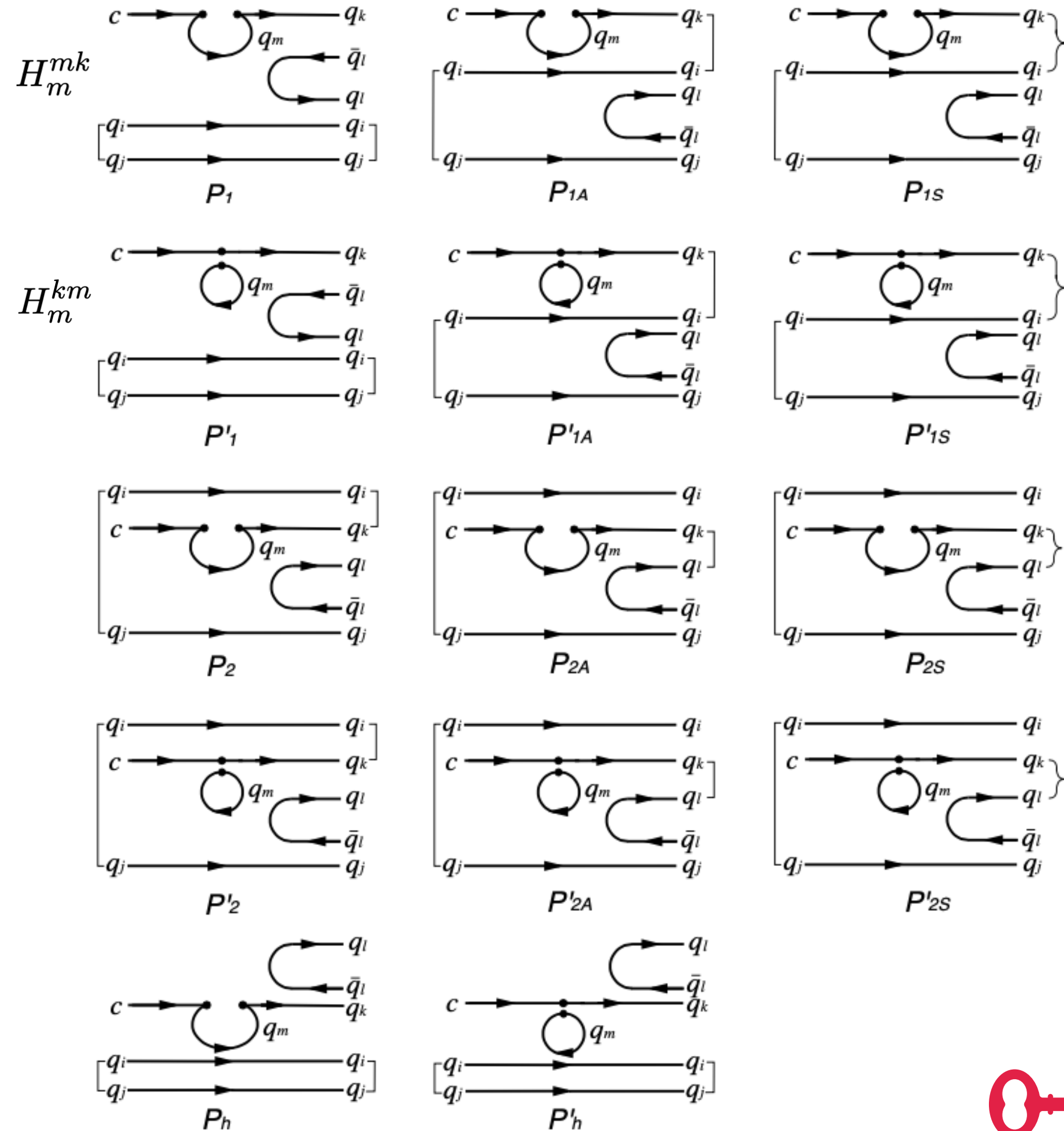
- further reduction to 5 by redefinition

$$\tilde{T} = T - E_{1S}, \quad \tilde{C} = C + E_{1S}, \quad \tilde{C}' = C' - 2E_{1S},$$

$$\tilde{E}_1 = E_{1A} + E_{1S} - E_3, \quad \tilde{E}_h = E_h + 2E_{1S}.$$

TOPOLOGICAL DIAGRAM @ PENGUIN

H.Y. Cheng, FX, H. L. Zhong, PRD 111 (2025), 034011



Similar to tree diagrams

$$\begin{aligned} \mathcal{A}_{\text{TDA}} = & P_h(\mathcal{B}_c)^{ij} H_m^{mk} (\overline{\mathcal{B}}_8)_{ijk} (P^\dagger)_l^l + P_1(\mathcal{B}_c)^{ij} H_m^{mk} (\overline{\mathcal{B}}_8)_{ijl} (P^\dagger)_k^l \\ & + P_{2A}(\mathcal{B}_c)^{ij} H_m^{mk} (\overline{\mathcal{B}}_8)_{kil} (P^\dagger)_j^l + P_{2S}(\mathcal{B}_c)^{ij} H_m^{mk} (P^\dagger)_j^l [(\overline{\mathcal{B}}_8)_{kli} + (\overline{\mathcal{B}}_8)_{ilk}] \\ & + P'_h(\mathcal{B}_c)^{ij} H_m^{km} (\overline{\mathcal{B}}_8)_{ijk} (P^\dagger)_l^l + P'_1(\mathcal{B}_c)^{ij} H_m^{km} (\overline{\mathcal{B}}_8)_{ijl} (P^\dagger)_k^l \\ & + P'_{2A}(\mathcal{B}_c)^{ij} H_m^{km} (\overline{\mathcal{B}}_8)_{kil} (P^\dagger)_j^l + P'_{2S}(\mathcal{B}_c)^{ij} H_m^{km} (P^\dagger)_j^l [(\overline{\mathcal{B}}_8)_{kli} + (\overline{\mathcal{B}}_8)_{ilk}] \end{aligned}$$



$$H_k^{ij} = \frac{1}{2} \left[(H_{15})_k^{ij} + (H_{\overline{6}})_k^{ij} \right] + \delta_k^j \left(\frac{3}{2} (H_{3p})^i - \frac{1}{2} (H_{3t})^i \right) + \delta_k^i \left(\frac{3}{2} (H_{3t})^j - \frac{1}{2} (H_{3p})^j \right)$$

$$\begin{aligned} \mathcal{A}_{\text{TDA}}^{\lambda_b} = & \tilde{b}_1(\mathcal{B}_c)_i (H_3)^j (\overline{\mathcal{B}}_8)_j^i (P^\dagger)_l^l + \tilde{b}_2(\mathcal{B}_c)_i (H_3)^j (\overline{\mathcal{B}}_8)_j^l (P^\dagger)_l^i + \tilde{b}_3(\mathcal{B}_c)_i (H_3)^i (\overline{\mathcal{B}}_8)_j^l (P^\dagger)_l^j \\ & + \tilde{b}_4(\mathcal{B}_c)_i (H_3)^l (\overline{\mathcal{B}}_8)_j^i (P^\dagger)_l^j + \tilde{b}_5(\mathcal{B}_c)_i (H_{15^b})_l^{jk} (\overline{\mathcal{B}}_8)_j^i (P^\dagger)_k^l, \end{aligned}$$

$$\tilde{b}_1 = \frac{1}{4}(\tilde{T} - 3\tilde{C}) - \frac{1}{2}\tilde{C}' - \frac{1}{2}\tilde{E}_h - 2\tilde{P}_h - 2\tilde{P}_{2S},$$

$$\tilde{b}_2 = \frac{1}{2}\tilde{C}' + 2\tilde{P}_{2S},$$

$$\tilde{b}_3 = -\frac{1}{2}\tilde{C}' + \frac{1}{2}\tilde{E}_{1A} - \tilde{P}_{2A} - \tilde{P}_{2S},$$

$$\tilde{b}_4 = -\frac{1}{4}(3\tilde{T} - \tilde{C}) + \frac{1}{2}\tilde{C}' + \frac{1}{2}\tilde{E}_1 - 2\tilde{P}_1 + \tilde{P}_{2A} + \tilde{P}_{2S},$$

$$\tilde{b}_5 = \tilde{T} + \tilde{C},$$

$$\tilde{P}_1 = P_1 + E_{1S} - \frac{1}{2}E_3, \quad \tilde{P}_h = P_h - E_{1S},$$

$$\tilde{P}_{2S} = P_{2S} + E_{1S}, \quad \tilde{P}_{2A} = P_{2A} + E_{1S} - E_3,$$

$$\tilde{P}'_1 = P'_1 - E_{1S} + \frac{1}{2}E_3, \quad \tilde{P}'_h = P'_h + E_{1S},$$

$$\tilde{P}'_{2S} = P'_{2S} - E_{1S}, \quad \tilde{P}'_{2A} = P'_{2A} - E_{1S} + E_3.$$

← contribute to CPV

EQUIVALENCE BETWEEN TDA & IRA

X.G. He, Y. J. Shi and W. Wang, EPJC 80 (2020), 359

• TDA

$$\mathcal{A}_{\text{TDA}} = \mathcal{A}_{\text{TDA}}^{\text{tree}} + \mathcal{A}_{\text{TDA}}^{\lambda_b}$$

H.Y. Cheng, FX, H. L. Zhong, PRD 111 (2025), 034011; 2505.07150

$$\begin{aligned} \mathcal{A}_{\text{TDA}}^{\text{tree}} = & (T + C)(\mathcal{B}_c)_i (H_{15})_m^{jl} (\bar{\mathcal{B}}_8)_j^i (P^\dagger)_l^m - E_h(\mathcal{B}_c)_i (H_{\bar{6}})_l^{ij} (\bar{\mathcal{B}}_8)_j^l (P^\dagger)_m^m \\ & + (T - C - C' - 2E_{1S})(\mathcal{B}_c)_i (H_{\bar{6}})_m^{jl} (\bar{\mathcal{B}}_8)_j^i (P^\dagger)_l^m - C'(\mathcal{B}_c)_i (H_{\bar{6}})_m^{ij} (\bar{\mathcal{B}}_8)_j^l (P^\dagger)_l^m \\ & + (E_{1A} - E_{1S} - E_3)(\mathcal{B}_c)_i (H_{\bar{6}})_j^{il} (\bar{\mathcal{B}}_8)_m^j (P^\dagger)_l^m + 2E_{1S}(\mathcal{B}_c)_i (H_{\bar{6}})_m^{jl} (\bar{\mathcal{B}}_8)_j^m (P^\dagger)_l^i \end{aligned}$$

$$\begin{aligned} \mathcal{A}_{\text{TDA}}^{\lambda_b} = & \tilde{b}_1(\mathcal{B}_c)_i (H_3)^j (\bar{\mathcal{B}}_8)_j^i (P^\dagger)_l^l + \tilde{b}_2(\mathcal{B}_c)_i (H_3)^j (\bar{\mathcal{B}}_8)_j^l (P^\dagger)_l^i + \tilde{b}_3(\mathcal{B}_c)_i (H_3)^i (\bar{\mathcal{B}}_8)_j^l (P^\dagger)_l^j \\ & + \tilde{b}_4(\mathcal{B}_c)_i (H_3)^l (\bar{\mathcal{B}}_8)_j^i (P^\dagger)_l^j + \tilde{b}_5(\mathcal{B}_c)_i (H_{15^b})_l^{jk} (\bar{\mathcal{B}}_8)_j^i (P^\dagger)_k^l, \end{aligned}$$

$$\begin{aligned} \tilde{b}_1 &= \tilde{f}_3^a, & \tilde{b}_2 &= \tilde{f}_3^b, & \tilde{b}_3 &= \tilde{f}_3^c, \\ \tilde{b}_4 &= \tilde{f}_3^d, & \tilde{b}_5 &= \tilde{f}_3^e. \end{aligned}$$

• IRA

$$\begin{aligned} \mathcal{A}_{\text{IRAb}}^{\text{tree}} = & \tilde{f}^a (\bar{\mathcal{B}}_c)^{ik} (H_{\bar{6}})_{ij} (\mathcal{B}_8)_k^j (P^\dagger)_l^l + \tilde{f}^b (\mathcal{B}_c)^{ik} (H_{\bar{6}})_{ij} (\bar{\mathcal{B}}_8)_k^l (P^\dagger)_l^j + \tilde{f}^c (\mathcal{B}_c)^{ik} (H_{\bar{6}})_{ij} (\bar{\mathcal{B}}_8)_l^j (P^\dagger)_k^l \\ & + \tilde{f}^d (\mathcal{B}_c)^{kl} (H_{\bar{6}})_{ij} (\bar{\mathcal{B}}_8)_k^i M_l^j + \tilde{f}^e (\mathcal{B}_c)_j (H_{15})_l^{ik} (\bar{\mathcal{B}}_8)_i^j (P^\dagger)_k^l, \end{aligned}$$

C.Q. Geng, X.G. He, X.N. Jin, C.W. Liu and C. Yang, PRD 109, L071302

$$\begin{aligned} \mathcal{A}_{\text{IRAb}}^{\lambda_b} = & \tilde{f}_3^a (\mathcal{B}_c)_i (H_3)^j (\bar{\mathcal{B}}_8)_j^i (P^\dagger)_k^k + \tilde{f}_3^b (\mathcal{B}_c)_k (H_3)^i (\bar{\mathcal{B}}_8)_i^l (P^\dagger)_l^k + \tilde{f}_3^c (\mathcal{B}_c)_i (H_3)^i (\bar{\mathcal{B}}_8)_j^l (P^\dagger)_l^j \\ & + \tilde{f}_3^d (\mathcal{B}_c)_i (H_3)^l (\bar{\mathcal{B}}_8)_j^i (P^\dagger)_l^j + \tilde{f}_3^e (\mathcal{B}_c)_i (H_{15^b})_l^{jk} (\bar{\mathcal{B}}_8)_j^i (P^\dagger)_k^l. \end{aligned}$$

X.G. He, C.W. Liu, Sci. Bull. 70 (2025) 2598

FINAL-STATE RESCATTERING

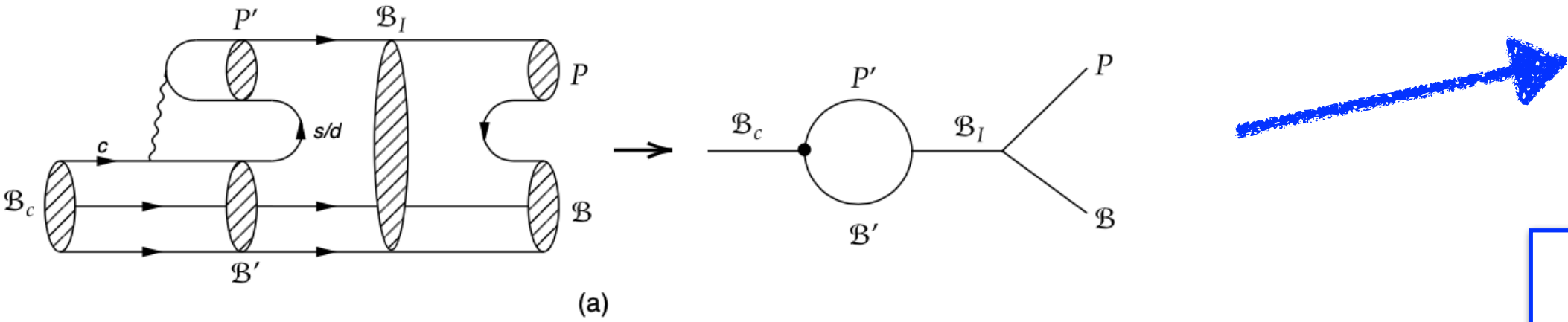
Factorizable contribution:

$$\mathcal{L}_{\mathcal{B}_c\mathcal{B}P} = \sum_{P,\mathcal{B},\mathcal{B}_c} F_{\mathcal{B}_c\mathcal{B}P} P^\dagger \overline{\mathcal{B}} \mathcal{B}_c = (P^\dagger)_j^k (\overline{\mathcal{B}})_i^l \left(\tilde{F}_V^+(H_+)^{ij}_k + \tilde{F}_V^-(H_-)^{ij}_k \right) (\mathcal{B}_c)_l,$$

X.G. He, C.W. Liu, Sci. Bull. 70 (2025) 2598

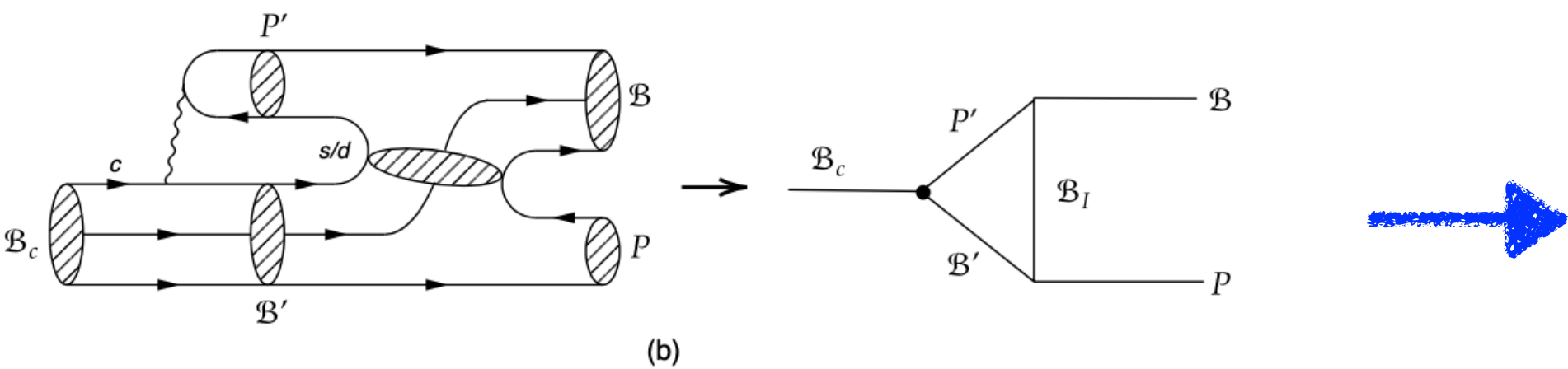
Non-factorizable contribution: FSR

s-channel



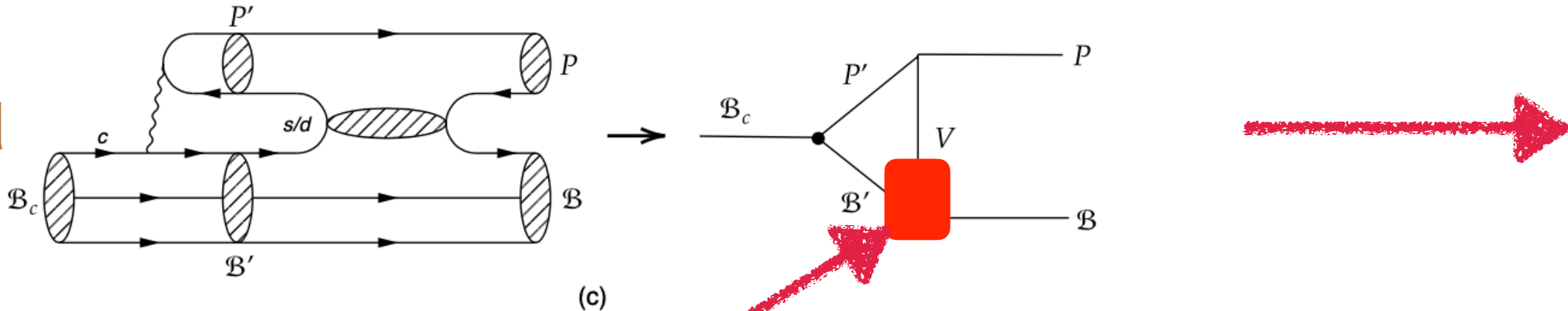
$$A^s = S^- \left[-\frac{1}{3}(r_- + 4)\tilde{f}^b - \frac{1}{3}(r_-^2 + 4r_-)\tilde{f}^c - \frac{1}{6}(7r_- - 2)\tilde{f}_3^b + \frac{1}{18}(r_- + 1)(7r_- - 2)\tilde{f}_3^c - \frac{1}{6}r_-(7r_- - 2)\tilde{f}_3^d \right],$$

t-channel



$$A^t = \sum_{\lambda=\pm} T_\lambda^- \left[-\frac{1}{9}(2r_\lambda^2 + 4r_\lambda + 11)\tilde{f}^a + \frac{1}{3}(2r_\lambda^2 - r_\lambda)\tilde{f}^b + \frac{1}{3}(r_\lambda^2 - 2r_\lambda + 3)\tilde{f}^c + \frac{1}{3}(2r_\lambda^2 - 2r_\lambda - 4)\tilde{f}^d - \frac{1}{18}(10r_\lambda^2 + 2r_\lambda + 1)\tilde{f}_3^a + \frac{1}{3}(r_\lambda^2 - \frac{5}{2}r_\lambda + 1)\tilde{f}_3^b - \frac{1}{18}(r_\lambda^2 + 11r_\lambda + 1)\tilde{f}_3^c + \frac{1}{6}(r_\lambda + 1)^2\tilde{f}_3^d \right],$$

u-channel



$$\mathcal{L}_{V\mathcal{B}'\mathcal{B}} = f_{V\mathcal{B}'\mathcal{B}}\text{Tr}(\overline{\mathcal{B}'}\gamma_\mu V^\mu \mathcal{B}) + \frac{\tilde{f}_{V\mathcal{B}'\mathcal{B}}}{m_{\mathcal{B}'} + m_{\mathcal{B}}}\text{Tr}(\overline{\mathcal{B}'}\sigma_{\mu\nu}\mathcal{B}\partial^\mu V^\nu).$$

tensor coupling dominated

$$A^u = U^- \left[\frac{2}{3}(1 - 2\rho_+)\tilde{f}^b + \tilde{f}^d - \frac{1}{2}\tilde{f}_3^a + \frac{1}{6}(1 + 2\rho_+)\tilde{f}_3^b - \frac{1}{18}(1 + 10\rho_+)\tilde{f}_3^c + \frac{1}{6}(1 - 2\rho_+)\tilde{f}_3^d \right],$$

1. LCSR indicates tensor coupling one order of magnitude larger

T.M. Aliev et. al., PRD 80 (2009), 016010
2. Realistic calculation in $B_c \rightarrow BV$ implies two traces are of the same order

C. P. Jia et. al., JHEP 11, 072 (2024)
- H.Y. Cheng, FX, H.L. Zhong, PRD 112 054022 (2505)

MATCHING FSR & IRA (TDA)

$$A^{\text{FSR}} = A^{\text{IRA}}$$

$$\begin{aligned}\tilde{f}^a &= -\frac{1}{3}(2r_-^2 + 4r_- + 11)T^- \\ \tilde{f}^b &= \tilde{F}_V^- - (r_- + 4)S^- - 2(2\rho_+ - 1)U^- + (2r_-^2 - r_-)T^-, \\ \tilde{f}^c &= -r_-(r_- + 4)S^- + (r_-^2 - 2r_- + 3)T^-, \\ \tilde{f}^d &= \tilde{F}_V^- + 3U^- + (2r_-^2 - 2r_- - 4)T^-, \\ \tilde{f}^e &= \tilde{F}_V^+, \\ \tilde{f}_3^a &= \frac{1}{4}(-\tilde{F}_V^+ + 2\tilde{F}_V^-) - \frac{3}{2}U^- - \frac{1}{6}(10r_-^2 + 2r_- + 1)T^-, \\ \tilde{f}_3^b &= -\frac{1}{2}(7r_- - 2)S^- + \frac{1}{2}(1 + 2\rho_+)U^- + \frac{1}{2}(2r_-^2 - 5r_- + 2)T^-, \\ \tilde{f}_3^c &= \frac{1}{6}(r_- + 1)(7r_- - 2)S^- - \frac{1}{6}(r_-^2 + 11r_- + 1)T^- - \frac{1}{6}(10\rho_+ + 1)U^-, \\ \tilde{f}_3^d &= -\frac{1}{4}(\tilde{F}_V^+ + 2\tilde{F}_V^-) - \frac{1}{2}r_-(7r_- - 2)S^- + \frac{1}{2}U^- + \frac{1}{2}(r_- + 1)^2T^-, \end{aligned}$$

$$r_- = r_+ = 2.5 \pm 0.8 \quad \text{X.G. He, C.W. Liu, Sci. Bull. 70 (2025) 2598}$$

$$\tilde{\rho}_+ = -21 \quad \text{T.M. Aliev et. al., PRD 80 (2009), 016010}$$

$$A^{\text{FSR}} = A^{\text{TDA}}$$

$$\begin{aligned}\tilde{T} &= \frac{1}{2}(\tilde{F}_V^+ + \tilde{F}_V^-) - \frac{1}{2}(r_- + 4)S^- + \frac{1}{2}(2r_-^2 - r_-)T^- - (2\rho_+ - 1)U^-, \\ \tilde{C} &= \frac{1}{2}(\tilde{F}_V^+ - \tilde{F}_V^-) + \frac{1}{2}(r_- + 4)S^- - \frac{1}{2}(2r_-^2 - r_-)T^- + (2\rho_+ - 1)U^-, \\ \tilde{C}' &= -(r_- + 4)S^- + (r_- + 4)T^- - (4\rho_+ + 1)U^-, \\ \tilde{E}_1 &= r_-(r_- + 4)S^- - (r_-^2 - 2r_- + 3)T^-, \\ \tilde{E}_h &= -\frac{1}{3}(2r_-^2 + 4r_- + 11)T^-, \end{aligned}$$

$$\begin{aligned}\tilde{b}_1 &= \frac{1}{4}(-\tilde{F}_V^+ + 2\tilde{F}_V^-) - \frac{1}{6}(10r_-^2 + 2r_- + 1)T^- - \frac{3}{2}U^-, \\ \tilde{b}_2 &= -\frac{1}{2}(7r_- - 2)S^- + \frac{1}{2}(2r_-^2 - 5r_- + 2)T^- + \frac{1}{2}(2\rho_+ + 1)U^-, \\ \tilde{b}_3 &= \frac{1}{6}(r_- + 1)(7r_- - 2)S^- - \frac{1}{6}(r_-^2 + 11r_- + 1)T^- - \frac{1}{6}(10\rho_+ + 1)U^-, \\ \tilde{b}_4 &= -\frac{1}{4}(\tilde{F}_V^+ + 2\tilde{F}_V^-) - \frac{1}{2}r_-(7r_- - 2)S^- + \frac{1}{2}(r_- + 1)^2T^- + \frac{1}{2}U^-, \end{aligned}$$

FIT STRATEGY

- Experimental inputs: 44 BFs and Lee-Yang parameters (updated to 2025/03)

$$\mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ K_S^0) = (1.94 \pm 0.90) \times 10^{-3},$$

$$\mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 \pi^+) = (7.19 \pm 3.23) \times 10^{-3}, \quad \text{Belle and Belle-II, 2503.17643}$$

$$\mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 K^+) = (4.9 \pm 2.3) \times 10^{-4}.$$

- To fit 19 parameters corresponding to 5 sets of parameters tree level topological diagrams

$$|\tilde{T}|_S e^{i\delta_S^{\tilde{T}}}, \quad |\tilde{C}|_S e^{i\delta_S^{\tilde{C}}}, \quad |\tilde{C}'|_S e^{i\delta_S^{\tilde{C}'}} , \quad |\tilde{E}_1|_S e^{i\delta_S^{\tilde{E}_1}}, \quad |\tilde{E}_h|_S e^{i\delta_S^{\tilde{E}_h}},$$

$$|\tilde{T}|_P e^{i\delta_P^{\tilde{T}}}, \quad |\tilde{C}|_P e^{i\delta_P^{\tilde{C}}}, \quad |\tilde{C}'|_P e^{i\delta_P^{\tilde{C}'}} , \quad |\tilde{E}_1|_P e^{i\delta_P^{\tilde{E}_1}}, \quad |\tilde{E}_h|_P e^{i\delta_P^{\tilde{E}_h}},$$

- To extract 5 sets of FSR parameters (F^+, F^-, S^-, T^-, U^-) and penguin coefficients

$$|\tilde{b}_1|_S e^{i\delta_S^{\tilde{b}_1}}, \quad |\tilde{b}_2|_S e^{i\delta_S^{\tilde{b}_2}}, \quad |\tilde{b}_3|_S e^{i\delta_S^{\tilde{b}_3}}, \quad |\tilde{b}_4|_S e^{i\delta_S^{\tilde{b}_4}},$$

$$|\tilde{b}_1|_P e^{i\delta_P^{\tilde{b}_1}}, \quad |\tilde{b}_2|_P e^{i\delta_P^{\tilde{b}_2}}, \quad |\tilde{b}_3|_P e^{i\delta_P^{\tilde{b}_3}}, \quad |\tilde{b}_4|_P e^{i\delta_P^{\tilde{b}_4}}.$$

INPUTS

Observable	PDG [55]	BESIII	Belle	LHCb	Average
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+)$	1.29 ± 0.05				1.29 ± 0.05
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+)$	1.27 ± 0.06				1.27 ± 0.06
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0)$	1.24 ± 0.09				1.24 ± 0.09
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \eta)$	0.32 ± 0.05				0.32 ± 0.05
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \eta')$	0.41 ± 0.08				0.41 ± 0.08
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Xi^0 K^+)$	0.55 ± 0.07				0.55 ± 0.07
$10^4 \mathcal{B}(\Lambda_c^+ \rightarrow \Lambda^0 K^+)$	6.42 ± 0.31				6.42 ± 0.31
$10^4 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 K^+)$	3.70 ± 0.31				3.70 ± 0.31
$10^4 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ K_S)$	4.7 ± 1.4				4.7 ± 1.4
$10^4 \mathcal{B}(\Lambda_c^+ \rightarrow n \pi^+)$	6.6 ± 1.3				6.6 ± 1.3
$10^4 \mathcal{B}(\Lambda_c^+ \rightarrow p \pi^0)$	< 0.8	$1.56^{+0.75}_{-0.61}$			$1.56^{+0.75}_{-0.61}$
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow p K_S)$	1.59 ± 0.07				1.59 ± 0.07
$10^3 \mathcal{B}(\Lambda_c^+ \rightarrow p \eta)$	1.57 ± 0.12	1.63 ± 0.33			1.58 ± 0.11
$10^4 \mathcal{B}(\Lambda_c^+ \rightarrow p \eta')$	4.8 ± 0.9				4.8 ± 0.9
$10^2 \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$	1.43 ± 0.27		1.80 ± 0.52		1.80 ± 0.52
$10^2 \frac{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- K^+)}{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)}$	2.75 ± 0.57				2.75 ± 0.57
$10^2 \frac{\mathcal{B}(\Xi_c^0 \rightarrow \Lambda K_S^0)}{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)}$	22.5 ± 1.3				22.5 ± 1.3
$10^2 \frac{\mathcal{B}(\Xi_c^0 \rightarrow \Sigma^0 K_S^0)}{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)}$	3.8 ± 0.7				3.8 ± 0.7
$10^2 \frac{\mathcal{B}(\Xi_c^0 \rightarrow \Sigma^+ K^-)}{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)}$	12.3 ± 1.2				12.3 ± 1.2
$10^3 \mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \pi^0)$			6.9 ± 1.6		6.9 ± 1.6
$10^3 \mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta)$			1.6 ± 0.5		1.6 ± 0.5
$10^3 \mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \eta')$			1.2 ± 0.4		1.2 ± 0.4
$10^2 \mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 \pi^+)$	1.6 ± 0.8		0.719 ± 0.323 [24]		0.84 ± 0.48
$\alpha(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+)$	-0.755 ± 0.006			-0.782 ± 0.010	-0.762 ± 0.006
$\alpha(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+)$	-0.466 ± 0.018				-0.466 ± 0.018
$\alpha(\Lambda_c^+ \rightarrow p K_S)$	0.18 ± 0.45			-0.744 ± 0.015	-0.743 ± 0.028
$\alpha(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0)$	-0.484 ± 0.027				-0.484 ± 0.027
$\alpha(\Lambda_c^+ \rightarrow \Sigma^+ \eta)$	-0.99 ± 0.06				-0.99 ± 0.06
$\alpha(\Lambda_c^+ \rightarrow \Sigma^+ \eta')$	-0.46 ± 0.07				-0.46 ± 0.07
$\alpha(\Lambda_c^+ \rightarrow \Lambda^0 K^+)$	-0.585 ± 0.052			-0.569 ± 0.065	-0.579 ± 0.041
$\alpha(\Lambda_c^+ \rightarrow \Sigma^0 K^+)$	-0.54 ± 0.20				-0.54 ± 0.20
$\alpha(\Lambda_c^+ \rightarrow \Xi^0 K^+)$		0.01 ± 0.16			0.01 ± 0.16
$\alpha(\Xi_c^0 \rightarrow \Xi^- \pi^+)$	-0.64 ± 0.05				-0.64 ± 0.05
$\alpha(\Xi_c^0 \rightarrow \Xi^0 \pi^0)$			-0.90 ± 0.27		-0.90 ± 0.27
$\beta(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+)$				0.368 ± 0.021	0.368 ± 0.021
$\beta(\Lambda_c^+ \rightarrow \Lambda^0 K^+)$				0.35 ± 0.13	0.35 ± 0.13
$\gamma(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+)$				0.502 ± 0.017	0.502 ± 0.017
$\gamma(\Lambda_c^+ \rightarrow \Lambda^0 K^+)$				-0.743 ± 0.071	-0.743 ± 0.071
$10^2 \mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ K_S^0)$		0.194 ± 0.090 [24]			0.194 ± 0.090
$10^2 \mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 \pi^+)$		0.719 ± 0.323 [24]			0.719 ± 0.323
$10^2 \mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 K^+)$		0.049 ± 0.023 [24]			0.049 ± 0.023
$10^4 \mathcal{B}(\Xi_c^+ \rightarrow p K_S^0)$		7.16 ± 3.25 [25]			7.16 ± 3.25
$10^4 \mathcal{B}(\Xi_c^+ \rightarrow \Lambda \pi^+)$		28.452 ± 2.09 [25]			4.52 ± 2.09
$10^4 \mathcal{B}(\Xi_c^+ \rightarrow \Sigma^0 \pi^+)$		1.20 ± 0.55 [25]			1.20 ± 0.55

TABLE I: The decay amplitudes of SCS processes in the TDA.

Channel	TDA
$\Lambda_c^+ \rightarrow \Lambda K^+$	$\frac{1}{\sqrt{6}} [\frac{1}{2}(\lambda_d - \lambda_s)(4\tilde{T} - \tilde{C}' + 2\tilde{E}_1) + \lambda_b(\tilde{b}_2 - 2\tilde{b}_4 + \frac{1}{2}\tilde{b}_5)]$
$\Lambda_c^+ \rightarrow \Sigma^0 K^+$	$\frac{1}{\sqrt{2}} [\frac{1}{2}(\lambda_d - \lambda_s)\tilde{C}' + \lambda_b\tilde{b}_2]$
$\Lambda_c^+ \rightarrow \Sigma^+ K^0$	$\frac{1}{2}(\lambda_d - \lambda_s)\tilde{C}' + \lambda_b\tilde{b}_2$
$\Lambda_c^+ \rightarrow p \pi^0$	$\frac{1}{\sqrt{2}} [\frac{1}{2}(\lambda_d - \lambda_s)(-2\tilde{C} - \tilde{C}' - \tilde{E}_1) + \lambda_b(\tilde{b}_4 + \frac{3}{4}\tilde{b}_5)]$
$\Lambda_c^+ \rightarrow p \eta_8$	$\frac{1}{\sqrt{6}} [\frac{1}{2}(\lambda_d - \lambda_s)(6\tilde{C} + \tilde{C}' - \tilde{E}_1) + \lambda_b(-2\tilde{b}_2 + \tilde{b}_4 + \frac{3}{4}\tilde{b}_5)]$
$\Lambda_c^+ \rightarrow p \eta_1$	$\frac{1}{\sqrt{3}} [\frac{1}{2}(\lambda_d - \lambda_s)(\tilde{C}' - \tilde{E}_1 + 3\tilde{E}_h) + \lambda_b(3\tilde{b}_1 + \tilde{b}_2 + \tilde{b}_4)]$
$\Lambda_c^+ \rightarrow n \pi^+$	$\frac{1}{2}(\lambda_d - \lambda_s)(2\tilde{T} - \tilde{C}' - \tilde{E}_1) + \lambda_b(\tilde{b}_4 - \frac{1}{4}\tilde{b}_5)$
$\Xi_c^0 \rightarrow \Lambda \pi^0$	$\frac{1}{2\sqrt{3}} [\frac{1}{2}(\lambda_d - \lambda_s)(-2\tilde{C} - 2\tilde{C}' + \tilde{E}_1) + \lambda_b(\tilde{b}_2 + \tilde{b}_4 + \frac{3}{4}\tilde{b}_5)]$
$\Xi_c^0 \rightarrow \Lambda \eta_8$	$\frac{1}{6} [\frac{1}{2}(\lambda_d - \lambda_s)(6\tilde{C} + 3\tilde{E}_1) + \lambda_b(\tilde{b}_2 + 6\tilde{b}_3 + \tilde{b}_4 + \frac{3}{4}\tilde{b}_5)]$
$\Xi_c^0 \rightarrow \Lambda \eta_1$	$\frac{1}{3\sqrt{2}} [\frac{1}{2}(\lambda_d - \lambda_s)(3\tilde{C}' - 3\tilde{E}_1 + 9\tilde{E}_h) + \lambda_b(3\tilde{b}_1 + \tilde{b}_2 + \tilde{b}_4)]$
$\Xi_c^0 \rightarrow \Sigma^0 \pi^0$	$\frac{1}{2} [\frac{1}{2}(\lambda_d - \lambda_s)(-2\tilde{C} - \tilde{E}_1) + \lambda_b(\tilde{b}_2 + 2\tilde{b}_3 + \tilde{b}_4 + \frac{3}{4}\tilde{b}_5)]$
$\Xi_c^0 \rightarrow \Sigma^0 \eta_8$	$\frac{1}{2\sqrt{3}} [\frac{1}{2}(\lambda_d - \lambda_s)(6\tilde{C} + 2\tilde{C}' + \tilde{E}_1) + \lambda_b(\tilde{b}_2 + \tilde{b}_4 + \frac{3}{4}\tilde{b}_5)]$
$\Xi_c^0 \rightarrow \Sigma^0 \eta_1$	$\frac{1}{\sqrt{6}} [\frac{1}{2}(\lambda_d - \lambda_s)(-\tilde{C}' + \tilde{E}_1 - 3\tilde{E}_h) + \lambda_b(3\tilde{b}_1 + \tilde{b}_2 + \tilde{b}_4)]$
$\Xi_c^0 \rightarrow \Sigma^+ \pi^-$	$\frac{1}{2}(\lambda_d - \lambda_s)(-\tilde{E}_1) + \lambda_b(\tilde{b}_2 + \tilde{b}_3)$
$\Xi_c^0 \rightarrow \Sigma^- \pi^+$	$\frac{1}{2}(\lambda_d - \lambda_s)(2\tilde{T}) + \lambda_b(\tilde{b}_3 + \tilde{b}_4 - \frac{1}{4}\tilde{b}_5)$
$\Xi_c^0 \rightarrow \Xi^0 K^0$	$\frac{1}{2}(\lambda_d - \lambda_s)(-\tilde{C}' - \tilde{E}_1) + \lambda_b\tilde{b}_3$
$\Xi_c^0 \rightarrow \Xi^- K^+$	$\frac{1}{2}(\lambda_d - \lambda_s)(-2\tilde{T}) + \lambda_b(\tilde{b}_3 + \tilde{b}_4 - \frac{1}{4}\tilde{b}_5)$
$\Xi_c^0 \rightarrow p K^-$	$\frac{1}{2}(\lambda_d - \lambda_s)\tilde{E}_1 + \lambda_b(\tilde{b}_2 + \tilde{b}_3)$
$\Xi_c^0 \rightarrow n \bar{K}^0$	$\frac{1}{2}(\lambda_d - \lambda_s)(\tilde{C}' + \tilde{E}_1) + \lambda_b\tilde{b}_3$
$\Xi_c^+ \rightarrow \Lambda \pi^+$	$\frac{1}{\sqrt{6}} [\frac{1}{2}(\lambda_d - \lambda_s)(-2\tilde{T} + 2\tilde{C}' - \tilde{E}_1) + \lambda_b(-\tilde{b}_2 - \tilde{b}_4 + \frac{1}{4}\tilde{b}_5)]$
$\Xi_c^+ \rightarrow \Sigma^0 \pi^+$	$\frac{1}{\sqrt{2}} [\frac{1}{2}(\lambda_d - \lambda_s)(2\tilde{T} + \tilde{E}_1) + \lambda_b(-\tilde{b}_2 + \tilde{b}_4 - \frac{1}{4}\tilde{b}_5)]$
$\Xi_c^+ \rightarrow \Sigma^+ \pi^0$	$\frac{1}{\sqrt{2}} [\frac{1}{2}(\lambda_d - \lambda_s)(2\tilde{C} - \tilde{E}_1) + \lambda_b(\tilde{b}_2 - \tilde{b}_4 - \frac{3}{4}\tilde{b}_5)]$
$\Xi_c^+ \rightarrow \Sigma^+ \eta_8$	$\frac{1}{\sqrt{6}} [\frac{1}{2}(\lambda_d - \lambda_s)(-6\tilde{C} - 2\tilde{C}' - \tilde{E}_1) + \lambda_b(-\tilde{b}_2 - \tilde{b}_4 - \frac{3}{4}\tilde{b}_5)]$
$\Xi_c^+ \rightarrow \Sigma^+ \eta_1$	$\frac{1}{\sqrt{3}} [\frac{1}{2}(\lambda_d - \lambda_s)(\tilde{C}' - \tilde{E}_1 + 3\tilde{E}_h) + \lambda_b(-3\tilde{b}_1 - \tilde{b}_2 - \tilde{b}_4)]$
$\Xi_c^+ \rightarrow \Xi^0 K^+$	$\frac{1}{2}(\lambda_d - \lambda_s)(2\tilde{T} - \tilde{C}' - \tilde{E}_1) + \lambda_b(-\tilde{b}_4 + \frac{1}{4}\tilde{b}_5)$
$\Xi_c^+ \rightarrow p \bar{K}^0$	$\frac{1}{2}(\lambda_d - \lambda_s)\tilde{C}' + \lambda_b(-\tilde{b}_2)$

FITTED PARAMETERS

	$ X_i _S$ ($10^{-2}G_F$ GeV ²)	$ X_i _P$ (GeV ²)	$\delta_S^{X_i}$ (in radian)	$\delta_P^{X_i}$
$\tilde{T}$	4.31 ± 0.11	12.11 ± 0.31	–	2.39 ± 0.04
$\tilde{C}$	3.23 ± 0.48	11.35 ± 0.93	3.10 ± 0.11	-0.72 ± 0.16
$\tilde{C}'$	5.84 ± 0.35	17.74 ± 0.92	0.02 ± 0.04	2.27 ± 0.11
$\tilde{E}_1$	2.79 ± 0.19	10.41 ± 0.47	-2.81 ± 0.06	1.83 ± 0.09
$\tilde{E}_h$	4.30 ± 0.50	13.26 ± 1.83	2.70 ± 0.11	-1.85 ± 0.20
$\tilde{F}_V^+$	1.10 ± 0.43	0.83 ± 0.46	0.13 ± 0.35	2.01 ± 1.37
$\tilde{F}_V^-$	0.48 ± 0.43	7.01 ± 1.79	0.69 ± 1.02	-2.81 ± 0.15
S^-	0.12 ± 0.01	0.92 ± 0.05	-2.20 ± 0.11	1.66 ± 0.08
T^-	0.39 ± 0.04	1.19 ± 0.16	-0.45 ± 0.11	1.29 ± 0.20
U^-	0.04 ± 0.00	0.22 ± 0.01	0.17 ± 0.08	2.43 ± 0.08
$\tilde{b}_1$	4.57 ± 0.78	16.04 ± 2.43	2.89 ± 0.13	-2.00 ± 0.18
$\tilde{b}_2$	0.49 ± 0.11	10.00 ± 0.22	1.30 ± 0.25	-1.11 ± 0.07
$\tilde{b}_3$	1.38 ± 0.30	10.89 ± 1.13	2.93 ± 0.19	2.47 ± 0.11
$\tilde{b}_4$	3.16 ± 0.37	11.94 ± 0.93	0.23 ± 0.10	-0.96 ± 0.15
$\tilde{b}_5$	1.10 ± 0.43	0.83 ± 0.46	0.13 ± 0.35	2.01 ± 1.37

fit directly

	Case I		Case II	
	TDA	IRA	TDA	IRA
χ^2	65.47	65.47	89.53	90.08
$\chi^2/d.o.f.$	3.27	3.27	3.73	3.75

$$|T^-| > |S^-| > |U^-|$$

- ▶ S^- is not negligible
- ▶ U^- is not negligible: compensated by large ρ^+
- ▶ $\tilde{b}_1$ is not negligible: hairpin is contained.

PREDICTION 1: CPV

TABLE III: The predicted CP observables calculated with $\rho_+ = -21$ and $r_- = 2.5$ in both TDA (upper) and IRA (lower).

Channel	$10^3 \mathcal{B}$	$10^4 A_{CP}$	$10^4 A_{CP}^S$	$10^4 A_{CP}^P$	$10^4 A_\alpha$	$10^4 R_\beta$
$\Lambda_c^+ \rightarrow \Lambda^0 K^+$	0.64 ± 0.03	0.49 ± 0.87	5.07 ± 2.27	-0.30 ± 0.83	6.89 ± 0.82	-7.66 ± 1.67
	0.64 ± 0.03	0.49 ± 0.91	5.07 ± 2.30	-0.30 ± 0.87	6.89 ± 0.82	-7.66 ± 1.68
$\Lambda_c^+ \rightarrow \Sigma^0 K^+$	0.39 ± 0.02	-1.33 ± 0.27	-1.04 ± 0.25	-1.73 ± 0.64	-4.68 ± 0.97	2.93 ± 0.50
	0.39 ± 0.02	-1.33 ± 0.28	-1.04 ± 0.25	-1.73 ± 0.67	-4.68 ± 0.98	2.93 ± 0.50
$\Lambda_c^+ \rightarrow \Sigma^+ K_S$	0.39 ± 0.02	-1.33 ± 0.27	-1.04 ± 0.25	-1.73 ± 0.64	-4.68 ± 0.97	2.93 ± 0.50
	0.39 ± 0.02	-1.33 ± 0.28	-1.04 ± 0.25	-1.73 ± 0.67	-4.68 ± 0.98	2.93 ± 0.50
$\Lambda_c^+ \rightarrow p \pi^0$	0.19 ± 0.03	-7.88 ± 2.92	-0.86 ± 4.46	-13.13 ± 2.76	6.97 ± 9.17	-0.78 ± 2.09
	0.19 ± 0.03	-7.88 ± 2.95	-0.86 ± 4.57	-13.13 ± 2.75	6.97 ± 9.78	-0.78 ± 2.21
$\Lambda_c^+ \rightarrow p \eta$	1.63 ± 0.09	2.55 ± 0.23	2.55 ± 0.58	2.54 ± 0.24	0.45 ± 0.31	-0.77 ± 0.44
	1.63 ± 0.09	2.55 ± 0.22	2.55 ± 0.59	2.54 ± 0.24	0.45 ± 0.31	-0.77 ± 0.44
$\Lambda_c^+ \rightarrow p \eta'$	0.52 ± 0.08	-14.40 ± 1.26	-17.31 ± 4.08	-13.57 ± 1.78	-1.81 ± 2.62	-0.69 ± 2.33
	0.52 ± 0.08	-14.40 ± 1.26	-17.31 ± 4.06	-13.57 ± 1.78	-1.81 ± 2.76	-0.69 ± 2.35
$\Lambda_c^+ \rightarrow n \pi^+$	0.61 ± 0.06	-5.16 ± 0.90	-0.65 ± 0.84	-17.36 ± 2.48	-5.55 ± 3.86	-2.99 ± 1.43
	0.61 ± 0.06	-5.16 ± 0.90	-0.65 ± 0.86	-17.36 ± 2.54	-5.55 ± 3.89	-2.99 ± 1.47
$\Xi_c^0 \rightarrow \Lambda^0 \pi^0$	0.07 ± 0.01	1.76 ± 2.31	0.92 ± 0.41	29.36 ± 57.57	51.88 ± 308.2	-23.28 ± 31.01
	0.07 ± 0.02	1.76 ± 2.32	0.92 ± 0.41	29.36 ± 57.67	51.88 ± 69.69	-23.28 ± 16.56
$\Xi_c^0 \rightarrow \Lambda^0 \eta$	0.40 ± 0.05	3.97 ± 0.79	1.79 ± 0.55	9.15 ± 2.95	-4.07 ± 0.87	3.52 ± 1.69
	0.40 ± 0.05	3.97 ± 0.82	1.79 ± 0.54	9.15 ± 3.17	-4.07 ± 0.88	3.52 ± 1.71
$\Xi_c^0 \rightarrow \Lambda^0 \eta'$	0.63 ± 0.08	-3.96 ± 0.38	-4.29 ± 1.00	-3.84 ± 0.61	0.32 ± 0.52	-0.51 ± 0.52
	0.63 ± 0.08	-3.96 ± 0.38	-4.29 ± 1.01	-3.84 ± 0.62	0.32 ± 0.53	-0.51 ± 0.52
$\Xi_c^0 \rightarrow \Sigma^0 \pi^0$	0.34 ± 0.03	-3.26 ± 0.72	-2.55 ± 0.68	-5.40 ± 2.71	-0.40 ± 0.58	-2.45 ± 2.90
	0.34 ± 0.03	-3.26 ± 0.74	-2.55 ± 0.69	-5.40 ± 2.85	-0.40 ± 0.59	-2.45 ± 3.07
$\Xi_c^0 \rightarrow \Sigma^0 \eta$	0.17 ± 0.03	11.58 ± 1.55	5.20 ± 1.37	16.52 ± 4.41	-0.60 ± 2.46	-14.72 ± 37.07
	0.17 ± 0.03	11.58 ± 1.62	5.20 ± 1.41	16.52 ± 4.58	-0.60 ± 2.64	-14.72 ± 25.20
$\Xi_c^0 \rightarrow \Sigma^0 \eta'$	0.18 ± 0.03	4.73 ± 0.96	5.56 ± 2.11	4.07 ± 1.86	-1.29 ± 1.72	0.54 ± 0.48
	0.18 ± 0.03	4.73 ± 0.96	5.56 ± 2.17	4.07 ± 1.98	-1.29 ± 1.79	0.54 ± 0.48
$\Xi_c^0 \rightarrow \Sigma^+ \pi^-$	0.26 ± 0.02	1.61 ± 1.04	-5.19 ± 1.02	5.64 ± 1.44	-41.25 ± 68.74	-1.19 ± 0.62
	0.26 ± 0.02	1.61 ± 1.07	-5.19 ± 1.02	5.64 ± 1.52	-41.26 ± 361.3	-1.19 ± 0.64
$\Xi_c^0 \rightarrow \Sigma^- \pi^+$	1.80 ± 0.05	-1.63 ± 0.46	-1.47 ± 0.39	-1.80 ± 0.68	-1.28 ± 0.34	1.46 ± 0.30
	1.80 ± 0.05	-1.63 ± 0.47	-1.47 ± 0.40	-1.80 ± 0.70	-1.28 ± 0.34	1.46 ± 0.30
$\Xi_c^0 \rightarrow \Xi^0 K_{S/L}$	0.38 ± 0.01	1.50 ± 0.50	-0.18 ± 1.31	1.81 ± 0.59	-5.86 ± 0.48	4.38 ± 1.48
	0.38 ± 0.01	1.50 ± 0.53	-0.18 ± 1.32	1.81 ± 0.62	-5.86 ± 0.48	4.38 ± 1.48
$\Xi_c^0 \rightarrow \Xi^- K^+$	1.31 ± 0.04	1.59 ± 0.43	1.47 ± 0.39	1.80 ± 0.68	1.32 ± 0.34	-1.43 ± 0.32
	1.31 ± 0.04	1.59 ± 0.44	1.47 ± 0.40	1.80 ± 0.70	1.32 ± 0.34	-1.43 ± 0.32
$\Xi_c^0 \rightarrow p K^-$	0.31 ± 0.02	-2.61 ± 1.11	5.19 ± 1.02	-5.64 ± 1.44	41.98 ± 67.85	2.19 ± 0.69
	0.31 ± 0.02	-2.61 ± 1.16	5.19 ± 1.02	-5.64 ± 1.52	41.98 ± 307.3	2.19 ± 0.71
$\Xi_c^0 \rightarrow n K_{S/L}$	0.83 ± 0.04	-1.67 ± 0.54	0.18 ± 1.31	-1.82 ± 0.59	6.03 ± 0.51	-4.21 ± 1.52
	0.83 ± 0.04	-1.67 ± 0.57	0.18 ± 1.32	-1.81 ± 0.62	6.02 ± 0.51	-4.21 ± 1.52
$\Xi_c^+ \rightarrow \Lambda^0 \pi^+$	0.21 ± 0.04	3.15 ± 3.95	1.26 ± 0.60	81.20 ± 171.2	62.14 ± 548.6	-53.44 ± 76.39
	0.21 ± 0.04	3.15 ± 4.09	1.26 ± 0.59	81.20 ± 91.49	62.15 ± 180.3	-53.44 ± 188.1
$\Xi_c^+ \rightarrow \Sigma^0 \pi^+$	3.16 ± 0.09	0.16 ± 0.65	-1.32 ± 0.81	0.56 ± 0.73	-3.66 ± 0.46	2.85 ± 0.47
	3.16 ± 0.09	0.16 ± 0.68	-1.32 ± 0.83	0.56 ± 0.76	-3.66 ± 0.46	2.85 ± 0.47
$\Xi_c^+ \rightarrow \Sigma^+ \pi^0$	2.57 ± 0.11	0.03 ± 0.74	-4.80 ± 2.96	0.65 ± 0.68	-6.20 ± 1.82	6.64 ± 4.41
	2.57 ± 0.11	0.03 ± 0.73	-4.80 ± 2.96	0.65 ± 0.68	-6.20 ± 1.94	6.64 ± 4.66
$\Xi_c^+ \rightarrow \Sigma^+ \eta$	1.02 ± 0.17	11.59 ± 1.55	5.20 ± 1.37	16.52 ± 4.41	-0.62 ± 2.45	-14.73 ± 37.07
	1.02 ± 0.16	11.59 ± 1.62	5.20 ± 1.41	16.52 ± 4.58	-0.62 ± 2.64	-14.73 ± 25.21
$\Xi_c^+ \rightarrow \Sigma^+ \eta'$	1.09 ± 0.17	4.73 ± 0.96	5.56 ± 2.11	4.07 ± 1.86	-1.29 ± 1.73	0.54 ± 0.48
	1.09 ± 0.18	4.73 ± 0.96	5.56 ± 2.17	4.07 ± 1.98	-1.29 ± 1.79	0.54 ± 0.48
$\Xi_c^+ \rightarrow \Xi^0 K^+$	1.15 ± 0.10	3.02 ± 0.73	0.65 ± 0.84	17.35 ± 2.48	7.70 ± 3.62	5.12 ± 1.62
	1.15 ± 0.10	3.02 ± 0.73	0.65 ± 0.86	17.35 ± 2.53	7.70 ± 3.64	5.12 ± 1.69
$\Xi_c^+ \rightarrow p K_{S/L}$	1.52 ± 0.08	-1.47 ± 0.39	-1.04 ± 0.25	-1.73 ± 0.64	-4.54 ± 1.07	3.07 ± 0.59
	1.52 ± 0.08	-1.47 ± 0.40	-1.04 ± 0.25	-1.73 ± 0.67	-4.54 ± 1.09	3.07 ± 0.59

► Large CPV modes: promising to be measured

$$\mathcal{A}_{CP}(\Lambda_c \rightarrow p \pi^0) = -(0.8 \pm 0.3) \times 10^{-3}, \quad \mathcal{A}_{CP}(\Lambda_c \rightarrow p \eta') = (1.4 \pm 0.1) \times 10^{-3},$$

$$\mathcal{A}_{CP}(\Xi_c^0 \rightarrow \Sigma^0 \eta) = (1.2 \pm 0.2) \times 10^{-3}, \quad \mathcal{A}_{CP}(\Xi_c^+ \rightarrow \Sigma^+ \eta) = (1.2 \pm 0.2) \times 10^{-3}.$$

► The large CPV modes containing one of the neutral pseudo- scalar, π^0, η, η' , some of which indicate a significant role of hairpin diagram E_h .

► U-spin symmetry relation also observed as:

$$\mathcal{A}_{CP}(\Lambda_c^+ \rightarrow n \pi^+) = -\mathcal{A}_{CP}(\Xi_c^+ \rightarrow \Xi^0 K^+),$$

$$\mathcal{A}_{CP}(\Lambda_c^+ \rightarrow \Sigma^+ K_{S,L}) = -\mathcal{A}_{CP}(\Xi_c^+ \rightarrow p K_{S,L}),$$

$$\mathcal{A}_{CP}(\Xi_c^0 \rightarrow \Xi^0 K_{S,L}) = -\mathcal{A}_{CP}(\Xi_c^0 \rightarrow n K_{S,L}),$$

$$\mathcal{A}_{CP}(\Xi_c^0 \rightarrow \Sigma^- \pi^+) = -\mathcal{A}_{CP}(\Xi_c^0 \rightarrow \Xi^- K^+),$$

$$\mathcal{A}_{CP}(\Xi_c^0 \rightarrow \Sigma^+ \pi^-) = -\mathcal{A}_{CP}(\Xi_c^0 \rightarrow p K^-),$$

X.G. He, Y.J. Shi, W. Wang,
EPJC 80 (2020), 359

D. Wang, EPJC 79 (2020), 429

PREDICTION II: STRONG PHASE

BESIII, PRL 132 (2024), 031801
2309.02744

(1) two sets for amplitudes

$$\text{I. } \begin{cases} |A| = 1.6_{-1.6}^{+1.9} \pm 0.4, \\ |B| = 18.3 \pm 2.8 \pm 0.7, \end{cases} \quad \text{II. } \begin{cases} |A| = 4.3_{-0.2}^{+0.7} \pm 0.4, \\ |B| = 6.7_{-6.7}^{+8.3} \pm 1.6, \end{cases}$$

$$\gamma = \frac{|A|^2 - \kappa^2 |B|^2}{|A|^2 + \kappa^2 |B|^2},$$

γ : relative size of partial waves

(2) ambiguity in sign of phase-shift

$$(\delta_S^{X_i}, \delta_P^{X_i}) \rightarrow (-\delta_S^{X_i}, -\delta_P^{X_i})$$

$$\delta_P - \delta_S = -1.55 \pm 0.25 \pm 0.05 \quad \text{or} \quad 1.59 \pm 0.25 \pm 0.05 \text{ rad.}$$

$$\beta = \frac{2\kappa |A^* B| \sin(\delta_P - \delta_S)}{|A|^2 + \kappa^2 |B|^2},$$

β : relative sign of phase-shift

$$\Gamma = \frac{p_c}{8\pi} \frac{(m_i + m_f)^2 - m_P^2}{m_i^2} (|A|^2 + \kappa^2 |B|^2)$$

$$\alpha = \frac{2\kappa |A^* B| \cos(\delta_P - \delta_S)}{|A|^2 + \kappa^2 |B|^2},$$

only had BF and α in 2024/01, could not solve the problems!

H. L. Zhong, FX, H.Y. Cheng, PRD 109 (2024), 114027; 2401.15926

PREDICTION II: STRONG PHASE

(1) two sets for amplitudes

I.
$$\begin{cases} |A| = 1.6^{+1.9}_{-1.6} \pm 0.4, \\ |B| = 18.3 \pm 2.8 \pm 0.7, \end{cases}$$

II.
$$\begin{cases} |A| = 4.3^{+0.7}_{-0.2} \pm 0.4, \\ |B| = 6.7^{+8.3}_{-6.7} \pm 1.6, \end{cases}$$

(2) sign ambiguity of phase-shift

$$(\delta_S^{X_i}, \delta_P^{X_i}) \rightarrow (-\delta_S^{X_i}, -\delta_P^{X_i})$$

$$\delta_P - \delta_S = -1.55 \pm 0.25 \pm 0.05 \text{ or } 1.59 \pm 0.25 \pm 0.05 \text{ rad.}$$



 solution selected

Decay	α	β	γ	Δ (radian)
$\Lambda_c^+ \rightarrow \Lambda \pi^+$	$-0.782 \pm 0.009 \pm 0.004$	$0.368 \pm 0.019 \pm 0.008$	$0.502 \pm 0.016 \pm 0.006$	$0.633 \pm 0.036 \pm 0.013$
$\Lambda_c^+ \rightarrow \Lambda K^+$	$-0.569 \pm 0.059 \pm 0.028$	$0.35 \pm 0.12 \pm 0.04$	$-0.743 \pm 0.067 \pm 0.024$	$2.70 \pm 0.17 \pm 0.04$
$\Lambda_c^+ \rightarrow p K_S^0$	$-0.744 \pm 0.012 \pm 0.009$	–	–	–

LHCb, PRL 133 (2024), 261804
 2409.02759



Fit 10/2024

$$|A| = 2.76 \pm 0.18, |B| = 9.71 \pm 0.47,$$

$$\alpha_{\Xi^0 K^+} = -0.04 \pm 0.12,$$

$$\beta_{\Xi^0 K^+} = -0.98 \pm 0.02$$

$$\delta_P - \delta_S = -1.61 \pm 0.12 \text{ rad}$$

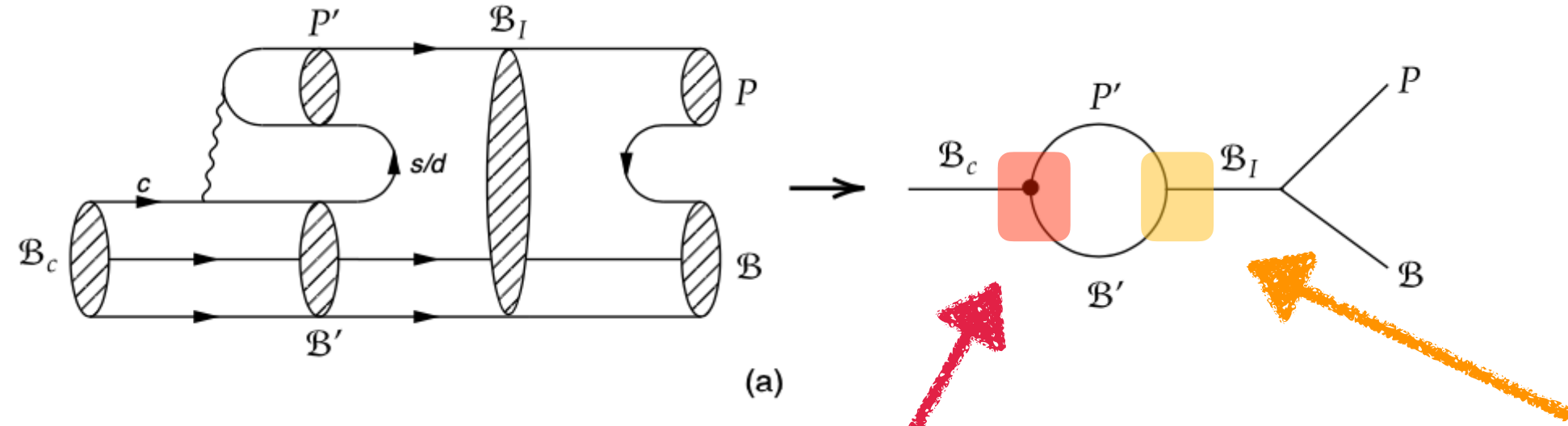
H.Y. Cheng, FX, H. L. Zhong, PRD 111 (2025), 034011

SUMMARY

- TDA & IRA both work for CPV of charmed baryons.
- FSR u, s, t channels are calculated.
- FSR plays a role to connect tree and penguin parameters in TDA.
- CPV of about 30 channels are calculated, 4 of them found to have large CPV of $\mathcal{O}(10^{-3})$: $\Lambda_c \rightarrow p\pi^0, \Lambda_c \rightarrow p\eta, \Xi_c^0 \rightarrow \Sigma^0\eta, \Xi_c^+ \rightarrow \Sigma^+\eta$.
- The sign ambiguity of β, γ reported in BESIII has been clarified by recent precise measurement provided by LHCb.

Backup

AN EXAMPLE: S-CHANNEL CALCULATION



$$\mathcal{L}_{\mathcal{B}_c \mathcal{B} P} = \sum_{P, \mathcal{B}, \mathcal{B}_c} F_{\mathcal{B}_c \mathcal{B} P} P^\dagger \bar{\mathcal{B}} \mathcal{B}_c = (P^\dagger)_j^k (\bar{\mathcal{B}})_i^l \left(\tilde{F}_V^+(H_+)^{ij}_k + \tilde{F}_V^-(H_-)^{ij}_k \right) (\mathcal{B}_c)_l,$$

$$\begin{aligned} \mathcal{L}_{\mathcal{B} \mathcal{B}' P} &= \sum_{\mathcal{B}', P} \left[\sum_{\mathcal{B}_-} g_{\mathcal{B}_- \mathcal{B}' P} P \bar{\mathcal{B}}' \mathcal{B}_- + \sum_{\mathcal{B}_+} g_{\mathcal{B}_+ \mathcal{B}' P} P \bar{\mathcal{B}}' i \gamma_5 \mathcal{B}_+ \right] \\ &= g_- \left[(P)_j^i (\bar{\mathcal{B}}')_k^j (B_-)_i^k + r_- (P)_k^j (\bar{\mathcal{B}}')_j^i (B_-)_i^k \right] \\ &\quad + g_+ \left[(P)_j^i (\bar{\mathcal{B}}')_k^j i \gamma_5 (B_+)_i^k + r_+ (P)_k^j (\bar{\mathcal{B}}')_j^i i \gamma_5 (B_+)_i^k \right] \end{aligned}$$

$$A^s \propto \sum_{\mathcal{B}_I} g_{\mathcal{B}_I \mathcal{B} P} \left(\sum_{\mathcal{B}', P'} g_{\mathcal{B}_I \mathcal{B}' P'} F_{\mathcal{B}_c \mathcal{B}' P'} \right)$$

$$\begin{aligned} A^s &= \sum_{\mathcal{B}_I, \mathcal{B}', P'} \bar{u}_{\mathcal{B}} \left(\int \frac{d^4 q}{(2\pi)^4} g_{\mathcal{B}_I \mathcal{B} P} \frac{p_{\mathcal{B}_c}^\mu \gamma_\mu + m_I}{p_{\mathcal{B}_c}^2 - m_I^2} g_{\mathcal{B}_I \mathcal{B}' P'} \frac{q^\mu \gamma_\mu + m_{\mathcal{B}'}}{q^2 - m_{\mathcal{B}'}^2} \frac{1}{(q - p_{\mathcal{B}_c})^2 - m_{P'}^2} F_{\mathcal{B}_c \mathcal{B}' P'} \right) u_{\mathcal{B}_c} \\ &= \bar{u}_{\mathcal{B}} \left[\int \frac{d^4 q}{(2\pi)^4} \left(\sum_{\mathcal{B}_I, \mathcal{B}', P'} g_{\mathcal{B}_I \mathcal{B} P} g_{\mathcal{B}_I \mathcal{B}' P'} F_{\mathcal{B}_c \mathcal{B}' P'} \right) I(q^2) \right] u_{\mathcal{B}_c} \end{aligned}$$

$$\sum_{\mathcal{B}', P'} g_{\mathcal{B}_I \mathcal{B}' P'} F_{\mathcal{B}_c \mathcal{B}' P'} \propto \sum_{\mathcal{B}', P'} \left((P')_{j_2}^{i_2} (\mathcal{B}_I)_{k_2}^{j_2} (\mathcal{B}')_{i_2}^{k_2} + r_- (P')_{k_2}^{j_2} (\mathcal{B}_I)_{j_2}^{i_2} (\mathcal{B}')_{i_2}^{k_2} \right) \left((P')_i^k (\mathcal{B}')_j^l (H_-)^{ij}_k (\mathcal{B}_c)_l \right)$$

$$\begin{aligned} A^s &= \tilde{f}^b + r_- \tilde{f}^c + \left(\frac{8r_- + 2}{8 + 2r_-} - \frac{1}{2} \right) \tilde{f}_3^b + \frac{1}{6} (1 + r_-) \left(1 - \frac{8r_- + 2}{r_- + 4} \right) \tilde{f}_3^c + r_- \left(\frac{4r_- + 1}{r_- + 4} - \frac{1}{2} \right) \tilde{f}_3^d \\ &= \tilde{f}^b + r_- \tilde{f}^c + \frac{7r_- - 2}{8 + 2r_-} \tilde{f}_3^b + \frac{(1 + r_-)(2 - 7r_-)}{24 + 6r_-} \tilde{f}_3^c + \frac{r_-(7r_- - 2)}{8 + 2r_-} \tilde{f}_3^d \end{aligned}$$