重介子四体半轻衰变理论进展

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Overview

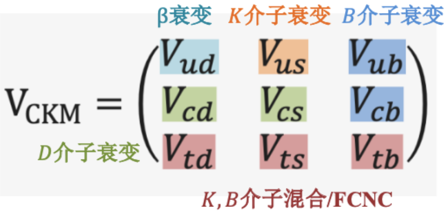
I: $|V_{ub}|$ 疑难和 FCNC 反常

II: B 介子四体半轻衰变的初步探索

III: 总结和展望

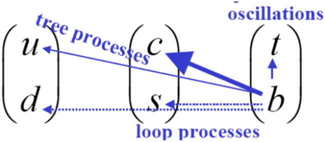
$|V_{ub}|$ 疑难和 FCNC 反常

$|V_{ub}|$ 疑难和 FCNC 反常



- $VV^\dagger = V^\dagger V = I_3$ in the Standard Model
- $VV^\dagger \neq V^\dagger V \neq I_3$ New Physics
- $|V_{ub}|/|V_{cb}|$ contributes to *CPV* measurement in B decays

- CKM matrix elements are mainly measured via **the charged current processes**, i.e, $b \to ul^- \bar{\nu}, b \to cl^- \bar{\nu}, c \to sl^+ \nu$
- **Flavor changing neutral current precesses** are sensitive to new physical contributions, i.e, $b \to sl^+ l^-, b \to dl^+ l^-$



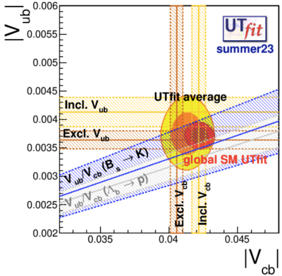
$|V_{ub}|$ 疑难问题

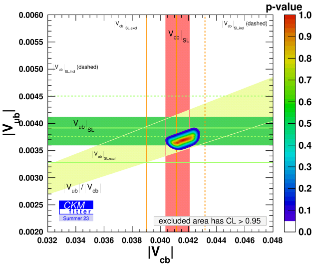
- $|V_{ub}|$ tension $\quad |V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ [PDG 2024]

‡ $\sim 2.5\sigma$ tension between $(4.13 \pm 0.25) \times 10^{-3}$ and $(3.70 \pm 0.16) \times 10^{-3}$ measured via the $B \to X_u l^- \bar{\nu}$ and $B \to \pi l^- \bar{\nu}$ processes, respectively.

- $|V_{cb}|$ tension $\quad |V_{cb}| = (41.1 \pm 1.2) \times 10^{-3}$ [PDG 2024]

‡ $\sim 2.5\sigma$ tension between $(42.2 \pm 0.5) \times 10^{-3}$ and $(39.8 \pm 0.6) \times 10^{-3}$ measured via the $B \to X_c l^- \bar{\nu}$ and $B \to D^{(*)} l^- \bar{\nu}$ processes, respectively.

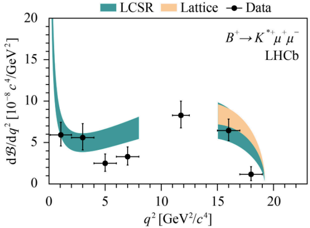


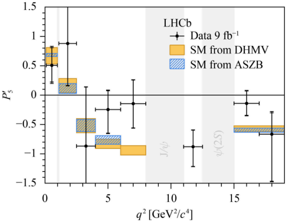


B 介子半轻衰变的反常现象

- LFU in $b \to c l^- \bar{\nu}$ processes $\quad R_{D^{(*)}} = \mathcal{B}(B \to D^{(*)}\tau^-\bar{\nu})/\mathcal{B}(B \to D^{(*)}\mu^-\bar{\nu})$
  - $R_D = 0.407 \pm 0.046$, $R_{D^*} = 0.306 \pm 0.015$ Average with [Belle PRL124, 161803 (2020)]
  - $2.1\sigma, 3.0\sigma$ derivations from the SM predictions of $R_D = 0.298 \pm 0.004$, $R_{D^*} = 0.254 \pm 0.005$ [HFLAV]
  - $R_D = 0.441 \pm 0.089$, $R_{D^*} = 0.281 \pm 0.030$ [LHCb PRL131,111802 (2023)]
  - would make the CKM measurements more complicated if confirmed

- Anomalies in FCNC processes $B \to K^*\mu^+\mu^-$
  - 3.6σ derivation from SM of $d\mathcal{B}(B \to K^*\mu^+\mu^-)/dq^2$ in $q^2 \in [1, 6]$ GeV^2
  - 1.9σ derivation from SM of $p_5' = S_5/\sqrt{F_L(1-F_L)}$ in $q^2 \in [4, 8]$ GeV^2





[Heavy Flavour Physics and CP Violation at LHCb: a Ten-Year Review, Front. Phys.18,44601(2023)]

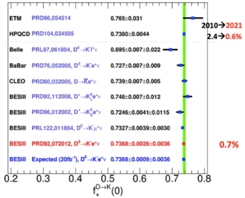
解决方案　在传统过程继续奋斗

- **更精确的测量和格点计算**

  $|V_{cs}|$ issue　$|V_{cs}| = 0.975 \pm 0.006$ [PDG 2022, 24]

  ∗ 0.972 ± 0.007 and 0.984 ± 0.012 measured via the $D \to Kl\nu$ and $D_s \to \mu^+\nu_\mu$ processes $\sim 3\sigma \to \sim 1.5\sigma$

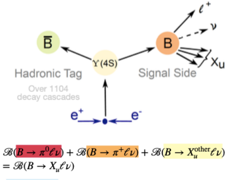
  理论与实验联合研讨的成功典范



- **更全面更系统的物理分析方法**

  $|V_{ub}|$ result from Belle collaboration with Simultaneous Determination in excl. and incl. processes [Belle PRL131, 211801 (2023)]

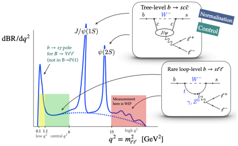
  ∗ $(3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3}$ and $(3.88 \pm 0.20 \pm 0.31 \pm 0.09) \times 10^{-3}$



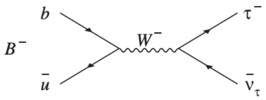
- **精细结构、丰富的 QCD 效应**

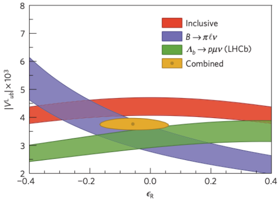
  high order QCD corrections, more structures

  [AK, TM, YMW, JHEP 02 (2013) 010]
  [AK, TM, AAP, YMW, JHEP 09 (2010) 089]



解决方案　寻找新的增长极





‡ $|V_{ub}|f_B$ in **pure leptonic decay**

∗ 0.72 ± 0.09 MeV from Belle, 1.01 ± 0.14 MeV from BABAR, 0.77 ± 0.12 MeV average [FLAG2021]

‡ $|V_{ub}|$ in **baryon decay**

[LHCb Nature Physics 11, 743-747 (2015)]

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{\mathcal{B}(\Lambda_b^0 \to p\mu^-\bar{\nu})}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+\mu^-\bar{\nu})} R_{\rm FF} = 0.68 \pm 0.07 \Downarrow$$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.06 \xrightarrow{|V_{cb}|} |V_{ub}| = (3.72 \pm 0.23) \times 10^{-3}$$

∗ consistent with determinations in exclusive $B \to \pi l \bar{\nu}$ decay

confirms the existing incompatibility with the inclusive sample

‡ $|V_{ub}|/|V_{cb}|$ in $\mathcal{B}(B_s \to K^-\mu^+\nu)/\mathcal{B}(B_s \to D_s^-\mu^+\nu)$ [LHCb PRL126, 081804 (2021)]

‡ $|V_{cb}|$ in $B_s \to D_s\mu^+\nu$, $\frac{d\mathcal{B}(B\to K^*\mu^+\mu^-)}{dq^2}$ and p_5' in $\Lambda_b^0 \to \Lambda\mu^+\mu^-$

解决方案

- 以上过程都只涉及到基态粒子
- 含有激发态粒子的过程也可以提供独立的测量
- $|V_{ub}|$ in the $B \to \rho l \nu$ channel the same $b \to u l \nu$ transition as in the golden channel
- simultaneous measurements of the $\frac{d\mathcal{B}}{dq^2}$ for $B \to \pi^- l^+ \nu_l$ and $B \to \rho^0 l^+ \nu_l$
  [Belle-II PRD 111. 112009(2025)] [HFLAV 2023] [P. Bharucha and et.al., JHEP 1608. 098]

$$|V_{ub}|_{B\to\pi l\nu} = (3.73 \pm 0.10 \pm 0.16|_{\mathrm{LQCD+LCSRs}}) \times 10^{-3},$$
$$|V_{ub}|_{B\to\rho l\nu} = (3.19 \pm 0.22 \pm 0.26|_{\mathrm{LCSRs}}) \times 10^{-3}.$$

- $B \to V$ FFs updated via B-meson LCSRs [Gao, et.al., PRD 101. 074035(2020)]

$$|V_{ub}|_{B\to\rho l\nu} = \left(3.05^{+1.34}_{-1.30}|_{\mathrm{theo}}\,{}^{+0.19}_{-0.20}|_{\mathrm{data}}\right) \times 10^{-3},$$
$$|V_{ub}|_{B\to\omega l\nu} = \left(2.54^{+1.09}_{-1.05}|_{\mathrm{theo}}\,{}^{+0.18}_{-0.19}|_{\mathrm{data}}\right) \times 10^{-3}$$

- a notably smaller value is obtained in $B \to \rho$ transition
- $|V_{ub}|_{B_s\to K l\nu} = 3.58(9) \times 10^{-3}$ is consistent with the "golden" channel
  [PRD104. 114041 (2021)]

- the uniformity of $|V_{ub}|$ determinations across different exclusive channels ?

B 介子四体半轻衰变的初步探索

B 介子四体半轻衰变的探索

- ρ is an unstable resonance that decays into $\pi\pi$ via the strong interaction

* the signal channel in a $B \to \rho l\nu$-type decay is $B \to \pi\pi l\nu$ (B_{l4})
* the $\pi\pi$ spectra in $[0.554, 0.996]$GeV serves as the candidate region for ρ

  [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]

  [S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van. Dyk PRD 89. 014015 (2014)]

* the QCD studies usually treat ρ as a stable single particle
* mismatch between experimental measurements and theoretical calculations, particularly in accounting for the finite width and nonresonant backgrounds

- how to address the finite width of the ρ mesons, nonresonant QCD backgrounds, and the effects of different partial waves in the calculation of $B \to \pi\pi$ form factor?

* usually small ($< 10\%$) in three-body D decays, while large or even dominate in the penguin dominated three-body B decays (such as $B \to K\pi\pi$, KKK) [PRL 111.101801(2013) LHCb]

- $|V_{cb}|$ in the $B \to D^* l\nu$ processes, B anomalies in $B \to K^* l^+ l^-$ processes

$B \to \pi\pi$ 形状因子

$$
\begin{aligned}
i\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = {} & F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma \\
& + F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right) \\
& + F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right)
\end{aligned}
$$

† $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function

† $q\cdot k = (m_B^2 - q^2 - k^2)/2$ and $q\cdot\bar{k} = \sqrt{\lambda}\beta_\pi(k^2)\cos\theta_\pi/2 = \sqrt{\lambda}\,(2\zeta - 1)$

† $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$, θ_π is the angle between the 3-momenta of the neutral pion and the B-meson in the dipion rest frame

$B \to \pi\pi$ 形状因子

- **LQCD** (Lattice QCD) in the ρ resonance region with a simple BW model
  - [L. Leskovec and et.al, PRL 134.161901 (2025), Editors' Suggestion]
- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large q^2 by taking dispersive methods in terms of Omnés functions
  - [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]

  in the full phase-space by a novel parameterization with unitarity
  - [F. Herren, B. Kubis and R. van Tonder, PRD 112, 014037 (2025),Editors' Suggestion]
- **QCDF** (QCD factorization) in the large dipion mass
  - [P. Böer, T. Feldmann and D. van Dyk, JHEP02, 133(2017)]

    $T_I \propto F_{Bto\pi} \otimes \phi_\pi$, $T_{II} \propto \phi_B \otimes \phi_\pi \otimes \phi_\pi$
- **LCSRs** (Light-cone sum rules) in the small and intermediate q^2
  - [S. Cheng, A. Khodjamirian and J. Virto, JHEP 05, 157(2017)] B-meson LCSRs
  - [C. Hambrock and A. Khodjamirian, NPB 905(2016)379-390] 2πDAs LCSRS of $F_{\parallel,\perp}$
  - [S. Cheng, A. Khodjamirian and J. Virto, PRD(R) 96 (2017)051901] timelike-helicity FF F_t and F_0
  - [S. Cheng, PRD 99 (2019) 053005] 2πDAs updates and $B \to [\pi\pi]_{\rm S,P}$ FFs
  - [S. Cheng and J.M Shen, EPJC(2020)6:554, S. Cheng and S.L Zhang, EPJC (2024)84:379] Pheno
  - [S. Cheng, arXiv: 2502.07333[hep-ph]] first study of twist-three 2πDAs and $|V_{ub}|$ extraction
  - [S. Cheng, L.Y. Dai, J.M. Shen and S.L. Zhang, to be appear] $D_s \to [\pi\pi]_{\rm S}\, e\nu$, minor contribution from $q\bar{q}$ component

$B \to \pi\pi$ 形状因子

- $B \to \pi\pi$ form factors from the B-meson LCSRs

$$F_{\mu\nu}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_\mu(x), j_\nu^{V-A}(0)\}|\bar{B}^0(q+k)\rangle$$
$$\equiv \quad \varepsilon_{\mu\nu\rho\sigma} q^\rho k^\sigma F_{(\varepsilon)}(k^2,q^2) + i g_{\mu\nu} F_{(g)}(k^2,q^2) + i q_\mu k_\nu F_{(qk)}(k^2,q^2) + \cdots$$

- $B \to \pi\pi$ form factors from the 2πDAs LCSRs

$$F_\mu(k_1,k_2,q) = i\int d^4x e^{iq\cdot x}\langle \pi^+(k_1)\pi^-(k_2)|\mathrm{T}\{j_\mu^{V-A}(x), j_5(0)\}|0\rangle$$
$$\equiv \quad \varepsilon_{\mu\nu\rho\sigma} q^\nu k_1^\rho k_1^\sigma F^V + q_\mu F^{(A,q)} + k_\mu F^{(A,k)} + \bar{k}_\mu F^{(A,\bar{k})}$$

- Advantages of the 2πDAs LCSRs

  * the QCD calculation does not rely on any resonant model, however, encounter a inverse problem in B-meson LCSRs

$$2\,\mathrm{Im}F_{\mu\nu}(k,q) = \int d\tau_{2\pi}\, \langle 0|\bar{d}\gamma_\mu u\,|\pi^+\pi^-\rangle\langle \pi^+\pi^-|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0\rangle + \cdots$$

  * provides a unifies framework to predict contributions from different partial-waves
  * theoretical uncertainties are better controlled, higher-power corrections are suppressed by $\mathcal{O}(1/m_b)$ when k^2 is not too large

2πDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\overline{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

‡ Three independent kinematic variables

- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z,\zeta,k^2,\mu) = 6z(1-z) \sum_{n=0,\mathrm{even}}^{\infty} \sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu) C_n^{3/2}(2z-1) C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z) \sum_{n=1,\mathrm{odd}}^{\infty} \sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu) C_n^{3/2}(2z-1) C_\ell^{1/2}(2\zeta-1)$$

- How to describe **the evolution from $4m_\pi^2$ to large invariant mass k^2** ?

‡ Watson theorem of π-π scattering amplitudes

$$B_{n\ell}^I(k^2) = B_{n\ell}^I(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^m}{dk^{2m}} \ln B_{n\ell}^I(0) + \frac{k^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds\, \frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

△ **2πDAs in a wide range of energies is given by δ_ℓ^I and a few subtraction constants**

2πDAs

- The subtraction constants of $B_{n\ell}(s)$ at low s (around the threshold)

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ \ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ \ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^{π}, a_2^{ρ} [**SC** 2019, 2023]

- All the above discussions are at leading twist
- twist three 2πDAs are studied recently [S. Cheng, arXiv:2502.07333[hep-ph]]

2πDAs widely used in the three-body B decays studied from pQCD and QCDF are the asymptotic formula

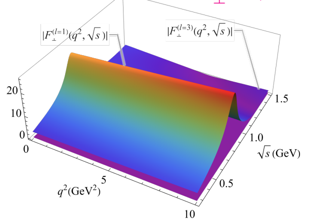
[J. Chai, S. Cheng and A.J Ma PRD 105 (2022) 033003]

normalized to unit as $\Gamma^{I=1}_{M_1M_2}(0) = 1$. When the invariant mass of dimeson system is small, the higher $\mathcal{O}(s)$ terms in the expansion of coefficient $B_{nl}(s,\mu)$ around the resonance pole can be safely neglected due to the large suppression $\mathcal{O}(s/m_b^2)$ in contrast to the energetic dimeson system in B decay, so the relation $B_{n1}(s,\mu) \to a_n(\mu)\Gamma^{I=1}_{M_1M_2}(s)$ can be obtained in the lowest partial wave approximation. This argument induces the basic assumption in PQCD that the energetic dimeson DAs can be deduced from the DAs of resonant meson by replacing the decay constant by the timelike form factor.

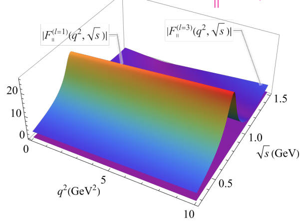
$B \to \pi\pi$ 形状因子

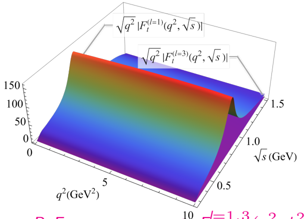
[S Cheng, 2502.07333]

P, F-wave components $F_\perp^{l=1,3}(q^2, k^2)$

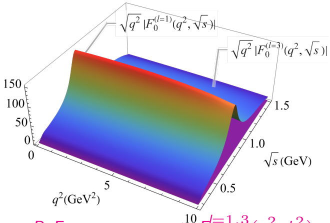


P, F-wave components $F_\parallel^{l=1,3}(q^2, k^2)$





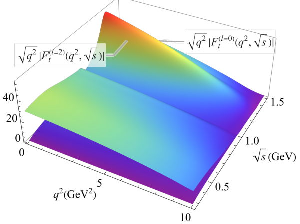
P, F-wave components $F_t^{l=1,3}(q^2, k^2)$



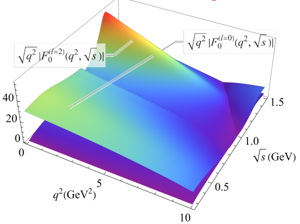
P, F-wave components $F_0^{l=1,3}(q^2, k^2)$

$B \to \pi\pi$ 形状因子

S, D-wave components $F_t^{l=0,2}(q^2, k^2)$



S, D-wave components $F_0^{l=0,2}(q^2, k^2)$



- twist-three contribution vanish in $F_{\perp,\parallel}^{(l=1,3,\cdots)}$
- persist in $F_{t,0}^{(l=0,1,\cdots)}$, give a significant correction $\sim 40\%$ to leading twist result
- ρ resonance is dominate in $F_{\perp,\parallel,t,0}^{(l=1)}(q^2, k^2)$, F-wave contributions are negligible
- S-wave component is dominate in the small k^2, D-wave component is comparable in the large k^2 and small q^2

$|V_{ub}|$ extraction from B_{l4} decay

- 2D partial differential decay width

$$\frac{d^2\Gamma}{dq^2\,dk^2} = G_F^2|V_{ub}|^2 \frac{\beta_\pi \sqrt{\lambda} q^2}{3\,(4\pi)^5\, m_B^3}\left[\left(|F_0^{(S)}|^2 + |F_0^{(P)}|^2\right) + \beta_\pi^2\left(|F_\parallel^{(P)}|^2 + |F_\perp^{(P)}|^2\right) + \cdots\right]$$

- Partial branching fractions $\triangle\mathcal{B}^i$ (in unit of 10^{-5}) in different bins

take the PDG average value of $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$

bins	$\sqrt{s}$	q^2	$\triangle\mathcal{B}^i$	$\triangle\mathcal{B}^i$ [Belle 2021]
1	$[4m_\pi^2, 0.6]$	$[0, 8]$	$0.27 \pm 0.03 \pm 0.06$	$0.84^{+0.39}_{-0.32} \pm 0.18$
2	$(0.6, 0.9]$	$[0, 4]$	$1.91 \pm 0.21 \pm 0.38$	$2.39^{+0.53}_{-0.47} \pm 0.32$
3	$(0.6, 0.9]$	$(4, 8]$	$1.54 \pm 0.17 \pm 0.27$	$2.16^{+0.47}_{-0.42} \pm 0.23$
4	$(0.9, 1.2]$	$[0, 4]$	$0.65 \pm 0.07 \pm 0.12$	$0.70^{+0.32}_{-0.25} \pm 0.20$
5	$(0.9, 1.2]$	$(4, 8]$	$0.41 \pm 0.04 \pm 0.08$	$0.64^{+0.28}_{-0.22} \pm 0.11$
6	$(1.2, 1.5]$	$[0, 4]$	$0.57 \pm 0.05 \pm 0.10$	$0.91^{+0.35}_{-0.28} \pm 0.12$
7	$(1.2, 1.5]$	$(4, 8]$	$0.16 \pm 0.02 \pm 0.02$	$0.64^{+0.32}_{-0.26} \pm 0.08$

- $|V_{ub}|$ extraction in the regions of ρ and f_0 resonances

$$\boxed{|V_{ub}|_{B^+\to[\rho^0\to]\pi^+\pi^-l^+\nu_l} = (4.27 \pm 0.49|_{\rm Data} \pm 0.55|_{\rm LCSRs}) \times 10^{-3}}$$

$$\boxed{|V_{ub}|_{B^+\to[f_0\to]\pi^+\pi^-l^+\nu_l} = (3.96 \pm 0.47|_{\rm Data} \pm 0.52|_{\rm LCSRs}) \times 10^{-3}}$$

总结和展望

总结和展望

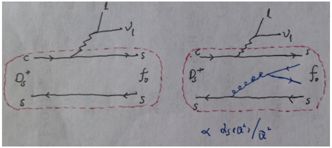
- $|V_{ub}|$ “疑难”是重味物理研究的一个核心问题
- B 介子四体半轻衰变（B_{l4}）提供了一个新的解决方案
- 两介子系统光锥分布振幅（2πDAs）是 B_{l4} 研究的关键部件

- 首次在三扭度水平研究了 2πDAs，实现了在 B_{l4} 过程抽取 $|V_{ub}|$
- 解释了通过 $B \to \rho l \nu$ 过程测量的较小结果
- 当前的理论误差和实验误差都比较大，但是都可以被进一步减小
  - ‡ Belle-II 积分亮度未来五年有望达到 $3ab^{-1}$，实验精度将至少提高 3 倍
  - ‡ LCSRs 和 LQCD 对形状因子的联合分析，至少可以将理论误差减小一半

- B_{l4} 过程将在 $|V_{ub}|$ 测量和味物理反常检验中扮演重要角色
  - ‡ $B^0 \to \pi^- \pi^0 l^+ \nu_l$ 和 $B^+ \to \pi^0 \pi^0 l^+ \nu_l$ 过程的测量将进一步帮助理解 2πDAs
- B_{l4} 过程的研究正在进入高精度时代

Thank you for your patience.

Backup slides $D_s \to [\pi\pi]_{\rm S}\, e\nu$



- ⋆ in SL B_s decays
- ⋆ tetraquark contribution is suppressed doubly by strong coupling and power
- ⋆ FSI is weak too

$$|f_0(980)\rangle = \psi_{q\bar q}|q\bar q\rangle + \psi_{q\bar q g}|q\bar q g\rangle + \psi_{q\bar q q\bar q}|q\bar q q\bar q\rangle + \cdots$$

$$|\,[\pi\pi]_{\rm S}\rangle = \psi_{q\bar q}|q\bar q\rangle + \psi_{q\bar q g}|q\bar q g\rangle + \psi_{q\bar q q\bar q}|q\bar q q\bar q\rangle + \cdots$$

$$\psi^n_{f_0}(x_i, k_{\perp i}, \lambda_i) = \langle n, x_i, k_{\perp i}, \lambda_i | f_0\rangle$$

- ⋆ physical observables are usually written in a QCD convolution

$$\frac{d\sigma}{d\Omega} = \sum_t \int_0^1 dx_i\, \mathcal{H}^t(x_i, Q)\psi^t(x_i, \mu)$$

- ⋆ ψ^t is universal, however H^t is process dependent, hence different observables might highlike the contributions from different components

- **how about the energetic $q\bar q$ picture $f_0(980)$ in D_s decays ?**
- ⋆ this talk focus on the contributions from lowest Fock state $|q\bar q\rangle$

$D_s \to [f_0, \cdots \to] \pi\pi e\nu$

- Semileptonic $D_{(s)}$ decays provide a clean environment to study scalar mesons
  - $D_s \to f_0 e^+ \nu$ [CLEO '09], $D_{(s)} \to a_0 e^+ \nu$ [BESIII '18, '21], $D^+ \to f_0/\sigma e^+ \nu$[BESIII '19]
  - $D_s \to f_0(\to \pi^0\pi^0, K_s K_s) e^+ \nu$ [BESIII 22], $D_s \to f_0(\to \pi^+\pi^-) e^+ \nu$[BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0) e^+ \nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-) e^+ \nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$
$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- single particle (narrow width limit) $\quad D_s \to f_0 e^+ \nu$

$$\frac{d\Gamma(D_s^+ \to f_0 l^+ \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2, \ D_s \to f_0 \ \mathrm{FF}$$

- improvement with the width effect by resonant model $\quad D_s \to [f_0 \to] \pi\pi e^+ \nu$

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_{\mathrm{S}} l^+ \nu)}{ds dq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1 \beta_\pi(s)}{|m_{\mathrm{S}}^2 - s + i(g_1\beta_\pi(s)) + g_2\beta_K(s))\,|^2}, \ \mathrm{BESIII}$$

- calculate directly the signal channel $D_s \to [\pi\pi]_{\mathrm{S}}\, e^+ \nu$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_{\mathrm{S}} l^+ \nu)}{dk^2 dq^2} = \frac{G_F^2 |V_{cs}|^2\ \beta_{\pi\pi}(k^2) \sqrt{\lambda_{D_s}} q^2}{3(4\pi)^5 m_{D_s}^3} \sum_{\ell=0}^{\infty} |F_0^{(\ell)}(q^2, k^2)|^2, \ D_s \to \pi\pi \ \mathrm{FF}$$

$D_s \to f_0$ form factor and $D_s^+ \to (f_0, [\pi\pi]_S)e^+\nu_e$ decay

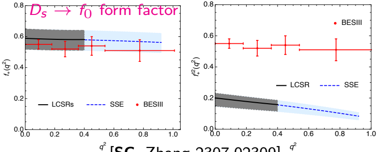
- $\{M^2, s_0\} = \{5.0 \pm 0.5, 6.0 \pm 0.5\}\mathrm{GeV}^2$ *see Hai-bing Fu's talk* for the LCSRs

this work	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	$0.86/0.90$	0.30 ± 0.03

- the BESIII result in the $\pi^+\pi^-$ system $f_+(0) = 0.518 \pm 0.018 \pm 0.036$ [BESIII 23]

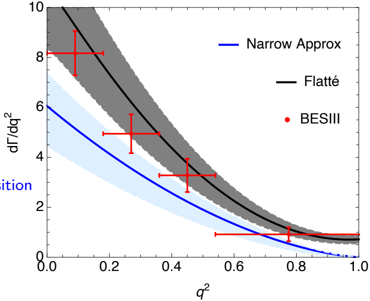
  different input of the decay constant $\tilde{f}_{f_0} = 335$ MeV, much larger than 180 MeV in LCSRs(10)
  we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$s\bar{s}$-$n\bar{n}$ mixing scenario of f_0 with $\theta = 20^\circ \pm 10^\circ$



[**SC**, Zhang 2307.02309]

- Twist-3 LCDAs give dominate contribution in $D_s \to f_0$ transition
  - the uncertainty estimation is conservative
  - without NLO correction
  - we need a model independent calculation
  - not only for the QCD understanding
  - but also for the future partial-wave measurement



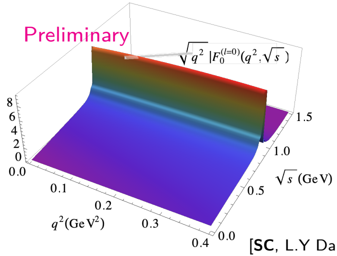
Differential decay width of $D_s^+ \to (f_0, [\pi\pi]_S)e^+\nu_e$

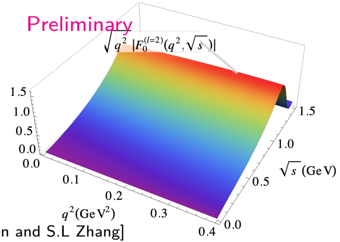
$D_s \to [\pi\pi]_S$ form factor and $D_s \to [\pi\pi]_S\, e^+\nu$ decay

- The LCSRs ℓ'-wave $D_s \to [\pi\pi]_S$ form factors ($\ell' = \mathrm{even}$ & $\ell' \leqslant n+1$)

$$\sqrt{q^2}F_0^{(\ell')}(q^2,k^2) = \frac{m_c(m_c+m_s)\sqrt{q^2}\sqrt{\lambda_{D_s}}}{m_{D_s}^2 f_{D_s}} \sum_{n=1,\mathrm{odd}}^{\infty} \frac{\beta_\pi(k^2)}{\sqrt{2\ell'+1}} J_n^0(q^2,k^2,M^2,s_0)\, B_{n\ell,\parallel}^{I=0}(k^2)\, I_{\ell\ell'}$$

- Near threshold, B_{nl} can be determined from the low-energy effective theory of pions interacting with massive "constituent" quarks, based on the instanton model of the QCD vacuum
- For the resonant regions, Watson's theorem yields the k^2-dependence via Omnés solutions with the phase shifts mplemented through subtracted dispersion relations



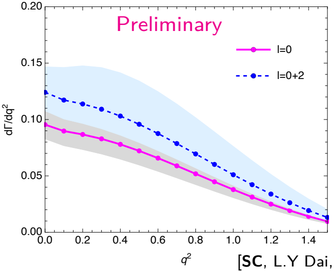


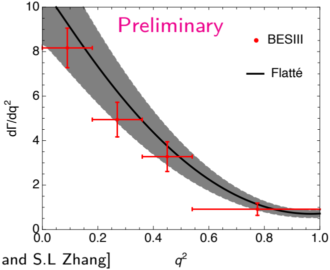
[**SC**, L.Y Dai, J.M Shen and S.L Zhang]

- S- and D-wave FFs: $\sqrt{q^2}|F_0^{(I=0)}(\sqrt{s},q^2)|$ and $\sqrt{q^2}|F_0^{(I=2)}(\sqrt{s},q^2)|$

$D_s \to [\pi\pi]_{\rm S}$ form factor and $D_s \to [\pi\pi]_{\rm S}\, e^+\nu$ decay

- The differential decay width on momentum transfers





[**SC**, L.Y Dai, J.M Shen and S.L Zhang]

- Differential widths $d\Gamma/dq^2$ is two-order in magnitude smaller than the data
- the FFs at the two-particle level are one-order lower than the required
- the conventional $q\bar{q}$ is not the dominate component in the charm decays
- we have to go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g$, $q\bar{q}q\bar{q}$)
- much different in B decays leading twist dominated [**SC**, arXiv: 2502.07333]