

第二届重味物理前沿论坛研讨会

2025.9.12-15, 武汉

Two-boson semi-leptonic tau decays in chiral EFT



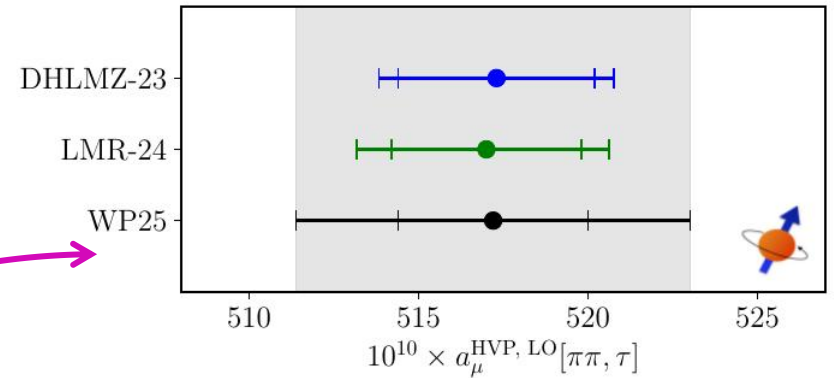
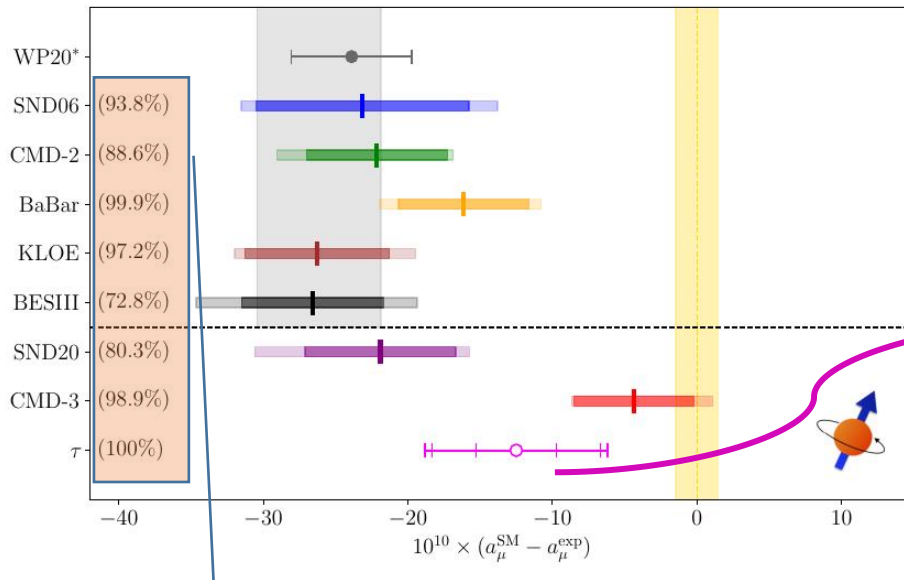
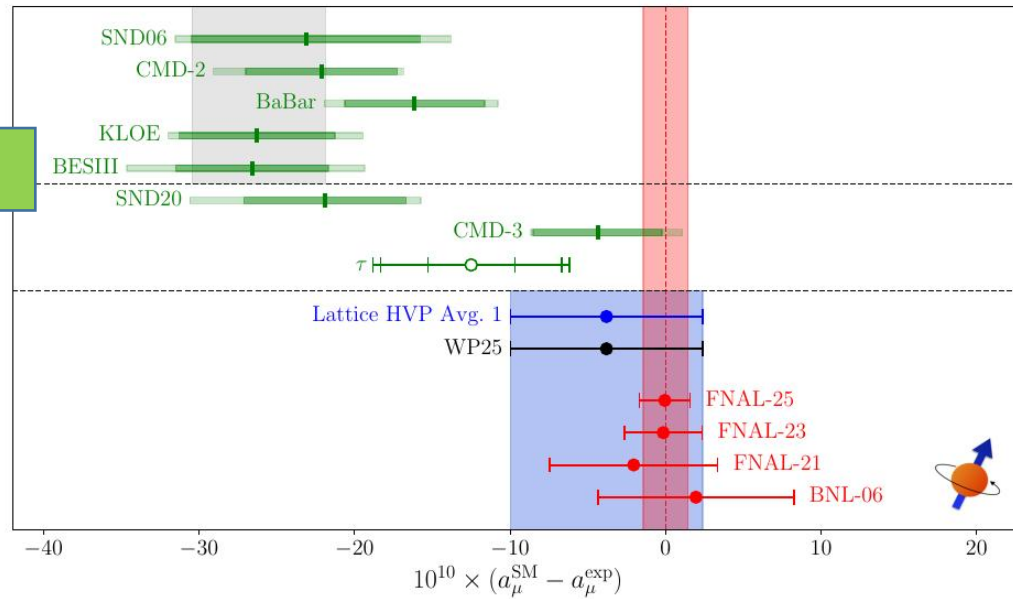
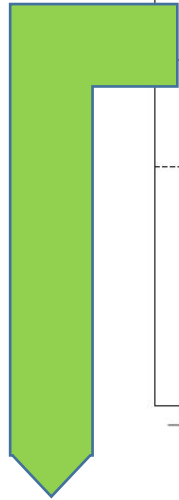
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Current status on muon g-2

[2505.21476, White Paper 25]

- $e^+e^- \rightarrow \pi^+\pi^-$ from each indicated Exp
- HVP-LO beyond $\pi\pi$ from WP20
- Others, such as HLbL, EW, QED, from WP25



$\pi\pi$ contribution from each
Exp to HVP integral

Alternative way to address HVP from $\pi\pi$

$$a_\mu^{\text{HVP,LO}} = \frac{1}{4\pi^3} \int_{4M_2^2}^{t_{\text{max}}} dt K(t) \sigma_{e^+e^- \rightarrow \text{hadrons}}^0(t)$$

Known kernel function
(enhanced contribution
from energy below 1GeV)

$$\sigma_{e^+e^- \rightarrow \pi^+\pi^-}^0 = \frac{\pi\alpha^2}{3t} \beta_{\pi^+\pi^-} \left| F_{\pi\pi}^{(0)}(t) \right|^2 \quad \frac{d\Gamma(\tau_{2\pi})}{dt} = \frac{G_F^2 |V_{ud}|^2 m_\tau^3 S_{\text{EW}}}{384\pi^3} \left(1 - \frac{t}{m_\tau^2}\right)^2 \left(1 + \frac{2t}{m_\tau^2}\right) \beta_{\pi^-\pi^0} \left| F_{\pi\pi}^{(-)}(t) \right|^2$$

$$\tau \rightarrow \pi \pi^0 \nu_\tau : \langle \pi^- \pi^0 | \bar{d} \gamma_\mu u | 0 \rangle \sim F_{\pi\pi}^{(-)}(t) \quad [I=1, I_3=-1]$$

$$e^+e^- \rightarrow \pi^+\pi^- : \langle \pi^+\pi^- | \bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d | 0 \rangle \sim F_{\pi\pi}^{(0)}(t) \quad [I=1, I_3=0]$$

Isospin limit



Conserved vector current
(CVC)

$$F_{\pi\pi}^{(0)}(t) = F_{\pi\pi}^{(-)}(t)$$

$$\sigma_{e^+e^- \rightarrow \pi^+\pi^-}^0 = \frac{K_\sigma(t)}{K_\Gamma(t)} \frac{\beta_{\pi^+\pi^-}}{S_{\text{EW}} \beta_{\pi^-\pi^0}} \frac{d\Gamma(\tau_{2\pi})}{dt}$$

$$K_\sigma(t) = \frac{\pi\alpha^2}{3t}, \quad K_\Gamma(t) = \frac{G_F^2 |V_{ud}|^2 m_\tau^3}{384\pi^3} \left(1 - \frac{t}{m_\tau^2}\right)^2 \left(1 + 2\frac{t}{m_\tau^2}\right)$$

➤ Isospin breaking (IB) effects become **CRUCIAL** at the sub-percent level.

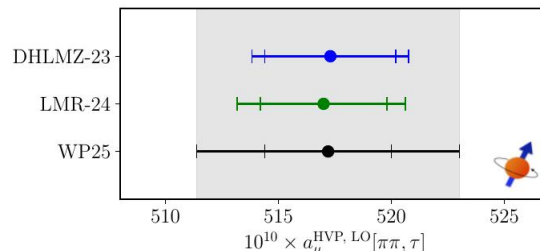
➤ Full control of all the IB terms is yet to be reached.

Results on the estimation of a_μ based on the tau data in WP25 are based on:

[Davier, et al., (DHLMZ), EPJC'24] [Lopez Castro, et al., (LMR) PRD'25]

+

$\tau \rightarrow \nu_\tau \pi\pi$ data (Belle, ALEPH, CLEO, OPAL)



$$a_\mu^{\text{HVP,LO}}[\pi\pi, \tau] = 517.2(2.8)_{\text{exp}}(5.1)_{\text{th}} \times 10^{-10}$$

Overview of tau decays

分支比概览

➤ $\text{Br}(\tau \rightarrow e \nu_\tau \bar{\nu}_e) : 17.8\%$

$\text{Br}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu) : 17.4\%$

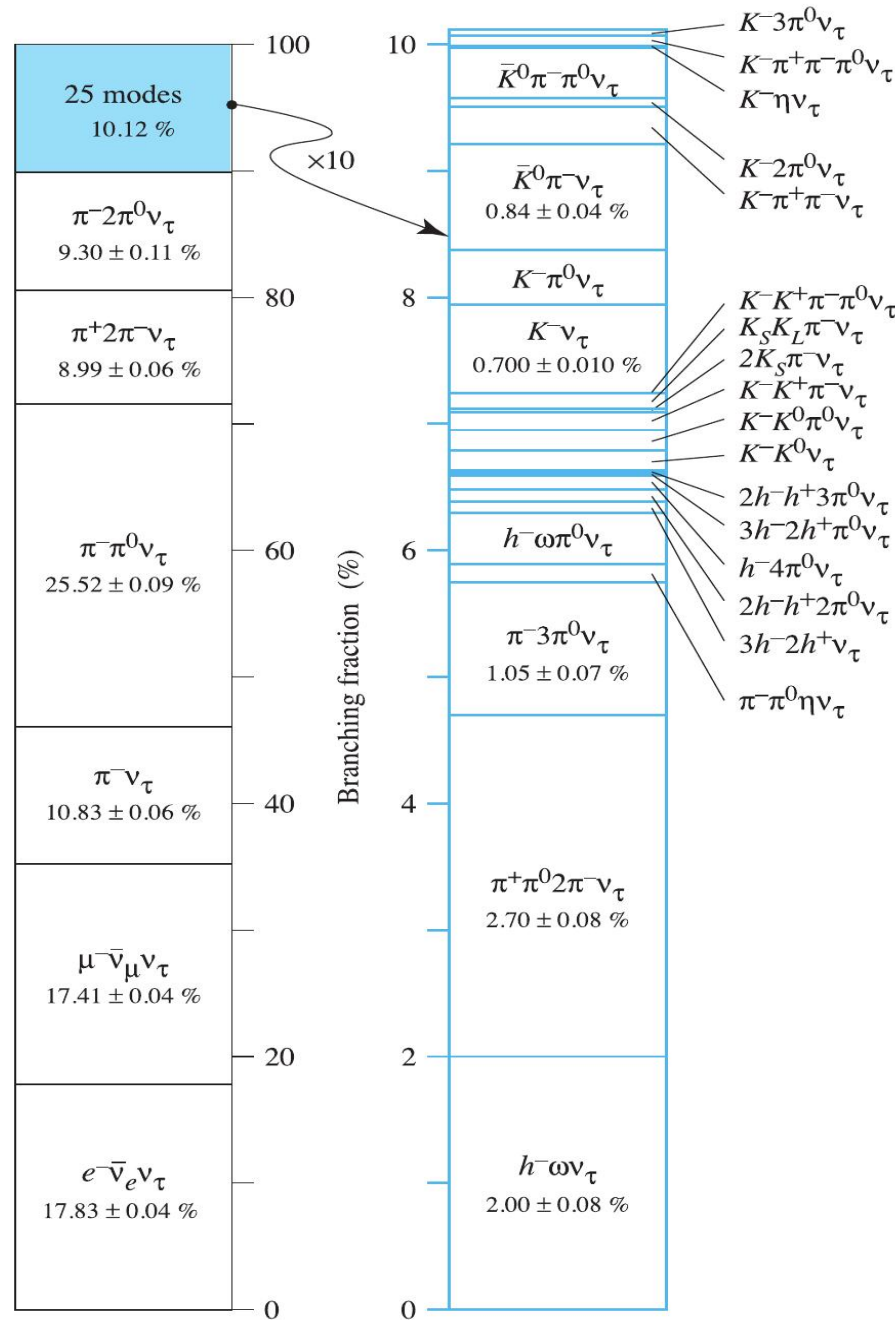
➤ $\text{Br}(\tau \rightarrow \nu + \text{Cabbibo allowed hadrons}) \sim 62\%$

➤ $\text{Br}(\tau \rightarrow \nu + \text{Cabbibo suppressed hadrons}) : \sim 3\%$

□ $\text{Br}(\tau \rightarrow \nu \pi \pi) \sim 25\%$, 单举衰变中分支比最大

□ tau的衰变末态只有轻味强子, 不涉及重味粒子 ($m_\tau < m_D$)

□ 在重子数守恒的假设下, tau不能衰变至含有重子的末态($m_\tau < 2m_N$)



强衰变过程

tau的强衰变可以给我们提供什么信息？

- **inclusive**衰变：（某类）所有的强子末态

$$\tau^- \rightarrow \nu_\tau (\bar{u}d, \bar{u}s)$$



可以用来研究标准模型的基本参数： $\alpha_S, V_{us}, \dots$

- **exclusive**衰变：衰变至特定的强子末态

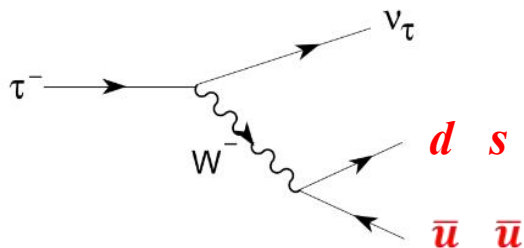
$$\tau^- \rightarrow \nu_\tau (P, \textcolor{red}{PP'}, P_1 P_2 P_3, \dots)$$



- 强作用形状因子，强子共振态，手征对称性，...
- CPV、轻子味道破坏、...

基础理论回顾

标准模型的带电流



$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W_{\mu}^{\dagger} (V_{uD} \bar{u}_L \gamma^{\mu} D_L + \bar{\nu}_L \gamma^{\mu} \tau_L) + h.c.$$

$$\bullet M_{\tau \rightarrow H \nu_{\tau}} = \frac{g^2 V_{uD}^*}{2} \frac{L_{\mu} H^{\mu}}{s - M_W^2} \approx -\frac{g^2 V_{uD}^*}{2 M_W^2} L_{\mu} H^{\mu}$$

$$H_{\mu} = \langle n | \bar{d}_L \gamma_{\mu} u | 0 \rangle, L_{\mu} = \bar{u}_L \gamma_{\mu} u_L, \tau$$

因为 $m_{\tau} \ll m_W$ ，另外如果在电弱能标下没有质量轻的新粒子，四费米子等价相互作用可以用来很好地描述tau衰变（标准模型或者新物理模型均可）。

tau强衰变相关的标准模型下的四费米子作用可总结为：

$$\mathcal{L}_{CC}^{SM} = -\frac{G_F}{\sqrt{2}} V_{uD} \bar{l}_{\nu_{\tau}} \gamma^{\mu} (1 - \gamma_5) l_{\tau} \bar{u} \gamma_{\mu} (1 - \gamma_5) D, \quad [D = d, s]$$

CKM:
 V_{ud}/V_{us}

轻子流

夸克流

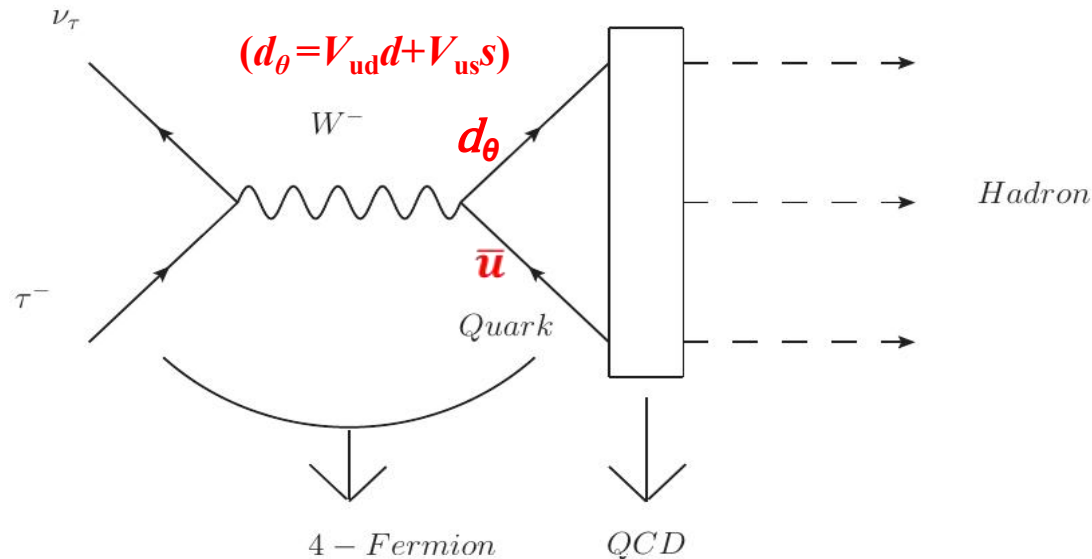
$$G_F = \frac{g^2}{4\sqrt{2}M_W^2}$$

- **SMEFT → LEFT** [Cirigliano et al, '10] [Y.Liao et al., '21] [J.H.Yu et al., '21][F.Z.Chen et al, '22] ...

$$\mathcal{L}_{\text{eff}} = -\frac{G_{\mu} V_{uD}}{\sqrt{2}} \left[(1 + \epsilon_L^{D\ell}) \bar{\ell} \gamma_{\mu} (1 - \gamma_5) \nu_{\ell} \cdot \bar{u} \gamma^{\mu} (1 - \gamma_5) D + \epsilon_R^{D\ell} \bar{\ell} \gamma_{\mu} (1 - \gamma_5) \nu_{\ell} \cdot \bar{u} \gamma^{\mu} (1 + \gamma_5) D \right. \\ \left. + \bar{\ell} (1 - \gamma_5) \nu_{\ell} \cdot \bar{u} [\epsilon_S^{D\ell} - \epsilon_P^{D\ell} \gamma_5] D + \frac{1}{4} \hat{\epsilon}_T^{D\ell} \bar{\ell} \sigma_{\mu\nu} (1 - \gamma_5) \nu_{\ell} \cdot \bar{u} \sigma^{\mu\nu} (1 - \gamma_5) D \right] + h.c.,$$

ϵ_X parameterize various new physics at high energy scale

Hadronic decays: a unique feature for tau lepton



Hadronic V-A currents

$$\mathbf{H}_\mu = \langle H^- | \bar{u} \gamma_\mu (1 - \gamma_5) d_\theta e^{iL_{QCD}} | 0 \rangle$$

Chiral EFT is the low energy realization of QCD:

$$e^{iZ(v_\mu, a_\mu, s, p)} = \int \mathcal{D}q \mathcal{D}\bar{q} \mathcal{D}G_\mu e^{i \int d^4x \mathcal{L}_{QCD}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}u e^{i \int d^4x \mathcal{L}_{EFT}(v_\mu, a_\mu, s, p)}$$

$$\mathcal{L}^{QCD} = \mathcal{L}_0^{QCD} + \bar{q} \gamma^\mu (v_\mu + a_\mu \gamma_5) q - \bar{q} (s - i \gamma_5 p) q$$

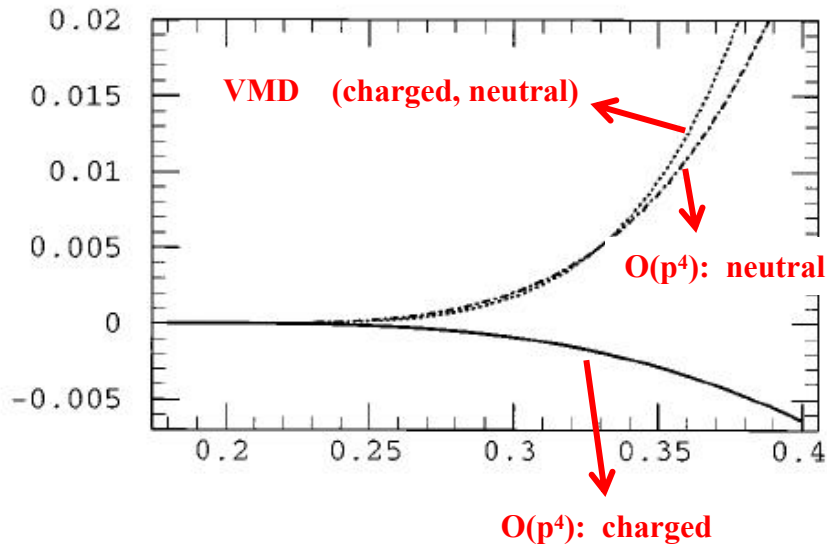
v_μ, a_μ, s, p are the external source fields .

Chiral symmetry is RELEVANT to tau decays

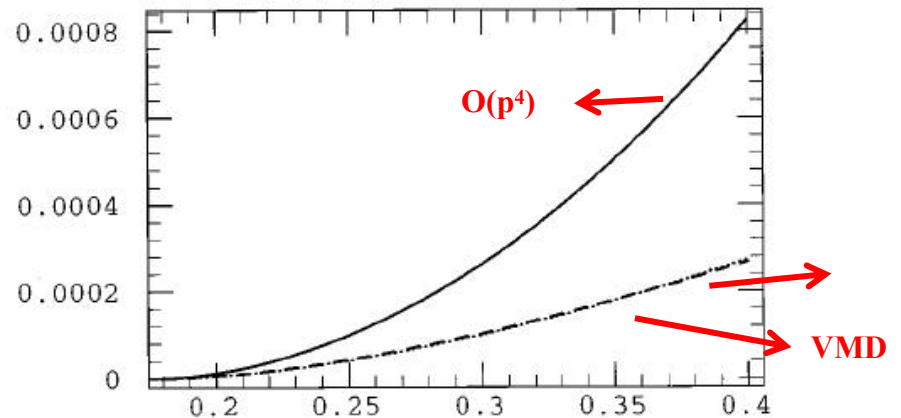
Example: $\tau \rightarrow \nu_\tau \pi \pi \pi$ transition amplitudes in the low energy region
VMD models do not automatically respect chiral symmetry.

$$J_\alpha = -i \frac{2\sqrt{2}}{3f_\pi} \text{BW}_a(Q^2) (B_\rho(s_2) V_{1\alpha} + B_\rho(s_1) V_{2\alpha}) \quad [\text{Kuhn, Santamaria, ZPC'90}]$$

W_D structure function



W_{SA} structure function (neutral channel)



[Colangelo, et al., PRD'96]

➤ Resonance chiral theory implements the constraint of chiral symmetry from the very beginning in the construction of the Lagrangians.

Resonance chiral theory (R χ T)

Chiral group: $G = SU(3)_L \times SU(3)_R$, $H = SU(3)_V$, $u(\phi) = G/H$

Resonances : $R \xrightarrow{G} h R h^\dagger$, $h \in H$

pNGB and external sources : $X = u_\mu, \chi_\pm, f_\pm^{\mu\nu}, h_{\mu\nu}$

Operators	P	C	h.c.	chiral order
u	$u^\dagger$	u^T	$u^\dagger$	1
Γ_μ	Γ^μ	$-\Gamma_\mu^T$	$-\Gamma_\mu$	p
u_μ	$-u^\mu$	u_μ^T	u_μ	p
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$	$\pm\chi_\pm$	p^2
$f_{\mu\nu} \pm$	$\pm f_\pm^{\mu\nu}$	$\mp f_{\mu\nu}^T$	$f_{\mu\nu} \pm$	p^2
$h_{\mu\nu}$	$-h^{\mu\nu}$	$h_{\mu\nu}^T$	$h_{\mu\nu}$	p^2

Operators	P	C	h.c.
$V_{\mu\nu}$	$V^{\mu\nu}$	$-V_{\mu\nu}^T$	$V_{\mu\nu}$
$A_{\mu\nu}$	$-A^{\mu\nu}$	$A_{\mu\nu}^T$	$A_{\mu\nu}$
S	S	S^T	S
P	$-P$	P^T	P

Operators beyond minimal

[Cirigliano, et al., '04]:

$$\mathcal{L}_{VAP} = \lambda_1^{VA} \langle [V^{\mu\nu}, A_{\mu\nu}] \chi_- \rangle + \dots,$$

[Ruiz-Femenia, Pich and Portolés, '03]

$$\mathcal{L}_{VVP} = d_1 \varepsilon_{\mu\nu\rho\sigma} \langle \{V^{\mu\nu}, V^{\rho\alpha}\} \nabla_\alpha u^\sigma \rangle + \dots,$$

$$\mathcal{L}_{VJP} = \frac{c_1}{M_V} \varepsilon_{\mu\nu\rho\sigma} \langle \{V^{\mu\nu}, f_+^{\rho\alpha}\} \nabla_\alpha u^\sigma \rangle + \dots,$$

$$d_4 \varepsilon_{\mu\nu\rho\sigma} \langle \{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\} u_\alpha \rangle$$

Minimal R χ T Lagrangian [Ecker, et al., '89]

$$\mathcal{L}_{2V} = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

$$\mathcal{L}_{2A} = \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle,$$

$$\mathcal{L}_{2S} = c_d \langle S u_\mu u^\mu \rangle + c_m \langle S \chi_+ \rangle,$$

$$\mathcal{L}_{2P} = id_m \langle P \chi_- \rangle.$$

QCD dynamics in R χ T

- Low energy QCD: implemented from the construction of R χ T
- Intermediate energy: explicit resonance states
- **High energy information:** to match the same physical objects in R χ T and QCD, $\langle J(x_n) \cdots J(0) \rangle^{\text{R}\chi\text{T}} = \langle J(x_n) \cdots J(0) \rangle^{\text{QCD}}$.

For example: $\pi\pi$ vector form factor

$$\begin{aligned} [\mathcal{F}_{\pi\pi}^{\nu}(q^2)]^{\text{R}\chi\text{T}} &= 1 + \frac{F_V G_V}{F^2} \frac{q^2}{M_V^2 - q^2}, \\ [\mathcal{F}_{\pi\pi}^{\nu}(q^2)]^{\text{QCD}} &\rightarrow 0, \quad \text{for } q^2 \rightarrow \infty \end{aligned}$$

This leads to

$$[\mathcal{F}_{\pi\pi}^{\nu}(q^2)]^{\text{R}\chi\text{T}} = [\mathcal{F}_{\pi\pi}^{\nu}(q^2)]^{\text{QCD}} \implies F_V G_V = F^2$$

Unified study of $\tau \rightarrow P_1 P_2 + \nu_\tau$

[Hao, Duan, ZHG, 2507.00383]

$$\langle P_1 P_2 | \bar{D} \gamma^\mu u | 0 \rangle = \left[(p_2 - p_1)^\mu - \frac{\Delta_{P_2 P_1}}{s} q^\mu \right] F_+^{P_1 P_2}(s) + \frac{\Delta_{Du}}{s} q^\mu \hat{F}_0^{P_1 P_2}(s)$$

(vector FF) (scalar FF)

$$\Delta_{P_2 P_1} = m_{P_2}^2 - m_{P_1}^2, \quad \Delta_{Du} = B_0(m_D - m_u), \quad q_\mu = (p_1 + p_2)_\mu, \quad s = q^2.$$

• Invariant-mass distribution of $P_1 P_2$

$$\frac{d\Gamma_{\tau \rightarrow P_1 P_2 \nu_\tau}}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{48\pi^3 s} S_{EW} |V_{uD}|^2 \left(1 - \frac{s}{M_\tau^2}\right) \left\{ \left(1 + \frac{2s}{M_\tau^2}\right) q_{P_1 P_2}^3(s) \left|F_+^{P_1 P_2}(s)\right|^2 + \frac{3\Delta_{Du}^2}{4s} q_{P_1 P_2}(s) \left|\hat{F}_0^{P_1 P_2}(s)\right|^2 \right\}$$

• Forward-Backward (FB) asymmetry distribution

$$A_{FB}(s) = \frac{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau \rightarrow P_1 P_2 \nu_\tau}}{d\sqrt{s} d\cos\alpha} - \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau \rightarrow P_1 P_2 \nu_\tau}}{d\sqrt{s} d\cos\alpha}}{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau \rightarrow P_1 P_2 \nu_\tau}}{d\sqrt{s} d\cos\alpha} + \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau \rightarrow P_1 P_2 \nu_\tau}}{d\sqrt{s} d\cos\alpha}} = \frac{\Delta_{Du} q_{P_1 P_2}(s) \Re \left[F_+^{P_1 P_2}(s) \hat{F}_0^{P_1 P_2*}(s) \right]}{\frac{2\sqrt{s}}{3} \left(1 + \frac{2s}{M_\tau^2}\right) q_{P_1 P_2}^2(s) \left|F_+^{P_1 P_2}(s)\right|^2 + \frac{\Delta_{Du}^2}{2\sqrt{s}} \left|\hat{F}_0^{P_1 P_2}(s)\right|^2}$$

α : angle between the momenta of P_1 and τ in the $P_1 P_2$ rest frame

$\tau \rightarrow \pi^- \pi^0 \nu_\tau$: $\Delta_{PP'} \rightarrow 0$ (同位旋破坏项), 标量 F_0 项可忽略, 矢量 F_+ 绝对主导!

$\tau \rightarrow K \pi \nu_\tau$: $\Delta_{PP'} \neq 0$, 标量形状因子 F_0 项以及矢量形状因子 F_+ 项都有贡献!

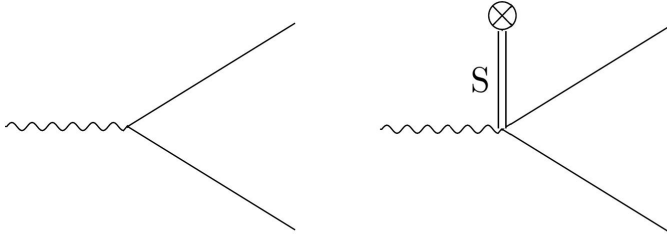
Calculation of $\mathbf{P_1P_2}$ form factors

$$\mathcal{L}^{\text{LO}} = \frac{F^2}{4} \langle u_\mu u^\mu \rangle + \frac{F^2}{4} \langle \chi_+ \rangle + \frac{F^2}{12} M_0^2 X^2$$

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle ,$$

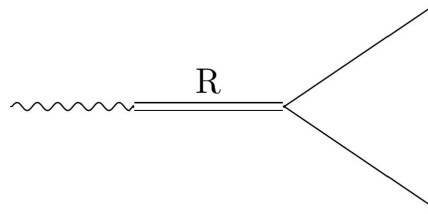
$$\mathcal{L}_\Lambda^{\text{NLO,U(3)}} = -\frac{F^2 \Lambda_1}{12} D^\mu X D_\mu X - \frac{F^2 \Lambda_2}{12} X \langle \chi_- \rangle$$

$$\mathcal{L}_S = c_d \langle S u_\mu u^\mu \rangle + c_m \langle S \chi_+ \rangle ,$$

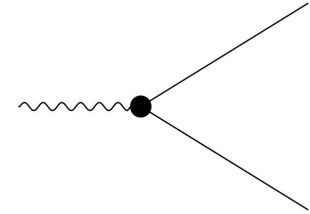


(a)

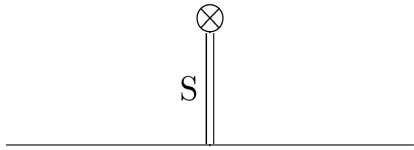
(b)



(c)



(d)



(a)



(b)

Mixing relations of π^0 - η - η' -a (axion)

[Gao,ZHG,Oller,Zhou, JHEP'23]

[Gao,Hao,ZHG,Oller,Zhou, EPJC'25]

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = \begin{pmatrix} 1 + z_{11} & c_\theta(-v_{12} + z_{12}) + s_\theta(-v_{13} + z_{13}) & -s_\theta(-v_{12} + z_{12}) + c_\theta(-v_{13} + z_{13}) & -v_{14} + z_{14} \\ v_{12} + z_{21} & c_\theta(1 + z_{22}) + s_\theta(z_{23} - v_{23}) & -s_\theta(1 + z_{22}) + c_\theta(z_{23} - v_{23}) & -v_{24} + z_{24} \\ v_{13} + z_{31} & c_\theta(z_{32} + v_{23}) + s_\theta(1 + z_{33}) & -s_\theta(z_{32} + v_{23}) + c_\theta(1 + z_{33}) & -v_{34} + z_{34} \\ v_{41} + z_{41} & c_\theta(v_{42} + z_{42}) + s_\theta(v_{43} + z_{43}) & -s_\theta(v_{42} + z_{42}) + c_\theta(v_{43} + z_{43}) & 1 + v_{44} + z_{44} \end{pmatrix} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix}$$

$\mathbf{v_{ij}}$: LO terms

$\mathbf{z_{ij}}$: NLO terms ($\mathbf{L_5, L_8, \Lambda_1, \Lambda_2}$) or [$\mathbf{(L_5, L_8) \sim (c_d c_m, c_m c_m)/M^2_S, \Lambda_1, \Lambda_2}$]

➤ **Scalar form factor** $\langle P_1 P_2 | (m_D - m_u) \bar{D} u | 0 \rangle$

$$\Delta_{du} = B_0(m_d - m_u) = \Delta_{du}^{\text{Phy}} \left\{ 1 + \frac{m_K^2}{F^2} \left[\frac{16c_m(c_d - c_m)}{M_S^2} + \frac{16c'_m(c'_d - c'_m)}{M_{S'}^2} \right] \right\}$$

$$\Delta_{du}^{\text{Phy}} = m_{K^0}^2 - m_{K^+}^2 - (m_{\pi^0}^2 - m_{\pi^+}^2)$$



$$(m_d - m_u) \langle \pi^- P | \bar{d} u | 0 \rangle = \Delta_{du}^{\text{Phy}} F_0^{\pi^- P}(s) \qquad \hat{F}_0^{\pi^- P}(s) = \frac{\Delta_{du}^{\text{Phy}}}{\Delta_{du}} F_0^{\pi^- P}(s)$$



$$\langle \pi^- P | \bar{d} \gamma^\mu u | 0 \rangle = \left[(p_P - p_\pi)^\mu - \frac{\Delta_{P\pi}}{s} q^\mu \right] F_+^{\pi^- P}(s) + \frac{\Delta_{du}^{\text{Phy}}}{s} q^\mu F_0^{\pi^- P}(s)$$

➤ **Finiteness of the above expression at $s = 0$ requires**

$$F_+^{\pi^- (\eta, \eta', a)}(0) = \frac{\Delta_{du}^{\text{Phy}}}{\Delta_{(\eta, \eta', a)\pi}} F_0^{\pi^- (\eta, \eta', a)}(0)$$

✓ **Non-trivial confirmation is verified in chiral EFT!**

Some explicit expressions for Form Factors

- VFF- $\pi\pi$**

$$F_+^{\pi^- \pi^0}(s) = -\frac{\sqrt{2}}{F^2} G_{\text{LO}+\rho \text{Ex}}(s)$$

$$G_{\text{LO}+\rho \text{Ex}}(s) = \frac{G_V F_V s + F^2 (M_\rho^2 - s)}{M_\rho^2 - s - i M_\rho \Gamma_\rho(s)} - \frac{G'_V F'_V s}{M_{\rho'}^2 - s - i M_{\rho'} \Gamma_{\rho'}(s)} - \frac{G''_V F''_V s}{M_{\rho''}^2 - s - i M_{\rho''} \Gamma_{\rho''}(s)},$$

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi F_\pi^2} \left[\sigma_{\pi\pi}^3(s) + \frac{1}{2} \sigma_{KK}^3(s) \right], \quad \Gamma_{\rho', \rho''}(s) = \Gamma_{\rho', \rho''} \frac{s}{M_{\rho', \rho''}^2} \frac{\sigma_{\pi\pi}^3(s)}{\sigma_{\pi\pi}^3(M_{\rho', \rho''}^2)}, \quad \sigma_{P_1 P_2}(s) = \frac{2q_{P_1 P_2}(s)}{\sqrt{s}} \theta[s - (m_{P_1} + m_{P_2})^2]$$

- VFF- $\pi\eta/\pi\eta'/\pi a$**

$$F_+^{\pi^- \eta}(s) = -\frac{\sqrt{2}v_{12}}{F^2} G_{\text{LO}+\rho \text{Ex}}(s) - \sqrt{2} \left(y_{12} - v_{13} y_{23}^{(0)} \right),$$

$$F_+^{\pi^- \eta'}(s) = -\frac{\sqrt{2}v_{13}}{F^2} G_{\text{LO}+\rho \text{Ex}}(s) - \sqrt{2} \left(y_{13} + v_{12} y_{23}^{(0)} \right),$$

$$F_+^{\pi^- a}(s) = -\frac{\sqrt{2}v_{41}}{F^2} G_{\text{LO}+\rho \text{Ex}}(s) - \sqrt{2} \left(y_{14} + v_{12} y_{24}^{(0)} + v_{13} y_{34}^{(0)} \right)$$

• **SFF- $\pi\eta/\pi\eta'/\pi a$**

$$F_0^{\pi^-\eta}(s) = \sqrt{\frac{2}{3}}(c_\theta - \sqrt{2}s_\theta) + \frac{1}{\sqrt{3}}(\Lambda_1 - 2\Lambda_2)s_\theta - \frac{1}{\sqrt{3}}y_{23}(2c_\theta + \sqrt{2}s_\theta) + 4\sqrt{\frac{2}{3}}\frac{c_\theta - \sqrt{2}s_\theta}{F^2} \left\{ \right. \\ \left. \left[\frac{c_m(c_m - c_d)2m_\pi^2 + c_m c_d(s + m_\pi^2 - m_\eta^2)}{M_{a_0}^2 - s - iM_{a_0}\Gamma_{a_0}(s)} - \frac{2c_m(c_m - c_d)(2m_K^2 - m_\pi^2)}{M_S^2} \right] \right. \\ \left. + \left[c_{m,d}, M_{a_0}, \Gamma_{a_0}, M_S \rightarrow c'_{m,d}, M_{a'_0}, \Gamma_{a'_0}, M_{S'} \right] \right\},$$

$$F_0^{\pi^-\eta}(s) = \sqrt{\frac{2}{3}}(c_\theta - \sqrt{2}s_\theta) + \frac{1}{\sqrt{3}}(\Lambda_1 - 2\Lambda_2)s_\theta - \frac{1}{\sqrt{3}}y_{23}(2c_\theta + \sqrt{2}s_\theta) + 4\sqrt{\frac{2}{3}}\frac{c_\theta - \sqrt{2}s_\theta}{F^2} \left\{ \right. \\ \left. \left[\frac{c_m(c_m - c_d)2m_\pi^2 + c_m c_d(s + m_\pi^2 - m_\eta^2)}{M_{a_0}^2 - s - iM_{a_0}\Gamma_{a_0}(s)} - \frac{2c_m(c_m - c_d)(2m_K^2 - m_\pi^2)}{M_S^2} \right] \right. \\ \left. + \left[c_{m,d}, M_{a_0}, \Gamma_{a_0}, M_S \rightarrow c'_{m,d}, M_{a'_0}, \Gamma_{a'_0}, M_{S'} \right] \right\},$$

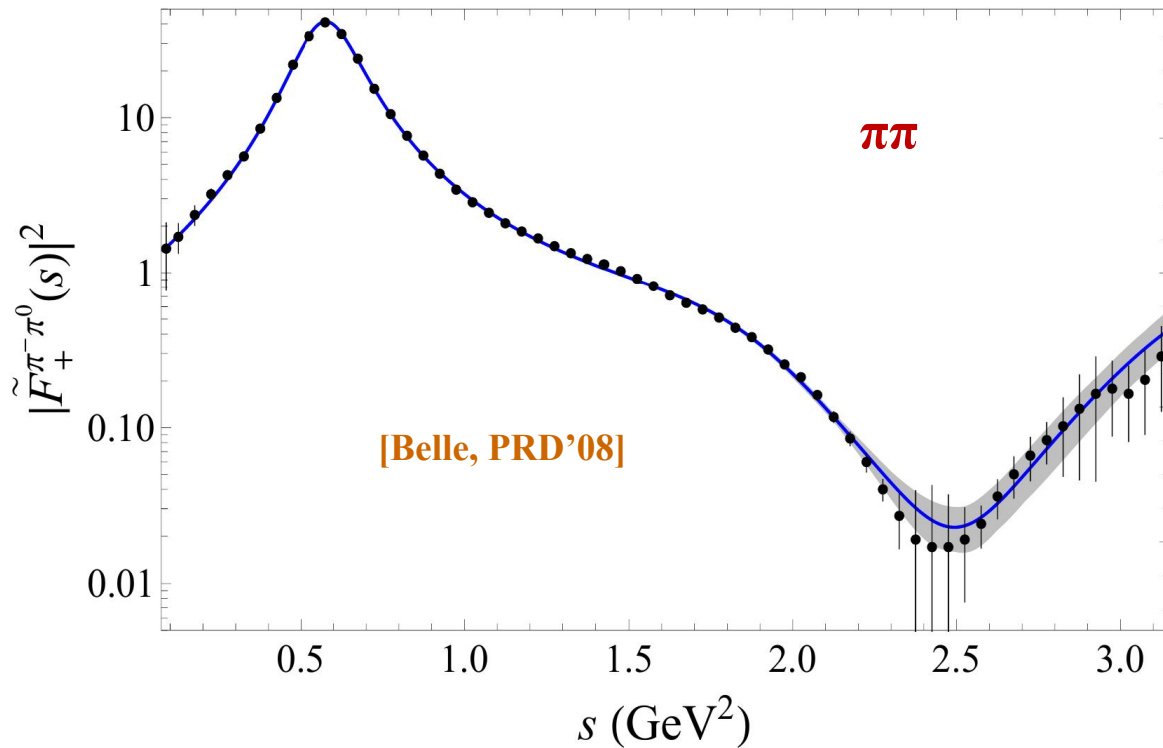
$$F_0^{\pi^-a}(s) = \frac{(\sqrt{2}c_\theta - 2s_\theta)}{\sqrt{3}}v_{24}^{(0)} + \frac{(2c_\theta + \sqrt{2}s_\theta)}{\sqrt{3}}v_{34}^{(0)} - \frac{2}{\sqrt{3}} \left(s_\theta v_{24}^{(0)} - c_\theta v_{34}^{(0)} + \frac{F}{\sqrt{6}f_a} \right) (\Lambda_2 - \frac{1}{2}\Lambda_1) \\ + \frac{(\sqrt{2}c_\theta - 2s_\theta)}{\sqrt{3}}y_{24}^{(0)} + \frac{(2c_\theta + \sqrt{2}s_\theta)}{\sqrt{3}}y_{34}^{(0)} + \frac{4(\sqrt{2}c_\theta - 2s_\theta)v_{24}^{(0)} + 4(2c_\theta + \sqrt{2}s_\theta)v_{34}^{(0)}}{\sqrt{3}F^2} \left\{ \right. \\ \left. \left[\frac{2c_m^2 m_\pi^2 + c_m c_d (s - m_a^2 - m_\pi^2)}{M_{a_0}^2 - s - iM_{a_0}\Gamma_{a_0}(s)} + \frac{2c_m(c_d - c_m)(2m_K^2 - m_\pi^2)}{M_S^2} \right] \right. \\ \left. + \left[c_{m,d}, M_{a_0}, \Gamma_{a_0}, M_S \rightarrow c'_{m,d}, M_{a'_0}, \Gamma_{a'_0}, M_{S'} \right] \right\}.$$

Fits to experimental spectra and BRs

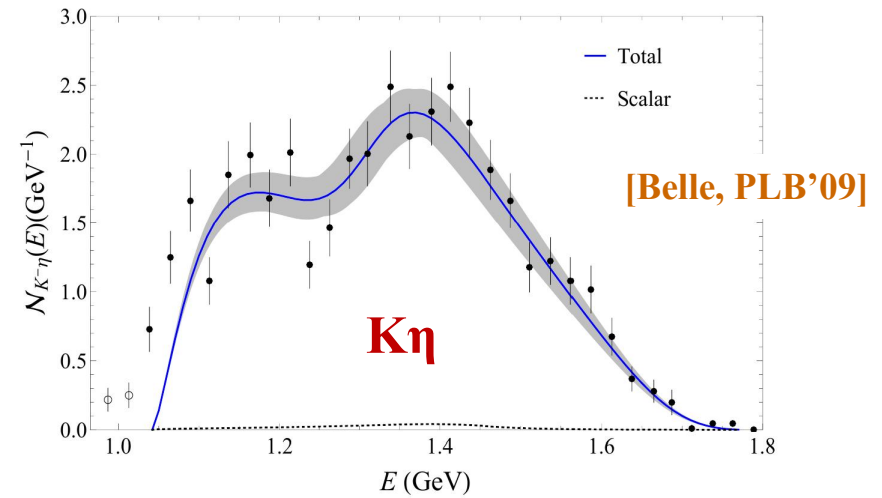
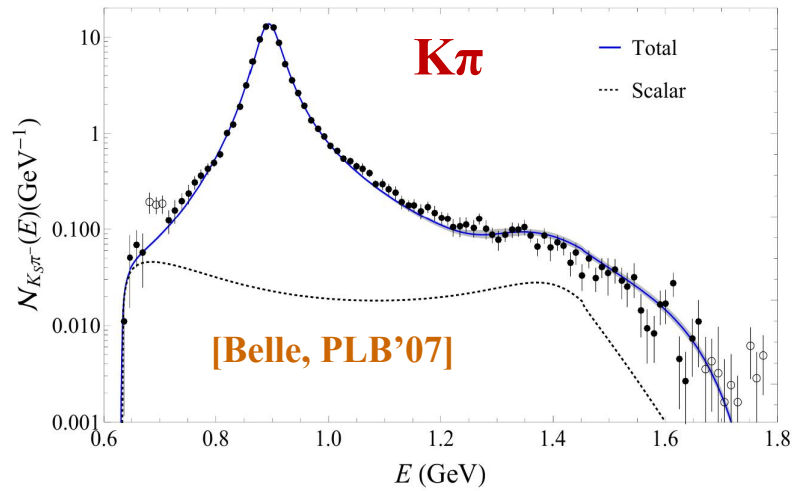
$G_V F_V(\text{GeV}^2) \times 10^3$	$10.26^{+0.01}_{-0.01}$	$G'_V F'_V(\text{GeV}^2) \times 10^3$	$0.64^{+0.03}_{-0.02}$
$G''_V F''_V(\text{GeV}^2) \times 10^3$	$-0.94^{+0.05}_{-0.05}$	$M_\rho(\text{GeV})$	$0.7738^{+0.0003}_{-0.0003}$
$M_{\rho'}(\text{GeV})$	$1.409^{+0.004}_{-0.004}$	$\Gamma_{\rho'}(\text{GeV})$	$0.338^{+0.012}_{-0.010}$
$M_{\rho''}(\text{GeV})$	$1.842^{+0.012}_{-0.013}$	$\Gamma_{\rho''}(\text{GeV})$	$0.268^{+0.025}_{-0.026}$
$c'_m(\text{GeV})$	$0.053^{+0.007}_{-0.009}$	$M_{K^*}(\text{GeV})$	$0.8956^{+0.0002}_{-0.0002}$
$\Gamma_{K^*}(\text{GeV})$	$0.0477^{+0.0005}_{-0.0005}$	$M_{K^{*'}}(\text{GeV})$	$1.339^{+0.009}_{-0.009}$
$\overline{B}_{K_S\pi^-} \times 10^3$	$3.98^{+0.04}_{-0.04}$	$\overline{B}_{K-\eta} \times 10^4$	$1.34^{+0.04}_{-0.04}$
$\chi^2/\text{d.o.f}$	$271.5/(182 - 14) = 1.61$		

$$c_m c_d + c'_m c'_d = \frac{F^2}{4} \quad c_m = 27 \text{ MeV}, c_d = 15 \text{ MeV} \quad [\text{ZHG, Oller, PRD'11}]$$

Parameters for $a_0(980)/a_0(1450)/K_0^*(700)/K_0^*(1430)$ are fixed to their pole positions .



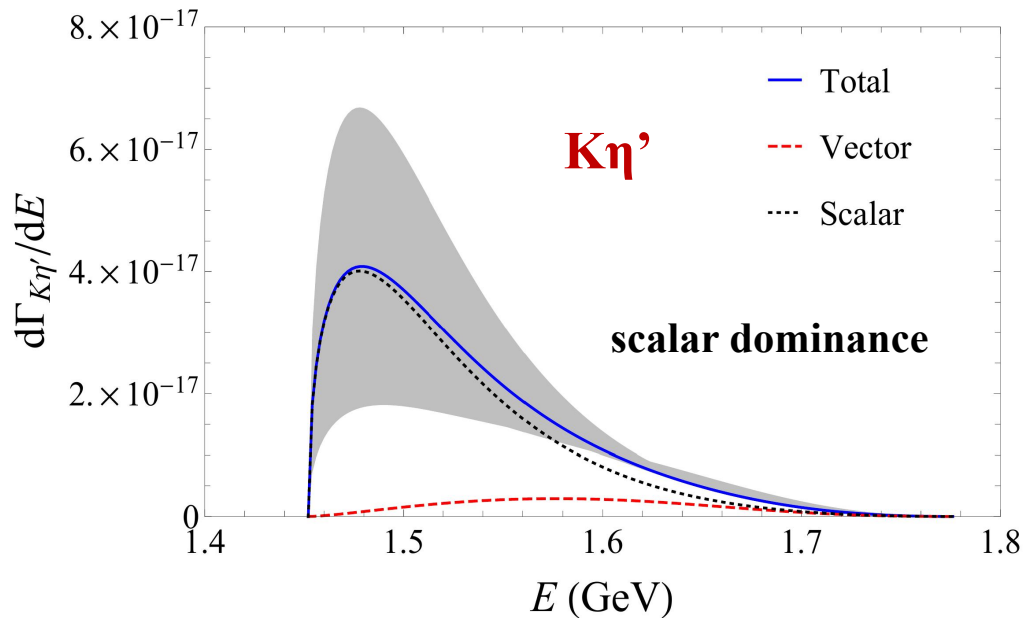
- **Crucial inputs to address muon g-2**
- **Most precise spectra is from Belle;**
but most precise BR is from ALEPH: 25.47(13)%
- **Coherent precise measurements of both spectra and BR from one Exp would be invaluable!**



$F_+^{KP}: K^*, K^*(1410)$

$F_0^{KP}: \kappa, K^*_0(1430)$

prediction to $K\eta'$



$$\text{BR}(K^- \eta')^{\text{Theo}} = (2.0 \pm 1.0) \times 10^{-6}$$

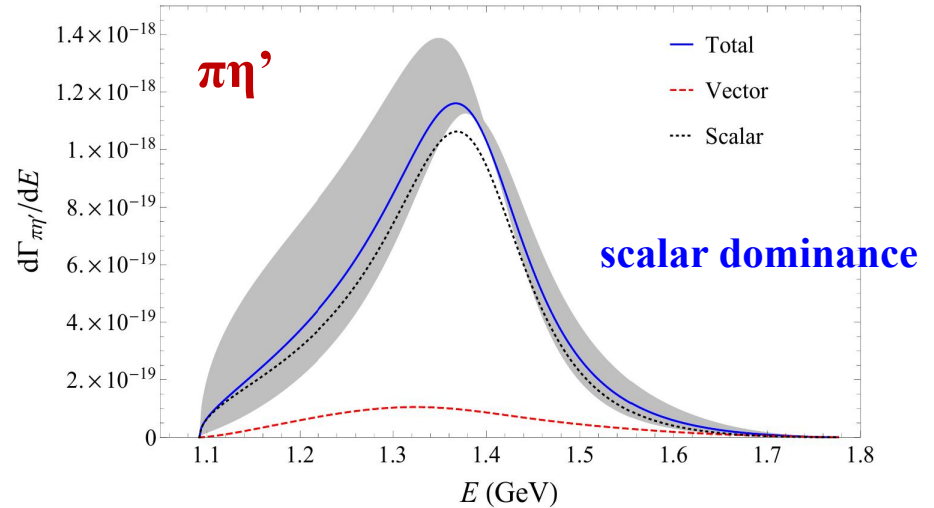
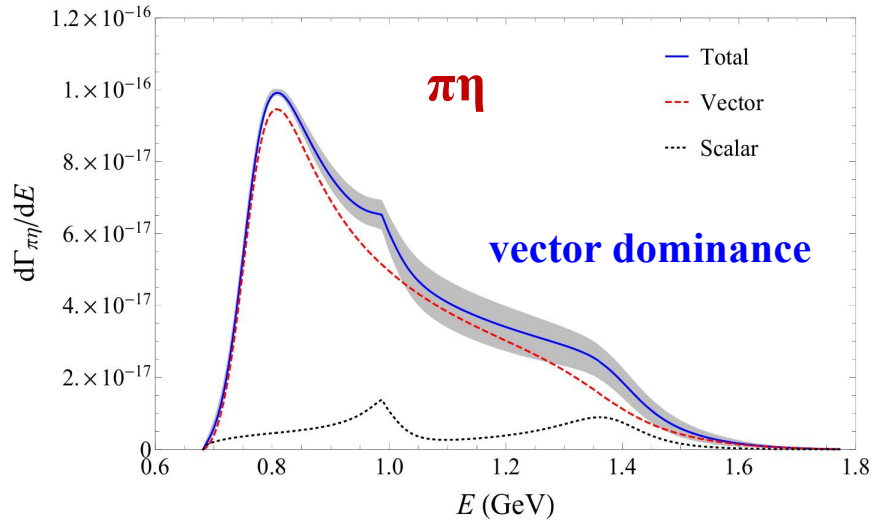
$$\text{BR}(K^- \eta')^{\text{Exp, BaBar}} < 2.4 \times 10^{-6}$$

Predictions to $\tau^- \rightarrow \pi^- \eta^{(\prime)} \nu_\tau$ (Cabibbo allowed): second-class currents

若 $\pi\eta$ 的 $J=0$ ，则其 $P=+1$ （V-A型的流不允许）；若 $J=1$ ，则 $P=-1$ （矢量流）

但是对于SM来讲，1-矢量流对应的G宇称为正，而 $\pi\eta$ 的G宇称为负，表明这是一个破坏G宇称的过程（second-class current），因此可能是寻找新物理的一个有效途径。

$$\langle \pi^- P | \bar{d} \gamma^\mu u | 0 \rangle = \left[(p_P - p_\pi)^\mu - \frac{\Delta_{P\pi}}{s} q^\mu \right] F_+^{\pi^- P}(s) + \frac{\Delta_{du}^{\text{Phy}}}{s} q^\mu F_0^{\pi^- P}(s)$$

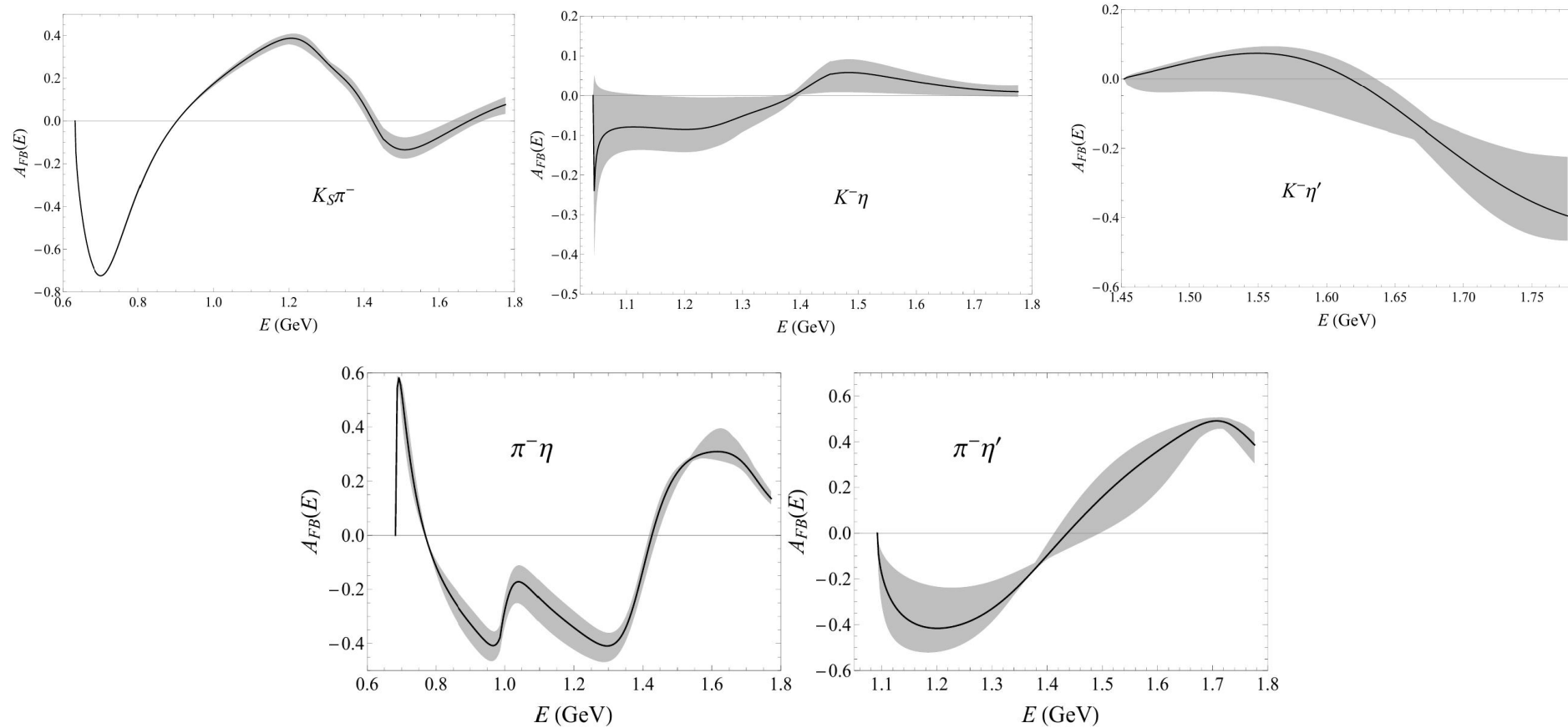


Channel	Total	Vector	Scalar	Exp Limits
$\tau^- \rightarrow \pi^- \eta \nu_\tau$ ($\times 10^5$)	$1.63^{+0.14}_{-0.14}$	$1.43^{+0.18}_{-0.21}$	$0.20^{+0.07}_{-0.04}$	< 9.9 (BaBar) [69] < 7.3 (Belle) [70]
$\tau^- \rightarrow \pi^- \eta' \nu_\tau$ ($\times 10^7$)	$1.17^{+0.36}_{-0.07}$	$0.14^{+0.09}_{-0.08}$	$1.03^{+0.44}_{-0.16}$	< 40 (BaBar) [71]

[Hao, Duan, ZHG, 2507.00383]

Predictions to Forward-Backward asymmetries

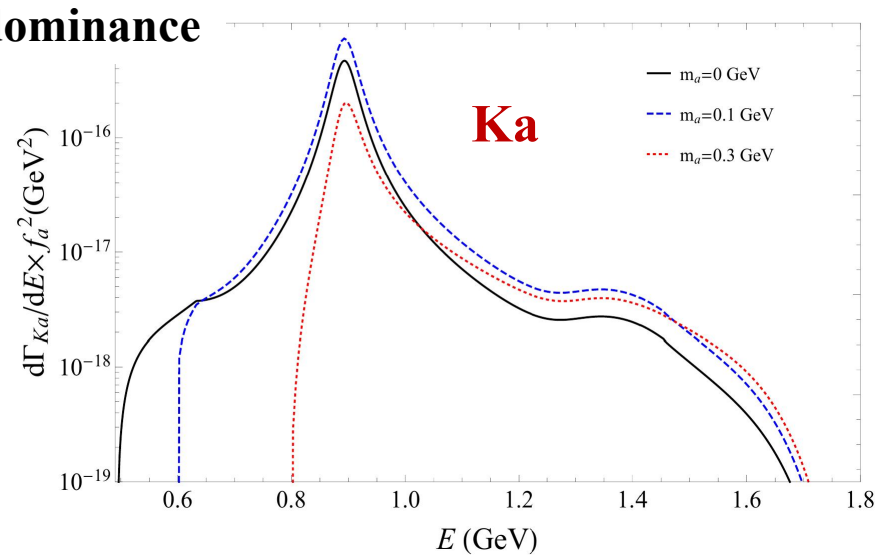
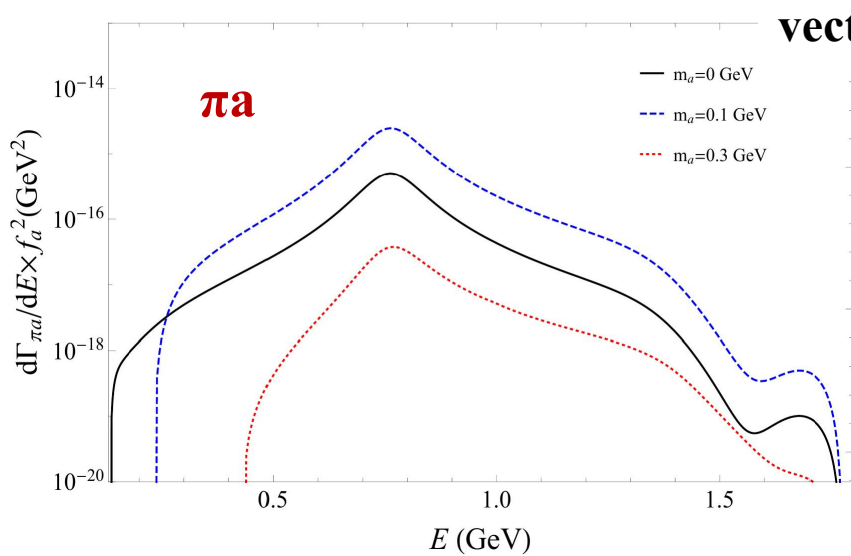
$$\frac{\Delta_{Du} q_{P_1 P_2}(s) \Re \left[F_+^{P_1 P_2}(s) \hat{F}_0^{P_1 P_2*}(s) \right]}{\frac{2\sqrt{s}}{3} \left(1 + \frac{2s}{M_\tau^2} \right) q_{P_1 P_2}^2(s) \left| F_+^{P_1 P_2}(s) \right|^2 + \frac{\Delta_{Du}^2}{2\sqrt{s}} \left| \hat{F}_0^{P_1 P_2}(s) \right|^2}$$



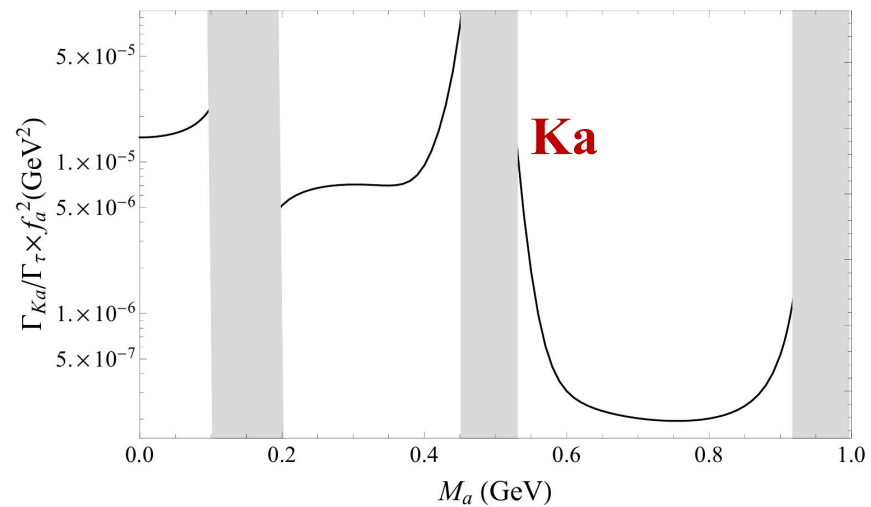
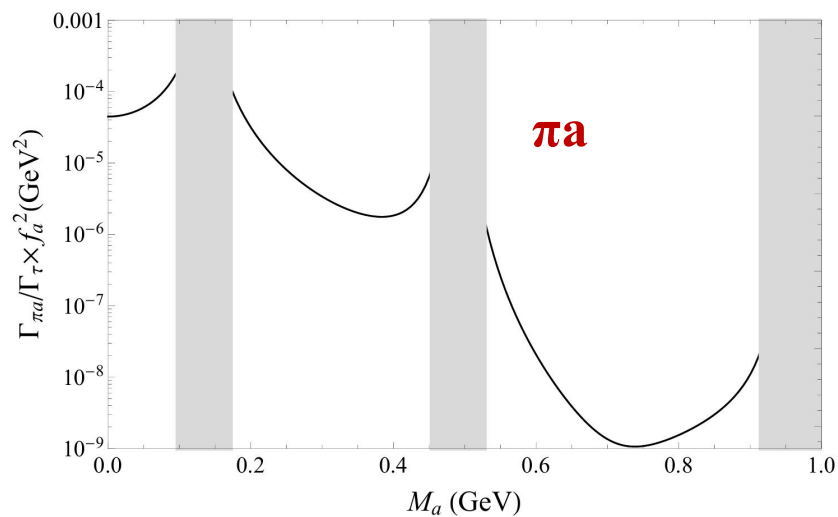
➤ Measurement on A_{FB} can determine the crucial inputs for CPV

Predictions to axion-meson production in tau decays

Spectra



BRs



结语 **Tau**的两体玻色子衰变包含丰富有趣的物理:

- $\tau^- \rightarrow \nu_\tau \pi^0 \pi^-$ [为muon g-2提供关键输入]
- $\tau^- \rightarrow \nu_\tau (K\pi, K\eta, K\eta')^-$ [V_{us} , 奇异轻强子态, 可能的CPV现象]
- 第二类流主导的过程: $\tau^- \rightarrow \nu_\tau \pi^- \eta/\eta'$
- τ 衰变中的前后不对称性的测量
- $\tau^- \rightarrow \nu_\tau a K^-/\pi^-$ 衰变的可能性
-

谢谢大家!