从第一性原理到重子光锥分布振幅

PRD 111.034510(2025), arXiv:2508.08971

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Cor & 王伟、白颢阳、张沐华、张家璐 …

重味物理前沿论坛

2025 09/14 @华中师范大学

OUT LINE

Motivation About Meson & Baryon

- **Meson — ubiquitous messengers**

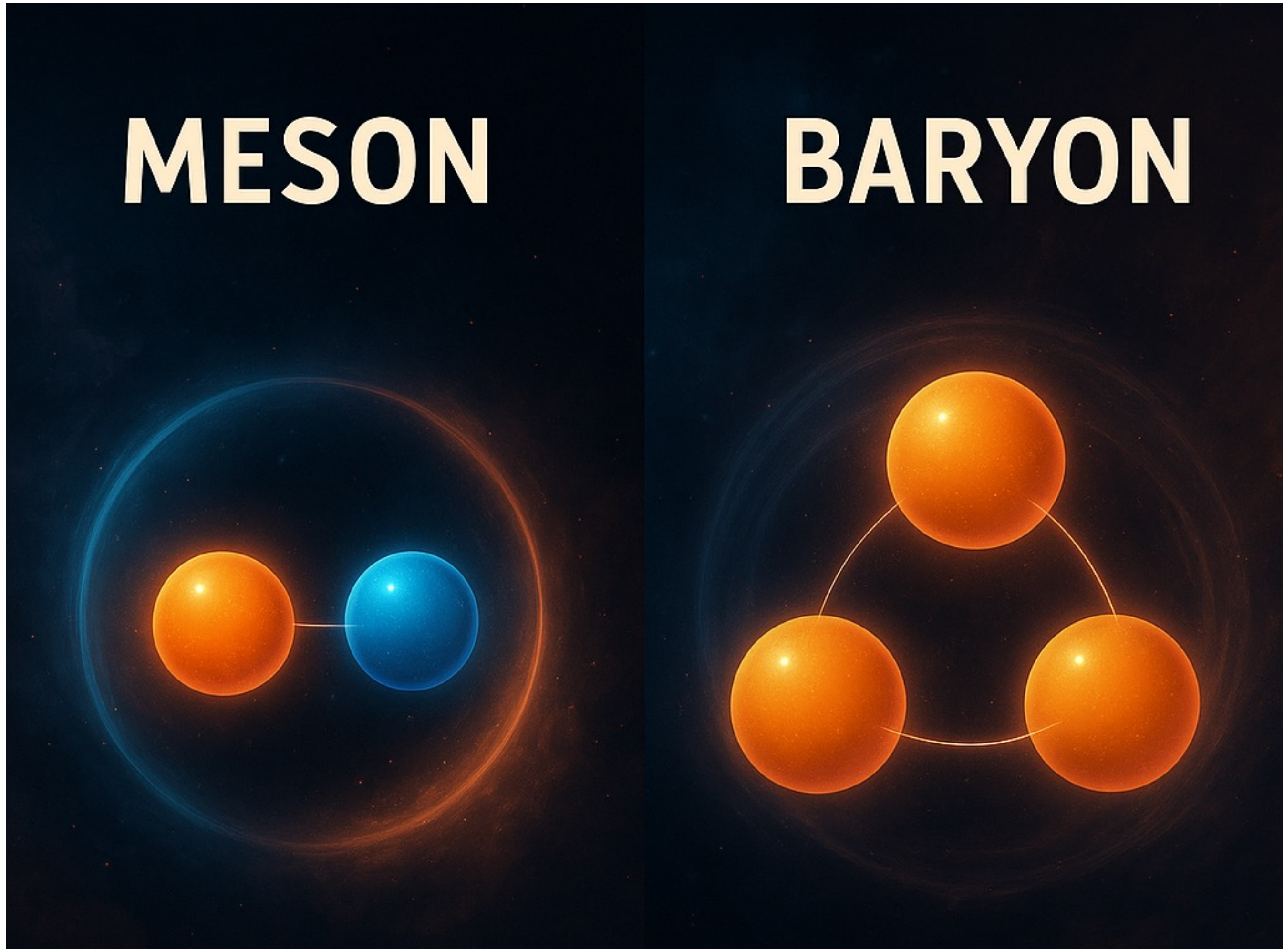
  Light Meson $SU(3)_{flavor}$ $8 \otimes 1$

  Heavy Meson $SU(4)_{flavor}$ …

- **Baryon — cornerstone particles**

  Octet baryons

  Decuplet baryons



Motivation About Meson & Baryon

Meson and Baryon in Flavor Physics

- 1956, Parity violation in weak interaction
- 1964, Observation of CP violation in **Kaon meson**
- 1973, Kobayashi-Maskawa mechanism
- 2004, Observation of direct CPV in **B meson**
- 2019, Observation of direct CPV in **D meson**
- 2025, Observation of direct CPV in **Baryon $\Lambda_b^0 \to p$**

↓

physicsworld
TOP 10
BREAKTHROUGH
2019

LHCb, Nature (2025)； Theoretical: *J.J.Han, et.al. PRL 134 221801(2025)*

Motivation About Meson & Baryon

Meson and Baryon in Flavor Physics

- New physics beyond the SM (LHCb, Belle II)

PTEP

Prog. Theor. Exp. Phys. **2019**, 123C01 (654 pages)
DOI: 10.1093/ptep/ptz106

The Belle II Physics Book

2.3. Flavor physics questions to be addressed by Belle II

Further study of the quark sector is necessary to reveal NP at high mass scales, even beyond the direct reach of the LHC, that may manifest in flavor observables. There are several important questions that can only be addressed by further studies of flavor physics, as described below. Belle II will access a large number of new observables to test for NP in flavor transitions in the quark and lepton sectors.

- *Does nature have a left–right symmetry, and are there flavor-changing neutral currents beyond the SM?* Approaches include measurements of time-dependent CP violation in $B \to K^{*0}(\to K_S^0\pi^0)\gamma$, triple-product CP violation asymmetries in $B \to VV$ decays, and semileptonic decays

- FCNC: $b \to sl^+l^-$
  - $B \to K^{*0}(\to K_S^0\pi^0)\gamma$
  - $\Lambda_b \to \Lambda(\to p\pi)l^+l^-;\ \Lambda_b \to \Lambda\gamma$

  LHCb JHEP 09 146(2018)
  L.S.Lu, C.D.Lv, Y.L.Shen & Y.B.Wei arXiv:2506.21419

- Nucleon Structure:
  - Nucleon electromagnetic form factors

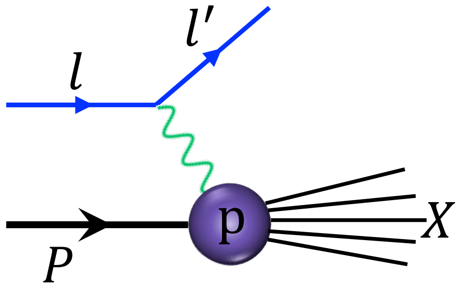
  L.B.Chen, et.al. arXiv:2406,19994

Motivation About LCDA

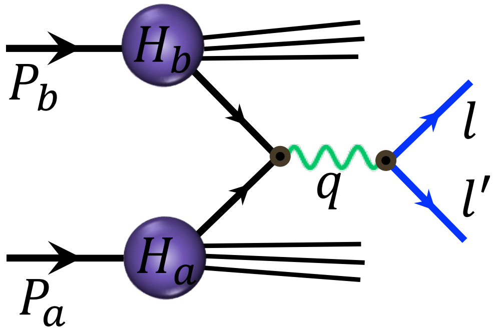
- One try to probe internal structure of nucleons → **Inclusive beam-target collision**

Defined PDFs

DIS



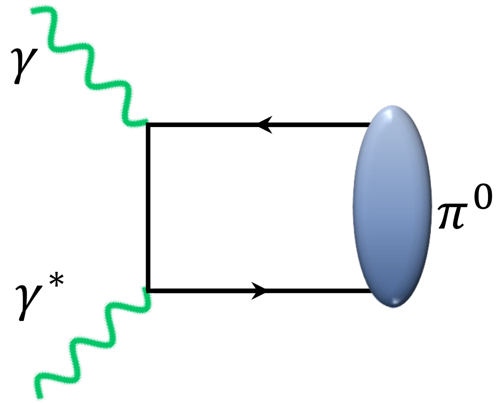
Drell-Yan



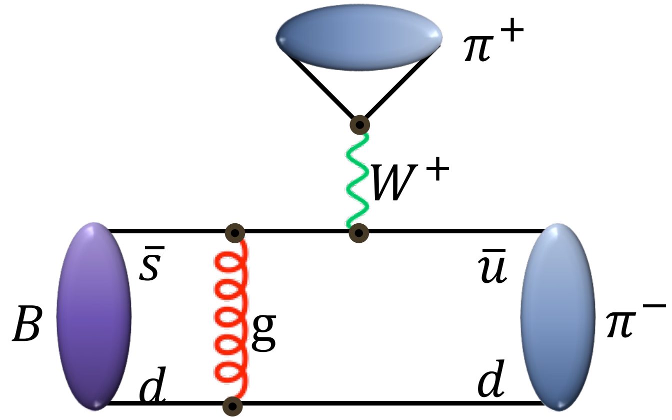
- One try to obtain richer QCD dynamical information → **Exclusive scattering**

Defined LCDAs

$\gamma\gamma^* \to \pi$



$B \to \pi^+\pi^-$



Motivation About LCDA

Light Meson LCDAs: (1977 - now)

- Asymptotic form

  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; ……
- QCD Sum rules

  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; ……
- Dyson-Schwinger Equation

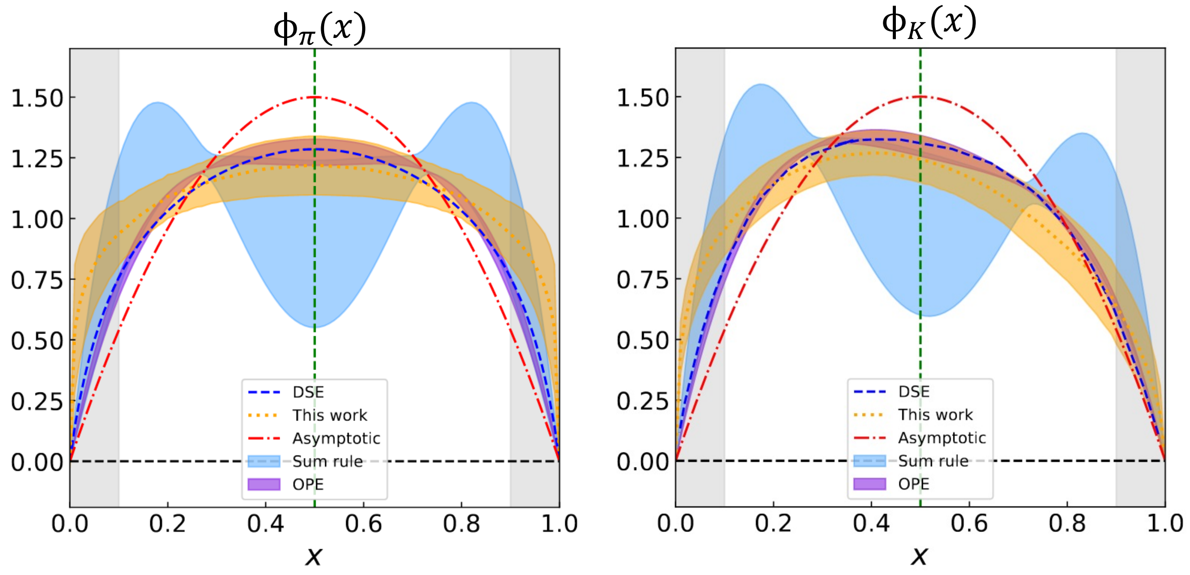
  Chang, Cloet, et.al, 2013; Gao, Chang, et.al, 2014; ……
- Global Fits

  Cheng, et.al, 2020; Hua, Li, Lu, Wang, Xing, 2021; ……
- Models

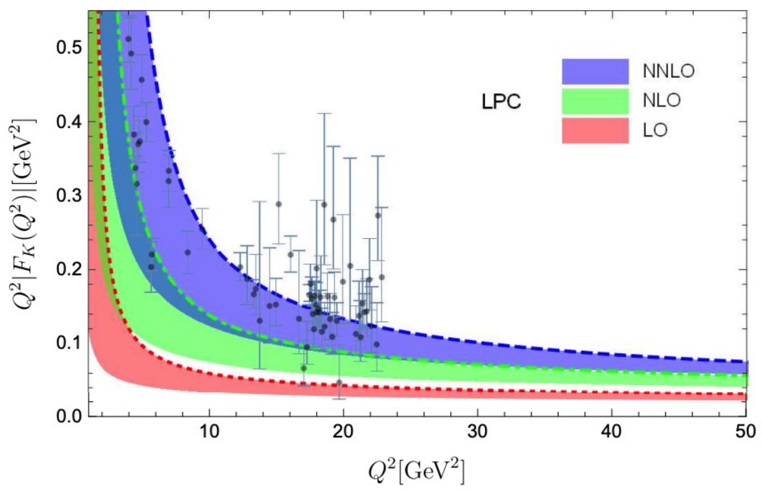
  Arriola, Broniowski, 2002; Zhong, Zhu, et.al, 2021; ……
- Lattice QCD with OPE

  Braun, et al., 2016; RQCD collaboration, 2019, 2020; ……
- Lattice QCD with LaMET

  LP3, 2019; LPC, 2021, 2022; ……



Application:



L.B.Chen, W.Chen, F.Feng & Y.Jia. PRL 132 201901, arXiv:2407,21120

Motivation About LCDA

Light Baryon LCDAs: (1983 - now)

- Asymptotic form

  Chernyak, Zhitnitsky, 1983 ; ……
- QCD Sum rules

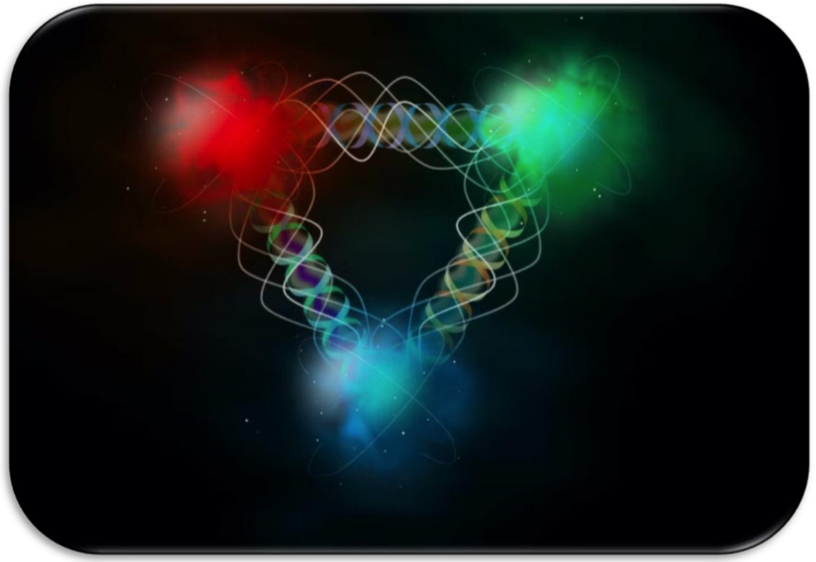
  King, Sachrajda,1987; Stefanis, Bergmann, 1993; Braun, et.al, 2000,2006; ……
- Models parametrization

  Bell, Feldmann, Wang, Matthew 2013; ……
- Lattice QCD with OPE

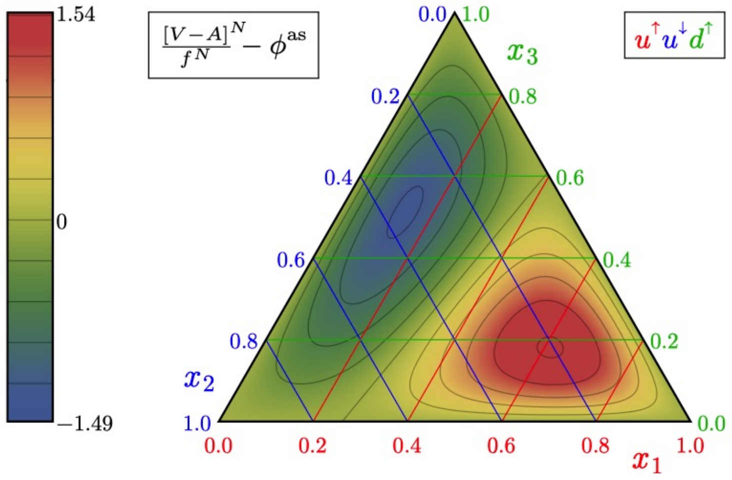
  QCDSF, 2008, 2009; RQCD, 2016, 2019(2025) ……
- Lattice QCD with LaMET

  LPC, 2024,2025; ……

  PRD 111, 034510, arXiv:2508.08971



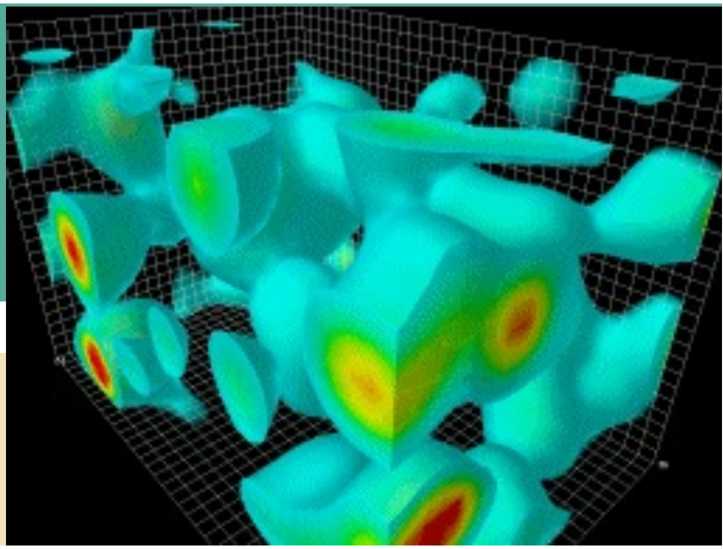
More is Different



Motivation Moments & LaMET

Lattice QCD formulate a Feynman path integral on discrete 4D Euclidean grid.

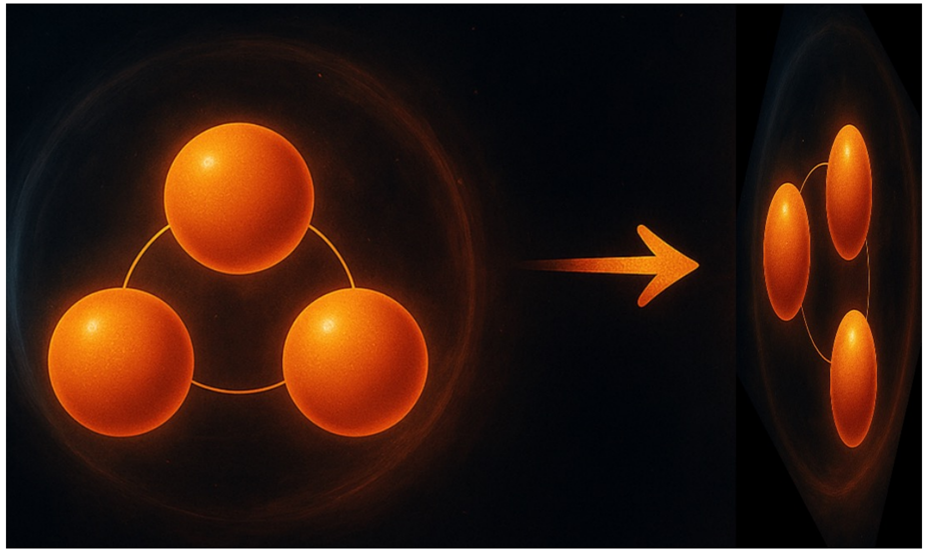
Numerical simulations based on a QCD Lagrangian with discrete from:



$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} \quad \xrightarrow{t = i\tau} \quad \mathcal{L}^E = \bar{\psi}(i\gamma^\mu D_\mu + m)\psi + \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu}$$

$$S_E^{\text{latt}} = -\sum_{\square} \frac{6}{g^2} \text{Retr}_N \left(U_{\square,\mu\nu}\right) - \sum_q \bar{q} \left(D_\mu^{\text{lat}} \gamma_\mu + a m_q\right) q$$

- Lost of real-time correlation !
- Longitudinal correlations compress to a point



Motivation Moments & LaMET

Operator Product Expansion (OPE): the local moments are consistent between light-cone coordinate or Euclidean space

Limited for only few moments:

- First two moments for light mesons
- First moments for light baryons

Inverse problem from moments to LCDA/PDF

Light meson:

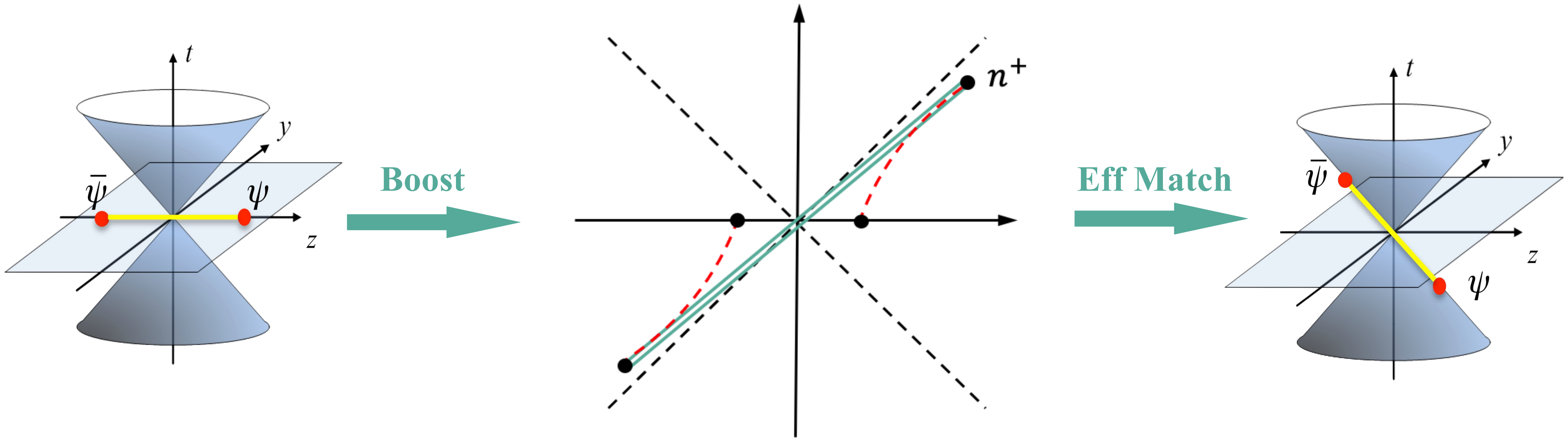
Braun, et al., 2016; RQCD collaboration, 2019, 2020; ……

Light baryon:

QCDSF, 2008, 2009; RQCD, 2016, 2019(2025) ……

Motivation Moments & LaMET

Large momentum effective theory (LaMET): the light-cone non-local operator correlated to Euclidean non-local operator with a large momentum boost



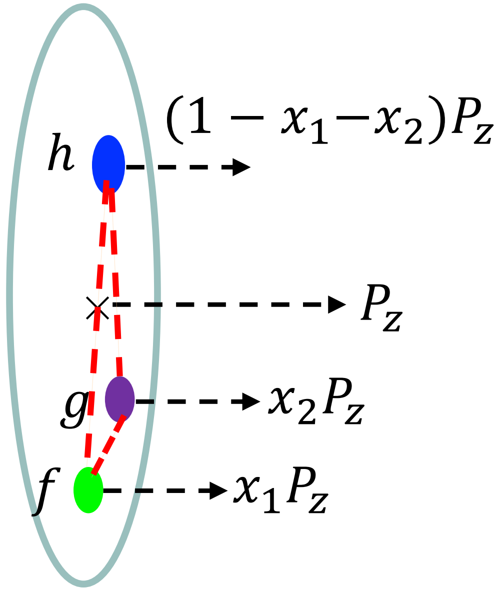
LaMET factorization:

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C(x, y, P^z, \mu)\, q(y, \mu) + \mathcal{O}(\frac{\Lambda^2, M^2}{(P^z)^2})$$

- Light meson: LP3, 2019; LPC, 2021, 2022; ANL,2024; ……
- Heavy meson: LPC, 2024,2025; ……
- Light baryon: LPC, 2025; ……

Baryon LCDA on Lattice LCDA

- **Definition of baryon LCDA:**



$$M_L(\xi_1,\xi_2;P)=\langle 0|\epsilon^{ijk} f_\alpha^{i'}(\xi_1 n)W_{i'i}(\xi_1,\xi_0)g_\beta^{j'}(\xi_2 n)W_{j'j}(\xi_2,\xi_0)h_\gamma^k(\xi_0 n)|B(P,\lambda)\rangle$$

$$\Phi(x_1,x_2,\mu)=\int\frac{\mathrm{d}P^+\xi_1}{2\pi}\frac{\mathrm{d}P^+\xi_2}{2\pi}\mathrm{e}^{\mathrm{i}x_1P^+\xi_1+\mathrm{i}x_2P^+\xi_2}\frac{M_L(\xi_1,\xi_2;P,\mu)}{M_L(0,0;P,\mu)}$$

C.Han et.al. JHEP 07019 (2024);
V.L.C & I.R.Z NPB 24652(1984); G.R.Farrar et.al. NPB 311585(1989)

- **Leading twist octet baryon LCDA:**

Octet	n	p	Λ
fgh	d d u	u u d	u d s

$$\langle 0|f_\alpha(z_1n)g_\beta(z_2n)h_\gamma(z_3n)|B(P_B,\lambda)\rangle$$
$$=\frac{1}{4}f_V\left[(\not P_BC)_{\alpha\beta}(\gamma_5u_B)_\gamma V^B(z_in\cdot P_B)+(\not P_B\gamma_5C)_{\alpha\beta}(u_B)_\gamma A^B(z_in\cdot P_B)\right]$$
$$+\frac{1}{4}f_T(i\sigma_{\mu\nu}P_B^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5u_B)_\gamma T^B(z_in\cdot P_B),$$

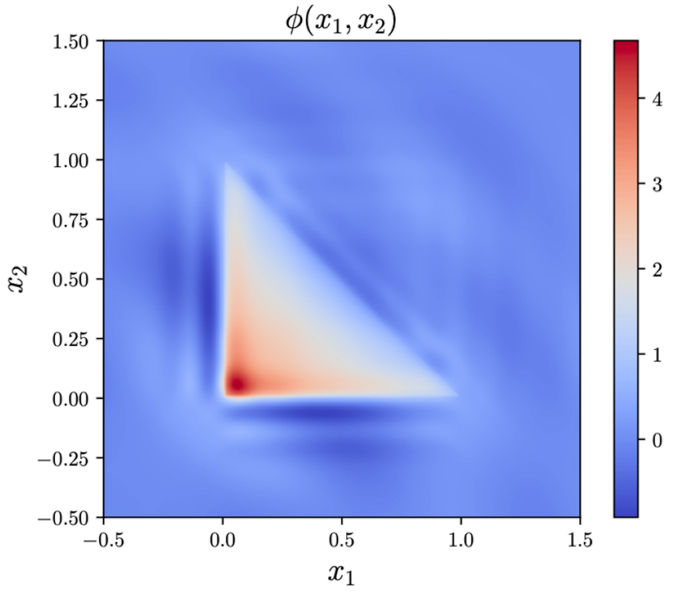
Baryon LCDA on Lattice LaMET

- **Determining Baryon LCDA on Lattice**

Quasi-DA 2pt in E space → **Renormaliza-tion** → **Fourier Transform** → **Effective Matching**

- **Improvement**

Lattice setup Dynamic enhance	Self renorm Hybrid scheme	Inverse Approach	High dimension Matching



- **Definition Baryon Quasi-DA on Euclidean lattice:**

$$M(z_1, z_2; P^z) = \langle 0|\epsilon^{ijk} f_\alpha^{i'}(z_1) W_{i'i}(z_1, 0) g_\beta^{j'}(z_2) W_{j'j}(z_2, 0) h_\gamma^k(0) |B(P^z, \lambda)\rangle$$

$$\tilde{\Phi}(x_1, x_2, P^z, \mu) = (P^z)^2 \int \frac{\mathrm{d}z_1}{2\pi} \frac{\mathrm{d}z_2}{2\pi} \mathrm{e}^{-x_1 P^z z_1 - x_2 P^z z_2} \frac{M(z_1, z_2; P^z, \mu)}{M(0, 0; P^z, \mu)}$$

- **The Leading twist (V,A,T terms) for octet baryon:**

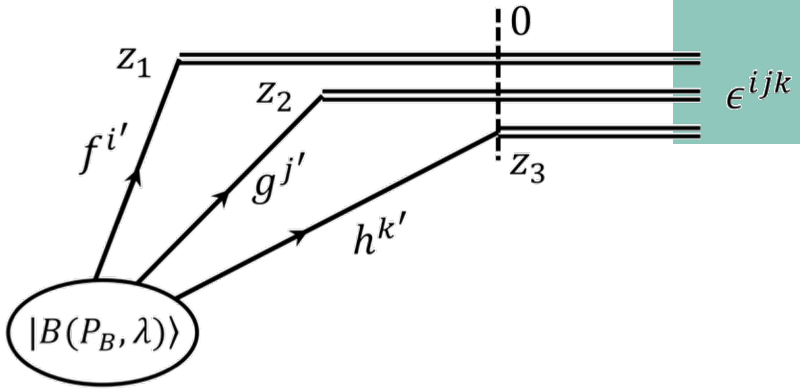
$$\left\langle 0 \left| f^T(z_1 n)(C\not{n}) g(z_2 n) h(z_3 n) \right| B \right\rangle = -f_V V^B(z_i n \cdot P_B) P_B^+ \gamma_5 u_B,$$

LPC, PRD 111, 034510 (2025) — Λ

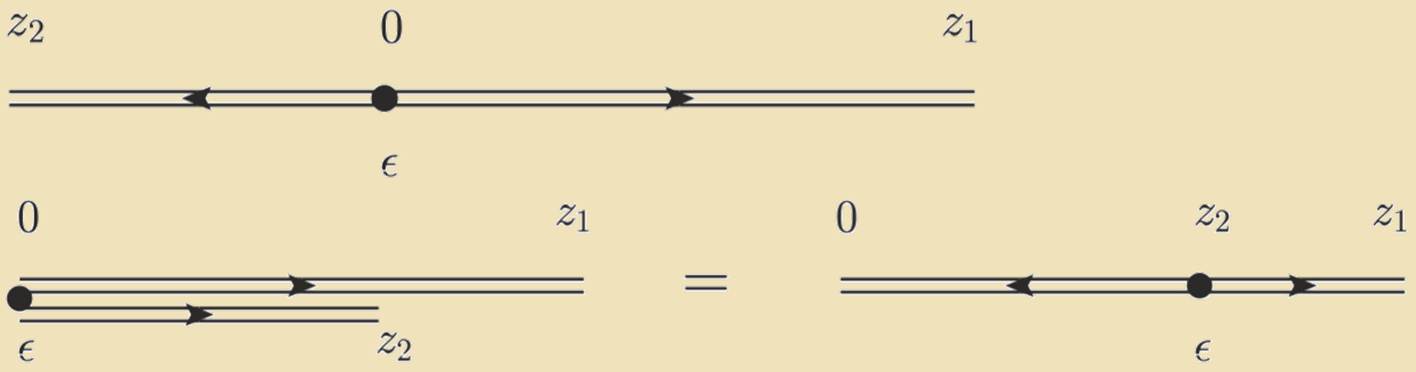
$$\left\langle 0 \left| f^T(z_1 n)(C\gamma_5\not{n}) g(z_2 n) h(z_3 n) \right| B \right\rangle = f_V A^B(z_i n \cdot P_B) P_B^+ u_B,$$

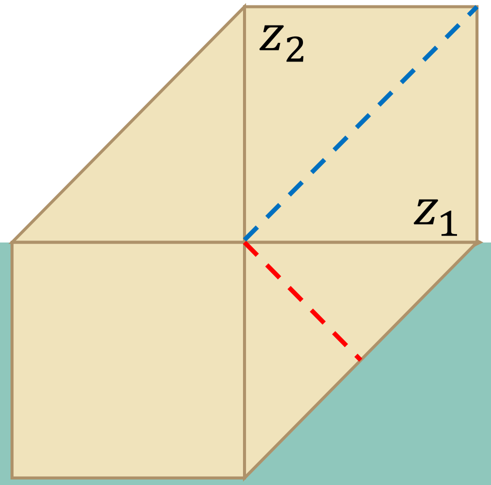
$$\left\langle 0 \left| f^T(z_1 n)(iC\sigma_{\mu\nu} n^\nu) g(z_2 n) \gamma^\mu h(z_3 n) \right| B \right\rangle = 2 f_T T^B(z_i n \cdot P_B) P_B^+ \gamma_5 u_B,$$

- **Nonlocal three-quark operators at light-like separations:**



$$H(z_1,z_2,z_3)_{\alpha\beta\gamma} = \epsilon^{ijk}\langle 0|f_\alpha^{i'}(z_1 n)W^{i'i}(z_1 n, z_0 n)\times g_\beta^{j'}(z_2 n)W^{j'j}(z_2 n, z_0 n) \times h_\gamma^{k'}(z_3 n)W^{k'k}(z_3 n, z_0 n)\,|B(P_B,\lambda)\rangle$$



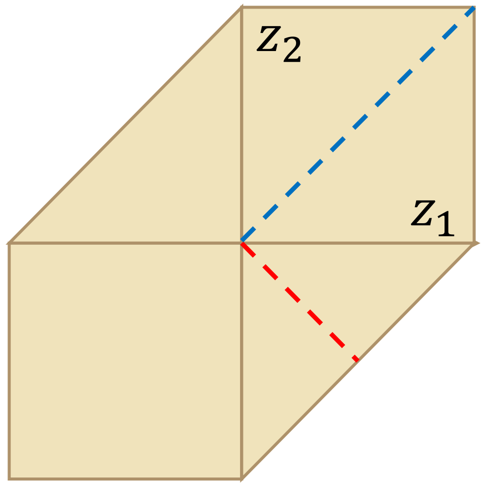


- **Effective length of Wilson Line** (SU(3) unitary, invariant symbol)

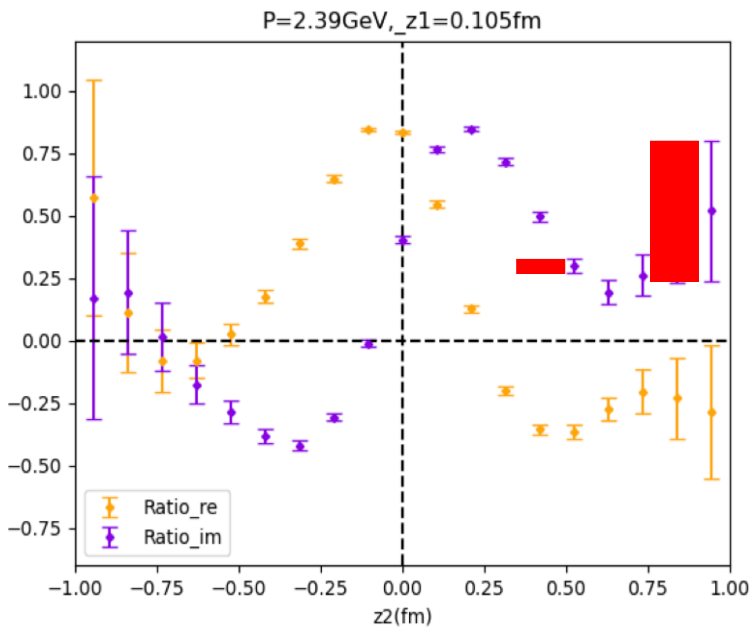
$$\tilde{z} = \begin{cases} |z_1 - z_2|, & z_1 z_2 < 0 \\ \max(|z_1|,|z_2|), & z_1 z_2 \geq 0. \end{cases}$$

C.Han et.al. JHEP 12044 (2023); LPC, PRD 111, 034510 (2025)

- More Symmetries
- Hard for Lattice simulation



- **From effective Wilson length → Signal to Noise Ratio**



- Signal to Noise Ratio * 8
- Statistics * 64

基于不同翻倍周期，以下是性能提升64倍所需的时间和目标日期：

— By Deep Seek

翻倍周期	每次翻倍所需时间	翻倍6次所需总时间	目标日期（从2025–07–19计算）	说明
乐观场景	18个月（1.5年）	6 × 1.5 = 9 年	2034年07月19日	基于摩尔定律的经典假设（快速技术进步）。
基准场景	24个月（2年）	6 × 2 = 12 年	2037年07月19日	当前较常见的估计（平衡模型）。
保守场景	36个月（3年）	6 × 3 = 18 年	2043年07月19日	近年趋势（摩尔定律明显放缓）。

How about 64*20 ?

- **Based on CLQCD Ensembles**
  - ☐ Three lattice spacing for $a \to 0$ limit
  - ☐ Three momentum for $P_z \to \infty$ limit

Ensemble	Volume	Lattice spacing	π mass	measurement	P^z
C24P29	$24^3 \times 72$	0.105 fm	293 MeV	864*4*8	1.96, 2.45, 2.94, 3.43 GeV
F32P30	$32^3 \times 96$	0.077 fm	303 MeV	777*4*8	1.99, 2.50, 2.99, 3.49 GeV
G36P29	36^3x108	0.068 fm	295MeV	656*6*8	2.01, 2.53, 3.03, 3.54 GeV
H48P32	$48^3 \times 144$	0.052 fm	317 MeV	550*6*8	1.99, 2.48, 2.98, 3.48GeV

- **Two point correlation of baryon**

$$C_2(z_a, z_2; t, \vec{P}) = \int \mathrm{d}^3x e^{-\mathrm{i}\vec{P}\cdot\vec{x}} \langle 0| \mathcal{O}_{\mathrm{Sink}}(\vec{x}, t; z_1, z_2) \bar{\mathcal{O}}_{\mathrm{Src}}(0, 0; 0, 0) \mathcal{T} |0\rangle$$

Determined by DA   Up to choice

- ☐ Enhanced interpolator $\bar{O}_{src}$: $N_{\gamma 5} = \epsilon^{ijk}(f_i^T C\gamma_5 g_j)h_k \rightarrow \epsilon^{ijk}(f_i^T C\gamma_5 \gamma_\mu g_j)h_k$
- ☐ Enhanced Projection: $\frac{1+\gamma_t}{2} \rightarrow more$

Inspired by R.Zhang et.al, arXiv: :2501.00729 ;

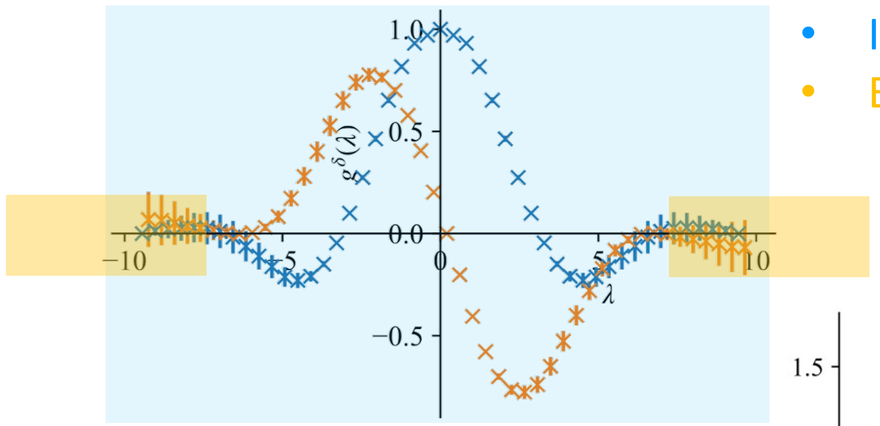
- **Dynamic enhance:** **SNR 200% improve (eff 400%) !**
- **Coding in Pyquda:** **Computing efficiency 800% !**

Thanks for X.Y.Jiang, arXiv: 2411.08461

Same Compute, Superior Performance !

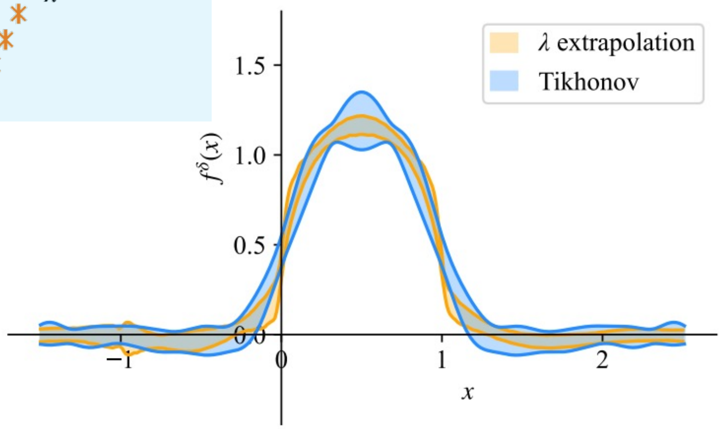
- **Quasi-DA in momentum space related to matrix elements in coordinate space with a limited Fourier transform**

$$\tilde{\psi}(x,\mu) \equiv \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{ix\lambda} \tilde{h}(\lambda,\mu) \quad \rightarrow \quad \tilde{\psi}(x,\mu) \equiv \int_{-\lambda_{cut}}^{\lambda_{cut}} \frac{d\lambda}{2\pi} e^{ix\lambda} \tilde{h}(\lambda,\mu)$$



Inverse problem ?

H.Dutrieux et.al, arXiv: :2504.17706;
H.Dutrieux et.al, arXiv: 2506.24037;
J.W.Chen, J.Hua, et.al. arXiv:2505.14619; — can be circumvented
A.S.Xiong , J.Hua, et.al. arXiv:2506.16689; — can be addressed

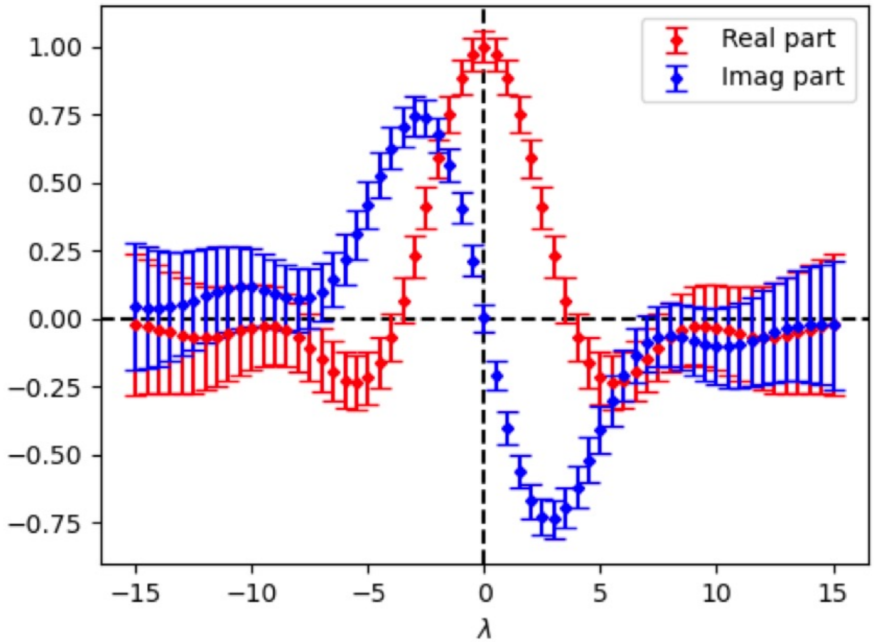


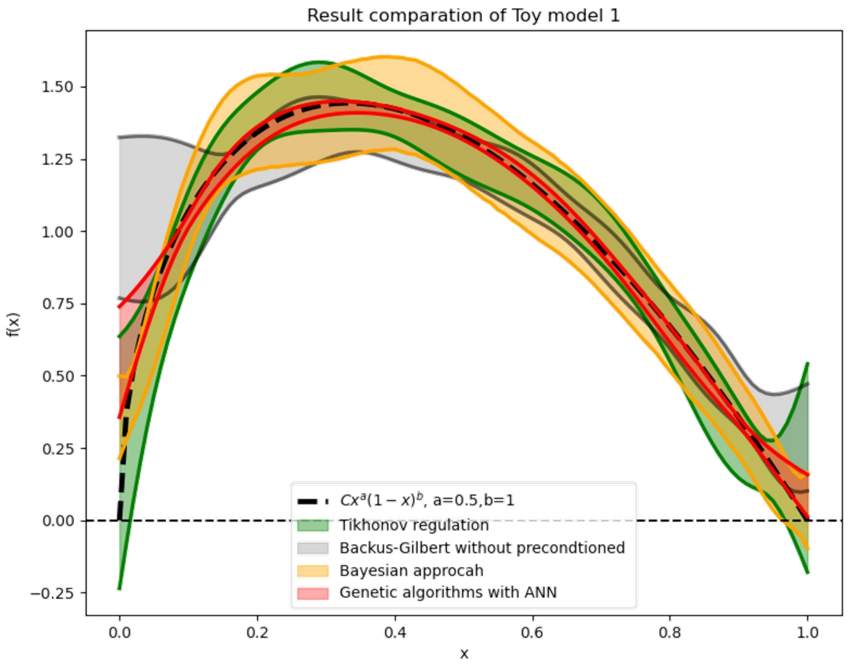
Reduce SNR requirement at large z (200%, expected)

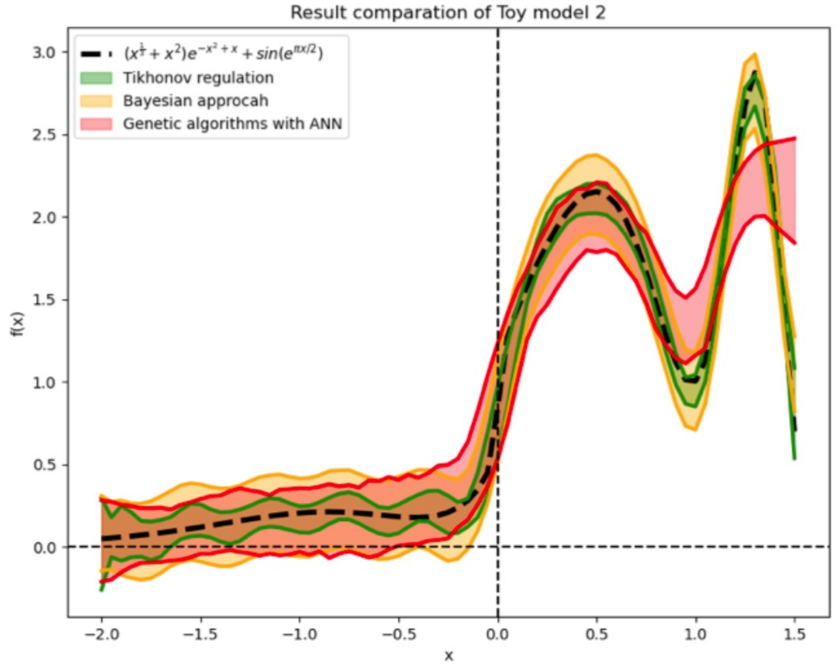
- **Baryon LCDA is more complicated 2D FT**

More alternatives: — *arXiv:25XXX*

- **Tikhonov Regularization**
- **Backus-Gilbert Method**
- **Bayesian Approach**
- **Genetic Algorithms with ANNs**

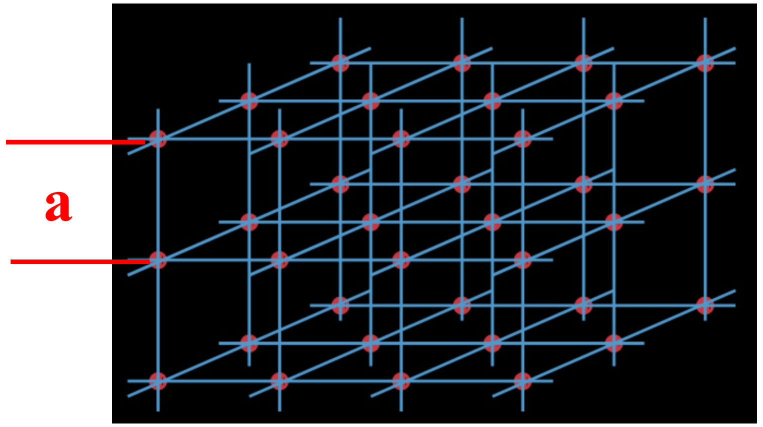




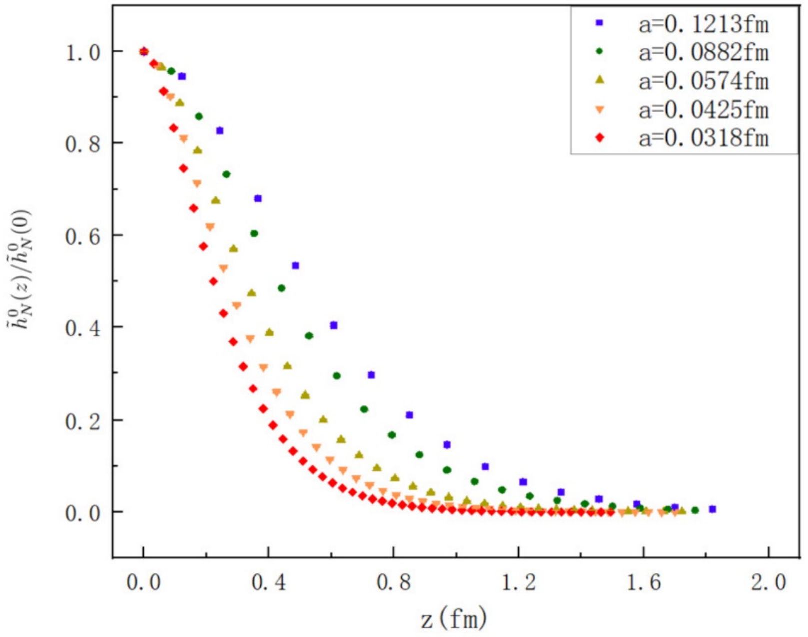


- ## Renormalization on Lattice

$$\frac{1}{a}\,cut \quad \rightarrow \quad \overline{\mathrm{MS}}$$



- ## Linear divergence on Lattice



- **RI/MOM Scheme**

  Alexandrou et.al, NPB 2017, Stewart, Zhao, PRD 2018

- **Ratio Scheme**

  Radyushkin et.al, PRD 2017

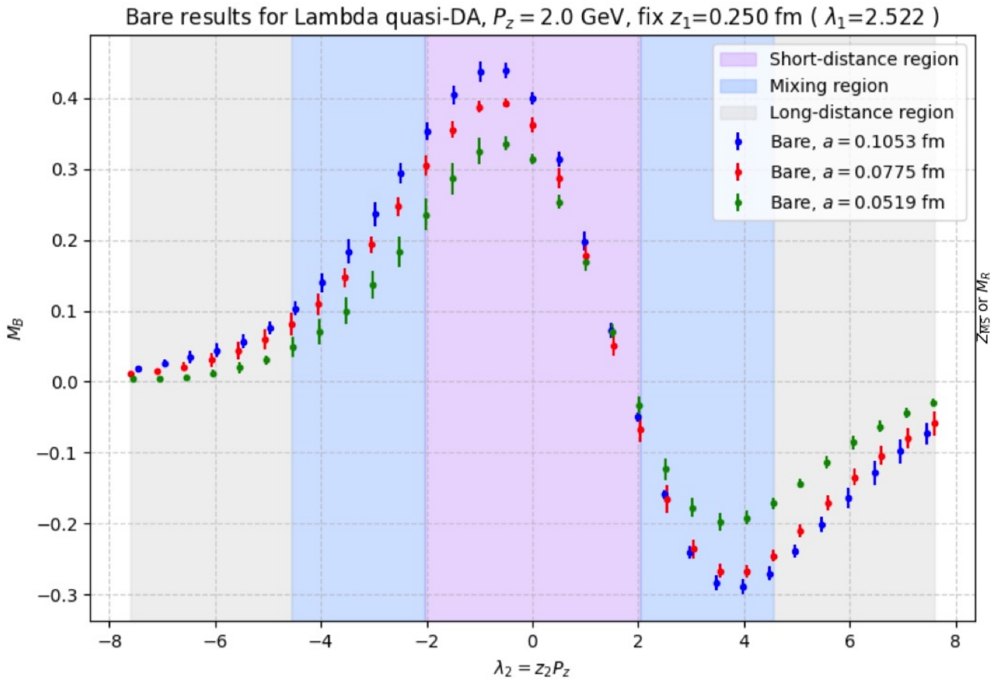
- **Hybrid Scheme and Self Renormalization**

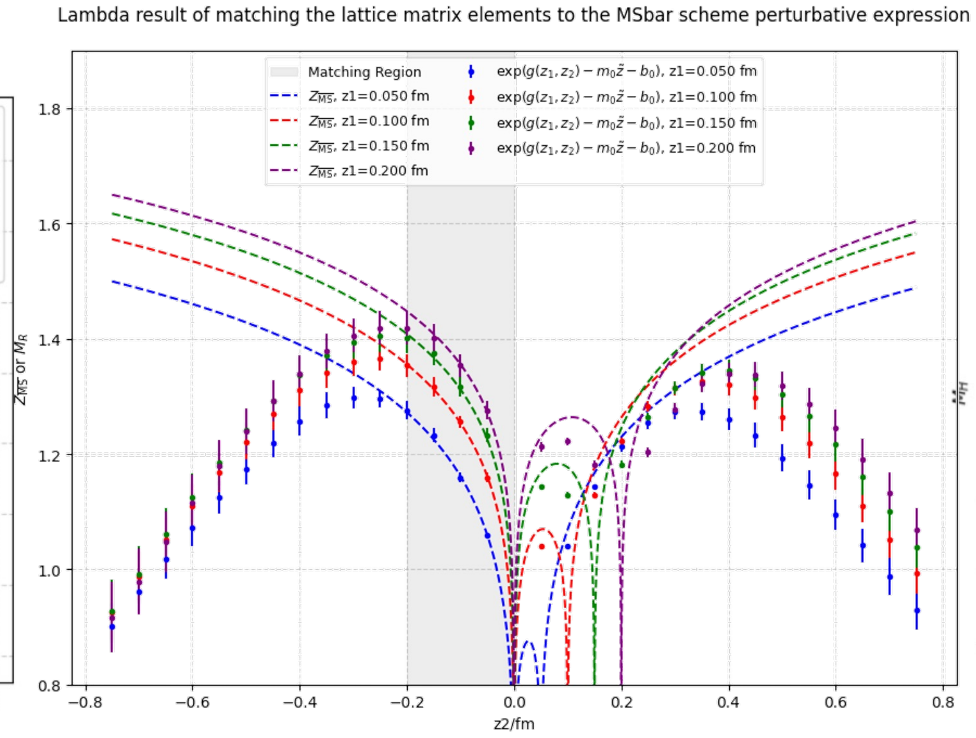
  Ji et.al, NPB 2021, Huo et.al, NPB 2021

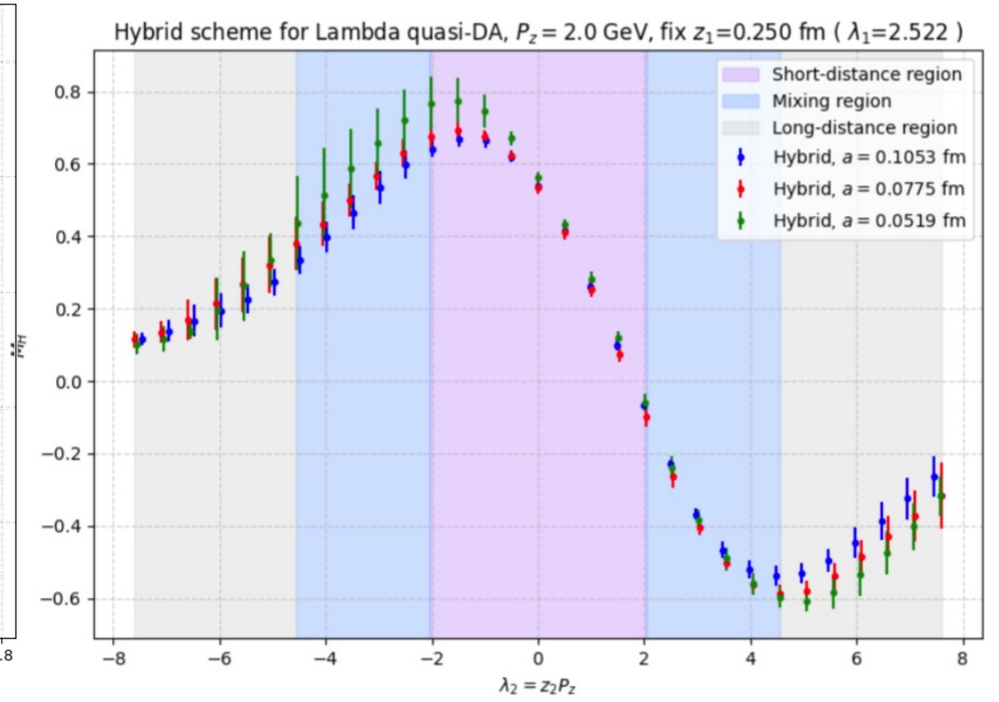
  —— The only solution

• Hybrid (based on self renormalization) scheme for baryon quasi-DA

- Bare quasi-DA
- Self renormalization
- Hybrid renormalized quasi-DA



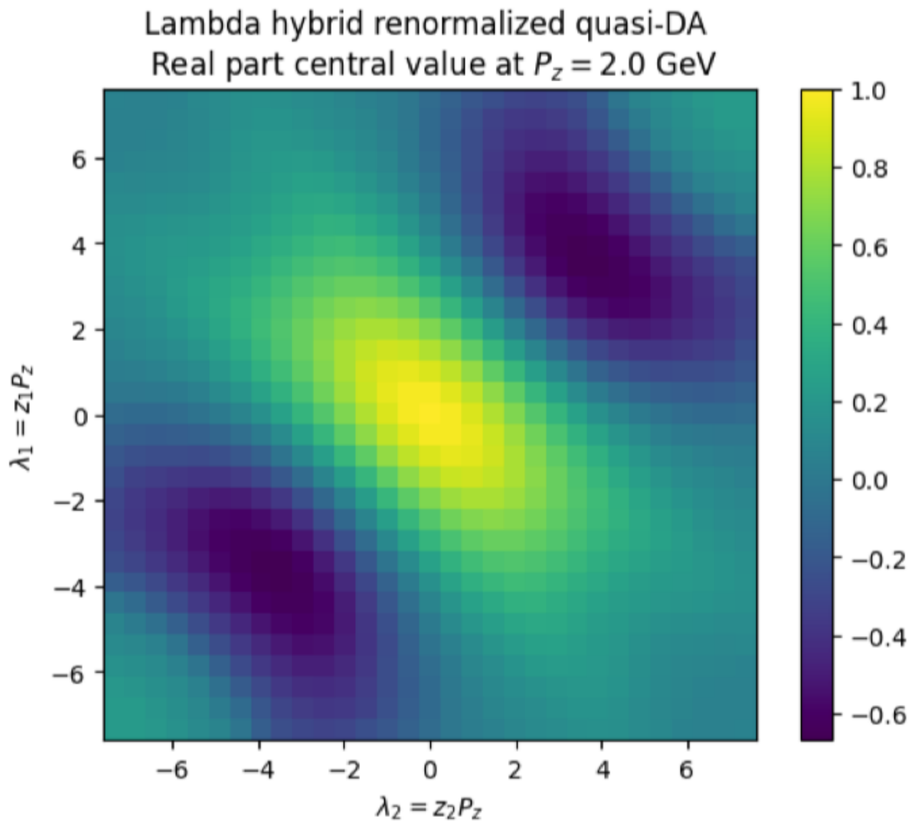


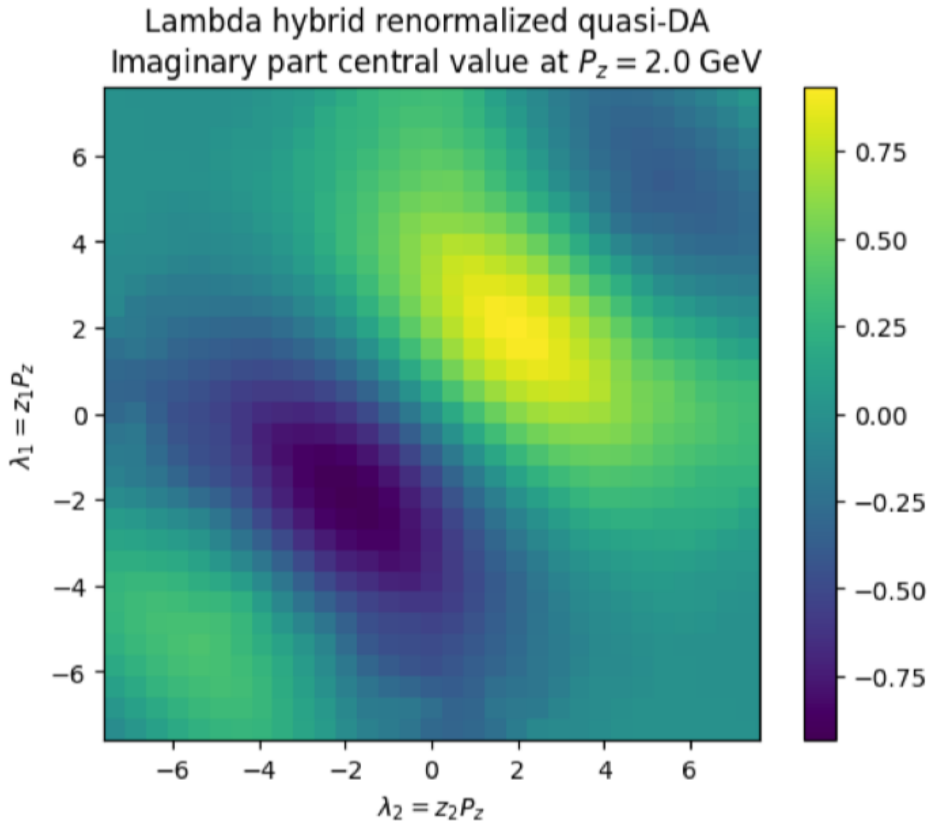


- Meson: $a \uparrow \rightarrow uncertainty \uparrow$
- Baryon: $\boldsymbol{a \uparrow \rightarrow unable} \times$

- **Hybrid (based on self renormalization) scheme for baryon quasi-DA**

□ Renormalized quasi-DA



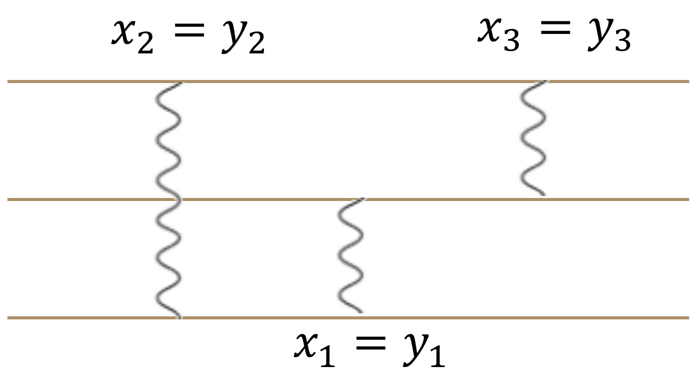


C.Han et.al. JHEP 12, 044 (2023)
H.Y.Bai et.al. arXiv:2508.08971 …

- **2D matching for baryon LCDA**

- **Meson:** $$\tilde{\phi}(x) = \int_0^1 dy C(x,y)\phi(y) + \mathcal{O}\left(\frac{1}{(xP^z)^2}, \frac{1}{[(1-x)P^z]^2}\right)$$
- **Baryon:** $$\tilde{\phi}(x_1,x_2) = \int_0^1 dy_1 \int_0^{1-y_1} dy_2 C(x_1,x_2,y_1,y_2)\phi(y_1,y_2) + \mathcal{O}\left(\frac{1}{(x_1P^z)^2}, \frac{1}{(x_2P^z)^2}, \frac{1}{[(1-x_1-x_2)P^z]^2}\right)$$

- Plus function → Double plus function
- 1-D discretize matching → 2-D



LPC, PRD 111, 034510 (2025)

Summary & Outlook

- We have extend the numerical computation on Lattice from light meson to light baryon in the LaMET framework.

- To calculate all leading twist structure Proton and Lambda LCDA, we improve:
  - ☐ lattice simulation $a \to 0$
  - ☐ simulation enhance
  - ☐ matching scheme
  - ☐ strategies for limited FT
  - ☐ renormalization scheme
  - ⊠ Twist analysis
  - ⊠ Excited contamination …

$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****

- ◆ Please stay tuned our results for all leading twists LCDA of Proton and Lambda
- ◆ High twists will be the next

Thanks for Your Attention !