◆ 重味物理前沿论坛研讨会

北京航空航天大学
BEIHANG UNIVERSITY

重味介子光锥分布振幅的格点QCD精确计算

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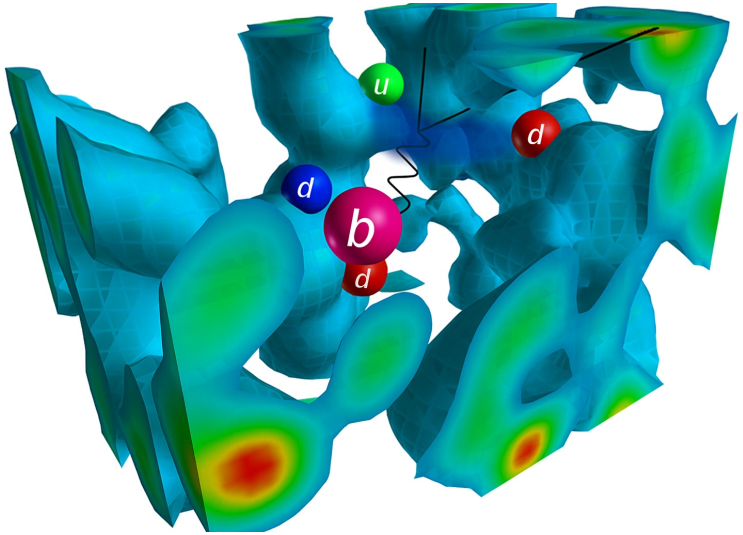
Collaborated with LPC Lattice Parton Collaboration,

W. Wang, J. Xu, S. Zhao, *et.al.*

09/14/2025 @ 华中师范大学，武汉

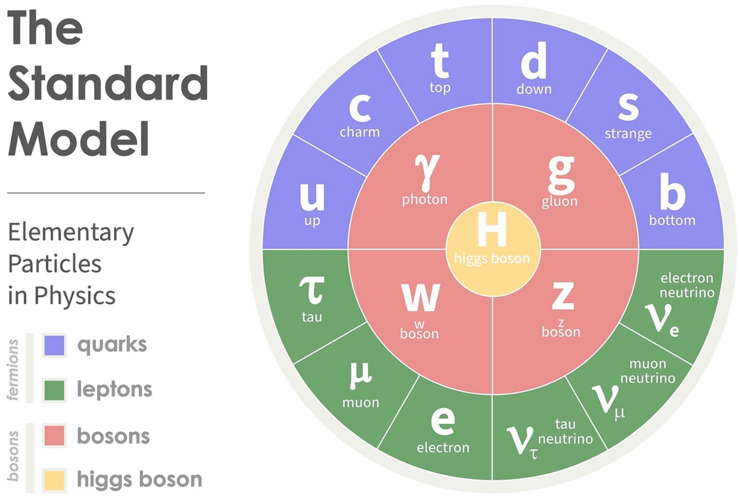


Outline

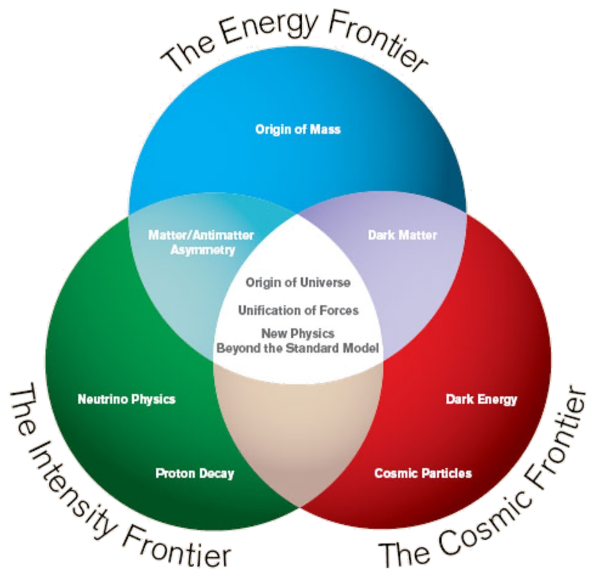
- Motivation
- Theoretical Framework
- Lattice QCD Calculation
- Perturbative Matching in LaMET and boosted-HQET
- Rebuild the full distribution of heavy meson HQET LCDA
- Summary and Outlook

Heavy Flavor Physics

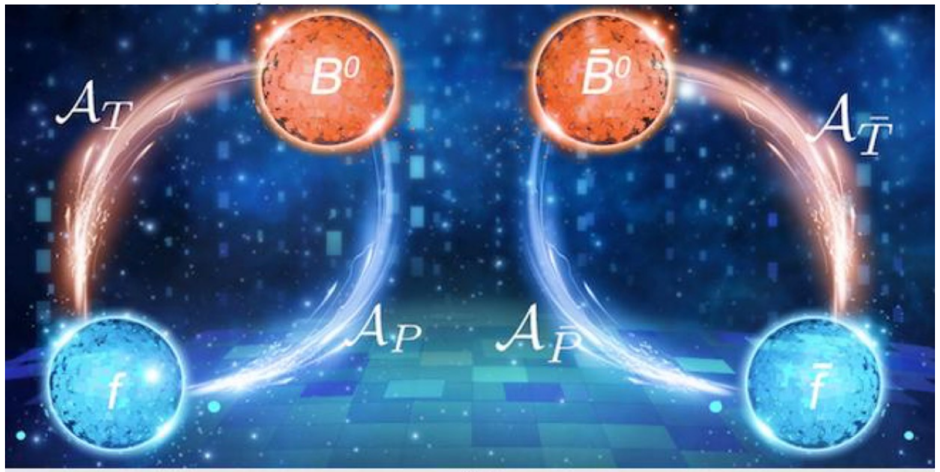
- **Precisely testing standard model**



- **Indirect search for new physics**



- **Study on CP violation**

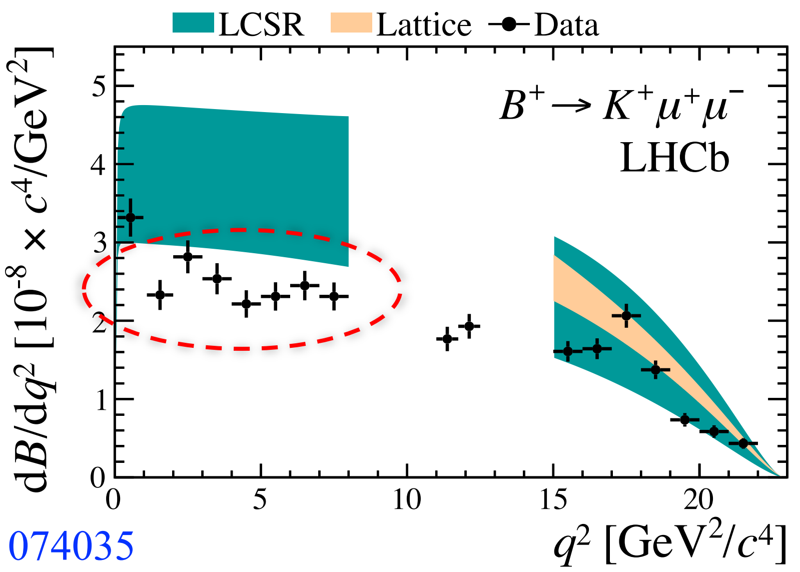


$$\mathcal{A} = \mathcal{A}_T e^{i(\delta_1+\phi_1)} + \mathcal{A}_P e^{i(\delta_2+\phi_2)} \overset{\text{CP}}{\leftrightarrow} \bar{\mathcal{A}} = \bar{\mathcal{A}}_T e^{i(\delta_1-\phi_1)} + \bar{\mathcal{A}}_P e^{i(\delta_2-\phi_2)}$$

Indirect search for new physics

➢ FCNC processes: $b \to s\ell\ell$ rare decays

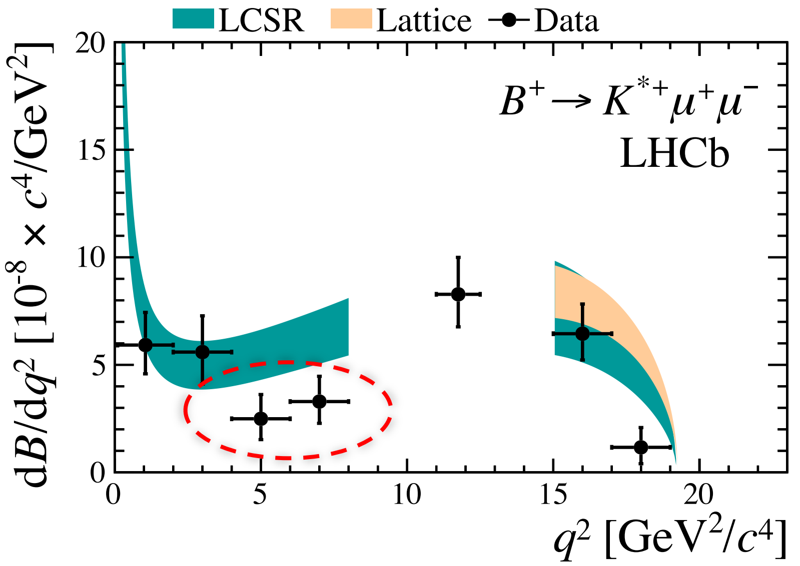
- Small SM amplitude ⇒ Excellent place to search for NP
- Significant deviations between exp. & theo.
- Ubiquitous: $B \to K\mu\mu$, $B \to K^*\mu\mu$, $B_s \to \phi\mu\mu$, $\Lambda_b \to \Lambda\mu\mu$



➢ Theoretical prediction:

Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

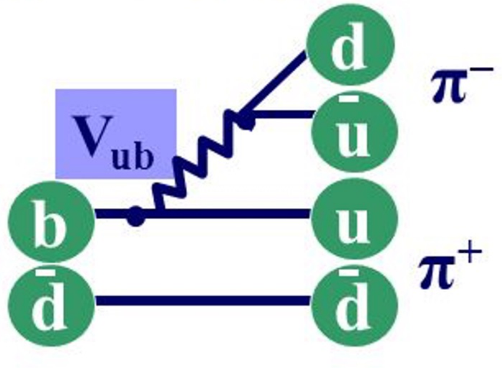


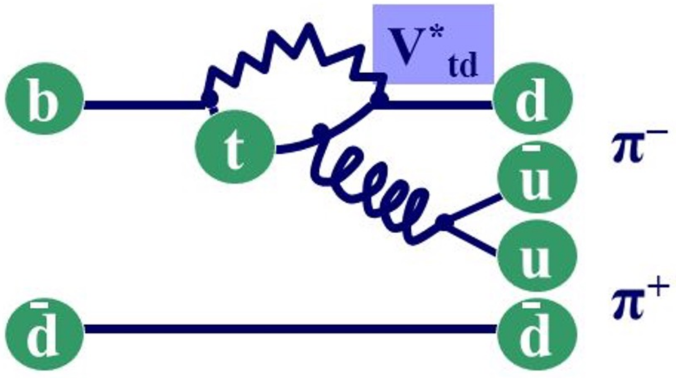
λ_B, σ_1: Inverse moment of heavy meson LCDA

$\varphi_\pm$: systematic error from model uncertainty

Study on CP violation

- **Nonleptonic decays: *e.g.* $B^0 \to \pi\pi$**
  - **The penguin diagrams are not small and have different weak phase;**
  - **Both direct and indirect CPV are significant**





- **Theoretical prediction:**

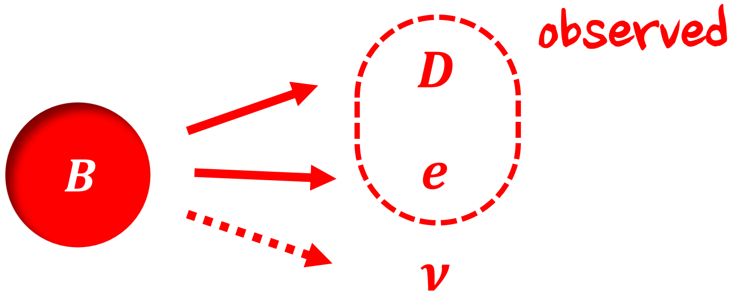
$$f^0_{B\to\pi}(0) = 0.122 \times \left[1 \pm 0.07\Big|_{S_0^\pi} \pm 0.11\Big|_{\Lambda_q} \pm 0.02\Big|_{\lambda_E^2/\lambda_H^2} {}^{+0.05}_{-0.06}\Big|_{M^2} \pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2} {}^{+0.06}_{-0.10}\Big|_{\mu_h} \pm 0.04\Big|_{\mu} {}^{+1.36}_{-0.56}\Big|_{\lambda_B} {}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\right]$$

Cui, Huang, Shen, Wang, JHEP 03 (2023) 140

Light-cone Distribution Amplitudes (LCDAs) of Heavy Mesons

LCDAs contribute to the exclusive decays of heavy meson, *e.g.* $B \to D\ell\nu$ process:

$$F_i^{B\to D}(q^2) = C_i^{(A0)}(q^2)\xi_a(q^2) + \int_0^\infty \frac{d\omega}{\omega}\int_0^1 dv\, T_i(\ln\omega, v, q^2)\, \varphi_\pm^B(\omega,\mu)\phi_D(v,\mu)$$



- **HQET LCDA**: quark distribution in **initial-state** heavy mesons $\boldsymbol{B} \to D\ell\nu$:

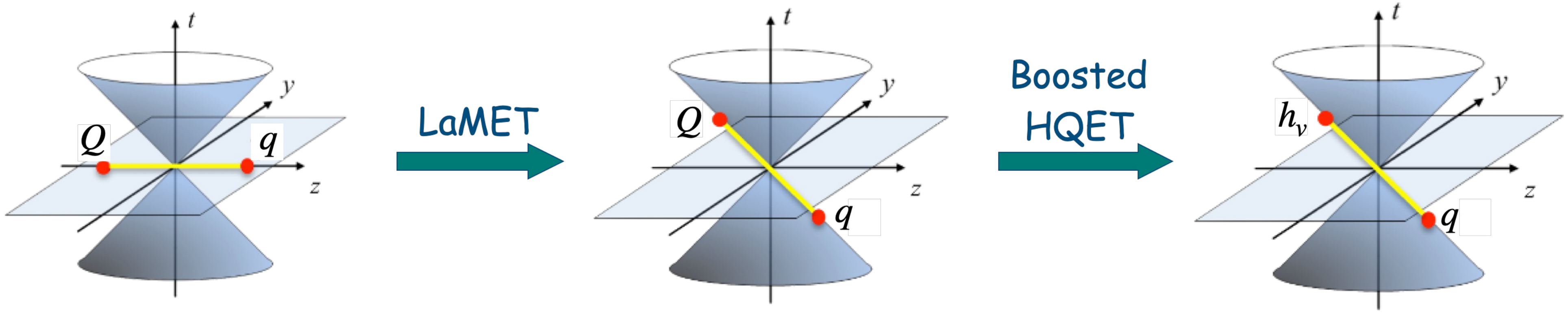
$$\varphi^+(\omega,\mu) = \frac{1}{i\,\tilde{f}_{H_Q} m_{H_Q} n_+\cdot v}\int \frac{dt}{2\pi} e^{-i\omega t n_+\cdot v} \langle 0\,|\,\bar{q}(tn_+)\not{n}_+\gamma_5 W_c(tn_+,0)h_v(0)\,|\,H_Q(v)\rangle$$

- **QCD LCDA**: quark distribution in **final-state** heavy mesons $B \to \boldsymbol{D}\ell\nu$:

$$\phi(x,\mu) = \frac{1}{i\,f_{H_Q}}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \langle H_Q(P_{H_Q})\,|\,\bar{q}(\xi^-)\not{n}_+\gamma_5 W_c(\xi^-,0)Q(0)\,|\,0\rangle$$

Sequential Effective Theory

- **We adopt a two-step factorization to combine LaMET and bHQET:**



Equal-time correlators

Light-like correlators with QCD fields

Light-like correlators with HQET fields

- **Scales: $\Lambda_{\mathrm{QCD}}, m_Q, P^z$**
- **LQCD calculable**

LaMET:

- $\Lambda_{\mathrm{QCD}}, m_Q \ll P^z$
- **Integrate out P^z**

bHQET:

- $\Lambda_{\mathrm{QCD}} \ll m_Q$
- **Integrate out m_Q**

Liu, Wang, Xu, **QAZ**, Zhao, PRD 99, 094036 (2019)

Han, **QAZ**, *et.al.*, PRD111, 034503, (2025)

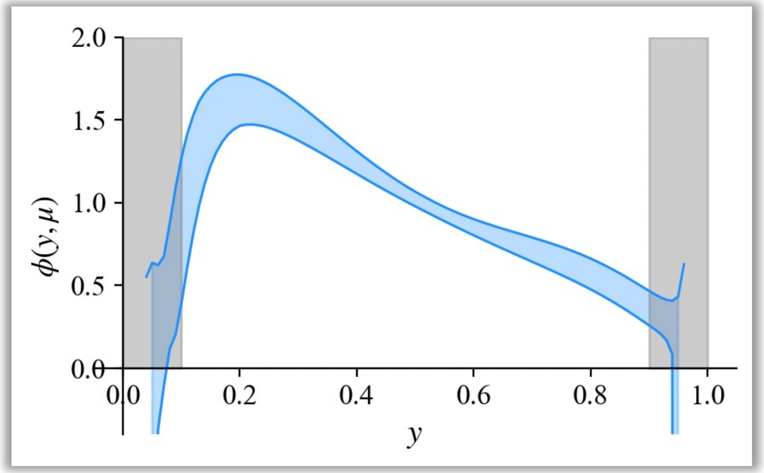
Ishaq, Jia, Xiong, Yang, PRL125, 132001 (2020)

Zhao, PRD 101, 071503 (2020)

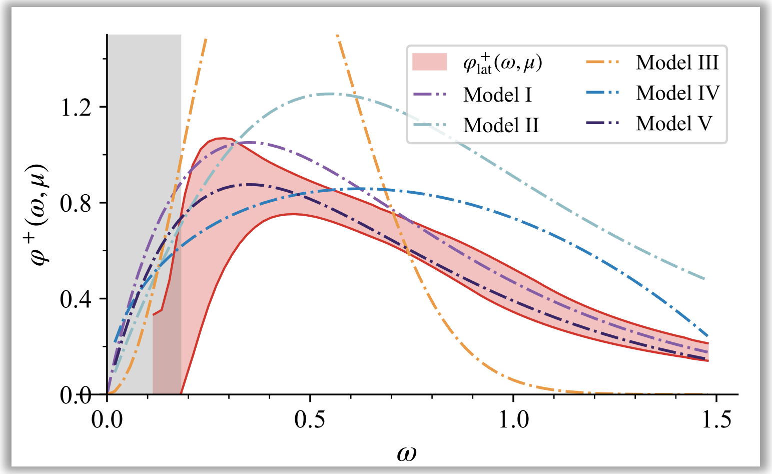
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

A Preliminary Extrapolation

D meson QCD LCDA



HQET LCDA φ^+



Han, QAZ, *et.al.*, PRD111, L111503 (2025); PRD111, 034503 (2025)

- **Exploratory** calculations on the CLQCD ensembles, with single lattice spacing and non-physical hadron masses;
- The QCD LCDA of *D* meson, as well as the HQET LCDA for heavy meson, have been obtained **as expected**;
- Based on this results, we can obtain:

  More accurate extraction of the inverse moments;

  More precise prediction for the $B \to K^*$ form factors.

This result inspires us to conduct further studies...

Towards High-Precision Calculation

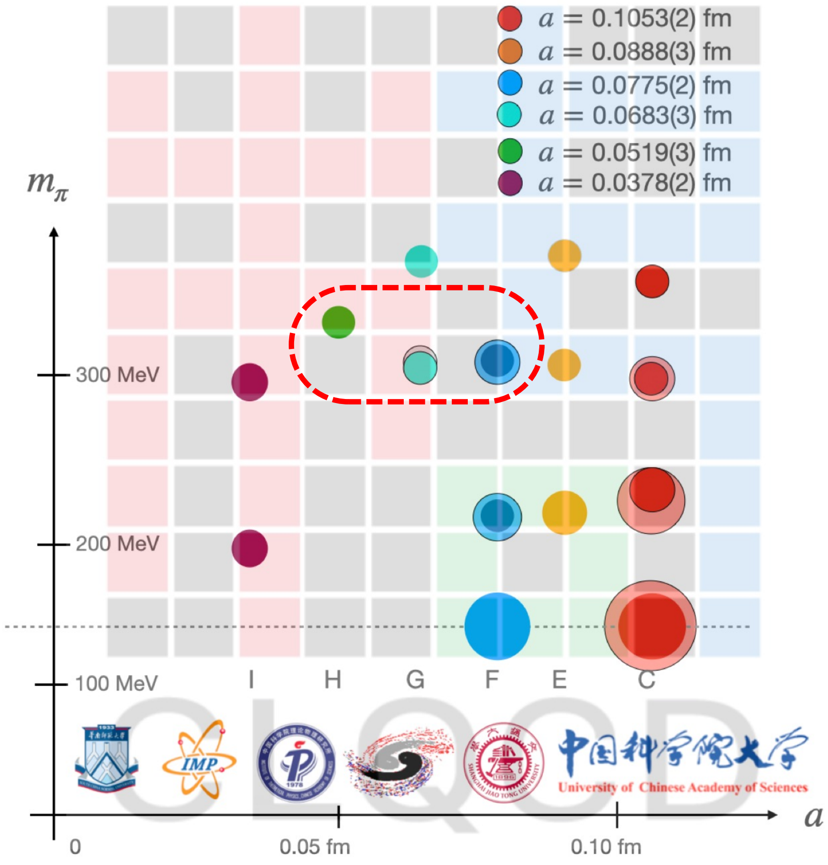
Simulation setup:

- 2+1 flavor QCD ensembles with stout smeared clover fermion and Symanzik gauge actions, generated by the **CLQCD collaboration**

  Hu *et.al.*, PRD109, 054507 (2024)
  Du *et.al.*, PRD111, 054504 (2025)

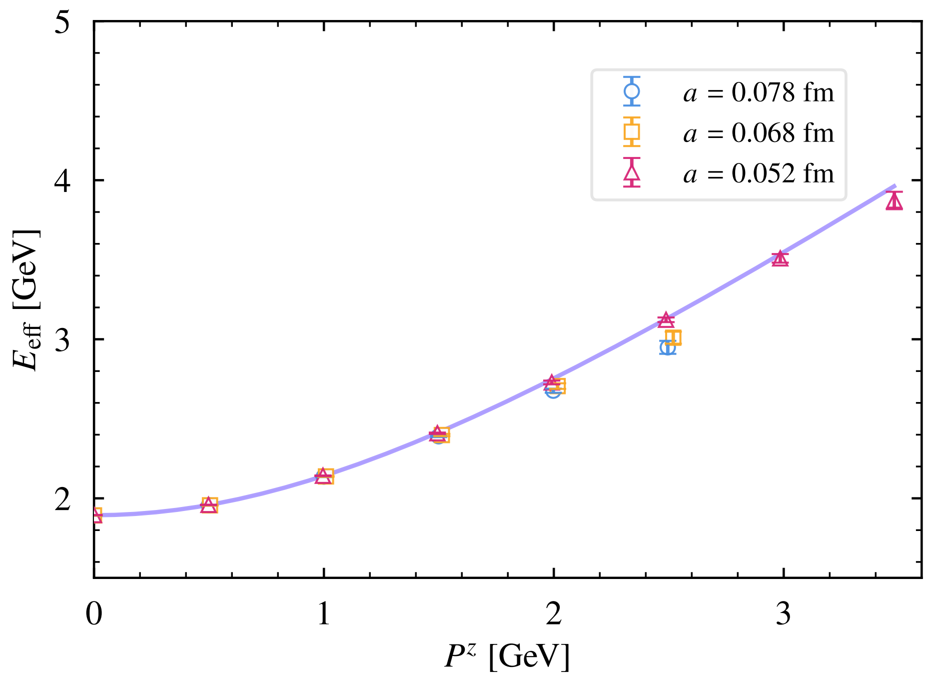
- Three lattice spacings $a = \{0.078, 0.068, 0.052\}$fm, enables continuum extrapolation.

- Multiple momenta **up to 3.5GeV** to carry out the infinite momentum extrapolation.



Towards High-Precision Calculation

- Power counting $\Lambda_{\text{QCD}} \ll m_H \ll P^z$ constrains the heavy quark mass, we take **D meson** as the benchmark.
- The dispersion relation demonstrates that discretization effects can be **well controlled**.
- Over **100,000 measurements** on each ensemble are accumulated.



Ensemble	$n_s^3 \times n_t$	a [fm]	m_π [GeV]	m_c	P^z [GeV]	N_{stats}
F32P30	$32^3 \times 96$	0.07750(18)	303.2(1.3)	0.1968(21)	$\{0, 0.5, \cdots, 2.5\}$	183600
G36P29	$36^3 \times 108$	0.06826(27)	295.1(1.2)	0.1378(28)	$\{0, 0.5, \cdots, 2.5\}$	112832
H48P32	$48^3 \times 144$	0.05187(26)	317.2(0.9)	0.0533(24)	$\{0, 0.5, \cdots, 3.5\}$	108000

Bare matrix elements

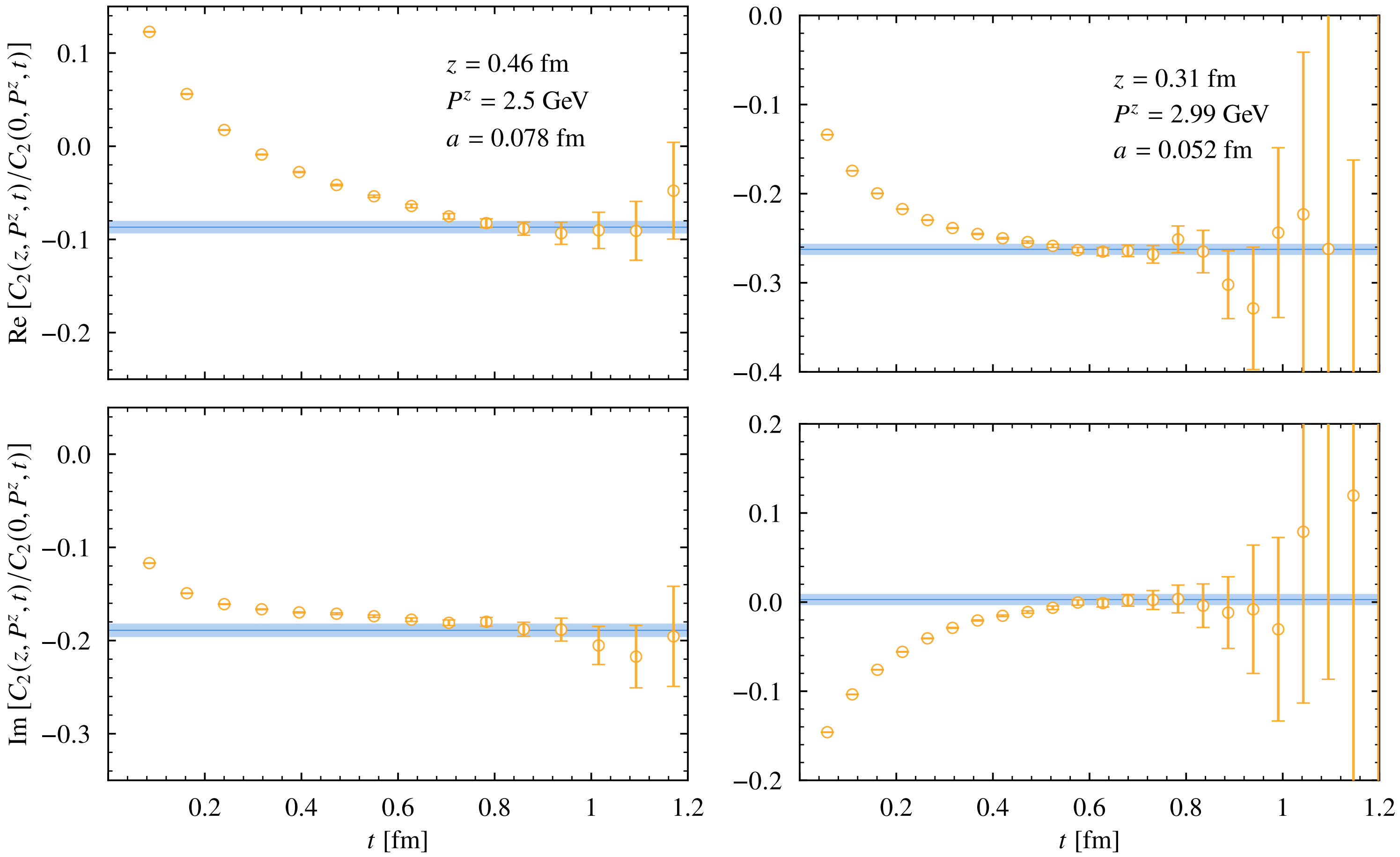
- Nonlocal 2-point correlation function:

$$C_2(z,P^z,t) = \sum_{x^3} e^{iP^z x^3} \langle G_q(x^3+z,t,0)\gamma^z\gamma_5 \times W_c(x^3+z,x^3)\gamma_5 G_Q^\dagger(x^3,t,0)\gamma_5 \rangle,$$

- Beside high statistics, **HYP smearing** is used to improve the signal.

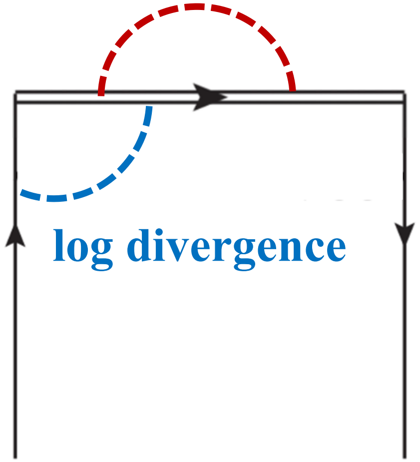
- Bare matrix elements $\widetilde{M}^B(z,P^Z)$:

$$\frac{C_2(z,P^z,t)}{C_2(0,P^z,t)} = \tilde{M}^B(z,P^z)e^{-E_0 t} \times \left[\sum_{n=0}^{N_{\mathrm{stat}}-1} A_n e^{-(E_n-E_0)t}\right].$$



Nonperturbative Renormalization in Hybrid Scheme

- $\tilde{M}^B(z, P^z)$ contains both **linear** and **logarithmic** divergences, can be multiplicatively renormalized.



- **Hybrid renormalization scheme**:

$$\tilde{M}^R(z, P^z) = \frac{\tilde{M}^{(0)}(z, P^z, 1/a)}{\tilde{M}^{(0)}(z, P^z_{\rm ref}, 1/a)}\theta(z_s - |z|) + \frac{\tilde{M}^{(0)}(z, P^z, 1/a)}{Z_R(z, 1/a)}\theta(|z| - z_s),$$

  - Short distance (perturbative region): **ratio scheme**

$$\langle 0|\bar{q}(z)\gamma^z\gamma_5 W_c(z,0)Q(0)|H(P^z)\rangle \propto P^z \quad \Rightarrow \quad P^z_{\rm ref} = 0.5\text{GeV}$$

  - Long distance (nonperturbative region): **self renormalization scheme**

$$Z_R(z, 1/a) = \exp\left\{\frac{kz}{a\ln[a\Lambda_{\rm QCD}]} + m_0 z + f(z)a + \frac{3C_F}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{\rm QCD})]}{\ln[\mu/\Lambda_{\rm QCD}]}\right] + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\rm QCD})}\right]\right\}.$$

Nonperturbative Renormalization in Hybrid Scheme

- **Hybrid renormalization scheme**:
  - **Short distance**:

    Lattice **agree well** with perturbation results (NLO, LRR+RGR-improvement)

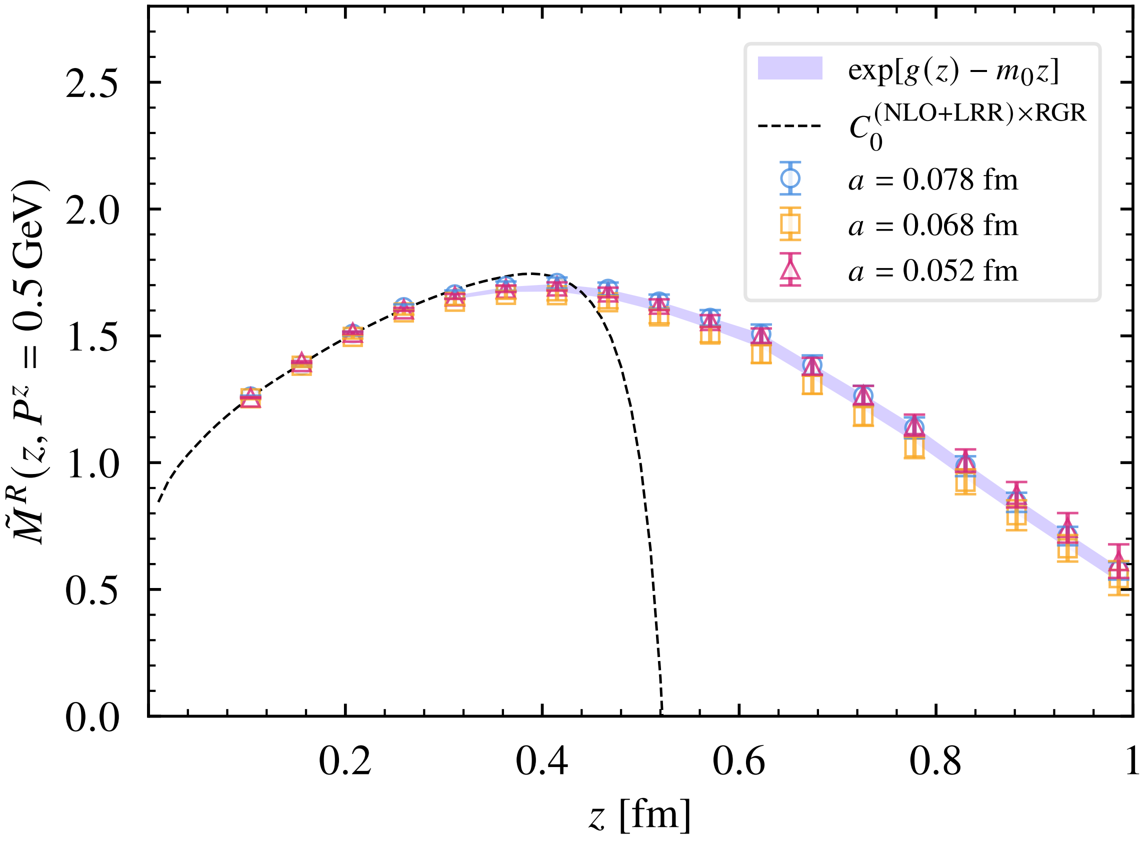
  - **Long distance**:

$$Z_R(z,1/a) = \exp\left\{\frac{kz}{a\ln[a\Lambda_{\rm QCD}]} + m_0 z + f(z)a + \frac{3C_F}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{\rm QCD})]}{\ln[\mu/\Lambda_{\rm QCD}]}\right] + \ln\left[1+\frac{d}{\ln(a\Lambda_{\rm QCD})}\right]\right\}.$$

Fit results: $(\chi^2/d.o.f = 0.92)$

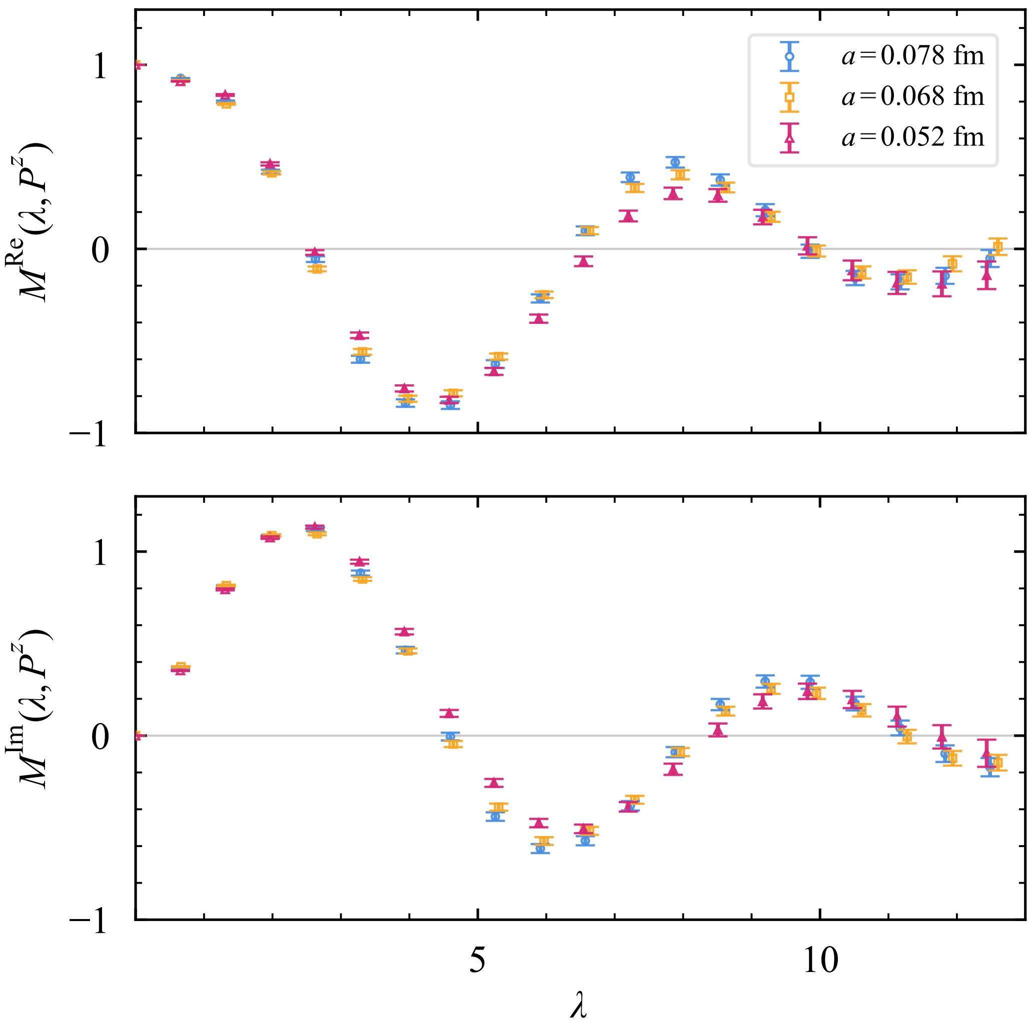
$k = 0.802(35)$, $\Lambda_{\rm QCD} = 0.192(75)$GeV,

$m_0 = 0.241(98)$GeV, $d = -0.18(10)$

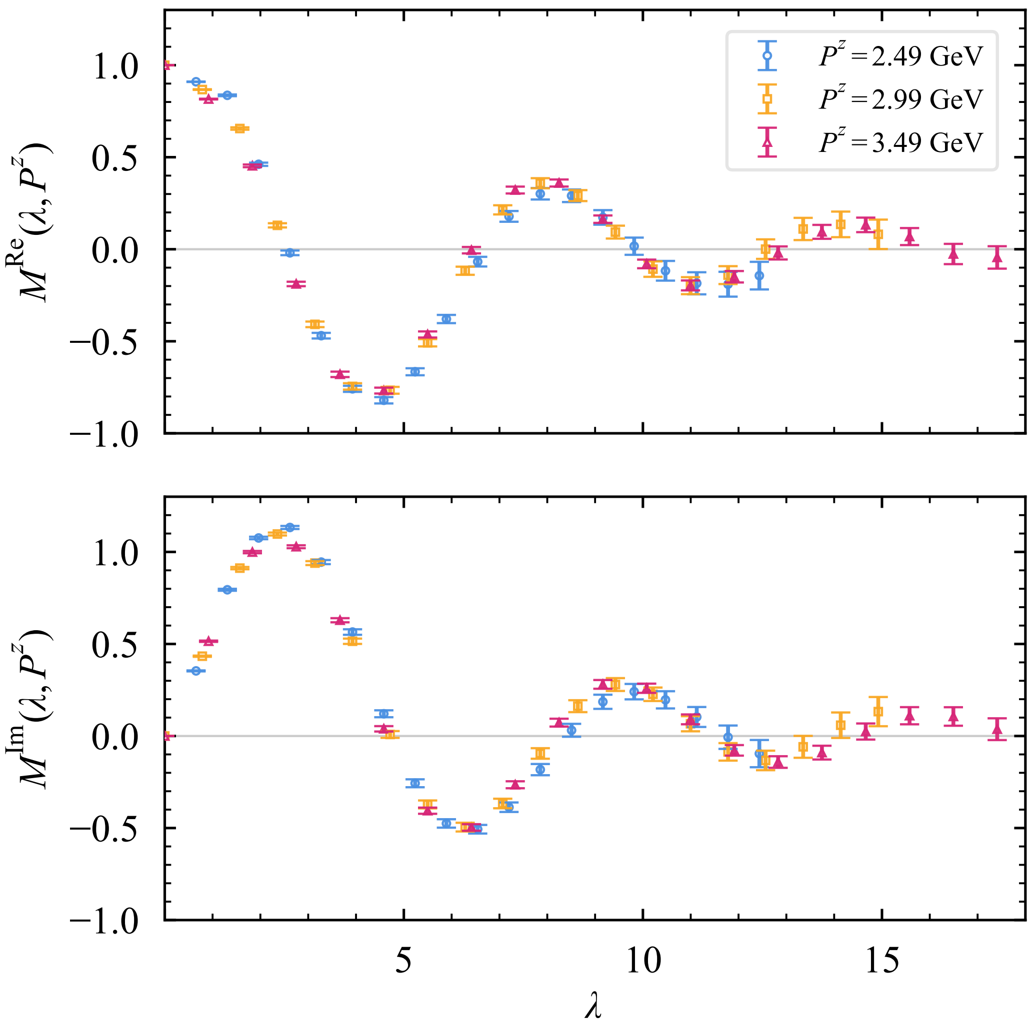


Renormalized matrix elements

- a**-dependence,** $P^z = 2.5\text{GeV}$



- P^z**-dependence,** $a = 0.052\text{fm}$



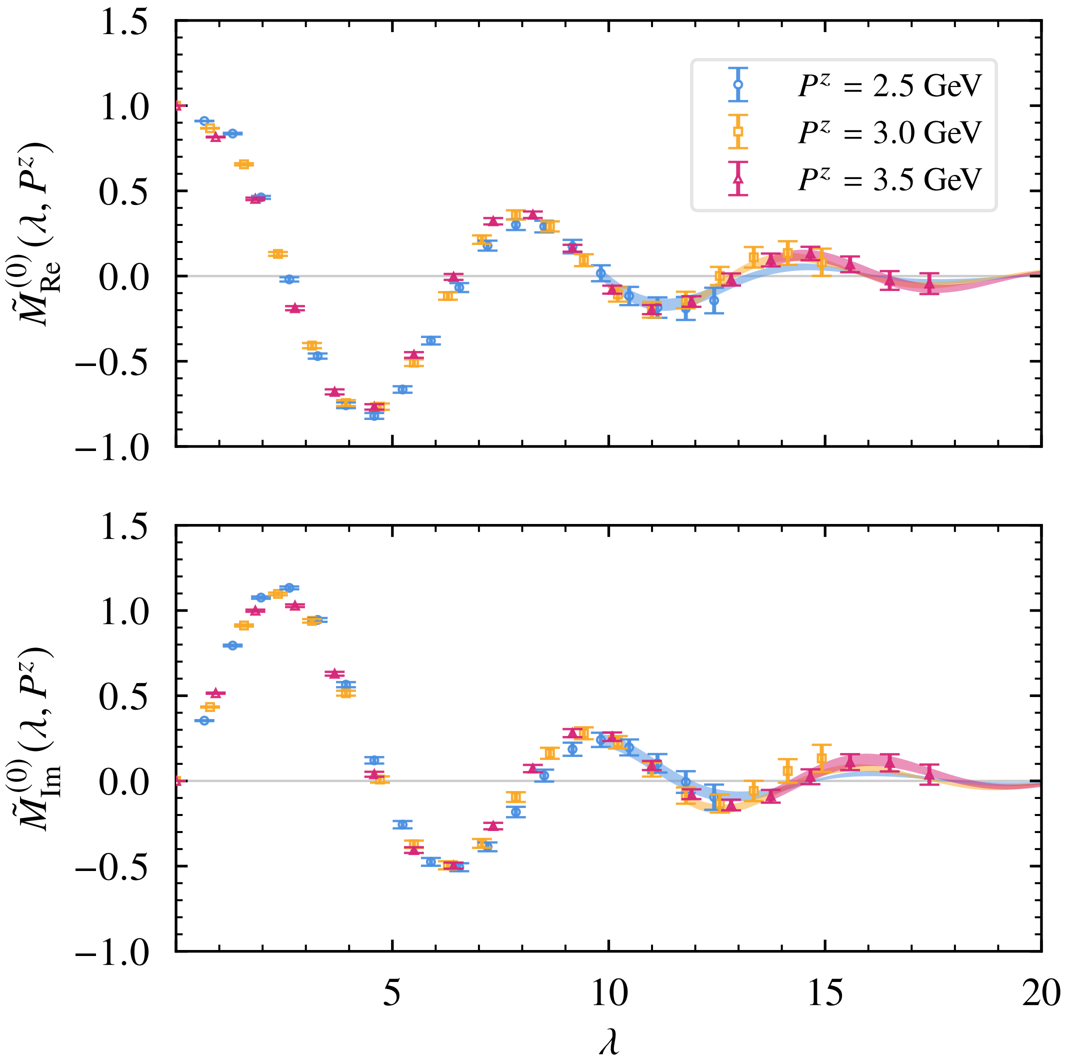
A brute-fore truncation at large λ will lead to strong oscillation after FT…

λ Extrapolation

We rebuild the long-range correlations from the λ-extrapolation:

$$\tilde{h}_{\text{extra}}(\lambda) = \left[\frac{c_1}{(-i\lambda)^{n_1}} + e^{i\lambda}\frac{c_2}{(i\lambda)^{n_2}}\right]e^{-\lambda/\lambda_0}$$

- end point power-law behavior $x^a(1-x)^b$;
- correlation function has a finite correlation length λ_0.



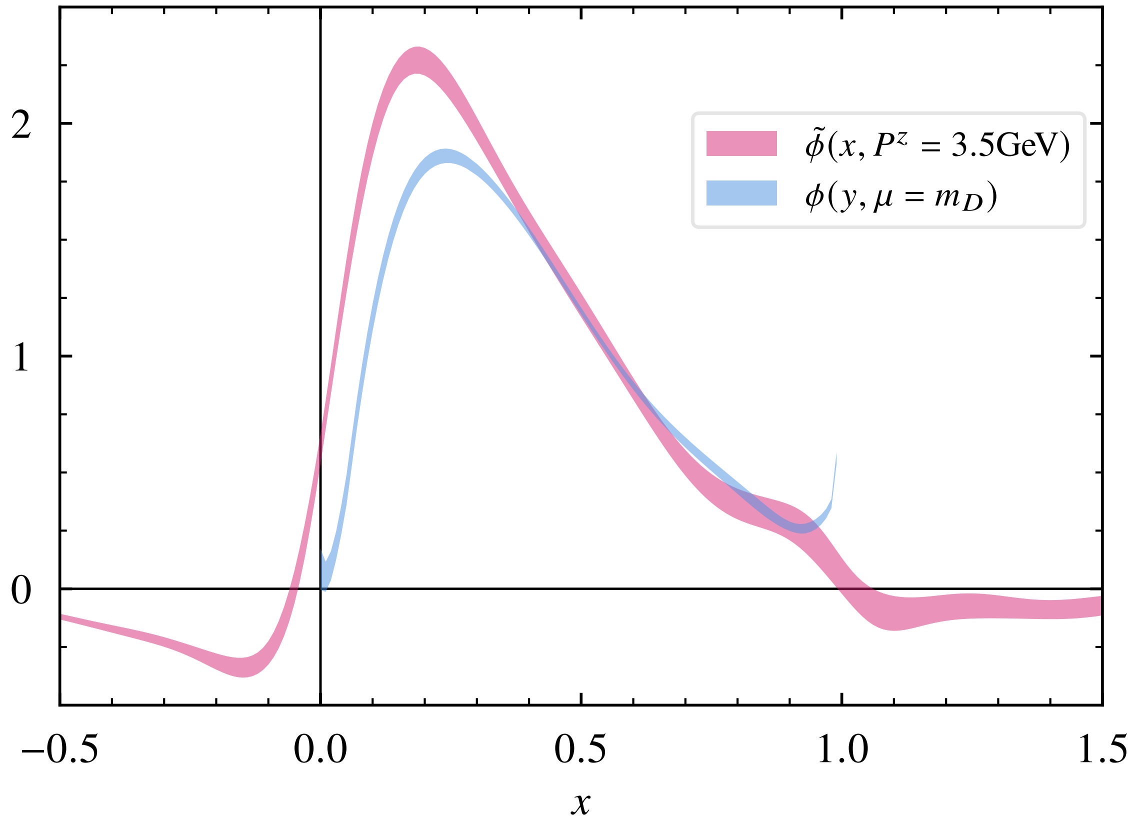
Matching in LaMET

Matching from quasi-DA to QCD LCDA:

$$\phi(y,\mu) = \int_{-\infty}^{+\infty} \frac{dx}{|x|}\, \mathcal{C}\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \tilde{\phi}(x, P^z) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(yP^z, (1-y)P^z)^2}\right).$$

Liu, Wang, Xu, **QAZ**, Zhao, PRD 99, 094036 (2019)

Han, **QAZ**, *et.al.*, PRD111, 034503, (2025)



- Power corrections:

$\frac{\Lambda_{\rm QCD}^2}{(yP^z, (1-y)P^z)^2}$: Contribute to end-point region.

$\frac{m_H^2}{(P^z)^2}$: All region, can be estimated by predictions without LaMET.

⇒ The first few moments are sufficient.

Moments of *D* meson as benchmark

- Gegenbauer moments from LaMET:

$$\phi(x,\mu^2) \;=\; 6x(1-x)\left[1+\sum_{n=1}^{\infty} a_n(\mu^2)\, C_n^{3/2}(2x-1)\right],$$

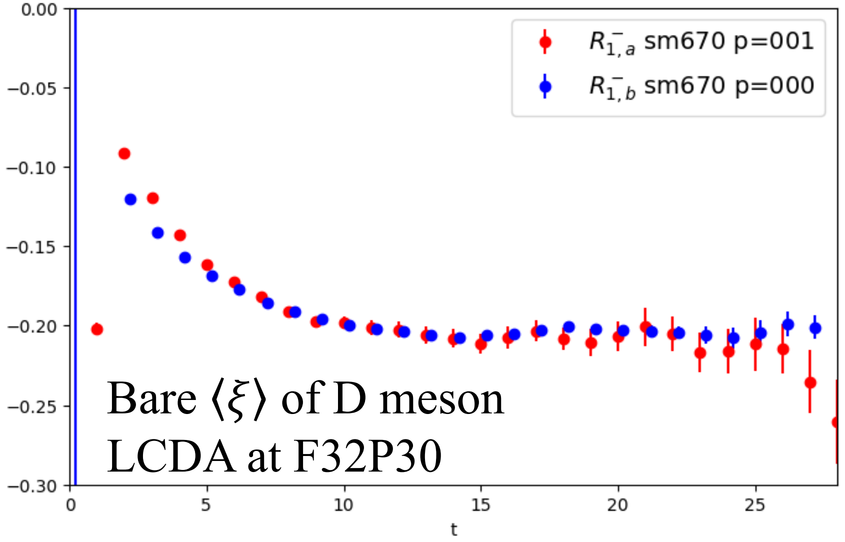
Fitting from data excluding the end-point region:

n	a_1	a_2	a_3	a_4
2	-0.397(18)	0.118(10)		
4	-0.412(20)	0.134(16)	-0.016(11)	0.005(8)
6	-0.391(22)	0.111(18)	0.026(20)	-0.019(15)
8	-0.376(24)	0.109(19)	0.041(22)	-0.034(18)

- Moments from local operators:

$$\bar{q}(z_2 n)\not{n}\gamma_5 W_c(z_2 n, z_1 n)\, q'(z_1 n) = \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k!l!} n^\rho n^{\mu_1}\ldots n^{k+l} \mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} + \cdots,$$

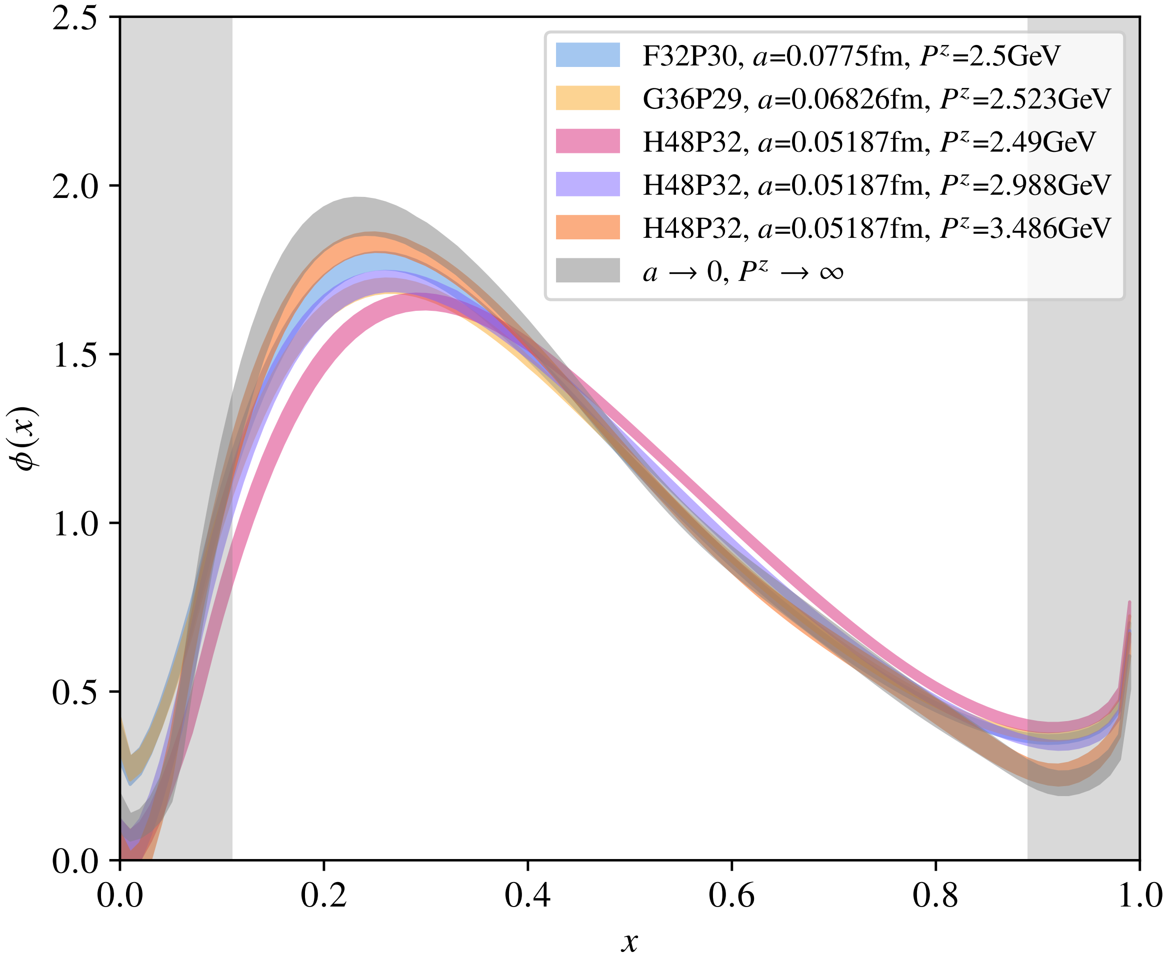
$$\mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} \;=\; \bar{q}(0)\overleftarrow{D}_{(\mu_1}\ldots\overleftarrow{D}_{\mu_k}\overrightarrow{D}_{\mu_{k+1}}\ldots\overrightarrow{D}_{\mu_{k+l}}\gamma_{\rho)}\gamma_5 q'(0).$$



QAZ, *et.al.*, in progress

QCD LCDA of D meson

Continuum and infinite-momentum extrapolation: $\phi(x,\mu;a,P^z) = \phi(x,\mu)\left[1 + c_1(x)a^2 + \frac{c_2(x)}{(P^z)^2}\right]$

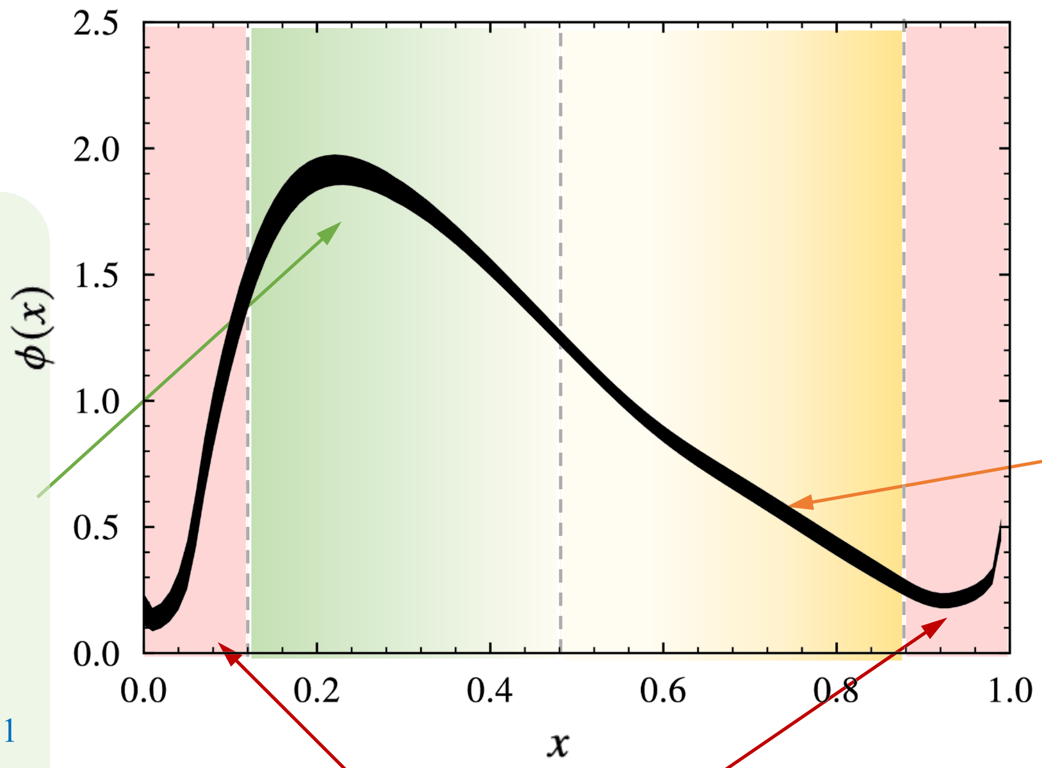


QCD LCDA of D meson

Peak region: $y \sim \frac{\Lambda_{\text{QCD}}}{m_H}$

- Light quark carries small momentum fraction;
- Related to the HQET LCDA.

Ishaq, Jia, Xiong, Yang, PRL125(2020)132001
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)



Tail region: $y \sim 1$

- Contain only hard-collinear physics;
- Suppressed in LCDA.

End-point region:

- LaMET matching kernel suffer large power corrections.
- Lattice QCD predictions fail

Prediction for HQET LCDA

- Leading twist HQET LCDA:

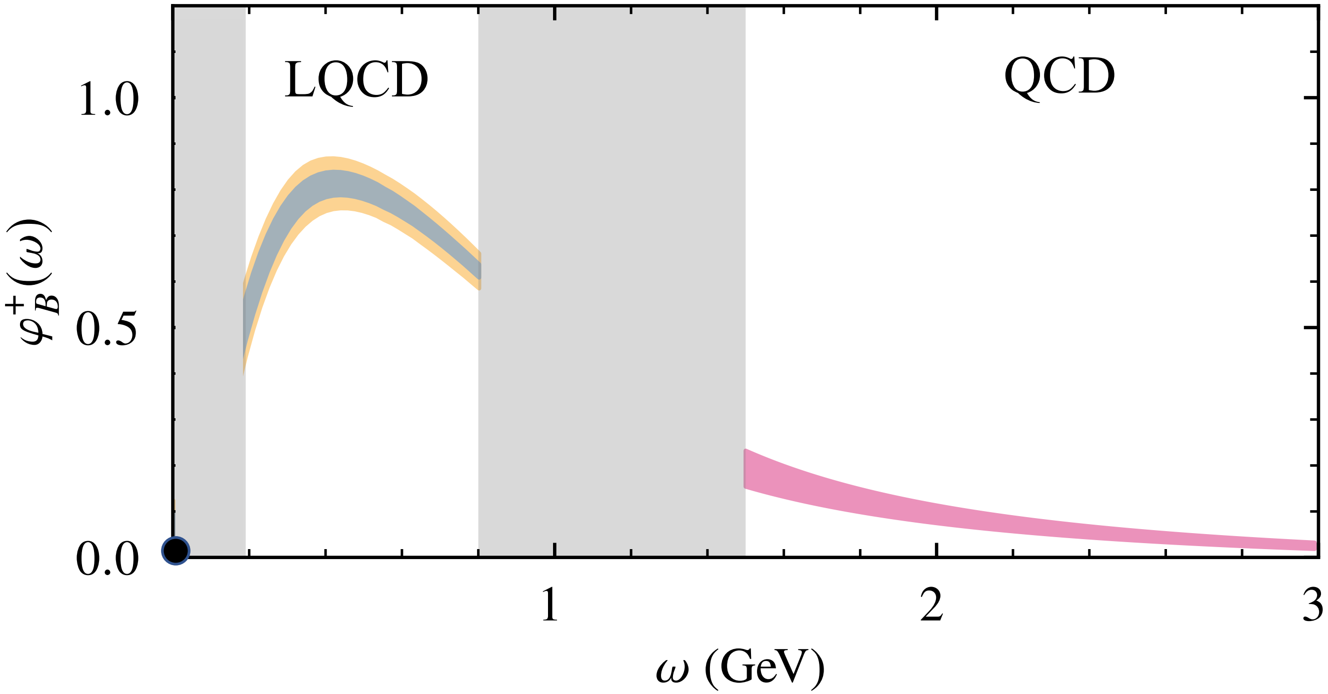
QCD LCDA, nonperturbative

$$\varphi^+(\omega,\mu) = \begin{cases} \frac{1}{m_H}\left[1-\frac{\alpha_s C_F}{4\pi}\left(\frac{1}{2}\ln^2\frac{\mu^2}{m_H^2}+\frac{1}{2}\ln\frac{\mu^2}{m_H^2}+\frac{3}{2}\ln\frac{\mu^2}{m_Q^2}+\frac{\pi^2}{12}+4\right)+\mathcal{O}\left(\alpha_s^2\right)\right]\phi\left(\frac{\omega}{m_H},\mu\right), & \omega\sim\Lambda_{\rm QCD} \\ \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2}-\ln\frac{\omega}{\mu}\right)+\frac{4\bar\Lambda}{3\omega}\left(2-\ln\frac{\omega}{\mu}\right)\right]+\mathcal{O}\left(\alpha_s^2\right), & \omega\sim m_H \end{cases}$$

Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

Lee, Neubert, PRD 72, 094028 (2005)

Prediction for HQET LCDA from first-principle:

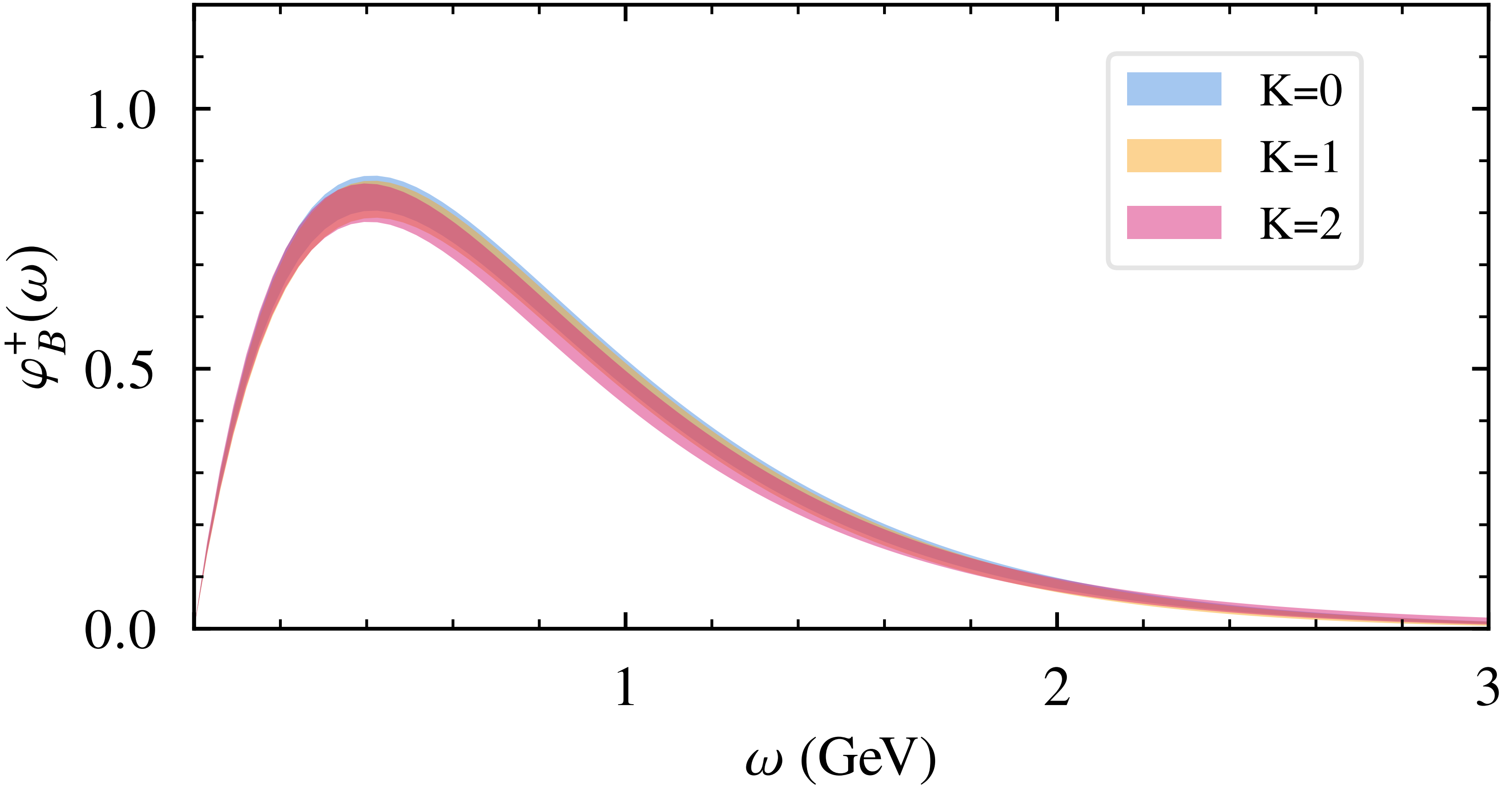


Prediction for HQET LCDA

- Rebuild the full distribution of the HQET LCDA from a **model-independent parametrization**:

Feldmann, Lughausen, Dyk, JHEP10, 162 (2020)

$$\varphi^{+}(\omega,\mu)=\frac{\omega e^{-\omega/\omega_0}}{\omega_0^2}\sum_{k=0}^{K}\frac{a_k(\mu)}{1+k}L_k^{(1)}(2\omega/\omega_0),$$



Inverse and inverse-logarithmic moments

Predictions of inverse and inverse-logarithmic moments from the full distribution of HEQT LCDA:

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\varphi^+(\omega,\mu),$$

$$\sigma_B^{(n)}(\mu) = \lambda_B(\mu)\int_0^\infty \frac{d\omega}{\omega}\ln\left(\frac{\mu}{\omega}\right)^{(n)}\varphi^+(\omega,\mu).$$

	λ_B (GeV)	$\sigma_B^{(1)}$
Our work	0.414 (16)	1.451 (19)
PRD98 (2018)	> 0.24	
JHEP (2020)	0.383 (153)	
PRD72 (2005)	0.48 (11)	1.6 (2)
PRD69 (2004)	0.46 (11)	1.4 (4)
PRD55 (1997)	0.35 (15)	
PRD101 (2020)	$0.343^{+0.064}_{-0.079}$	
PLB848 (2024)	0.338 (68)	

Summary and Outlook

- ✓ We present a **first-principles precision determination** of the QCD and HQET LCDA for heavy mesons at leading power.
- ✓ Systematic uncertainties from non-perturbative renormalization, scale conversion, continuum and infinite-momentum extrapolation etc., are comprehensively addressed.

- Power correction in LaMET ($\propto m_H/P^z$) can be estimated from **local moments**, this work is currently ongoing.
- Power correction in bHQET ($\propto \Lambda_{\mathrm{QCD}}/m_Q$) can be estimated by **difference between spin-split heavy mesons**, this is our goal for next stage.

Thanks for your attention