Impact of structure-dependent QED effects on $|V_{ub}|$ extraction

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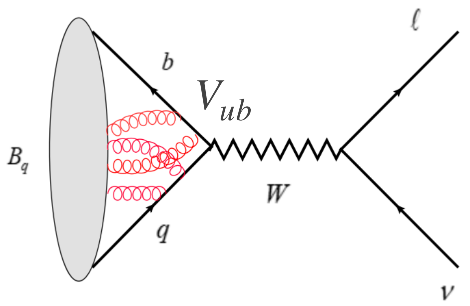
Based on:

Impact of structure-dependent QED effects on $|V_{ub}|$ extraction

in preparation with **Y.L. Shen (沈月龙) , C. Wang (王超) and Y.B. Wei (魏焰冰)**

$B_u \to \ell \nu$ serves as an excellent channel to extract $|V_{ub}|$

- Determination of $|V_{ub}|$ largely unaffected by hadronic uncertainties



$$\langle 0 | \bar{u} \gamma^\mu \gamma_5 b | B_u(p) \rangle = i f_B p^\mu$$

- Recently, Belle II measured its branching fraction

$$\mathscr{B}^{(\exp)}(B_u \to \tau\nu) = \left(1.24 \pm 0.41(stat.) \pm 0.19(syst.)\right) \times 10^{-4}$$

and updated the value $|V_{ub}| = \left(4.41^{+0.74}_{-0.89}\right) \times 10^{-3}$ *[arXiv: 2502.04885]*

- This result is derived from the SM prediction of branching fraction

$$\mathscr{B}(B \to \tau\nu_\tau) = \frac{m_B m_\tau^2 G_F^2}{8\pi} \left[1 - \frac{m_\tau^2}{m_B^2}\right]^2 |V_{ub}|^2 f_B^2 \tau_B$$

2

in the absence of QED corrections

Why do we need to know the QED corrections?

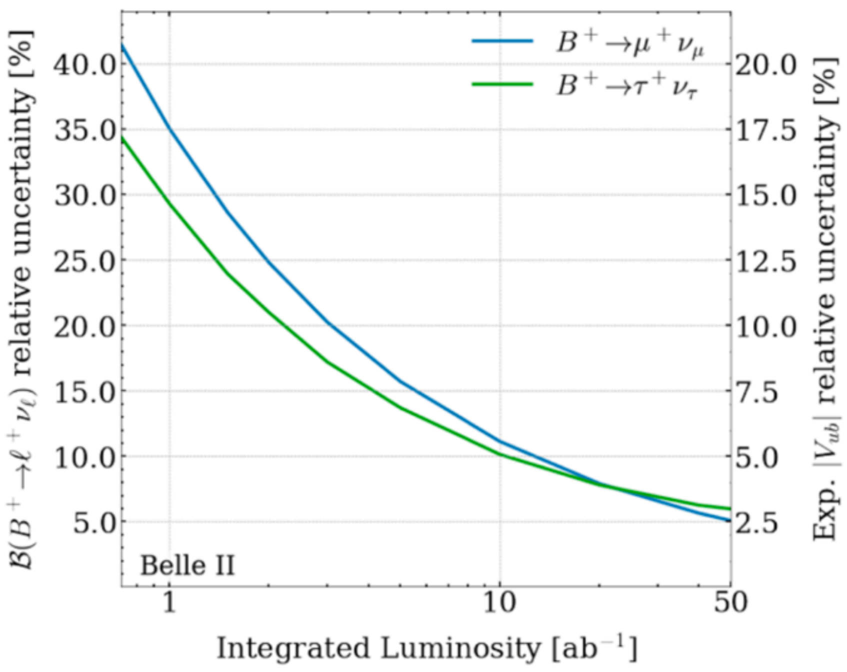
- QCD matrix element is known with **<1% accuracy**

$$\langle 0 | \bar{u}\, \gamma^\mu \gamma_5\, b \,|\, B_u(p) \rangle \;=\; i\, f_B\, p^\mu \quad \text{with} \quad f_B = (190.0 \pm 1.3)\text{MeV}$$ *[FLAG 2024]*

QED corrections can be of similar magnitude or even larger, due to presence of **large logarithms** $\alpha \ln(m_b^2/m_\ell^2)$ and $\alpha \ln(m_\ell/E_\gamma) \ln(m_b/m_\ell)$

→ **be comparable to QCD uncertainties**

- Belle II will measure the τ, μ channels with **$5-7$ % uncertainty**

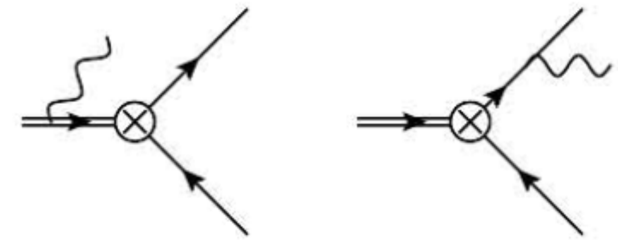


[Belle II Physics Book]

→ **compete with experimental uncertainties**

Beyond ultra-soft photon approximation

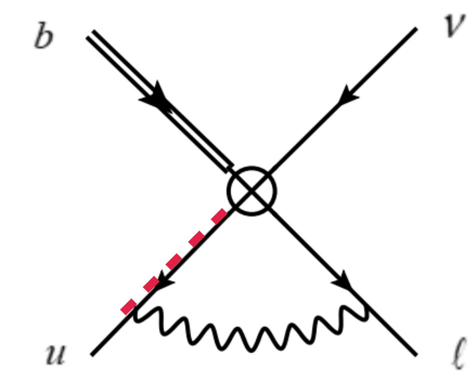
- In most cases, analyses focused solely on **ultrasoft-photon emissions**, where photons were treated as extremely low-energy (“ultrasoft”).



e.g. $\bar{B} \to \bar{K}\ell^+\ell^-$ *[G. Isidori, etc. JHEP 12,104 (2020)]*

- scale $\Lambda_{\text{QCD}} < \mu < m_b$ and photon emitting from lepton can recoil against the light spectator quark, and the light spectator is then delocalized along the light cone

$$\frac{1}{n_- \ell_q} \sim \frac{1}{\Lambda_{\text{QCD}}}$$



→ Power enhanced effects

$B_s \to \mu^+\mu^-$ *[M. Beneke, etc. 17 & 19]*

$B_s \to \tau^+\tau^-$ *[Y.K.Huang, Y.L.Shen, X.C.Zhao, SHZ 23]*

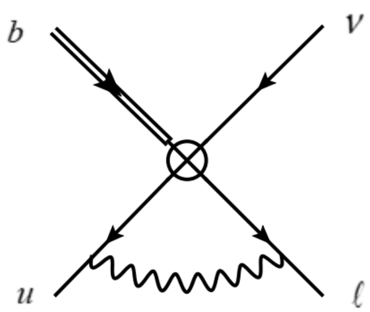
→ The hadronic currents become non-local, the corresponding hadronic matrix element

$$\langle 0 | \bar{q}_s(vn_-)\, Y(vn_-,0) \frac{\not{n}_+}{2} P_L\, h_v(0) | B \rangle$$

is light cone distribution $\sim \phi_B(\omega)$, and demonstrates explicitly **the structure-dependent QED effects**

Main challenges in formulating a factorization theorem

- Quark current $\bar{q}_s \gamma^\mu P_L h_v$ is not gauge invariant under QED ☹️



$$\bar{q}_s \gamma^\mu P_L h_v \longrightarrow \bar{q}_s \gamma^\mu P_L h_v S_{n_-}^{(\ell)\dagger}$$

add a **Wilson line** $S_{n_-}^{(\ell)\dagger}$ to account for soft photon interactions with charged lepton
→ anomalous dimension sensitive to **IR regulators**

- Beyond leading power convolutions have **endpoint divergences** ☹️

[Feldmann, Gubernari, Huber, Neubert, Seitz 2022; Hurth, Neubert, Szafron 2023]

cannot be dealt with using standard renormalization techniques and require appropriate subtractions.

e.g. "refactorization-based subtraction (RBS) scheme" in $B_u \to \mu\,\nu$

QED corrections for $B_u \to \tau\nu$

A muti-scale process

focus on $B_u \to \tau\nu$ new scales appear in the present of QED effects

m_W

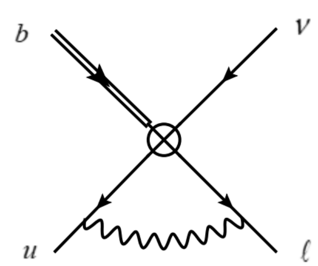
$m_b \sim \mu_h$

$\sqrt{m_b \Lambda_{\rm QCD}} \sim \mu_{hc}$

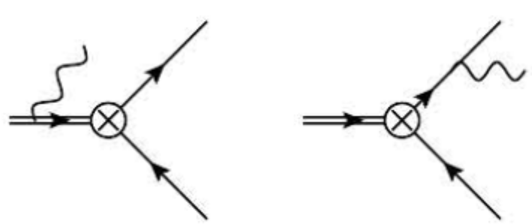
$\Lambda_{\rm QCD} \sim \mu_s, \mu_{sc}$

$E_\gamma \sim \mu_{us}$

$\frac{m_\tau}{m_b} E_\gamma \sim \mu_{usc}$

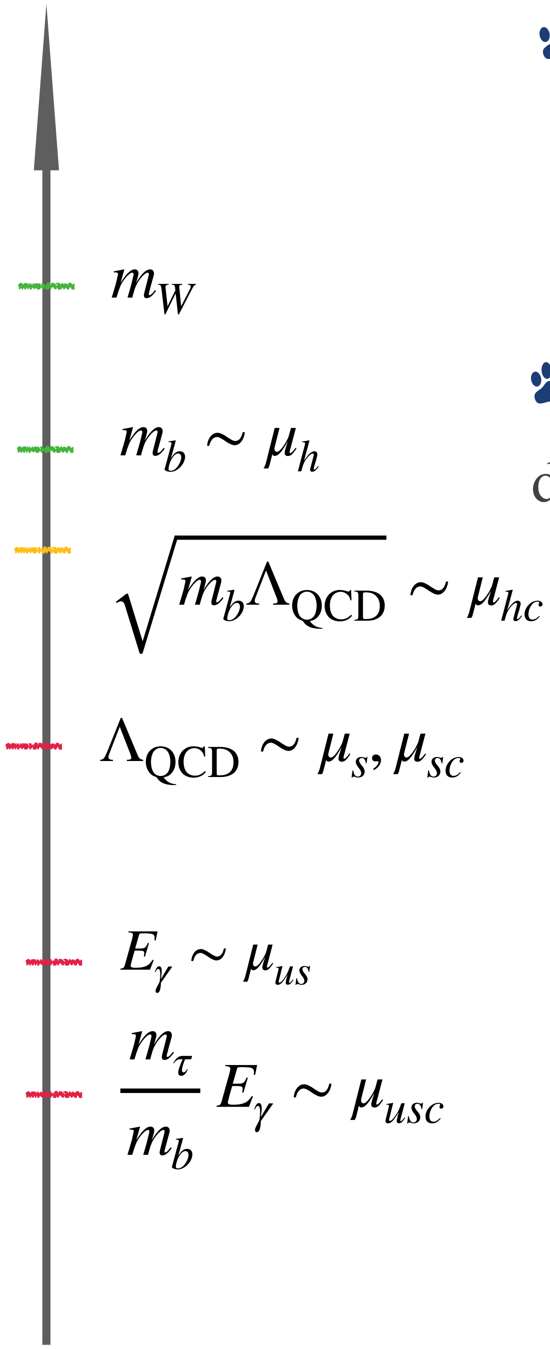


- relevant modes for virtual QED corrections

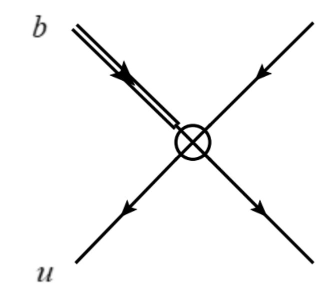


- relevant modes for real QED corrections

A muti-scale process

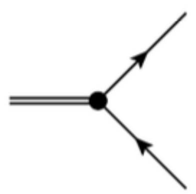


- QED for $\mu > m_b$ included in Effective weak Hamiltonian



$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} K_{\rm EW}(\mu) V_{ub}\, (\bar{u}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell)$$

- Ultrasoft photons $\mu \ll \Lambda_{\rm QCD}$ see B meson as point-like particle, description as a Yukawa theory



[Isidori, Nabeebaccus, Zwicky 2020; Zwicky 2021; Dai, Kim, Leibovich 2021]

A muti-scale process

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$m_b \sim \mu_h$

$\sqrt{m_b \Lambda_{\rm QCD}} \sim \mu_{hc}$

$\Lambda_{\rm QCD} \sim \mu_s, \mu_{sc}$

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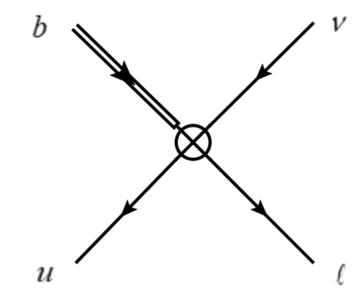

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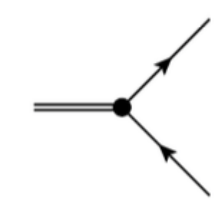
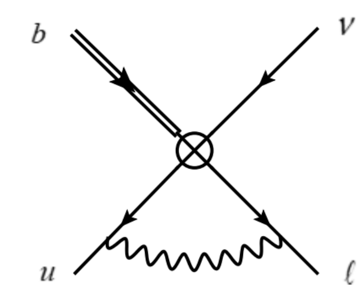
[Isidori, Nabeebaccus, Zwicky 2020; Zwicky 2021; Dai, Kim, Leibovich 2021]



- Intermediate scale $\Lambda_{\rm QCD} < \mu < m_b$, virtual photons can **resolve the structure of *B* meson**



different scale hierarchies require different effective field-theory constructions

In this talk, we focus on the **virtual QED corrections to** $B_u \to \tau \nu$

modes

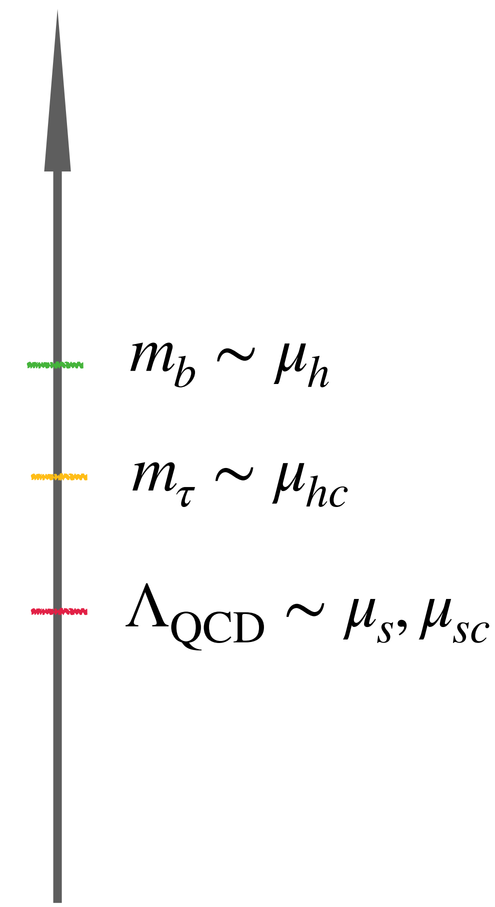
- Relevant modes $p \sim (n_+ p, n\_p, p_\perp)$ for **virtual** QED corrections:

Expansion parameters:

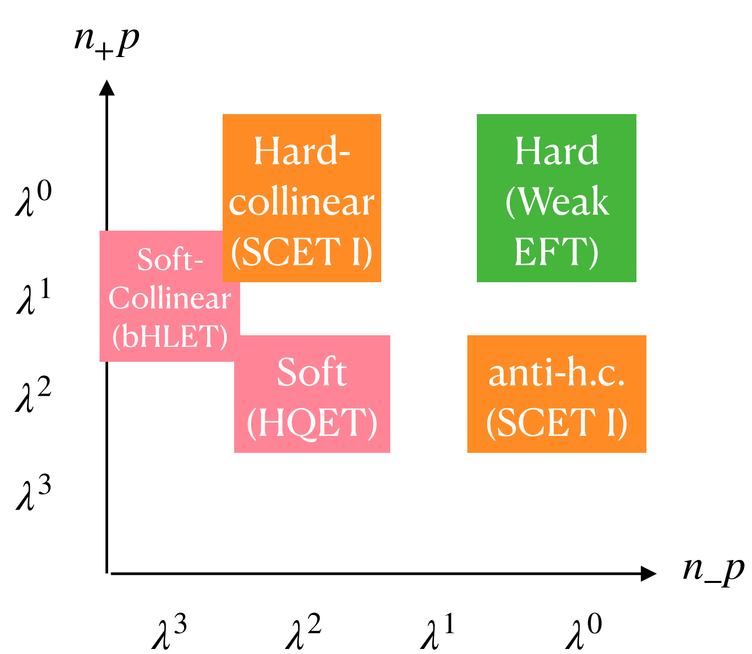
- Hard $(1, 1, 1)$
- Hard-collinear $(1, \lambda^2, \lambda)$
- Soft $(\lambda^2, \lambda^2, \lambda^2)$
- Soft-collinear $\lambda^2 (1/b, b, 1) \sim (\lambda, \lambda^3, \lambda^2)$

$$\lambda^2 = \frac{\Lambda_{\mathrm{QCD}}}{m_b}$$

$$b = \frac{m_\tau}{m_b} \sim \lambda$$



- Factorization requires a number of different EFT constructions (HQET, SCET I, bHLET…)



- The factorized amplitude can be expressed as

$$\mathscr{A}^{\mathrm{virtual}} = \sum_i H_i J_i S_i + \sum_j H_j \otimes_u J_j \otimes_\omega S_j$$

From Fermi theory to HQET × SCET_I

m_W

$m_b \sim \mu_h$ ☑ Fermi Theory

$m_\tau \sim \mu_{hc}$ ☐ …

$\Lambda_{\text{QCD}} \sim \mu_s$

$E_\gamma \sim \mu_{us}$

$\frac{m_\tau}{m_b} E_\gamma \sim \mu_{usc}$

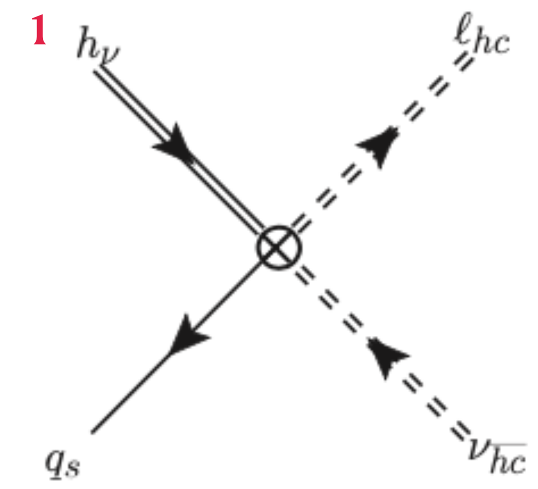
$$\mathscr{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ub} [\bar{q}\, \gamma_\mu (1-\gamma_5)\, b][\bar{\ell}\, \gamma^\mu (1-\gamma_5)\, \nu]$$

$$\equiv -\frac{G_F}{\sqrt{2}} V_{ub}\, Q_1 \,,$$

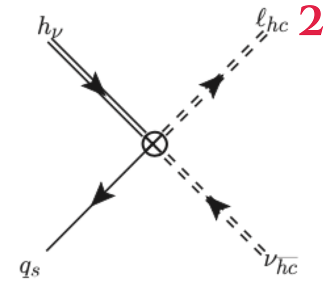
Fermi theory → HQET × SCET$_\mathrm{I}$

- The ***b* quark** can be described by a soft HQET field

$$b(x) \to e^{-im_b v\cdot x}(1 + \mathcal{O}(\lambda^2))\, h_v(x)$$



Fermi theory → HQET × SCET_I



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$$b(x) \to e^{-im_b v\cdot x}(1 + \mathcal{O}(\lambda^2))\, h_v(x)$$

- **Leptonic** fields can have large momenta, but small invariant mass, needs SCET

Relevant modes $p \sim (n_+p, n_-p, p_\perp)$

with expansion parameters:

$$\lambda^2 = \frac{\Lambda_{\text{QCD}}}{m_b}$$

- Hard-collinear $p \sim (1, \lambda^2, \lambda)$

  →given by the lepton virtuality

- Soft $p \sim (\lambda^2, \lambda^2, \lambda^2)$

  →given by the spectator virtuality

Fermi theory $\rightarrow$ HQET $\times$ SCET$_{\rm I}$

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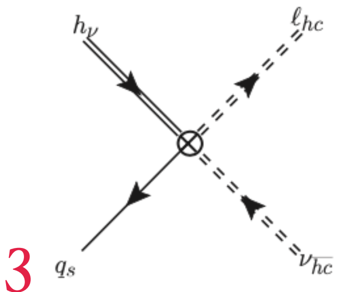

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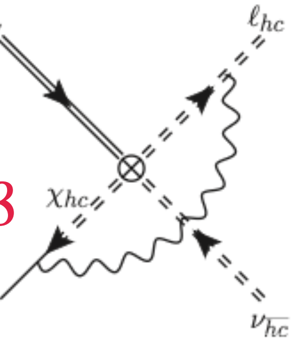

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- **Leptonic** fields can have large momenta, but small invariant mass, needs SCET

  In SCET$_{\rm I}$, subleading power description for the different modes of **the spectator and the lepton** :

$$\ell(x) \rightarrow \left(1 + \frac{i\, D_\perp + m_\ell}{in_+ D_C}\frac{\not{n}_+}{2}\right)\xi_C^{(\ell)}(x) + \left(1 + \frac{1}{in_- D_s}Q_q\ A_{C\perp}\frac{\not{n}_-}{2}\right)\ell_s(x)$$

$$q(x) \rightarrow \left(1 + \frac{i\, D_\perp}{in_+ D_C}\frac{\not{n}_+}{2}\right)\xi_C^{(q)} + \left(1 + \frac{1}{in_- D_s}Q_q\ A_{C\perp}\frac{\not{n}_-}{2}\right)q_s(x)$$

Relevant modes $p \sim (n_+p, n_-p, p_\perp)$

with expansion parameters:

$$\lambda^2 = \frac{\Lambda_{\rm QCD}}{m_b}$$

- Hard-collinear $\quad p \sim (1, \lambda^2, \lambda)$

  $\rightarrow$given by the lepton virtuality

- Soft $\quad p \sim (\lambda^2, \lambda^2, \lambda^2)$

  $\rightarrow$given by the spectator virtuality

fields with power counting parameter

$$h_v, q_s \sim \lambda^3$$

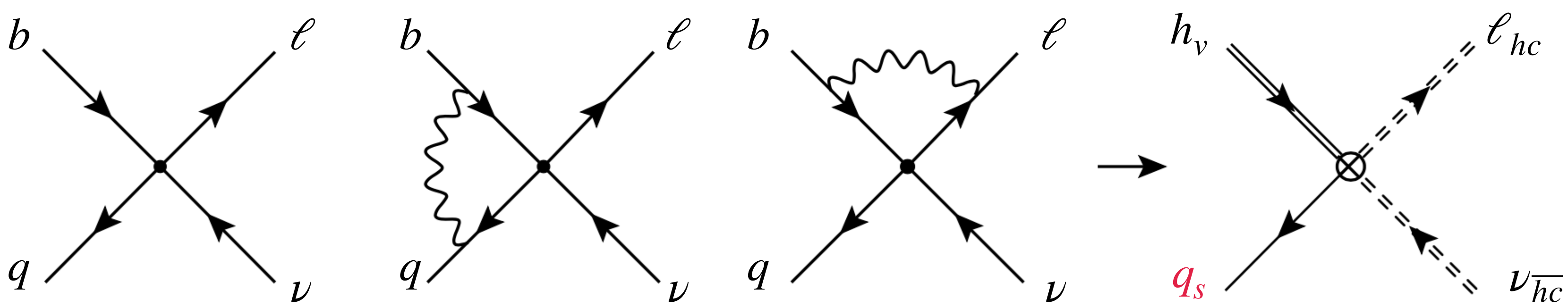
$$\ell_{hc}, \nu_{\overline{hc}}, \chi_{hc}^{(q)} \sim \lambda$$

$$\mathscr{A}_{hc}^{\perp} \sim \lambda$$

Construction of HQET × SCET_I operator

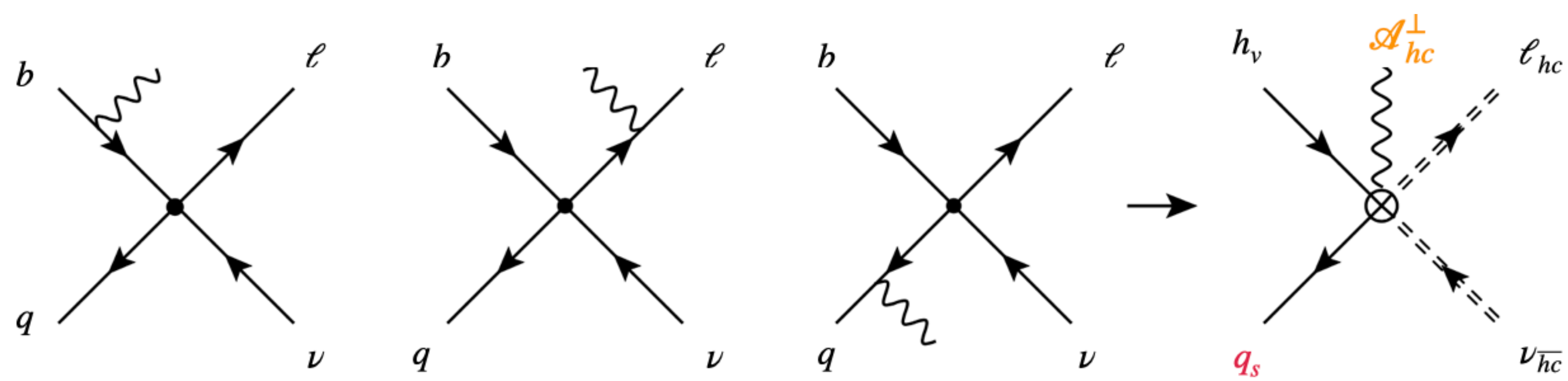
- **three classes operator are relevant**

1. local operator with soft spectator $\mathcal{O}_A$



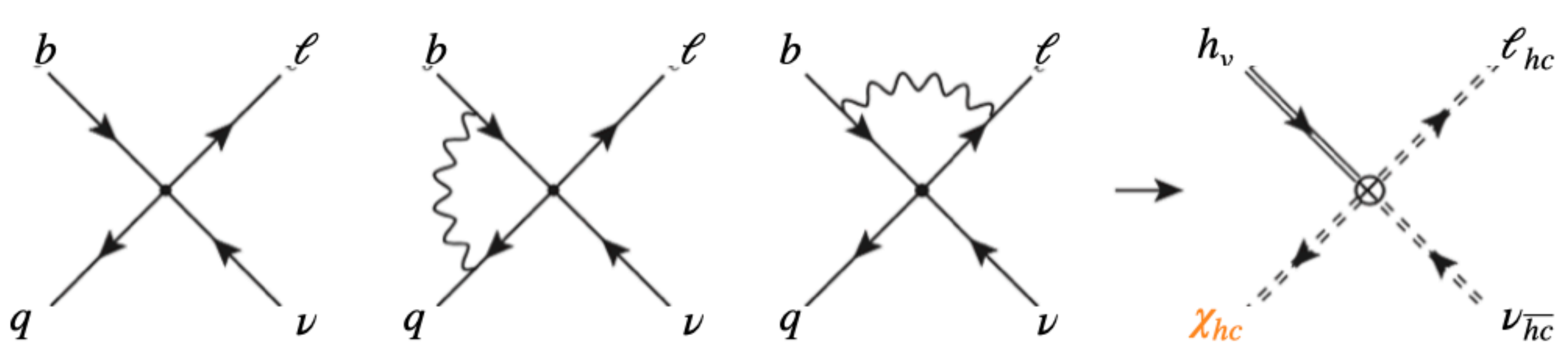
$$\mathcal{O}_A = [\bar{q}_s \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]$$

2. local operator with soft spectator and hard-collinear photon $\mathcal{O}_B$



$$\mathcal{O}_B = [\bar{q}_s \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}] \quad \mathcal{A}_{hc}^{\perp}$$

3. nonlocal operator with hard-collinear spectator $\mathcal{O}_C$



$$\mathcal{O}_C = [\ \chi_{hc} \ \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]$$

Hard function at $\mu \sim m_b$

m_W

$m_b \sim \mu_h$ ☑ Fermi Theory

$m_\tau \sim \mu_{hc}$ ☑ $\mathrm{HQET} \times \mathrm{SCET_I}$

$\Lambda_{\mathrm{QCD}} \sim \mu_s$ ☐ $\ldots$

SCET$_\mathrm{I}$ → bHLET $\qquad \mu \sim m_b \Lambda_{\mathrm{QCD}}$

lower the virtuality to remove the hard-collinear mode to reach to bHLET

- heavy tau filed become to a soft-collinear (sc) field in boosted HLET after integrating m_τ

$$\ell_{hc} \to \mathrm{e}^{-i m_\ell v_\ell \cdot x}\left(1 + b\,\frac{\not{n}_+}{2}\right)\ \ell_{sc}$$

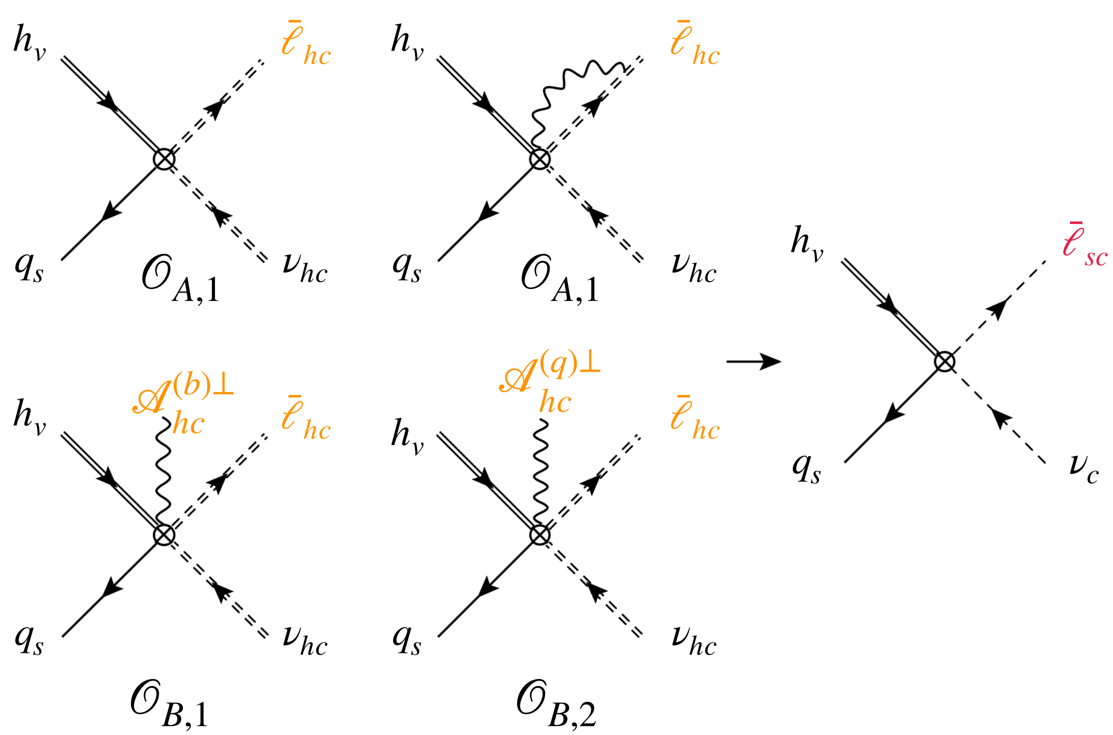
boosted parameters:

$$b = \frac{m_\tau}{m_b} \sim \lambda \qquad \lambda^2 = \frac{\Lambda_{\mathrm{QCD}}}{m_b}$$

- Soft-collinear $\quad p \sim \lambda^2\left(\frac{1}{b}, b, 1\right)$

→soft scale to τ boosted in the B frame

$$p' \sim (\lambda^2, \lambda^2, \lambda^2)$$



$$\mathcal{J}_m^{A,B} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v]\, S_{n_-}^{(\ell)\dagger}\, [\bar{\ell}_{sc} P_L \nu_{\bar{c}}]$$

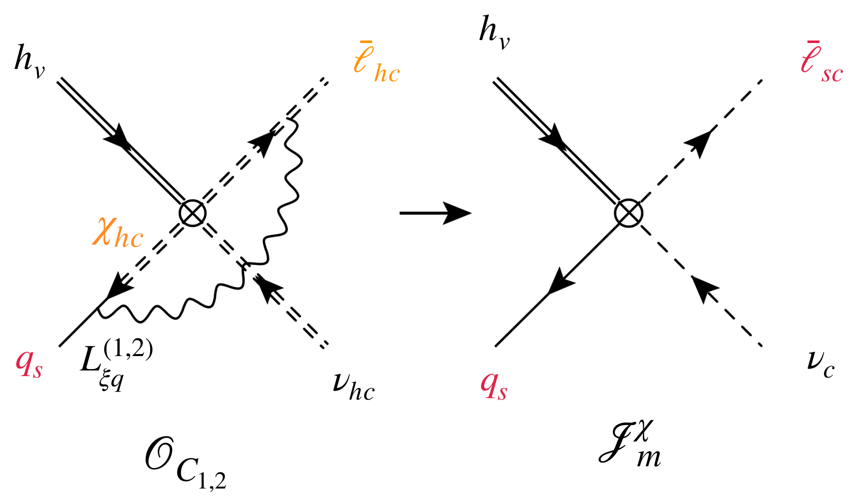
$\text{SCET}_{\text{I}} \to \text{HQET} \times \text{bHLET}$

- $\chi_{hc}^{(q)} \to q_s$ by inserting the following NLP and NNLP interactions

$$\mathcal{O}_C = [\ \chi_{hc}^{(q)} \ \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]$$

$$L_{\xi q}^{(1)}(x) = \bar{q}_s(x_-)[W_{\xi C} W_C]^\dagger(x)\, i \not{D}_{C\perp}\, \xi_C(x) + \text{h.c.}$$

$$L_{\xi q}^{(2)}(x) = \bar{q}_s(x_-)\left[W_{\xi,hc} W_{hc}\right]^\dagger(x)\ \left(i\,n_- D_{hc} + i \not{D}_{hc\perp}(i\,n_+ D_{hc})^{-1} i \not{D}_{hc\perp}\right) \frac{\not{n}_+}{2}\, \xi_{hc}(x) + \ldots$$



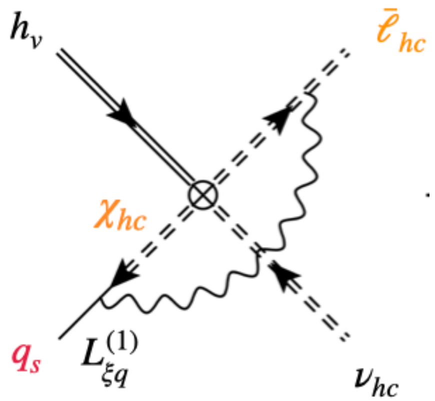
intermediate propagators introduce
non-local operators

$$\mathcal{J}_m^\chi(v) = [\bar{q}_s(v\,n_-)\, Y(v\,n_-,0)\, \frac{\not{n}_+}{2} P_L\, h_v(0)]\, S_{n_-}^{(\ell)\dagger}(0)\, [\bar{\ell}_{sc}(0)\, P_L\, \nu_{\bar{c}}(0)]$$

The helicity suppression be relaxed or not

$$\mathcal{J}_m^{\chi}(v) = [\bar{q}_s(v\,n_-)\,Y(v\,n_-,0)\,\frac{\not{n}_+}{2}\,P_L\,h_v(0)]\,S_{n_-}^{(\ell)\dagger}(0)\,[\bar{\ell}_{sc}(0)\,P_L\,\nu_{\bar{c}}(0)]$$

Nonlocal annihilation can **probe the meson structure**, and **possibly overcome** the **helicity suppression**



$$\langle 0|\,q_s\,\frac{1}{in_-\partial_s}\ldots h_v\,|B\rangle \sim \frac{1}{\lambda_B} \sim \frac{1}{\Lambda_{\rm QCD}}$$

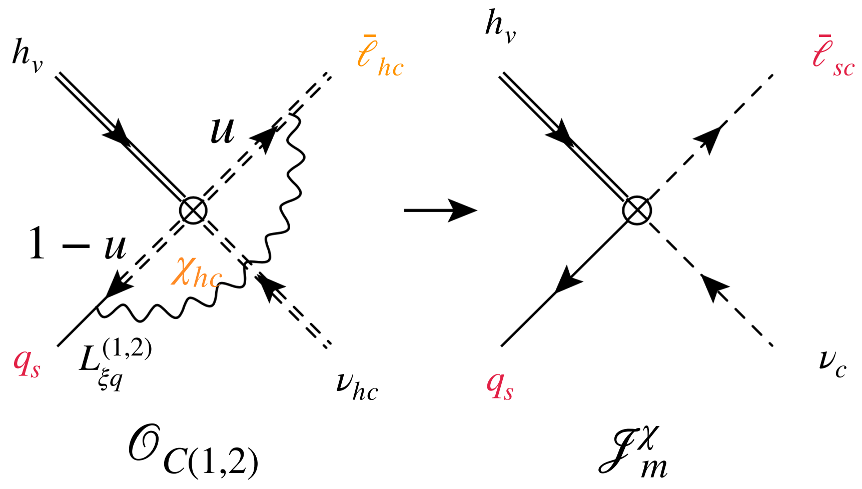
Happens for $B_s \to \ell^+\ell^-$, but not for $B_u \to \ell\,\nu$ with left-handed currents

[Beneke, Bobeth, Szafron 2017 & 2019, Y.K.Huang, Y.L.Shen, X.C.Zhao, SHZ 2023]

$$[\bar{u}\,\frac{\not{n}_+}{2}\,\gamma_\perp^\mu\gamma_\perp^\nu\,P_L\,b][\bar{\ell}\,\gamma_{\perp\mu}\gamma_{\perp\nu}\,(\frac{v-a\,\gamma_5}{2})\,\nu] = 2\,(v\,-\,a)\,[\bar{u}\,\frac{\not{n}_+}{2}\,P_L\,b][\bar{\ell}\,P_R\,\nu]$$

$\rightarrow$ No power enhancement in $B_u \to \ell\,\nu$!

$\text{SCET}_\text{I} \to \text{HQET} \times \text{bHLET}$



$$J_{\chi,1}^{(1)} = \frac{\alpha_{\text{em}}}{\pi} Q_\ell Q_u \frac{m_\ell}{n_+ p_\ell} u \left[\ln \frac{\mu^2}{\bar{u}^2 m_b n_- p_\ell} - \frac{1+r}{r} \ln(1+r) \right] \theta(u)\theta(\bar{u})$$

$$J_{\chi,2}^{(1)} = \frac{\alpha_{\text{em}}}{\pi} Q_\ell Q_u \frac{m_\ell}{n_+ p_\ell} \bar{u} \left[\ln \frac{\mu^2}{\bar{u}^2 m_b n_- p_\ell} - \frac{1+r}{r} \ln(1+r) + \frac{1}{2\bar{u}^2 r} \ln(1+r) \right] \theta(u)\theta(\bar{u})$$

$$r = \frac{u}{\bar{u}} \frac{\omega m_B}{m_\ell^2}$$

$\to$ **No endpoint div.** ($1/u \to \infty$, when u$\to$ 0) in $B_u \to \tau \nu$ when convoluting to hard function!

subtractions scheme independence

Factorization Formula

- $m_b \sim \mu_h$ ☑ Fermi Theory
- $m_\tau \sim \mu_{hc}$ ☑ HQET $\times$ SCET$_\mathrm{I}$
- $\Lambda_\mathrm{QCD} \sim \mu_s$ ☑ HQET $\times$ bHLET

$$A^{\text{virtual}}_{B\to\tau\nu} \sim H_{A,B}\, J_{A,B}\, \langle \tau^- \nu \,|\, \mathcal{J}^{A,B}_m \,|\, \bar{B}_u \rangle + \int_0^1 du\, H_\chi(u) \int_0^\infty d\omega\, J_\chi(u;\omega)\, \langle \tau^- \nu \,|\, \mathcal{J}^{\chi}_m \,|\, \bar{B}_u \rangle$$

SCET$_\mathrm{I}$ operators with soft spectator (A-type and B-type)

SCET$_\mathrm{I}$ operators with hc spectator (C-type)

HQET $\times$ bHLET operators

$$\mathcal{J}^{A,B}_m = [\bar{q}_s \frac{\not{n}_+}{2} P_L\, h_v]\, S^{(\ell)\dagger}_{n_-}\, [\bar{\ell}_{sc}\, P_L\, \nu_{\bar{c}}]$$

$$\mathcal{J}^{\chi}_m(v) = [\bar{q}_s(v\, n_-)\, Y(v\, n_-, 0) \frac{\not{n}_+}{2} P_L\, h_v(0)]\, S^{(\ell)\dagger}_{n_-}(0)\, [\bar{\ell}_{sc}(0)\, P_L\, \nu_{\bar{c}}(0)]$$

Generalized decay constant and LCDA

- **Modified** B-meson decay constant and LCDA

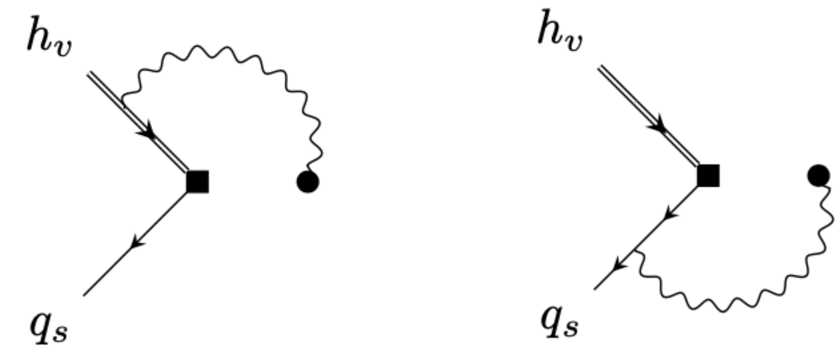
$$\mathcal{J}_m^{A,B} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v] S_{n_-}^{(\ell)\dagger} [\bar{\ell}_{sc} P_L \nu_{\bar{c}}]$$

$$\mathcal{J}_m^{\chi}(v) = [\bar{q}_s(v n_-) Y(v n_-,0) \frac{\not{n}_+}{2} P_L h_v(0)] S_{n_-}^{(\ell)\dagger}(0) [\bar{\ell}_{sc}(0) P_L \nu_{\bar{c}}(0)]$$

- Factorization anomaly

$$\mathcal{J}_S^{A,B} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v] S_{n_-}^{(\ell)\dagger}$$

$$\mathcal{J}_S^{\chi}(v) = [\bar{q}_s(v n_-) Y(v n_-,0) \frac{\not{n}_+}{2} P_L h_v(0)] S_{n_-}^{(\ell)\dagger}(0)$$

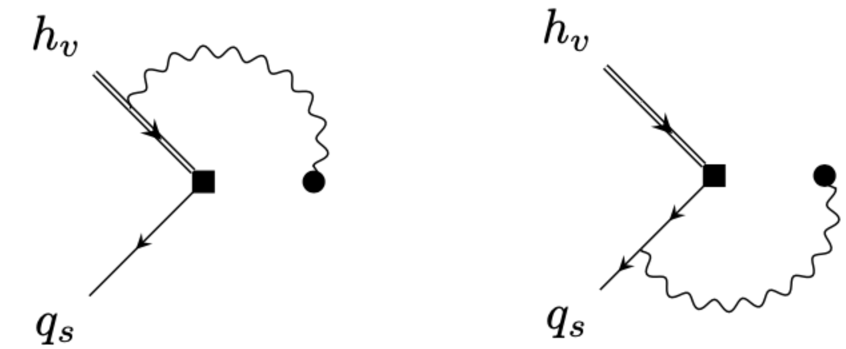


Soft photon decoupling from lepton

Additional QED soft Wilson lines

$$S_r^{(i)}(x) = \exp\left[-i\,e\,Q_i \int_0^\infty ds\, r\cdot A_s(x+s\cdot r)\right]$$

Generalized decay constant and LCDA



Soft photon decoupling from lepton

- **Modified** B-meson decay constant and LCDA

$$\mathcal{J}_m^{A,B} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v] \, S_{n_-}^{(\ell)\dagger} \, [\bar{\ell}_{sc} P_L \nu_{\bar{c}}]$$

$$\mathcal{J}_m^{\chi}(v) = [\bar{q}_s(v n_-) \, Y(v n_-, 0) \frac{\not{n}_+}{2} P_L h_v(0)] \, S_{n_-}^{(\ell)\dagger}(0) \, [\bar{\ell}_{sc}(0) P_L \nu_{\bar{c}}(0)]$$

Additional QED soft Wilson lines

$$S_r^{(i)}(x) = \exp\left[-i e Q_i \int_0^\infty ds \, r \cdot A_s(x + s \cdot r) \right]$$

- Factorization anomaly

$$\mathcal{J}_S^{A,B} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v] \, S_{n_-}^{(\ell)\dagger}$$

$$\mathcal{J}_S^{\chi}(v) = [\bar{q}_s(v n_-) \, Y(v n_-, 0) \frac{\not{n}_+}{2} P_L h_v(0)] \, S_{n_-}^{(\ell)\dagger}(0)$$

- Refactorization

$$\mathcal{F}_B \equiv \frac{\langle 0 | \mathcal{J}_S^{A,B} | B \rangle}{\langle 0 | [S_{v_B}^{(B)}(0) \, S_{n_-}^{(\ell)\dagger}(0)] | 0 \rangle}$$

$$\mathcal{F}_B \Phi_B(v) \equiv \frac{\langle 0 | \mathcal{J}_S^{\chi}(v) | B \rangle}{\langle 0 | [S_{v_B}^{(B)}(0) \, S_{n_-}^{(\ell)\dagger}(0)] | 0 \rangle}$$

- For $\alpha_{\rm em} \to 0$, $\mathcal{F}_B$ and $\mathcal{F}_B \Phi_B$ reduces to the standard HQET decay constant and LCDA
- For $\alpha_{\rm em} \neq 0$, high order $\mathcal{F}_B$ and $\mathcal{F}_B \Phi_B$ are **new nonperturbative hadronic parameters**.

Lattice determination ? QCD SR estimate ?

Decay amplitude including virtual QED corrections at NLP+NLO

$$i\,\mathscr{A}^{\text{virtual}} = -\frac{i}{4}\, m_{B_u}\, \bar{u}_{sc}\,(p_\ell)\;P_L\, v_{\bar{c}}\,(p_\nu)\;\left[\sum_{i=A,1}^{B,(1,2)} H_i(\mu)\, J_i(\mu)\, \mathscr{F}_{B_u}(\mu)\; + \right.$$

$$\left. \sum_{j=\chi,1}^{\chi,2} \int_0^1 du\, H_j(u,\mu) \int_0^\infty d\omega\, J_j(u;\omega,\mu)\, \mathscr{F}_{B_u}(\mu)\, \Phi_B(\omega,\mu) \right]$$

$$\mathcal{A}^{\text{virtual}} = \frac{G_F}{\sqrt{2}} V_{ub}\, m_B\, f_B\, \bar{u}_{sc}\, P_L\, v_{\bar{c}} \times \frac{\alpha}{4\pi} Q_\ell\, b \left\{ 2\,Q_b \left[(L - 2\ln s - s - 1)\, L - \frac{1}{2} L^2 + \right.\right.$$
$$2\ln^2 s + \frac{s^2-2}{s-1}\ln s + 2\,\text{Li}_2(1-s) - s - 1 + \left.\frac{\pi^2}{12}\right] - 6\,(2\,Q_b - Q_\ell) -$$
$$Q_\ell \left(\frac{1}{2} L'^2 + \frac{\pi^2}{12} \right) + 4 \int_0^1 du\, (\bar{u}\, Q_b + Q_u) \ln\frac{\mu^2}{\bar{u}^2\, m_\tau^2} -$$
$$\left. 4\,Q_u \int_0^1 du \int_0^\infty d\omega\, \phi_B^+(\omega)\, (1+\bar{u}) \left[\ln\frac{\mu^2}{\bar{u}^2\, m_\tau^2} - \frac{1+r}{r}\ln(1+r) + \frac{1}{(1+\bar{u})\,\bar{u}\, r}\ln(1+r) \right] \right\}$$

- Large (double) logarithms L, L'
- $\phi_B^+(\omega)$ demonstrates explicitly the structure-dependent QED effects

Numerical prediction

- **The non-radiative QED corrections to branching fraction of $B_u \to \tau\nu$ for central values of the parameters**

$$\mathrm{Br}^{(0)}(B_u \to \tau\nu) = \left(0.89_{\mathrm{(LO)}} - 0.01_{\mathrm{(NLO)}}\right) \times 10^{-4}$$

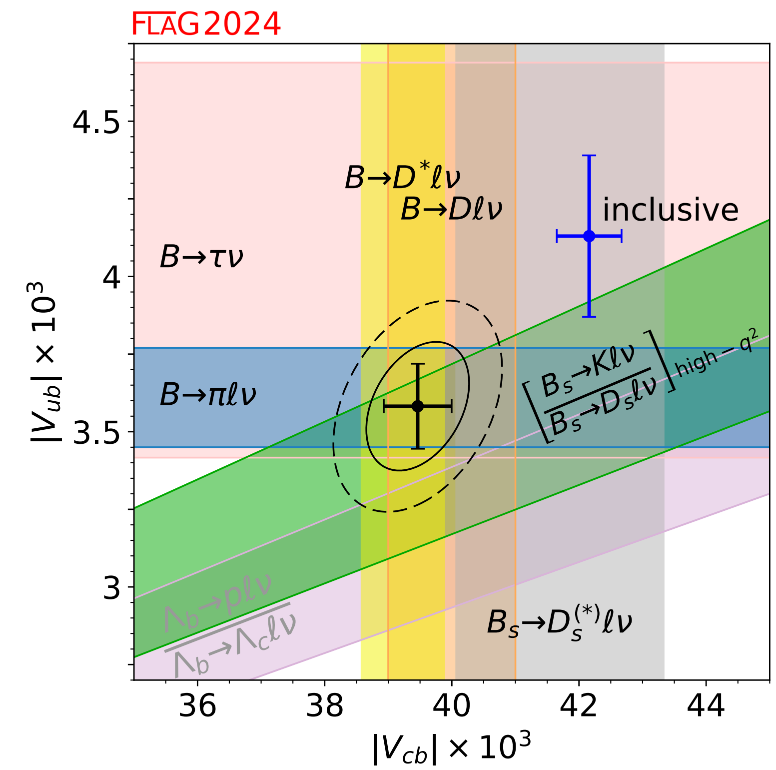
NLP+**NLO**+**LL** QED virtual correction changes the branching fraction by: $\sim 1\,\%$

compete with QCD uncertainties $\delta f_B^2 \sim 1.4\,\%$

- **Determining the CKM matrix element using the latest data from Belle II**

$$|V_{ub}| = \left[\left(4.41 + 0.01_{[\delta_{\mathrm{QED}}]}\right) \pm 0.03(\mathrm{th.})^{+0.73}_{-0.91}(\mathrm{exp.})\right] \times 10^{-3}$$

the experimental uncertainty would reduce to $\sim 0.08 \times 10^{-3}$ based on $\sim$ 50 ab^{-1} of electron-positron collision data



Summary

- We have analyzed exclusive leptonic decay $B_u \to \tau\nu$, which is an important mode for extraction of $|V_{ub}|$.

- Subleading power factorization formula for QED corrections to $B_u \to \tau\nu$ derived in SCET, HQET and bHLET

  no endpoint divergences in this factorization at NLP (due to tau mass) $\to$ **subtractions scheme independence**

  Structure depended QED corrections arising from hard, hard-collinear photons exchange $\to$ important source of **large logarithmic corrections**

  Structure depended QED corrections from leptonic field decoupling produce **generalized *B* decay constant and LCDA** $\to$ new hadronic parameters

- **NLP+NLO+LL** QED virtual correction changes the branching fraction by $\sim 1\ \%$, are critical for $|V_{ub}|$ determination.

Thank you

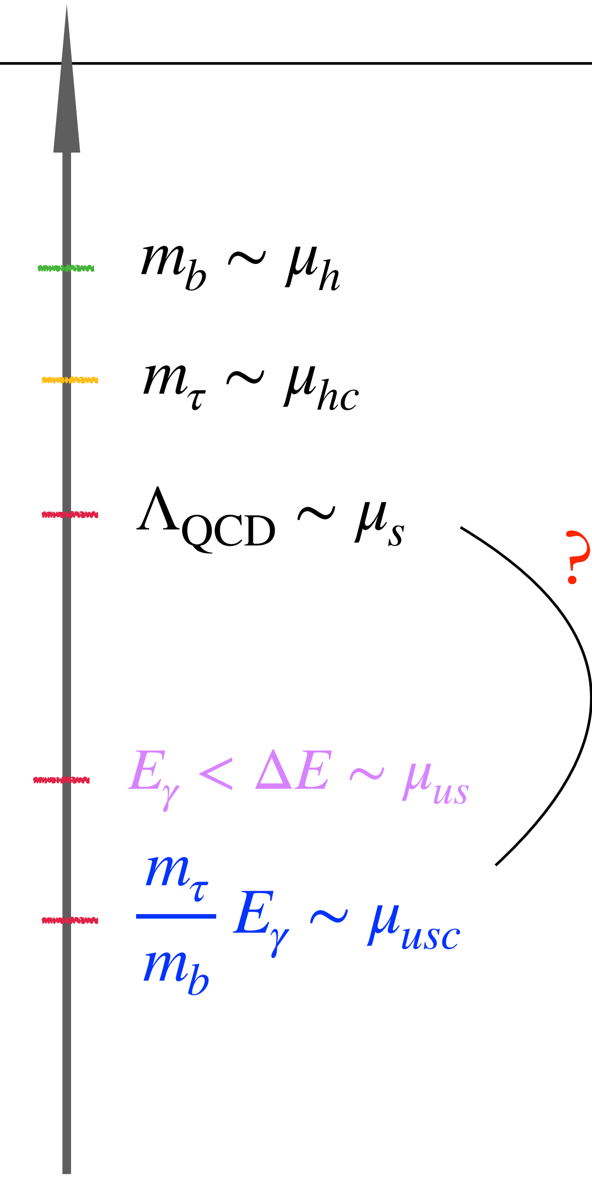
Backup slides

HQET × bHLET → **Low-energy theory ($\mu < \mu_s, \mu_{sc}$)**

power parameters: $\lambda_E^2 = \dfrac{E_\gamma}{m_b} \sim \lambda^4$

- Ultra-soft $p \sim (\lambda_E^2, \lambda_E^2, \lambda_E^2)$
- Ultra-soft-collinear $p \sim \lambda_E^2 (1, b^2, b)$

→ultrosoft scale to τ boosted in the B frame



HQET × bHLET → **Low-energy theory** ($\mu < \mu_s, \mu_{sc}$)

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→ultrosoft scale to τ boosted in the B frame

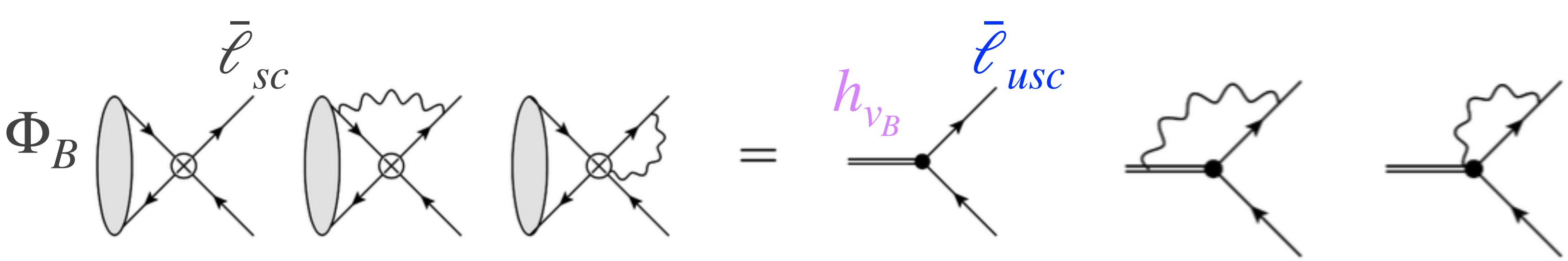
- $\mu < \Lambda_{\rm QCD}$, the hadronic B meson can be described as a heavy scalar effective theory (HSET)

$$\Phi_B(x) \to e^{-im_B v_B \cdot x}\, h_{v_B}(x) \qquad m_B v_B \sim \mu_s$$

- $\mu < \mu_{sc}$, soft-coll. (sc) field in bHLET turned into ultra-soft-coll. one (usc) in bHLET-2

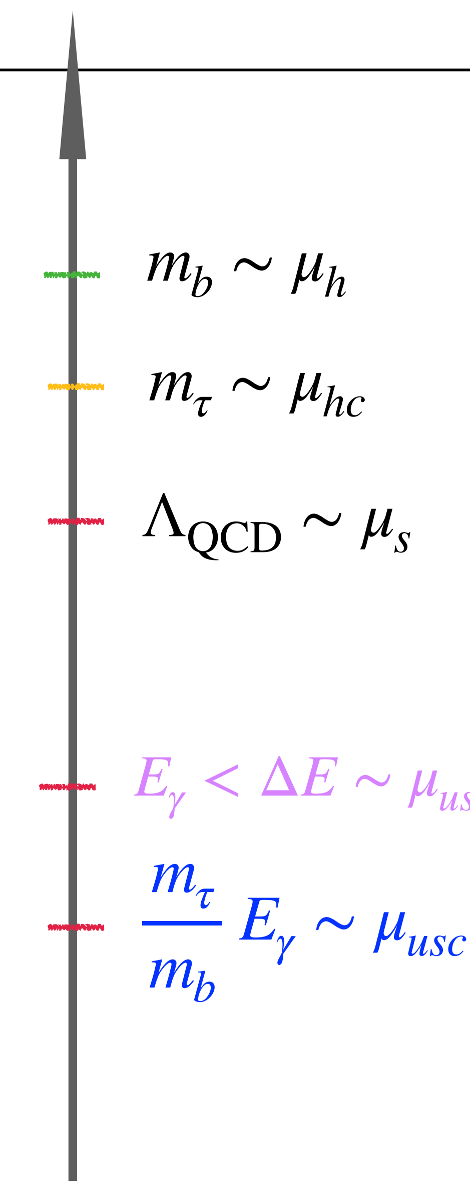
$$\ell_{sc} \to e^{-im_\ell v'_\ell \cdot x}\, \ell_{usc} \qquad m_\ell v'_\ell \sim \mu_{sc}$$

- HQET × bHLET → HSET × bHLET$_{\rm II}$ $\quad \mu \sim \Lambda_{\rm QCD}$



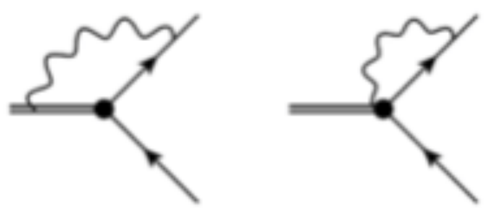
nonperturbative hadronic matrix element before decoupling

$$= y_B\, h_{v_B} [\bar{\ell}_{usc} P_L \nu_{\bar{c}}]$$



Real correction to $\mathrm{HSET} \times \mathrm{bHLET}_{\mathrm{II}}$

- all interactions of the B and the tauon with ultra-soft **and** ultra-soft-collinear photons can be decoupled into **Wilson lines** via field redefinitions, so that the low-energy theory is a theory of only **Wilson lines**



$$h_{v_B}(x) \to S^{(B)}_{v_B} \; C^{(B)}_{n_+} \; h^{(0)}_{v_B}(x)$$

$$\chi^{(\ell)}_{usc}(x) \to S^{(\ell)}_{n_-} \; C^{(\ell)}_{v'_\ell} \; \chi^{(\ell,0)}_{usc}(x)$$

- **Real corrections** are matrix elements of these Wilson lines

$$S(E_\gamma, \mu) = \int_0^\infty d\,\omega_{us} \int_0^\infty d\,\omega_{usc}\, \theta\Big(\frac{E_\gamma}{2} - \omega_{us} - \omega_{usc}\Big) \; W_{us}(\omega_{us}, \mu) \; W_{usc}(\omega_{usc}, \mu)$$

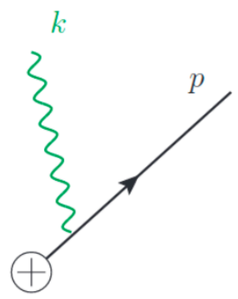
- Real emissions are factorized at the level of the decay rate

$$\Gamma[B_u \to \tau\nu] \sim |A^{\mathrm{virtual}}_{B\to\tau\nu}|^2 \otimes S(E_\gamma, \mu)$$

$$A^{\mathrm{virtual}}_{B\to\tau\nu} \sim H_i \otimes J_i \otimes S \otimes SC$$

- Ultra-soft photons (under the assumption that $\Delta E \ll \Lambda_{\rm QCD}$)

  Based on eikonal approximation,

$$\varepsilon_\mu(k)\bar{u}(p)\gamma^\mu \frac{\not{p}+\not{k}+m}{(k+p)^2-m^2} \to \frac{\varepsilon_\mu(k)p^\mu}{p\cdot k}\bar{u}(p)\,,$$

  note $k^\mu \ll p^\mu, m$

$$\delta_{\rm QED} \sim \frac{\alpha}{\pi}\ln^2\frac{m_B}{m_\ell}$$

  Large logarithmic enhancements can mimic lepton-flavor universality violation

The ultrasoft contribution $\mathcal{S}(v_\ell, v_{\bar{\ell}}, \Delta E)$ is

$$\mathcal{S}(v_\ell, v_{\bar{\ell}}, \Delta E) = \sum_{X_s}\left|\left\langle X_s \left| S^\dagger_{v_\ell}(0)\, S_{v_{\bar{\ell}}}(0)\right| 0\right\rangle\right|^2 \theta(\Delta E - E_{X_s})\,, \tag{5.5}$$

with the one-loop ultrasoft function for massive final particles given in [16] ,

$$S^{(1)}(v_\ell,\, v_{\bar{\ell}},\, \Delta E) = 8\left(1+\frac{1}{2}\ln\frac{m_\tau^2}{m_B^2}\right)\ln\frac{\mu}{2\,\Delta E} - \left(2+\ln\frac{m_\tau^2}{m_B^2}\right)\ln\frac{m_\tau^2}{m_B^2} + 4 - \frac{2}{3}\pi^2, \tag{5.6}$$

The resummed soft function can be achieved by using the QED exponentiation theorem as a approximate, e.g. full soft function can be considered as the exponent of the one-loop result,

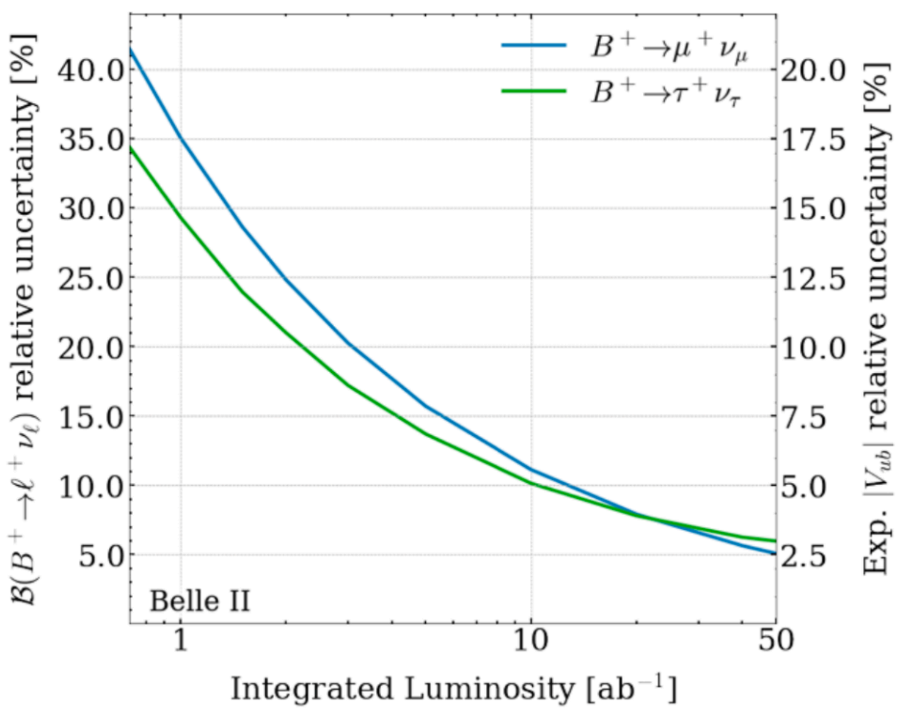
$$\mathcal{S}(v_\ell,\, v_{\bar{\ell}},\, \Delta E) = \exp\left[\frac{\alpha_{\rm em}}{4\,\pi}\, Q_\ell^2\, S^{(1)}(v_\ell,\, v_{\bar{\ell}},\, \Delta E)\right]. \tag{5.7}$$

- $\phi_B^+(\omega)$:

$$\phi_+^B(\omega,\mu) = N\,\frac{\omega}{\omega_0^2}\,e^{-\omega/\omega_0} + \theta(\omega-\omega_t)\,\frac{C_F\alpha_s}{\pi\omega} \times \left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}_{\rm DA}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right], \quad (29)$$

which exhibits a negative radiation tail
for $\omega \gg \mu$

Seung J. Lee and Matthias Neubert hep-ph/ 0509350



[Belle II Physics Book]

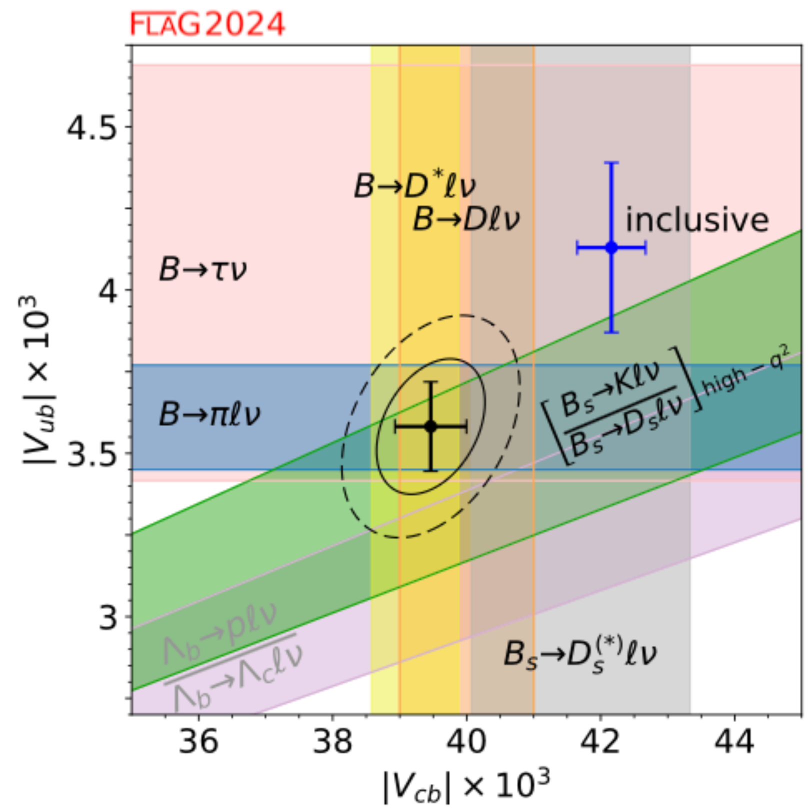


Figure 1: Projection of uncertainties on the branching fractions $\mathcal{B}(B^+ \to \mu^+ + \nu_\mu)$ and $\mathcal{B}(B^+ \to \tau^+ + \nu_\tau)$. The corresponding uncertainty on the experimental value of $|V_{ub}|$ is shown on the right-hand vertical axis.

TAB. I. Published results for $\mathcal{B}(B^+ \to \tau^+ \nu_\tau)$ by Belle, *BABAR* and the PDG average.

Experiment	Tag	$\mathcal{B}(10^{-4})$
Belle	Hadronic	$0.72^{+0.27}_{-0.25} \pm 0.11$
BABAR	Hadronic	$1.83^{+0.53}_{-0.49} \pm 0.24$
Belle	Semileptonic	$1.25 \pm 0.28 \pm 0.27$
BABAR	Semileptonic	$1.8 \pm 0.8 \pm 0.2$
PDG		1.09 ± 0.24

modes

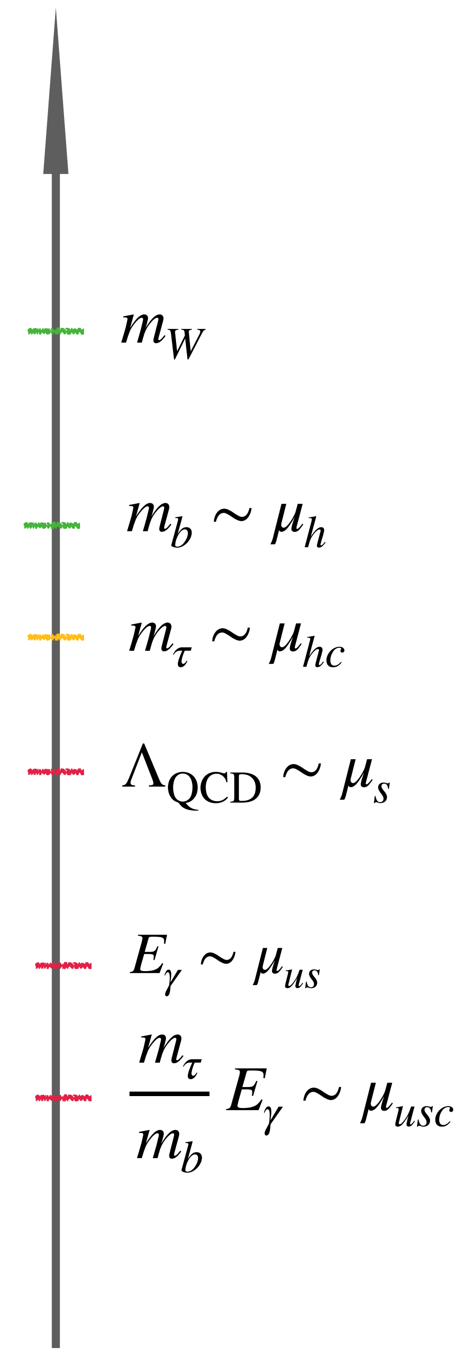
- Relevant modes $k \sim (n_+k, n\_k, k_\perp)$ for **virtual** QED corrections:

Expansion parameters:

- Hard $(1,1,1)$
- Hard-collinear $(1,\lambda^2,\lambda)$
- Soft $(\lambda^2,\lambda^2,\lambda^2)$
- Soft-collinear $\lambda^2(1,b^2,b) \sim (\lambda^2,\lambda^4,\lambda^3)$

$$\lambda^2 = \frac{\Lambda_{\mathrm{QCD}}}{m_b}$$

$$b = \frac{m_\tau}{m_b} \sim \lambda$$



- Relevant modes for **real** QED corrections:
  - ultrasoft $(\lambda_E^2,\lambda_E^2,\lambda_E^2)$
  - ultrasoft soft-collinear $\lambda_E^2(1,b^2,b)$

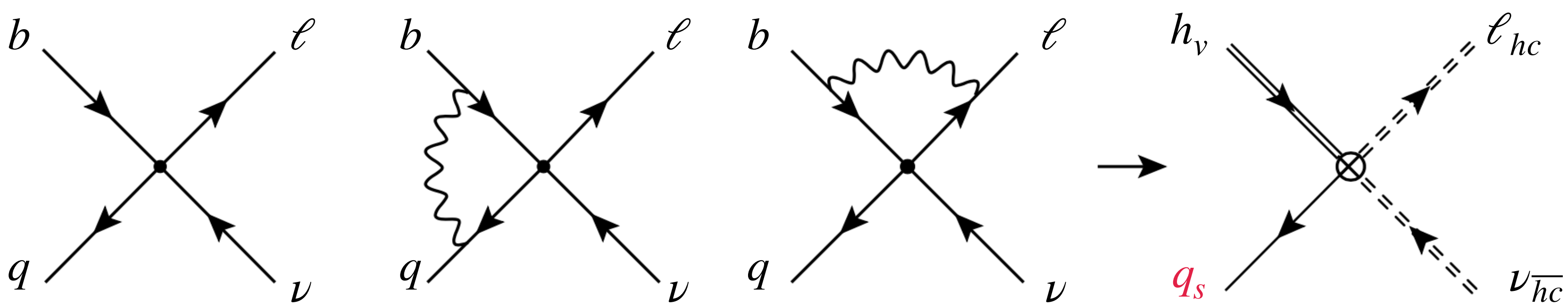
$$\lambda_E^2 = \frac{E_\gamma}{m_b} \sim \lambda^4$$

Construction of HQET $\times$ SCET$_\mathrm{I}$ operator

only two irreducible Dirac structures

1. local operator with soft spectator $\mathcal{O}_A$

$$[\bar{\ell}_{hc}\,\Gamma_\ell\,P_L \nu_{\overline{hc}}] \text{ with } \Gamma_\ell = 1, \gamma_\mu^\perp$$



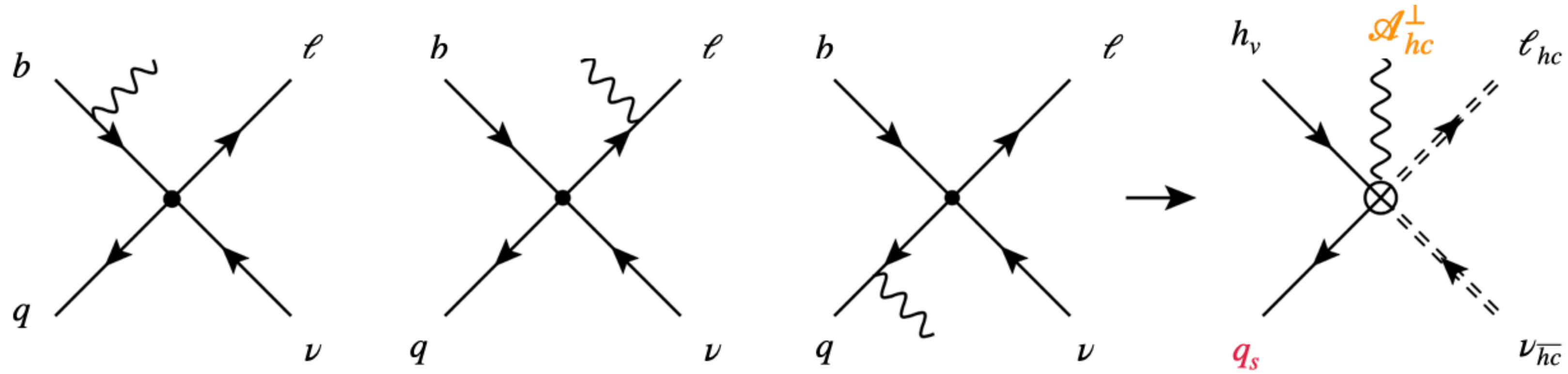
$$\mathcal{O}_A^{(\lambda)} = [\bar{q}_s \ldots h_v]\,[\bar{\chi}_{hc}^{(\ell)} \ldots \chi_{\overline{hc}}^{(\nu)}]$$

$$\mathcal{O}_{A,1}^{(9)} = m_\ell\,[\bar{q}_s \frac{\not{n}_+}{2} P_L\, h_v]\,[\bar{\ell}_{hc} \frac{1}{i\,n_+ \overleftarrow{\partial}_{hc}} P_L\, \nu_{\overline{hc}}]$$

$$\mathcal{O}_{A,2}^{(8)} = [\bar{q}_s \gamma_{\mu\perp} P_L\, h_v]\,[\bar{\ell}_{hc} \gamma_\perp^\mu P_L\, \nu_{\overline{hc}}]$$ ✗

Construction of HQET $\times$ SCET$_{\mathrm{I}}$ operator

2. local operator with soft spectator and hard-collinear photon $\mathcal{O}_B$



$$\mathcal{O}_B^{(\lambda)} = [\bar{q}_s \dots h_v]\,[\bar{\ell}_{hc} \dots \nu_{\overline{hc}}]\ \mathcal{A}^{\perp}_{hc}$$

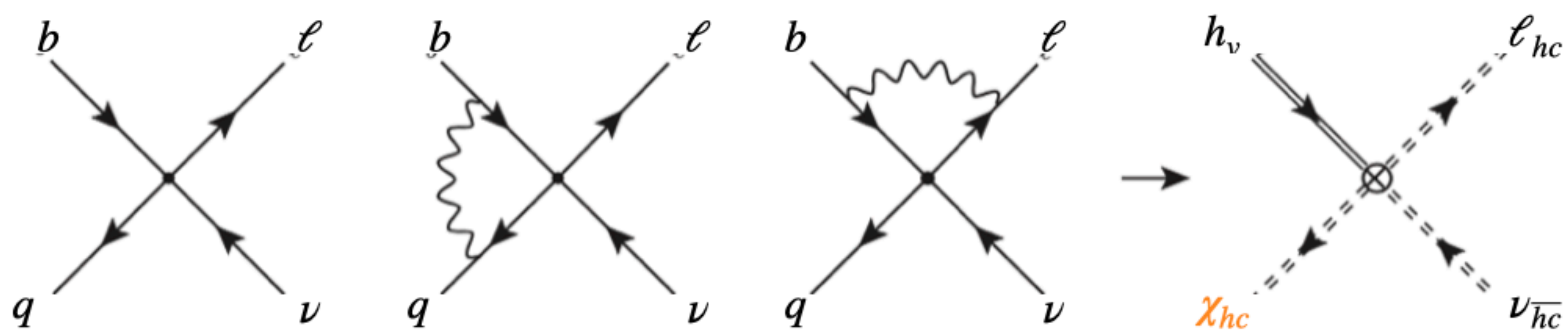
$$\mathcal{O}^{(9)}_{B,1} = \frac{1}{i\,n_+\partial_{hc}}\,[\bar{q}_s \frac{\not{n}_-}{2}\gamma_{\mu\perp}\ \not{\mathcal{A}}^{(b)}_{hc\perp} P_L h_v]\,[\bar{\ell}_{hc}\gamma^{\mu}_{\perp} P_L \nu_{\overline{hc}}]$$

$$\mathcal{O}^{(9)}_{B,2} = [\bar{q}_s\ \not{\mathcal{A}}^{(q)}_{hc\perp}\frac{1}{i\,n_+\overleftarrow{\partial}_{hc}}\frac{\not{n}_+}{2}\gamma_{\mu\perp} P_L h_v]\,[\bar{\ell}_{hc}\ \gamma^{\mu}_{\perp} P_L \nu_{\overline{hc}}]$$

$$\mathcal{O}^{(9)}_{B,3} = \frac{1}{i\,n_+\partial_{hc}}\,[\bar{q}_s \frac{\not{n}_+}{2} P_L h_v]\,[\bar{\ell}_{hc}\ \ \not{\mathcal{A}}^{(\ell)}_{hc\perp} P_L \nu_{\overline{hc}}]$$

Construction of HQET $\times$ SCET$_{\mathrm{I}}$ operator

3. nonlocal operator with hard-collinear spectator $\mathcal{O}_C$



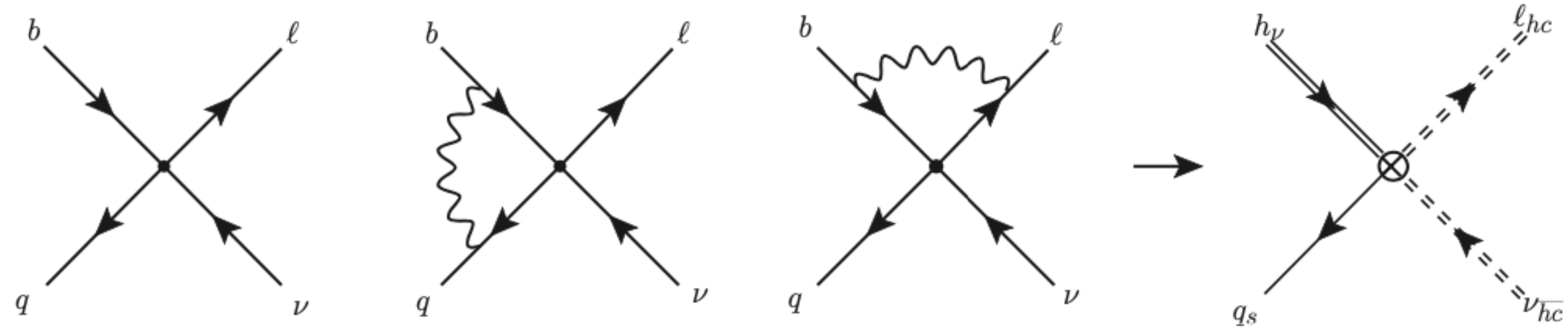
$$\mathcal{O}_C = [\ \chi_{hc} \quad \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]$$

$$\mathcal{O}^{(6)}_{C,1}(s,t) = [\bar{\chi}^{(q)}_{hc}(sn_+)\,\gamma_{\mu\perp}\,P_L\,h_v(0)]\,[\bar{\ell}_{hc}(tn_+)\,\gamma^{\mu}_{\perp}\,P_L\,\nu_{\overline{hc}}(0)]$$

$$\mathcal{O}^{(7)}_{C,2}(s,t) = m_\ell\,[\bar{\chi}^{(q)}_{hc}(sn_+)\,\frac{\not{n}_+}{2}\,P_L\,h_v(0)]\,[\bar{\ell}_{hc}(tn_+)\,\frac{1}{i\,n_+\overleftarrow{\partial}_{hc}}\,P_L\,\nu_{\overline{hc}}(0)]$$

C operators are **power-enhanced** with respect to A and B ones, but hard-collinear quark needs to be converted to a soft field through SCET$_{\mathrm{I}}$ **power-suppressed** soft-collinear interactions.

Fermi theory $\to$ HQET $\times$ SCET$_\mathrm{I}$



$$Q_1 = [\bar{u}\,\gamma^\mu P_L\, b]\,[\bar{\ell}\,\gamma_\mu P_L\, \nu]$$

$$E_1 = [\bar{u}\,\gamma^\mu \gamma^\nu \gamma^\rho P_L\, b]\,[\bar{\ell}\,\gamma_\mu \gamma_\nu \gamma_\rho P_L\, \nu] - 16\, Q_1$$

evanescent operator

$$\mathcal{O}^{(9)}_{A,1} = m_\ell\,[\bar{q}_s \frac{\not{n}_+}{2} P_L\, h_v]\,[\bar{\ell}_{hc} \frac{1}{i\,n_+\overleftarrow{\partial}_{hc}} P_L\, \nu_{\overline{hc}}]$$

$$\mathcal{O}_E = [\bar{q}_s \frac{\not{n}_+}{2} \gamma_\perp^\mu \gamma_\perp^\nu P_L\, h_v]\,[\bar{\ell}_{hc}\,\gamma_{\perp\nu}\gamma_{\perp\mu} P_L\, \nu_{\overline{hc}}]$$

$$\langle Q_1 \rangle = \langle \mathcal{O}_{A,1} \rangle$$

$$\langle E_1 \rangle = 6\,(D-4)\,\langle \mathcal{O}_{A,1} \rangle - 3\,\langle \mathcal{O}_E \rangle \sim \mathcal{O}(\epsilon)\; A^{(0)}_{E1,A1}$$

$$H^{(1)}_{A,1} = A^{(1)}_{1,(A,1)} + Z^{(1)}_{ext} A^{(0)}_{1,(A,1)} + Z^{(1)}_{(A,1)j} A^{(0)}_{j,(A,1)} - H^{(0)}_{A,1}(\mu_b)\; Z^{(1)}_{(A,1)(A,1)}$$

$$Z_{A1,E1} = \frac{1}{2\,\epsilon} Q_\ell\,(Q_\ell + 2\,Q_u)$$

$$Y(x,y) = \exp\left[i\,e\,Q_q \int_y^x dz_\mu\, A_s^\mu(z)\right] \mathcal{P} \exp\left[i\,g_s \int_y^x dz_\mu\, G_s^\mu(z)\right] ,$$

$$Y_\pm(x) = \exp\left[-i\,e\,Q_\ell \int_0^\infty ds\, n_\mp A_s\,(x + s n_\mp)\right] .$$

$$S_r^{\prime(i)}(x) \;=\; \exp\left[-i\,e\,Q_i \int_0^\infty ds\, r \cdot A_{us}\,(x + s \cdot r)\right]$$

$$C_r^{\prime(i)}(x) \;=\; \exp\left[-i\,e\,Q_i \int_0^\infty ds\, r \cdot A_{usc}\,(x + s \cdot r)\right]$$