

Nonrelativistic QCD as a novel probe of fully-heavy exotic hadrons

Hong-Fei Zhang
Guizhou University of Finance and Economics

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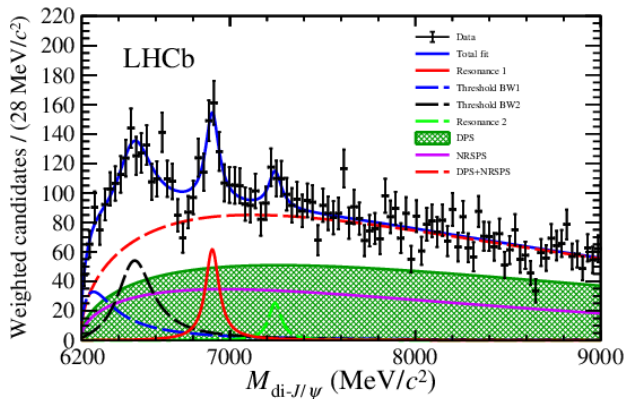
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Background

Background

Double J/ψ Resonances

- Double J/ψ resonances observed at the LHC



Possible candidates (S-wave)

$T_{cc\bar{c}\bar{c}}$	$^1S_0(^1S_0, ^1S_0)$	$^1S_0(^3S_1, ^3S_1)$	$^5S_2(^3S_1, ^3S_1)$
$M_{J/\psi J/\psi}$	1S_0	5S_2	

- Mass splitting: $m_c v^4 \approx 100\text{MeV}$

Questions

- How to determine its J^{PC} ? (Solved!)
 - From first principle
 - Robust!
- How to explore the structures of these resonances?
 - Compact tetraquarks or molecules?

NRQCD Framework

NRQCD Framework

NRQCD Factorization

- Cross Section Factorization

$$d\sigma(H) = \sum_n d\hat{\sigma}(n) \langle \mathcal{O}^H(n) \rangle$$

- n : intermediate state
- $\hat{\sigma}(n)$: Short-distance coefficient
- $\langle \mathcal{O}^H(n) \rangle$: Long-distance Matrix Element
- Difficulty: $\langle \mathcal{O}^H(n) \rangle \sim m_c^9 \alpha_s^9$, no available data!

Replacement

- For J/ψ

$$v_i(p_b)\bar{u}_j(p_a) \rightarrow \Pi_{J/\psi} \equiv \frac{1}{2\sqrt{2}} \frac{1}{\sqrt{N_c}} \delta_{ij} \not{\epsilon} (\not{p} + m_{J/\psi})$$

$$d\Phi_a d\Phi_b \rightarrow d\Phi_{J/\psi} \equiv \frac{d^3p}{(2\pi)^3 2p_0} \frac{2}{m_{J/\psi}} |\psi_{J/\psi}(0)|^2$$

Replacement

- For Molecule

$$\epsilon^\mu \otimes \epsilon^\nu \rightarrow \epsilon_{ss_z}^{\mu\nu}$$

$$\epsilon_{00}^{\mu\nu} = \sqrt{\frac{1}{3}}(-g^{\mu\nu} + \frac{P^\mu P^\nu}{M^2}) \equiv \sqrt{\frac{1}{3}}\Pi^{\mu\nu}$$

$$\sum_{s_z} \epsilon_{2s_z}^{\mu\nu} \epsilon_{2s_z}^{\alpha\beta*} = \frac{1}{2}(\Pi^{\mu\alpha} \Pi^{\nu\beta} + \Pi^{\mu\beta} \Pi^{\nu\alpha}) - \frac{1}{3}\Pi^{\mu\nu} \Pi^{\alpha\beta}$$

Replacement

- For Genuine Tetraquark
 - CParity Transformation:

$$\begin{aligned} & \bar{u}_i^\alpha(p_b)(\not{k}_1 + m_1) \dots (\not{k}_n + m_n) v_j^\beta(p_{(c,d)}) \\ &= \bar{u}_j^\beta(p_{(c,d)})(-\not{k}_n + m_n) \dots (-\not{k}_1 + m_1) v_i^\alpha(p_b) \end{aligned}$$

- Replacement

$$\begin{aligned} & v_{i_b}(p_b) \bar{u}_{i_a}(p_a) \otimes v_{i_{(c,d)}}(p_{(c,d)}) \bar{u}_{i_{(d,c)}}(p_{(d,c)}) \\ & \rightarrow \frac{1}{32} \varepsilon^{\mu\nu}_{ss_z} \frac{\delta_{i_a i_c} \delta_{i_b i_d} - \delta_{i_a i_d} \delta_{i_b i_c}}{\sqrt{12}} \times \gamma^\mu(\not{p} + M) \otimes \gamma^\nu(\not{p} + M) \\ & v_{i_b}(p_b) \bar{u}_{i_a}(p_a) \otimes v_{i_{(c,d)}}(p_{(c,d)}) \bar{u}_{i_{(d,c)}}(p_{(d,c)}) \\ & \rightarrow \frac{1}{32} \text{sgn} \frac{\delta_{i_a i_c} \delta_{i_b i_d} + \delta_{i_a i_d} \delta_{i_b i_c}}{\sqrt{24}} \times \gamma^5(\not{p} + M) \otimes \gamma^5(\not{p} + M) \end{aligned}$$

Prediction of the Spin

Prediction of the Spin

Quantity to explore the spin

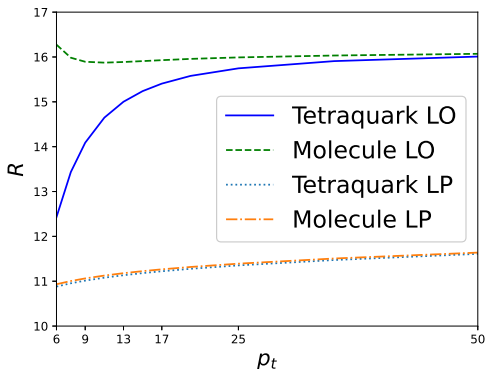
- Ratio (R) of cross sections for spin-2 to spin-0

$$R(T) \equiv \frac{d\sigma(T_{cc\bar{c}\bar{c}}[2^{++}])}{d\sigma(T_{cc\bar{c}\bar{c}}[0^{++}])} = \frac{5d\hat{\sigma}(cc\bar{c}\bar{c}[2^{++}])}{d\hat{\sigma}(cc\bar{c}\bar{c}[0^{++}])}$$
$$R(M) \equiv \frac{d\sigma(M_{\psi\psi}[2^{++}])}{d\sigma(M_{\psi\psi}[0^{++}])} = \frac{5d\hat{\sigma}(\psi\psi[2^{++}])}{d\hat{\sigma}(\psi\psi[0^{++}])}$$

- Free of nonperturbative parameters!

Ratio versus Transverse Momentum

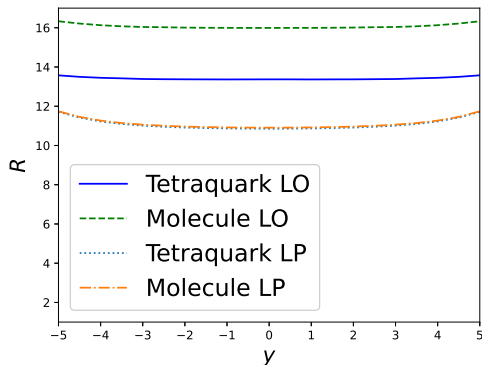
- Ratio of cross sections for spin-2 to spin-0¹



¹HFZ, Y.-Q. Ma, W.-L. Sang, Sci. Bull. 70, 1915 (2025)

Ratio versus Rapidity

- Ratio of cross sections for spin-2 to spin-0²



²HFZ, Y.-Q. Ma, W.-L. Sang, Sci. Bull. 70, 1915 (2025)

Conclusion

- The observed resonances are likely spin-2
- Confirmed by the CMS measurement

Ratio to Explore the Structure

Ratio to Explore the Structure

Ratio to distinguish between compact tetraquarks and molecules

- Ratio $\mathcal{R}$ to distinguish between compact tetraquarks and molecules

$$\mathcal{R} \equiv \frac{\mathcal{B}(T_{cc\bar{c}\bar{c}} \rightarrow c\bar{c})}{\mathcal{B}(T_{cc\bar{c}\bar{c}} \rightarrow J/\psi\gamma)}$$

- Free of nonperturbative parameters!

Results of $\mathcal{R}$

- Results of $\mathcal{R}$ for compact tetraquarks and molecules:

$$\mathcal{R}(T) \approx 3.4$$

$$\mathcal{R}(M) \approx 780$$

- The two structures can be perfectly distinguished!
- The structures of the resonances can be explored even by measuring the upper or lower limit of $\mathcal{R}$.

Conclusion

Conclusion

Conclusion

- NRQCD successfully predicted the spin of the di- J/ψ resonances.
- NRQCD can access the structure of the fully-heavy exotic states.

Thanks

Thanks!

Appendix

Appendix: New Method for Solving Multibody Systems

Hartree-Fock Method

- Wave function ansatz: Slater determinant

$$\begin{vmatrix} \psi_1(\mathbf{x}_1) & \psi_2(\mathbf{x}_1) & \dots & \psi_n(\mathbf{x}_1) \\ \psi_1(\mathbf{x}_2) & \psi_2(\mathbf{x}_2) & \dots & \psi_n(\mathbf{x}_2) \\ \dots & \dots & \dots & \dots \\ \psi_1(\mathbf{x}_n) & \psi_2(\mathbf{x}_n) & \dots & \psi_n(\mathbf{x}_n) \end{vmatrix}$$

- Hartree-Fock Equation

$$\begin{aligned} & -\frac{\hbar^2}{2m_i} \nabla_i^2 \psi_i(\mathbf{x}_i) + \int d^3x_1 \dots d^3x_{i-1} d^3x_{i+1} \dots d^3x_n \\ & \times |\psi_1(\mathbf{x}_1)|^2 \dots |\psi_{i-1}(\mathbf{x}_{i-1})|^2 |\psi_{i+1}(\mathbf{x}_{i+1})|^2 \dots |\psi_n(\mathbf{x}_n)|^2 \\ & \times V(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n) \psi_i(\mathbf{x}_i) = E_i \psi_i(\mathbf{x}_i) \end{aligned}$$

Drawbacks of the Hartree-Fock Method

- Incapable of strongly correlated systems
 - Slater determinant ansatz
- Low precision
 - fail in solving hydrogen ground-state energy
- Kinetic energy of the center of mass cannot be naturally eliminated
 - each ψ_i has kinetic energy

New Ansatz

- Construct an eigenstate of angular momentum
- Spatial wave function

$$\psi = \sum_{mm' m_1 m_2} C_{LM}^{mm'} Y_{lm}(\theta, \vartheta) R_{nl}(r) C_{l'm'}^{m_1 m_2} \\ \times Y_{l_1 m_1}(\theta_1, \vartheta_1) R_{1n_1 l_1}(r_1) Y_{l_2 m_2}(\theta_2, \vartheta_2) R_{2n_2 l_2}(r_2)$$

- Functions to be solved: R_{nl} , $R_{1n_1 l_1}$, $R_{2n_2 l_2}$

Hartree-Fock Equations

- Hartree-Fock Equation via new ansatz

$$\begin{aligned} & \left[-\frac{\hbar^2}{2m_q} \left(\frac{1}{r} \frac{d^2}{dr^2} r - \frac{l(l+1)}{r^2} \right) + V_0(r) - \mathcal{E}_0 \right] R_{nl}(r) = 0, \\ & \left[-\frac{\hbar^2}{m_q} \left(\frac{1}{r_1} \frac{d^2}{dr_1^2} r_1 - \frac{l_1(l_1+1)}{r_1^2} \right) + V_{qq}(r_1) + V_1(r_1) - \mathcal{E}_1 \right] R_{1m_1 l_1}(r_1) = 0, \\ & \left[-\frac{\hbar^2}{m_q} \left(\frac{1}{r_2} \frac{d^2}{dr_2^2} r_2 - \frac{l_2(l_2+1)}{r_2^2} \right) + V_{qq}(r_2) + V_2(r_2) - \mathcal{E}_2 \right] R_{2m_2 l_2}(r_2) = 0 \end{aligned}$$

$$V_{q\bar{q}}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \sum_{lm l_1 m_1 l_2 m_2} V_{lm l_1 m_1 l_2 m_2}(r, r_1, r_2) Y_{lm}(\theta, \vartheta) Y_{l_1 m_1}(\theta_1, \vartheta_1) Y_{l_2 m_2}(\theta_2, \vartheta_2)$$

$$V_0(r) = \int dr_1 dr_2 \varrho_1(r_1) \varrho_2(r_2) V_{lm l_1 m_1 l_2 m_2}(r, r_1, r_2),$$

$$V_1(r_1) = \int dr dr_2 \varrho(r) \varrho_2(r_2) V_{lm l_1 m_1 l_2 m_2}(r, r_1, r_2),$$

$$V_2(r_2) = \int dr dr_1 \varrho(r) \varrho_1(r_1) V_{lm l_1 m_1 l_2 m_2}(r, r_1, r_2)$$

Thanks!