Systematic analysis of $D_{(s)}$ meson semi-leptonic decays in the covariant light-front quark model

张志清

河南工业大学物理与新能源学院

第二届重味物理前沿论坛研讨会 @ 华中师范大学，武汉

2025 年 9 月 14 日

Outline

Introduction

- The $D_{(s)}$ meson semi-leptonic decays serve as a crucial platform for testing SM, precisely measuring the CKM matrix elements, and searching for NP.
- More and more data on the $D_{(s)} \to (P, S, V, A)\ell\nu_\ell$ decays have been collected by BESIII, CLEO, Belle and BaBar collaborations, particularly some scalar and axial-vector mesons have been observed in $D_{(s)}$ meson semi-leptonic decays by BESIII.

Introduction

- In theory, accurate calculations of the $D_{(s)} \to (P, S, V, A)\ell\nu_\ell$ decays require knowledge of the related transition form factors, but this depends on a non-perturbative QCD understanding of the internal structures of the final-state hadrons.
- Current theoretical models show reasonable agreement for the form factors of the transitions $D_{(s)} \to P, V$, but significant discrepancies in those of the transitions $D_{(s)} \to S, A$.

Introduction

At the end of the twentieth century, Jaus put forward the covariant light-front quark model (CLFQM). The CLFQM has some unique advantages:

- The light-front wave functions descrbing the hadron through quark and gluon degrees of freedom can preserve a Lorentz invariant formalism.
- The final state meson at $q^2 = 0$ is usually relativistic. The CLFQM with relativistic effects involved is suitable to study hadronic transtion form factors.

The CLFQM general calculation procedure

The CLFQM general calculation procedure

The Bauer-Stech-Wirbel (BSW) transition form factors are more frequently used and defined by,

$$\left\langle S\left(P''\right)|A_\mu|D_{(s)}\left(P'\right)\right\rangle = \left(P_\mu - \frac{m^2_{D_{(s)}} - m^2_S}{q^2} q_\mu\right) F_1^{D_{(s)}S}\left(q^2\right) + \frac{m^2_{D_{(s)}} - m^2_S}{q^2} q_\mu F_0^{D_{(s)}S}\left(q^2\right), \quad (1)$$

$$\begin{aligned}\left\langle A\left(P'', \varepsilon^{\mu *}\right)|V_\mu|D_{(s)}\left(P'\right)\right\rangle &= -i\left\{\left(m_{D_{(s)}} - m_A\right)\varepsilon^*_\mu V_1^{D_{(s)}A}\left(q^2\right) - \frac{\varepsilon^* \cdot P}{m_{D_{(s)}} - m_A} P_\mu V_2^{D_{(s)}A}\left(q^2\right)\right. \\ &\left. -2m_A \frac{\varepsilon^* \cdot P}{q^2} q_\mu \left[V_3^{D_{(s)}A}\left(q^2\right) - V_0^{D_{(s)}A}\left(q^2\right)\right]\right\}, \quad (2)\end{aligned}$$

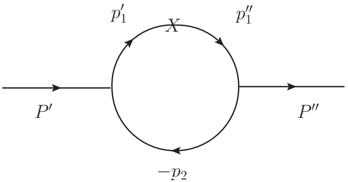
$$\left\langle A\left(P'', \varepsilon^{\mu *}\right)|A_\mu|D_{(s)}\left(P'\right)\right\rangle = -\frac{1}{m_{D_{(s)}} - m_A}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}P^\alpha q^\beta A^{D_{(s)}A}\left(q^2\right), \quad (3)$$

where $P = P' + P''$, $q = P' - P''$, ϵ is the polarization vector. The four-momentum of the initial (final) meson is $P' = p'_1 + p_2$ $(P'' = p''_1 + p_2)$.

The CLFQM general calculation procedure

The decay amplitude in the lowest order for the transition $D_{(s)} \to M$

$$\mathcal{B}_{\mu}^{D_{(s)}M} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{h'_{D_{(s)}} (h''_M)}{N_1' N_1'' N_2} S_{\mu\nu}^{D_{(s)}M} \varepsilon^{*\nu}, \tag{4}$$



- $N_1^{\prime(\prime\prime)} = p_1^{\prime(\prime\prime)2} - m_1^{\prime(\prime\prime)2}, N_2 = p_2^2 - m_2^2$ arise from the quark propagators.

The CLFQM general calculation procedure

The trace $S_{\mu\nu}^{D_{(s)}M}$ can be obtained directly using Lorentz contraction,

$$
\begin{aligned}
S_{\mu}^{D_{(s)}P} &= \mathrm{Tr}\left[\gamma_5\left(\not p_1'' + m_1''\right)\gamma_\mu\left(\not p_1' + m_1'\right)\gamma_5\left(-\not p_2 + m_2\right)\right], \\
S_{\mu}^{D_{(s)}S} &= \mathrm{Tr}\left[(-i)\left(\not p_1'' + m_1''\right)\gamma_\mu\gamma_5\left(\not p_1' + m_1'\right)\gamma_5\left(-\not p_2 + m_2\right)\right], \\
S_{\mu\nu}^{D_{(s)}V} &= \left(S_V^{D_{(s)}\ V} - S_A^{D_{(s)}\ V}\right)_{\mu\nu} \\
&= \mathrm{Tr}\left[\left(\gamma_\nu - \frac{1}{W_V''}\left(p_1'' - p_2\right)_\nu\right)\left(\not p_1'' + m_1''\right)\left(\gamma_\mu - \gamma_\mu\gamma_5\right)\left(\not p_1' + m_1'\right)\gamma_5\left(-\not p_2 + m_2\right)\right], \\
S_{\mu\nu}^{D_{(s)}\ {}^3A} &= \left(S_V^{D_{(s)}\ {}^3A} - S_A^{D_{(s)}\ {}^3A}\right)_{\mu\nu} \\
&= \mathrm{Tr}\left[\left(\gamma_\nu - \frac{1}{W_{^3A}''}\left(p_1'' - p_2\right)_\nu\right)\left(\not p_1'' - m_1''\right)\left(\gamma_\mu\gamma_5 - \gamma_\mu\right)\left(\not p_1' + m_1'\right)\gamma_5\left(-\not p_2 + m_2\right)\right], \\
S_{\mu\nu}^{D_{(s)}\ {}^1A} &= \left(S_V^{D_{(s)}\ {}^1A} - S_A^{D_{(s)}\ {}^1A}\right)_{\mu\nu} \\
&= \mathrm{Tr}\left[\left(-\frac{1}{W_{^1A}''}\left(p_1'' - p_2\right)_\nu\right)\left(\not p_1'' - m_1''\right)\left(\gamma_\mu\gamma_5 - \gamma_\mu\right)\left(\not p_1' + m_1'\right)\gamma_5\left(-\not p_2 + m_2\right)\right].
\end{aligned}
\tag{5}
$$

The CLFQM general calculation procedure

The covariant vertex function h''_M are defined as

$$h''_P = h''_V = (M''^2 - M''^2_0)\sqrt{\frac{x_1 x_2}{N_c}}\frac{1}{\sqrt{2}\widetilde{M}''_0}\varphi,$$
$$h''_S = \sqrt{\frac{2}{3}}h''_{3A} = (M''^2 - M''^2_0)\sqrt{\frac{x_1 x_2}{N_c}}\frac{1}{\sqrt{2}\widetilde{M}''_0}\frac{\widetilde{M''^2_0}}{2\sqrt{3}M''_0}\varphi_p,$$
$$h''_{1A} = (M''^2 - M''^2_0)\sqrt{\frac{x_1 x_2}{N_c}}\frac{1}{\sqrt{2}\widetilde{M}''_0}\varphi_p, \qquad (6)$$

with

$$M''^2_0 = (e''_1 + e_2)^2 = \frac{p'^2_\perp + m''^2_1}{x_1} + \frac{p^2_\perp + m^2_2}{x_2}, \quad \widetilde{M}''_0 = \sqrt{M''^2_0 - (m''_1 - m_2)^2}. \qquad (7)$$

The CLFQM general calculation procedure

The phenomenological Gaussian-type wave function $\varphi_{(p)}$ depicts the light-front momentum distribution amplitude for the S(P)-wave mesons,

$$\begin{aligned} \varphi &= \varphi\left(x_2, p_\perp\right) = 4\left(\frac{\pi}{\beta^2}\right)^{\frac{3}{4}} \sqrt{\frac{dp_z}{dx_2}} \exp\left(-\frac{p_z^2 + p_\perp^2}{2\beta^2}\right), \\ \varphi_p &= \varphi_p\left(x_2, p_\perp\right) = \sqrt{\frac{2}{\beta^2}}\varphi, \end{aligned} \tag{8}$$

- β is a phenomenological parameter and can be fixed by fitting the corresponding decay constant.

Transition Form Factors

Input parameters:

- The constituent quark masses(GeV): $m_c = 1.4$, $m_s = 0.37$, $m_{u,d} = 0.25$;
- The masses of the initial and the final mesons(GeV): $m_{D^0} = 1.865$,

  $m_{D^\pm} = 1.87$, $m_{D_s^\pm} = 1.968$, $m_{K^\pm} = 0.494$, $m_{K^0} = 0.498$, $m_{\pi^\pm} = 0.140$,

  $m_{\pi^0} = 0.135$, $m_\eta = 0.548$, $m_{\eta'} = 0.958$, $m_{K^*(892)} = 0.89$, $m_\rho = 0.775$,

  $m_\omega = 0.783$, $m_{a_0(980)} = 0.98$, $m_{f_0(980)} = 0.99$, $m_{a_0(1450)} = 1.439$,

  $m_{K_0^*(1430)} = 1.425$, ,$m_{K_1(1270)} = 1.253$, $m_{K_1(1400)} = 1.403$, $m_{K_{1A}} = 1.31$,

  $m_{K_{1B}} = 1.34$, ,$m_{a_1(1260)} = 1.209$, $m_{b_1(1235)} = 1.23$;
- The CKM matrix elements: $V_{cd} = 0.221 \pm 0.004$, $V_{cs} = 0.975 \pm 0.006$,

  $V_{ud} = 0.97373 \pm 0.00031$;

Transition Form Factors

Input parameters:

- The shape parameters fitted by the decay constants:

$$\beta_D = 0.465^{+0.012}_{-0.012},\ \beta_{D_s} = 0.545^{+0.013}_{-0.013},\ \beta_\pi = 0.328^{+0.002}_{-0.004},$$

$$\beta_K = 0.394^{+0.003}_{-0.003},\ \beta_{\eta_q} = 0.374^{+0.038}_{-0.045},\ \beta_{\eta_s} = 0.404^{+0.030}_{-0.032},$$

$$\beta_{K^*} = 0.279,\ \beta_\rho = 0.26,\ \beta_\omega = 0.252,\ \beta_\phi = 0.322,$$

$$\beta_{K_{1A}} = 0.246^{+0.017}_{-0.018},\ \beta_{K_{1B}} = 1.783^{+0.266}_{-0.265},\ \beta_{a_1(1260)} = 0.229^{+0.016}_{-0.017};$$

- Full widths(GeV): $\Gamma_{D^0} = (1.583 \pm 0.004) \times 10^{-12}$,

$$\Gamma_{D^\pm} = (6.288 \pm 0.03) \times 10^{-13},\ \Gamma_{D_s^\pm} = (1.296 \pm 0.006) \times 10^{-12}.$$

Transition Form Factors

- All the calculations are carried out within the $q^+ = 0$ reference frame, where the form factors can only be obtained at spacelike momentum transfers $q^2 = -q_\perp^2 \leq 0$,
- The parameterized form factors are extrapolated from the space-like region to the time-like region by using

$$F\left(q^2\right) = \frac{F(0)}{\left(1 - q^2/m^2\right)\left[1 - a\left(q^2/m^2\right) + b\left(q^2/m^2\right)^2\right]}. \tag{9}$$

- $F(q^2)$ denotes different form factors, such as $F_1(q^2)$, $F_0(q^2)$, $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$.

Transition Form Factors

- The form factors of the transitions $D_{(s)} \to K, \pi, \eta^{(\prime)}$ at $q^2 = 0$.

	$D \to K$	$D \to \pi$	$D_s \to K$	–
This work	$0.789^{+0.002}_{-0.001}$	$0.679^{+0.003}_{-0.004}$	$0.714^{+0.004}_{-0.002}$	–
CLFQM'11	0.79	0.66	0.66	–
CCQM'19	0.77	0.63	0.60	–
RQM'19	0.716	0.640	0.674	–
BESIII'15	0.737	0.637	–	–
BABAR'07	0.727	–	–	–
	$D \to \eta$	$D \to \eta'$	$D_s \to \eta$	$D_s \to \eta'$
This work	$0.558^{+0.019}_{-0.025}$	$0.456^{+0.015}_{-0.021}$	$0.490^{+0.014}_{-0.017}$	$0.599^{+0.017}_{-0.021}$
CLFQM'11	0.55	0.45	0.48	0.59
LCSR'13	0.552	0.458	0.432	0.520
LCSR'15	0.429	0.292	0.495	0.558
CCQM'19	0.36	0.36	0.49	0.59
RQM'19	0.547	0.538	0.443	0.559
LQCD'14	–	–	0.564	0.437
BESIII'19	–	–	0.4576	0.490

- The relations $F_0^{D\to\eta} > F_0^{D\to\eta'}$, $F_0^{D_s\to\eta} < F_0^{D_s\to\eta'}$ are helpful explain the data of related semi-leptonic decays.

- $$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \tag{10}$$

  where the mixing angle θ has been well determined as $\theta = 39.3^\circ \pm 1.0^\circ$.

- Precise experimental measurements of these form factors are helpful to test the mixing mechanism and probe the $\eta - \eta'$ mixing angle.

- The $D \to K^*, \rho$ transition form factors at $q^2 = 0$.

Transition	Reference	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$
$D \to K^*$	This work	$0.947^{+0.010}_{-0.011}$	$0.635^{+0.002}_{-0.003}$	$0.656^{+0.004}_{-0.004}$	$0.574^{+0.000}_{-0.001}$
	CLFQM'03	0.94	0.69	0.65	0.57
	CLFQM'11	0.98	0.78	0.72	0.60
	LCSR'06	0.791	0.571	0.345	−0.723
	CCQM'19	0.90	2.08	0.68	−0.90
	RQM'19	0.927	0.655	0.608	0.520
	MS'08	1.03	0.76	0.66	0.49
	QSR'93	1.1	0.4	0.5	0.6
	BSW'85	1.23	0.73	0.88	1.15
$D \to \rho$	This work	$0.845^{+0.009}_{-0.009}$	$0.546^{+0.002}_{-0.002}$	$0.571^{+0.003}_{-0.003}$	$0.485^{+0.001}_{-0.001}$
	CLFQM'03	0.86	0.64	0.58	0.48
	CLFQM'11	0.88	0.69	0.60	0.47
	LCSR'06	0.801	0.599	0.372	−0.719
	CCQM'19	0.76	1.47	0.57	−0.74
	RQM'19	0.979	0.712	0.682	0.640
	MS'08	0.90	0.66	0.59	0.49
	QSR'93	1.00	0.6	0.5	0.4
	BSW'85	1.23	0.67	0.78	0.92

Transition Form Factors

- The $D_s \to K^*, \phi$ transition form factors at $q^2 = 0$.

Transition	Reference	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$
$D_s \to K^*$	This work	$0.860^{+0.010}_{-0.010}$	$0.531^{+0.002}_{-0.002}$	$0.549^{+0.003}_{-0.003}$	$0.485^{+0.001}_{-0.002}$
	CLFQM'11	0.87	0.61	0.56	0.46
	LCSR'06	0.771	0.589	0.345	-0.675
	CCQM'19	0.80	1.53	0.57	-0.82
	RQM'19	0.959	0.629	0.596	0.540
$D_s \to \phi$	This work	$1.009^{+0.011}_{-0.011}$	$0.634^{+0.002}_{-0.002}$	$0.656^{+0.003}_{-0.004}$	$0.564^{+0.002}_{-0.002}$
	CLFQM'11	0.98	0.72	0.69	0.59
	LCSR'06	0.778	0.569	0.304	-0.757
	CCQM'19	0.91	2.13	0.67	-0.95
	RQM'19	0.999	0.713	0.643	0.492

The ratios of the form factors $r_V = \frac{V(0)}{A_1(0)}$ and $r_2 = \frac{A_2(0)}{A_1(0)}$, show good agreement with the data.

For the $D \to K^*$ transition, the ratios $r_V = 1.44$ and $r_2 = 0.86$, which are consistent with $r_V = 1.46$ and $r_2 = 0.67$ (BESIII). For the $D \to \rho$ transition, the ratios are $r_V = 1.49$ and $r_2 = 0.86$, which are comparable with the $r_V = 1.70$ and $r_2 = 0.85$ (BESIII).

Transition Form Factors

- The $D_{(s)} \to a_0(980), f_0(980)$ and $D_{(s)} \to a_0(1450), K_0^*(1430)$ transition form factors.

	$D \to a_0(980)$	$D \to f_0(980)$	$D_s \to f_0(980)$
This work	$0.515^{+0.042}_{-0.032}$	$0.283^{+0.023}_{-0.017}$	$0.431^{+0.007}_{-0.007}$
LCSR'21	0.85	–	–
QCDSR'24	0.53	–	–
CCQM'20	0.55	0.45	0.39
LCSR'10,17	1.75	0.32	0.30
LFQM'09	–	0.22	0.43
DR'09	–	0.22	0.46
	$D \to a_0(1450)$	$D \to K_0^*(1430)$	$D_s \to K_0^*(1430)$
This work	$0.515^{+0.017}_{-0.015}$	$0.498^{+0.022}_{-0.015}$	$0.537^{+0.027}_{-0.020}$
CLFQM'03	–	0.48	–
CLFQM'11	0.51	0.47	0.55
LCSR'21,23,24	0.94	0.60	0.65
QCDSR'06,24	0.28	0.57	0.51

Transition Form Factors

- The transition form factors $D \to a_1(1260), b_1(1235)$ at $q^2 = 0$, together with other theoretical results.

Transitions	References	$A(0)$	$V_0(0)$	$V_1(0)$	$V_2(0)$
$D \to a_1(1260)$	This work	$0.159^{+0.012}_{-0.014}$	$0.307^{+0.006}_{-0.007}$	$1.349^{+0.028}_{-0.043}$	$0.048^{+0.004}_{-0.004}$
	CLFQM'03	0.20	0.31	1.54	0.06
	CLFQM'11	0.19	0.32	1.51	0.05
	LCSR'19	0.04	0.10	0.26	-0.02
	LCSR'21	0.34	0.24	2.63	0.34
	QCDSR'16	0.314	-0.114	0.039	0.112
	QCDSR'17	0.13	0.217	1.898	0.228
$D \to b_1(1235)$	This work	$0.120^{+0.001}_{-0.002}$	$0.498^{+0.020}_{-0.021}$	$1.401^{+0.029}_{-0.037}$	$-0.104^{+0.012}_{-0.011}$
	CLFQM'03	0.11	0.49	1.37	-0.10
	CLFQM'11	0.12	0.50	1.39	-0.10
	LCSR'19	-0.28	-0.23	-0.16	0.15
	LCSR'21	-0.24	-0.16	-1.78	-0.24

Transition Form Factors

- The transition form factors $D \to K_{1A}, K_{1B}$ at $q^2 = 0$, together with other theoretical results.

Transitions	References	$A(0)$	$V_0(0)$	$V_1(0)$	$V_2(0)$
$D \to K_{1A}$	This work	$0.133^{+0.012}_{-0.012}$	$0.311^{+0.004}_{-0.005}$	$1.566^{+0.036}_{-0.040}$	$0.019^{+0.003}_{-0.003}$
	CLFQM'03	0.98	0.34	2.02	0.03
	CLFQM'11	0.15	0.28	1.60	0.01
	LCSR'19	0.06	0.11	0.32	-0.03
	QCDSR'08	0.11	0.04	0.02	-0.01
$D \to K_{1B}$	This work	$0.010^{+0.005}_{-0.003}$	$0.091^{+0.037}_{-0.025}$	$0.239^{+0.102}_{-0.066}$	$-0.036^{+0.010}_{-0.014}$
	CLFQM'03	0.10	0.44	1.53	-0.09
	CLFQM'11	0.10	0.48	1.58	-0.13
	LCSR	-0.47	-0.42	-0.26	0.29
	QCDSR'08	-0.75	-0.13	-0.16	0.08

- It is noticed that in the calculations of the $D \to K_{1A}, K_{1B}$ transition form factors, the shape parameters for K_{1A} and K_{1B} are determined from their respective decay constants, it is different in the previous CLFQM work.

Transition Form Factors

- The transition form factors $D_s \to K_{1A}, K_{1B}$ at $q^2 = 0$, together with other theoretical results.

Transitions	References	$A(0)$	$V_0(0)$	$V_1(0)$	$V_2(0)$
$D_s \to K_{1A}$	This work	$0.155^{+0.011}_{-0.012}$	$0.293^{+0.005}_{-0.006}$	$1.435^{+0.035}_{-0.041}$	$0.053^{+0.003}_{-0.003}$
	CLFQM'11	0.19	0.29	1.68	0.07
	LCSR'19	0.05	0.10	0.28	-0.01
	QCDSR'08	0.16	0.03	0.05	-0.02
$D_s \to K_{1B}$	This work	$0.017^{+0.007}_{-0.004}$	$0.148^{+0.057}_{-0.039}$	$0.339^{+0.135}_{-0.091}$	$-0.055^{+0.014}_{-0.020}$
	CLFQM'11	0.10	0.51	1.50	-0.12
	LCSR'19	-0.40	-0.41	-0.22	0.24
	QCDSM'08	-0.84	-0.26	-0.30	0.14

- It is noticed that in the calculations of the $D_s \to K_{1A}, K_{1B}$ transition form factors, the shape parameters for K_{1A} and K_{1B} are determined from their respective decay constants, it is different in the previous CLFQM work.

Semi-leptonic Decays

Semi-leptonic Decays

The differential widths of the decays $D_{(s)} \to P(S)\ell\nu_\ell$,

$$\frac{d\Gamma}{dq^2} = (\frac{q^2 - m_l^2}{q^2})^2 \frac{\sqrt{\lambda(m_{D_{(s)}}^2, m_{P(S)}^2, q^2)}\, G_F^2 |V_{cq}|^2}{384 m_{D_{(s)}}^3 \pi^3} \Big\{ (2 + \frac{m_l^2}{q^2}) \lambda(m_{D_{(s)}}^2, m_{P(S)}^2, q^2) F_1^2(q^2) + 3\frac{m_l^2}{q^2} (m_{D_{(s)}}^2 - m_{P(S)}^2)^2 F_0^2(q^2) \Big\}, \tag{11}$$

where V_{cq} with $q = s$ or d is the CKM matrix element,

$\lambda(m_{D_{(s)}}^2, m_{P(S)}^2, q^2) = (m_{D_{(s)}}^2 + m_{P(S)}^2 - q^2)^2 - 4m_{D_{(s)}}^2 m_{P(S)}^2$ and m_ℓ is the mass of the lepton ℓ.

The differential widths of the decays $D_{(s)} \to V\ell\nu_\ell$ is listed as

$$\begin{aligned}
\frac{d\Gamma_L}{dq^2} &= (\frac{q^2-m_l^2}{q^2})^2 \frac{\sqrt{\lambda(m_{D_{(s)}}^2, m_V^2, q^2)}G_F^2|V_{cq}|^2}{384 m_{D_{(s)}}^3 \pi^3} \times \frac{1}{q^2}\left\{3m_l^2\lambda(m_{D_{(s)}}^2, m_V^2, q^2)A_0^2(q^2)\right. \\
&\left. + \frac{m_l^2+2q^2}{4m_V^2}\left|(m_{D_{(s)}}^2 - m_V^2 - q^2)(m_{D_{(s)}} + m_V)A_1(q^2) - \frac{\lambda(m_{D_{(s)}}^2, m_V^2, q^2)}{m_{D_{(s)}} + m_V}A_2(q^2)\right|^2\right\}, \qquad (12)
\end{aligned}$$

$$\begin{aligned}
\frac{d\Gamma_\pm}{dq^2} &= (\frac{q^2-m_l^2}{q^2})^2 \frac{\sqrt{\lambda(m_{D_{(s)}}^2, m_V^2, q^2)}G_F^2|V_{cq}|^2}{384 m_{D_{(s)}}^3 \pi^3} \\
&\times\left\{(m_l^2+2q^2)\lambda(m_{D_{(s)}}^2, m_V^2, q^2)\left[\frac{V(q^2)}{m_{D_{(s)}}+m_V} \mp \frac{(m_{D_{(s)}}+m_V)A_1(q^2)}{\sqrt{\lambda(m_{D_{(s)}}^2, m_V^2, q^2)}}\right]^2\right\}. \qquad (13)
\end{aligned}$$

Semi-leptonic Decays

The combined transverse and total differential decay widths

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_+}{dq^2} + \frac{d\Gamma_-}{dq^2}, \tag{14}$$

The decay width for the decays $D_{(s)} \to A\ell\nu_\ell$ can be obtained from the equation on the previous page through the subsequent substitutions:

$$\left\{V\left(q^2\right), A_0\left(q^2\right), A_1\left(q^2\right), A_2\left(q^2\right)\right\} \longrightarrow \left\{A\left(q^2\right), V_0\left(q^2\right), V_1\left(q^2\right), V_2\left(q^2\right)\right\} \tag{15}$$

$$m_V \longrightarrow m_A, \tag{16}$$

$$m_{D_{(s)}} \pm m_V \longrightarrow m_{D_{(s)}} \mp m_A. \tag{17}$$

Semi-leptonic Decays

- The branching ratios of the semi-leptonic decays $D^+_{(s)} \to \eta^{(\prime)} \ell^+ \nu_\ell$.

	$10^{-3} \times \mathcal{B}r(D^+ \to \eta e^+ \nu_e)$	$10^{-3} \times \mathcal{B}r(D^+ \to \eta \mu^+ \nu_\mu)$	$10^{-4} \times \mathcal{B}r(D^+ \to \eta' e^+ \nu_e)$	$10^{-4} \times \mathcal{B}r(D^+ \to \eta' \mu^+ \nu_\mu)$
This work	$1.06^{+0.01+0.00+0.07}_{-0.01-0.00-0.09}$	$0.98^{+0.00+0.00+0.07}_{-0.00-0.00-0.09}$	$1.69^{+0.01+0.00+0.12}_{-0.01-0.00-0.15}$	$1.48^{+0.01+0.00+0.10}_{-0.01-0.00-0.13}$
LCSR'06	0.86	0.84	–	–
CCQM'19	0.94	0.91	2.00	1.90
RQM'19	1.24	1.21	2.25	2.11
CLFQM'17	1.20	1.20	1.80	1.70
LCSR'15	1.42	–	1.52	–
BESIII'25	0.98	0.91	1.79	1.92
CLEO'11	1.14	–	2.16	–
PDG'24	1.11	1.04	2.0	–
	$10^{-2} \times \mathcal{B}r(D_s^+ \to \eta e^+ \nu_e)$	$10^{-2} \times \mathcal{B}r(D_s^+ \to \eta \mu^+ \nu_\mu)$	$10^{-2} \times \mathcal{B}r(D_s^+ \to \eta' e^+ \nu_e)$	$10^{-2} \times \mathcal{B}r(D_s^+ \to \eta' \mu^+ \nu_\mu)$
This work	$2.14^{+0.01+0.01+0.13}_{-0.01-0.02-0.14}$	$2.00^{+0.01+0.01+0.12}_{-0.01-0.01-0.14}$	$0.88^{+0.00+0.01+0.05}_{-0.00-0.01-0.06}$	$0.78^{+0.00+0.01+0.05}_{-0.00-0.01-0.05}$
LCSR'06	1.27	1.25	–	–
CCQM'19	2.24	2.18	0.83	0.79
RQM'19	2.37	2.32	0.87	0.83
LCSR'17	2.40	–	0.79	–
PDG'24	2.27	2.24	0.81	0.80
BESIII'23,24	2.255	2.235	0.81	0.801
CLEO'15	2.28	–	0.68	–

Semi-leptonic Decays

- The branching fraction ratios $\frac{\mathcal{B}r(D^+\to\eta\ell^+\nu_\ell)}{\mathcal{B}r(D^+\to\eta'\ell^+\nu_\ell)} \approx 6(5.6)$ and $\frac{\mathcal{B}r(D_s^+\to\eta\ell^+\nu_\ell)}{\mathcal{B}r(D_s^+\to\eta'\ell^+\nu_\ell)} \approx 2.4(2.8)$.
- The decays $D \to \eta\ell\nu_\ell$ have a larger final-state phase space compared with that of the decays $D \to \eta'\ell\nu_\ell$. Furthermore, $F_0^{D\eta} > F_0^{D\eta'}$.
- The discrepancy between the branching ratios of the decays $D_s \to \eta\ell\nu_\ell$ and $D_s \to \eta'\ell\nu_\ell$ is narrowed by the form factors, since $F_0^{D_s\eta} < F_0^{D_s\eta'}$.

Semi-leptonic Decays

- The branching ratios of the decays $D_{(s)} \to (a_0(980), f_0(980), f_0(500))\ell\nu_\ell$, their values correspond to the mixing angle $\theta = 34^\circ$ between $f_0(980)$ and $f_0(500)$.

	$10^{-4} \times \mathcal{B}r(D^0 \to a_0^- e^+\nu_e)$	$10^{-4} \times \mathcal{B}r(D^0 \to a_0^- \mu^+\nu_\mu)$	$10^{-4} \times \mathcal{B}r(D^+ \to a_0 e^+\nu_e)$	$10^{-4} \times \mathcal{B}r(D^+ \to a_0 \mu^+\nu_\mu)$
This work	$1.49^{+0.00+0.06+0.12}_{-0.00-0.00-0.06}$	$1.29^{+0.00+0.05+0.10}_{-0.00-0.00-0.05}$	$1.92^{+0.01+0.07+0.23}_{-0.01-0.00-0.29}$	$1.67^{+0.01+0.06+0.21}_{-0.01-0.00-0.25}$
LCSR'21	1.36	1.21	1.79	1.59
CCQM'20	1.68	1.63	2.18	2.12
LCSR'22	1.57	–	1.98	–
SU3'23	1.18	0.98	1.55	1.28
BESIII'18,24	1.08	–	2.08	–
	$10^{-5} \times \mathcal{B}r(D^+ \to f_0 e^+\nu_e)$	$10^{-5} \times \mathcal{B}r(D^+ \to f_0 \mu^+\nu_\mu)$	$10^{-3} \times \mathcal{B}r(D_s^+ \to f_0 e^+\nu_e)$	$10^{-3} \times \mathcal{B}r(D_s^+ \to f_0 \mu^+\nu_\mu)$
This work	$5.73^{+0.03+0.23+0.69}_{-0.03-0.01-0.87}$	$4.95^{+0.02+0.19+0.61}_{-0.02-0.00-0.75}$	$4.00^{+0.02+0.15+0.16}_{-0.02-0.00-0.67}$	$3.52^{+0.02+0.00+0.14}_{-0.02-0.14-0.00}$
CCQM'20	7.78	7.87	2.1	2.1
LCSR'10	–	–	2	–
LFQM'09	5.7	–	1.32	–
SU3'23	3.92	3.23	2.3	1.95
BESIII'18,23	< 5.28	–	3.25	–
CLEO'09	3.77	–	–	–
	$10^{-4} \times \mathcal{B}r(D^+ \to \sigma e^+\nu_e)$	$10^{-4} \times \mathcal{B}r(D^+ \to \sigma \mu^+\nu_\mu)$	$10^{-3} \times \mathcal{B}r(D_s^+ \to \sigma e^+\nu_e)$	$10^{-3} \times \mathcal{B}r(D_s^+ \to \sigma \mu^+\nu_\mu)$
This work	$5.36^{+0.03+0.02+0.05}_{-0.03-0.02-0.05}$	$4.91^{+0.02+0.02+0.05}_{-0.02-0.02-0.05}$	$6.55^{+0.03+0.03+0.01}_{-0.04-0.03-0.00}$	$6.04^{+0.03+0.03+0.01}_{-0.04-0.03-0.00}$
SU3'23	4.05	3.69	6.73	6.21

Semi-leptonic Decays

- The branching ratio of the decay $D^0 \to a_0(980)^- e^+ \nu_e$, $a_0(980)^- \to \eta\pi^-$ was determined as $(0.86 \pm 0.17 \pm 0.05) \times 10^{-4}$ by BESIII , and $Br(a_0(980)^- \to \eta\pi^-) = 0.80 \pm 0.38$ can be obtained from $\Gamma_{\gamma\gamma} Br(a_0(980) \to \eta\pi) = 0.24 \pm 0.08$ keV and $\Gamma_{\gamma\gamma} = 0.30 \pm 0.10$ keV . So the branching raito of the decay $D^0 \to a_0(980)^- e^+ \nu_e$ can be estimated to be $(1.08 \pm 0.56) \times 10^{-4}$ under the narrow width approximation.
- Our predictions for $D^+ \to f_0 e^+ \nu_e$ exceed the experimental upper limits reported by BESIII and the measured values from CLEO. However, when accounting for the uncertainty in the mixing angle $(25^\circ < \theta < 40^\circ)$, the value of $Br(D^+ \to f_0(980) e^+ \nu_e)$ falls in the range of $(3.28 \sim 7.58) \times 10^{-5}$, which is able to cover the experimental range.

Semi-leptonic Decays

- The branching ratios of the decays $D_{(s)} \to (K_0^*(1430), a_0(1450))\ell^+\nu_\ell$.

	$10^{-4} \times \mathcal{B}r(D^+ \to \bar{K}_0^* e^+\nu_e)$	$10^{-4} \times \mathcal{B}r(D^+ \to \bar{K}_0^* \mu^+\nu_\mu)$	$10^{-5} \times \mathcal{B}r(D_s^+ \to K_0^* e^+\nu_e)$	$10^{-5} \times \mathcal{B}r(D_s^+ \to K_0^* \mu^+\nu_\mu)$
This work	$3.34^{+0.02+0.00+0.14}_{-0.02-0.13-0.39}$	$2.32^{+0.01+0.00+0.07}_{-0.01-0.09-0.25}$	$2.48^{+0.01+0.00+0.09}_{-0.01-0.03-0.27}$	$1.85^{+0.01+0.00+0.08}_{-0.01-0.00-0.20}$
LCSR'24,23	4.59	3.52	3.6	3.1
QCDSR'06	4.6	4.6	2.4	2.4
CLFQM'17	2.9	2.2	2.7	2.2
	$10^{-6} \times \mathcal{B}r(D^0 \to a_0^- e^+\nu_e)$	$10^{-6} \times \mathcal{B}r(D^0 \to a_0^- \mu^+\nu_\mu)$	$10^{-6} \times \mathcal{B}r(D^+ \to a_0 e^+\nu_e)$	$10^{-6} \times \mathcal{B}r(D^+ \to a_0 \mu^+\nu_\mu)$
This work	$5.80^{+0.01+0.00+0.46}_{-0.01-0.00-0.23}$	$3.98^{+0.01+0.12+0.28}_{-0.01-0.04-0.19}$	$8.01^{+0.04+0.00+0.31}_{-0.04-0.30-0.61}$	$5.48^{+0.03+0.00+0.21}_{-0.03-0.21-0.41}$
LCSR'21	3.14	2.01	4.28	2.76
CLFQM'17	–	–	5.4	3.8

Semi-leptonic Decays

- The branching ratios of the decays $D_{(s)} \to A\ell\nu_\ell$ with $A = a_1(1260), b_1(1235), f_1 = f_1(1285), f_1' = f_1(1420)$.

	$10^{-5}\times\mathcal{B}r(D^0\to a_1^- e^+\nu_e)$	$10^{-5}\times\mathcal{B}r(D^0\to a_1^- \mu^+\nu_\mu)$	$10^{-5}\times\mathcal{B}r(D^+\to a_1 e^+\nu_e)$	$10^{-5}\times\mathcal{B}r(D^+\to a_1 \mu^+\nu_\mu)$
This work	$4.39^{+0.01+0.23+0.16}_{-0.01-0.11-0.22}$	$3.88^{+0.01+0.19+0.13}_{-0.01-0.12-0.19}$	$5.80^{+0.03+0.30+0.21}_{-0.03-0.14-0.29}$	$5.13^{+0.02+0.26+0.18}_{-0.02-0.15-0.26}$
LCSR'19	2.85	–	3.76	–
LCSR'21	6.90	6.27	9.38	8.52
QCDSR'16	1.11	–	1.47	–
QCDSR'21	5.261	4.732	6.673	6.002
CLFQM'15	4.1	3.6	–	–
	$10^{-5}\times\mathcal{B}r(D^0\to b_1^- e^+\nu_e)$	$10^{-5}\times\mathcal{B}r(D^0\to b_1^- \mu^+\nu_\mu)$	$10^{-5}\times\mathcal{B}r(D^+\to b_1 e^+\nu_e)$	$10^{-5}\times\mathcal{B}r(D^+\to b_1 \mu^+\nu_\mu)$
This work	$5.52^{+0.01+0.08+0.35}_{-0.01-0.06-0.34}$	$4.74^{+0.01+0.05+0.30}_{-0.01-0.03-0.29}$	$7.29^{+0.03+0.08+0.46}_{-0.03-0.10-0.45}$	$6.27^{+0.03+0.05+0.40}_{-0.03-0.07-0.37}$
LCSR'19	1.88	–	2.47	–
LCSR'21	4.85	4.40	6.58	6.00
CLFQM'17	–	–	7.4	6.4
	$10^{-5}\times\mathcal{B}r(D^+\to f_1 e^+\nu_e)$	$10^{-5}\times\mathcal{B}r(D^+\to f_1 \mu^+\nu_\mu)$	$10^{-4}\times\mathcal{B}r(D_s^+\to f_1 e^+\nu_e)$	$10^{-4}\times\mathcal{B}r(D_s^+\to f_1 \mu^+\nu_\mu)$
This work	$3.75^{+0.02+0.09+0.00}_{-0.02-0.06-0.06}$	$3.21^{+0.02+0.07+0.00}_{-0.02-0.06-0.05}$	$2.58^{+0.01+0.10+0.03}_{-0.01-0.09-0.03}$	$2.22^{+0.01+0.09+0.03}_{-0.01-0.07-0.02}$
CLFQM'17	3.7	3.2	$0.6-3.6$	$0.52-3.06$
QCDSR'16	1.07	–	–	–
	$10^{-7}\times\mathcal{B}r(D^+\to f_1' e^+\nu_e)$	$10^{-7}\times\mathcal{B}r(D^+\to f_1' \mu^+\nu_\mu)$	$10^{-4}\times\mathcal{B}r(D_s^+\to f_1' e^+\nu_e)$	$10^{-4}\times\mathcal{B}r(D_s^+\to f_1' \mu^+\nu_\mu)$
This work	$6.95^{+0.03+0.15+0.00}_{-0.03-0.11-0.16}$	$5.70^{+0.03+0.11+0.00}_{-0.03-0.11-0.11}$	$3.49^{+0.02+0.15+0.04}_{-0.02-0.12-0.04}$	$2.89^{+0.01+0.12+0.03}_{-0.01-0.09-0.03}$
CLFQM'17	$2-14$	$2-12$	2.5	2.1
QCDSR'16	1.22	–	–	–

Semi-leptonic Decays

- The branching ratios of the semi-leptonic decays $D_{(s)} \to K_1^{(\prime)} \ell^+ \nu_\ell$.

	$10^{-3} \times \mathcal{B}r(D^0 \to K_1^- e^+ \nu_e)$	$10^{-3} \times \mathcal{B}r(D^0 \to K_1^- \mu^+ \nu_\mu)$	$10^{-5} \times \mathcal{B}r(D^0 \to K_1^{\prime -} e^+ \nu_e)$	$10^{-5} \times \mathcal{B}r(D^0 \to K_1^{\prime -} \mu^+ \nu_\mu)$
This work	$1.49^{+0.00+0.03+0.13}_{-0.00-0.06-0.17}$	$1.24^{+0.00+0.03+0.10}_{-0.00-0.04-0.14}$	$5.53^{+0.01+0.50+0.61}_{-0.01-0.35-0.88}$	$4.50^{+0.01+0.41+0.48}_{-0.01-0.26-0.69}$
CLFQM'21	1.56	1.3	3.7	2.9
BESIII'24,25	1.05	0.78	–	–
	$10^{-3} \times \mathcal{B}r(D^+ \to \bar{K}_1^0 e^+ \nu_e)$	$10^{-3} \times \mathcal{B}r(D^+ \to \bar{K}_1^0 \mu^+ \nu_\mu)$	$10^{-4} \times \mathcal{B}r(D_s^+ \to K_1^0 e^+ \nu_e)$	$10^{-4} \times \mathcal{B}r(D_s^+ \to K_1^0 \mu^+ \nu_\mu)$
This work	$3.89^{+0.02+0.07+0.33}_{-0.02-0.14-0.46}$	$3.26^{+0.02+0.06+0.28}_{-0.02-0.10-0.37}$	$1.86^{+0.01+0.04+0.22}_{-0.01-0.04-0.24}$	$1.63^{+0.01+0.04+0.20}_{-0.01-0.04-0.21}$
CLFQM'17	3.2	2.6	1.7	1.5
CLFQM'21	3.97	3.31	–	–
BESIII'23,24,25	1.29	2.36	< 4.1	–
	$10^{-4} \times \mathcal{B}r(D^+ \to \bar{K}_1^{\prime 0} e^+ \nu_e)$	$10^{-4} \times \mathcal{B}r(D^+ \to \bar{K}_1^{\prime 0} \mu^+ \nu_\mu)$	$10^{-6} \times \mathcal{B}r(D_s^+ \to K_1^{\prime 0} e^+ \nu_e)$	$10^{-6} \times \mathcal{B}r(D_s^+ \to K_1^{\prime 0} \mu^+ \nu_\mu)$
This work	$1.48^{+0.01+0.11+0.09}_{-0.01-0.11-0.24}$	$1.21^{+0.01+0.09+0.08}_{-0.01-0.09-0.19}$	$8.26^{+0.04+0.40+0.17}_{-0.04-0.64-0.21}$	$7.22^{+0.03+0.41+0.15}_{-0.03-0.54-0.18}$
CLFQM'17	$0.05 - 0.20$	$0.04 - 0.17$	$0.5 - 1.4$	$0.5 - 1.2$
CLFQM'21	0.94	0.74	–	–

Semi-leptonic Decays

-

$$K_1(1270) = K_{1A}\sin\theta_{K_1} + K_{1B}\cos\theta_{K_1},$$
$$K_1(1400) = K_{1A}\cos\theta_{K_1} - K_{1B}\sin\theta_{K_1}. \quad (18)$$

- With the implementation of the new form factors for K_{1A} and K_{1B}, the predicted branching ratios for the decays $D^0 \to K_1^- \ell^+ \nu_\ell$ and $D^+ \to \bar{K}_1^0 \ell^+ \nu_\ell$ show no significant difference from previous results.
- The theoretical predictions for the decay $D^+ \to \bar{K}_1^0 e^+ \nu_e$ is much larger than the data.
- The branching ratio of the decay $D^+ \to \bar{K}_1^0 e^+ \nu_e$ should larger than that of the decay $D^+ \to \bar{K}_1^0 \mu^+ \nu_\mu$ due to the larger phase space.

Summary

- The form factors of the transitions $D_{(s)} \to P, S, V, A$ are systematically investigated within the CLFQM framework.
- For the form factors of the transitions $D_{(s)} \to P, V$, our predictions are consistent well with those given by other theoretical approaches, such as LCSR, CCQM, QCDSR and RQM.
- Significant discrepancies emerge in some $D_{(s)} \to S, A$ transitions, such as $D \to a_0(980), a_0(1450)$ and $D_{(s)} \to K_{1B}$, which highlight uncertainties in the internal structures of the scalar and axial-vector mesons.
- Pronounced differences occur in the decays $D_{(s)} \to (S, A)\ell\nu_\ell$ among different theoretical predictions. We look forward to measuring more of these decay channels at BESIII.
- A comprehensive and systematic study of $D_{(s)}$ meson semileptonic decays in both theory and experiment plays a crucial role in testing SM and searching for NP in charm physics.

Thank you for your attention!

表: The branching ratios of the semi-leptonic decays $D_{(s)} \to (\pi, K)\ell\nu_\ell$.

	$10^{-3}\times\mathcal{B}r(D^0\to\pi^- e^+\nu_e)$	$10^{-3}\times\mathcal{B}r(D^0\to\pi^-\mu^+\nu_\mu)$	$10^{-3}\times\mathcal{B}r(D^+\to\pi^0 e^+\nu_e)$	$10^{-3}\times\mathcal{B}r(D^+\to\pi^0\mu^+\nu_\mu)$
This work	$2.69^{+0.01+0.02+0.02}_{-0.01-0.03-0.04}$	$2.57^{+0.01+0.02+0.02}_{-0.01-0.02-0.04}$	$3.46^{+0.02+0.02+0.02}_{-0.02-0.03-0.05}$	$3.30^{+0.02+0.02+0.02}_{-0.02-0.03-0.05}$
LCSR	2.78	2.75	3.52	3.49
CCQM	2.2	2.2	2.9	2.8
RQM	2.78	2.74	3.53	3.47
CLFQM	–	–	4.1	4.1
HMXT	2.7	–	3.3	–
PDG	2.93	2.67	3.72	3.50
BESIII	–	2.72	3.50	–
Belle	–	2.31	–	–
	$10^{-2}\times\mathcal{B}r(D^0\to K^- e^+\nu_e)$	$10^{-2}\times\mathcal{B}r(D^0\to K^-\mu^+\nu_\mu)$	$10^{-2}\times\mathcal{B}r(D^+\to\bar{K}^0 e^+\nu_e)$	$10^{-2}\times\mathcal{B}r(D^+\to\bar{K}^0\mu^+\nu_\mu)$
This work	$3.70^{+0.01+0.00+0.01}_{-0.01-0.01-0.02}$	$3.45^{+0.01+0.00+0.01}_{-0.01-0.01-0.02}$	$9.39^{+0.05+0.02+0.02}_{-0.04-0.00-0.05}$	$8.75^{+0.04+0.00+0.02}_{-0.04-0.02-0.04}$
LCSR	3.20	3.15	8.12	7.98
CCQM	3.63	3.53	9.28	9.02
RQM	3.56	3.49	9.02	8.85
CLFQM	–	–	10.32	10.07
HMXT	3.4	–	8.4	–
PDG	3.542	3.41	8.73	8.76
Belle	–	3.45	–	–
BESIII	3.521	3.419	8.864	8.665
	$10^{-3}\times\mathcal{B}r(D_s^+\to K^0 e^+\nu_e)$	$10^{-3}\times\mathcal{B}r(D_s^+\to K^0\mu^+\nu_\mu)$	–	–
This work	$2.65^{+0.01+0.02+0.02}_{-0.01-0.02-0.02}$	$2.49^{+0.02+0.02+0.02}_{-0.01-0.01-0.02}$	–	–
LCSR	3.90	3.83	–	–
CCQM	2.0	2.0	–	–
RQM	4.0	3.9	–	–
CLFQM	2.7	2.6	–	–
LCSR	3.379	3.351	–	–
CQM	3.18	3.18	–	–
PDG	3.9	–	–	–
CLEO	3.9	–	–	–
BESIII	2.98	–	–	–

Semi-leptonic Decays

表: The branching ratios of the semi-leptonic decays $D \to (K^*, \rho, \omega)\ell\nu_\ell$.

	$10^{-2} \times \mathcal{B}r(D^0 \to K^{*-} e^+ \nu_e)$	$10^{-2} \times \mathcal{B}r(D^0 \to K^{*-} \mu^+ \nu_\mu)$	$10^{-2} \times \mathcal{B}r(D^+ \to \bar{K}^{*0} e^+ \nu_e)$	$10^{-2} \times \mathcal{B}r(D^+ \to \bar{K}^{*0} \mu^+ \nu_\mu)$
This work	$2.59^{+0.01+0.03}_{-0.01-0.04}$	$2.44^{+0.01+0.03}_{-0.01-0.04}$	$6.65^{+0.03+0.10}_{-0.03-0.10}$	$6.26^{+0.03+0.43}_{-0.03-0.22}$
LCSR	2.12	2.01	5.37	5.10
CCQM	2.96	2.80	7.61	7.21
RQM	1.92	1.82	4.87	4.62
CLFQM	–	–	7.5	7.0
CQM	2.46	–	6.24	–
UChPT	2.15	1.98	5.56	5.12
PDG	2.15	1.89	5.40	5.27
BESIII	2.033	–	–	–
CLEO	2.16	–	–	5.27
	$10^{-3} \times \mathcal{B}r(D^0 \to \rho^- e^+ \nu_e)$	$10^{-3} \times \mathcal{B}r(D^0 \to \rho^- \mu^+ \nu_\mu)$	$10^{-3} \times \mathcal{B}r(D^+ \to \rho^0 e^+ \nu_e)$	$10^{-3} \times \mathcal{B}r(D^+ \to \rho^0 \mu^+ \nu_\mu)$
This work	$1.69^{+0.00+0.02}_{-0.00-0.02}$	$1.60^{+0.00+0.02}_{-0.00-0.02}$	$2.17^{+0.01+0.03}_{-0.01-0.03}$	$2.07^{+0.01+0.03}_{-0.01-0.03}$
LCSR	1.81	1.73	2.29	2.20
CCQM	1.62	1.55	2.09	2.01
RQM	1.96	1.88	2.49	2.39
CLFQM	–	–	2.3	2.2
UChPT	1.97	1.84	2.54	2.37
PDG	1.77	1.89	2.18	2.4
BESIII	1.439	1.35	1.86	–
CLEO	1.77	–	2.17	–
	$10^{-3} \times \mathcal{B}r(D^+ \to \omega e^+ \nu_e)$	$10^{-3} \times \mathcal{B}r(D^+ \to \omega \mu^+ \nu_\mu)$	–	–
This work	$2.02^{+0.01+0.01}_{-0.01-0.05}$	$2.00^{+0.01+0.08}_{-0.01-0.13}$	–	–
LCSR	1.93	1.85	–	–
CCQM	1.85	1.78	–	–
RQM	2.17	2.08	–	–
CLFQM	2.1	2.0	–	–
UChPT	2.46	2.29	–	–
PDG	1.93	1.85	–	–
CLEO	1.82	–	–	–
BESIII	1.68	1.77	–	–