Nuclear matrix elements for $0\nu\beta\beta$ decay

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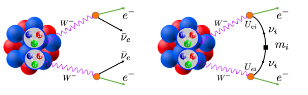
Outline

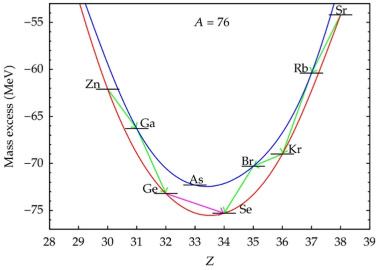
1. Significance of the NMEs for $0\nu\beta\beta$ decay
2. Ab initio computations of the NMEs of $0\nu\beta\beta$ decay
3. A strategy to enhance the sensitity of $0\nu\beta\beta$ decay
4. A proposal to reduce the uncertainty of the NMEs
5. Summary

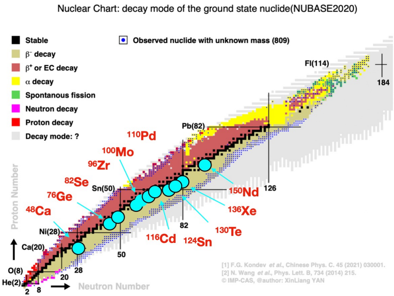
An introduction to $0\nu\beta\beta$ decay

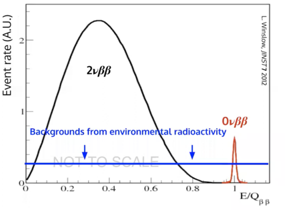
- The two modes of $\beta^-\beta^-$ decay:

$$(A, Z) \rightarrow (A, Z+2) + 2e^- + (2\bar{\nu}_e)$$









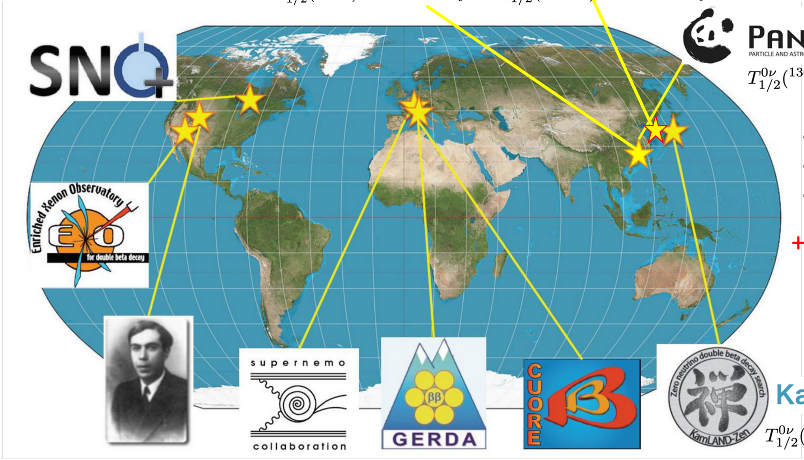
Wordwide experiments searching for $0\nu\beta\beta$ decay

CDEX-1T
China dark-matter experiment

$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 1.0 \times 10^{23}\mathrm{yr}$

AMoRE experiment
Advanced Mo-based Rare process Experiment

$T^{0\nu}_{1/2}(^{100}\mathrm{Mo}) > 2.9 \times 10^{24}\mathrm{yr}$



PANDA X
PARTICLE AND ASTROPHYSICAL XENON TPC

$T^{0\nu}_{1/2}(^{136}\mathrm{Xe}) > 2.1 \times 10^{24}\mathrm{yr}$

- NvDEX
- CUPID-China
- JUNO

+several others

KamLAND-Zen

$T^{0\nu}_{1/2}(^{136}\mathrm{Xe}) > 2.3 \times 10^{26}\mathrm{yr}$

$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 1.8 \times 10^{26}\mathrm{yr}$

Most sensitive measurements and limits on neutrino mass

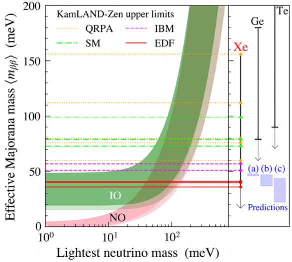
Isotope	$G_{0\nu}$ $[10^{-14}\ \mathrm{yr}^{-1}]$	$M^{0\nu}$ [min, max]	$T_{1/2}^{0\nu}$ [yr]	$\langle m_{\beta\beta}\rangle$ [meV]	Experiments with best sensitivity References
^{48}Ca	2.48	[0.85, 2.94]	$> 5.8 \cdot 10^{22}$	[2841, 9828]	CANDLES: PRC78, 058501 (2008)
^{76}Ge	0.24	[2.38, 6.64]	$> 1.8 \cdot 10^{26}$	[73, 180]	GERDA: PRL125, 252502(2020)
^{82}Se	1.01	[2.72, 5.30]	$> 4.6 \cdot 10^{24}$	[277, 540]	CUPID-0: PRL129, 111801 (2023)
^{96}Zr	2.06	[2.86, 6.47]	$> 9.2 \cdot 10^{21}$	[3557, 8047]	NPA847, 168 (2010)
^{100}Mo	1.59	[3.84, 6.59]	$> 2.9 \cdot 10^{24}$	[210, 610]	AMoRE-I: PRL134, 082501 (2025)
^{116}Cd	0.48	[3.29, 5.52]	$> 2.2 \cdot 10^{23}$	[1766, 2963]	PRD 98, 092007 (2018)
^{130}Te	1.42	[1.37, 6.41]	$> 3.5 \cdot 10^{25}$	[70, 250]	CUORE: Science (2025)
^{136}Xe	1.46	[1.11, 4.77]	$> 3.8 \cdot 10^{26}$	[28, 122]	KamLAND-Zen: arXiv:2406.11438v1 (2024)
^{150}Nd	6.30	[1.71, 5.60]	$> 2.0 \cdot 10^{22}$	[1593, 5219]	NEMO-3: PRD 94, 072003 (2016)

If $0\nu\beta\beta$ decay is driven by exchanging light Majorana neutrinos,

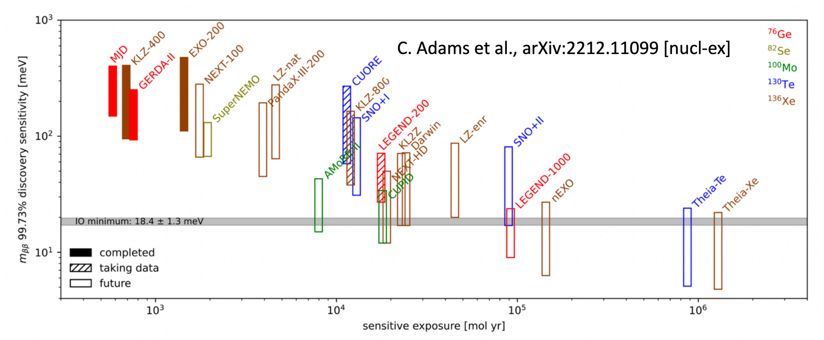
$$\langle m_{\beta\beta}\rangle \equiv \left|\sum_{j=1}^{3} U_{ej}^2 m_j\right| = \left[\frac{m_e^2}{g_A^4 G_{0\nu} T_{1/2}^{0\nu} \left|M^{0\nu}\right|^2}\right]^{1/2}$$

Accurate values of the NMEs $M^{0\nu} = \langle 0_F^+|\hat{O}^{0\nu}|0_I^+\rangle$ are crucial for designing and interpreting those experiments, as they link the observed decay rate to the effective neutrino mass $\langle m_{\beta\beta}\rangle$.

KamLAND-Zen: PRL130, 051801(2023)



Current and future experimental measurements



- Lifetime sensitivity of the future ton-scale experiments: $T^{0\nu}_{1/2} > 10^{28}$yr.
- Whether or not the ton-scale experiments are able to cover the entire parameter space for the IO case depends strongly on the employed NME, with a large systematic uncertainty.

Status of studies on nuclear matrix elements

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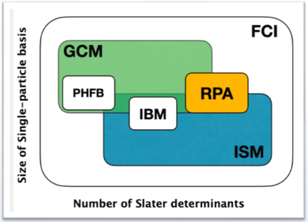
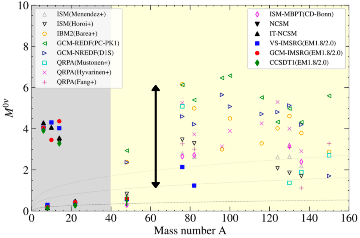


JMY et al., PPNP 126, 103965 (2022)



Problems to be resolved

- **Uncertainty from nuclear forces**
  - EDFs: parameters of pairing force and tensor force, etc.
  - ISM: parameters of the interaction matrix elements
- **Uncertainty from truncations in model space**
  - EDF: large s.p basis, missing many-body correlation
  - ISM: full many-body correlation, small model space
- **Uncertainty from transition operators:**
  - axial-vector coupling g_A : quenching factor
  - Higher-order operators: two-body currents, etc.

Large model dependence in predicted NMEs, difficult to constrain or reduce.

Ab initio studies of nuclear weak processes

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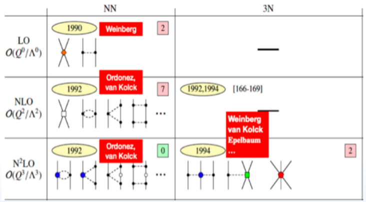
❑ Basic idea and advantages

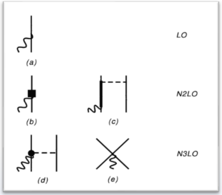
- Nuclear forces & transition operators: consistently constructed within **chiral EFT** (N, π)
- The theoretical uncertainty: controlled by the power-counting $(Q/\Lambda)^n$ with $Q \approx 140$ MeV, $\Lambda \approx 700$ MeV)
- Nuclear wave functions: solved using **exact or systematically improvable many-body methods**

Nuclear forces



Phys. Rep. 503, 1 (2011); 890,1 (2020)

Axial currents



Phys. Rev. C 95, 059902 (2017)
Few Body Syst. 62, 114 (2021)

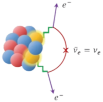
● Nuclear ab initio methods

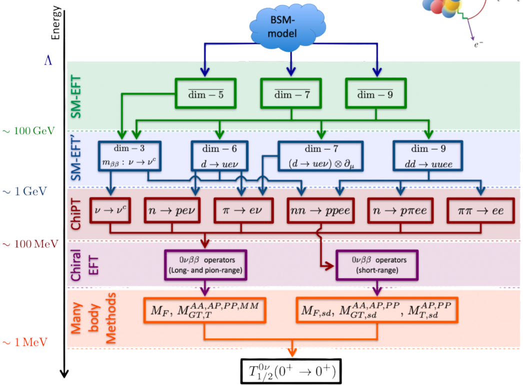
- ✓ 无芯壳模型 (NCSM)
- ✓ 介质相似重整化群方法 (IMSRG)
- ✓ 格林函数蒙特卡罗方法 (GFMC)
- ✓ 耦合团簇理论 (CC)
- ✓ 多体微扰论方法 (MBPT)
- ✓ Brueckner Hartree-Fock (BHF)
- ✓ ...

Frontiers in Physics 8, 379 (2020)

Lepton-number violating operators for neutrinoless double-beta decay

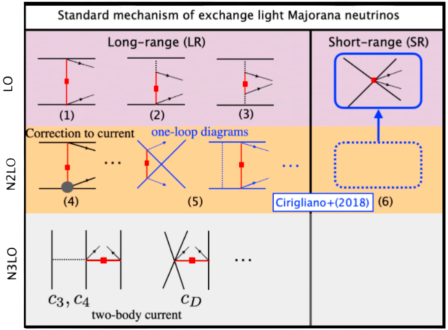
$$(A, Z) \to (A, Z+2) + 2e^- + (2\bar{\nu}_e)$$





The 0νββ decay operators in the chiral EFT

Power Counting: $\nu = 2A + 2L - 2 + \sum_i \left(\frac{n_f}{2} + d - 2 + n_e\right)_i$



G. Prézeau, M. Ramsey-Musolf, P. Vogel, PRD68, 034016 (2003); V. Cirigliano et al., JHEP 12, 097(2018)

- Multi-reference in-medium generator coordinate method (IM-GCM)

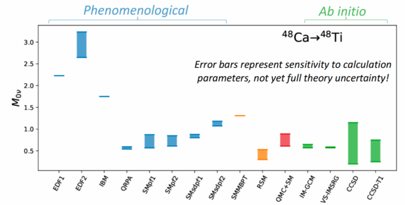
  JMY et al., PRL124, 232501 (2020)

- Valence-space shell model+IMSRG (VS-IMSRG)

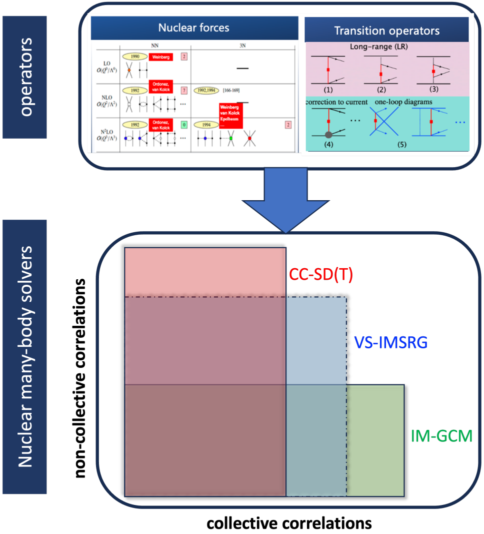
  A. Belley et al., PRL126, 042502 (2021)

- Coupled-cluster with singlets, doublets, and partial triplets (CCSDT1) .

  S. Novario et al., PRL126, 182502 (2021)



arXiv:2212.11099, adapted from J. Phys. G: Nucl. Part. Phys. 49, 120502 (2022)



Another ab initio study of $0\nu\beta\beta$ decay in ^{48}Ca

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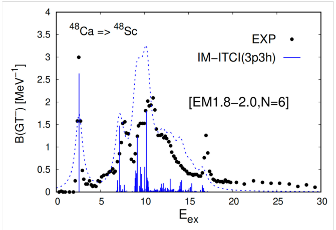
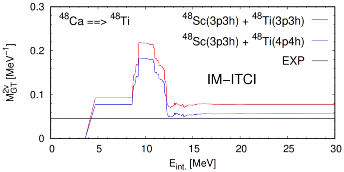
IMSRG + Configuration interaction (IMCI-Q) method

连新





More many-body correlations reduce the NME for both 2nu and 0nu DBD

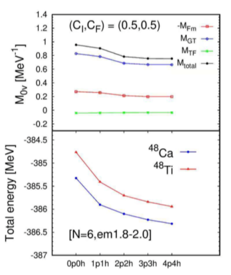


Table: $0\nu\beta\beta$ matrix elements for ^{48}Ca. ($\hbar\omega = 12$)

Force	Reference state	Cut off	$M^{0\nu}_F$	$M^{0\nu}_{GT}$	$M^{0\nu}_T$	$M^{0\nu}$
EM1.8/2.0	^{48}Ca (HF)	N=6,IM-ITCI(4p4h)	-0.376	1.035	-0.088	1.180
EM1.8/2.0	^{48}Ti (HFB)	N=6,IM-ITCI(4p4h)	-0.291	0.740	-0.088	0.833

A consistent NME is obtained between the two methods

In comparison with the IM-GCM calculation

		NME		
Interaction	$\hbar\omega$	$e_{max} = 6$	$e_{max} = 8$	$e_{max} = 10$
EM1.8/2.0	12	0.85	0.70	0.64
EM1.8/2.0	16	1.03	0.78	0.66

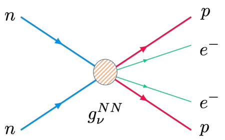
JMY et al., PRL124, 232501 (2020)

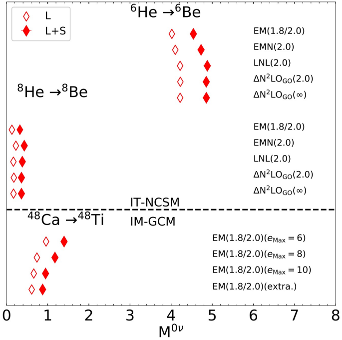
X. Lian, C. R. Ding, C.L. Bai, JMY, in preparation (2025)

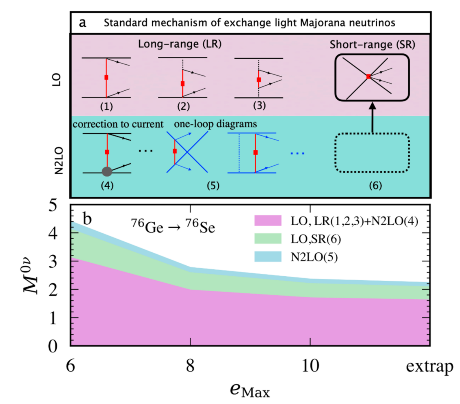
Promoting the contact transition operator to LO

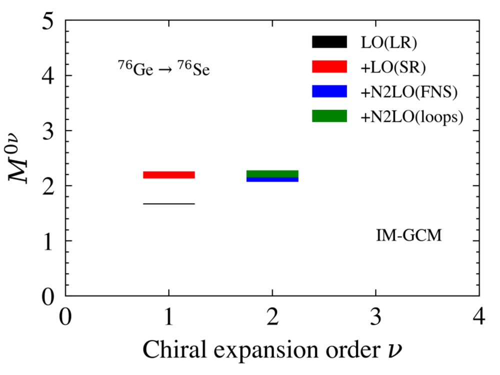
- A contact transition operator which could either enhance or quench the $0\nu\beta\beta$ decay, is needed to be promoted to LO to ensure renormalizibility. V. Cirigliano et al., PRL120, 202001 (2018)
- We determine the unknown LEC g_ν^{NN} of the contact operator, consistent with the employed chiral interaction (EM1.8/2.0), based on the synthetic data for the process $2n \to 2p + 2e^-$. V. Cirigliano et al., PRL126, 172002 (2021)
- The contact term turns out to enhance the NME for ^{48}Ca by 43(7)%, thus reducing the half-life $T_{1/2}^{0\nu}$ significantly.

R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)









- The contact term enhances the NME by about 30%
- The two parts of the contributions at N2LO (about 5%) almost cancel out.

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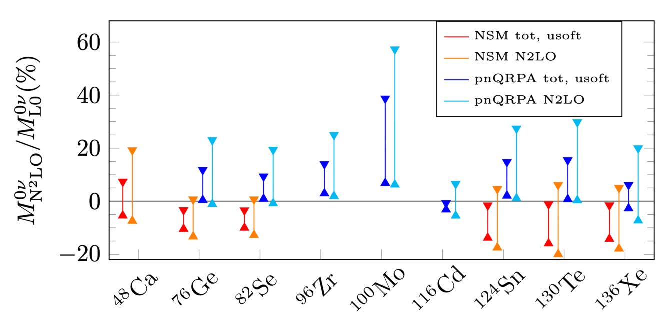
Table 2: Long- and short-range $0\nu\beta\beta$-decay matrix elements $M_L^{0\nu}$ and $M_S^{0\nu}$ calculated with the pnQRPA and nuclear shell model (NSM) for several nuclei. The ranges cover results for neutrino potentials with the couplings and regulators in Table 1, combined with Argonne and CD-Bonn short-range correlations.

	pnQRPA			NSM		
Nucleus	$M_L^{0\nu}$	$M_S^{0\nu}$	$M_S^{0\nu}/M_L^{0\nu}(\%)$	$M_L^{0\nu}$	$M_S^{0\nu}$	$M_S^{0\nu}/M_L^{0\nu}(\%)$
^{48}Ca				0.96 – 1.05	0.22 – 0.65	23 – 62
^{76}Ge	4.72 – 5.22	1.49 – 3.80	32 – 73	3.34 – 3.54	0.52 – 1.49	15 – 42
^{82}Se	4.20 – 4.61	1.27 – 3.24	30 – 70	3.20 – 3.38	0.48 – 1.38	15 – 41
^{96}Zr	4.22 – 4.63	1.24 – 3.19	29 – 69			
^{100}Mo	3.40 – 3.95	1.66 – 4.26	49 – 108			
^{116}Cd	4.24 – 4.57	1.10 – 2.80	26 – 61			
^{124}Sn	4.72 – 5.29	1.69 – 4.28	36 – 81	3.20 – 3.41	0.54 – 1.58	17 – 46
^{128}Te	3.92 – 4.50	1.37 – 3.45	35 – 77	3.56 – 3.80	0.61 – 1.76	17 – 46
^{130}Te	3.46 – 3.89	1.18 – 3.05	34 – 77	3.26 – 3.48	0.57 – 1.64	17 – 47
^{136}Xe	2.53 – 2.80	0.76 – 1.95	30 – 70	2.62 – 2.79	0.45 – 1.31	17 – 47

Lotta Jokiniemi, Pablo Soriano, Javier Menéndez, Phys.Lett.B 823, 136720(2021)

Convergence of the NMEs with the expansion order in pheno. m

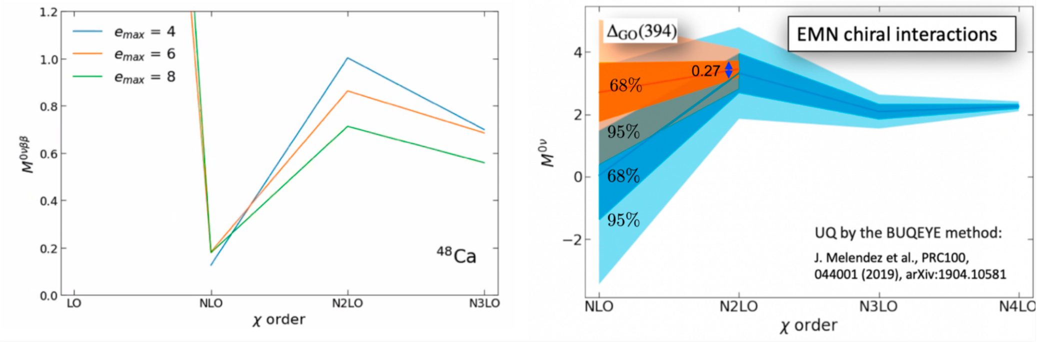


Daniel Castillo, Lotta Jokiniemi, Pablo Soriano, Javier Menéndez, Phys.Lett.B 860, 139181(2025)

Overall N2LO contribution (exception: 100Mo):

- Shell model: around 5% – 10%; pnQRPA: around 10% – 15%
- Uncertainties: scale dependence of the ultrasoft NMEs and short-range part of the loop NMEs.
- Caveat: nuclear forces are not derived from the chiral EFT in pheno. models. Inconsistence between nuclear forces and transition operators.

$^{48}\mathrm{Ca} \rightarrow ^{48}\mathrm{Ti} + 2e^-$ $^{76}\mathrm{Ge} \rightarrow ^{76}\mathrm{Se} + 2e^-$



- The NME converges with the chiral expansion order ν of nuclear forces.
- The EFT truncation error is shrinking with the increase of ν.

truncation of transition operators

Emulator error

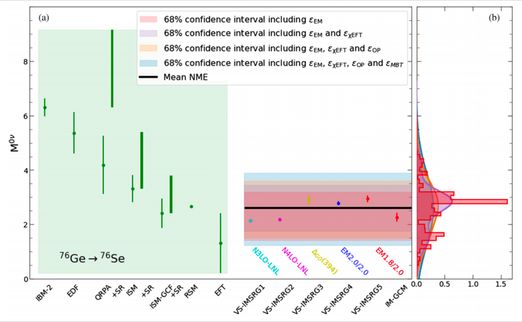
$$M^{0\nu} = M^{0\nu}_{k} + \epsilon_{\chi\mathrm{EFT}} + \epsilon_{\mathrm{MBT}} + \epsilon_{\mathrm{OP}} + \epsilon_{\mathrm{EM}},$$

truncation of nuclear forces

many-body truncation

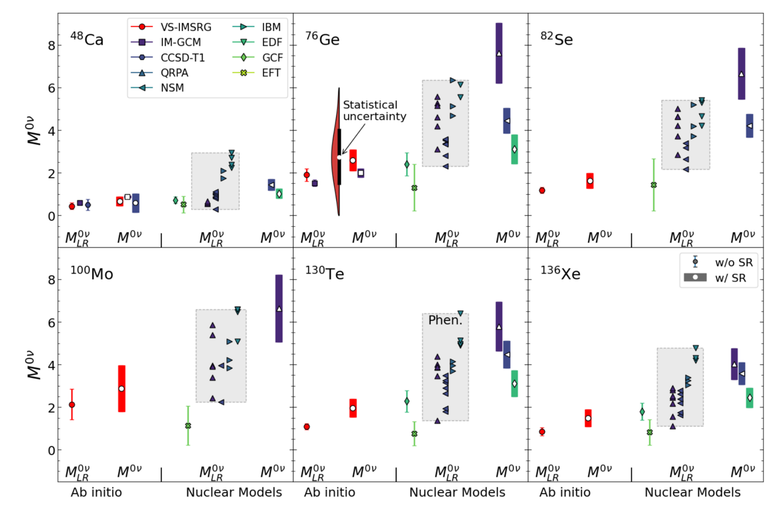
TABLE I. The recommended value for the total NME of $0\nu\beta\beta$ decay in ^{76}Ge, together with the uncertainties from different sources.

$M^{0\nu}$	ϵ_{LEC}	$\epsilon_{\chi\mathrm{EFT}}$	ϵ_{MBT}	ϵ_{OP}	ϵ_{EM}
$2.60^{+1.28}_{-1.36}$	0.75	0.3	0.88	0.47	< 0.06

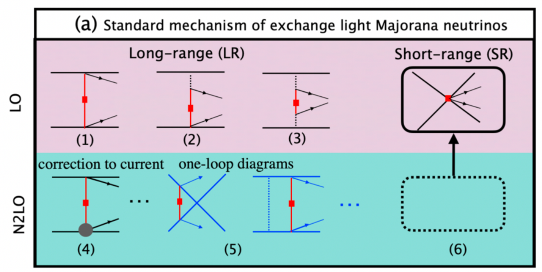


- Our recommended value $M^{0\nu} = 2.60^{+1.28}_{-1.36}$.
- Together with the best half-life limit: $> 1.8 \times 10^{26}$ yr, it sets the upper limit $\langle m_{\beta\beta} \rangle = 187^{+205}_{-62}$ meV, and the sensitivity of the next-generation experiment $\langle m_{\beta\beta} \rangle = 22^{+24}_{-7}$ meV, covering almost the entire range of IO hierarchy.

A. Belley, JMY et al., PRL132, 182502 (2024)



A. Belley, PhD thesis (2025)



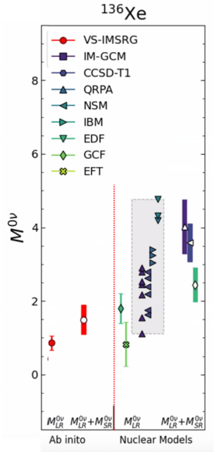
IM-GCM calcualtion for each contribution to the NME of 136Xe

Interaction	LO(LR) (1-3)	LO (SR) (6)	Part of N^2LO (4)	Residual N^2LO (5)	Total (1-6)
EM1.8–2.0	1.52	0.57(8)	−0.67	0.21	1.63(8)
N2LOGO394	1.56	0.50(7)	−0.86	0.21	1.41(7)

- The NMEs vary from 1.4 to 1.7 by two different nuclear chiral forces
- The contact term enhances the NME by about 30%-40%
- The net N2LO quenches the NME about 25%

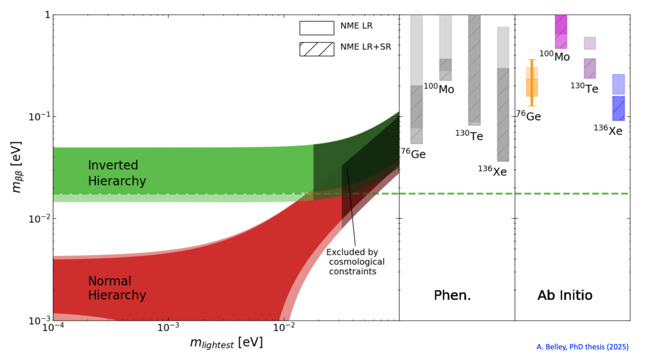
C. R. Ding, JMY, et al. in preparation (2025)

The NME by the VS-IMSRG: [1.08-1.90]
a factor of about two uncertainty

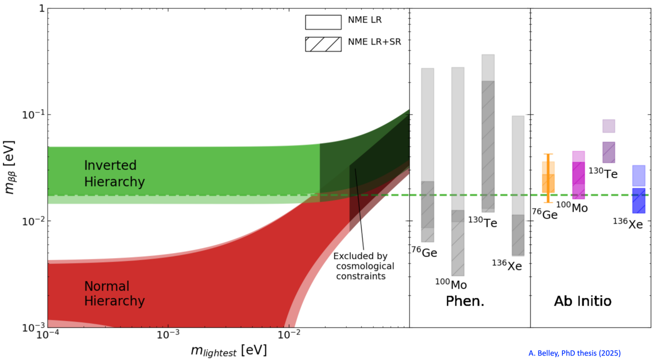


A. Belley et al., arXiv2307.15156 [nucl-th]

Effective neutrino mass from current $0\nu\beta\beta$ decay



- Using the NMEs from the phenomenological models, current ^{136}Xe exp touches the parameter region for the IH case.
- Using the NMEs from ab initio nuclear models, none of the four candidate nuclei touches the region.



A. Belley, PhD thesis (2025)

- Using the NMEs from the phenomenological models with the contribution of short-range transition operator, future ^{100}Mo and ^{136}Xe can cover the IH case.
- Using the NMEs from ab initio nuclear models, none of the four candidate nuclei could fully cover the IH case on the ton-scale exp.

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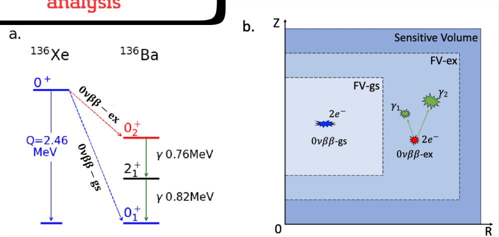
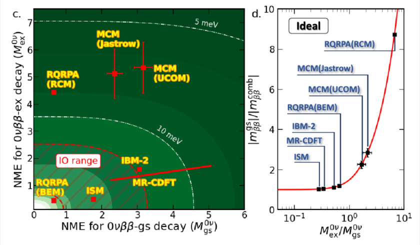
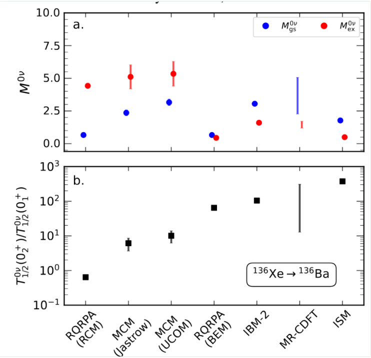
Combined multi-transition analysis





10-year data acquisition on XLZD and the PandaX-xT

Comparison between the NMEs for both decay modes



Some models predicted a larger NME for the decay to the excited 0+ state

Ding, Han, Wang, JMY, arXiv2508.17413 [nucl-th]

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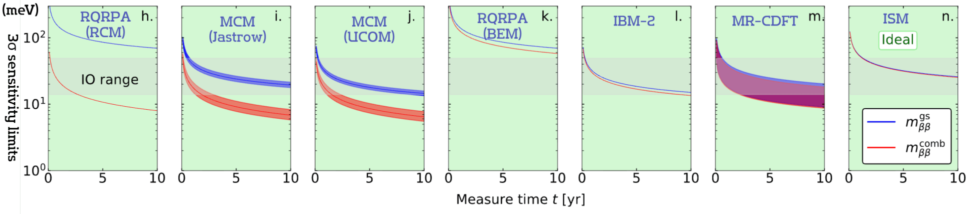
Combined multi-transition analysis



丁晨蓉

- The combined analysis: can enhance the sensitivity of next-generation experiments, without requiring a larger detector.
- The effectiveness strongly depends on the NMEs, which currently exhibit significant discrepancies across different nuclear models.
- It underscores the importance of achieving theoretically accurate and systematically quantified NME calculations for the NLDBD.

Ding, Han, Wang, JMY, arXiv2508.17413 [nucl-th]

A proposal to reduce the uncertainty of the NMEs

- angular distribution of heavy-ion collision

$$\frac{dN_{ch}}{d^2\mathbf{p}} \propto \frac{dN_{ch}}{p_T dp_T}\left(1+\sum_{n>1} 2 v_n \cos n(\phi-\phi_n)\right)$$

where v_2 is strongly correlated with the initial-state ellipticity parameter, ε_2

- observables of interest

$$\Delta(v_2\{2\}) \equiv \frac{1}{2}\Delta\left(\langle v_2^2\rangle\right),$$

$$\Delta(\delta[p_T]) \equiv \frac{1}{2}\Delta\left(\langle(\delta[p_T])^2\rangle\right),$$

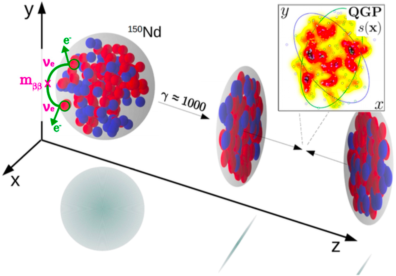
$$\Delta\left(\mathrm{cov}(v_2^2,[p_T])\right) \equiv \frac{1}{3}\Delta\left(\langle v_2^2\delta[p_T]\rangle\right),$$

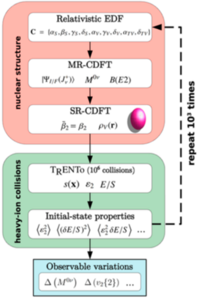
$$\Delta(v_2\{4\}) \equiv \frac{1}{4}\Delta\left(2\langle v_2^2\rangle^2-\langle v_2^4\rangle\right)$$

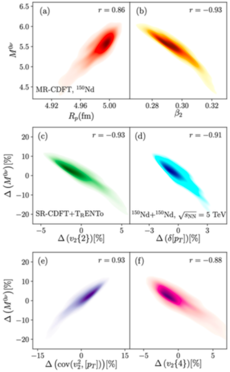
- Relative variation of an observable

$$\Delta(O) = \frac{O-\langle O\rangle_C}{|\langle O\rangle_C|}$$

李义







Y. Li, X. Zhang, G. Giacalone, JMY, PRL135, 022301 (2025)

Open questions: model-dependence? nucleus-dependence?

A precise calculation of the NMEs for $0\nu\beta\beta$ decay is crucial to the design and intepretation of current and next-generation experiments.

- A large uncertainty exists in the NMEs prediced by pheno. nuclear models. The inclusion of the contact transition operator can increase the NMEs overally by 20%-80%.
- Ab initio methods for the NMEs of $0\nu\beta\beta$ decay: systematically smaller than those by the pheno. nuclear models.
- Even though large uncertainties remain in the NMEs by ab initio methods, there is in principle a way to reduce them.
- A combined analysis of decays to both ground state and excited state was proposed to enhance the sensitivity of the next-generation Xe exp.
- A strategy of using the HI collision exp to reduce the uncertainty in the NMEs was proposed, though more comprehensive analysis remains to be done.

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Collaborators

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Thank you for your attention!