

Cosmological Signatures of Neutrino Seesaw mechanism

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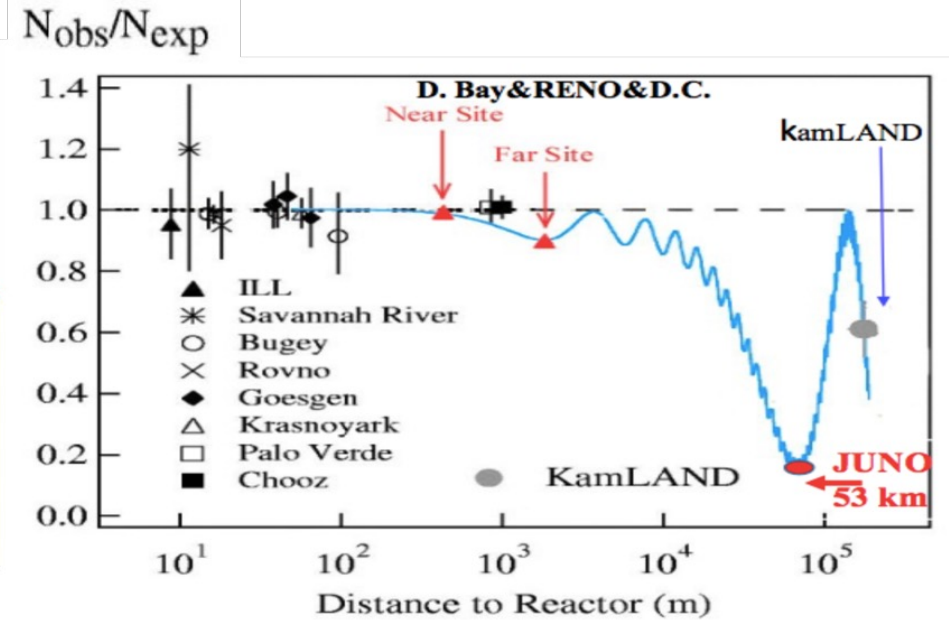
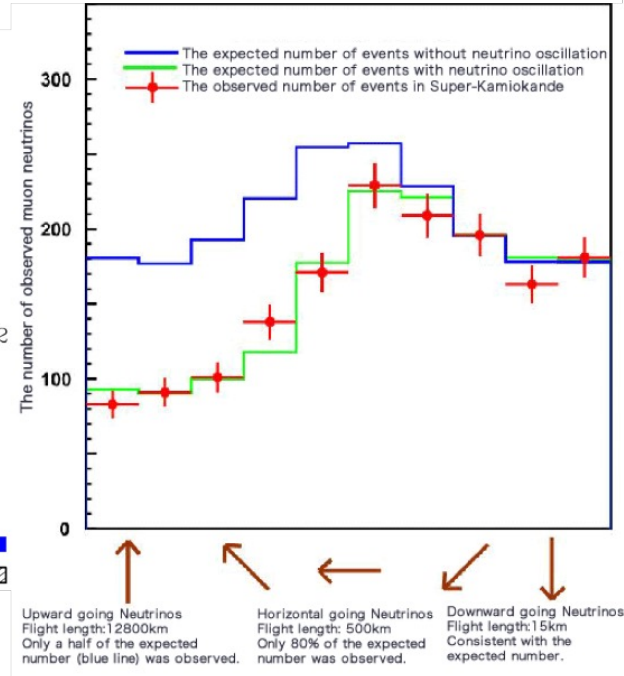
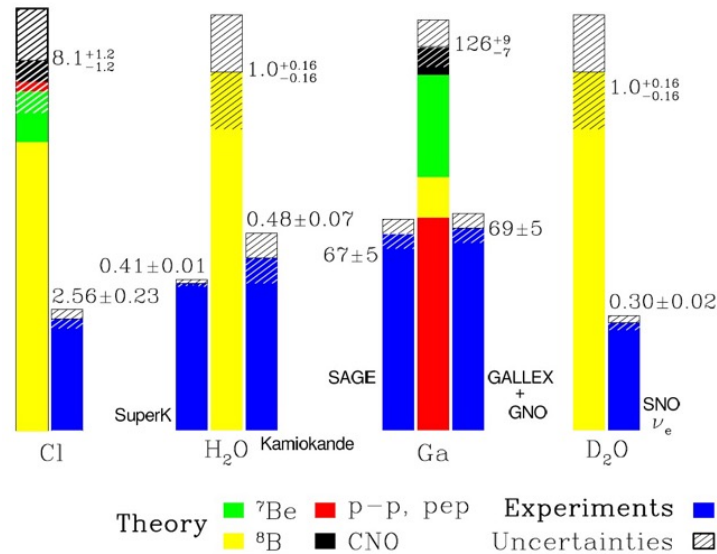
**With Hongjian He, Linghao Song, Jingtao You, arXiv: 2412.21045(PRDL),
2412.16033(PRD)**

Workshop on Neutrinos and related New Physics Research, NuPhyR

2025.11.29

Neutrino masses

Neutrino oscillation indicates massive neutrinos



Solar Neutrino oscillations

$$\theta_{12}$$

$$\Delta m_{21}^2 \simeq 7.42 \times 10^{-5} \text{ eV}^2$$

Atmospheric Neutrino Oscillations

$$\theta_{23}$$

$$|\Delta m_{13}^2| \approx |\Delta m_{23}^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2$$

Reactor Neutrino Oscillations

$$\theta_{13}$$

Cosmological limit

Table 26.2: Summary of $\sum m_\nu$ constraints.

PDG

	Model	95% CL (eV)	Ref.
CMB alone			
Pl18[TT+lowE]	$\Lambda\text{CDM}+\sum m_\nu$	< 0.54	[24]
Pl18[TT,TE,EE+lowE]	$\Lambda\text{CDM}+\sum m_\nu$	< 0.26	[24]
CMB + probes of background evolution			
Pl18[TT,TE,EE+lowE] + BAO	$\Lambda\text{CDM}+\sum m_\nu$	< 0.13	[49]
Pl18[TT,TE,EE+lowE] + BAO	$\Lambda\text{CDM}+\sum m_\nu+5 \text{ params.}$	< 0.515	[25]
CMB + LSS			
Pl18[TT+lowE+lensing]	$\Lambda\text{CDM}+\sum m_\nu$	< 0.44	[24]
Pl18[TT,TE,EE+lowE+lensing]	$\Lambda\text{CDM}+\sum m_\nu$	< 0.24	[24]
Pl18[TT,TE,EE+lowE]+ ACT[lensing]	$\Lambda\text{CDM}+\sum m_\nu$	< 0.12	[50]
CMB + probes of background evolution + LSS			
Pl18[TT,TE,EE+lowE] + BAO + RSD	$\Lambda\text{CDM}+\sum m_\nu$	< 0.10	[49]
Pl18[TT,TE,EE+lowE+lensing] + BAO + RSD + Shape	$\Lambda\text{CDM}+\sum m_\nu$	< 0.082	[51]
Pl18[TT+lowE+lensing] + BAO + Lyman- α	$\Lambda\text{CDM}+\sum m_\nu$	< 0.087	[52]
Pl18[TT,TE,EE+lowE] + BAO + RSD + SN + DES-Y1	$\Lambda\text{CDM}+\sum m_\nu$	< 0.12	[49]
Pl18[TT,TE,EE+lowE] + BAO + RSD + SN + DES-Y3	$\Lambda\text{CDM}+\sum m_\nu$	< 0.13	[53]

The heavy neutrino mass should be around 0.05 eV(IO)-0.06eV(NO)

Seesaw mechanism

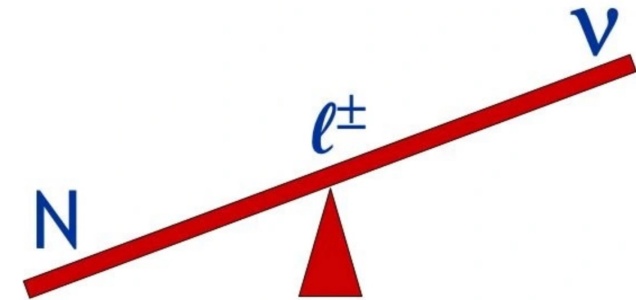
Origin of neutrino masses: seesaw mechanism

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + y_\nu \tilde{H} \bar{L} N_R - \frac{1}{2} M_R \bar{N}_R^c N_R + h.c.$$

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

$$m_\nu \sim \frac{m_D^2}{M_R} = \frac{y_\nu^2 \langle h \rangle^2}{2M_R}$$

P. Minkowski ; T. Yanagida; S. L. Glashow;
M. Gell-Mann, P. Ramond and R. Slansky



- Natural prediction of small neutrino masses
- Explain the baryon asymmetry of the universe: leptogenesis

Baryogenesis Without Grand Unification, Fukugita and Yanagida, 1986'

Seesaw mechanism

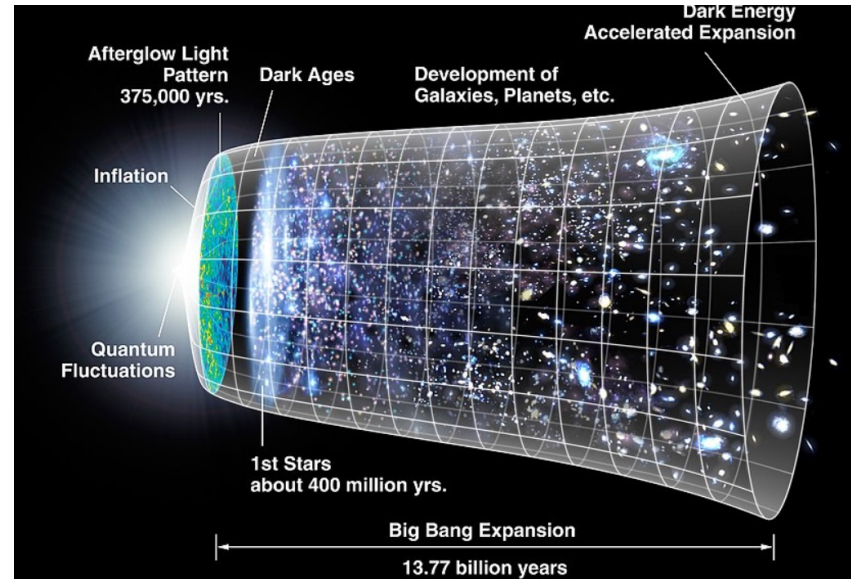
$$m_\nu \sim \frac{m_D^2}{M_R} = \frac{y_\nu^2 \langle h \rangle^2}{2M_R}$$

If the Yukawa coupling is $O(1)$ (as predicted by the GUT), the seesaw scale M_R should be around 10^{13-14} GeV, which is much beyond the reach of particle experiments.

How to test such high scale seesaw?

Inflation

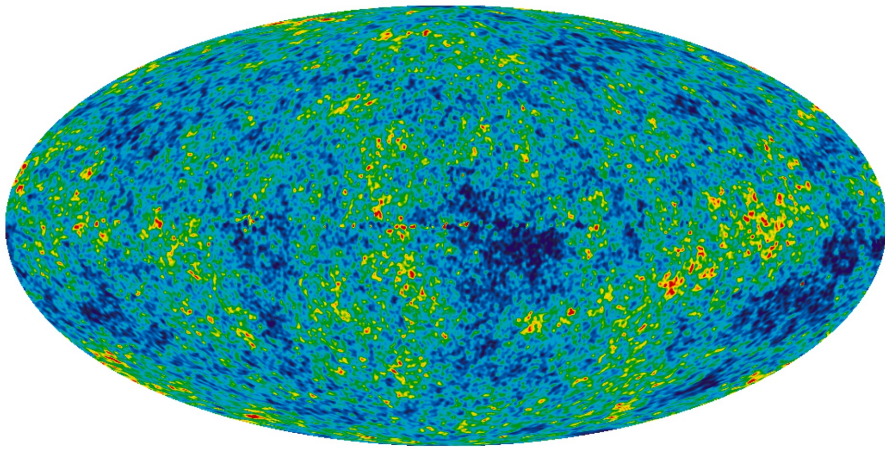
Rapid expansion of the universe in the early time



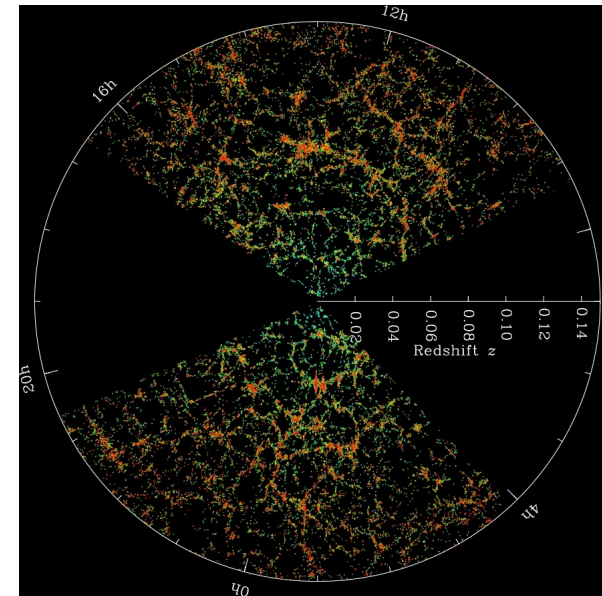
- Flatness problem
- Horizon problem
- Seeding the primordial anisotropies in CMB

Inflation

Stretching quantum fluctuations to large scale



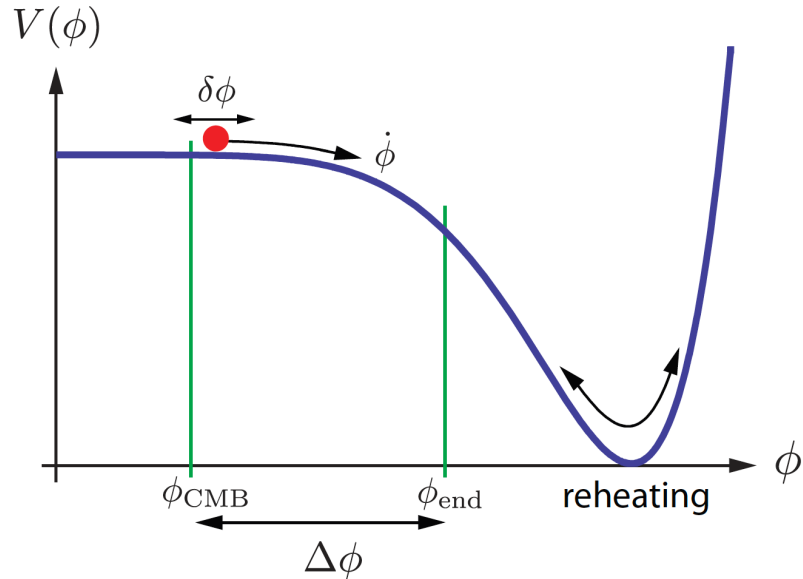
$$\frac{\delta T}{T} \sim 10^{-5}$$



Such small fluctuations finally develop the large structure of our universe

Slow-roll Inflation

Inflation is driven by a scalar field (inflaton)



$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$
$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right)$$

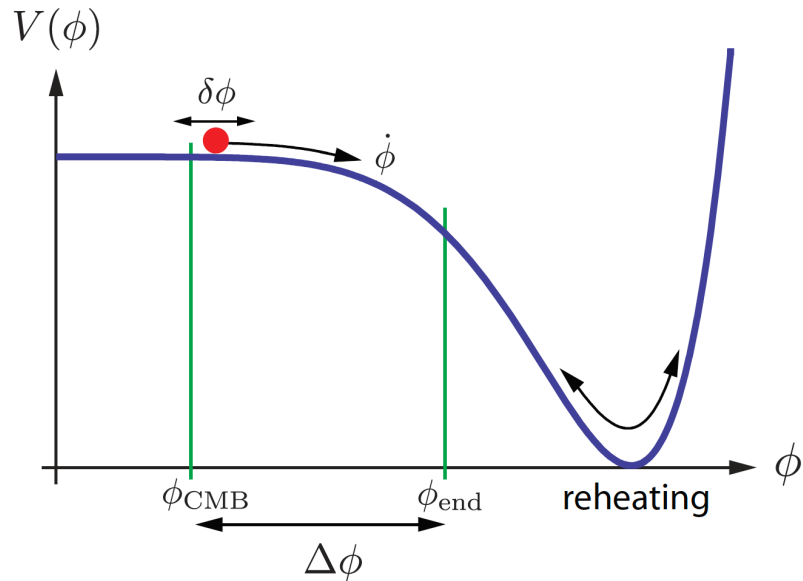
Slow roll condition (epsilon, eta $\ll 1$)

$$\epsilon_v(\phi) \equiv \frac{M_{\text{pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \quad \eta_v(\phi) \equiv M_{\text{pl}}^2 \frac{V_{,\phi\phi}}{V}$$

- Hubble parameter is nearly constant (de Sitter spacetime)
- After inflation, inflaton oscillates at the bottom of the potential and finally decays into SM particles, then reheats the universe (**still no clear how it occurs**)

Slow-roll Inflation

In a de Sitter universe, scalar fields get quantum fluctuation



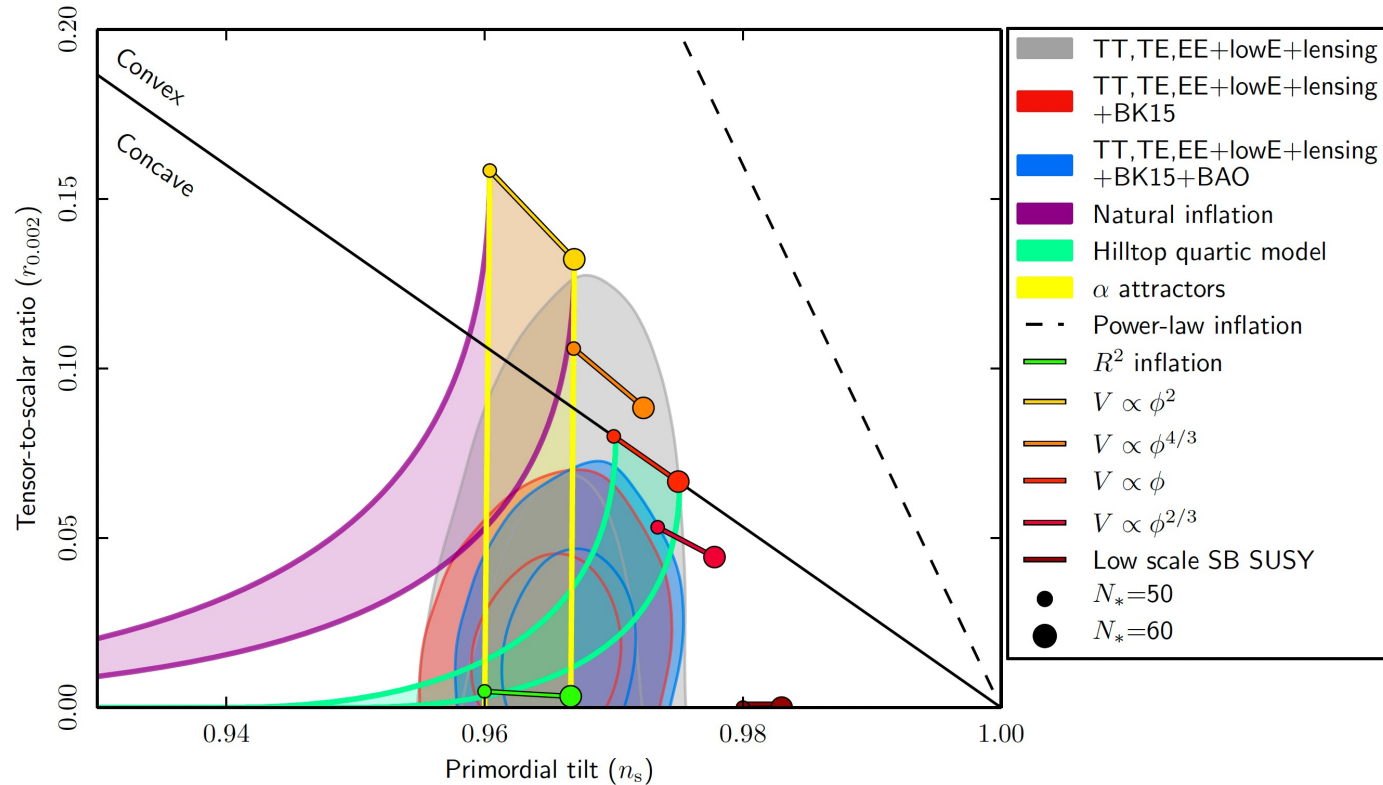
$$\langle \delta\phi_{\mathbf{k}} \delta\phi_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} \left(\frac{H}{2\pi} \right)^2$$

$$\Delta_s^2(k) \approx \frac{1}{24\pi^2} \frac{V}{M_{\text{pl}}^4} \frac{1}{\epsilon_v} \bigg|_{k=aH} \quad \zeta = -\frac{H}{\dot{\phi}} \delta\phi$$

$$n_s - 1 \equiv \frac{d \ln \Delta_s^2}{d \ln k} = 2\eta_v - 6\epsilon_v$$

- Quantum fluctuation of inflaton induces CMB anisotropies(or curvature perturbations)
- In the single field inflation, the fluctuations should be nearly gaussian and adiabatic, close to scale invariant

Inflation



$$\Delta_t^2(k) \approx \frac{2}{3\pi^2} \frac{V}{M_{\text{Pl}}^4} \Big|_{k=aH}$$

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon_V$$

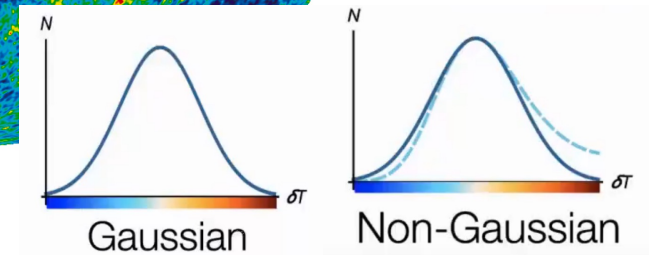
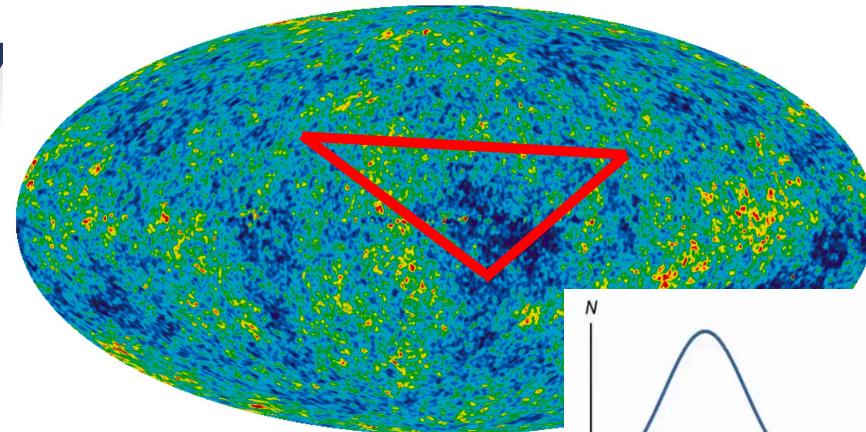
$$\epsilon_V < 0.0097$$

$$\eta_V = -0.010^{+0.007}_{-0.011}$$

$$\frac{H_*}{M_{\text{Pl}}} < 2.5 \times 10^{-5}$$

- Inflaton potential should be flat enough(shift-symmetry?)
- Hubble scale could be as high as $6 \cdot 10^{13}$ GeV(close to seesaw scale), providing access to the high scale physics

Non-Gaussianity



Non-Gaussianity is sensitive to new physics

- New physics could induce large non-Gaussianity : multi-field inflation models, modulated reheating, curvaton scenario...
- Current limit from Planck on local type $f_{\text{NL}} \sim \mathcal{O}(10)$, future CMB observations, CMB S4, large scale structure observations DESI $\mathcal{O}(1)$, 21 cm tomography $\mathcal{O}(0.01-0.1)$
- Non-Gaussianity could provide information to the new particle mass, spin, interactions: cosmological collider signals

Nima Arkani-Hamed, Juan Maldacena, arXiv:1503.08043

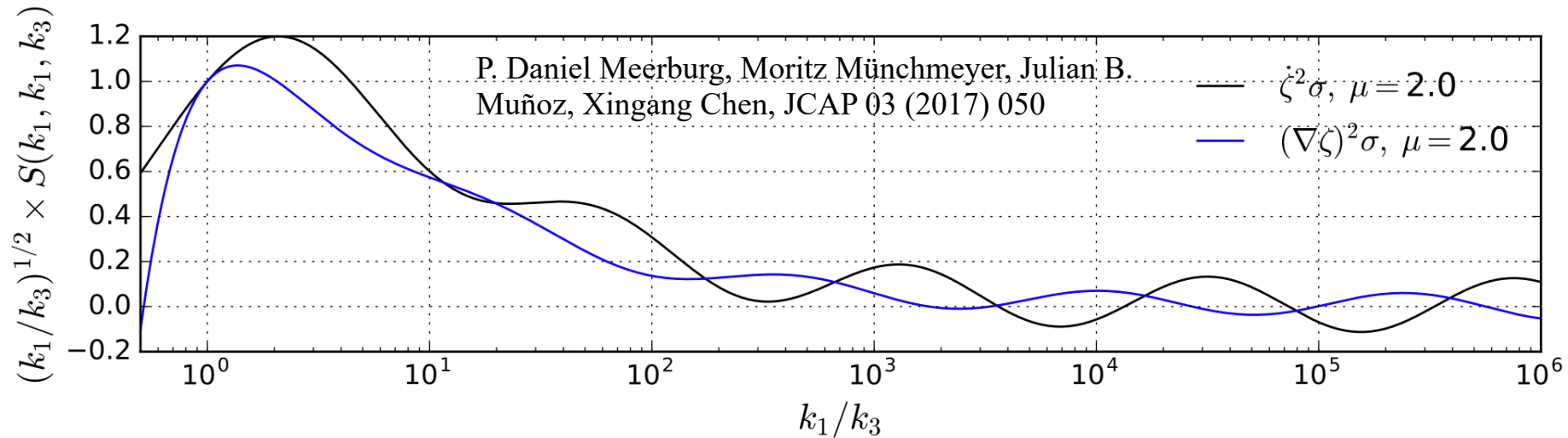
Xingang Chen, Yi Wang, JCAP 04 (2010) 027

Cosmological collider signals

Bispectrum $\langle \zeta^3 \rangle \equiv (2\pi)^3 \delta_D(\mathbf{k}_{123}) \frac{A^2}{(k_1 k_2 k_3)^2} S(k_1, k_2, k_3) \quad P_\zeta(k) = A/k^3$

Massive particle coupling to the inflaton could induce

$$S_{\text{squeezed}} \propto \left(\frac{k_{\text{long}}}{k_{\text{short}}} \right)^{1/2 \pm i\mu} \quad \mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$



- Probing new particles with mass around Hubble scale
- Signature would be highly suppressed when mass is large

A minimal model

Minimal model incorporates inflation and seesaw

$$\Delta\mathcal{L} = \sqrt{-g} \left[-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + \bar{N}_R i \not{\partial} N_R + \frac{1}{\Lambda} \partial_\mu \phi \bar{N}_R \gamma^\mu \gamma^5 N_R \right. \\ \left. + \left(-\frac{1}{2} M \bar{N}_R^c N_R - y_\nu \bar{L}_L \tilde{H} N_R + \text{H.c.} \right) \right]$$

- $V(\phi)$ is the potential for inflation is unknown but denominated by the mass term after inflation
- Derivative coupling to keep the flatness of the inflaton potential(shift-symmetry, dim-4 coupling should be suppressed, otherwise the induced ϕ^4 potential would destroy the flatness of the potential)
- $\Lambda > 60$ Hubble to keep perturbative unitarity
- After inflation, inflaton oscillates at the bottom of the potential until decays into heavy neutrinos ($m_\phi > 2 m_N$). The heavy neutrinos quickly decay into SM particles and reheat the universe

Seesaw mechanism

Consequence of the seesaw mechanism

$$\mathcal{L} \supset \frac{1}{2} \bar{\psi}_L \mathbf{M}_\nu \psi_R + \text{h.c.}, \quad \mathbf{M}_\nu = \begin{pmatrix} 0 & \frac{y_\nu h}{\sqrt{2}} \\ \frac{y_\nu h}{\sqrt{2}} & M \end{pmatrix}$$

$$m_\nu \simeq -\frac{y_\nu^2 h^2}{2M}, \quad M_N \simeq M + \frac{y_\nu^2 h^2}{2M}$$

- Light neutrino gets a mass
- Heavy neutrino mass are get lifted (h dependent)

Decay rate of the inflaton is h dependent:

$$\Gamma \simeq \frac{m_\phi M^2}{4\pi\Lambda^2} \left[1 + \frac{1}{4} \left(\frac{y_\nu h}{M} \right)^2 \right]$$

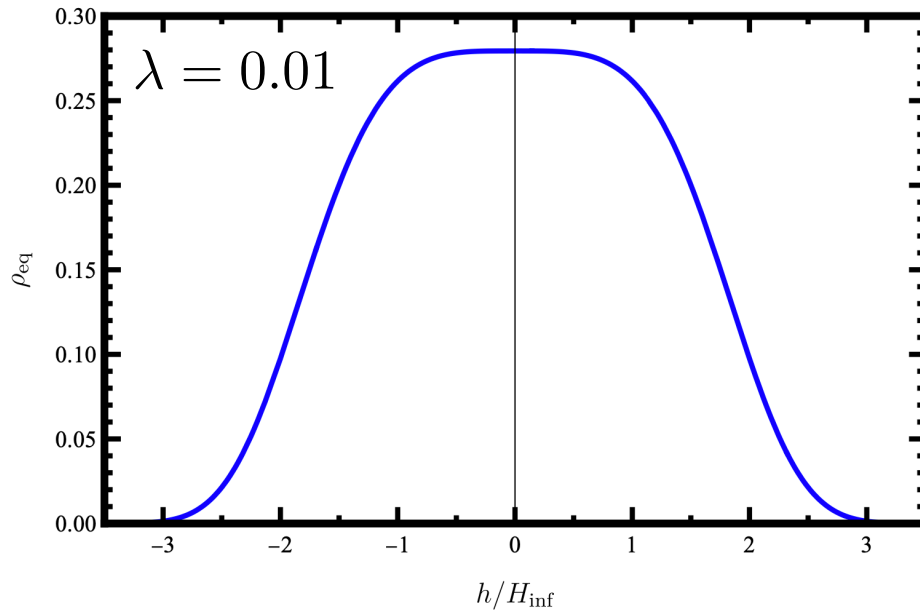
What happens to h in the early universe?

Higgs during inflation

Alexei A. Starobinsky, Jun'ichi Yokoyama,
Phys.Rev.D 50 (1994) 6357-6368

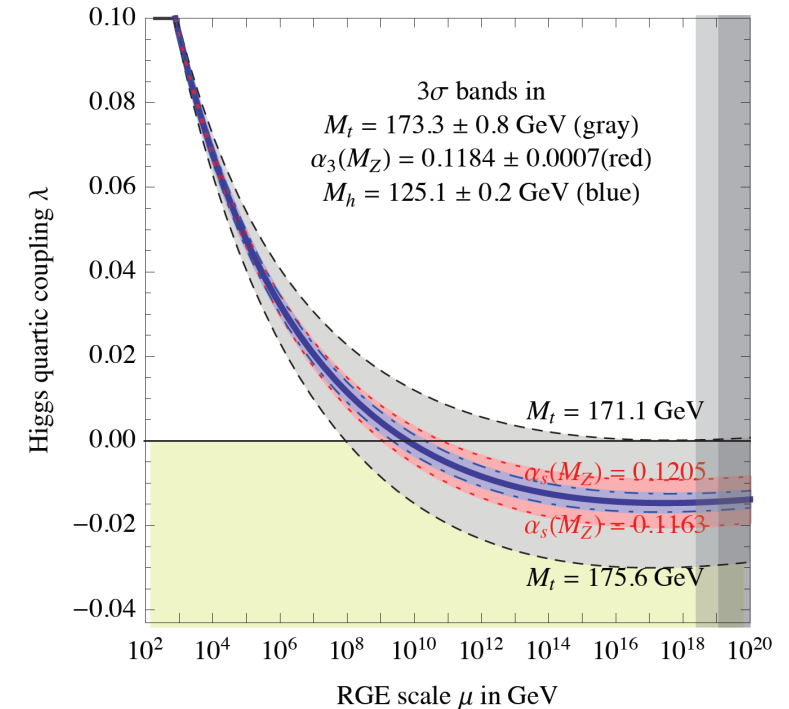
- During inflation(de-Sitter universe), Higgs also gets quantum fluctuations
- The fluctuations reach a equilibrium state(F-P equation)

$$\rho_{\text{eq}}(h) = \frac{2\lambda^{1/4}}{\Gamma(1/4)} \left(\frac{2\pi^2}{3}\right)^{1/4} \exp\left(\frac{-2\pi^2\lambda h^4}{3H_{\text{inf}}^4}\right)$$



$$\bar{h} = \sqrt{\langle h^2 \rangle} = \left[\int_{-\infty}^{+\infty} dh h^2 \rho_{\text{eq}}(h) \right]^{1/2} \simeq 0.363 \left(\frac{H_{\text{inf}}}{\lambda^{1/4}} \right)$$

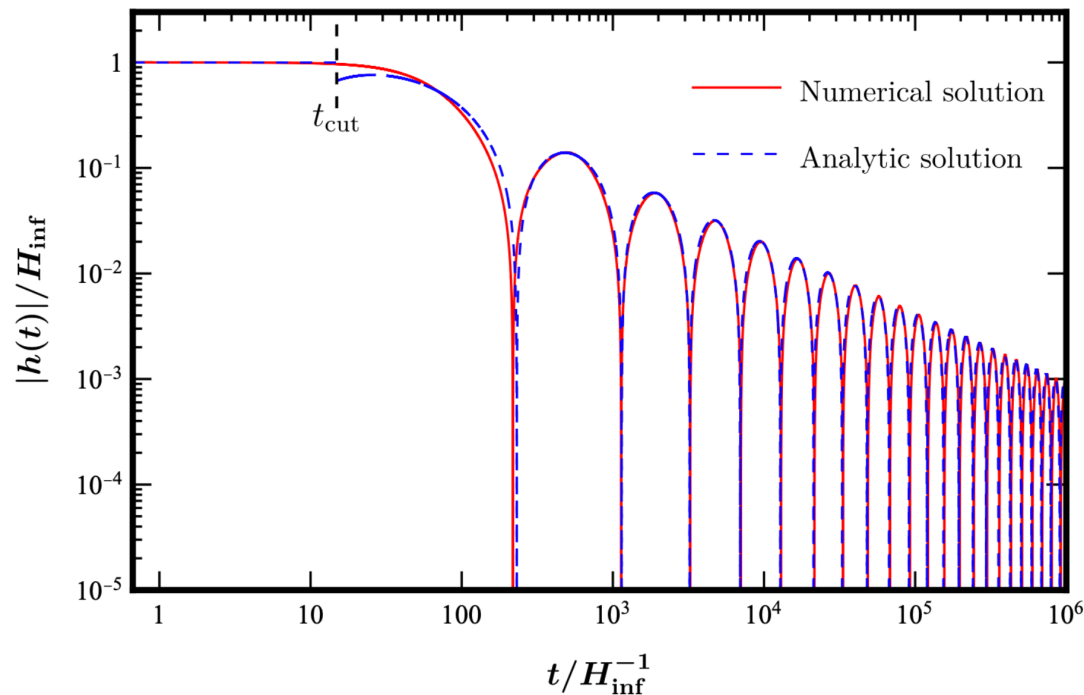
D. Buttazzo, et al arXiv:1307.3536



Higgs after inflation

Inflaton oscillates at the bottom potential. If the inflaton potential is dominated by the mass term, the Universe is matter-dominated

$$\ddot{h}(t) + \frac{2}{t}\dot{h}(t) + \lambda h^3(t) = 0$$



$$h(t) = \begin{cases} h_{\text{inf}}, & t \leq t_{\text{cut}} \\ AH_{\text{inf}} \left(\frac{h_{\text{inf}}}{H_{\text{inf}} \lambda} \right)^{\frac{1}{3}} (H_{\text{inf}} t)^{-\frac{2}{3}} \cos \left(\lambda^{\frac{1}{6}} |h_{\text{inf}}|^{\frac{1}{3}} \omega t^{\frac{1}{3}} + \theta \right), & t > t_{\text{cut}} \end{cases}$$

$$t_{\text{cut}} = \frac{\sqrt{2}}{3\sqrt{\lambda} h_{\text{inf}}}$$

$$A = 2^{1/3} 3^{-2/3} 5^{1/4} \simeq 0.9$$

$$\omega = \frac{\Gamma^2(3/4)}{\sqrt{\pi}} 2^{1/3} 3^{1/3} 5^{1/4} \simeq 2.3$$

$$\theta = -3^{-1/3} 2^{1/6} \omega - \arctan 2 \simeq -2.9$$

Higgs value would oscillate and decrease

Higgs modulated reheating

Considering decay rate of the inflaton is h dependent

$$\Gamma \simeq \frac{m_\phi M^2}{4\pi\Lambda^2} \left[1 + \frac{1}{4} \left(\frac{y_\nu h}{M} \right)^2 \right]$$

Gia Dvali, Andrei Gruzinov, Matias Zaldarriaga,
Phys.Rev. D69 (2004) 023505

- Different patches of the universe reheat differently (modulated reheating)
- The curvature perturbation is generated by Higgs field
- Delta N formalism ($\zeta = \delta N \sim N - \langle N \rangle$)

$$\begin{aligned} \zeta_h(t > t_{\text{reh}}, \mathbf{x}) &= \delta N(\mathbf{x}) = N(\mathbf{x}) - \langle N(\mathbf{x}) \rangle \\ &= -\frac{1}{6} \left[\ln(\Gamma_{\text{reh}}) - \langle \ln(\Gamma_{\text{reh}}) \rangle \right] \end{aligned}$$

Higgs modulated reheating

Curvature perturbation contains two parts

$$\zeta = \zeta_\phi + \zeta_h \qquad \mathcal{P}_\zeta^{(\phi)} = \left(\frac{H}{\dot{\phi}}\right)^2 \mathcal{P}_\phi = \left(\frac{H}{\dot{\phi}}\right)^2 \frac{H^2}{4\pi^2}$$

Taylor expansion of the curvature perturbation

$$\zeta_h(\mathbf{x}) = -\frac{1}{6} \left[\frac{\Gamma'_0}{\Gamma_0} \delta h_{\text{inf}}(\mathbf{x}) + \frac{\Gamma_0 \Gamma''_0 - \Gamma'_0 \Gamma'_0}{2\Gamma_0^2} \delta h_{\text{inf}}^2(\mathbf{x}) \right] \equiv z_1 \delta h_{\text{inf}}(\mathbf{x}) + \frac{1}{2} z_2 \delta h_{\text{inf}}^2(\mathbf{x})$$

$$\mathcal{P}_\zeta^{(h)} = z_1^2 \mathcal{P}_{\delta h} = \frac{z_1^2 H^2}{4\pi^2} \qquad R = \left(\frac{\mathcal{P}_\zeta^{(h)}}{\mathcal{P}_\zeta^{(o)}} \right)^{1/2} = |z_1| \left(\frac{\mathcal{P}_{\delta h}}{\mathcal{P}_\zeta^{(o)}} \right)^{1/2}$$

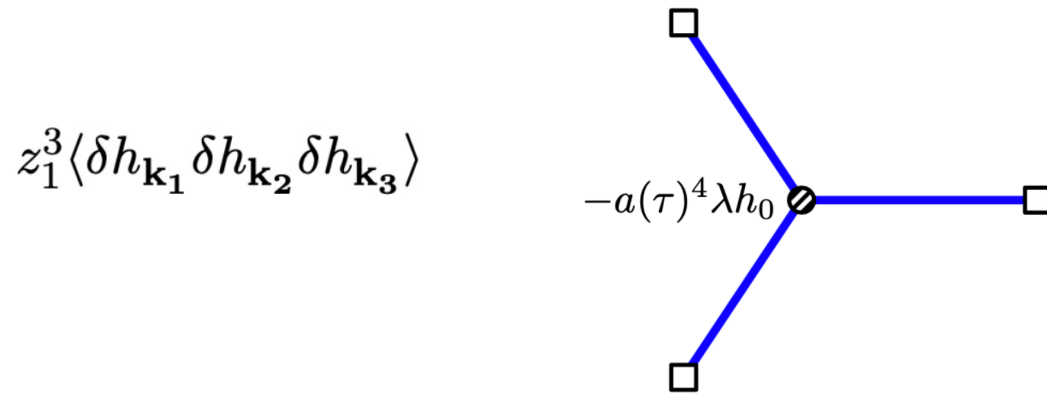
R should be less than 1

Bispectrum

Considering the three point correlation function

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle_h = z_1^3 \langle \delta h_{\mathbf{k}_1} \delta h_{\mathbf{k}_2} \delta h_{\mathbf{k}_3} \rangle + z_1^2 z_2 \langle \delta h^4 \rangle(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

First term is from Higgs self-coupling



Calculated by in-in formalism/Schwinger-Keldysh formalism

Steven Weinberg, Phys.Rev.D 72 (2005) 043514, Phys.Rev.D 74 (2006) 023508

Xingang Chen, Yi Wang, Zhong-Zhi Xianyu, JCAP 1712 (2017) 006

Bispectrum

$$\langle \delta h_{\mathbf{k}_1} \delta h_{\mathbf{k}_2} \delta h_{\mathbf{k}_3} \rangle' = 12\lambda \bar{h} \text{Im} \left(\int_{-\infty}^{\tau_f} a^4 \prod_{i=1}^3 G_+(\mathbf{k}_i, \tau) d\tau \right)$$

$$\begin{aligned} & \text{Im} \left(\int_{-\infty}^{\tau_f} a^4 \prod_{i=1}^3 G_+(\mathbf{k}_i, \tau) d\tau \right) \\ &= \text{Im} \int_{-\infty}^{\tau_f} \frac{d\tau}{(H\tau)^4} \cdot \frac{H^6}{8k_1^3 k_2^3 k_3^3} \left(\prod_{i=1}^3 (1 - ik_i \tau) \right) e^{i(k_1 + k_2 + k_3)\tau} \\ &= \frac{H^2}{24k_1^3 k_2^3 k_3^3} \cdot \left\{ (k_1^3 + k_2^3 + k_3^3) [\log(k_t |\tau_f|) + \gamma - \frac{4}{3}] + k_1 k_2 k_3 - \sum_{a \neq b} k_a^2 k_b \right\} \end{aligned}$$

$$N_e = \log\left(\frac{a_{\text{end}}}{a_k}\right) = \log\left(-\frac{1}{H\tau_f} \frac{k_t}{H}\right) = -\log(k_t |\tau_f|) \sim 60$$

Bispectrum

Second term is from non-linear evolution of the Higgs

$$\begin{aligned} z_1^2 z_2 \langle \delta h^4 \rangle(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) &= \frac{z_1^2 z_2}{2} \int \frac{d^3 \mathbf{k}_0}{(2\pi)^3} \langle \delta h(\mathbf{k}_1) \delta h(\mathbf{k}_2) \delta h(\mathbf{k}_0) \delta h(\mathbf{k}_3 - \mathbf{k}_0) \rangle + (2 \text{ perm.}) \\ &= \frac{z_1^2 z_2}{2} \left[\int \frac{d^3 \mathbf{k}_0}{(2\pi)^3} \langle \delta h(\mathbf{k}_1) \delta h(\mathbf{k}_0) \rangle \langle \delta h(\mathbf{k}_2) \delta h(\mathbf{k}_3 - \mathbf{k}_0) \rangle + (\mathbf{k}_1 \leftrightarrow \mathbf{k}_2) \right] + (2 \text{ perm.}) \\ &= \frac{z_1^2 z_2}{2} \left[\int d^3 \mathbf{k}_0 (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_0) \delta^3(\mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}_0) \frac{H^4}{4k_1^3 k_2^3} + (\mathbf{k}_1 \leftrightarrow \mathbf{k}_2) \right] + (2 \text{ perm.}) \\ &= (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) z_1^2 z_2 \left[\frac{H^4}{4k_1^3 k_2^3} + (2 \text{ perm.}) \right]. \end{aligned}$$

Local type non-gaussianity

The local type non-gaussianity which is defined by Bardeen Potential $\Phi \equiv \frac{3}{5}\zeta$

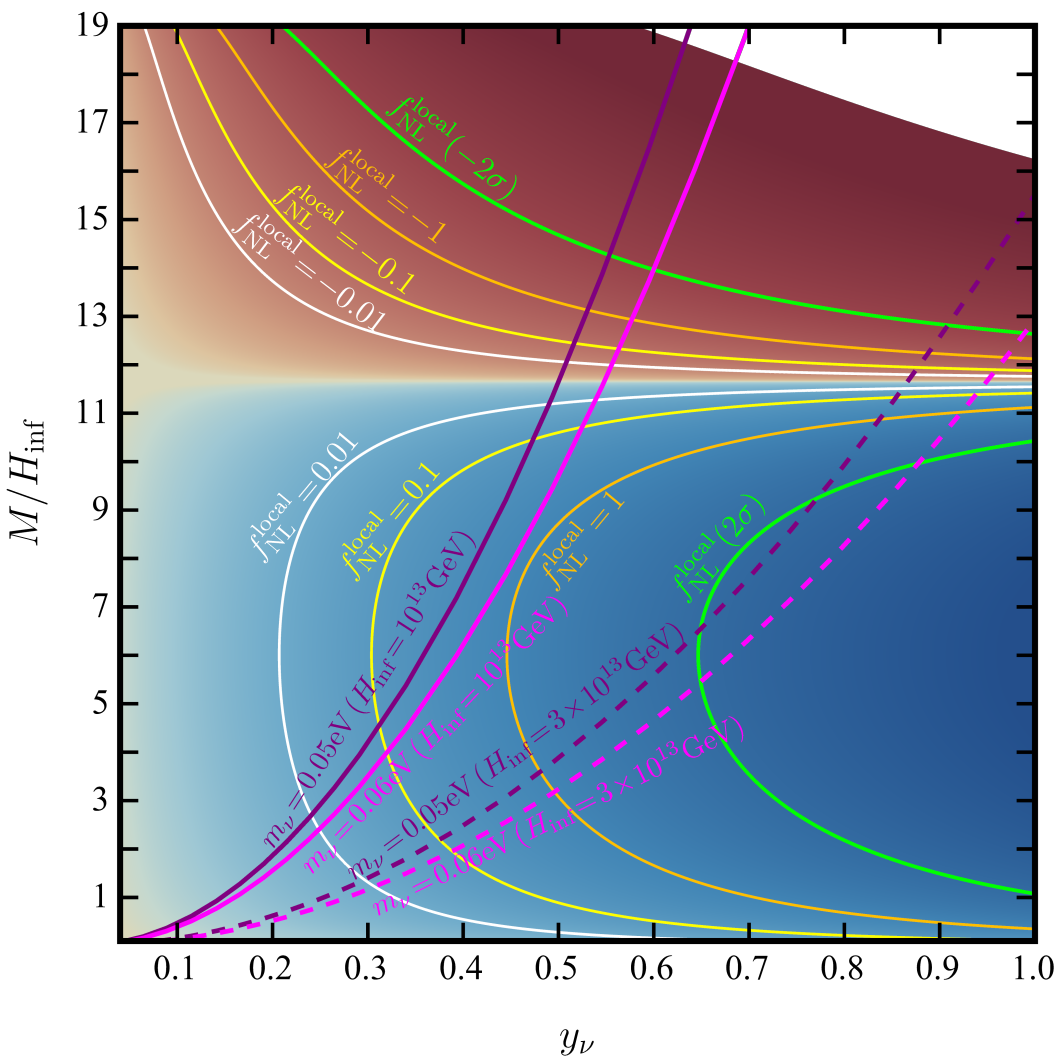
$$\langle \Phi_{\mathbf{k}_1} \Phi_{\mathbf{k}_2} \Phi_{\mathbf{k}_3} \rangle'_{\text{local}} = 2A^2 f_{\text{NL}}^{\text{local}} \left\{ \frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3} \right\}$$

In the limit $k_1 \sim k_2 \gg k_3$, we find

$$f_{\text{NL}}^{\text{local}} \simeq -\frac{10}{3} \frac{z_1^3 H^3}{(2\pi)^4 \mathcal{P}_\zeta^2} \left(\frac{\lambda \bar{h}}{2H} N_e - \frac{z_2 H}{4z_1} \right)$$

$$f_{\text{NL}}^{\text{local}} = -0.9 \pm 5.1 \quad (68\% \text{ C.L., Planck 2018})$$

Local type non-gaussianity

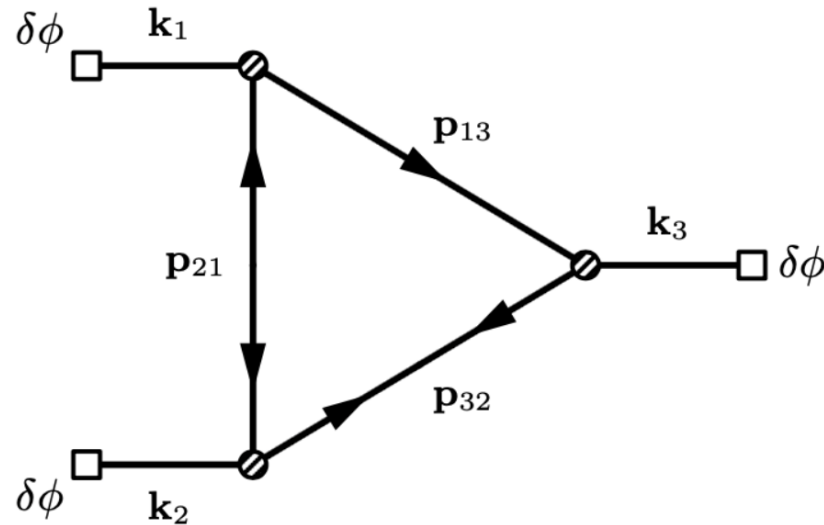


Parameters	\mathcal{P}_ζ	N_e	H_{inf}	m_ϕ	Λ	λ
Values	2.1×10^{-9}	60	$(1, 3) \times 10^{13} \text{ GeV}$	$40 H_{\text{inf}}$	$60 H_{\text{inf}}$	0.01

- Colored curves indicating future searches
- Parameter space with Yukawa O(1) could be probed by future observations
- The contribution from self-interaction and non-linear term are both important
- Interplaying with neutrino experiments(JUNO, DUNE for neutrino ordering)

Cosmological collider signal

Prediction of cosmological collider signal: help to identify the mass of the right-handed neutrino



Challenge for calculation of fermion loop, results appear soon

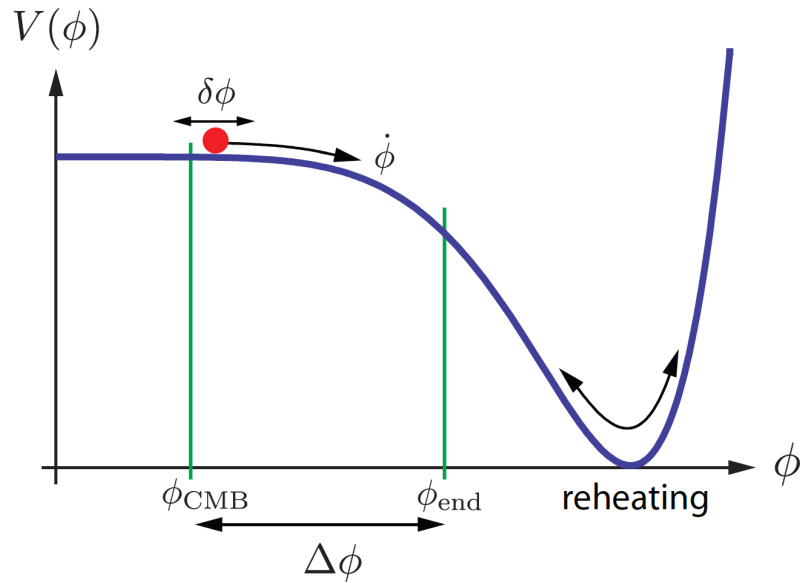
Summary

- We propose a minimal model incorporating inflation and seesaw
- It provides new mechanism of reheating
- High scale seesaw can be probed by the non-Gaussianity which could be observed in near future CMB or LSS
- Cosmological collider signal to determine the mass of N_R



Thanks!

Slow-roll Inflation



$$\epsilon_v(\phi) \equiv \frac{M_{\text{pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2$$

$$\eta_v(\phi) \equiv M_{\text{pl}}^2 \frac{V_{,\phi\phi}}{V}$$

$$\Delta_s^2(k) \approx \frac{1}{24\pi^2} \frac{V}{M_{\text{pl}}^4} \frac{1}{\epsilon_v} \Big|_{k=aH}$$

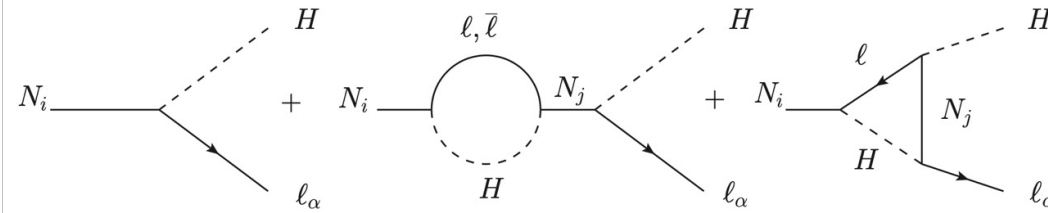
$$\Delta_t^2(k) \approx \frac{2}{3\pi^2} \frac{V}{M_{\text{pl}}^4} \Big|_{k=aH}$$

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon_v$$

Leptogenesis

Baryogenesis Without Grand Unification, Fukugita and Yanagida, 1986'

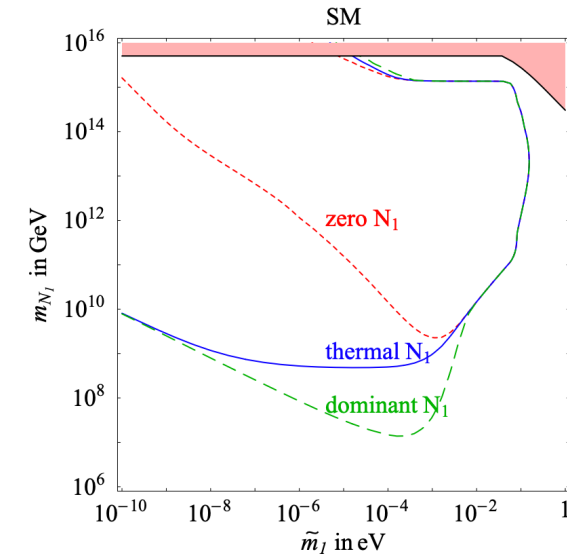
$$\mathcal{L}_I = \mathcal{L}_{SM} + i\overline{N_{Ri}}\not{\partial}N_{Ri} - \left(\frac{1}{2}M_i\overline{N_{Ri}^c}N_{Ri} + \epsilon_{ab}Y_{\alpha i}\overline{N_{Ri}}\ell_{\alpha}^aH^b + h.c. \right)$$



$$\epsilon_{i\alpha} = \frac{\gamma(N_i \rightarrow \ell_{\alpha}H) - \gamma(N_i \rightarrow \bar{\ell}_{\alpha}H^*)}{\sum_{\alpha} \gamma(N_i \rightarrow \ell_{\alpha}H) + \gamma(N_i \rightarrow \bar{\ell}_{\alpha}H^*)}$$

$$n_B = \frac{28}{79}(\mathcal{B} - \mathcal{L})_i$$

G.F. Giudice, et al,
Nucl.Phys.B 685 (2004) 89-149



Mass of the right-handed neutrino should be heavier than 10^7 GeV

S-K formalism

$$Q(\tau) \equiv \varphi^{A_1}(\tau, \mathbf{x}_1) \cdots \varphi^{A_N}(\tau, \mathbf{x}_N)$$

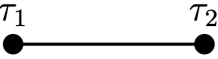
$$\langle Q(\tau) \rangle = \langle \Omega | \bar{F}(\tau, \tau_0) Q_I(\tau) F(\tau, \tau_0) | \Omega \rangle$$

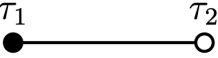
$$F(\tau, \tau_0) = \text{T exp} \left(-i \int_{\tau_0}^{\tau} d\tau_1 H_I(\tau_1) \right),$$

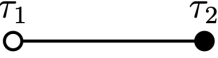
$$\bar{F}(\tau, \tau_0) = \bar{\text{T}} \exp \left(i \int_{\tau_0}^{\tau} d\tau_1 H_I(\tau_1) \right),$$

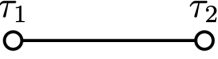
S-K formalism

$$\left\{ \begin{array}{l} G_{++}(\mathbf{k}; \tau_1, \tau_2) \equiv G_{>}(\mathbf{k}; \tau_1, \tau_2) \theta(\tau_1 - \tau_2) + G_{<}(\mathbf{k}; \tau_1, \tau_2) \theta(\tau_2 - \tau_1) \\ G_{+-}(\mathbf{k}; \tau_1, \tau_2) \equiv G_{<}(\mathbf{k}; \tau_1, \tau_2) \\ G_{-+}(\mathbf{k}; \tau_1, \tau_2) \equiv G_{>}(\mathbf{k}; \tau_1, \tau_2) \\ G_{--}(\mathbf{k}; \tau_1, \tau_2) \equiv G_{<}(\mathbf{k}; \tau_1, \tau_2) \theta(\tau_1 - \tau_2) + G_{>}(\mathbf{k}; \tau_1, \tau_2) \theta(\tau_2 - \tau_1) \end{array} \right.$$

$\tau_1 \quad \tau_2$

 $= G_{++}(k; \tau_1, \tau_2)$

$\tau_1 \quad \tau_2$

 $= G_{+-}(k; \tau_1, \tau_2)$

$\tau_1 \quad \tau_2$

 $= G_{-+}(k; \tau_1, \tau_2)$

$\tau_1 \quad \tau_2$

 $= G_{--}(k; \tau_1, \tau_2)$

$$G_{>}(k; \tau_1, \tau_2) \equiv u(\tau_1, k) u^*(\tau_2, k)$$

$$G_{<}(k; \tau_1, \tau_2) \equiv u^*(\tau_1, k) u(\tau_2, k)$$

$$\square u_{\mathbf{k}} = \ddot{u}_{\mathbf{k}} + 3H\dot{u}_{\mathbf{k}} + \frac{\mathbf{k}^2}{a^2(t)} u_{\mathbf{k}} = 0$$

$$u_{\mathbf{k}}(\tau) = \frac{H}{\sqrt{2k^3}} [1 + ik\tau] e^{-ik\tau}$$

$$\tau \quad \square \quad = \quad G_{+}(k; \tau) \equiv G_{++}(k; \tau, \tau_f)$$

$$\tau \quad \circ \quad = \quad G_{-}(k; \tau) \equiv G_{-+}(k; \tau, \tau_f)$$

S-K formalism

Bulk-to-Boundary propagator

$$G_{\pm}(\mathbf{k}, \tau) \equiv G_{\pm+}(\mathbf{k}; \tau, \tau_f)$$

$$\tau \bullet \text{---} \square = G_{+}(\mathbf{k}, \tau)$$

$$\tau \circ \text{---} \square = G_{-}(\mathbf{k}, \tau)$$

$$\tau \otimes \text{---} \square = G_{+}(\mathbf{k}, \tau) + G_{-}(\mathbf{k}, \tau)$$

$$G_{+}(\mathbf{k}, \tau) = \frac{H^2}{2k^3} [1 - ik(\tau - \tau_f) + k^2 \tau \tau_f] e^{ik(\tau - \tau_f)} \quad G_{-}(\mathbf{k}, \tau) \simeq \frac{H^2}{2k^3} [1 + ik\tau] e^{-ik\tau}$$
$$\simeq \frac{H^2}{2k^3} [1 - ik\tau] e^{ik\tau}$$

Higgs modulated reheating

Considering decay rate of the inflaton is h dependent

$$\Gamma \simeq \frac{m_\phi M^2}{4\pi\Lambda^2} \left[1 + \frac{1}{4} \left(\frac{y_\nu h}{M} \right)^2 \right]$$

Gia Dvali, Andrei Gruzinov, Matias Zaldarriaga,
Phys.Rev. D69 (2004) 023505

- Different patches of the universe reheat differently (modulated reheating)
- The curvature perturbation is generated by Higgs field
- Delta N formalism (from the end of inflation to the time after reheating completed)

$$\begin{aligned} N(\mathbf{x}) &= \int d \ln a(t) = \int_{t_{\text{inf}}}^{t_{\text{reh}}(\mathbf{x})} dt H(t) + \int_{t_{\text{reh}}(\mathbf{x})}^{t_f} dt H(t) \\ &= \int_{\rho_{\text{inf}}}^{\rho_{\text{reh}}(h(\mathbf{x}))} d\rho \frac{H}{\dot{\rho}} + \int_{\rho_{\text{reh}}(h(\mathbf{x}))}^{\rho_f} d\rho \frac{H}{\dot{\rho}}, \end{aligned}$$

Higgs modulated reheating

Equation of state: $\dot{\rho} + 3H(1 + \omega)\rho = 0$

From matter-dominated universe to radiation dominated universe

$$N(\mathbf{x}) = -\frac{1}{3} \ln \frac{\rho_{\text{reh}}(h(\mathbf{x}))}{\rho_{\text{inf}}} - \frac{1}{4} \ln \frac{\rho_f}{\rho_{\text{reh}}(h(\mathbf{x}))}$$

Reheating occurs $H_{\text{reh}} = \Gamma_{\text{reh}} \quad 3H^2 M_p^2 = \rho$

Curvature perturbation in terms of the decay rate

$$\begin{aligned} \zeta_h(t > t_{\text{reh}}, \mathbf{x}) &= \delta N(\mathbf{x}) = N(\mathbf{x}) - \langle N(\mathbf{x}) \rangle \\ &= -\frac{1}{6} [\ln(\Gamma_{\text{reh}}) - \langle \ln(\Gamma_{\text{reh}}) \rangle] \end{aligned}$$

Higgs modulated reheating

$$\zeta_h = -\frac{1}{6} [\ln(\Gamma_{\text{reh}}) - \langle \ln(\Gamma_{\text{reh}}) \rangle]$$

$$\Gamma_{\text{reh}} = \Gamma(h(t_{\text{reh}}))$$

$$h(t_{\text{reh}}) = h(h_{\text{inf}}, t_{\text{reh}})$$

$$h(t) = A \left(\frac{h_{\text{inf}}}{\lambda} \right)^{\frac{1}{3}} t^{-\frac{2}{3}} \cos \left(\lambda^{\frac{1}{6}} h_{\text{inf}}^{\frac{1}{3}} \omega t^{\frac{1}{3}} + \theta \right)$$

n-point correlation function of zeta changes into n-point correlation function of hinf

Higgs during inflation

During inflation(de-Sitter universe), Higgs also gets quantum fluctuations

暴胀期间，希格斯场可以分为长波和短波两部分

Alexei A. Starobinsky, Jun'ichi Yokoyama,
Phys.Rev.D 50 (1994) 6357-6368

$$h(\mathbf{x}, t) = h_L(\mathbf{x}, t) + \int \frac{d^3k}{(2\pi)^3} \theta(k - \epsilon a(t) H_{\text{inf}}) \left[a_{\mathbf{k}} h_{\mathbf{k}}(t) e^{-i\mathbf{k} \cdot \mathbf{x}} + a_{\mathbf{k}}^\dagger h_{\mathbf{k}}^*(t) e^{i\mathbf{k} \cdot \mathbf{x}} \right]$$

$$h_{\mathbf{k}} = \frac{H_{\text{inf}}}{\sqrt{2k^3}} (1 + ik\tau) e^{-ik\tau}$$

长波部分可以用郎之万方程描述

$$\dot{h}_L(\mathbf{x}, t) = -\frac{1}{3H_{\text{inf}}} \frac{\partial V}{\partial h_L} + f(\mathbf{x}, t)$$

$$\langle f(\mathbf{x}_1, t_1) f(\mathbf{x}_2, t_2) \rangle = \frac{H_{\text{inf}}^3}{4\pi^2} \delta(t_1 - t_2) j_0(\epsilon a(t_1) H_{\text{inf}} |\mathbf{x}_1 - \mathbf{x}_2|)$$

带自相互作用的随机游走模型