Modular TM_1 mixing in light of precision measurement in JUNO

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Outline

1. Introduction

Introduction

The origin of lepton masses and mixings remains an open question in neutrino theory (See also Talk by Yu-Feng Li): three mixing angles and two mass differences are observed, leaving the CP phase and mass ordering to be determined. In particular, JUNO just released their first measurement for normal ordering (NO) scenario:

$$\sin^2\theta_{12} = 0.3092 \pm 0.0087\,, \quad \Delta m^2_{21} = (7.50 \pm 0.12) \times 10^{-5}\,\text{eV}^2\,.$$

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}.$$

Motivation

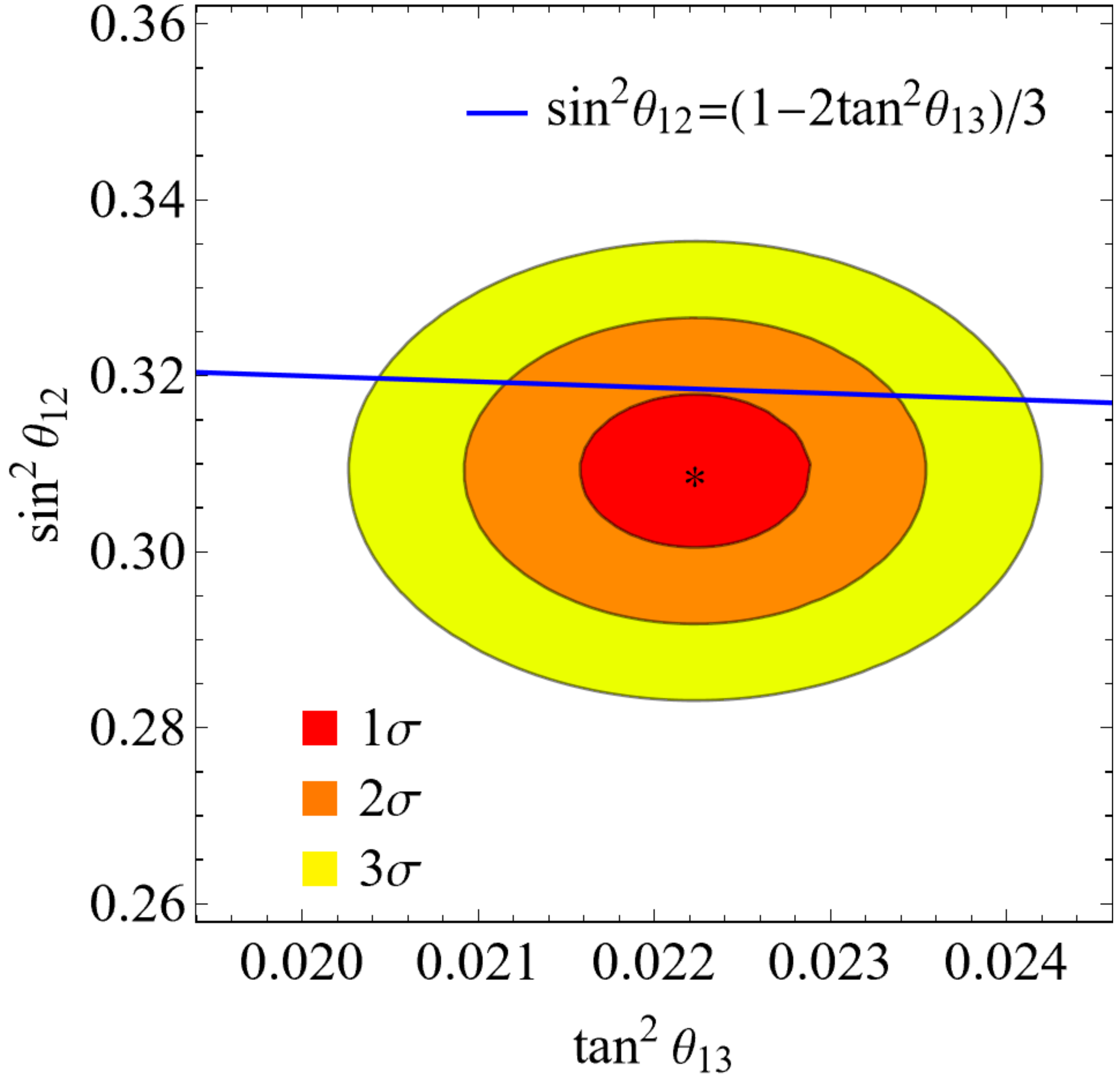
The origin of lepton masses and mixings remains an open question in neutrino theory (See also Talk by Yu-Feng Li): three mixing angles and two mass differences are observed, leaving the CP phase and mass ordering to be determined. In particular, JUNO just released their first measurement for normal ordering (NO) scenario:

$$\sin^2\theta_{12} = 0.3092 \pm 0.0087\,, \quad \Delta m^2_{21} = (7.50 \pm 0.12) \times 10^{-5}\,\mathrm{eV}^2\,.$$

The key question is: can we use symmetry to explain these parameters?

- To explain the observed neutrino oscillation parameters, various of simple mixing ansatzes for the lepton flavor mixing were proposed, such as Tri-bimaximal mixing (excluded, Harrison *et al*. ’02; Xing, ’02) and Trimaximal mixing (TM_1) proposed by (Xing & Zhou, ’06) and further discussed in (Lam, ’06; Albright & Rodejohann, ’08).
- The latest studies show that TM_1 is most favored, while the TM_2 is disfavored (Zhang, ’25; He, ’25).

TM_1 mixing pattern



TM_1 mixing (blue) and JUNO data
(S. Zhou) https://ihep.cas.cn/xwdt2022/gnxw/hotnews/2025/202511/t20251125_8016598.html

- The TM_1 is a partially constant mixing which inherits the first column the TBM form:

$$U_{\mathrm{TM}_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \times & \times \\ \frac{-1}{\sqrt{6}} & \times & \times \\ \frac{-1}{\sqrt{6}} & \times & \times \end{pmatrix} .$$

- It predicts the sum rule: $\sin^2\theta_{12} = 1 - \frac{2}{3(1-\sin^2\theta_{13})}$,

$$\theta_{23} \approx 45^\circ + \sqrt{2}\theta_{13}\cos\delta .$$

- It was showed that the spontaneous breaking of a non-Abelian discrete S_4 flavor symmetry can lead to TM_1 (de Medeiros Varzielas & Lavoura, '13; Luhn, '13).
- However, to realize the desired symmetry breaking pattern, many flavons with particular vev alignments are introduced.

Methodology

- The modular symmetry approach (See also Talk by Gui-Jun Ding) provides a new way to realize the TM_1 without introducing many flavons.

- The question we ask: how to realize TM_1 from S_4 symmetry with the least number of free parameters? Can these models well-fit the latest JUNO data?

Therefore, we investigates the landscape of models based on modular S_4 symmetry that predicts the TM_1 mixing pattern, and explores their parameter spaces with constraints from the latest high-precision measurement on θ_{12} and Δm_{21}^2 given by JUNO.

2. Realizing TM_1 mixing from S_4 flavor symmetry

Effective Lagrangian

- Let's assume the lepton sector is invariant under a flavor symmetry G_f at a high energy scale, which is broken spontaneously by either the VEV of a flavon or by the VEV of a modular field. We can consider the different residual flavor symmetries G_ℓ and G_ν as subgroups of G_f in the lepton sector and neutrino sector, respectively.

- After the spontaneous breaking of G_f, the effective Lagrangian at a low energy scale describing neutrino masses is

$$-\mathscr{L}_{\text{mass}} = \overline{\ell_L} M_\ell \ell_R + \frac{1}{2}\overline{\nu_L} M_\nu \nu_L^c + \text{h.c.}\,,$$

which transforms under G_ℓ and G_ν as:

$$\begin{cases} \ell_L \to \rho_L(g_\ell)\ell_L \\ \ell_R \to \rho_R(g_\ell)\ell_R \end{cases} \text{under } g_\ell \longrightarrow \overline{\ell_L} M_\ell \ell_R = \overline{\ell_L}\left(\rho_L^\dagger(g_\ell) M_\ell \rho_R(g_\ell)\right)\ell_R$$

$$\nu_L \to \rho_L(g_\nu)\nu_L \text{ under } g_\nu \longrightarrow \overline{\nu_L} M_\nu \nu_L^c = \overline{\nu_L}\left(\rho_L^\dagger(g_\nu) M_\nu \rho_L^*(g_\nu)\right)\nu_L^c$$

Charged lepton sector G_ℓ

- In the charged lepton sector, diagonalizing the Hermitian matrix $H_\ell \equiv M_\ell M_\ell^\dagger$ introduces an unitary matrix U_ℓ satisfying $U_\ell^\dagger H_\ell U_\ell = \hat{H}_\ell$. Applying the g_ℓ transformation $\rho_L^\dagger(g_\ell) H_\ell \rho_L(g_\ell) = H_\ell$ implies:

$$[U_\ell^\dagger \rho_L(g_\ell) U_\ell]^\dagger \, \hat{H}_\ell \, [U_\ell^\dagger \rho_L(g_\ell) U_\ell] = \hat{H}_\ell \, .$$

- To recover non-degenerate charged lepton masses, $U_\ell^\dagger \rho_L(g_\ell) U_\ell$ must be diagonal $U_\ell^\dagger \rho_L(g_\ell) U_\ell = \mathrm{diag}\{e^{i\alpha_1}, \, e^{i\alpha_2}, \, e^{i\alpha_3}\}$. Therefore, G_ℓ must be an Abelian symmetry and values of phases α_i depend on the selected residual symmetry.
- If the flavor symmetry is a S_4, it is convenient to choose the residual symmetry G_ℓ to be Z_3^T so that a diagonal U_ℓ is guaranteed. (Jiang, RO, Zhou ’25)

Neutrino sector G_ν

- Similarly, in the neutrino sector, a unitary U_ν diagonalizing the mass matrix $U_\nu^\dagger M_\nu U_\nu^* = \hat{M}_\nu$ implies

$$[U_\nu^\dagger \rho_L(g_\nu) U_\nu]^\dagger \hat{M}_\nu [U_\nu^\dagger \rho_L(g_\nu) U_\nu]^* = \hat{M}_\nu$$

- The condition requires $[U_\nu^\dagger \rho_L(g_\nu) U_\nu]$ to be real and diagonal, and thus the only choices could be

$$[U_\nu^\dagger \rho_L(g_\nu) U_\nu] = \mathrm{diag}\{(-1)^{k_1}, (-1)^{k_2}, (-1)^{k_3}\},$$

  where $k_{1,2,3} = 0,1$.

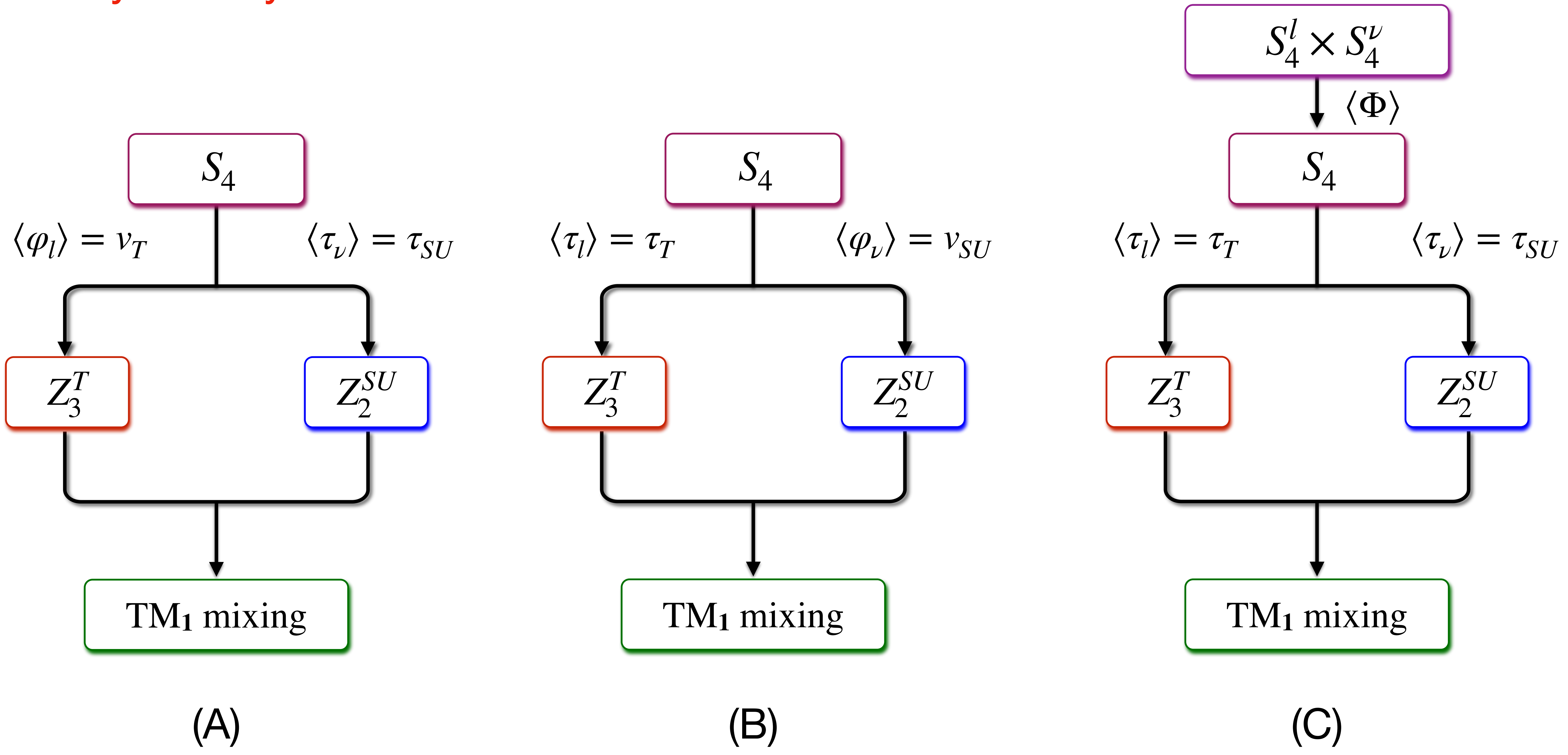
- This conclusion leaves only Z_2 or $Z_2 \times Z_2'$ allowed as residual symmetry G_ν in the Majorana neutrino sector.
- Therefore, there is a partial dependence of U_ℓ and U_ν on $\rho_L(g_\ell)$ and $\rho_L(g_\nu)$ which eventually restricts $U_{\mathrm{PMNS}} = U_\ell^\dagger U_\nu$.
- In the flavor basis $\rho_L(g_\ell)$ (and also U_ℓ) is diagonal, so U_{PMNS} appears to be the unitary matrix diagonalizing $\rho_L(g_\nu)$:

$$U_{\mathrm{PMNS}}^\dagger \rho_L(g_\nu) U_{\mathrm{PMNS}} = \mathrm{diag}\{(-1)^{k_1}, (-1)^{k_2}, (-1)^{k_3}\}.$$

- Different choices of G_ν clearly results in different mixing patterns. For S_4 symmetry, if we take $G_\nu = Z_2^{SU}$, we can find the representations $\rho_{3,3'}(SU)$ and its' eigenvector $(2, -1, -1)^T$ for the non-degenerate eigenvalues. (Jiang, RO, Zhou '25)

Realizing TM_1 mixing from S_4

- Therefore, the prediction of TM_1 mixing pattern is thus obtained via the structure of residual symmetries assigned as: $G_\ell = Z_3^T$ and $G_\nu = Z_2^{SU}$, which is independent of model details and regardless of realizations in traditional flavor symmetry or modular symmetry.



(Jiang, RO, Zhou '25)

3. Models based on modular S_4 symmetry

Modular symmetry

- Modular invariance can be used for lepton model building (Feruglio, '17), where the finite modular group $\Gamma(N)$ can be used which is isomorphic to a permutation group at the first few levels, i.e. $\Gamma_2 \simeq S_3$, $\Gamma_3 \simeq A_4$, $\Gamma_4 \simeq S_4$ for $N = 2,3,4$. (See also Talk by Gui-Jun Ding)

- Under the modular invariance Γ_N, a chiral superfield $\phi_i(\tau)$ transforms non-linearly as function of τ as

$$\phi_i(\tau) \to \phi_i(\gamma\tau) = (c\tau + d)^{-2k_i}\rho_{I_i}(\gamma)\phi_i(\tau)$$

- Due to the holomorphicity of the superpotential, the Yukawa coupling transforms as (where k_Y is non-negative):

$$Y_{I_Y}(\tau) \to Y_{I_Y}(\gamma\tau) = (c\tau + d)^{2k_Y}\rho_{I_Y}(\gamma)Y_{I_Y}(\tau)\,.$$

- The modulus τ can be stabilized at a fixed point ($\gamma\tau_\gamma = \tau_\gamma$) such that $\langle\tau\rangle = \tau_\gamma$. In this case, an Abelian residual symmetry generated by γ is preserved, leaving $Y_I(\gamma\tau_\gamma) = Y_I(\tau_\gamma)$:

$$\rho_I(\gamma)Y_I(\tau_\gamma) = (c\tau + d)^{-2k}Y_I(\tau_\gamma)\,.$$

- Given a representation matrix of S_4 and a fixed point τ_γ, we can determine the eigenvectors for $Y_I(\tau_\gamma)$.

Modular symmetry

For example, at the fixed point $\tau_\gamma = \tau_T = (-1 + i\sqrt{3})/2$, the modular forms are: (de Medeiros Varzielas, King, Zhou, '19)

$$Y^{(6j+2)}_{\mathbf{3}^{(\prime)}}(\tau_T) \propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad Y^{(6j+4)}_{\mathbf{3}^{(\prime)}}(\tau_T) \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad Y^{(6j+6)}_{\mathbf{3}^{(\prime)}}(\tau_T) \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$

- The advantage of using modular S_4 symmetry: it reduces physical degrees of freedom very efficiently.
- For example, in the minimal setup (Penedo & Petcov, '18; Novichkov, Penedo, Petcov, Titov, '18), it only needs one modular field, which has only two real degrees of freedom, acquiring a VEV, without the necessity of introducing many flavon fields aligned in some specific ways (e.g. King, Luhn, '13)

Model A

Fermions	S_4	$2k$	Scalars	S_4	$2k$	Modular forms	S_4	$2k$
e^c	$\mathbf{1}$	-5	φ_ℓ	$\mathbf{3}$	$+2$	$Y_{\mathbf{2}}(\tau_\nu)$	$\mathbf{2}$	$+2$
μ^c	$\mathbf{1}'$	-2	η	$\mathbf{2}$	-1	$Y_{\mathbf{3}'}(\tau_\nu)$	$\mathbf{3}'$	$+2$
τ^c	$\mathbf{1}$	-2	ξ	$\mathbf{1}'$	0			
L	$\mathbf{3}$	$+1$	$H_{u,d}$	$\mathbf{1}$	0			
ν^c	$\mathbf{3}$	-1						

- The superpotential is:

$$W = \frac{y_e}{\Lambda^2}(L\varphi_l^2)_{\mathbf{1}} e^c H_d + \frac{y_\mu}{\Lambda^2}(L(\varphi_l\eta)_{\mathbf{3}'})_{\mathbf{1}'}\, \mu^c H_d + \frac{y_\tau}{\Lambda^2}(L(\varphi_l\eta)_{\mathbf{3}})_{\mathbf{1}}\, \tau^c H_d$$
$$+ y_D L\nu^c H_u + \frac{1}{2}Y_{\mathbf{2}}(\tau_\nu)(\nu^c\nu^c)_{\mathbf{2}}\xi + \frac{1}{2}Y_{\mathbf{3}'}(\tau_\nu)(\nu^c\nu^c)_{\mathbf{3}}\xi\,.$$

- In the lepton sector, the flavon φ_l breaks S_4 to the residual symmetry Z_3^T by the vev $\langle\varphi_l\rangle = v_T\begin{pmatrix}1\\0\\0\end{pmatrix}$.
- In the neutrino sector, the modular field τ_ν stabilized at $\langle\tau_\nu\rangle = \tau_{SU} = -\frac{1}{2}+\frac{i}{2}$ breaks S_4 to the residual Z_2^{SU} symmetry.

Model A

- Expanding the superpotential gives the charged lepton mass matrix: $M_\ell = \frac{v_d v_T^*}{\Lambda^2}\begin{pmatrix} y_e v_T & 0 & 0 \\ 0 & y_\mu v_{\eta_1} & y_\tau v_{\eta_1} \\ 0 & -y_\mu v_{\eta_2} & y_\tau v_{\eta 2}\end{pmatrix}^*$,

  the Dirac mass matrix in the flavor basis: $M_D = y_D^* v_u U_\ell^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$,

  and the Majorana mass matrix: $M_R = a_2 \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + a_3 \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} - a_3\sqrt{6}\begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & -2 \end{pmatrix}$,

  where $a_2 = [\lambda_2 Y^{(2)}_{\mathbf{2},1}(\tau_{SU})]^*, a_3 = [\lambda_3 Y^{(2)}_{\mathbf{3}',1}(\tau_{SU})]^*$ are complex coefficients.

- The standard neutrino seesaw formula $M_\nu = -M_D M_R^{-1} M_D^T$ can then be used to find the mass matrix for (active) neutrinos and the $U_{\rm PMNS} = U_\ell^\dagger U_\nu$.

Model B

Fermions	S_4	$2k$	Scalars	S_4	$2k$	Modular forms	S_4	$2k$
e^c	$\mathbf{1}'$	-6	φ_ν	$\mathbf{3}$	0	$Y_e(\tau_l)$	$\mathbf{3}'$	+6
μ^c	$\mathbf{1}'$	-4	φ'_ν	$\mathbf{3}'$	0	$Y_\mu(\tau_l)$	$\mathbf{3}'$	+4
τ^c	$\mathbf{1}'$	-2	η	$\mathbf{1}'$	0	$Y_\tau(\tau_l)$	$\mathbf{3}'$	+2
L	$\mathbf{3}$	0	$H_{u,d}$	$\mathbf{1}$	0			
ν^c	$\mathbf{3}$	0						

- The superpotential reads:
$$W = Y_e(\tau_l)Le^cH_d + Y_\mu(\tau_l)L\mu^cH_d + Y_\tau(\tau_l)L\tau^cH_d + y_D(L\nu^c)_{\mathbf{1}}H_u + \frac{1}{2}M_1(\nu^c\nu^c)_{\mathbf{1}} + \frac{\lambda}{2}\varphi_\nu(\nu^c\nu^c)_{\mathbf{3}} + \frac{\lambda'}{2\Lambda}\varphi'_\nu(\nu^c\nu^c\eta)_{\mathbf{3}'}.$$
- In the lepton sector, the modular field τ_l stabilized at $\langle\tau_l\rangle = \tau_T$ preserves Z_3^T and diagonalizes charged leptons.
- In the neutrino sector, two flavons are required as one flavon only gives TBM (Luhn '13). To preserve the Z_2^{SU} with two flavons $\varphi_\nu \sim \mathbf{3}$ and $\varphi'_\nu \sim \mathbf{3}'$, their vevs must be aligned as:
$$\langle\varphi_\nu\rangle = v_{SU}\begin{pmatrix}1\\1\\1\end{pmatrix}, \qquad \langle\varphi'_\nu\rangle = v'_{SU}\begin{pmatrix}0\\1\\-1\end{pmatrix}.$$

Model B

- The charged lepton mass matrix $M_\ell = \begin{pmatrix} Y_e(\tau_T) & 0 & 0 \\ 0 & Y_\mu(\tau_T) & 0 \\ 0 & & Y_\tau(\tau_T) \end{pmatrix}^*$.

- The Dirac mass matrix $M_D = y_D^* v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

- The Majorana mass matrix $M_R = b_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + b_2 \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + b_3 \begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & -2 \end{pmatrix}$,

where $b_1 = M_1^*$, $b_2 = (\lambda v_{SU})^*$ and $b_3 = (\frac{1}{\Lambda}\lambda' v'_{SU})^*$. Due to the suppression of higher-dimensional operator, b_3 is in general much smaller than b_2.

Model C

- The superpotential is given by (King, Zhou '19):

$$W = Y_e(\tau_l)Le^cH_d + Y_\mu(\tau_l)L\mu^cH_d + Y_\tau(\tau_l)L\tau^cH_d \\ + \frac{y_\nu}{\Lambda}L\Phi H_u\nu^c + \frac{1}{2}M_1(\tau_\nu)(\nu^c\nu^c)_{\mathbf{1}} + \frac{1}{2}M_2(\tau_\nu)(\nu^c\nu^c)_{\mathbf{2}} + \frac{1}{2}M_3(\tau_\nu)(\nu^c\nu^c)_{\mathbf{3}}\,.$$

- The model is based on multiple modular symmetry (de Medeiros Varzielas, King, Zhou, '19), where the original modular symmetry $S_4^l \times S_4^\nu$ is broken to a diagonal subgroup S_4 by the vev of a flavon $\Phi \sim (\mathbf{3},\mathbf{3})$ assigned as

$$\langle\Phi\rangle = v_\Phi \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- The subgroup S_4 is then broken to two distinct residual symmetries $G_\ell = Z_3^T$ and $G_\nu = Z_2^{SU}$, by the modular field τ_l stabilized at $\langle\tau_l\rangle = \tau_T$ in the charged lepton sector, and the modular field τ_ν fixed at $\langle\tau_\nu\rangle = \tau_{SU}$ in the neutrino sector, respectively.

Fields	S_4^l	S_4^ν	$2k_l$	$2k_\nu$
e^c	$\mathbf{1}'$	$\mathbf{1}$	-6	-2
μ^c	$\mathbf{1}'$	$\mathbf{1}$	-4	-2
τ^c	$\mathbf{1}'$	$\mathbf{1}$	-2	-2
L	$\mathbf{3}$	$\mathbf{1}$	0	$+2$
ν^c	$\mathbf{1}$	$\mathbf{3}$	0	-2
Φ	$\mathbf{3}$	$\mathbf{3}$	0	0
$H_{u,d}$	$\mathbf{1}$	$\mathbf{1}$	0	0

(King, Zhou '19)

Model C

- This scenario gives the same Yukawa matrices as model B for charged leptons and thus will not be repeated.
- The Dirac mass matrix is given by $M_D = \frac{y_\nu^* v_\Phi^* v_u}{\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.
- The Majorana mass matrix is $M_R = c_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + c_3\sqrt{2} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} - c_3\sqrt{3} \begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & -2 \end{pmatrix}$,

where $c_1 = [M_{\mathbf{1}}(\tau_{SU})]^*$, $c_2 = [M_{\mathbf{2},1}(\tau_{SU})]^*$ and $c_3 = \frac{1}{\sqrt{2}}[M_{\mathbf{3},1}(\tau_{SU})]^*$.

4. Experimental predictions in light of JUNO data

Parameterization

- All three discussed models can be parameterized using the TBM matrix $U_{\rm TBM}$

$$U_{\rm TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$$

$$U_{\rm TBM}^T M_R U_{\rm TBM} = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \alpha & \gamma \\ 0 & \gamma & \beta \end{pmatrix},$$

where it can be shown that

$$\text{Model} \quad \text{A}: \eta = -\alpha + \beta\,, \gamma = \alpha - 2\beta, \quad \text{with} \quad \alpha = 2a_2, \beta = a_2 + 3a_3\,,$$

$$\text{Model} \quad \text{B}: \eta = 2\alpha + \beta, \quad \text{with} \quad \alpha = b_1, \beta = -b_1 + 3b_2\,, \gamma = \sqrt{6}b_3\,,$$

$$\text{Model} \quad \text{C}: \eta = -\beta - 2\gamma, \quad \text{with} \quad \alpha = c_1 + 2c_2, \beta = -c_1 + c_2 + 3\sqrt{2}c_3, \gamma = -3\sqrt{2}c_3\,.$$

- This parameterization allows us to express the analytical predictions for our models in closed forms.
- The seesaw formula gives $M_\nu = -m_0^2 U_\ell^\dagger P_{23} M_R^{-1} P_{23}^T U_\ell^*$, where $m_0^2 = (y_D^*)^2 v_u^2$.
- The M_ν can be diagonalized by the PMNS matrix $U_{\rm PMNS} = U_\ell^\dagger U_{{\rm TM}_1}$ where $U_{{\rm TM}_1} \equiv U_{\rm TBM} \begin{pmatrix} e^{-i\alpha_3'} & 0 & 0 \\ 0 & \cos\theta_R\, e^{i\alpha_1} & \sin\theta_R\, e^{-i\alpha_2} \\ 0 & -\sin\theta_R\, e^{i\alpha_2} & \cos\theta_R\, e^{-i\alpha_1} \end{pmatrix}$.

Universal Predictions

- All three models contain five free parameters, including two complex coefficients and a norm, which can be fit by three mixing angles and two mass differences.

- A straightforward result from the previous parameterization gives the predictions:

$$\sin\theta_{13} = \frac{\sin\theta_R}{\sqrt{3}},$$

$$\tan\theta_{12} = \frac{\cos\theta_R}{\sqrt{2}}.$$

- These correlations recover the TM_1 sum rules between mixing angles $\sin^2\theta_{12} = 1 - \frac{2}{3(1-\sin^2\theta_{13})}$!

Predictions for Model A

- For model A, we get the following correlations:

$$\sin\left(2\theta_R\right)\cos\left(\alpha_1-\alpha_2\right)=1+\frac{\left(5m_2^2-2m_1^2\right)m_3^2}{m_1^2\left(m_3^2-m_2^2\right)},$$

$$1=-\left[\frac{\Delta m_{21}^2}{4m_1^2}-\frac{\Delta m_{32}^2\left(2-5\sin^2\theta_R\right)}{8m_3^2}+\frac{1}{8}\right]^2+\frac{m_3^4}{m_1^4}\frac{20\left(\Delta m_{21}^2\right)^2+6m_1^4}{\left(\Delta m_{32}^2\right)^2\sin^2\left(2\theta_R\right)}$$
$$+\frac{m_3^2}{m_1^2}\frac{22m_3^2\Delta m_{21}^2+2\Delta m_{32}^2\left[2\Delta m_{21}^2-m_2^2\left(3\sin^2\theta_R-4\right)\right]}{\left(\Delta m_{32}^2\right)^2\sin^2\left(2\theta_R\right)}.$$

- This shows model A is over-constrained. Modifying to higher modular weight of τ_ν can fit the data with the price of introducing more free parameters, rendering model A less predictive and thus will not be considered.

Predictions for Model B & C

- For model B and C, the universal predictions are (King, Zhou '19):

$$\tan\theta_{23} = \left| \frac{\cos\theta_R + \sqrt{\frac{2}{3}} e^{i(\alpha_1-\alpha_2)}\sin\theta_R}{\cos\theta_R - \sqrt{\frac{2}{3}} e^{i(\alpha_1-\alpha_2)}\sin\theta_R} \right| ,$$

$$\delta = \arg[(5(\cos 2\theta_R + 1)\cos(\alpha_1-\alpha_2) - i(\cos 2\theta_R + 5)\sin(\alpha_1-\alpha_2)] .$$

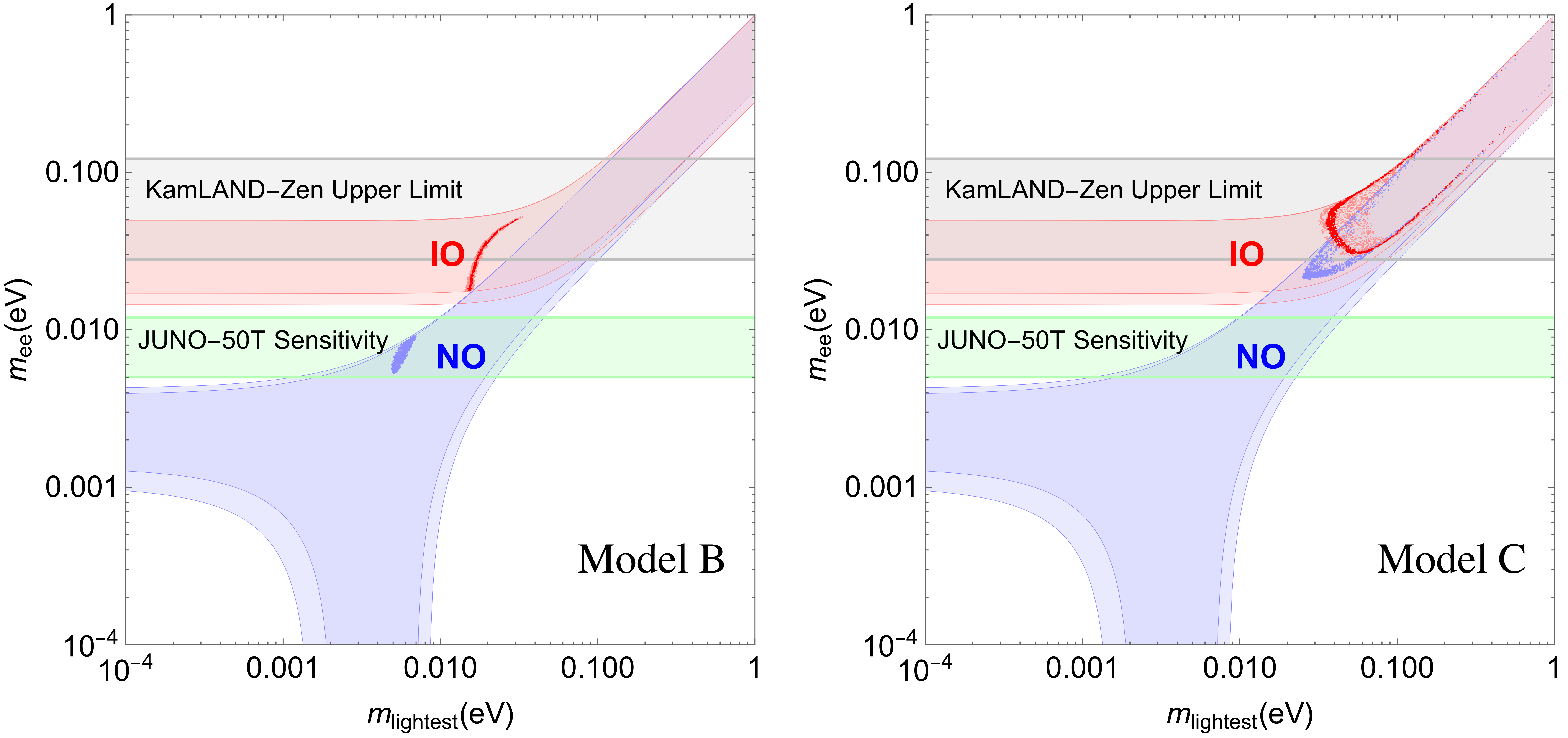
- In particular for model B, we found (Jiang, RO, Zhou '25):

$$\frac{1}{m_1} = \frac{1}{|2\alpha+\beta|} = \left| \frac{2e^{-2i\alpha_1}\cos^2\theta_R + e^{-2i\alpha_2}\sin^2\theta_R}{m_2} + \frac{e^{2i\alpha_1}\cos^2\theta_R + 2e^{2i\alpha_2}\sin^2\theta_R}{m_3} \right| .$$

$$m_{ee} = m_0^2 \left| \frac{2}{3(2\alpha+\beta)} + \frac{\beta}{3(\alpha\beta-\gamma^2)} \right| = \left| \frac{2m_2m_3}{3m_2(e^{2i\alpha_1}\cos^2\theta_R + 2e^{2i\alpha_2}\sin^2\theta_R) + 3m_3(e^{-2i\alpha_2}\sin^2\theta_R + 2e^{-2i\alpha_1}\cos^2\theta_R)} \right.$$

$$\left. -\frac{1}{3}\left(m_2 e^{2i\alpha_1}\cos^2\theta_R + m_3 e^{-2i\alpha_2}\sin^2\theta_R\right) \right| .$$

Numerical results



(Jiang, RO, Zhou ’25)

5. Summary

Summary

- We investigates the landscape of models based on modular S_4 symmetry that realizes TM$_1$ and compare them with the latest JUNO data.
- We reviewed how the TM$_1$ mixing pattern arises from the residual symmetries G_ℓ and G_ν after the spontaneous breaking of the flavor symmetry S_4.
- We show three different models that realize the TM$_1$ in three approaches with the same symmetry structure.
- The predictions of different models on the relations between the lightest neutrino mass and the effective Majorana mass in neutrinoless double beta decay are different, making them distinguishable.

Thank you for your attention!

More modular forms

- At the fixed point $\tau_\gamma = \tau_{SU} = -\frac{1}{2} + \frac{i}{2}$, one can solve for the eigenvector of $\rho_{\mathbf{3}'}(SU)$ with respect to the degenerate eigenvalue 1, to obtain the modular forms with weight ≤ 4 as (King, Zhou '19)

$$Y_{\mathbf{2}}^{(2)}(\tau_{SU}) \propto \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad Y_{\mathbf{3}'}^{(2)}(\tau_{SU}) \propto \begin{pmatrix} 1 \\ 1-\sqrt{6} \\ 1+\sqrt{6} \end{pmatrix},$$

$$Y_{\mathbf{3}'}^{(4)}(\tau_{SU}) \propto \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \quad Y_{\mathbf{3}}^{(4)}(\tau_{SU}) \propto \begin{pmatrix} \sqrt{2} \\ \sqrt{2}-\sqrt{3} \\ \sqrt{2}+\sqrt{3} \end{pmatrix}.$$

Model C predictions

- The predictions of model C is given by (King, Zhou ’19)

$$\frac{1}{m_1} = \frac{1}{|\beta + 2\gamma|} = \left| \frac{e^{-2i\alpha_2} \sin^2\theta_R + e^{-i(\alpha_1+\alpha_2)} \sin 2\theta_R}{m_2} + \frac{e^{2i\alpha_1} \cos^2\theta_R - e^{i(\alpha_1+\alpha_2)} \sin 2\theta_R}{m_3} \right| .$$

$$m_{ee} = m_0^2 \left| (M_{R,C}^{-1})_{(1,1)} \right| = m_0^2 \left| \frac{2}{3\beta + 2\gamma} - \frac{\beta}{3(\alpha\beta - \gamma^2)} \right|$$

$$= \left| \frac{2m_2 m_3}{3m_2(e^{i2\alpha_1} \cos^2\theta_R - e^{i(\alpha_1+\alpha_2)} \sin 2\theta_R) + 3m_3(e^{-i2\alpha_2} \sin^2\theta_R + e^{-i(\alpha_1+\alpha_2)} \sin 2\theta_R)} \right.$$

$$\left. + \frac{1}{3}\left(m_2 e^{2i\alpha_1} \cos^2\theta_R + m_3 e^{-2i\alpha_2} \sin^2\theta_R\right) \right| .$$