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NAWI Graz
Natural Sciences

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Fundação
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EXO HAD
EXOTIC HADRONS TOPICAL COLLABORATION

The few-body problem in QCD

(with functional methods)

Gernot Eichmann

University of Graz

2025 International Workshop and School on Hadron Structure
and Strong Interactions

Nanjing, China, October 13-17, 2025

1. Introduction
2. Spectrum & symmetries
3. Quark models
4. QFT toolbox
5. Functional methods
6. n-point functions & tensor bases
7. Multiquark states
8. Light-front wave functions

Lecture notes “QCD and hadron physics,”
<https://particle.uni-graz.at/en/quarks-hadrons-and-nuclei>

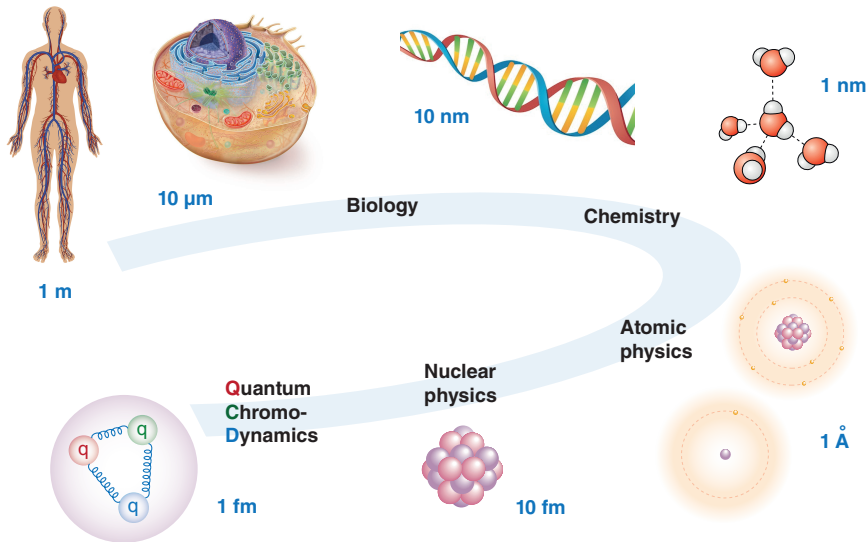
GE, “*Hadron physics with functional methods*,”
arXiv: 2503.10397 (for Encyclopedia of Particle Physics)

Burkert, GE, Klempt, “*The impact of γN and $\gamma^* N$ interactions on our understanding of nucleon excitations*,” arXiv:2506.16482

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer,
“*Baryons as relativistic three-quark bound states*”,
Prog. Part. Nucl. Phys. 91 (2016), arXiv: 1606.09602

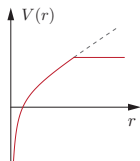
Barabanov, Bedolla, Brooks, Cates, Chen et al.,
“*Diquark correlations in hadron physics: Origin, impact and evidence*”, Prog. Part. Nucl. Phys. 116 (2021),
arXiv: 2008.07630

QCD binds us together

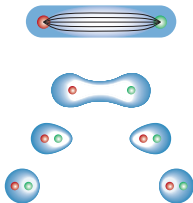


Confinement

- Quarks & gluons are **confined** in hadrons, cannot be observed in isolation



Microscopic origin still unclear – \$1M prize



- Only colorless **bound states** observable:



$$3 \times \bar{3} = \mathbf{1} + 8$$

Mesons



$$3 \times 3 \times 3 = \mathbf{1} + 8 + 8 + 10$$

Baryons

The hadron zoo

Mesons

$J^P(C)$

0^{-+}	0^{++}	1^{-+}	1^{--}	1^{++}	1^{+-}	2^{-+}	2^{++}	3^{--}
$\pi(140)$ $\pi(1300)$ $\pi(1800)$	$a_0(980)$ $a_0(1450)$ $a_0(1960)$	$\pi_1(1400)$ $\pi_1(1600)$	$\rho(770)$ $\rho(1450)$ $\rho(1870)$ $\rho(1700)$ $\rho(1900)$	$a_1(1260)$ $a_1(1420)$ $a_1(1640)$	$b_1(1235)$	$\pi_2(1670)$ $\pi_2(1880)$	$a_2(1320)$ $a_2(1700)$	$\rho_3(1690)$ $\rho_3(1990)$
$K(494)$ $K(1460)$ $K(1830)$	$K_0^*(800)$ $K_0^*(1430)$ $K_0^*(1950)$		$K^*(892)$ $K^*(1410)$ $K^*(1680)$	$K_1(1400)$ $K_1(1650)$	$K_1(1270)$	$K_3(1580)$ $K_3(1770)$ $K_3(1820)$	$K_2^*(1430)$ $K_2^*(1980)$	$K_3^*(1780)$
$\eta(548)$ $\eta'(958)$ $\eta(1296)$ $\eta(1405)$ $\eta(1478)$ $\eta(1760)$	$f_0(500)$ $f_0(980)$ $f_0(1370)$ $f_0(1600)$ $f_0(1710)$		$\omega(782)$ $\phi(1020)$ $\omega(1420)$ $\omega(1850)$ $\phi(1680)$	$f_1(1285)$ $f_1(1420)$ $f_1(1610)$	$h_1(1170)$ $h_1(1380)$ $h_1(1595)$	$\eta_2(1645)$ $\eta_2(1870)$	$f_2(1270)$ $f_2(1430)$ $f_2^*(1525)$ $f_2(1565)$ $f_2(1640)$ $f_2(1810)$ $f_2(1910)$ $f_2(1980)$	$\omega_3(1670)$ $\phi_3(1850)$

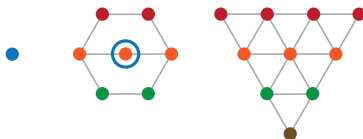


$$3 \times \bar{3} = 1 + 8$$

Baryons

J^P

$\frac{1}{2}^{+}$	$\frac{1}{2}^{-}$	$\frac{3}{2}^{+}$	$\frac{3}{2}^{-}$	$\frac{5}{2}^{+}$	$\frac{5}{2}^{-}$	$\frac{7}{2}^{+}$
$N(939)$ $N(1440)$ $N(1710)$ $N(1880)$	$N(1535)$ $N(1650)$ $N(1895)$	$N(1720)$ $N(1900)$	$N(1620)$ $N(1700)$ $N(1875)$	$N(1690)$ $N(1860)$ $N(2000)$	$N(1675)$	$N(1990)$
$\Delta(1910)$	$\Delta(1620)$ $\Delta(1900)$	$\Delta(1232)$ $\Delta(1600)$ $\Delta(1920)$	$\Delta(1700)$ $\Delta(1940)$	$\Delta(1905)$ $\Delta(2000)$	$\Delta(1930)$	$\Delta(1950)$
$\Lambda(1116)$ $\Lambda(1600)$ $\Lambda(1810)$	$\Lambda(1405)$ $\Lambda(1670)$ $\Lambda(1800)$	$\Lambda(1890)$	$\Lambda(1520)$ $\Lambda(1690)$	$\Lambda(1820)$	$\Lambda(1830)$	
$\Sigma(1189)$ $\Sigma(1680)$ $\Sigma(1880)$	$\Sigma(1760)$	$\Sigma(1385)$	$\Sigma(1670)$ $\Sigma(1940)$	$\Sigma(1915)$	$\Sigma(1775)$	
$\Xi(1315)$		$\Xi(1530)$	$\Xi(1820)$			
		$\Omega(1672)$				



$$3 \times 3 \times 3 = 1 + 8 + 8 + 10$$

The hadron zoo

Mesons

$J^P(C)$

0^{-+}

$\pi(140)$

$\pi(1300)$
 $\pi(1800)$

$\eta(548)$
 $\eta(782)$
 $\eta(958)$
 $\eta(1296)$
 $\eta(1405)$
 $\eta(1678)$
 $\eta(1780)$

1^{--}

$\rho(770)$

$\rho(1450)$
 $\rho(1670)$
 $\rho(1700)$
 $\rho(1800)$

$\omega(782)$
 $\phi(1020)$
 $\omega(1430)$
 $\omega(1630)$
 $\omega(1680)$
 $\phi(1880)$

$\pi_1(1400)$
 $\pi_2(1800)$

$\eta_1(1235)$
 $\eta_2(1670)$
 $\eta_2(1880)$

$\pi_3(1670)$
 $\pi_3(1880)$

$\pi_4(1335)$
 $\pi_4(1420)$
 $\pi_4(1640)$

$\pi_5(1260)$
 $\pi_5(1430)$
 $\pi_5(1610)$
 $\pi_5(1810)$
 $\pi_5(1900)$

$\pi_6(1330)$
 $\pi_6(1700)$
 $\pi_6(1990)$

$\pi_7(1330)$
 $\pi_7(1700)$
 $\pi_7(1990)$

$\pi_8(1330)$
 $\pi_8(1700)$
 $\pi_8(1990)$

$\pi_9(1330)$
 $\pi_9(1700)$
 $\pi_9(1990)$

$\pi_{10}(1330)$
 $\pi_{10}(1700)$
 $\pi_{10}(1990)$

$\pi_{11}(1330)$
 $\pi_{11}(1700)$
 $\pi_{11}(1990)$

$\pi_{12}(1330)$
 $\pi_{12}(1700)$
 $\pi_{12}(1990)$

Baryons

J^P

$\frac{1}{2}^{+}$

$N(939)$

$N(1440)$
 $N(1680)$
 $N(1710)$
 $N(1880)$

$\frac{1}{2}^{-}$

$\Lambda(1115)$

$\Lambda(1520)$
 $\Lambda(1670)$
 $\Lambda(1820)$
 $\Lambda(1830)$

$\frac{3}{2}^{+}$

$\Delta(1232)$

$\Delta(1600)$
 $\Delta(1680)$
 $\Delta(1890)$
 $\Delta(1900)$

$\frac{3}{2}^{-}$

$\Sigma(1385)$

$\Sigma(1670)$
 $\Sigma(1690)$
 $\Sigma(1840)$
 $\Sigma(1860)$

$\frac{5}{2}^{+}$

$\Sigma(1670)$

$\Sigma(1690)$
 $\Sigma(1840)$
 $\Sigma(1860)$

$\frac{5}{2}^{-}$

$\Sigma(1670)$

$\Sigma(1690)$
 $\Sigma(1840)$
 $\Sigma(1860)$

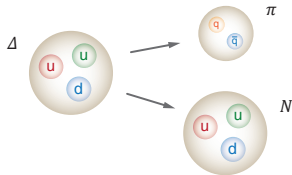
$\frac{7}{2}^{+}$

$\Sigma(1670)$

$\Sigma(1690)$
 $\Sigma(1840)$
 $\Sigma(1860)$

$\sim 1 \text{ GeV}$

Most hadrons are **resonances** and decay into other hadrons



And it doesn't stop there ...

Familiar states



Mesons



Baryons

Exotic states



Glueballs?



Hybrid mesons?

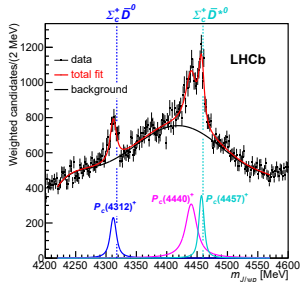


Tetraquarks?



Pentaquarks?

To understand QCD, we must understand
spectrum and interactions of hadrons!

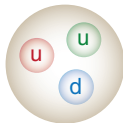


Aaij et al., Phys. Rev. Lett. 112 (2019) 222001

Mass generation

Three current quarks
do **not** make a proton:

$$2m_u + m_d \sim 10 \text{ MeV} \ll 1 \text{ GeV} ?$$



Higgs: Current-quark masses

QCD: Mass generation from
quark-gluon interactions,
“constituent-quark masses”

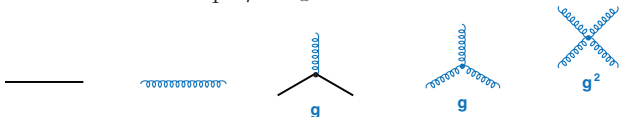
u	d	s	c	b	t
3	5	100	1000	4000	175000
350	350	350	350	350	350

[MeV]



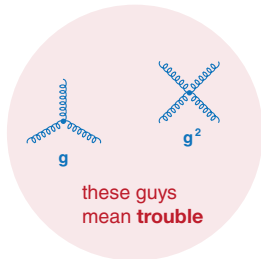
QCD Lagrangian

$$\mathcal{L} = \bar{\psi} (\not{\partial} + i \textcolor{red}{g} \not{A} + \textcolor{red}{m}) \psi + \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$



QCD Lagrangian

$$\mathcal{L} = \bar{\psi} (\not{\partial} + i \textcolor{red}{g} \not{A} + \textcolor{red}{m}) \psi + \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

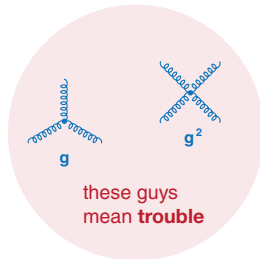
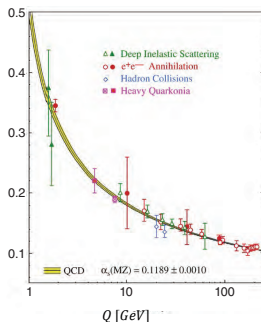


QCD Lagrangian

$$\mathcal{L} = \bar{\psi} (\not{\partial} + i \mathbf{g} \mathbf{A} + \mathbf{m}) \psi + \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$



$$\alpha = \frac{g^2}{4\pi}$$



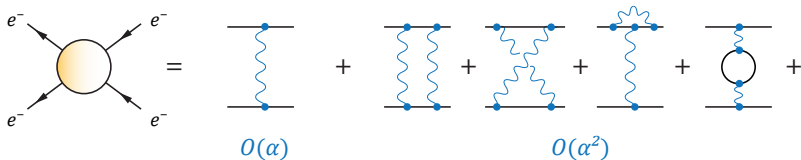
At large momenta, quarks & gluons behave as quasi-free particles:
asymptotic freedom



Gross, Politzer, Wilczek 1973

Perturbative vs. nonperturbative

Interaction between **two electrons**:



“Quantum
mechanics”

Perturbative QFT:
Feynman diagrams & loops



J. S. Schwinger



S. Tomonaga



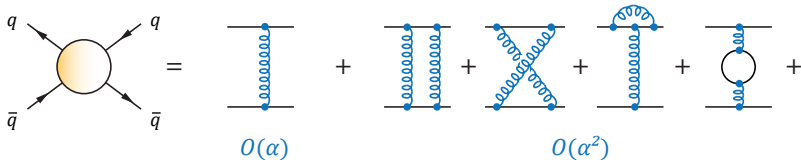
R. P. Feynman



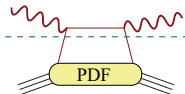
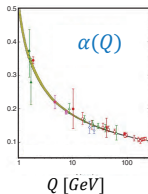
F. J. Dyson

Perturbative vs. nonperturbative

Interaction between **quark and antiquark**?



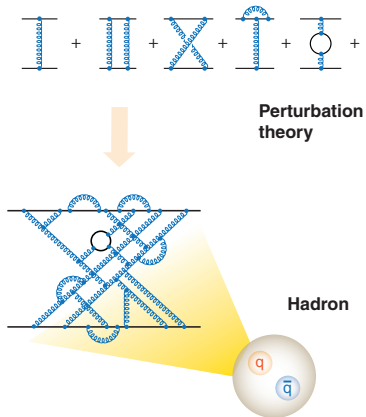
Answer still correct at large momenta:
high-energy scattering experiments



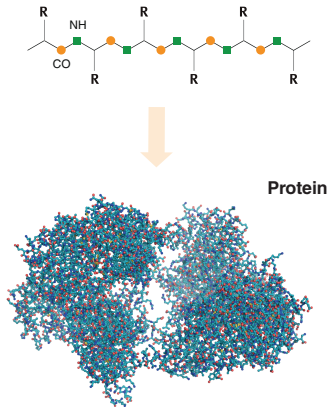
What about low momenta, confinement, mass generation, hadron physics?
→ need **nonperturbative methods!**

Emergent phenomena

Hadron physics:



Biology:



Tim Volders, Public Domain

Many open questions!

- **Understanding exotic hadrons:**

Hadron spectroscopy at [LHC](#), [Belle II](#), [BES III](#), [PANDA](#), [JLab](#), [ELSA](#), ...



Glueballs



Hybrid mesons

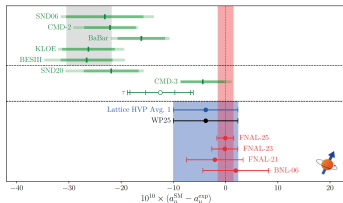


Tetraquarks



Pentaquarks

- **QCD contributions to BSM searches,**
e.g. muon g-2 at [Fermilab](#)



Alberti et al., Phys. Rept. 1143 (2025)

- **Mass generation and confinement?**

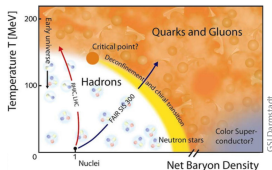
Higgs

QCD

- **Quark-gluon structure of hadrons and nuclei:**
[EIC](#), [JLab](#), [COMPASS/AMBER](#), ...



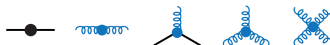
- **The phases of QCD:** [LHC](#), [RHIC](#), [CBM](#), ...



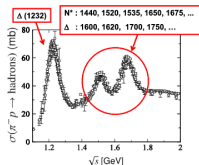
Theory tools

Functional methods

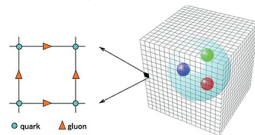
(Dyson-Schwinger & Bethe-Salpeter eqs, FRG, ...)



Amplitude analyses



Lattice QCD



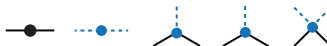
Phenomenological models

(Quark models, ...)



Effective theories

(Chiral perturbation theory, ...)



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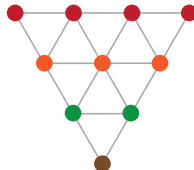
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“*Diquark correlations in hadron physics: Origin, impact and evidence*”, Prog. Part. Nucl. Phys. 116 (2021),
arXiv: 2008.07630

Symmetries in the spectrum

$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$
$N(939)$ $N(1440)$ $N(1710)$ $N(1880)$	$N(1535)$ $N(1650)$ $N(1895)$	$N(1720)$ $N(1900)$	$N(1520)$ $N(1700)$ $N(1875)$	$N(1680)$ $N(1860)$ $N(1875)$ $N(2000)$	$N(1675)$	$N(1990)$
$\Delta(1910)$	$\Delta(1620)$ $\Delta(1900)$	$\Delta(1232)$ $\Delta(1600)$ $\Delta(1920)$	$\Delta(1700)$ $\Delta(1940)$	$\Delta(1905)$ $\Delta(2000)$	$\Delta(1930)$	$\Delta(1960)$
$\Lambda(1115)$ $\Lambda(1600)$ $\Lambda(1810)$	$\Lambda(1405)$ $\Lambda(1670)$ $\Lambda(1800)$	$\Lambda(1890)$	$\Lambda(1520)$ $\Lambda(1690)$	$\Lambda(1820)$	$\Lambda(1830)$	
$\Sigma(1189)$ $\Sigma(1660)$ $\Sigma(1880)$	$\Sigma(1750)$	$\Sigma(1385)$	$\Sigma(1670)$ $\Sigma(1940)$	$\Sigma(1915)$	$\Sigma(1775)$	
$\Xi(1315)$		$\Xi(1530)$ $\Omega(1672)$	$\Xi(1820)$			



nnn

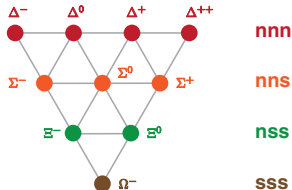
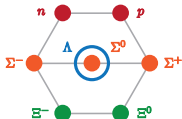
nns

nss

sss

Symmetries in the spectrum

$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$
$N(939)$	$N(1835)$	$N(1730)$	$N(1520)$	$N(1680)$	$N(1675)$	$N(1690)$
$N(1440)$	$N(1650)$	$N(1800)$	$N(1700)$	$N(1890)$		
$N(1710)$	$N(1890)$		$N(1875)$	$N(2000)$		
$N(1890)$						
$\Delta(1910)$	$\Delta(1620)$	$\Delta(1232)$	$\Delta(1700)$	$\Delta(1905)$	$\Delta(1990)$	$\Delta(1960)$
	$\Delta(1800)$	$\Delta(1600)$	$\Delta(1940)$	$\Delta(2000)$		
		$\Delta(1920)$				
$\Lambda(1115)$	$\Lambda(1405)$	$\Lambda(1890)$	$\Lambda(1830)$	$\Lambda(1820)$	$\Lambda(1830)$	
$\Lambda(1600)$	$\Lambda(1670)$		$\Lambda(1890)$			
$\Lambda(1810)$	$\Lambda(1800)$					
$\Sigma(1189)$	$\Sigma(1780)$	$\Sigma(1385)$	$\Sigma(1670)$	$\Sigma(1915)$	$\Sigma(1775)$	
$\Sigma(1640)$			$\Sigma(1640)$			
$\Sigma(1890)$						
$\Xi(1315)$		$\Xi(1530)$	$\Xi(1820)$			
		$\Omega(1672)$				



Symmetries of QCD

$$S = \int d^4x \mathcal{L}$$

$$\mathcal{L} = \bar{\psi} (i \not{D} + \mathbf{g} \not{A} - \mathbf{M}) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

Quark
mass matrix

$\psi_{\alpha,i,f}(x)$

Flavor: 1 ... N_f
Color: 1 ... 3
Dirac: 1 ... 4

Symmetries of QCD

$$S = \int d^4x \mathcal{L}$$

$$\mathcal{L} = \bar{\psi} (i \not{\partial} + g \not{A} - \mathbf{M}) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

Quark
mass matrix

$\psi_{\alpha,i,f}(x)$

Flavor: 1 ... N_f
Color: 1 ... 3
Dirac: 1 ... 4

- invariant under **local SU(3) gauge transformations**

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_{MA} \oplus 8_{MS} \oplus 1_A$$

⇒ Baryons are
color singlets

Decuplet

Octet

Singlet

Symmetries of QCD

$$S = \int d^4x \mathcal{L}$$

$$\mathcal{L} = \bar{\psi} (i \not{\partial} + g \not{A} - \mathbf{M}) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

Quark
mass matrix

$\psi_{\alpha,i,f}(x)$

Flavor: 1 ... N_f
Color: 1 ... 3
Dirac: 1 ... 4

- invariant under local SU(3) gauge transformations
- invariant under **Poincaré group**



Translations, rotations, boosts;

Two Casimir operators which label states:
M (mass), J (total angular momentum, “spin”)

Symmetries of QCD

$$S = \int d^4x \mathcal{L}$$

$$\mathcal{L} = \bar{\psi} (i \not{\partial} + g \not{A} - \mathbf{M}) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

Quark
mass matrix

$\psi_{\alpha,i,f}(x)$

Flavor: 1 ... N_f
Color: 1 ... 3
Dirac: 1 ... 4

- invariant under local SU(3) gauge transformations
- invariant under Poincaré group
- invariant under C, **P**, T

Parity labels states $\rightarrow M, J^P$

Symmetries of QCD

$$S = \int d^4x \mathcal{L}$$

$$\mathcal{L} = \bar{\psi} (i \not{\partial} + g \not{A} - \mathbf{M}) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

Quark
mass matrix

$\psi_{\alpha,i,f}(x)$

Flavor: 1 ... N_f
Color: 1 ... 3
Dirac: 1 ... 4

- invariant under local SU(3) gauge transformations
- invariant under Poincaré group
- invariant under C, P, T
- various global flavor (non-)symmetries:

$U(1)_V$

conserved: **baryon number** $B = \frac{1}{3} (n_u + n_d + n_s)$

$n_q \dots$ (# quarks) – (# antiquarks)

$$\psi' = e^{i\varepsilon} \psi$$

$\Rightarrow$ Baryons carry $B = 1$ (pentaquarks too)

Symmetries of QCD

$$S = \int d^4x \mathcal{L}$$

$$\mathcal{L} = \bar{\psi} (i \not{\partial} + g \not{A} - \mathbf{M}) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

Quark
mass matrix

$$\psi_{\alpha,i,f}(x)$$

- invariant under local SU(3) gauge transformations
- invariant under Poincaré group
- invariant under C, P, T
- various global flavor (non-)symmetries:

$U(1)_V$ conserved (baryon number)
 $SU(N_f)_V$ if $M = m$ (all quark masses equal)

$$\psi' = e^{i \sum_{a=1}^8 \epsilon_a t_a} \psi \quad \text{for 3 flavors}$$

Define **vector currents** and charges:

$$V_a^\mu = \bar{\psi} \gamma^\mu t_a \psi, \quad \partial_\mu V_a^\mu = i \bar{\psi} [M, t_a] \psi,$$

$$Q_a^V(t) = \int d^3x \psi^\dagger t_a \psi$$

Noether theorem: if symmetry preserved, divergence of current vanishes and charge is conserved. Can only work if $M = m$.

But the diagonal SU(3) generators t_3, t_8 commute with $M \rightarrow$ always conserved!

Isospin I_3 , **hypercharge** Y :

$$I_3 = \frac{1}{2} (n_u - n_d), \quad Y = \frac{1}{3} (n_u + n_d - 2n_s)$$

These are still good quantum numbers, even if flavor symmetry is broken!

Symmetries of QCD

$$S = \int d^4x \mathcal{L}$$

$$\mathcal{L} = \bar{\psi} (i \not{\partial} + g \not{A} - \mathbf{M}) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

Quark
mass matrix

$$\psi_{\alpha,i,f}(x)$$

- invariant under local SU(3) gauge transformations
- invariant under Poincaré group
- invariant under C, P, T
- various global flavor (non-)symmetries:

chiral symmetry

$U(1)_V$	conserved (baryon number)
$SU(N_f)_V$	if $M = m$ (all quark masses equal)
$SU(N_f)_A$	if $M = 0$ (but: spontaneously broken)

$$\psi' = e^{i\gamma_5 \sum_{a=1}^8 \varepsilon_a \mathbf{t}_a} \psi \quad \text{for 3 flavors}$$

Define **axialvector currents** and charges:

$$A_a^\mu = \bar{\psi} \gamma^\mu \gamma_5 \mathbf{t}_a \psi, \quad \partial_\mu A_a^\mu = i \bar{\psi} \{M, \mathbf{t}_a\} \gamma_5 \psi,$$

$$Q_a^A(t) = \int d^3x \psi^\dagger \gamma_5 \mathbf{t}_a \psi$$

Symmetry only if $M = 0$ (**chiral limit**)

But symmetry spontaneously broken in QFT:
dynamical chiral symmetry breaking

Symmetries of QCD

$$S = \int d^4x \mathcal{L} \quad \mathcal{L} = \bar{\psi} (i \not{\partial} + g \not{A} - \mathbf{M}) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

Quark mass matrix $\psi_{\alpha,i,f}(x)$

- invariant under local SU(3) gauge transformations
- invariant under Poincaré group
- invariant under C, P, T
- various global flavor (non-)symmetries:

chiral symmetry	{	$U(1)_V$	conserved (baryon number)	{	Only $M = 0$, but anomalous breaking in QFT (symmetry does not survive quantization)
		$SU(N_f)_V$	if $M = m$ (all quark masses equal)		
		$SU(N_f)_A$	if $M = 0$ (but: spontaneously broken)		
		$U(1)_A$	if $M = 0$ (but: anomalously broken)		

$\psi' = e^{i\gamma_5 \epsilon} \psi$

Symmetries of QCD

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⇒ **Quantum numbers**
for labelling baryons:

$$M, J^P, I_3, Y$$

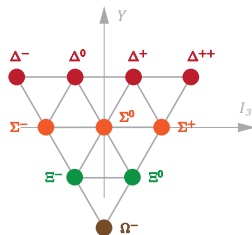
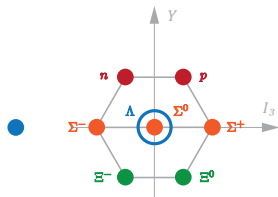
Symmetries of QCD

$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$
$N(939)$ $N(1440)$ $N(1710)$ $N(1880)$	$N(1535)$ $N(1650)$ $N(1895)$	$N(1720)$ $N(1900)$	$N(1520)$ $N(1700)$ $N(1875)$	$N(1680)$ $N(1860)$ $N(2000)$	$N(1675)$	$N(1990)$
$\Delta(1910)$	$\Delta(1620)$ $\Delta(1900)$	$\Delta(1232)$ $\Delta(1600)$ $\Delta(1920)$	$\Delta(1700)$ $\Delta(1940)$	$\Delta(1905)$ $\Delta(2000)$	$\Delta(1930)$	$\Delta(1950)$
$\Lambda(1115)$ $\Lambda(1600)$ $\Lambda(1810)$	$\Lambda(1405)$ $\Lambda(1670)$ $\Lambda(1800)$	$\Lambda(1890)$	$\Lambda(1520)$ $\Lambda(1690)$	$\Lambda(1820)$	$\Lambda(1830)$	
$\Sigma(1189)$ $\Sigma(1660)$ $\Sigma(1880)$	$\Sigma(1750)$	$\Sigma(1385)$	$\Sigma(1670)$ $\Sigma(1940)$	$\Sigma(1915)$	$\Sigma(1775)$	
$\Xi(1315)$	$\Xi(1530)$ $\Omega(1672)$	$\Xi(1820)$				

Quantum numbers
for labelling baryons:

$$M, J^P, I_3, Y$$

Symmetries of the spectrum
are the symmetries of $\mathcal{L}$!



nnn

nns

nss

sss

Symmetries of QCD

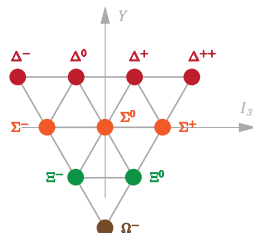
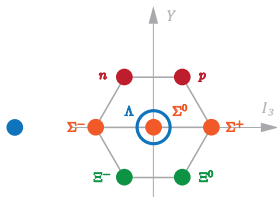
$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$
$N(939)$ $N(1440)$ $N(1710)$ $N(1880)$	$N(1535)$ $N(1650)$ $N(1895)$	$N(1720)$ $N(1900)$	$N(1520)$ $N(1700)$ $N(1875)$	$N(1680)$ $N(1860)$ $N(2000)$	$N(1675)$	$N(1990)$
$\Delta(1910)$	$\Delta(1620)$ $\Delta(1900)$	$\Delta(1232)$ $\Delta(1600)$ $\Delta(1920)$	$\Delta(1700)$ $\Delta(1940)$	$\Delta(1905)$ $\Delta(2000)$	$\Delta(1930)$	$\Delta(1950)$
$\Lambda(1115)$ $\Lambda(1600)$ $\Lambda(1810)$	$\Lambda(1405)$ $\Lambda(1670)$ $\Lambda(1800)$	$\Lambda(1890)$	$\Lambda(1520)$ $\Lambda(1690)$	$\Lambda(1820)$	$\Lambda(1830)$	
$\Sigma(1189)$ $\Sigma(1660)$ $\Sigma(1880)$	$\Sigma(1750)$	$\Sigma(1385)$	$\Sigma(1670)$ $\Sigma(1940)$	$\Sigma(1915)$	$\Sigma(1775)$	
$\Xi(1315)$		$\Xi(1530)$ $\Omega(1672)$	$\Xi(1820)$			

$SU(3)_f$ is broken:

- states inside multiplet are not mass-degenerate
- Casimirs of $SU(3)_f$ are not good quantum numbers (these distinguish multiplets)
- But I_3, Y still conserved: states with same I_3, Y can mix
- In practice: isospin symmetry $SU(2)_f$ still good (w/d masses) states with same I, I_3, Y can mix

→ octet-decuplet mixing (Σ, Ξ)

→ singlet-octet mixing (Λ)



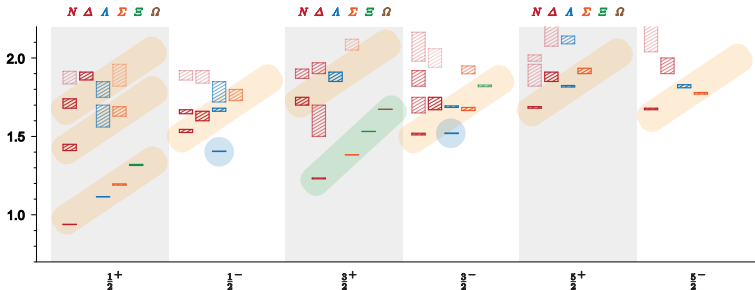
nnn

nns

nss

sss

Dynamics?

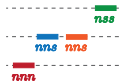


Expected patterns:

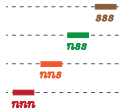
Singlet



Octet



Decuplet



1. Introduction

2. Spectrum & symmetries

3. Quark models

4. QFT toolbox

5. Functional methods

6. n-point functions & tensor bases

7. Multiquark states

8. Light-front wave functions

Lecture notes “QCD and hadron physics,”
<https://particle.uni-graz.at/en/quarks-hadrons-and-nuclei>

GE, “*Hadron physics with functional methods*,”
arXiv: 2503.10397 (for Encyclopedia of Particle Physics)

Burkert, GE, Klempt, “*The impact of γ N and γ^*N interactions on our understanding of nucleon excitations*,” arXiv:2506.16482

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer,
“*Baryons as relativistic three-quark bound states*”,
Prog. Part. Nucl. Phys. 91 (2016), arXiv: 1606.09602

Barabanov, Bedolla, Brooks, Cates, Chen et al.,
“*Diquark correlations in hadron physics: Origin, impact and evidence*”, Prog. Part. Nucl. Phys. 116 (2021),
arXiv: 2008.07630

Quark models

$$H \Psi = E \Psi \quad H = H_{\text{kin}} + \sum_{i < j} V(\mathbf{r}_{ij})$$



E.g. Breit-Fermi interaction:
non-relativistic limit of one-gluon exchange
de Rujula, Georgi, Glashow, PRD 12 (1975)

$$V(\mathbf{r}) \sim \alpha \left[-\frac{1}{r} + V_{\text{ss}}(\mathbf{r}) + V_{\text{so}}(\mathbf{r}) + \dots \right]$$

Coulomb

Spin-orbit

Hyperfine (spin-spin) interaction:
generates splittings between $S = \frac{1}{2}, \frac{3}{2}$
 $\Rightarrow$ e.g. $N-\Delta$ splitting

Reviews:

Hendry, Lichtenberg, Rep. Prog. Phys. 41 (1978)
Capstick, Roberts, Prog. Part. Nucl. Phys. 45 (2000)
Klempt, Richard, Rev. Mod. Phys. 82 (2010)
Richard, 1205.4326 [hep-ph]
Crede, Roberts, Rept. Prog. Phys. 76 (2013)

One-gluon exchange

de Rujula, Georgi, Glashow, PRD 12 (1975)
Isgur, Karl, PRD 20 (1979)
Godfrey, Isgur, PRD 32 (1985)
Capstick, Isgur, PRD 34 (1986)

Goldstone-boson exchange

Glozman, Riska, Phys. Rept. 268 (1996)
Glozman, Plessas, Varga, Wagenbrunn, PRD 58 (1998)
Melde, Plessas, Sengl, PRD 77 (2008)

Diquark models

Anselmino, Predazzi, Ekelin, Fredriksson, Lichtenberg,
Rev. Mod. Phys. 65 (1993)
Ebert, Faustov, Galkin, PRD 72 (2005), PLB 659 (2008)
Santopinto, Ferretti, PRC 92 (2015), FBS 57 (2016)

String models

Bijker, Iachello, Levitan, Annals Phys. 236 (1994),
Annals Phys. 284 (2000)

Large-Nc expansion

Goffy, Schat, Scoccola, PRD 66 (2002), PLB 564 (2003)
Matagne, Stancu, PRD 71 (2005), PRD 85 (2012)

Hypercentral quark model

Giannini, Santopinto, Vassallo, EPJ A 12 (2001)
Giannini, Santopinto, Chin. J. Phys. 53 (2015)

Light-front & holographic models

Brodsky, de Tera mond, Dosch, Erlich, Phys. Rept. 584 (2015)

BSE models

Loring, Kretzschmar, Metsch, Petry, EPJ A 10 (2001)
Loring, Metsch, Petry, EPJ A 10 (2001)
Metsch, EPJ A 35 (2008)

...

Quark potential models

Total wave function:

$$\Psi = \Psi_{\text{dynamics}} \otimes \Psi_{\text{flavor}} \otimes \Psi_{\text{color}}$$



Color part is totally antisymmetric:

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_{M_A} \oplus 8_{M_S} \oplus 1_A$$

Singlet

Flavor part:

Must find **combined irreducible reps.** of
permutation group S_3 (3 valence quarks)
and $SU(N_f)$ (N_f flavors)

$$\overbrace{\left(\begin{pmatrix} u \\ d \\ s \\ \vdots \end{pmatrix} \otimes \begin{pmatrix} u \\ d \\ s \\ \vdots \end{pmatrix} \otimes \begin{pmatrix} u \\ d \\ s \\ \vdots \end{pmatrix} \right)}^{S_3} \left. \vphantom{\begin{pmatrix} u \\ d \\ s \\ \vdots \end{pmatrix}} \right\} SU(N_f)$$

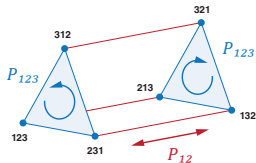
$\Rightarrow$ Nucleon:

$$\begin{aligned} \text{Proton:} & \quad \frac{1}{\sqrt{2}} \begin{bmatrix} udu - duu \\ -\frac{1}{\sqrt{3}}(udu + duu - 2wud) \end{bmatrix} \\ \text{Neutron:} & \quad \frac{1}{\sqrt{2}} \begin{bmatrix} udd - dud \\ \frac{1}{\sqrt{3}}(dud + udd - 2ddu) \end{bmatrix} \end{aligned}$$

Flavor

Permutation group S_3 consists of 6 group elements, can be reconstructed from **transposition** P_{12} ($1 \leftrightarrow 2$) and **cyclic permutation** P_{123} ($1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 1$):

$$\begin{array}{ll} 1 \psi_{123} = \psi_{123} & P_{12} \psi_{123} = \psi_{213} \\ P_{123} \psi_{123} = \psi_{231} & P_{12} P_{123} \psi_{123} = \psi_{132} \\ P_{123}^2 \psi_{123} = \psi_{312} & P_{12} P_{123}^2 \psi_{123} = \psi_{321} \end{array}$$



Cayley graph