Theory and phenomenology of Generalised Partons Distributions

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October 13^{th}, 2025

2025 International Workshop and School on Hadron Structure and Strong Interactions

Bibliography

No modern, complete review or lecture note on GPDs. However one can highlight:

- M. Diehl, Phys.Rept., 2003, 388, 41-277
- A. Belitsky and A. Radyushkin, Phys.Rept., 2005, 418, 1-387

which remains today the best review papers regarding GPDs.

A more pedestrian (but also far less complete) introduction can be found in

- C. Mezrag, Few Body Syst., 2022, 63, 62

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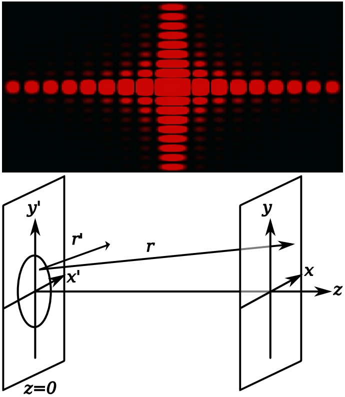
- Lecture 3: Experimental access: what can we really measure ?
- Lecture 4: Modelling of GPDs.

Of course, this may evolve by Friday !

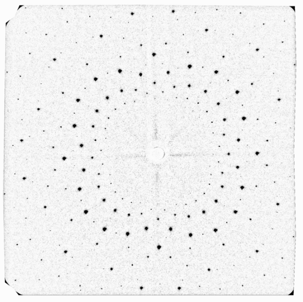
Motivation: Probing the internal structure of matter



source : Wikimedia Commons

- Far field diffraction $z >> x', y'$
- Monochromatic wavelenght $\lambda \approx 1\mu m$

$$U(x,y,z) \approx \frac{e^{ikz} e^{ik\frac{x^2+y^2}{z}}}{i\lambda z} \underbrace{\iint \mathrm{d}x'\mathrm{d}y' U(x',y',0) e^{-ik\left(\frac{x}{z}x' + \frac{y}{z}y'\right)}}_{\text{Fourier Transform of the aperture}}$$



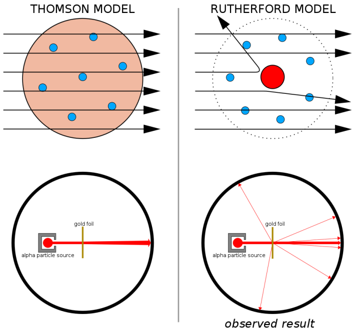
Silicium crystal diffractive pattern

source : UK's national synchrotron

- X-ray wavelength
  $\rightarrow \lambda \simeq$ typical size $\sim 1\mathrm{nm}$
- Bragg's Law
- Diffraction pattern
  $\rightarrow$ Fourier transform of electronic density
- Reminder, for a grating one gets

$$I(\theta) \propto \frac{\sin^2\left(k/2NS\sin\theta\right)}{\sin^2\left(k/2S\sin\theta\right)}$$

- Provide information on the cristal structure



source : Wikimedia Commons

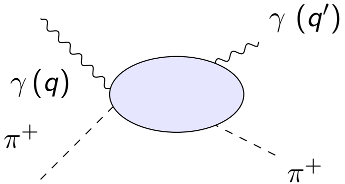
- α particles scattering on a gold foil
- Some of which are scattered at large angles
- Invalidate the Thomson Model (Plum Pudding)
- Allows to develop the Rutherford planetary model

A pattern a study matter

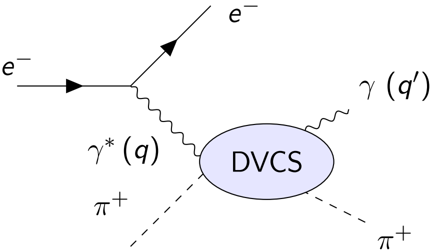
- Scattering without breaking
- Fourier transform relation between matter structure and diffraction figure
- Repeat itself for different orders of magnitude
- Can we extend that to hadron structure?

A pattern a study matter

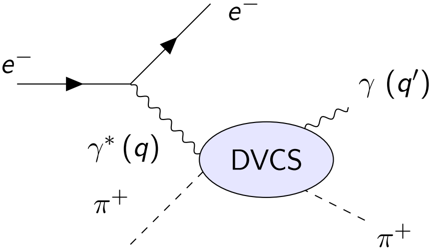
- Scattering without breaking
- Fourier transform relation between matter structure and diffraction figure
- Repeat itself for different orders of magnitude
- Can we extend that to hadron structure?
- Some order of magnitudes:
  - typical nucleon radius 1 fm
  - we thus want a photon wavelength smaller to resolve details within the nucleon
  - Photon minimal energy : $E = hc/\lambda \approx 1.24\mathrm{GeV}$ Highly energetic gamma ray
  - NB : shorter laser wavelength is 0.15 nm.



- No beam of $1 - 10\mathrm{GeV}$ photon



- No beam of $1 - 10\mathrm{GeV}$ photon
- We switch to electroproduction with $Q^2 = -q^2$ much larger than the other scales



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Kinematics:

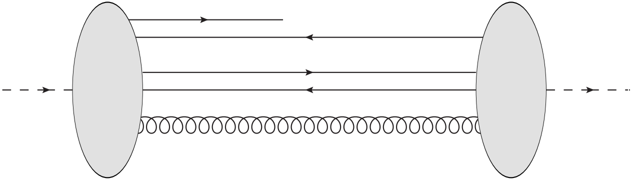
$$\pi^+_{in}\left(p = P - \frac{\Delta}{2}\right), \quad \pi^+_{out}\left(p' = P + \frac{\Delta}{2}\right)$$

$$\Delta^2 = t, \quad P \cdot \Delta = 0, \quad P^2 = M^2 - t/4$$

$$-q^2 = Q^2 >> M^2, t \quad q'^2 = 0$$

Deeply virtual Compton Scattering II

Leading contributions



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- all constituents (quarks and gluons) are moving colinearly

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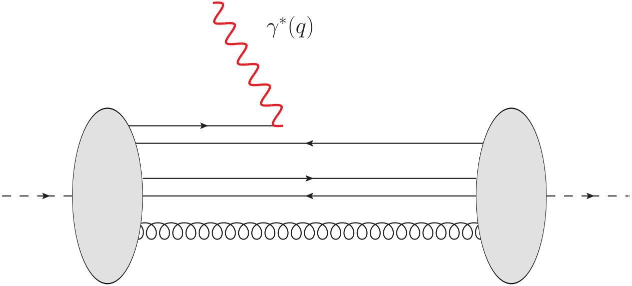

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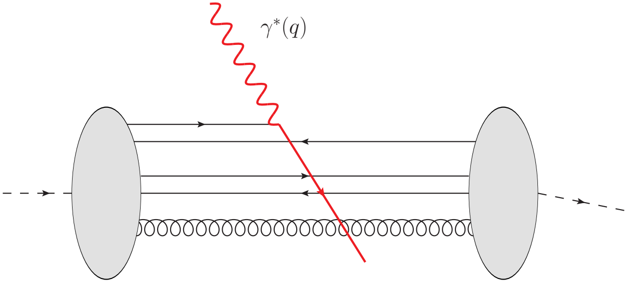

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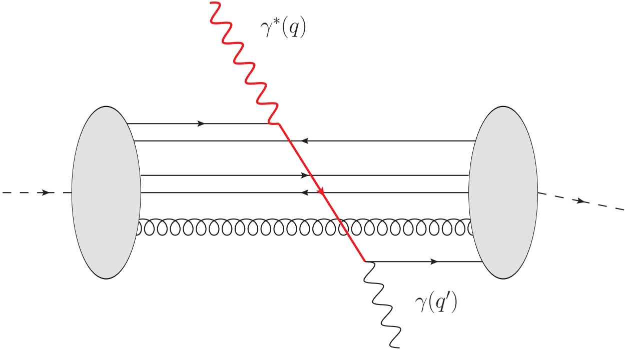

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- all constituents (quarks and gluons) are moving colinearly
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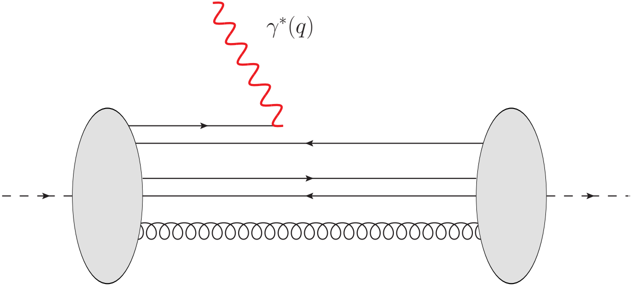
- the quark releases the energy before breaking the hadron

Deeply virtual Compton Scattering II

Subleading contributions



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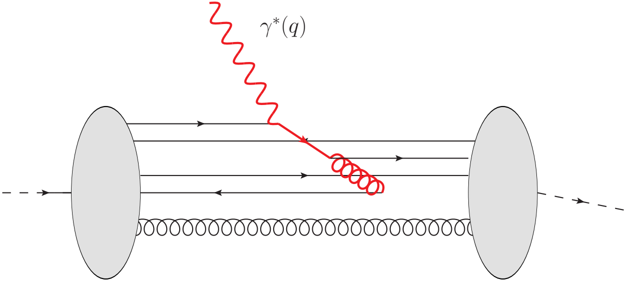

- this time, the quark releases energy through a gluon
- the gluon is absorbed by another quark transfering the energy

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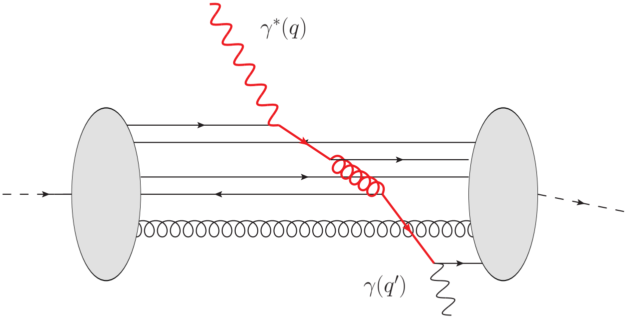

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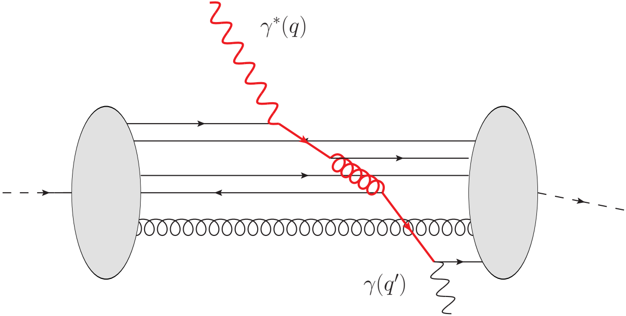

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- the gluon is absorbed by another quark transfering the energy
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Virtuality transfers between partons is power suppressed due to additional denominators (caveat : gauge link)

The amplitude can be seen as

$$\mathcal{A} \propto \epsilon^{\sigma}\gamma_{\sigma}S(k_2'+q')\gamma_{\mu}D^{\mu\nu}(k_1+q-k_1')\gamma_{\nu}S(k_1+q)\gamma_{\lambda}\epsilon^{\lambda}\mathcal{O}$$

where, in the lightcone gauge,

$$D^{\mu\nu}(k_1+q-k_1') = \frac{i\left(\eta^{\mu\nu} - \frac{n^{\mu}(k_1+q-k_1')^{\nu}+n^{\nu}(k_1+q-k_1')^{\mu}}{(k_1+q-k_1')\cdot n}\right)}{(k_1+q-k_1')^2+i\varepsilon}$$

$$= \frac{1}{Q^2}\frac{i\left(\eta^{\mu\nu} - \frac{n^{\mu}(k_1+q-k_1')^{\nu}+n^{\nu}(k_1+q-k_1')^{\mu}}{(k_1+q-k_1')\cdot n}\right)}{-1+\frac{2q\cdot(k_1-k_1')}{Q^2}+\frac{(k_1-k_1')^2}{Q^2}-i\varepsilon}$$

Deep Virtual Compton Scattering II

Virtuality transfer

The amplitude can be seen as

$$\mathcal{A} \propto \epsilon^\sigma \gamma_\sigma S(k_2' + q')\gamma_\mu D^{\mu\nu}(k_1 + q - k_1')\gamma_\nu S(k_1 + q)\gamma_\lambda \epsilon^\lambda \mathcal{O}$$

where, in the lightcone gauge,

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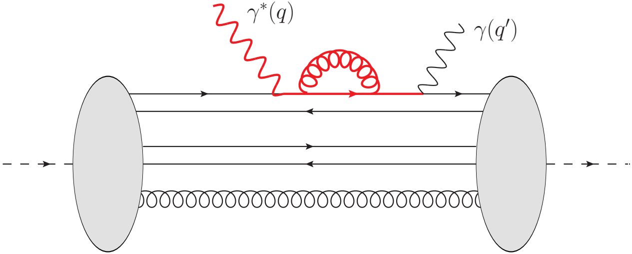
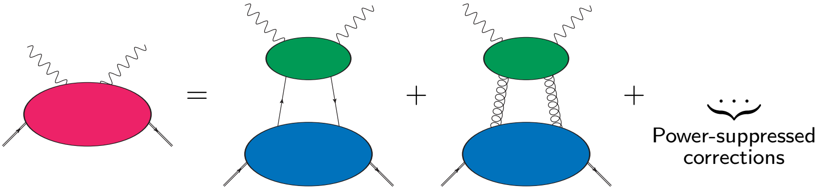
- The gluon propagator introduce a power suppression
- A complete proof requires the computation of the Dirac trace to ensure that no compensations appear at the numerator



- Loops are not power-suppressed, but provides logarithmic corrections.
- Reason : additional propagators are *integrated* over internal momenta.
- These loops are critical : “scaling violation” in DIS.

- When the photon is strongly virtual : $Q^2 = -q^2 >> M^2, t$



- Decomposition of DVCS between perturbative (green) and non-perturbative (blue) subparts.
- Perturbative part $\rightarrow$ description of the interaction between the probe and a parton inside hadron
- Non-perturbative part : description of a parton hadron amplitude called Generalised Partons Distributions (GPDs)
- GPDs is where the information on the hadrons structure lies.

All order proof of DVCS factorisation

The proof that DVCS does factorise can be found in the literature :
Ji, X.-D. and Osborne, J., Phys.Rev., 1998, D58, 094018
Collins, J. C. and Freund, A., Phys.Rev., 1999, D59, 074009

The discussion presented before should be seen as a handwaving argument to build some intuition of what is happening.

Generalised Parton Distributions

Definitions and some properties

We introduce two lightcone vectors n and $\tilde{n}$ such that :

$$n^2 = \tilde{n}^2 = 0 \quad n \cdot \tilde{n} = 1$$
$$n \cdot k = k^+ \quad \tilde{n} \cdot k = k^-$$
$$k = (k^+, k^-, k_\perp) \quad k^2 = 2k^+k^- - k_\perp^2$$

Formal Definition for the pion

$$H_\pi^q(x,\xi,t) = \frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$

$$H_\pi^g(x,\xi,t) = \frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|G^{+\mu}(-\frac{z}{2})G_\mu^+(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
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- $\xi = -\Delta \cdot n/(2P\cdot n)$ is the skewness parameter $\xi \simeq \frac{x_B}{2-x_B}$
- $t = \Delta^2$: the Mandelstam variable

$$H_\pi^q(x,\xi,t) = \frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$

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- $t = \Delta^2$: the Mandelstam variable
- Caveat ! In gauges other than the lightcone one, a Wilson line is necessary to make the GPDs gauge invariant

- In variable x, $H(x)$ is defined for $x \in [-1;1]$
  - Negative x is associated to antiquarks
  - The momentum fraction carried by a parton is smaller than 1 (momentum conservation on the lightcone)

Support and Parity properties

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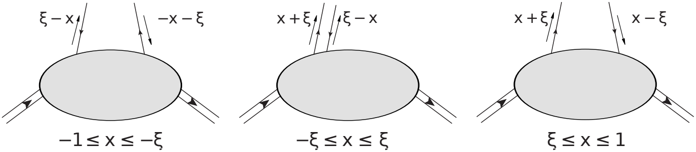
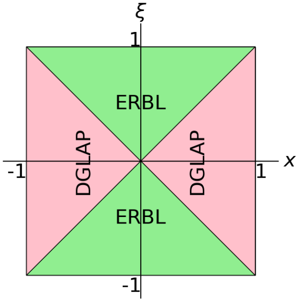
- GPDs can be continued for $|\xi| > 1$ into another type of distributions called Generalised Distribution Amplitudes (GDA).

Kinematical Range

Different values of (x, ξ) yields different lightfront interpretations:





- Modifies our understanding of what is probed
- Different type of contributions
- It determines two big regions
- Relevant for evolution equations
- $|\xi| > 1$ region of Generalised Distribution Amplitudes (GDA)

Connection with the PDF

Coming back to the definition:

$$H_\pi^q(x,\xi,t) = \frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$

$$H_\pi^g(x,\xi,t) = \frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|G^{+\mu}(-\frac{z}{2})G_\mu^+(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$

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When $\Delta \to 0$, then $(\xi = -\Delta\cdot n/(2P\cdot n); t = \Delta^2) \to (0,0)$

$$H_\pi^q(x,0,0) = q(x)\Theta(x) - \bar{q}(-x)\Theta(-x)$$
$$H_\pi^g(x,0,0) = xg(x)\Theta(x) - xg(-x)\Theta(-x)$$

In the limit $(\xi, t) \to (0,0)$, one recovers the PDFs.

Connection with the form factor

Looking at the quark definition:

$$H_\pi^q(x,\xi,t) = \frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$

we would recover the Form Factor if we could make the operator "local".

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we would recover the Form Factor if we could make the operator "local". Simple way to do that → integrate on Fourier conjugate variable:

$$\int \mathrm{d}x\, H_\pi^q(x,\xi,t) = \frac{1}{2}\int \delta(P^+z^-)\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(0)\gamma^+\psi^q(0)|P-\frac{\Delta}{2}\rangle$$

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We recover the pion electromagnetique Form Factor

Unpolarised nucleon GPDs

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$=\frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma^+u+E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u\right].$$

Polarised Nucleon GPDs

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\gamma_5\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
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Unpolarised nucleon GPDs

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- The number of GPDs depends on the hadron spin
- GPDs E and $\tilde{E}$ do not reduce to PDFs when $\Delta \to 0$

Unpolarised nucleon GPDs

$$\frac{1}{P^+}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|G^{+\mu}(-\frac{z}{2})G_\mu^+(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
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$$\frac{-i}{P^+}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|G^{+\mu}(-\frac{z}{2})\widetilde{G}_\mu^+(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$=\frac{1}{2P^+}\left[\tilde{H}^g(x,\xi,t)\bar{u}\gamma^+\gamma_5u+\tilde{E}^g(x,\xi,t)\bar{u}\frac{\gamma_5\Delta^+}{2M}u\right].$$

Gluon Nucleon GPDs

Unpolarised nucleon GPDs

$$\frac{1}{P^+}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|G^{+\mu}(-\frac{z}{2})G^{+}_{\mu}(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
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- Here again, no forward limit known for E^g and $\tilde{E}^g$

Probabilistic Interpretation of GPDs

This section mostly comes from
M. Diehl,Eur.Phys.J.C 25 (2002) 223-232

Localisation: Defining a center

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- under such boosts, the transverse plane obey galilean-like tranformations
- We define a center of longitudinal momentum $B_\perp$:

$$B_\perp = \frac{\sum_i k_i^+ b_\perp^i}{\sum_i k_i^+},$$

where k_i^+ is the longitudinal momentum of parton i and $b_\perp^i$ its position in the transverse plane.

We can define a hadron state with localised center of longitudinal momentum

$$|p^+, B_\perp\rangle = \int \frac{\mathrm{d}^{(2)} p_\perp}{16\pi^3} e^{-ip_\perp B_\perp} |p^+, p_\perp\rangle$$

Now defining the transverse position dependent operator:

$$O_{qq}(z_\perp) = \int \frac{\mathrm{d}z^-}{4\pi} e^{ixP^+z^-} \bar{q}(0, -\frac{z^-}{2}, z_\perp)\gamma^+ q(0, \frac{z^-}{2}, z_\perp),$$

One can show that in coordinate space:

$$\begin{aligned}\mathcal{M} &\propto \langle (p')^+, B'_\perp | O_{qq}(0_\perp) | p^+, B_\perp\rangle \\ &\propto \langle (p')^+, -\frac{\xi b_\perp}{1-\xi} | O_{qq}(b_\perp) | p^+, \frac{\xi b_\perp}{1+\xi}\rangle\end{aligned}$$

Understanding the result

$$\langle (p')^+, -\frac{\xi b_\perp}{1-\xi} | O_{qq}(b_\perp) | p^+, \frac{\xi b_\perp}{1+\xi} \rangle$$

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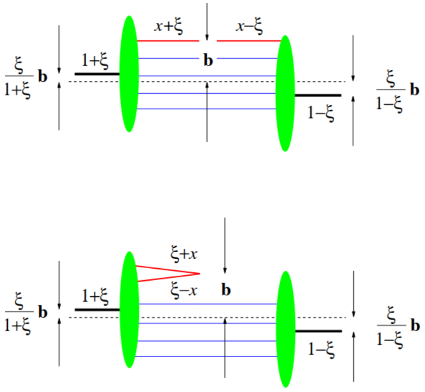
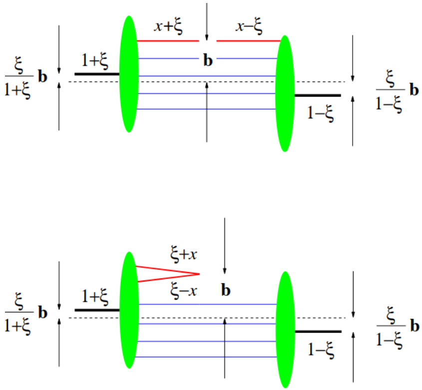


- The quark is now picked and put back in the hadron at position $b_\perp$ in the DGLAP region
- The center of longitudinal momentum is shifted by the skewness
- Thus for non-vanishing skewness, the transverse position of the quark respectively to the center **is** modified

figure from M. Diehl,Eur.Phys.J.C 25 (2002) 223-232

$$\langle (p')^+, -\frac{\xi b_\perp}{1-\xi} | O_{qq}(b_\perp) | p^+, \frac{\xi b_\perp}{1+\xi} \rangle$$



- In the ERBL region, $b_\perp$ is the position of the quark-antiquark pair annihilated

figure from M. Diehl,Eur.Phys.J.C 25 (2002) 223-232

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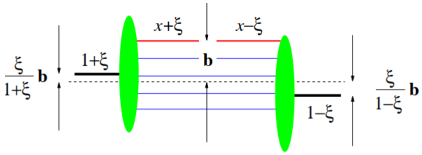

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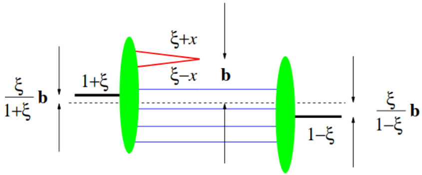

A probabilistic interpretation is recovered only for $\xi = 0$, where there is no ERBL region, and where the center of momentum is stable.

figure from M. Diehl,Eur.Phys.J.C 25 (2002) 223-232

$$\rho(x, b_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp b_\perp} H(x, 0, -\Delta_\perp^2)$$

M. Burkardt, PRD 62 (2000) 071503, PRD 66 (2002) 119903 (erratum)

Computations

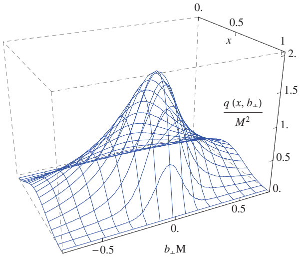

fig. from C. Mezrag *et al.*, PLB 741 (2015) 190-196

Extractions

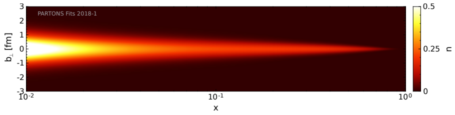

fig. from H. Moutarde *et al.*, EPJ C 78 (2018) 11, 890

Extractions require extrapolations and are model dependent.

$$\rho(x, b_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp b_\perp} H(x, 0, -\Delta_\perp^2)$$

Several important differences can be noticed compared to EFF:

- The Fourier transform of EFF provides the charge density in the transverse plane, here we found a parton number density
- In principle, one can perform flavour decomposition (where are s quark vs. u and d quarks), and obtain gluon number density as well
- One obtains correlations between x and $b_\perp$
- The interpretation picture is valid *on the lightcone*, not in a non-relativistic limit.
- One can define matter radius, interaction radius, valence radius, as second moments of the associated distributions.

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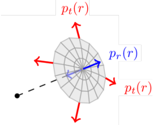
- GPDs are generalisation of the EM Form Factor measured in elastic scattering and of PDFs measured in inclusive processes (DIS).
- Finally, we demonstrated that the Fourier Transform of GPDs yield the 2+1D probability density to find a quark or a gluon with fixed momentum fraction at a given $b_\perp$ position in a hadron.

Connection with the Energy-Momentum Tensor

- In relativistic hydrodynamics → pressure for a anisotropic fluid enters the description of the EMT θ:

$$\theta^{\mu\nu}(r) = (\varepsilon + p_t)\frac{P^\mu P^\nu}{M^2} - p_t\eta^{\mu\nu} + (p_r - p_t)\frac{z^\mu z^\nu}{r^2}$$



Selcuk S. Bayin, Astrophys. J. 303, 101–110 (1986)
figure from C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89

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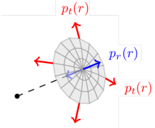

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- On can define isotropic pressure p and pressure anisotropy s:

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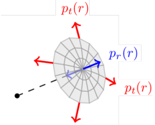

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Question

Can we obtain an analoguous definition within hadron physics?

Hadron EMT in QCD

In QCD, the energy momentum tensor of the nucleon is a correlator of the EMT operator, evaluated between two nucleon states:

$$\langle p', s' | T_{q,g}^{\{\mu\nu\}} | p, s \rangle = \bar{u} \left[P^{\{\mu}\gamma^{\nu\}} A_{q,g}(t;\mu) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{M} C_{q,g}(t;\mu) \right.$$
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- The total EMT is scale independent as it defines a conserved current
- Different definitions exist for the EMT, we stick to the one above
- 4 form factors are needed to parameterise the (symmetric) EMT correlator in the spin-1/2 case
- Constraints exist on some of these form factors:

$$A(0) = 1, \quad B(0) = 0, \quad \bar{C}(t) = 0$$

- Note that there is **no** constraint on C.

The quark sector of the EMT is given as:

$$T_q^{\mu\nu} = \bar{q}\gamma^{\{\mu} i \overleftrightarrow{D}^{\nu\}} q \quad \text{such that} \quad \overleftrightarrow{D}^{\mu} = \frac{1}{2}\left(\overrightarrow{D} - \overleftarrow{D}\right)$$

Working in the lightcone gauge where $D = \partial$ one can readily see that:

$$\frac{1}{2}\int_{-1}^{1} \mathrm{d}x x \int \frac{e^{ixP^+z^-}}{2\pi} \langle P + \frac{\Delta}{2} | \bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle \mathrm{d}z^- |_{z^+=0, z=0} \tag{1}$$

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\end{aligned}$$

Connection to GPDs II

Consequently, EMT Form Factors A, B and C are connected to GPDs H and E through:

$$\int_{-1}^{1} \mathrm{d}x x H^q(x,\xi,t) = A^q(t) + 4\xi^2 C^q(t)$$

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In principle, from GPDs extracted from experimental data, we would be able to get experimental information on these Form Factors.

Ji sum rule

The quark and gluon contributions to the angular momentum J are

$$
\begin{aligned}
2J^q =& A^q(0) + B^q(0) \\
=& \int \mathrm{d}x x \left(H^q(x,\xi,0) + E^q(x,\xi,0)\right) \\
2J^g =& A^g(0) + B^g(0) \\
=& \int \mathrm{d}x \left(H^g(x,\xi,0) + E^g(x,\xi,0)\right)
\end{aligned}
$$

X.D. Ji, Phys.Rev.Lett. 78 (1997) 610-613

And from them, extract pressure and shear forces following:

$$\varepsilon_a(r) = M \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\Delta\cdot r} \left\{ A_a(t) + \bar{C}_a(t) + \frac{t}{4M^2}\left[B_a(t) - 4\,C_a(t)\right]\right\},$$

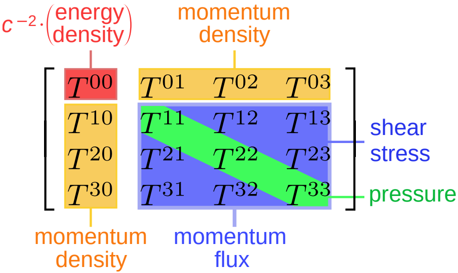
$$p_{r,a}(r) = M \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\Delta\cdot r} \left\{ -\bar{C}_a(t) - \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(t^{3/2}\, C_a(t)\right)\right\},$$

$$p_{t,a}(r) = M \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\Delta\cdot r} \left\{ -\bar{C}_a(t) + \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}}{\mathrm{d}t}\left[t\frac{\mathrm{d}}{\mathrm{d}t}\left(t^{3/2}\, C_a(t)\right)\right]\right\},$$

$$p_a(r) = M \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\Delta\cdot r} \left\{ -\bar{C}_a(t) + \frac{2}{3}\frac{t}{M^2}\, C_a(t)\right\},$$

$$s_a(r) = M \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\Delta\cdot r} \left\{ -\frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(t^{5/2}\, C_a(t)\right)\right\},$$

C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89



How energy, momentum, pressure are shared between quarks and gluons

Caveat: renormalization scheme and scale dependence

C. Lorcé *et al.*, PLB 776 (2018) 38-47,
M. Polyakov and P. Schweitzer, IJMPA 33 (2018) 26, 1830025
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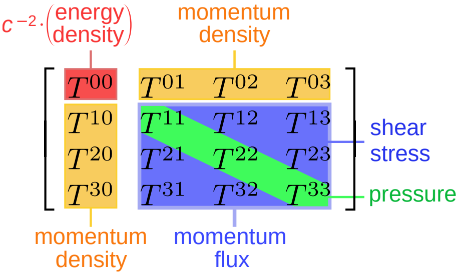

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$$\int_{-1}^{1} \mathrm{d}x\, x\, H_q(x,\xi,t;\mu) = A_q(t;\mu) + 4\xi^2 C_q(t;\mu)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E_q(x,\xi,t;\mu) = B_q(t;\mu) - 4\xi^2 C_q(t;\mu)$$

- Ji sum rule (nucleon)
- Fluid mechanics analogy

X. Ji, PRL 78, 610-613 (1997)
M.V. Polyakov PLB 555, 57-62 (2003)

Place of GPDs in the Hadron physics context

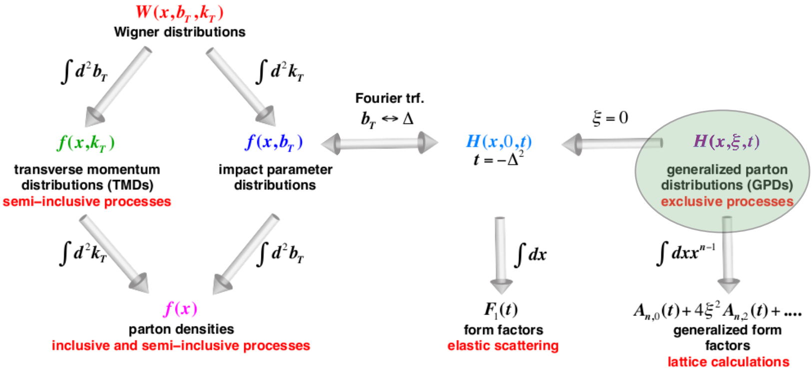

figure from A. Accardi *et al.*, Eur.Phys.J.A 52 (2016) 9, 268

Questions ?