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An introduction to TMD physics

lecture 3

International workshop and school on hadron structure and strong interactions

NJU, Nanjing
October 16, 2025

Plan of these lectures

1. **Breaking hadrons**
2. **Non-collinear partons**
3. **Symmetries & spin**
4. **Factorization, evolution, phenomenology**

3. Symmetries & spin

Gauge symmetry

Hard part
(perturbative)

Hadron structure
(non-perturbative)

Hadronization
(non-perturbative)

The idea of factorization: $$d\sigma = \mathcal{H} \otimes \Phi \otimes \Delta$$

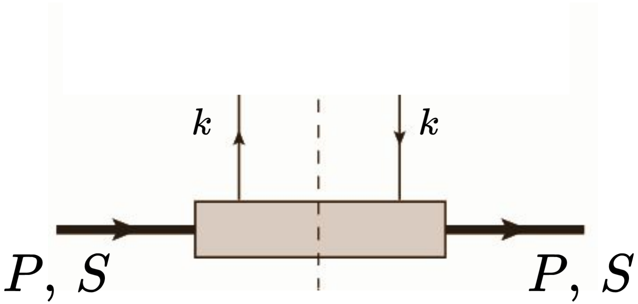
Observable, and thus
gauge invariant

$\Rightarrow$

Decomposition in
gauge invariant terms

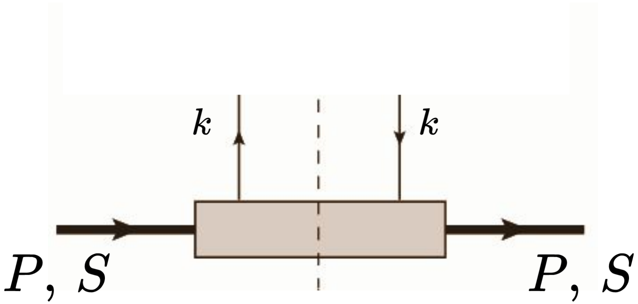
Quark correlator

$$\Phi_{ij}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\cdot\xi}\, \langle PS \big| \,\overline{\psi}_j(0)\, \psi_i(\xi) \big| PS \rangle$$



Quark correlator

$$\Phi_{ij}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\cdot\xi}\, \langle PS \big| \boxed{\overline{\psi}_j(0)\,\psi_i(\xi)} \big| PS\rangle$$



NOT GAUGE INVARIANT!

$$\mathcal{U}(x) = \; e^{i\,\alpha^a(x)\,t^a}$$

$$\overline{\psi}_j(0)\,\psi_i(\xi) \;\rightarrow\; \overline{\psi}_j(0)\,\mathcal{U}^\dagger(0)\,\mathcal{U}(\xi)\,\psi_i(\xi)$$

We need to “correct” the operator to make it gauge invariant

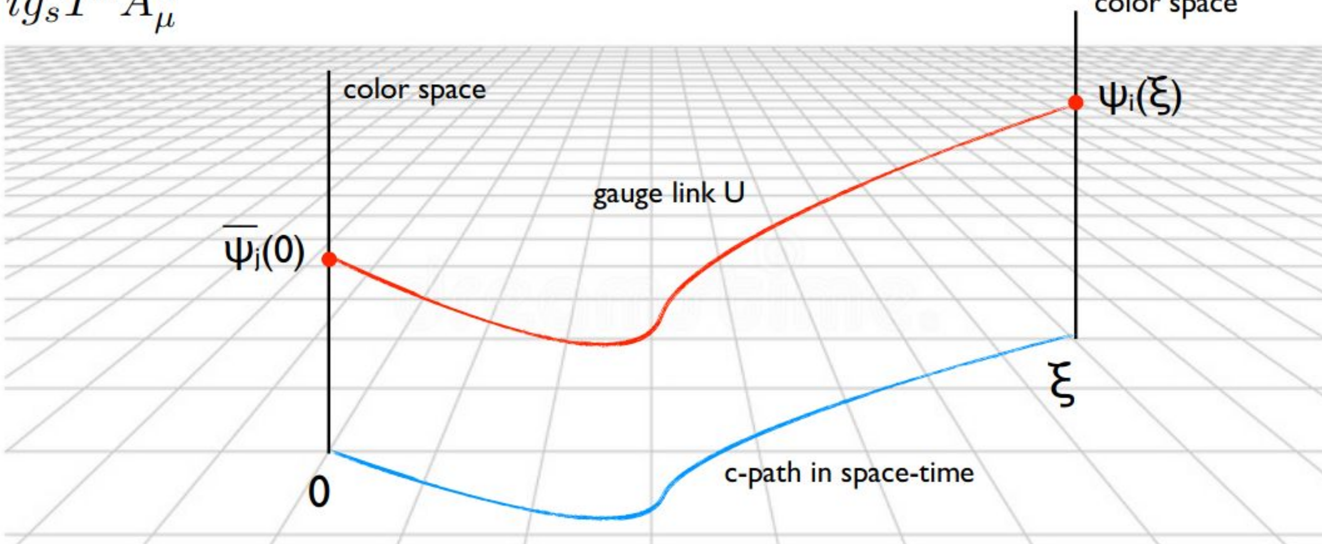
Close the non locality with a “**gauge link**” (or **Wilson line**)

Geometric interpretation

$$D_{\dot{c}}\ \psi(x(t)) = 0\ , \quad t \in I \subset \mathbb{R}$$

$$D_\mu = \partial_\mu - ig_s T^a A^a_\mu$$

"Parallel transport" to close the non-locality



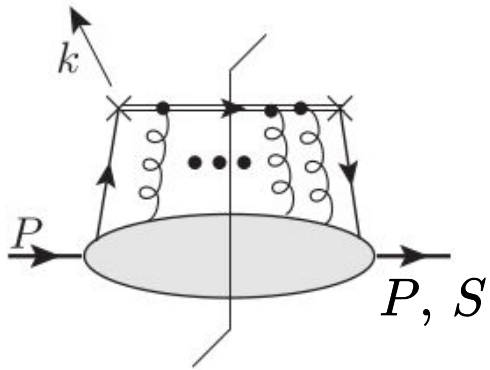
$$\psi'_\beta(x(t)) = \boxed{\mathbb{P}\exp\left\{ -ig\int_0^t ds \frac{dx^\mu}{ds} A^a_\mu(x(s)) T^a_{\beta\alpha} \right\}}\ \psi_\alpha(x(0))$$

$$\doteq \boxed{U_{\beta\alpha}(x(t), x(0))}\ \psi_\alpha(x(0))\ .$$

Gauge link U

Gauge invariant quark correlator

See Collins book



$$\Phi_{ij}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\cdot\xi}\,\langle PS|\boxed{\overline{\psi}_j(0)\,U(0,\xi)\,\psi_i(\xi)}|PS\rangle$$

GAUGE INVARIANT!

$$\mathcal{U}(x) = e^{i\,\alpha^a(x)\,t^a}$$

The Wilson line "bridges" the non-locality and makes the operator gauge invariant

$$U(0,\xi) \;\to\; \mathcal{U}(0)\,U(0,\xi)\,\mathcal{U}^\dagger(\xi)$$

$$\overline{\psi}_j(0)\,U(0,\xi)\,\psi_i(\xi) \;\to\; \overline{\psi}_j(0)\,\mathcal{U}^\dagger(0)\,\mathcal{U}(0)\,U(0,\xi)\,\mathcal{U}^\dagger(\xi)\,\mathcal{U}(\xi)\,\psi_i(\xi) \;=\; \overline{\psi}_j(0)\,U(0,\xi)\,\psi_i(\xi)$$

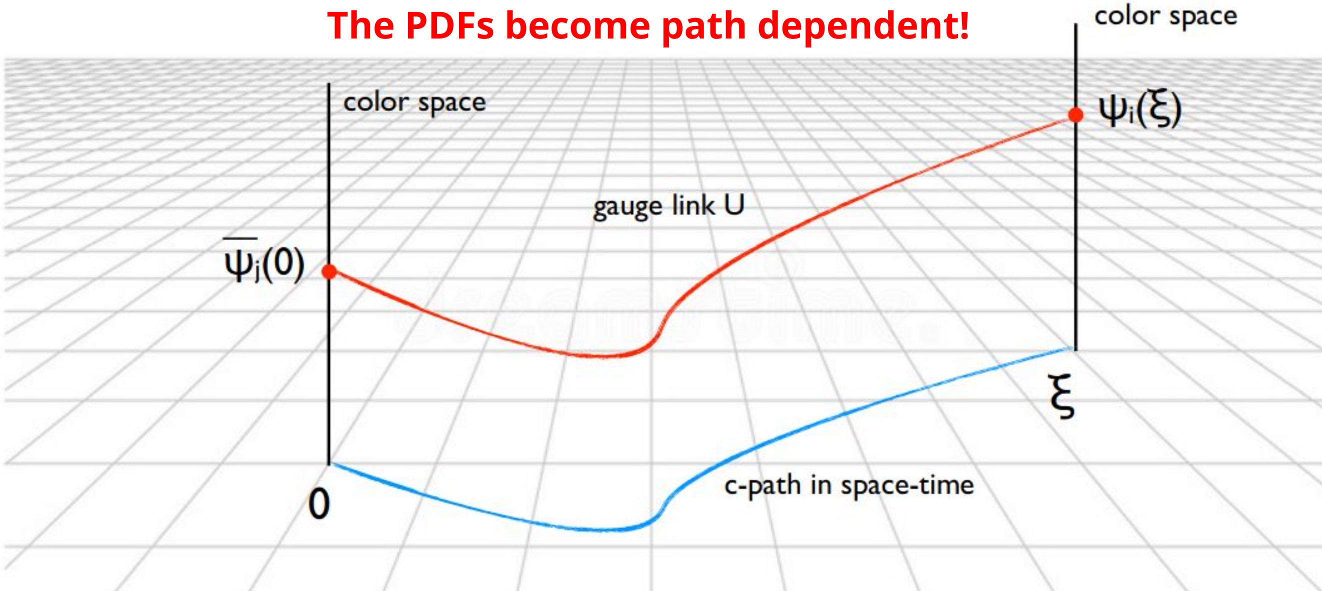
→ Eventually the correlator and the (TMD) PDFs **depend on the gauge link and its path** in spacetime

Generalized universality

Geometric structure

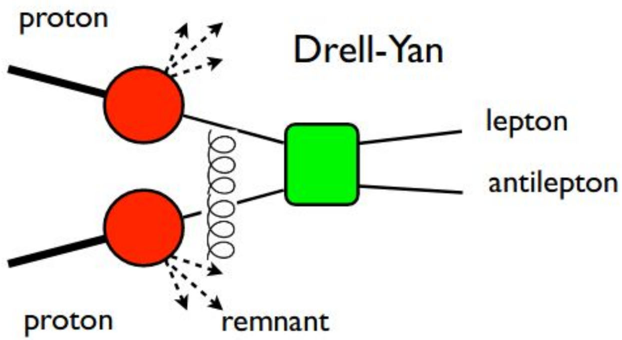
$$\Phi(k,P) = \mathrm{F.T.}\langle P|\overline{\psi_j}(0)\ U\ \psi_i(\xi)|P\rangle \longrightarrow f_1^{a\ [U]}(x,k_T^2)\ \not{P} + \cdots$$

The PDFs become path dependent!



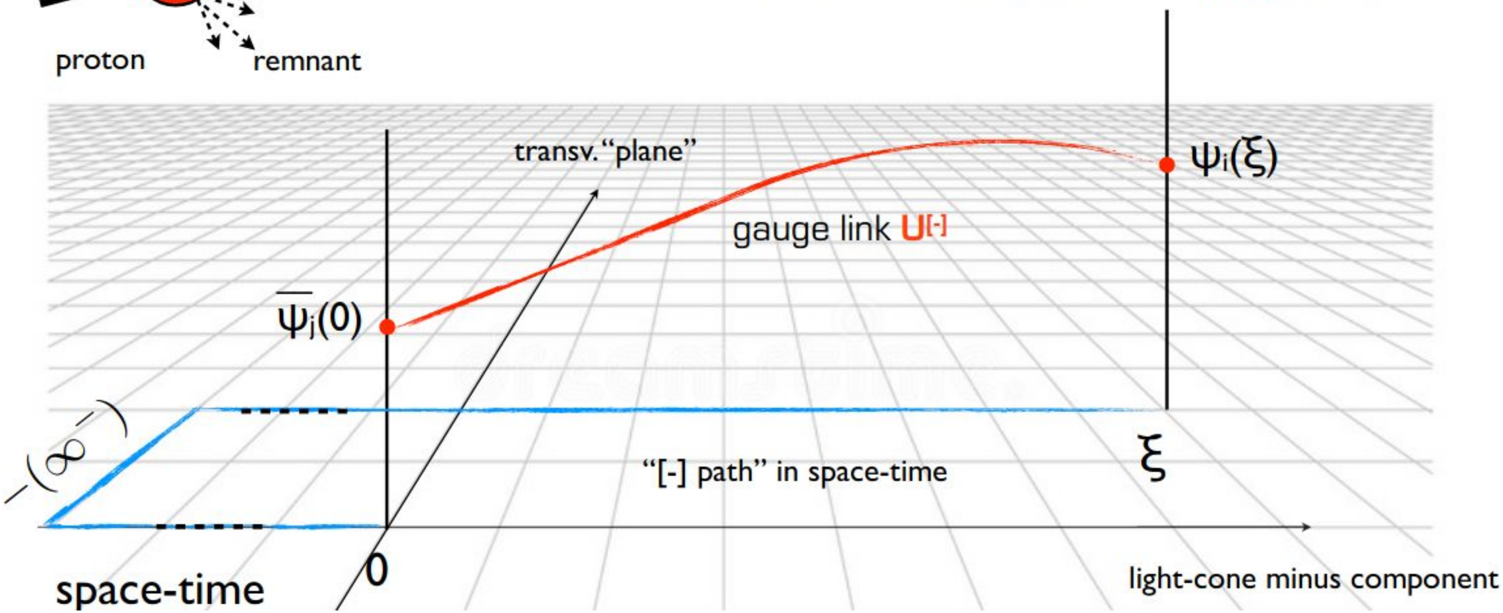
$$U(0,\xi) = \mathcal{P}\ \exp\left\{ -ig_s \int_{c(0,\xi)} ds_\mu A^{\mu a}(s) t^a \right\}$$

Is the path completely arbitrary?

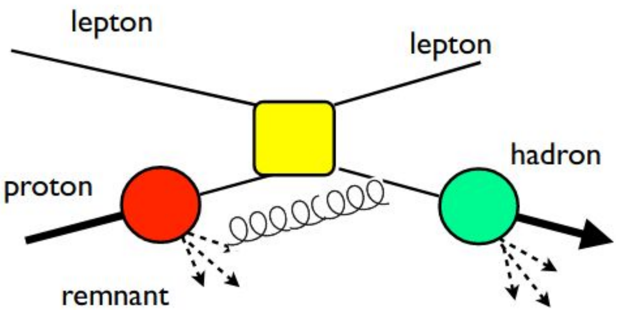


In **Drell-Yan** the **remnant** of the proton feels the color force of a **quark** in the **initial state**

$$\Phi(k, P) = \mathrm{F.T.}\langle P|\overline{\psi_j}(0)\ U^{[-]}(0,\xi)\ \psi_i(\xi)|P\rangle$$

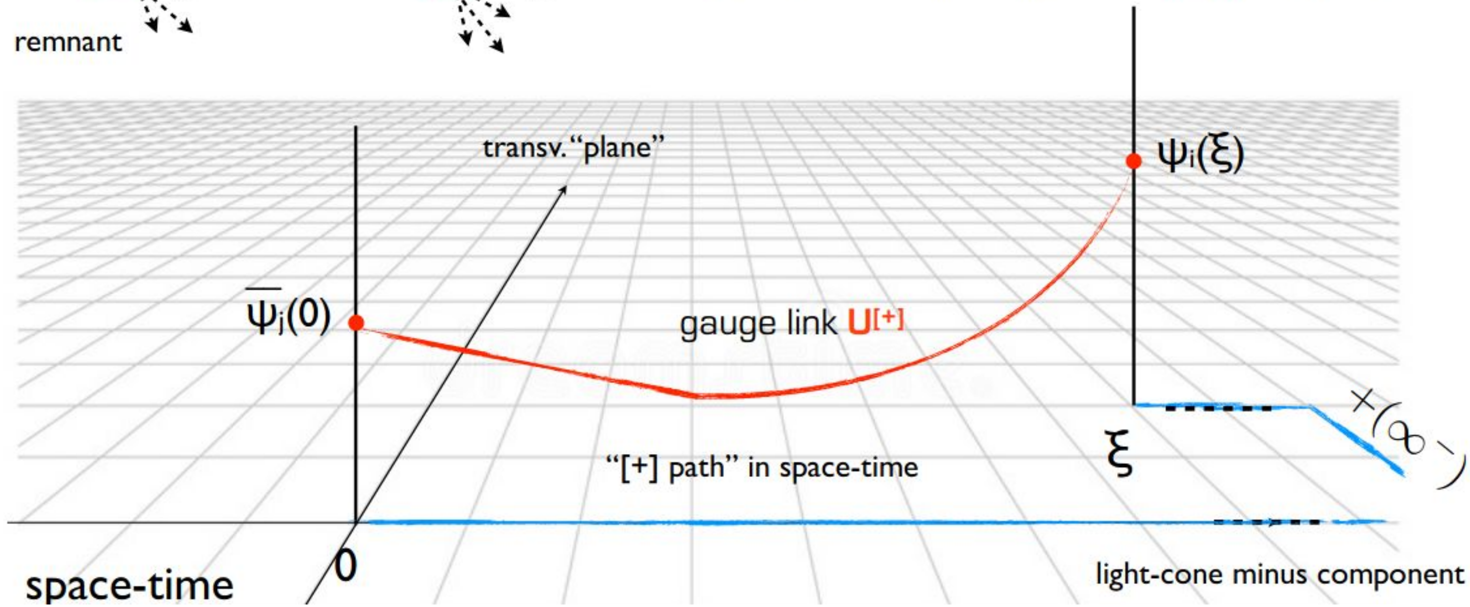


Distributions defined with U^- gauge link: $f_1^{[U^-]}\left(x, k_T^2\right)$



In **SIDIS** the **remnant** of the proton feels the color force of a **quark** in the **final state**

$$\Phi(k,P) = \mathrm{F.T.}\langle P|\overline{\psi_j}(0)\ U^{[+]}(0,\xi)\ \psi_i(\xi)|P\rangle$$

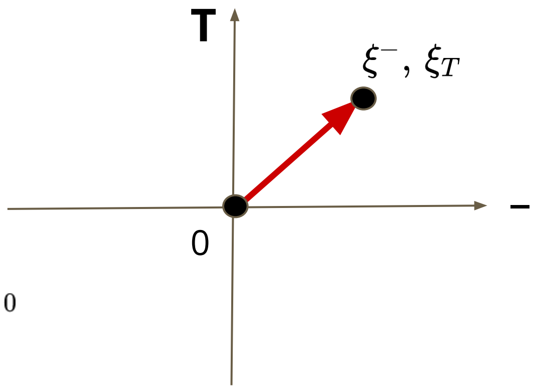


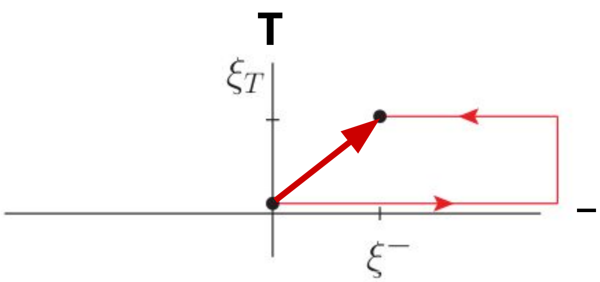
Distributions defined with ***U⁺*** gauge link: $f_1^{[U^+]}(x, k_T^2)$

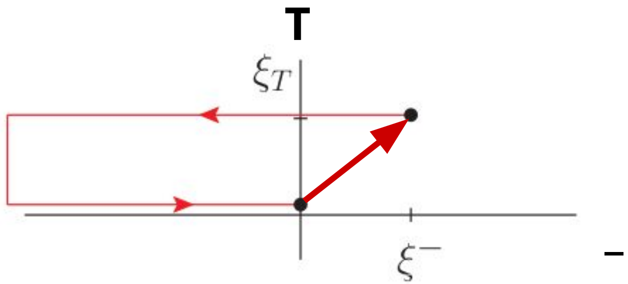
Gauge links for TMD PDFs

$$\Phi_{ij}^{[U]}(x,\, \mathbf{p}_T, S) = \int dp^+\, dp^-\, \delta\big(p^+ - xP^+\big)\, \Phi^{[U]}(p, P, S) =$$

$$= \int \frac{d\xi^-\, d^2\xi_T}{2\pi}\, e^{i\,p\cdot\xi}\, \langle PS \big| \,\overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)\, \big| PS \rangle_{\,\xi^+ = 0}$$







$U^{[+]}$ Future pointing (SIDIS)

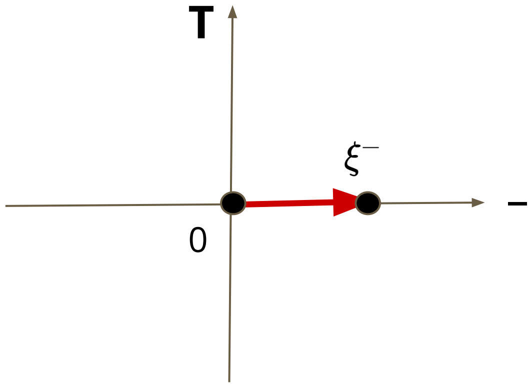
$U^{[-]}$ Past pointing (Drell-Yan)

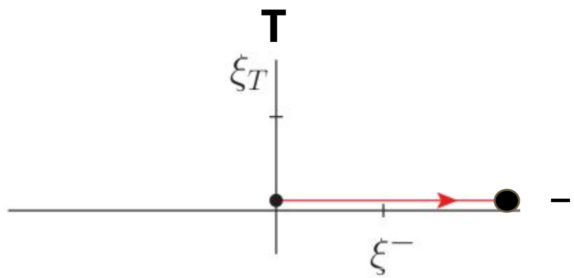
Different distributions: non-universality!

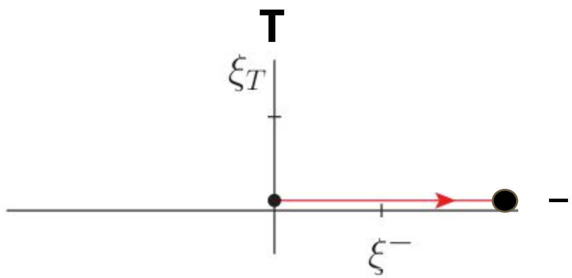
Gauge links for collinear PDFs (simpler)

$$\Phi_{ij}^{[U]}(x,S) = \int dk^+\, dk^-\, d^2\mathbf{k}_T\, \delta\big(k^+ - xP^+\big)\, \Phi^{[U]}(k,P,S) =$$

$$= \int \frac{d\xi^-}{2\pi}\, e^{i\,k\cdot\xi}\, \langle PS \big|\, \overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)\, \big| PS\rangle_{\,\xi^+ = \,\xi_T\, = 0}$$







In the **collinear limit** the two gauge links reduce to the same object:
universality!

Process dependence

The hard process determines the path of the link U,
and the **TMD distributions are process dependent**.

What happens to the concept of *universal* hadron structure?

The unpolarized and the Sivers TMD functions

$$\Phi_{\Gamma}^{[U]}(x,\vec{k}_T) = \frac{1}{2}\,\mathrm{Tr}\left[\Phi^{[U]}(x,\vec{k}_T)\,\Gamma\right]$$

projection $\Gamma = \gamma^+$

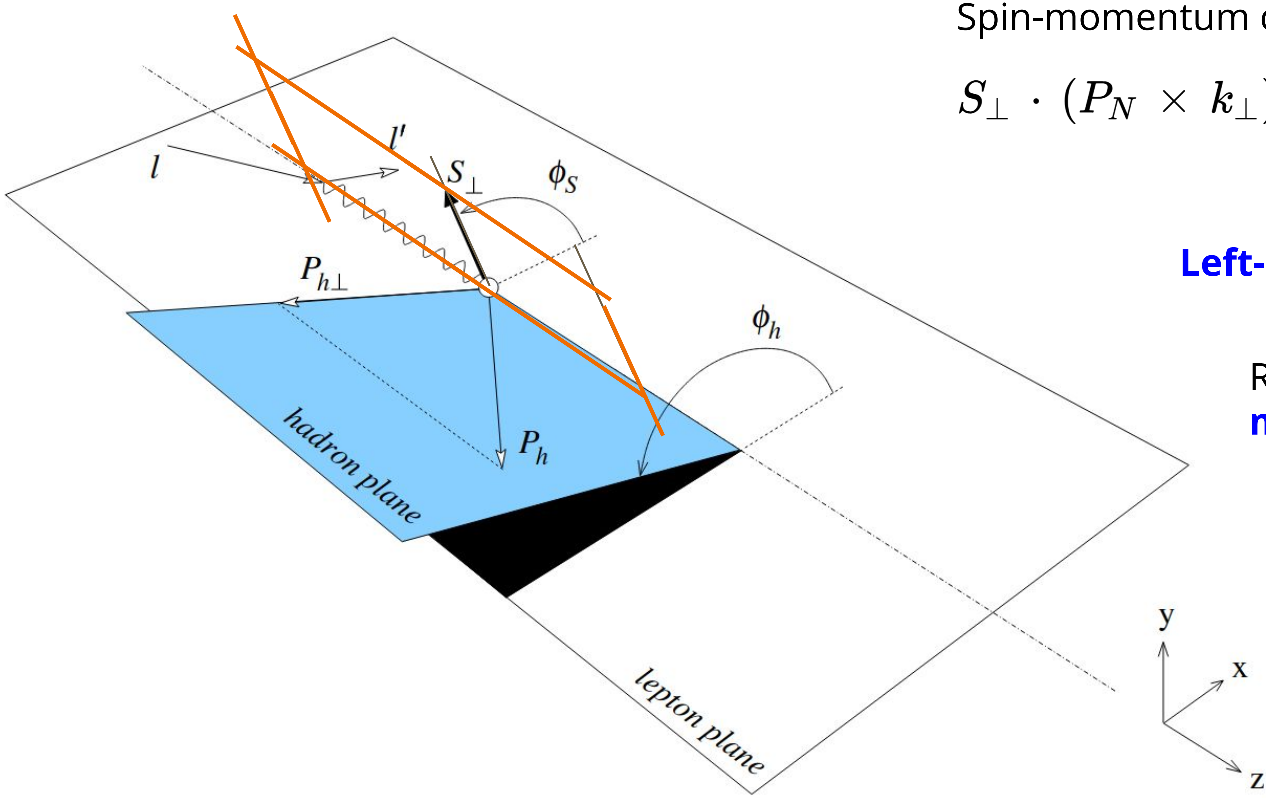
$$\Phi_{\gamma^+}^{[U]}(x,\vec{k}_T) = \frac{1}{2}\,f_1^{[U]}(x,k_T^2)\,\not{n}_+ +$$

Unpolarized TMD PDF

$$\frac{1}{2M}\,\boxed{f_{1T}^{[U]\perp}(x,k_T^2)}\,\underline{\epsilon_T^{\alpha\beta}\,S_{T\alpha}k_{T\beta}}\,\not{n}_+$$

Sivers function (**spin-dependent** term):
correlation between transverse spin and momentum

The Sivers effect



Spin-momentum correlation:

$$S_\perp \cdot (P_N \times k_\perp) f_{1T}^\perp(x, k_\perp^2)$$

Left-right asymmetry

Requires **quark transverse momentum**

Potential **QED** analogue (it's a geometric effect)

Process dependence: it's calculable!

The interplay between **time reversal** and **gauge symmetry** generates **relations** between the two **gauge link** configurations:

$$f_1^{a\ [+]}(x, k_T^2) = f_1^{a\ [-]}(x, k_T^2)$$

T-even distribution

striking consequence of the symmetries of QCD

$$f_{1T}^{a\perp\ [+]}(x, k_T^2) = -f_{1T}^{a\perp\ [-]}(x, k_T^2)$$

T-odd distribution

Sign-change relation for the Sivers function : **not yet confirmed** experimentally

Discrete symmetries: parity

$$a^\mu = \left(a^0,\, \vec{a}\right), \qquad \tilde{a}^\mu = \left(a^0,\, -\vec{a}\right)$$ ← let's consider this definition

$$z^\mu \longrightarrow \tilde{z}^\mu$$

$$P^\mu \longrightarrow \tilde{P}^\mu$$

$$S^\mu \longrightarrow S^\mu \equiv -\tilde{S}^\mu \quad (\text{since } S^\mu = (0, \vec{S}) \text{ by definition})$$

$$n_\pm \longrightarrow n_\mp$$

$$\psi(\xi) \longrightarrow \mathscr{P}\,\psi(\xi)\,\mathscr{P}^\dagger = \Lambda_{\mathscr{P}}\,\psi(\tilde{\xi})\,, \qquad \Lambda_{\mathscr{P}} = \gamma^0$$

$$\gamma^\mu \longrightarrow \mathscr{P}\,\gamma^\mu\,\mathscr{P}^\dagger = \Lambda_{\mathscr{P}}\,\gamma^\mu\,\Lambda^\dagger_{\mathscr{P}}$$

The action on the quark field is the one that leaves the QCD lagrangian invariant under parity transformation (symmetry)

Discrete symmetries: time reversal

$$a^\mu = \left(a^0,\, \vec{a}\right), \qquad \tilde{a}^\mu = \left(a^0,\, -\vec{a}\right)$$ ← let's consider this definition

$$z^\mu \longrightarrow -\tilde{z}^\mu$$

$$P^\mu \longrightarrow \tilde{P}^\mu$$

$$S^\mu \longrightarrow \tilde{S}^\mu$$

$$n_\pm \longrightarrow -n_\mp$$

$$\psi(\xi) \longrightarrow \mathscr{T}\,\psi(\xi)\mathscr{T}^\dagger = \Lambda_{\mathscr{T}}\,\psi(-\tilde{\xi})\,, \qquad \Lambda_{\mathscr{T}} = -i\gamma_5 C = i\gamma^1\gamma^3$$

$$\gamma^\mu \longrightarrow \mathscr{T}\,\gamma^\mu\,\mathscr{T}^\dagger = \Lambda_{\mathscr{T}}\,\gamma^\mu\,\Lambda^\dagger_{\mathscr{T}} = \gamma^*_\mu$$

The action on the quark field is the one that leaves the QCD lagrangian invariant under time reversal transformation (symmetry)

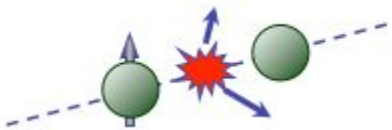
Implications of discrete symmetries

$$U_\pm(a,b)^\dagger = U_\pm(b,a)$$
$$\mathscr{P}U_\pm(a,b)\mathscr{P}^\dagger = U_\pm(\tilde{a},\tilde{b})$$
$$\mathscr{T}U_\pm(a,b)\mathscr{T}^\dagger = U_\mp(-\tilde{a},-\tilde{b})$$

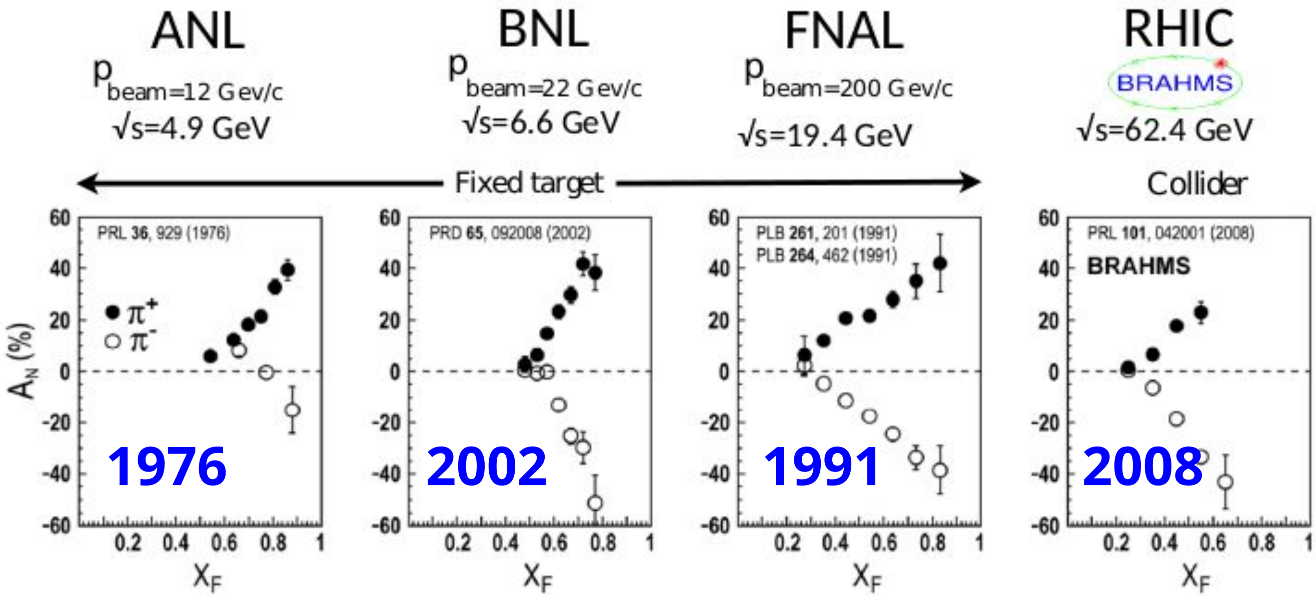
$$\begin{aligned}
\text{Hermiticity:} \qquad & \Phi^{[\pm]\dagger}(k;P,S) = \gamma^0\Phi^{[\pm]}(k;P,S)\gamma^0 \\
\text{Parity:} \qquad & \Phi^{[\pm]}(k;P,S) = \gamma^0\Phi^{[\pm]}(\tilde{k};\tilde{P},-\tilde{S})\gamma^0 \\
\text{Time reversal:} \qquad & \Phi^{[\pm]*}(k;P,S) = i\gamma^1\gamma^3\Phi^{[\mp]}(\tilde{k};\tilde{P},\tilde{S})i\gamma^1\gamma^3
\end{aligned}$$

Is it just a theoretical game?

Single-spin asymmetries



$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$



$$x_F = 2p_l/\sqrt{s}$$

$$A_N^{\mathrm{pQCD}} \sim \alpha_s\, m_q/\sqrt{s} \ll A_N^{\mathrm{exp}}$$

Need different mechanism: non-perturbative!
→ Sivers effect

See also https://inspirehep.net/literature/1410100 (review for asymmetries in pp collisions)

A brief history of the Sivers function

1990 D. Sivers introduces the “**Sivers mechanism**” to explain SSAs: https://inspirehep.net/literature/277567

1993 J. Collins argues that the Sivers effect is **incompatible with time-reversal symmetry**: https://inspirehep.net/literature/337381 (the Sivers function MUST be zero)

2002 Brodsky, Hwang, Schmidt show that in a spectator model of QCD the **final state interactions** (FSI) can generate a Sivers effect: https://inspirehep.net/literature/582417

2002 Collins amends his previous argument introducing the **Wilson lines** (FSI) and show the process dependence of the Sivers function: https://inspirehep.net/literature/584849

Sivers asymmetry: SIDIS and Drell-Yan

$$A_{UT}^{\sin(\phi_h-\phi_S)} \sim \quad F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\underline{f_{1T}^{\perp}D_1}\right],$$

$$\ell N^{\uparrow} \to \ell h X$$

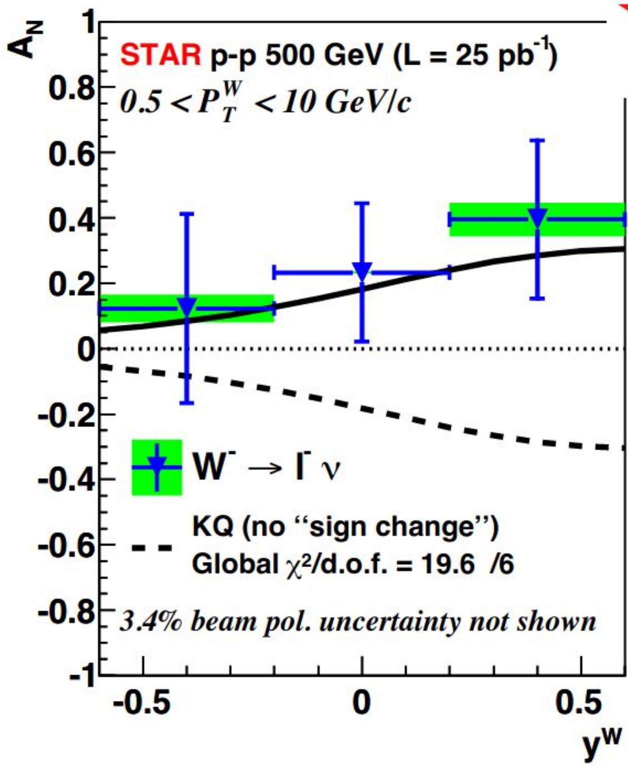
$$\frac{\sigma^{\uparrow}-\sigma^{\downarrow}}{\sigma^{\uparrow}+\sigma^{\downarrow}}$$

Do we need a sign change in the Sivers function between the two processes to describe **simultaneously** these asymmetries?

$$A_{UT} \sim \quad F_{UT}^{1} = \mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b}\underline{f_1\bar{f}_{1T}^{\perp}}\right],$$

$$pN^{\uparrow} \to \ell^{+}\ell^{-}X$$

Sign-change for Sivers function

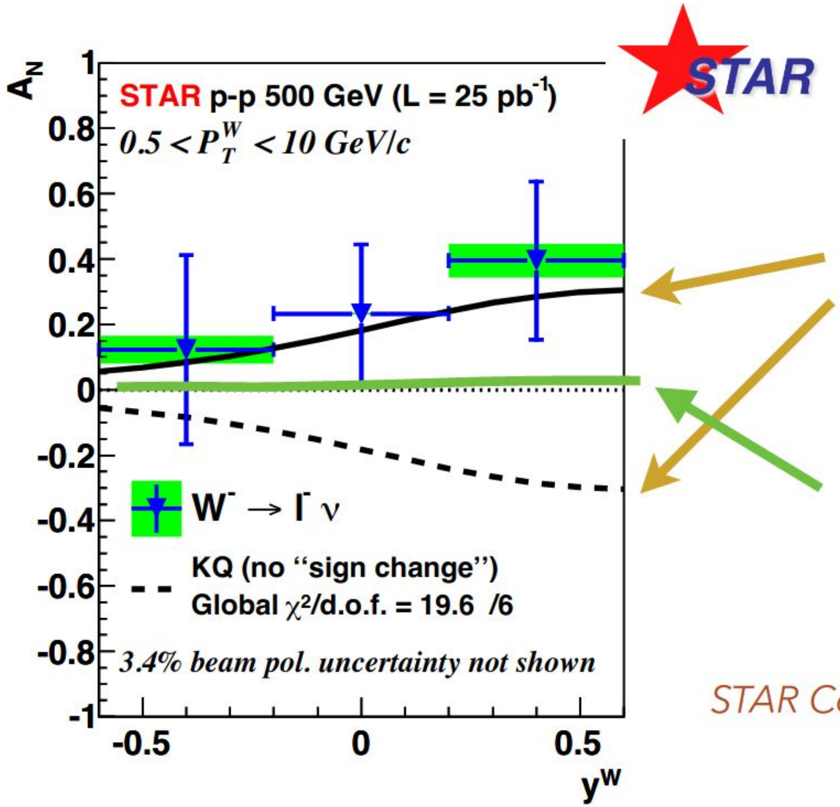


first evidence of sign change?

STAR Collab. arXiv:1511.06003

See also https://inspirehep.net/literature/1410100 (review for asymmetries in pp collisions)

Sign-change for Sivers function



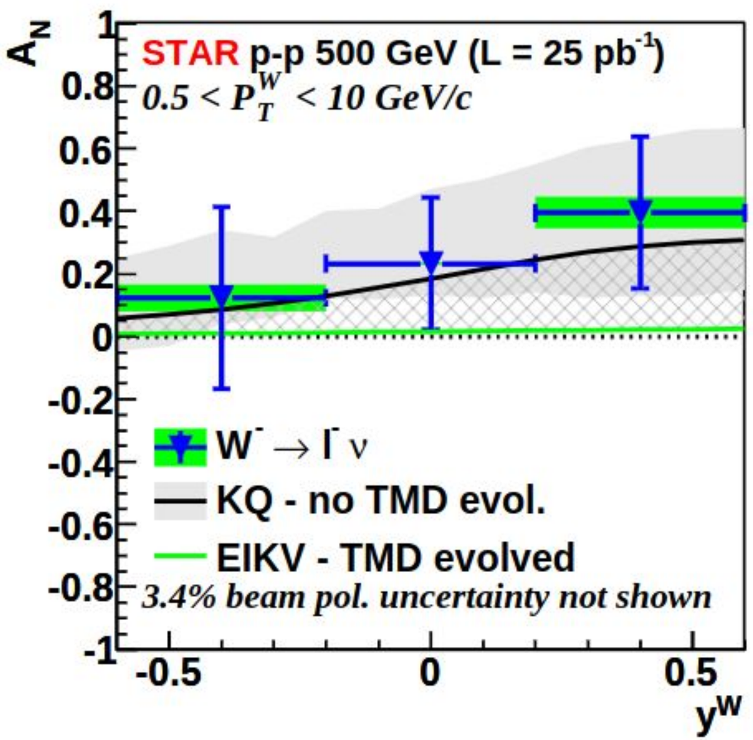
first evidence of sign change?

prediction with TMD evolution equations

STAR Collab. arXiv:1511.06003

See also https://inspirehep.net/literature/1410100 (review for asymmetries in pp collisions)

Sign-change for Sivers function



Uncertainties are still very large both in the theory and in the measurements

See also https://inspirehep.net/literature/1410100 (review for asymmetries in pp collisions)

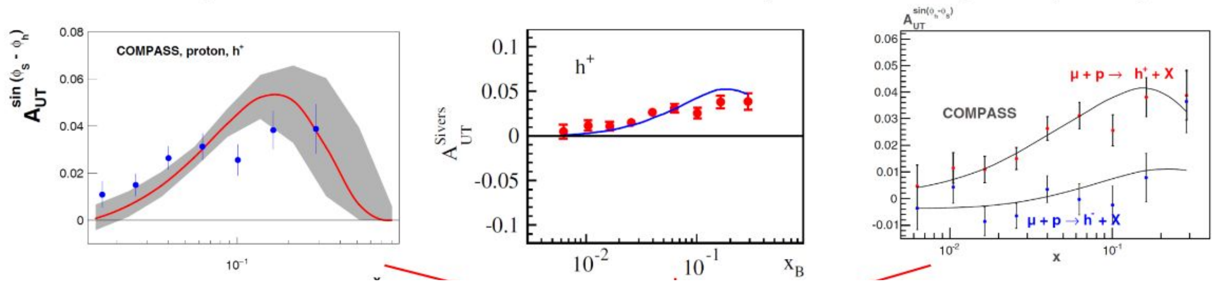
Sign-change for Sivers function

Sivers asymmetry in Semi-Inclusive DIS

DGLAP (2016) M. Anselmino et al., **arXiv:1612.06413**

TMD-1 (2014) M. G. Echevarria et al. **PRD89,074013**

TMD-2 (2013) P. Sun, F. Yuan, **PRD88, 114012**



Sign-change for Sivers function

Sivers asymmetry in Semi-Inclusive DIS

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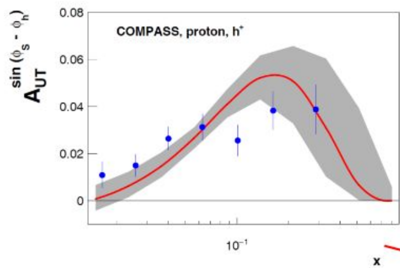

TMD-1 (2014)
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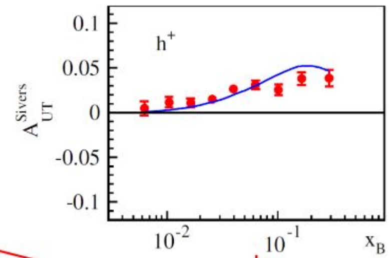
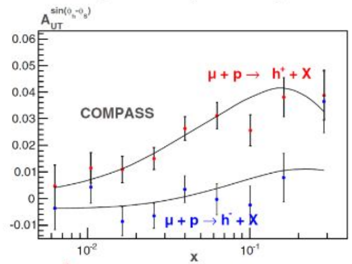
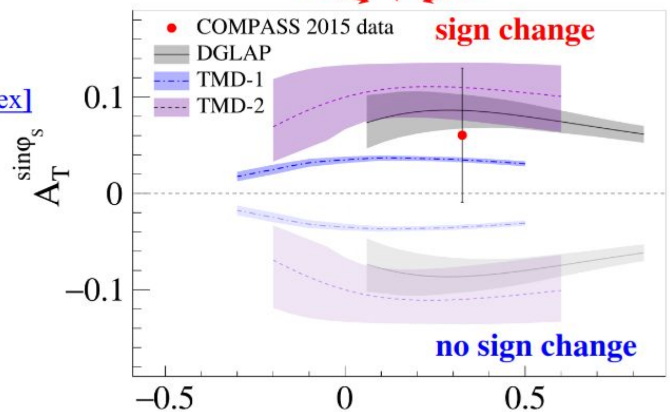


TMD-2 (2013)
P. Sun, F. Yuan, **PRD88, 114012**



New! 03 April 2017
COMPASS
[CERN-EP-2017-059]
[arXiv:1704.00488[hep-ex]]

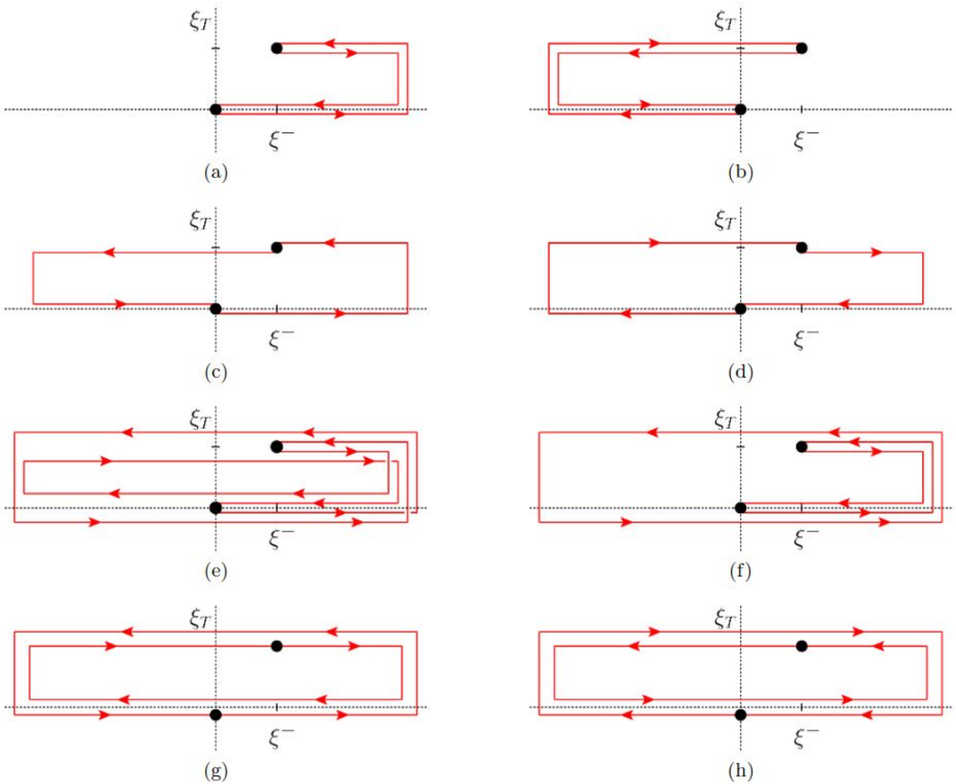


Sivers asymmetry in Drell-Yan

courtesy B. Parsamyan

Gauge links for gluon TMDs (more complicated)

$F^{\mu\nu}(0)\, U(0,\xi)\, F^{\rho\sigma}(\xi)\, U'(\xi,\, 0)$ ← more complicated operator with two gauge links



The process dependence for these TMDs amounts to more complicated relations than a minus sign (but still calculable!)

For more details see https://inspirehep.net/literature/1391461