Theory and phenomenology of Generalised Partons Distributions

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Reminder of lecture 1 and 2

- We have introduce GPDs as a way to encode the non-perturbative information contained in DVCS (at leading power)
- We have studied their properties and interpreted them as probability densities on the lightcone
- We have seen that they are connected to the EMT through their moments
- We have realised that the properties of QCD provide theoretical constraints on GPDs
  - Polynomiality
  - Positivity
- We had no time to show that positivity and polynomiality can be fulfilled together, but this is indeed the case.

Evolution properties of GPDs

- Coming back to our matrix element for arbirtary z:

$$\langle \pi, P + \frac{\Delta}{2} | \bar{\psi}\left(-\frac{z}{2}\right)\gamma^{+}\mathcal{W}\left(-\frac{z}{2};\frac{z}{2}\right)\psi\left(\frac{z}{2}\right)|\pi, P - \frac{\Delta}{2}\rangle$$

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Two approaches

- Renormalisation of local operators
- Renormalisation using "in partons" matrix elements

Operator Product Expansion

- The idea is to "Taylor expand" an operator:

$$\bar{\psi}\left(-\frac{z}{2}\right)\gamma^{+}\psi\left(\frac{z}{2}\right) = \sum_{N}^{\infty} c_N(z)\mathcal{O}^N(0)$$

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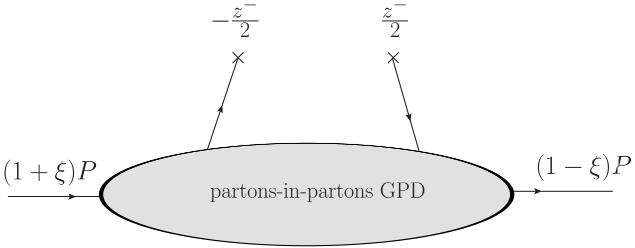
- But it requires to "resum" the renormalised local operators afterward: we saw already when talking about polynomiality that these operators are given by Mellin moment of GPDs $\rightarrow$ solve the inverse moment problem
- Caveat: operator mixing !

- Instead of moments, one can consider partons-in-partons GPDs



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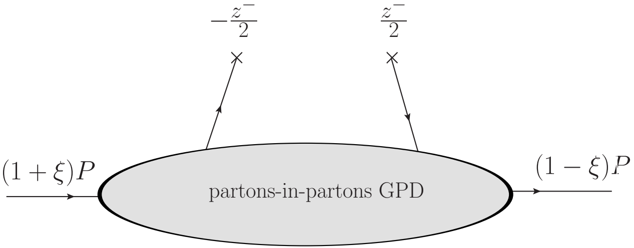

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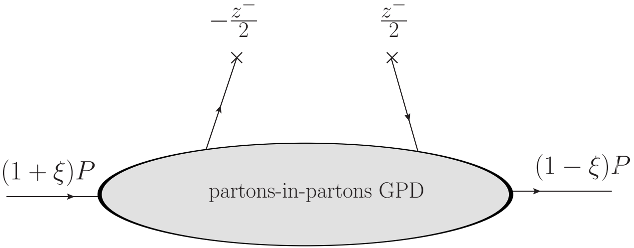

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For that purpose, $\overline{\mathrm{MS}}$ is well suited
GPDs (3D structure, pressure) become *scheme dependent* !

First order computation

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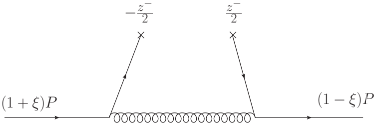

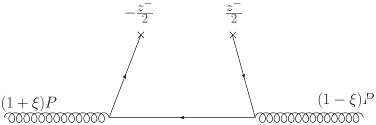

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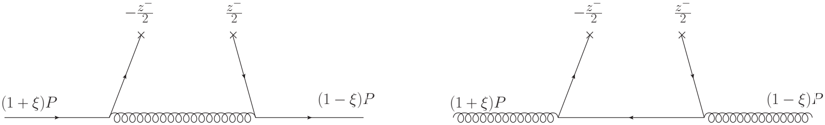

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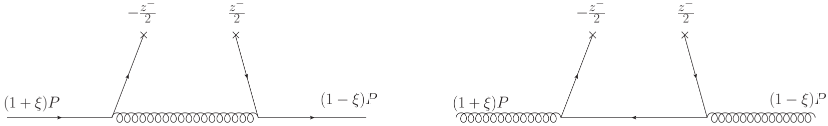

- Why are these graphs diverging, while there is no closed loop ?
- Because only k^+ is constraint by momentum conservation, k^- and $k_\perp$ are integrating out

Sketching the computation

- Write down the amplitude in terms of Fourier transform

$$\frac{g^2}{16\pi^2}F_{q\leftarrow q}(x,\xi) = \frac{\sqrt{1-\xi^2}}{2N_c}\int\frac{\mathrm{d}z^-}{2\pi}e^{i(1-x)z^-p^+}\int\frac{\mathrm{d}^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}}e^{-ik^+z^-}i\delta_{ab}D^{\mu\nu}(k)$$
$$\times g^2\mu^{2\epsilon}\mathrm{Tr}\left[t_a\gamma_\mu S((1+\xi)p^+-k)\gamma^+S(k-(1-\xi)p^+\gamma_\nu t_b \not{p}\right]$$

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Final result

$$H^i(x,\xi,t,\mu) = \int_{-1}^{1}\frac{\mathrm{d}y}{|y|}Z_{i,j}\left(\frac{x}{y},\frac{\xi}{x},\alpha_s(\mu),\epsilon\right)H^j_{reg}(y,\xi,t,\epsilon)$$

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Renormalisation Group

- Knowing the GPD at a scale μ we want to know how it behaves at $\mu + \mathrm{d}\mu$
- we describe perturbatively the impact of this $\mathrm{d}\mu$ leap

$$H(x, \xi, t, \mu + \mathrm{d}\mu) - H(x, \xi, t, \mu)$$

- we obtain like this a first-order integro-differential equation
- α_S becomes "exponentiated"

$$\mathcal{P}^{ij}\left(\frac{x}{z};\frac{\xi}{x};\alpha_s\right) = \lim_{\epsilon\to 0}\sum_{j=q,g}\int_{-1}^{1}\frac{\mathrm{d}y}{|y|}\frac{\mathrm{d}Z_{ij}\left(\frac{x}{y};\frac{\xi}{x};\alpha_s;\epsilon\right)}{\mathrm{d}\ln\mu^2}Z_{ij}^{-1}\left(\frac{y}{z};\frac{\xi}{y};\alpha_s;\epsilon\right)$$

Evolution equations for GPDs

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Non-Singlet Case

$$\frac{\mathrm{d}H^q_{NS}(x,\xi,t,\mu)}{\mathrm{d}\ln(\mu)}=\frac{\alpha_s(\mu)}{4\pi}\int_0^1\frac{\mathrm{d}y}{y}\mathcal{P}^0_{q\leftarrow q}\left(\frac{x}{y},\frac{\xi}{x}\right)H^q_{NS}(y,\xi,t,\mu)$$

Singlet Case

$$\begin{pmatrix}\frac{\mathrm{d}H^q_S(x,\xi,t,\mu)}{\mathrm{d}\ln(\mu)}\\ \frac{\mathrm{d}H^g(x,\xi,t,\mu)}{\mathrm{d}\ln(\mu)}\end{pmatrix}=\frac{\alpha_s(\mu)}{4\pi}\int_0^1\frac{\mathrm{d}y}{y}\begin{pmatrix}\mathcal{P}^0_{q\leftarrow q}\left(\frac{x}{y},\frac{\xi}{x}\right) & \mathcal{P}^0_{q\leftarrow g}\left(\frac{x}{y},\frac{\xi}{x}\right)\\ \mathcal{P}^0_{g\leftarrow q}\left(\frac{x}{y},\frac{\xi}{x}\right) & \mathcal{P}^0_{g\leftarrow g}\left(\frac{x}{y},\frac{\xi}{x}\right)\end{pmatrix}\begin{pmatrix}H^q_S(y,\xi,t,\mu)\\ H^g(y,\xi,t,\mu)\end{pmatrix}$$

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The $\mathcal{P}$ distributions can in principle be computed in pQCD

DGLAP connection

- Splitting function have been computed at:
  - LO (α_s)

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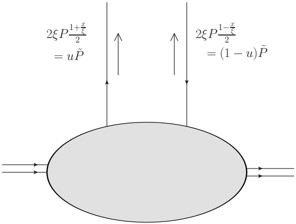
$$\lim_{\xi \to 0} \mathcal{P}\left(\frac{x}{y}, \frac{\xi}{x}\right) = P_{DGLAP}\left(\frac{x}{y}\right)$$

ERBL connection



- For $|x| \leq |\xi|$, a pair of quark-antiquark propagates along the lighcone in the t-channel sharing a fraction u of $q\bar{q}$ system momentum along the lightcone
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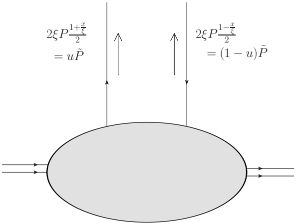

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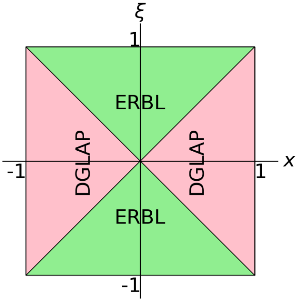
- For $|\xi| = 1$, this interpretation holds for the entire x-range
- We recover there, the so-called ERBL evolution equations

$$\lim_{\xi \to 1} \mathcal{P}\left(\frac{x}{y}, \frac{\xi}{x}\right) = P_{\mathrm{ERBL}}(x, y)$$



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Moments analysis I

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- Because they are defined with the *same* operator !

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- Same operator $\rightarrow$ same OPE $\rightarrow$ same renormalisation of local operators $\rightarrow$ same anomalous dimensions:

$$\gamma_n = 2C_F\left[-\frac{1}{2} + \frac{1}{(n+1)(n+2)} - 2\sum_{k=2}^{n+1}\frac{1}{k}\right]$$

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- Yet, evolution equations are written for *matrix elements*, not only operators.

  $\to$ therefore evolution equations *are* different !

Moments analysis II

Conformal Moments

- The ERBL LO kernel is diagonalised by the 3/2-Gegenbauer polynomials (non-singlet):

$$\int \mathrm{d}u V_{NS}(v,u) C_n^{\frac{3}{2}}(2u-1) \propto \gamma_n C_n(2v-1)$$

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  $\to$ we expect the $C_n^{3/2}(x/\xi)$ to play an important role w.r.t. the evolution kernel
- However we need them to be finite in the forward limit$\to$ rescaling $C_n^{3/2}(x/\xi) \to \xi^n C_n^{3/2}(x/\xi)$ so that $\lim_{\xi\to 0} \xi^n C_n^{3/2}(x/\xi) = x^n$

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$$\int \mathrm{d}u V_{NS}(v,u) C_n^{\frac{3}{2}}(2u-1) \propto \gamma_n C_n(2v-1)$$

- Remember, for GPD $u = \frac{1+\frac{x}{\xi}}{2} \to 2u - 1 = \frac{x}{\xi}$

  $\to$ we expect the $C_n^{3/2}(x/\xi)$ to play an important role w.r.t. the evolution kernel
- However we need them to be finite in the forward limit$\to$ rescaling $C_n^{3/2}(x/\xi) \to \xi^n C_n^{3/2}(x/\xi)$ so that $\lim_{\xi\to 0} \xi^n C_n^{3/2}(x/\xi) = x^n$
- In addition, in the forward limit, the Mellin moment do not mix:

$$\frac{\mathrm{d}}{\mathrm{d}\ln(\mu)}\left[\int \mathrm{d}x\, x^n q(x,\mu)\right] = \frac{\alpha_s(\mu)}{2\pi}\gamma_n \int \mathrm{d}x\, x^n q(x,\mu)$$

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GPD Conformal moments $\int \xi^n C_n^{3/2}(\frac{x}{\xi}) H(x,\xi)$
do not mix under LO evolution !

Other properties

- Charge conservation: $\gamma_0 = 0$
- Energy-Momentum Conservation: $\int \mathrm{d}x x(q(x) + g(x))$ is independent of μ
- Continuity at the crossover lines $|x| = |\xi|$

Solving evolution equations

Evolution in conformal space

- Conformal moments do not mix at LO $\rightarrow$ easy evolution

$$\xi^n \int_{-1}^{1} \mathrm{d}x C_n^{3/2}\left(\frac{x}{\xi}\right) H(x,\xi,\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{\gamma_n}{\beta_0}} \xi^n \int_{-1}^{1} \mathrm{d}x C_n^{3/2}\left(\frac{x}{\xi}\right) H(x,\xi,\mu_0)$$

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  $\rightarrow$ requires analytic continuation in the complex plane
  $\rightarrow$ solution is not unique

D. Mueller and A. Schafer, Nucl.Phys.B739 1-59, 2006

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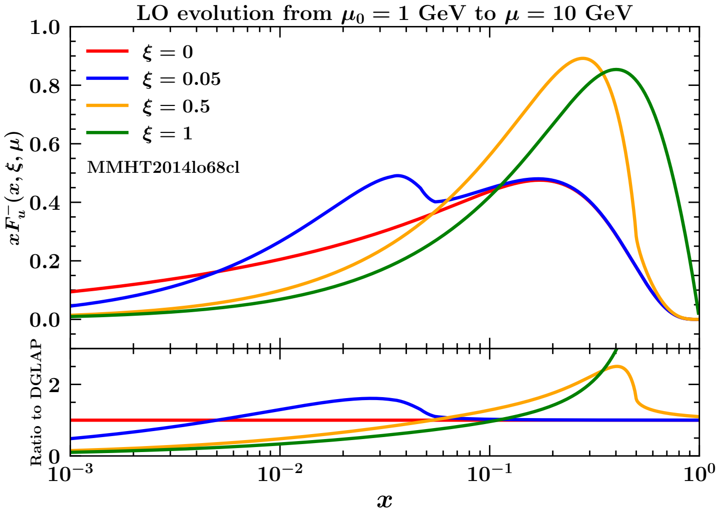
Evolution in x-space

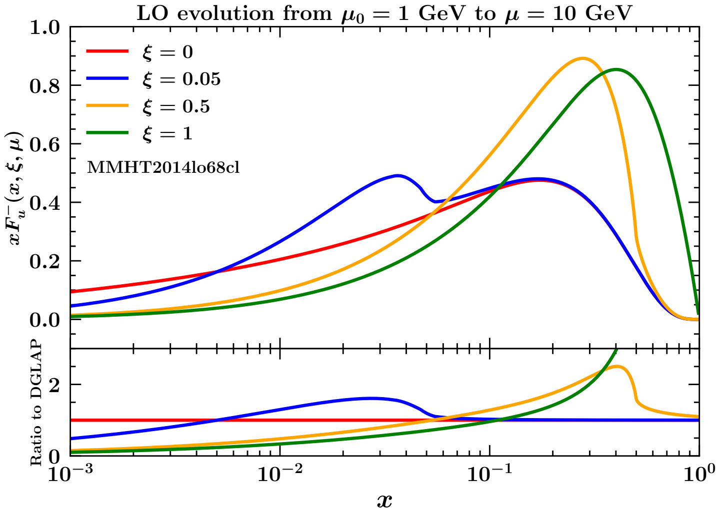
- Numerical solution of integro-differential equations
- Dedicated routines do it
- Splitting functions not easily available above one loop

Examples



LO evolution from $\mu_0 = 1$ GeV to $\mu = 10$ GeV
$\xi = 0$
$\xi = 0.05$
$\xi = 0.5$
$\xi = 1$
MMHT2014lo68cl
$xF_u^-(x,\xi,\mu)$
Ratio to DGLAP
1.0
0.8
0.6
0.4
0.2
0.0
2
0
10^{-3}
10^{-2}
10^{-1}
10^{0}
x

Examples



Evolution equations make the derivative of GPD discontinuous at $x = \xi$.

From evolution equations to evolution operator

Leading Ln resummation

Example on the α_s

- I believe everybody knows the RGE:

$$\frac{\mathrm{d}\alpha_s}{\mathrm{d}\ln\mu^2} = \beta(\alpha_s) = -b_0\alpha_s^2 - b_1\alpha_s^3 + O(\alpha_s^4)$$

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$$\alpha_s(Q^2) = \sum_{j=0}^{\infty} \frac{1}{j!} \underbrace{\ln^j\left(\frac{Q^2}{\mu^2}\right)}_{=L^j} \frac{\mathrm{d}^j\alpha_s}{\mathrm{d}\ln^j\mu^2}$$

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- and using the RGE reorganised as a power series of α_s:

$$\alpha_s(Q^2) = \alpha_s(\mu^2) + \sum_{j=1}^{\infty} \alpha_s^j(\mu^2)\left[L^{j-1}(-b_0)^{j-1} + \sum_{k=2}^{j-k} L^{j-k}\kappa_{j,k}\right]$$

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$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1+\alpha_s(\mu^2)Lb_0}$$

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Resumming leading ln is equivalent to solving LO RGE

GPD evolution operator

- We have derived the evolution equation for GPDs:

$$\frac{\mathrm{d}H^a(\mu^2)}{\mathrm{d}\ln\mu^2} = \sum_b \left[\alpha_s(\mu^2)\mathcal{P}^{ab,(0)} + \alpha_s^2(\mu^2)\mathcal{P}^{ab,(1)} + \ldots\right] \otimes H^b$$

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$$H(Q^2) = \sum_b \Gamma^{ab}(Q^2,\mu^2) \otimes H^b(\mu^2)$$

$$\Gamma(Q^2,\mu^2) = \delta_{ab} + \sum_j \alpha_s^j(\mu^2) \sum_{\ell}^{j-1} L^{j-\ell}\gamma_{j,l}^{ab}$$

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- We can now (partially) resum the leading $\alpha_s^{n+1}L^n$ terms consistently for experimental processes (DVCS)

Intermezzo : Ill-posed inverse problem

A tale of two eigenvalues

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- (a, b) is our experimental vector (measured), (x, y) is our unknown

A tale of two eigenvalues

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \epsilon \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

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- Now let's assume that $\lambda_1 \sim 1$ and $\lambda_2 = \epsilon << 1$

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- Let us put numbers everywhere : $a = 1.4$, $\delta = 0.1$, $\lambda_1 = 2$, $\epsilon = 10^{-3}$

$$x = 0.7 \pm 0.05, \quad y = 0 \pm 100$$

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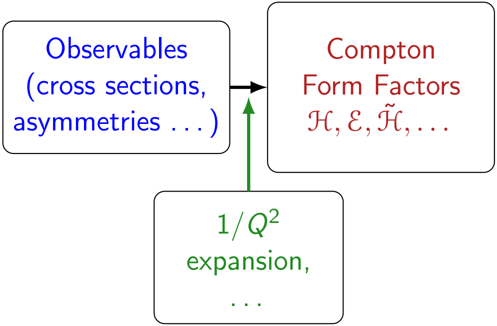
- even if $\rho_{\max} \simeq 10$, you gain an order of magnitude and theory **is** driving your knowledge of y.

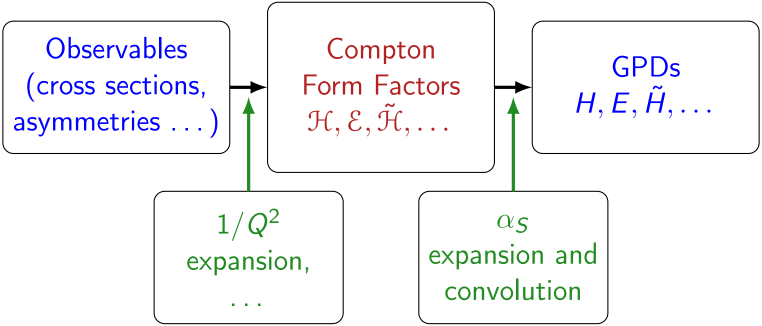
Probing GPDs through exclusive processes

Experimental connection to GPDs

Observables
(cross sections,
asymmetries . . .)


Observables
(cross sections,
asymmetries ...)
Compton
Form Factors
$\mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \ldots$
$1/Q^2$
expansion,
...

Experimental connection to GPDs



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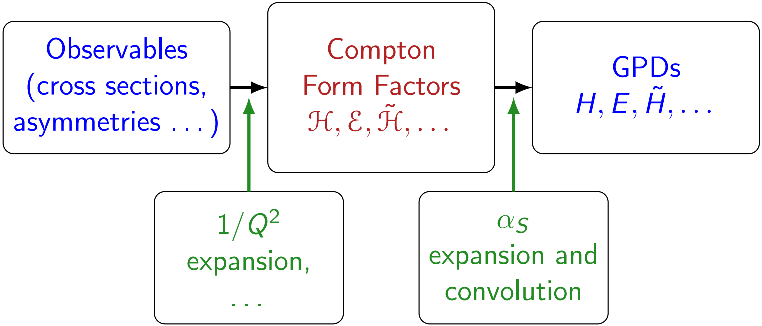
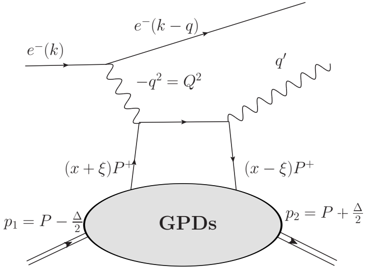


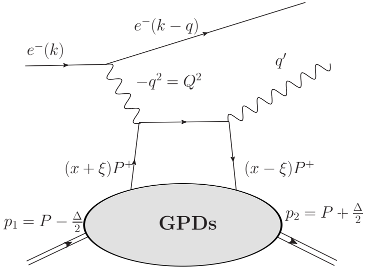
- CFFs play today a central role in our understanding of GPDs
- Extraction generally focused on CFFs

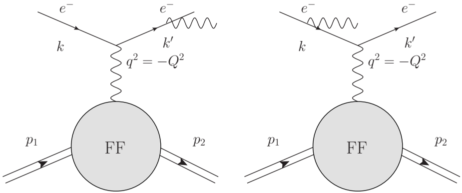
Deep Virtual Compton Scattering



- Best studied experimental process connected to GPDs

  $\rightarrow$ Data taken at Hermes, Compass, JLab 6, JLab 12





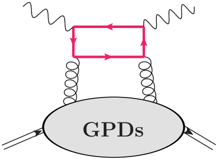
- Best studied experimental process connected to GPDs
  $\rightarrow$ Data taken at Hermes, Compass, JLab 6, JLab 12
- Interferes with the Bethe-Heitler (BH) process
  - Blessing: Interference term boosted w.r.t. pure DVCS one
  - Curse: access to the angular modulation of the pure DVCS part difficult

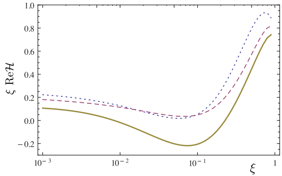
M. Defurne *et al.*, Nature Commun. 8 (2017) 1, 1408

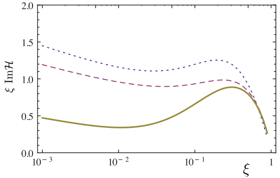
- At LO, the DVCS coefficient function is a QED one

QCD corrections to DVCS

- At LO, the DVCS coefficient function is a QED one
- At NLO, gluon GPDs play a significant role in DVCS







H. Moutarde *et al.*, PRD 87 (2013) 5, 054029

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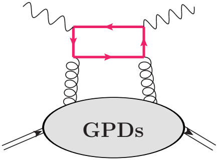
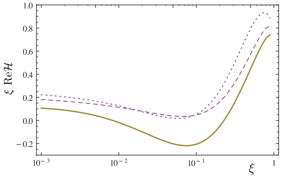
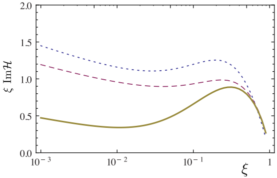




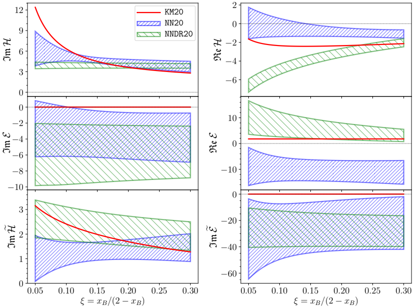


H. Moutarde *et al.*, PRD 87 (2013) 5, 054029

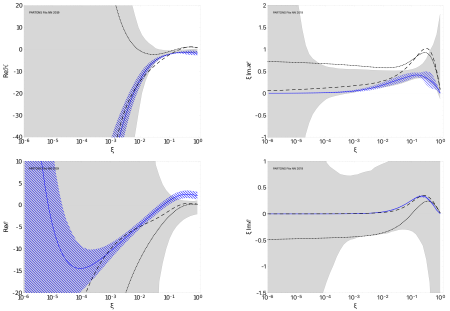
- Recent N2LO studies, impact needs to be assessed

V. Braun *et al.*, JHEP 09 (2020) 117

Recent CFF extractions



M. Cuič *et al.*, PRL 125, (2020), 232005



H. Moutarde *et al.*, EPJC 79, (2019), 614

- Recent effort on bias reduction in CFF extraction (ANN)

  additional ongoing studies, J. Grigsby *et al.*, PRD 104 (2021) 016001

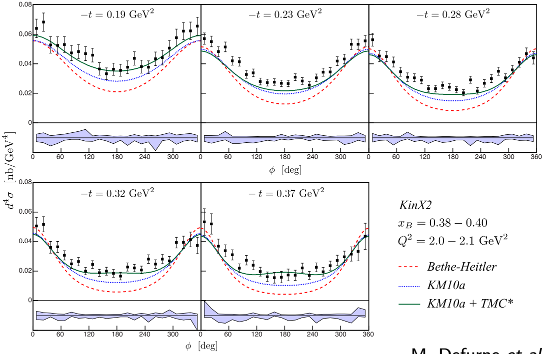
- Studies of ANN architecture to fulfil GPDs properties (dispersion relation,polynomiality,. . .)
- Recent efforts on propagation of uncertainties (allowing impact studies for JLAB12, EIC and EicC)

see *e.g.* H. Dutrieux *et al.*, EPJA 57 8 250 (2021)

Finite t corrections

Kinematical corrections in t/Q^2 and M^2/Q^2

V. Braun *et al.*, PRL 109 (2012), 242001



M. Defurne *et al.* PRC 92 (2015) 55202

- Sizeable even for $t/Q^2 \sim 0.1$
- Not currently included in global fits.

Dispersion relation and the D-term

- At all orders in α_S, dispersion relations relate the real and imaginary parts of the CFF.

I. Anikin and O. Teryaev, PRD 76 056007
M. Diehl and D. Ivanov, EPJC 52 (2007) 919-932
H. Dutrieux *et al.*, EPJC 85 (2025) 1, 105
V. Martinez Fernandez and C. Mezrag, arXiv:2509.05059

$$\mathcal{S}(t, Q^2) = \int_{-1}^{1} \mathrm{d}\omega \, T(\omega) D(\omega) = \Re\mathcal{H}(\xi) - \frac{2}{\pi} \fint_0^1 \frac{x^2 \Im\mathcal{H}(x)}{(\xi - x)(\xi + x)} \frac{\mathrm{d}x}{\xi}$$

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M.V. Polyakov PLB 555, 57-62 (2003)

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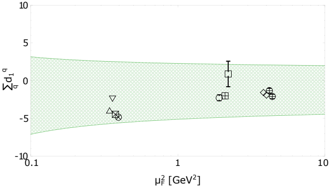


figure from H. Dutrieux *et al.*, Eur.Phys.J.C 81 (2021) 4

- First attempt from JLab 6 *GeV* data

Burkert *et al.*, Nature 557 (2018) 7705, 396-399

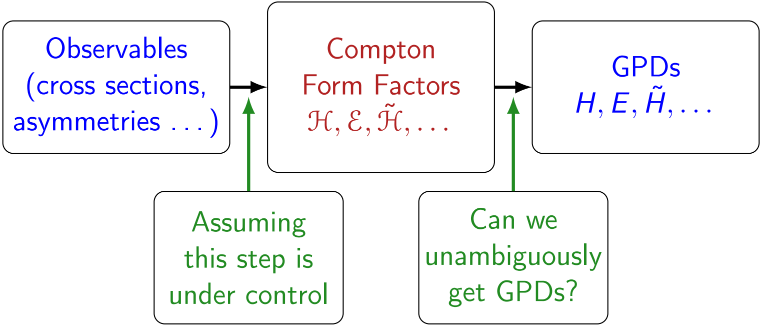
- Tensions with other studies
  $\rightarrow$ uncontrolled model-dependence

K. Kumericki, Nature 570 (2019) 7759, E1-E2
H. Moutarde *et al.*, Eur.Phys.J.C 79 (2019) 7, 614
H. Dutrieux *et al.*, Eur.Phys.J.C 81 (2021) 4

- Scheme/scale dependence

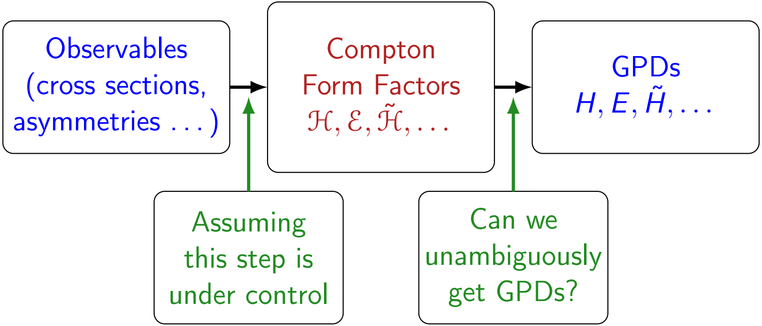
The DVCS deconvolution problem I

From CFF to GPDs



The DVCS deconvolution problem I

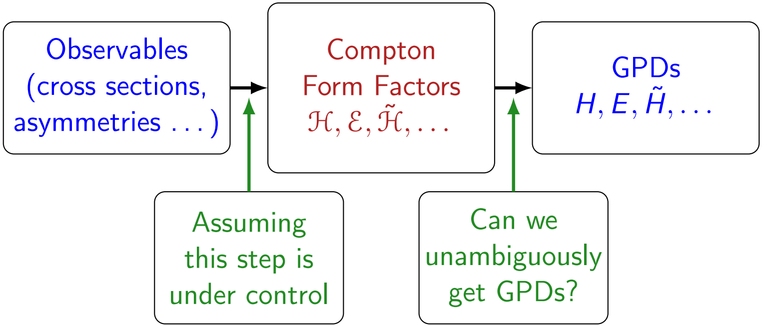
From CFF to GPDs



- It has been known for a long time that this is not the case at LO
  Due to dispersion relations, any GPD vanishing on $x = \pm\xi$ would not contribute to DVCS at LO (neglecting D-term contributions).

The DVCS deconvolution problem I

From CFF to GPDs



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  Due to dispersion relations, any GPD vanishing on $x = \pm\xi$ would not contribute to DVCS at LO (neglecting D-term contributions).
- Are QCD corrections improving the situation?